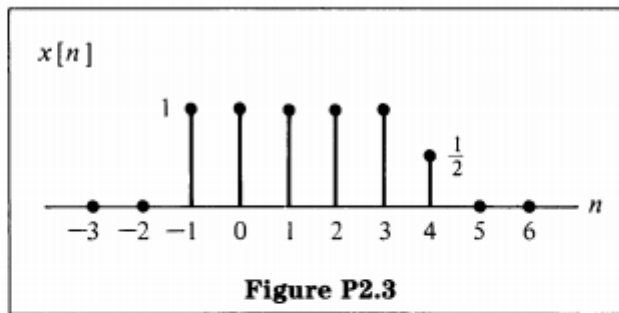# MIT OPENCOURSEWARE ÇÖZÜMLÜ SORULARI

Soru – 1: Verilen x[n] işaretini kullanarak (i), (ii) ve (iii) de verilen işaretleri çiziniz.

P2.3

(a)   A discrete-time signal $x[n]$ is shown in Figure P2.3.

Figure P2.3

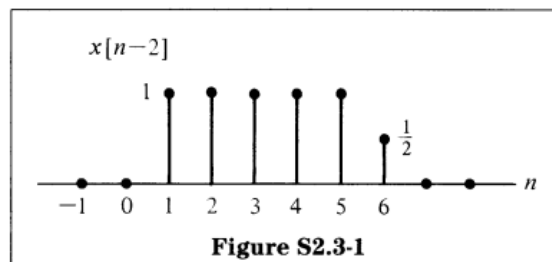Sketch and carefully label each of the following signals:

(i) $x[n - 2]$

(ii) $x[4 - n]$

(iii) $x[2n]$

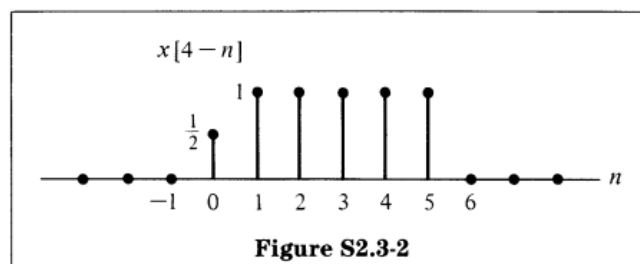**(b)** What difficulty arises when we try to define a signal as $x[n/2]$?
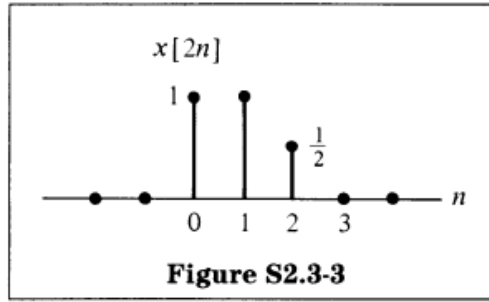
Çözüm -1:

S2.3

(a) (i)   This is just a shift to the right by two units.

Figure S2.3-1

(ii) $x[4 - n] = x[-(n - 4)]$, so we flip about the $n = 0$ axis and then shift to the right by 4.

Figure S2.3-2

(iii) $x[2n]$ generates a new signal with $x[n]$ for even values of n.


$x[2n]$

Figure S2.3-3

(b) The difficulty arises when we try to evaluate $x[n/2]$ at $n = 1$, for example (or generally for n an odd integer). Since $x[\frac{1}{2}]$ is not defined, the signal $x[n/2]$ does not exist.

Soru – 2: Aşağıdaki işaretlerin tek işaret mi çift işaret mi olduğunu belirtiniz.

P2.4

For each of the following signals, determine whether it is even, odd, or neither.

(a)

Figure P2.4-1

(b)

Figure P2.4-2

(c)

Figure P2.4-3

(d)

Figure P2.4-4
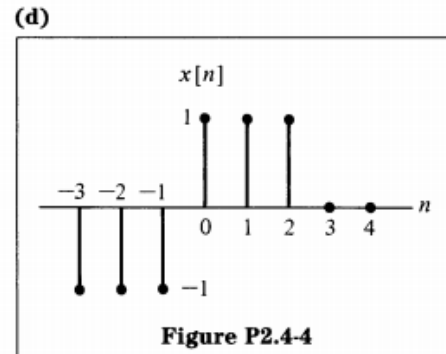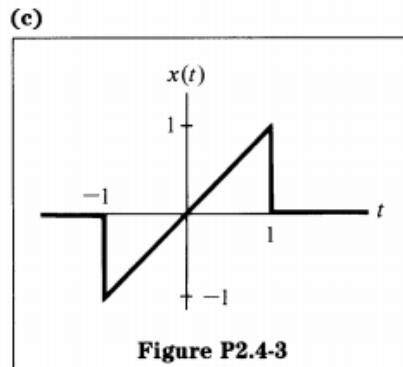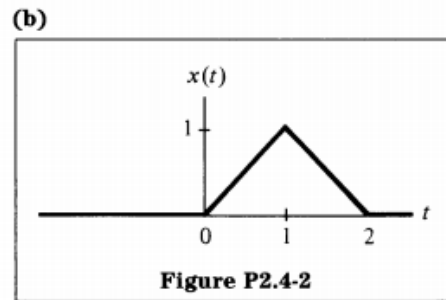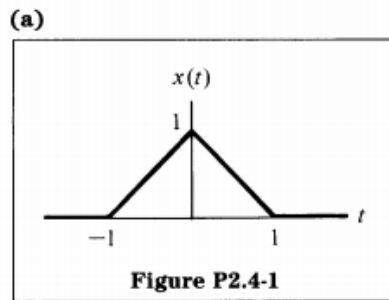
Çözüm -2:

S2.4

By definition a signal is even if and only if $x(t) = x(-t)$ or $x[n] = x[-n]$, while a signal is odd if and only if $x(t) = -x(-t)$ or $x[n] = -x[-n]$.

(a) Since $x(t)$ is symmetric about $t = 0$, $x(t)$ is even.

(b) It is readily seen that $x(t) \neq x(-t)$ for all t, and $x(t) \neq -x(-t)$ for all t; thus $x(t)$ is neither even nor odd.

(c) Since $x(t) = -x(-t)$, $x(t)$ is odd in this case.

Soru – 3: Aşağıdaki x[n] sinyallerini çiziniz.

P3.1

Sketch each of the following signals.

(a) $x[n] = \delta[n] + \delta[n-3]$

(b) $x[n] = u[n] - u[n-5]$

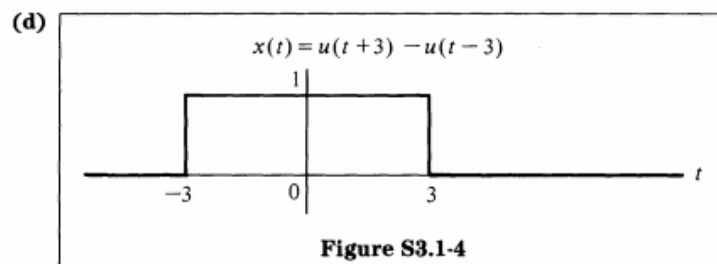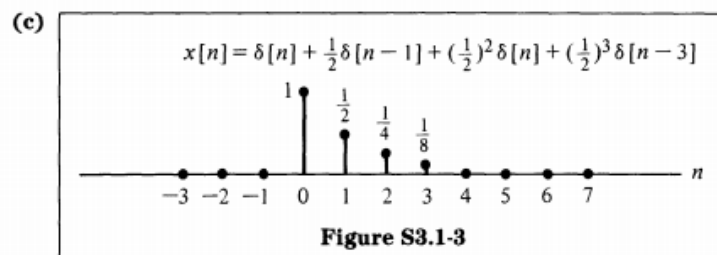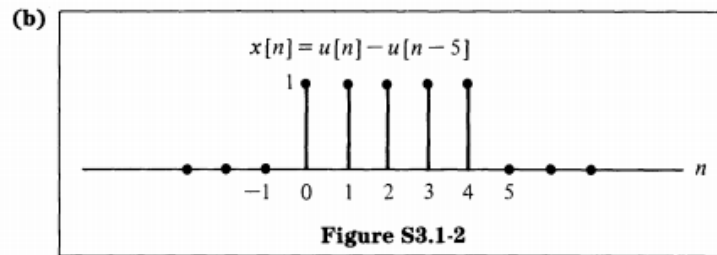(c) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + (\frac{1}{2})^2\delta[n-2] + (\frac{1}{2})^3\delta[n-3]$

(d) $x(t) = u(t+3) - u(t-3)$

(e) $x(t) = \delta(t+2)$

(f) $x(t) = e^{-t}u(t)$

Çözüm – 3:

S3.1

(a)

$x[n] = \delta[n] + \delta[n-3]$

Figure S3.1-1

(b)

$x[n] = u[n] - u[n-5]$

Figure S3.1-2

(c)

$x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + (\frac{1}{2})^2\delta[n] + (\frac{1}{2})^3\delta[n-3]$

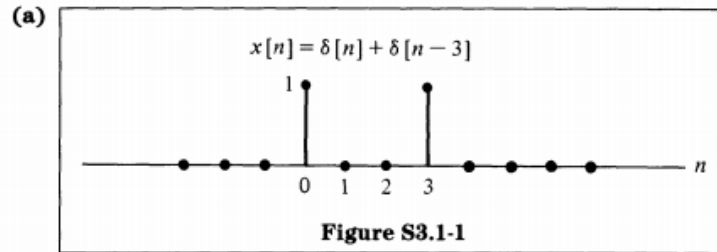**Figure S3.1-3**

(d)

$x(t) = u(t+3) - u(t-3)$

Figure S3.1-4

(e)

$$x(t) = \delta(t+2)$$

Figure S3.1-5

(f)

$$x(t) = e^{-t}u(t)$$
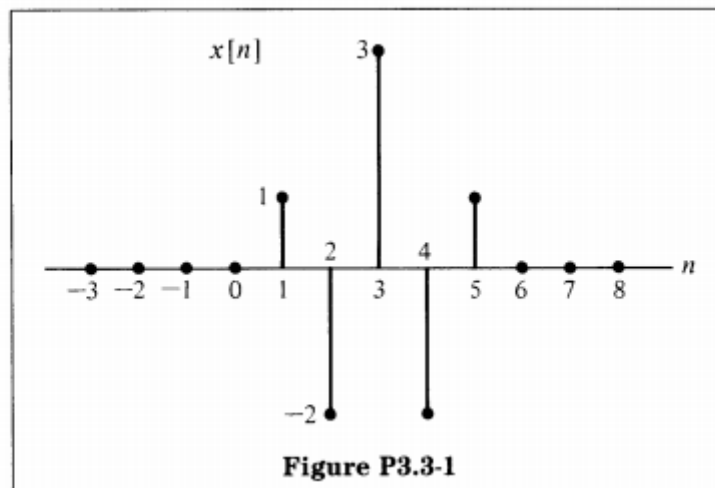
Figure S3.1-6
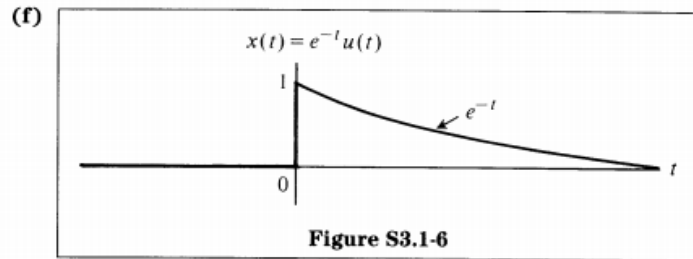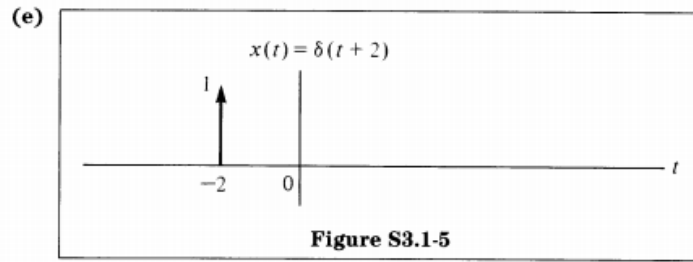
Soru – 4: Aşağıdaki işaretleri dürtü işaretlerinin toplamı şeklinde matematiksel olarak ifade ediniz.

<u>**P3.3**</u>

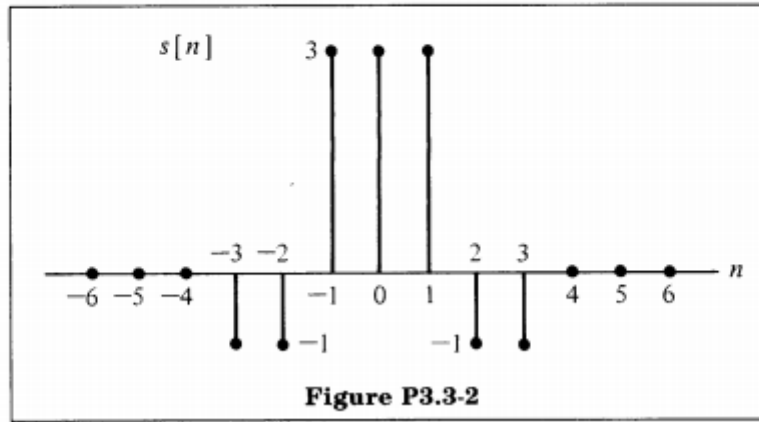**(a)** Express the following as sums of weighted delayed impulses, i.e., in the form

$$x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n-k]$$

Figure P3.3-1

(b) Express the following sequence as a sum of step functions, i.e., in the form

$$s[n] = \sum_{k=-\infty}^{\infty} a_k u[n-k]$$

Figure P3.3-2

Çözüm – 4:

S3.3

(a) $x[n] = \delta[n-1] - 2\delta[n-2] + 3\delta[n-3] - 2\delta[n-4] + \delta[n-5]$

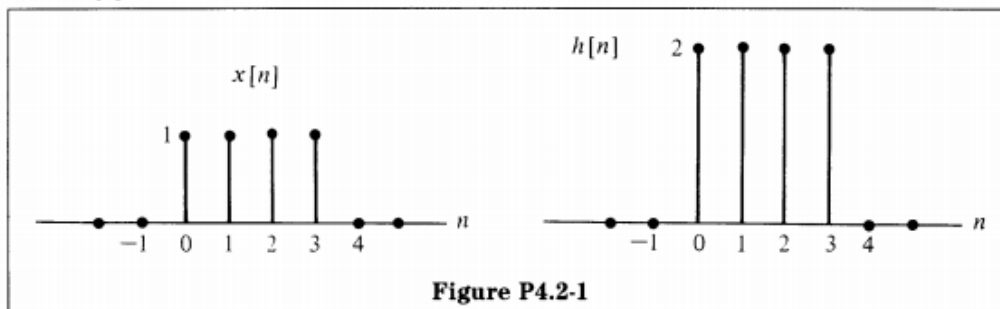(b) $s[n] = -u[n+3] + 4u[n+1] - 4u[n-2] + u[n-4]$

Soru – 5: Aşağıda verilen giriş işareti ve dürtü işaretini kullanarak çıkış işaretini bulunuz.

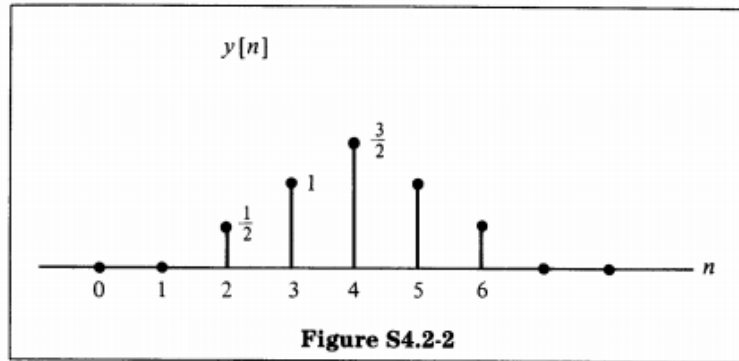**P4.2**

Determine the discrete-time convolution of $x[n]$ and $h[n]$ for the following two cases.

(a)

Figure P4.2-1

(b) By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure S4.2-2.
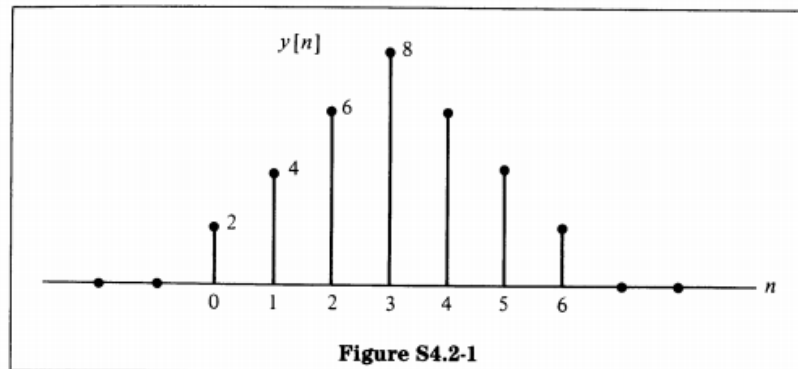

Figure S4.2-2

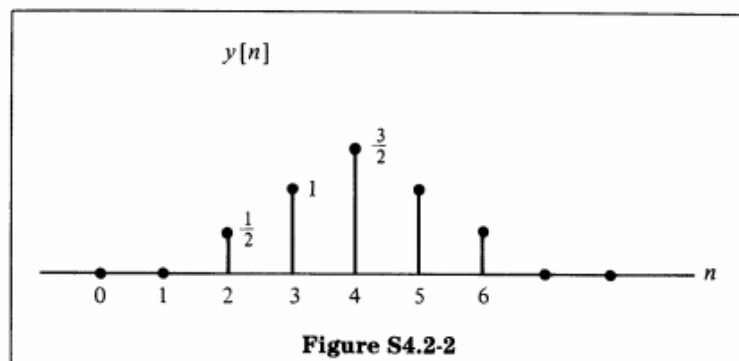Notice that $y[n]$ is a shifted and scaled version of $h[n]$.

Çözüm – 5:

S4.2

The required convolutions are most easily done graphically by reflecting $x[n]$ about the origin and shifting the reflected signal.

(a) By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure S4.2-1.

Figure S4.2-1

(b) By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure S4.2-2.

Figure S4.2-2

Notice that $y[n]$ is a shifted and scaled version of $h[n]$.

P4.3

Determine the continuous-time convolution of $x(t)$ and $h(t)$ for the following three cases:

(a)

Figure P4.3-1

(b)

Figure P4.3-2

(c)

Figure P4.3-3
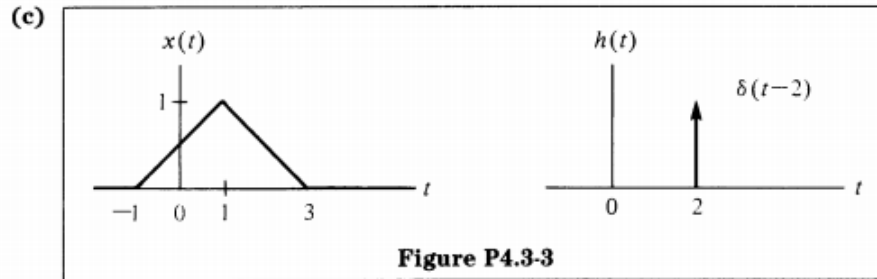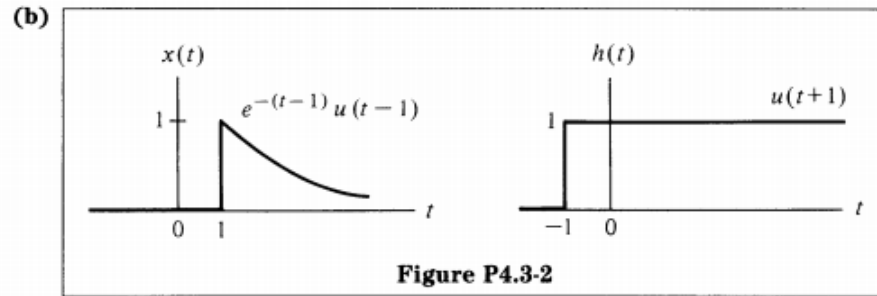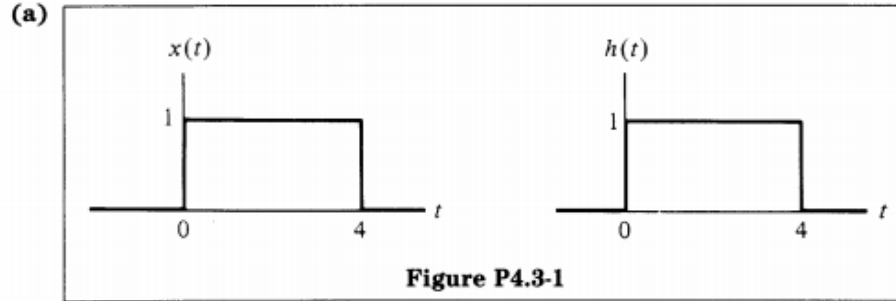
Çözüm – 6:

S4.3

(a) It is easiest to perform this convolution graphically. The result is shown in Figure S4.3-1.

$y(t) = x(t) * h(t)$

Figure S4.3-1
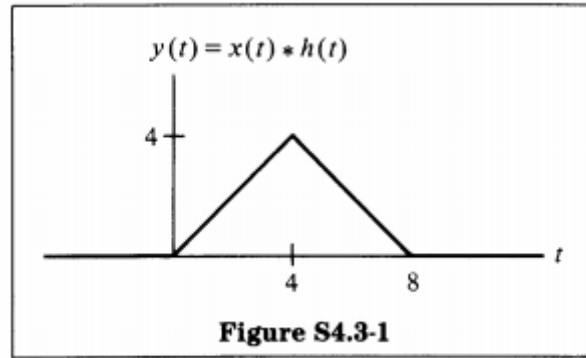
(b) The convolution can be evaluated by using the convolution formula. The limits can be verified by graphically visualizing the convolution.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h\,(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} e^{-(\tau-1)}u(\tau-1)u(t-\tau+1)d\tau$$

$$= \begin{cases} \cdot \displaystyle\int_{1}^{t+1} e^{-(\tau-1)}d\tau, & t > 0, \\ 0, & t < 0, \end{cases}$$

Let $\tau' = \tau - 1$. Then

$$y(t) = \begin{cases} \displaystyle\int_{0}^{t} e^{-\tau'}d\tau' \\ 0 \end{cases} = \begin{cases} 1 - e^{-t}, & t > 0, \\ 0, & t < 0 \end{cases}$$

(c) The convolution can be evaluated graphically or by using the convolution formula.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau - 2)\,d\tau = x(t - 2)$$

So $y(t)$ is a shifted version of $x(t)$.
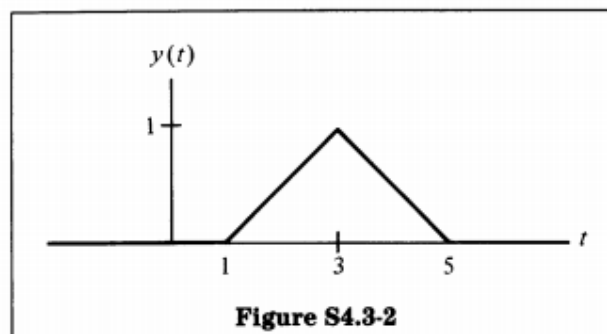
$y(t)$

Figure S4.3-2

P5.2

The first-order difference equation $y[n] - ay[n-1] = x[n]$, $0 < a < 1$, describes a particular discrete-time system initially at rest.

(a) Verify that the impulse response $h[n]$ for this system is $h[n] = a^n u[n]$.

(b) Is the system

    (i)   memoryless?

    (ii)   causal?

    (iii) stable?

    Clearly state your reasoning.

(c) Is this system stable if $|a| > 1$?

Çözüm – 7:

S5.2

(a) We want to show that

$$h[n] - ah[n-1] = \delta[n]$$

Substituting $h[n] = a^n u[n]$, we have

$$a^n u[n] - aa^{n-1}u[n-1] = a^n(u[n] - u[n-1])$$

But

$$u[n] - u[n-1] = \delta[n] \quad \text{and} \quad a^n\delta[n] = a^0\delta[n] = \delta[n]$$

(b) (i)   The system is not memoryless since $h[n] \neq k\delta[n]$.

    (ii)   The system is causal since $h[n] = 0$ for $n < 0$.

    (iii) The system is stable for $|a| < 1$ since

$$\sum_{n=0}^{\infty} |a|^n = \frac{1}{1 - |a|}$$

    is bounded.

(c) The system is not stable for $|a| > 1$ since $\sum_{n=0}^{\infty} |a|^n$ is not finite.

P5.3

The first-order differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

describes a particular continuous-time system initially at rest.

(a) Verify that the impulse response of this system is $h(t) = e^{-2t}u(t)$.

(b) Is this system

    (i)   memoryless?

    (ii)   causal?

    (iii) stable?

    Clearly state your reasoning.

S5.3

(a) Consider $x(t) = \delta(t) \rightarrow y(t) = h(t)$. We want to verify that $h(t) = e^{-2t}u(t)$, so

$$\frac{dy(t)}{dt} = -2e^{-2t}u(t) + e^{-2t}\delta(t), \quad \text{or}$$

$$\frac{dy(t)}{dt} + 2y(t) = e^{-2t}\delta(t),$$

(b) (i)   The system is not memoryless since $h(t) \neq k\delta(t)$.

(ii)   The system is causal since $h(t) = 0$ for $t < 0$.

(iii)   The system is stable since $h(t)$ is absolutely integrable.

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{0}^{\infty} e^{-2t}\, dt = -\tfrac{1}{2}e^{-2t}\Big|_{0}^{\infty}$$

$$= \frac{1}{2}$$