Probability — predictions the result of unknown events

Chance — olasılıkları tanımlamak için kullandığımız terimler bunlar

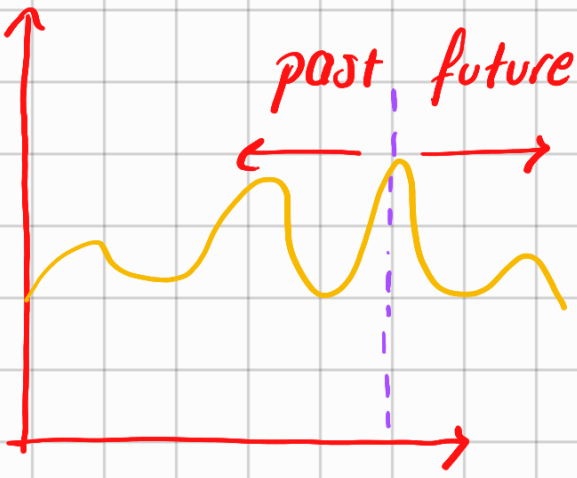
Uncertainity (Belirsizlik)

→ mesela ben yazı tura atıyorum ama ben bu olayı yaparken bütün parametreleri bilsem mesela parmağın kuvveti havanın direnç böyle olsaydı ben bu olayı hesaplayabilir miydim

Weather conditions - rain at 14.15 pm in Davutpaşa?

→ forecasts 80% rain 14-15 pm

Stock prices



Example: flipping a coin

1- Head
2- Tail

If I toss a coin what is the probability of heads?

{H, T}

P(T) = 1/2 = 50% = 0.5

P(H) = 1/2 = 50% = 0.5

100 → Kaç kere yazı tura atıldığı

if we have N equally likely outcomes

P(each) = 1/N

Sets

A set is an unordered collection of things (elements)

$A = \{1, 2, 3, 3\}$ $B = \{1, 2, 3\}$

Burdaki A ve B kümesi birbirine eşittir

Kümelerin içinde sayılar, semboller, karakterler olabilir

Subset (alt küme)

set A is a subset of B if every element of A is also an element of B

$A \subset B \equiv (x \in A) \Rightarrow (x \in B)$

↳ x bir a küme-

sinir elemanıysa
o zaman b kümesinin
bir elemanı olmalıdır

$\subset$ (subset)

$A = \{x^3 ; x = 2,3,4\} = \{2^3, 3^3, 4^3\}$

curly braces

; → such that

List

$\mathbb{N}$ natural numbers $= \{1, 2, 3 \dots\}$

$\mathbb{Z} = \{\dots -1, 0, 1, 2 \dots\}$

tam sayılar (integers)

Rational (ratio → oran)

→ Kesirli sayılar a/b

Symbol is $\mathbb{Q}$

$$\mathbb{Q} = \left\{ \frac{a}{b} \quad a \in \mathbb{Z} \ \& \ b \in \mathbb{Z} - \{0\} \right\}$$

Real numbers

↳ infinite numbers

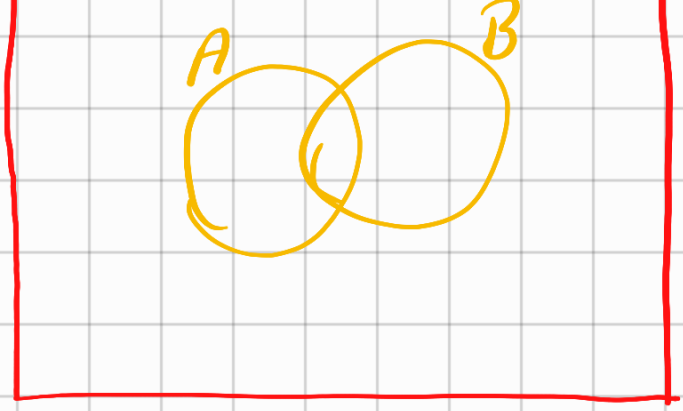
$\mathbb{R}$ → Reel numbers

Superset '$\supset$'

If A is a subset of set B, then B is superset of A

Kümeleri listelerle gösterebiliriz bir de venn diagramları ile gösterebiliriz

Venn diagrams



set

↳ S, Ω (omega)

A ⊂ Ω true

Set operations

Union (Birleşim) → the union of two sets A and B consists of all elements in A or B

$x \in (A \cup B) \iff (x \in A)$ or $(x \in B)$

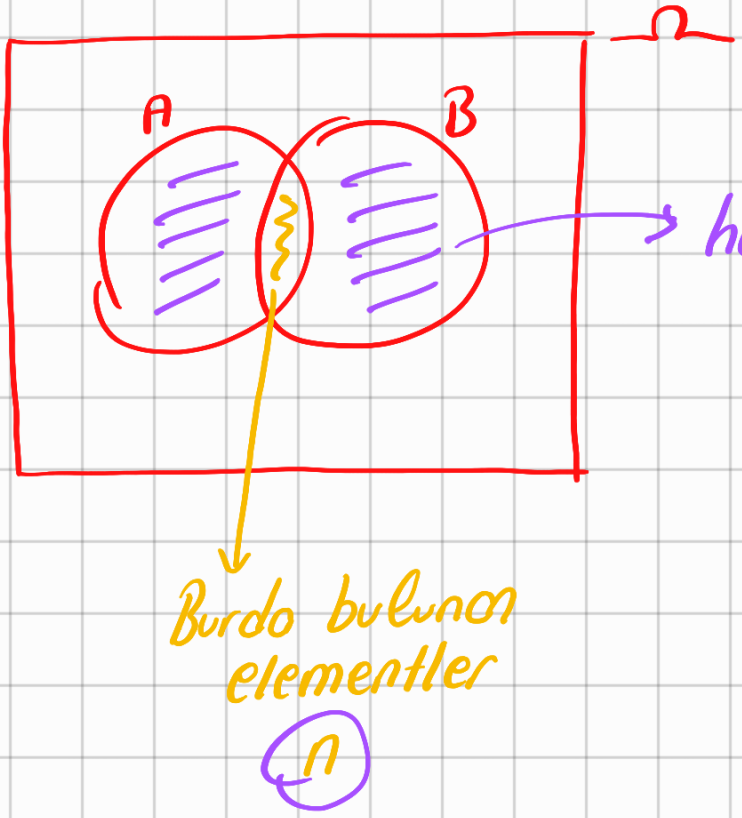
{1,2} {4} Ancak ve ancak $A \cup B = \{1,2,4\}$

Union'un matematiksel tanımı

intersection (Kesişim)

The intersection of two sets A and B consists of all elements in both

A and B

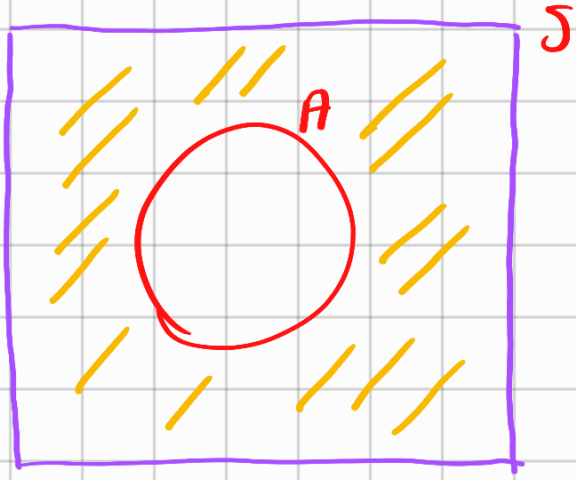


hepsi birleşimi gösterir
∪

Burda bulunan elementler
∩

Complement

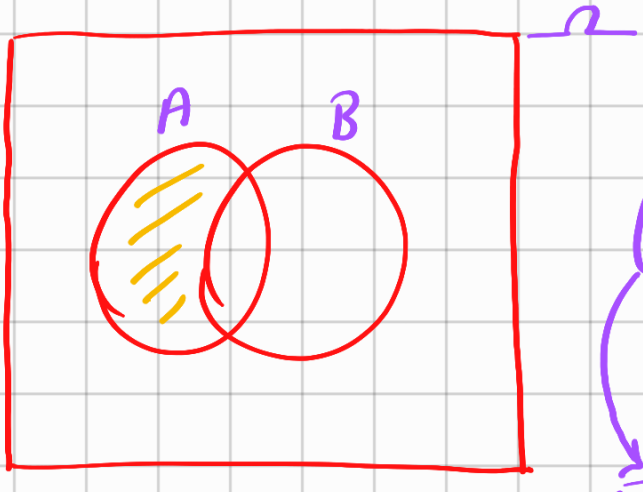
the complement of a set A is set of all elements in S that are not in A



A^C, A', $\bar{A}$

Difference (subtraction)

The difference of a set B from A is all elements in A that are not in B



$A-B$ = $x \in A$ and $x \notin B$

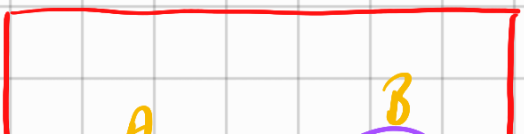
$A \setminus B$

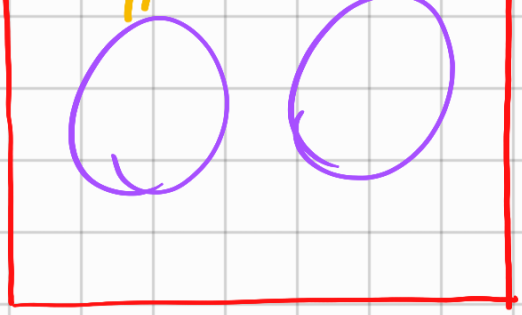
$A \cap B^c$

Disjoint sets (Ayrık küme) (Mutually exclusive set)

two sets A and B are disjoint $A \cap B = \emptyset$

$\emptyset$ → empty set (Boş küme elemanı olmayan bir kümedir





A and B are disjoint set

Partition:

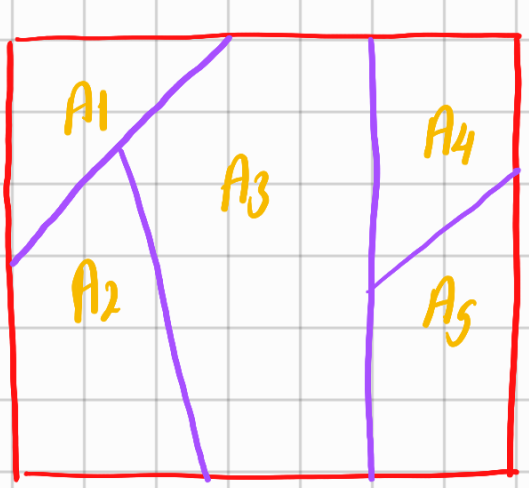
A collection of sets!

$A_1, A_2, \_\_\_ A_n$ is a partition set

if and only if

a- They are disjoint

b- $A_1 \cup A_2 \cup A_3 \_\_ \cup A_n = S$



→ Biz o zaman şöyle diyebiliriz $A_1, A_2 \_\_ A_5$ partition set'tir

De Morgan's law (De morgan kanunları)

$(A \cup B)^c = A \cup B = A^c \cap B^c$

Example

$S = \{1, 2, 3, 4, 5, 6\}$ $A = \{1, 2\}$ $B = \{2, 4, 6\}$

$A \cup B = \{1, 2, 4, 6\}$

$A \cap B = \{2\}$

$A^c = \{3, 4, 5, 6\}$

$B^c = \{1, 3, 5\}$

$(A \cup B)^c = \{1, 2, 4, 6\}^c = \{3, 5\}$

$(A \cup B)^c$ — De morgan's law

$A^c \cap B^c$

$\{3, 4, 5, 6\} \cap \{1, 3, 5\}$

$\{3, 5\}$

proof ?

Therom : Distribute law

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$4 * (2+17) \longrightarrow (4*2) + (4*17)$$

Functions

Fonksiyonlar bir kümeden diğerine eşlemedir
bi2 fonksiyonları nasıl gösteririz

fonksiyonlara genellikle f ismini veririz

$f : X \to Y$

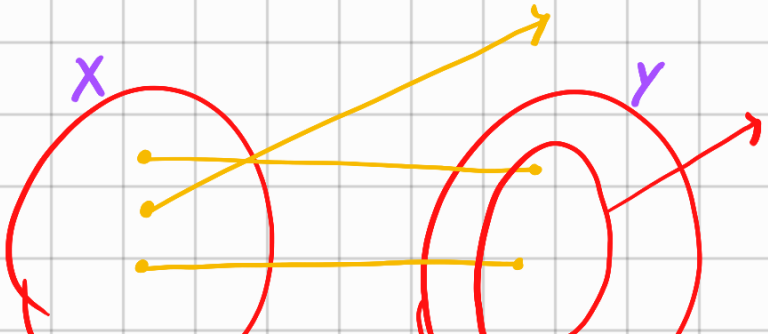
X → domain

Y → co-domain

Mesela bu tanımlı değil o zaman yapıyoruz $f : R - \{a\} \to R$

Böyle yapıyoruz

$\forall x \in X, f(x) \in Y$



domain

co-domain

Range : the set of all possible values of f(x)
(x'in tüm olası değerlerin kümesidir

Range $\subset Y$

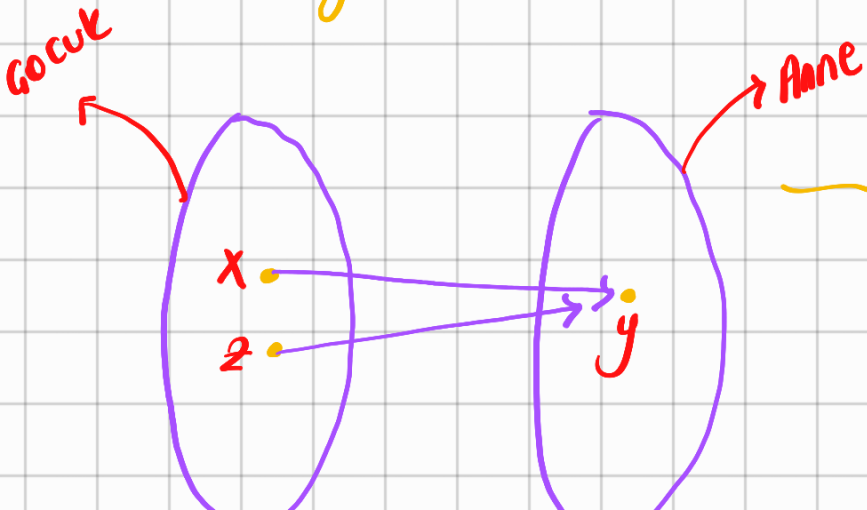
$f: R \longrightarrow R$ defined as $f(x) = x^2$

diyelim ki gerçek sayılardan gerçek sayılara f denilen fonksiyonumuz var

$X = Y = R$

$range(f) = R^+ = \{x \in R \mid x \geq 0\}$

Soru → x'teki 2 element y'deki tek elemente gidebilir mi



→ 2 çocuğun bir annesi olabilir
ama bir çocuğun 2 annesi olamaz

$f(x) = y$

$f(2) = y$

} Böyle bir şey olabilir mi

↓

Olabilir

Soru → x'teki bir element y'deki 2 element ile eşleşebilir mi

$f(2) = y$

$f(2) = \omega$

} Bu olabilir mi?

↓

Sadece bir sonuç olmalı bu belirsizlik anlamına gelir

↓

Olmaz

Countable and Uncountable sets

finite set

Yarlıkların tüm unsurları biliniyorsa

$A = \{1, 2, 4, 8, 9\}$

$x \in [4, 8)$ & $x \in \mathbb{Z}$

↓

4, 5, 6, 7

infinite set

$\mathbb{Z}^+ = \{1, 2, 3, 4 \dots\}$

Countable and Uncountable sets

$|A| \rightarrow$ cardinality

$A = \{1, 2, 3\}$

$|A| \rightarrow 3$

A set is finite of $|A| < \infty$

↳

finite set

A set Countable if it is finite or its elements can be enumerated or listed in a sequence

$$A = \bigcup_{k=1}^{\infty} \{a_k\}$$

↳ its means

$$A = \{a_1, a_2, a_3 \_ \_ \_ \_ \}$$

Lets Natural numbers

↳ $\{1, 2, 3 \_ \_ \_ \}$

it is an infinite set

N countable ?

↳ Bir kümenin elemanları sayılabilir is countable olur

↳ Yes it is countable

Uncountable sets

the element can not be enumarated

$R \rightarrow$ reel number

↳ Reel numbers is uncountable set
(Reel sayıları sayamadığın için uncountable olur)

↳ No enumaration

A set is countable infinite if it is in one to one correspondance with

$$N = \{1, 2, 3 \dots\} = \bigcup_{k=1}^{\infty} \{k\}$$

$Z \rightarrow$ countable ?

$Z \rightarrow \{\dots 0, 1, 2, \dots\}$

a_0 a_1 a_2

2 → infinite sets but it is countable sets

We can enumareted them

Q^+ (Rational numbers)

$\longrightarrow \{ a/b : a \text{ or } b > 0 \}$

infinite set

$$Q^+ = \bigcup_{i,j=1}^{\infty} \left\{ \frac{i}{j} \right\} \longrightarrow \bigcup_{i=1}^{\infty} \bigcup_{j=1}^{\infty}$$

$i/j \rightarrow 1 \ldots\ldots$ tekror başlıyor

Burası 2'ye geçtiği zaman aşağıdı 1'den

	0	1	2	3	4	5
0						
1	$0/1$	1	1	1	1	1
2	$0/2$	$1/2$	1	1	1	1
3	1	$1/3$		1	1	1
4	1	1	1	1	1	1
5	1	1	1	1	1	1

I can enumarete them

(reel sayılar)

$R \longrightarrow$ infinite $\longrightarrow$ Yes

Countable No

R is not countable

$[a,b]$ $b>a$

We can not enumorete them
so it is uncountable

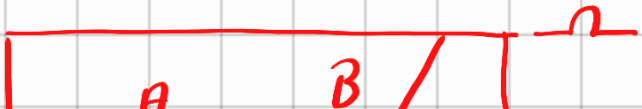
Power Set

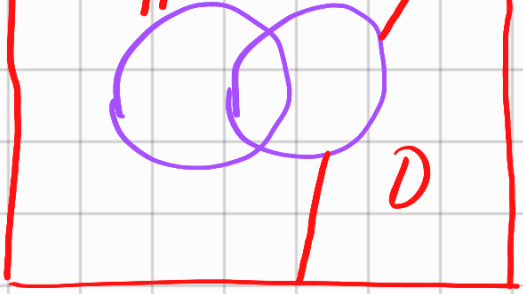
Set of all subsets of a set A

$A = \{1,2\}$

$P(A) = \{\{1\}, \{2\}, \{1,2\}, \{\emptyset\}\}$

Exhaustive Sets





Sets A, B, C, D are exhaustive if their union is Ω

$A \cup B \cup C \cup D = \Omega$

intersections are allowed

Probability theory

① random experiment → Bir sonuca olmayan fenomendir

A phenomenon whose outcome cannot be predicted with certainity

- Rolling a die
- flipping a coin
- Rolling a die 3 times

} random experiment

② Outcome :

outcome is a result of random exp.

→ Rolling a die ⟶ 1, 2, 3, 4, 5, 6

↳ 3

flipping a coin ⟶ H

Roll a die 2 times (6, 2)
(zar atmak)

③ Event
(olası sonuçların bir toplamıdır)

An event is a collection of possible outcomes

Rolling a die

$E_1 = \{1, 3, 5\}$

$E_2 = \{2, 4\}$

④ Sample space

Set of possible outcomes

Set of possible outcomes

↓

Ω (Bunuda gösterebiliriz)

Rolling a die

Ω = {1, 2, 3, 4, 5, 6}

Ω = {H, T}

Roll a die 3 times

Ω = {(1,1,1), (1,1,2) (1,3) ___ (6,6,6)}

↓

all of possible outcomes

↓

6 x 36 possible outcomes

↓

6 6 6 → 216

3. yer için 6 seçenek var

Birinci yer için 6 seçenek var

2. yer için 6 seçenek var

Random experiment sonucu olmayan bir fenomendir

Sample set tüm sonuçları içeren bir kümedir

→ Event is a set

Sample space = universal set

Outcome → element of a set

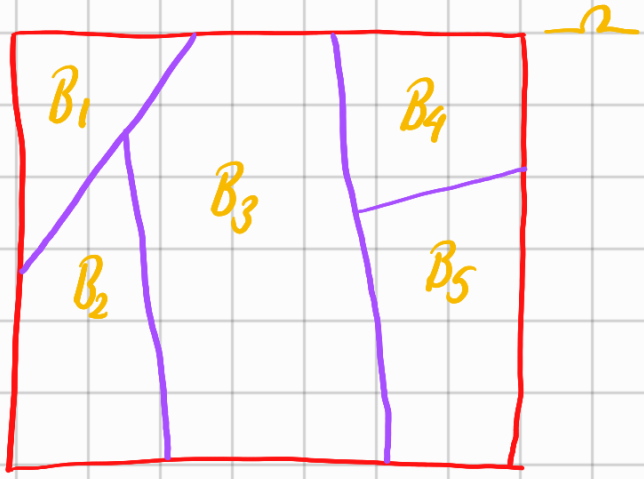
Partition

A partition is collectively exhaustive and mutually exclusive set of events

$A_1 A_2 \_\_\_ A_n$ is a partition

$A_1 \cup A_2 \cup \dots \cup A_n = \Omega$

$A_1 \cap A_2 = \emptyset \;,\; i \neq j$



Sample space → Tüm olası sonuçlardır

↳ universal sete çok benzer

event sample space ve universal setin alt kümesidir

We say that an event A is occured if the outcome of the experiment is an element of A

Probability

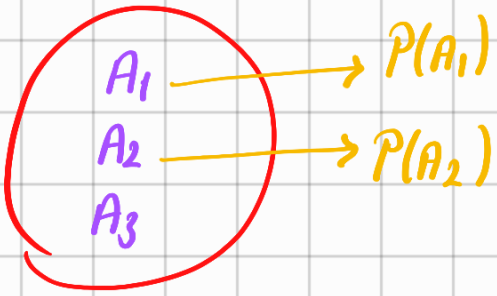
An event A → P(A) Probability of A

↗ Buna o olasılığı

denir

We assign a probobolity $P(A)$ to every event A

event



Axioms of probability

$P(\cdot)$ is a function that maps events in the sample space S to real numbers

① for any event A $P(A) \geq 0$

② Probability of the sample space Ω $P(\Omega) = 1$

③ for any countable collection $A_1, A_2, \ldots$ of disjoint events

$$P(A_1 \cup A_2 \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$$