BLM2041 Signals and Systems

Syllabus

The Instructors:

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Signals

- Typical think of signals in terms of communication and information
  - radio signal
  - broadcast or cable TV
  - audio
  - electric voltage or current in a circuit

- More generally, *any* physical or abstract quantity that can be measured, or influences one that can be measured, can be thought of as a signal.
  - tension on bike brake cable
  - roll rate of a spacecraft
  - concentration of an enzyme in a cell
  - the price of dollars in euros
  - the federal deficit

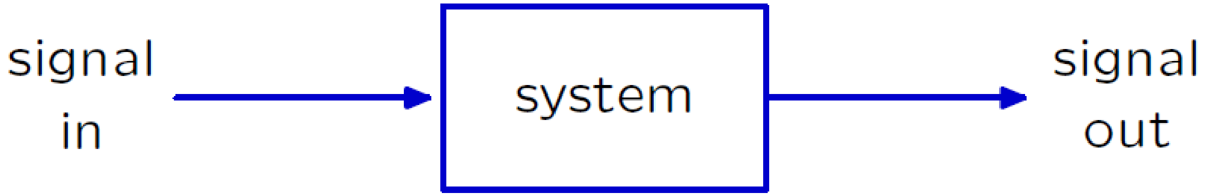
  Very general concept.

Systems

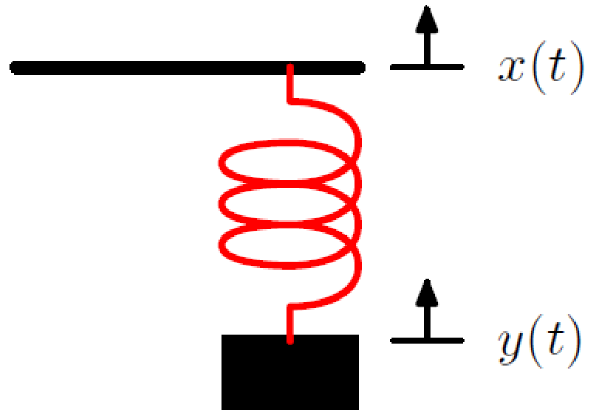
- Typical systems take a signal and convert it into another signal,
  - radio receiver
  - audio amplifier
  - modem
  - microphone
  - cell telephone
  - cellular metabolism
  - national and global economies
- Internally, a system may contain many different types of signals.
- The systems perspective allows you to consider all of these together.
- In general, a *system* transforms *input signals* into *output signals*.

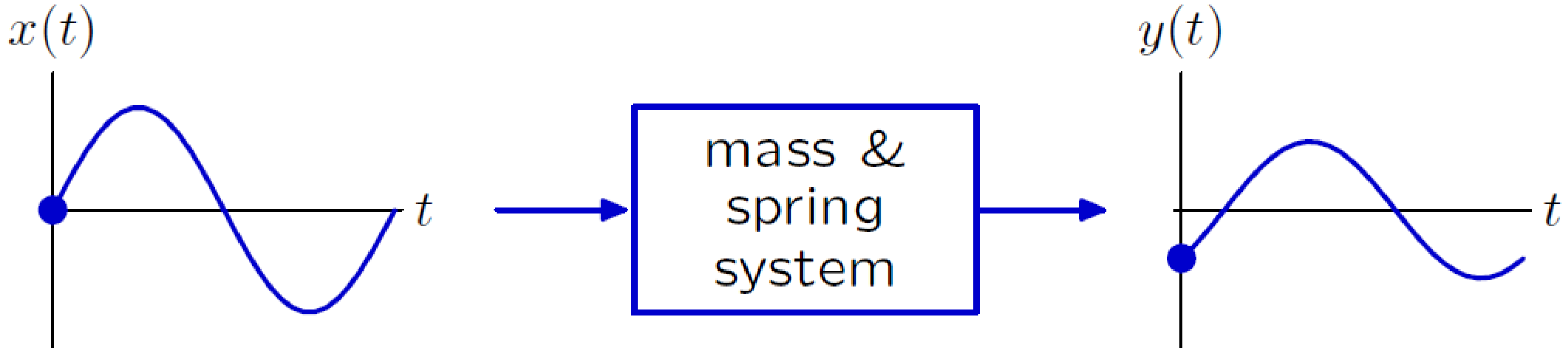
The Signals and Systems Abstraction

Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.

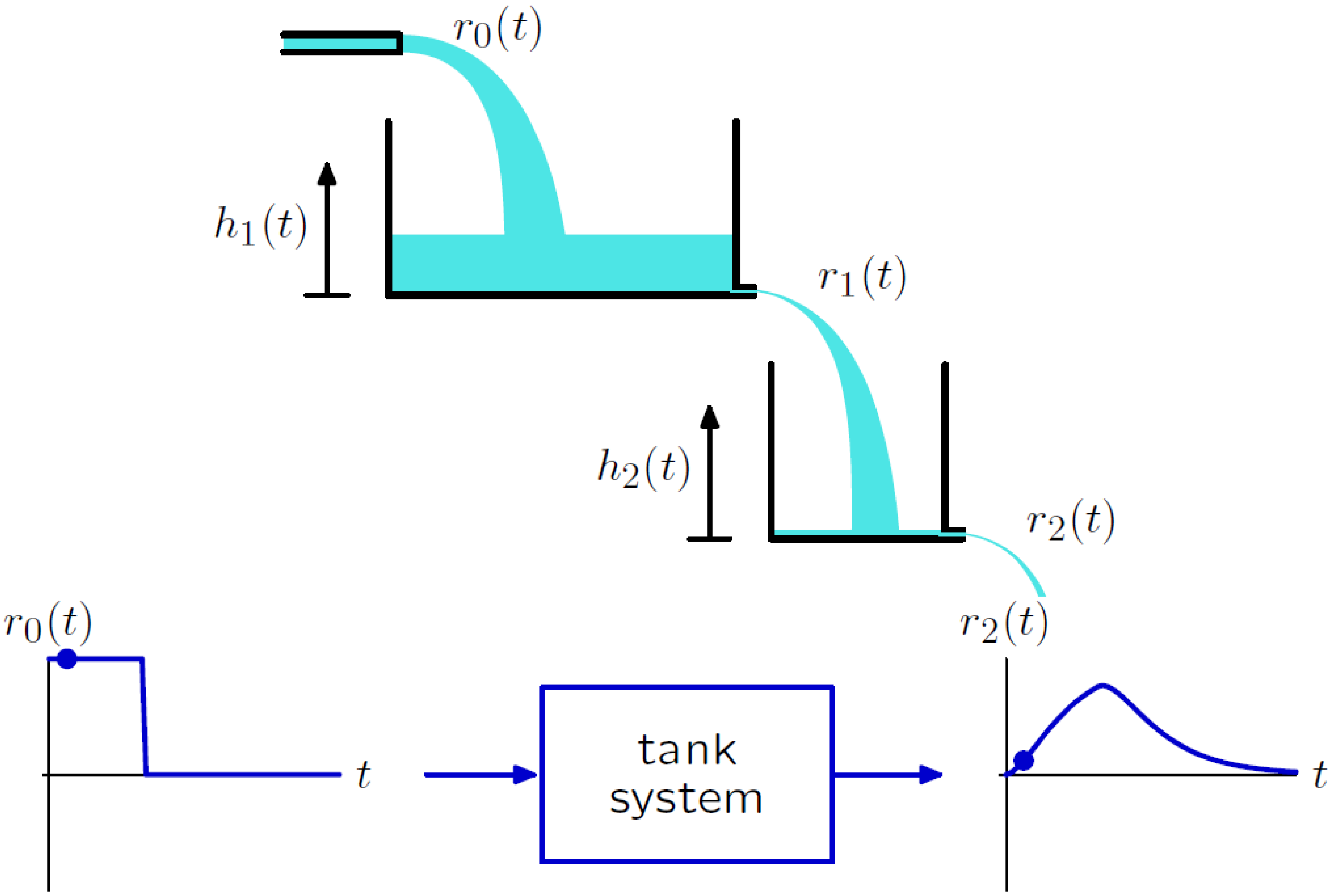


Example: Mass and Spring

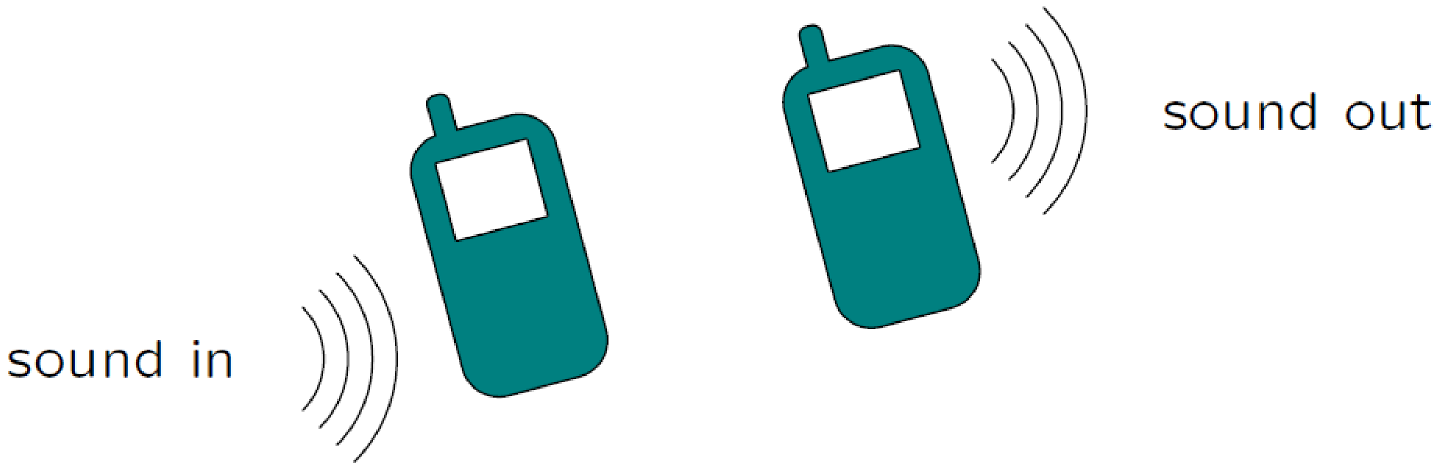


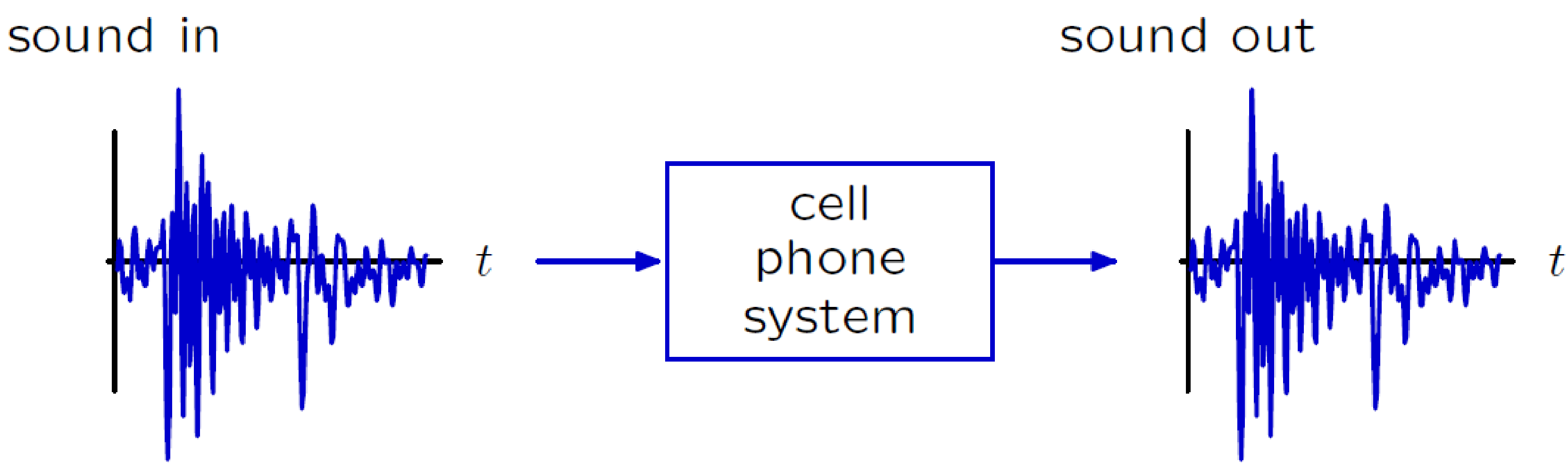


Example: Tanks



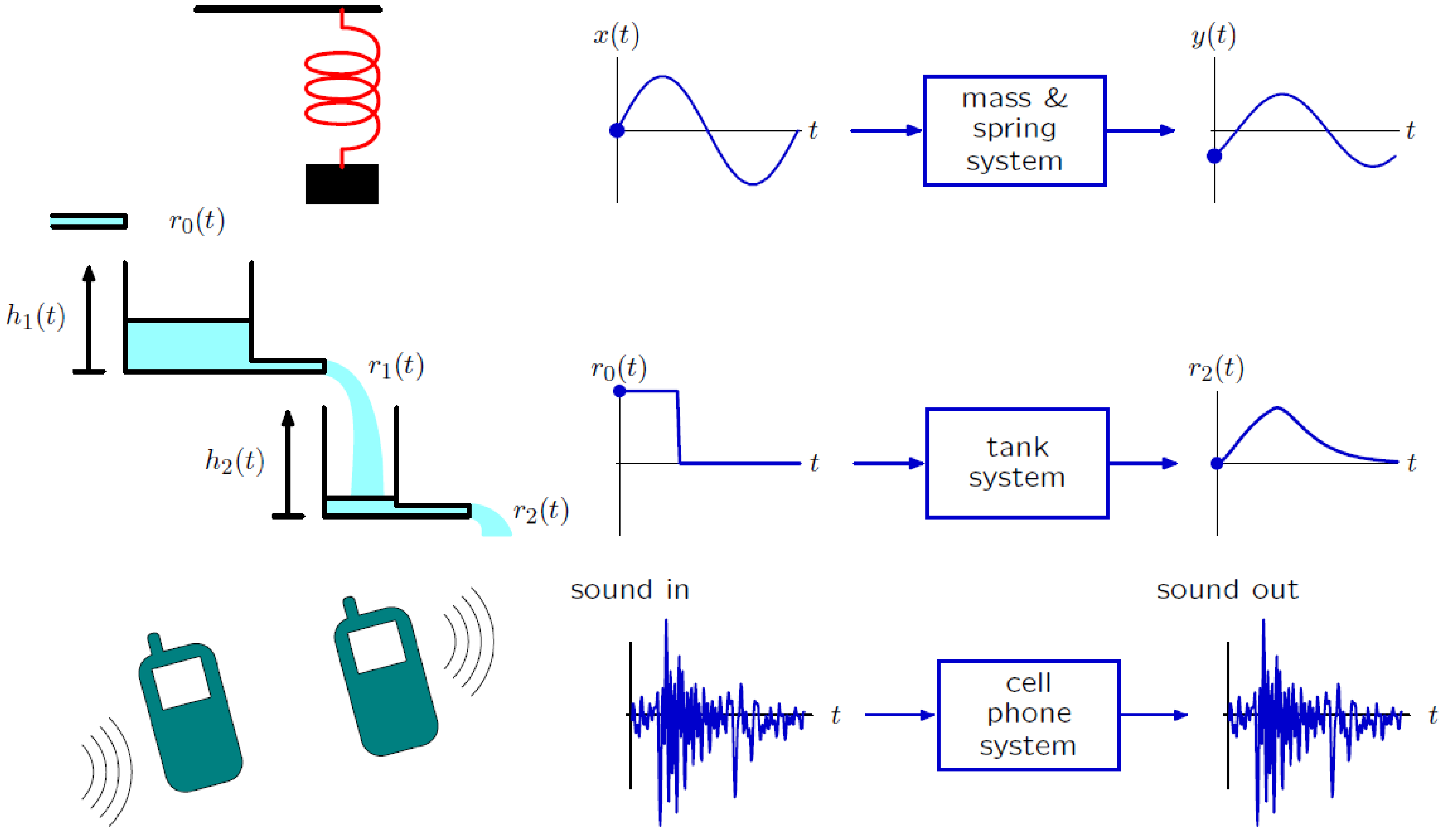
Example: Cell Phone System





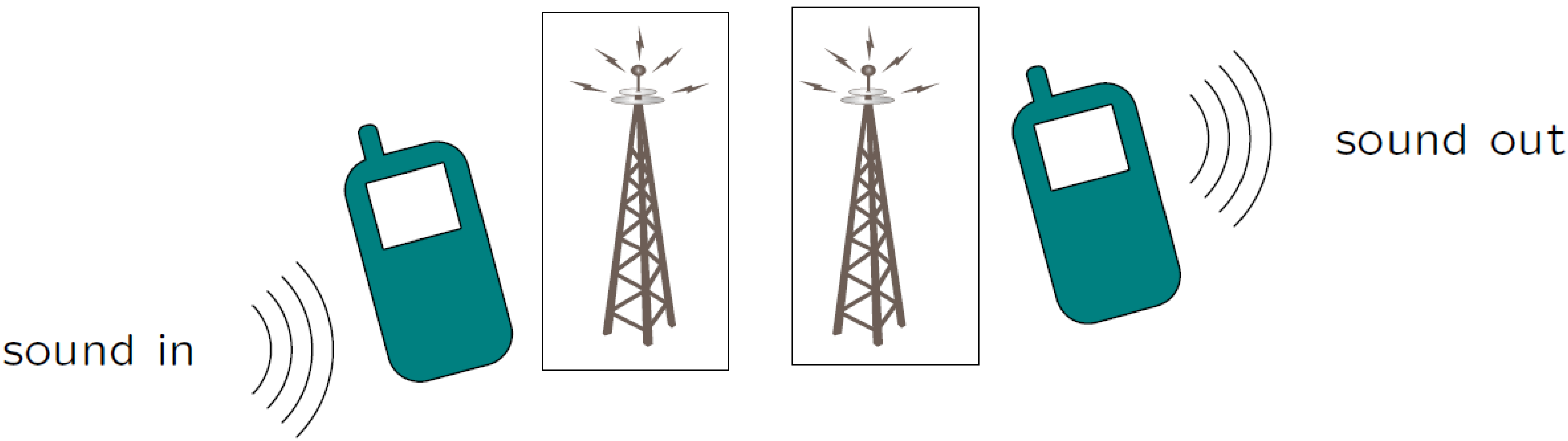
Signals and Systems: Widely Applicable

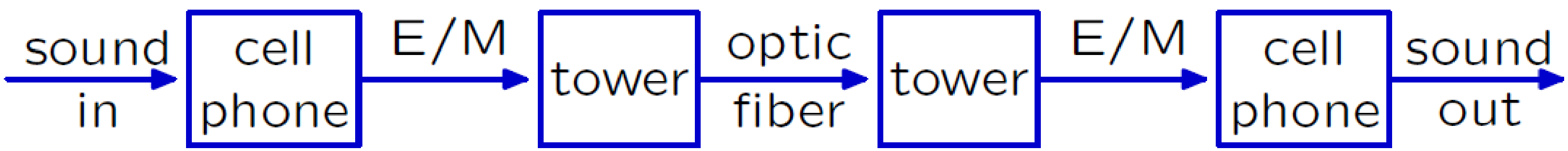
The Signals and Systems approach has broad application: electrical, mechanical, optical, acoustic, biological, financial, ...



Signals and Systems: Modular

The representation does not depend upon the physical substrate.



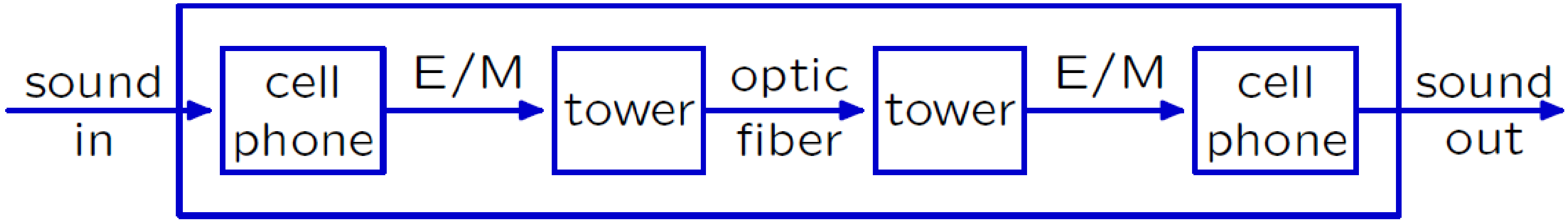


focuses on the flow of **information**, abstracts away everything else

Signals and Systems: Hierarchical

Representations of component systems are easily combined.

Example: cascade of component systems



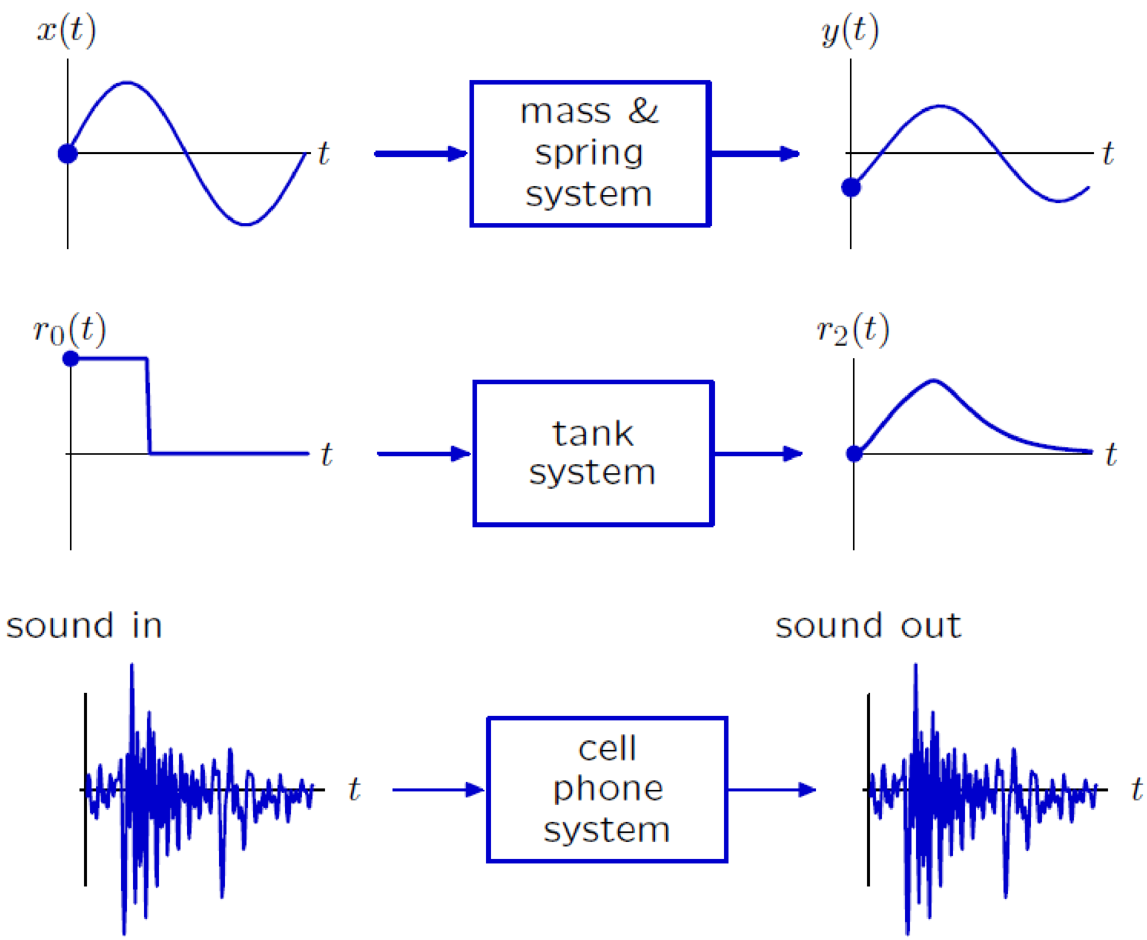
Composite system

Component and composite systems have the same form, and are analyzed with same methods.

Signals and Systems

Signals are mathematical functions.

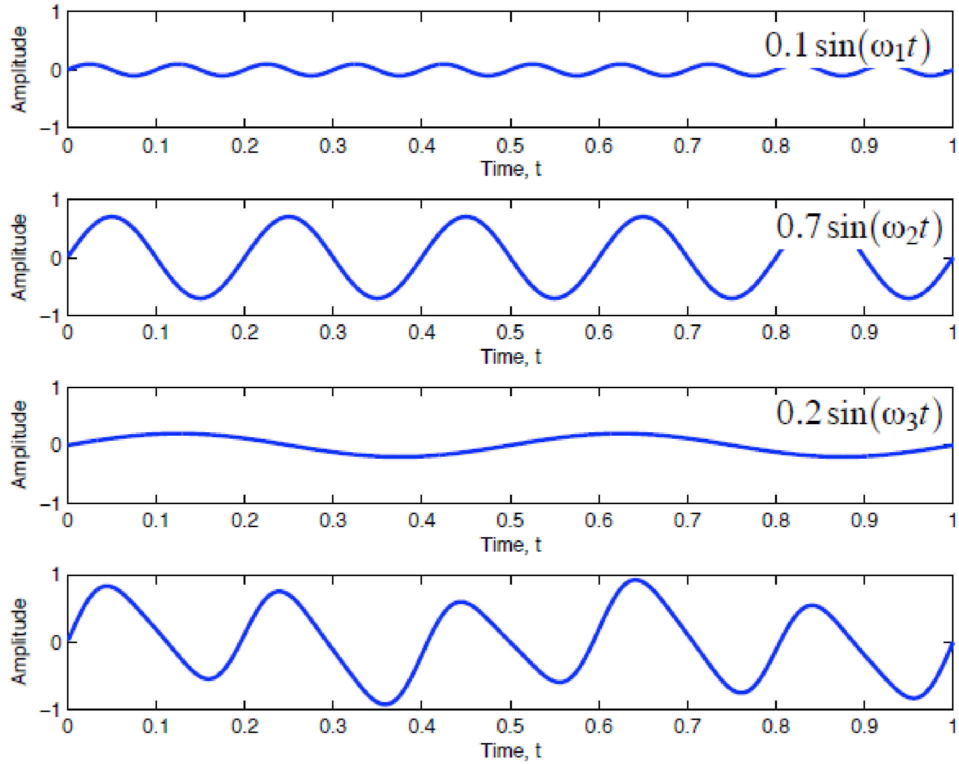
- independent variable = time
- dependent variable = voltage, flow rate, sound pressure



Why Frequency Domain?

Idea 1: Frequency Domain Representation of Signals

- Represent signal as a combination of sinusoids



$$f(t) = 0.1\sin(\omega_1 t) + 0.7\sin(\omega_2 t) + 0.2\sin(\omega_3 t)$$

Why Frequency Domain?

- This example is mostly a sinusoid at frequency ω_2, with small contributions from sinusoids at frequencies ω_1 and ω_3.
  - Very simple representation (for this case).
  - Not immediately obvious what the value is at any particular time.

- Why use frequency domain representation?
  - Simpler for many types of signals (AM radio signal, for example)
  - Many systems are easier to analyze from this perspective (Linear Systems).
  - Reveals the fundamental characteristics of a system.

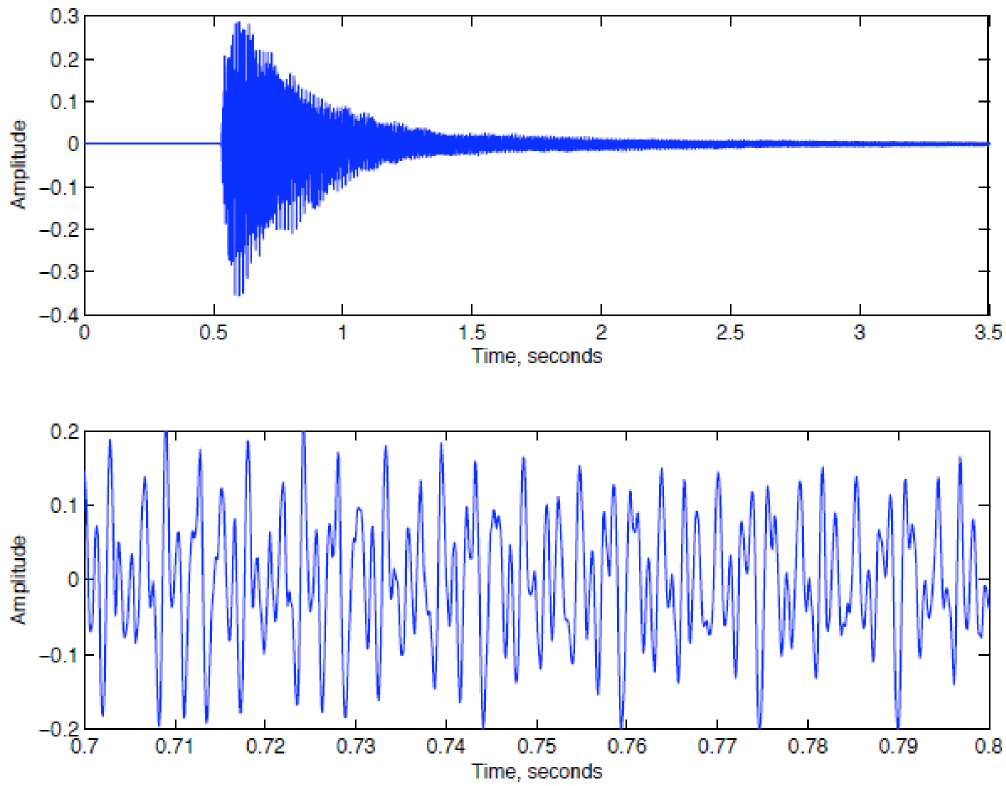
- *Rapidly becomes an alternate way of thinking about the world.*

Why Frequency Domain?

Demonstration: Piano Chord

- You are already a high sophisticated system for performing spectral analysis!

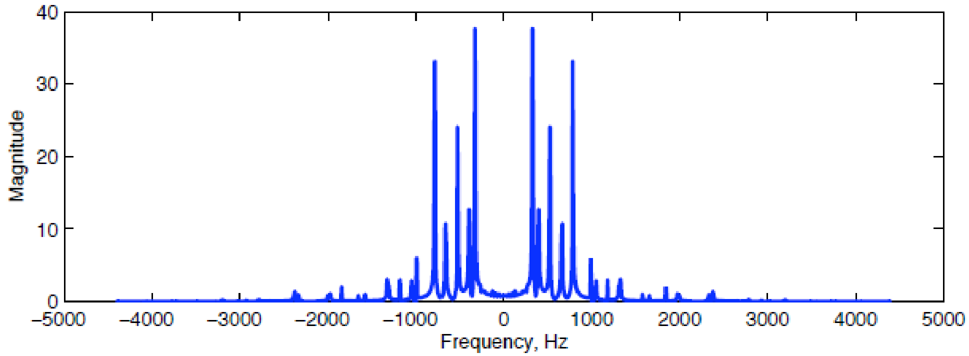
- Listen to the piano chord. You hear several notes being struck, and fading away. This is waveform is plotted below:

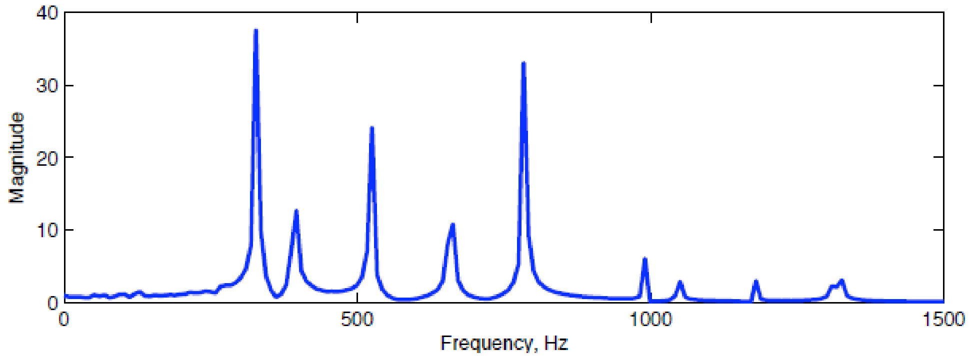


Why Frequency Domain?

- The time series plot shows the time the chord starts, and its decay, but it is difficult tell what the notes are from the waveform.

- If we represent the waveform as a sum of sinusoids at different frequencies, and plot the amplitude at each frequency, the plot is much simpler to understand.

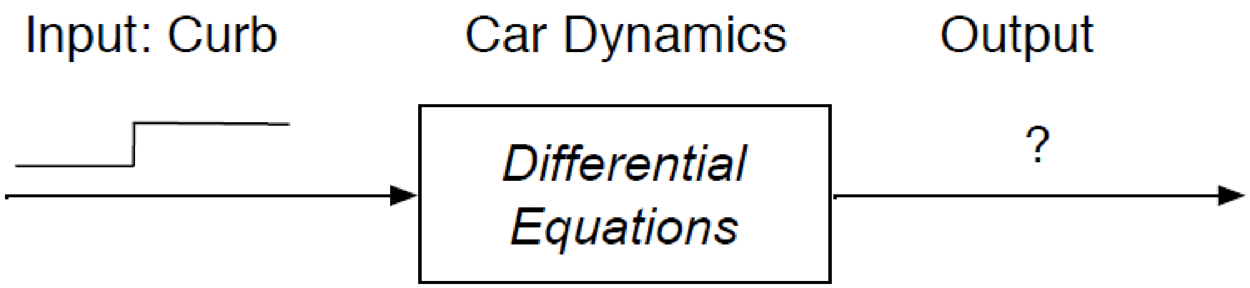




Why Frequency Domain?

Idea 2: Linear Systems are Easy to Analyze for Sinusoids

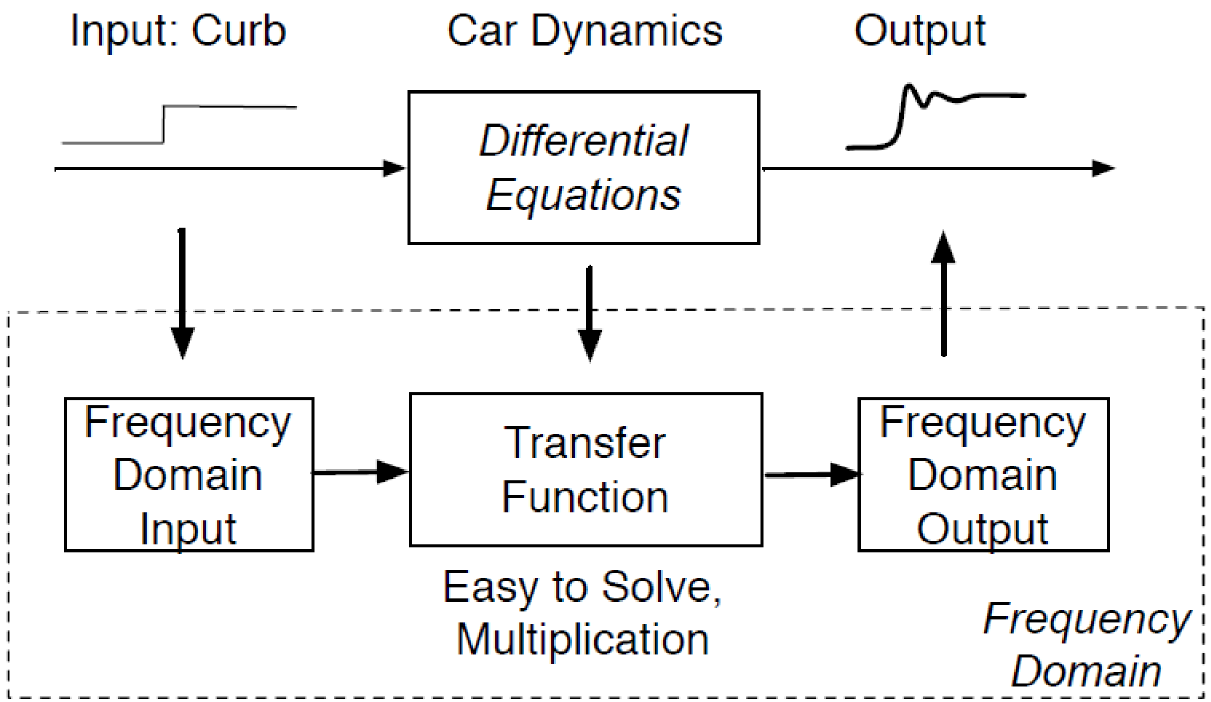
Example: We want to predict what will happen when we drive a car over a curb. The curb can be modelled as a "step" input. The dynamics of the car are governed by a set of differential equations, which are hard to solve for an arbitrary input (this is a linear system).



Hard to Solve Directly

Why Frequency Domain?

After transforming the input and the differential equations into the frequency domain,



Solving for the frequency domain output is easy. The time domain output is found by the inverse transform. We can predict what happens to the system.

Why Frequency Domain?

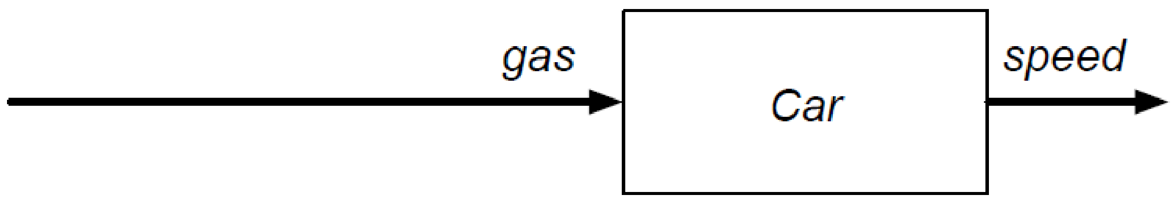
Idea 3: Frequency Domain Lets You Control Linear Systems

- Often we want a system to do something in particular automatically
  - Airplane to fly level
  - Car to go at constant speed
  - Room to remain at a constant temperature

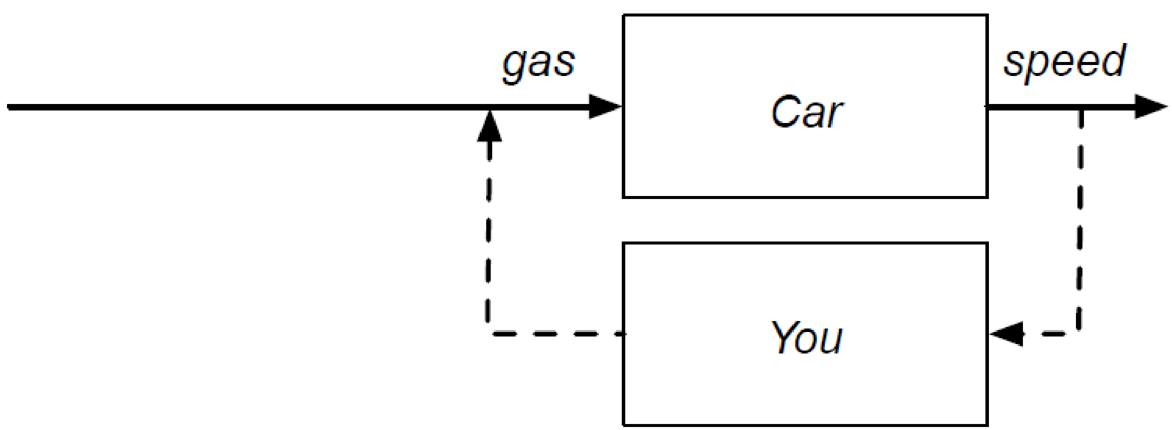
- This is not as trivial as you might think!

Why Frequency Domain?

Example: Controlling a car's speed. Applying more gas causes the car to speed up



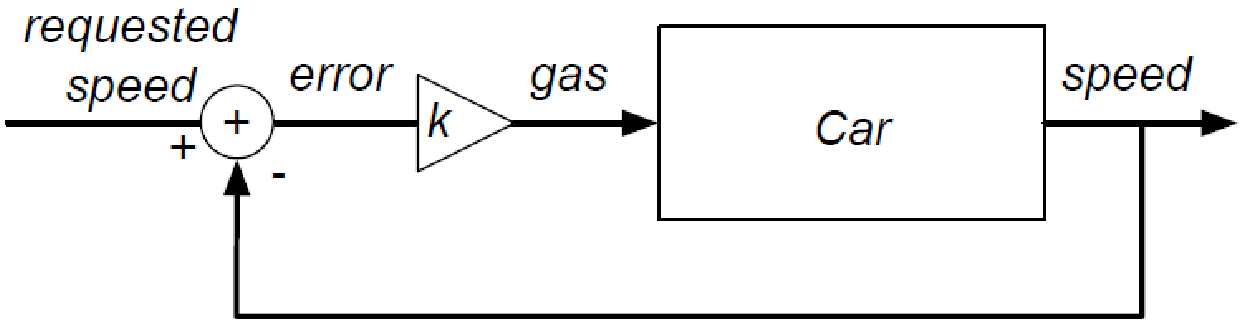
Normally you "close the loop"



How can you do this automatically?

Why Frequency Domain?

Use feedback by comparing the measured speed to the requested speed:

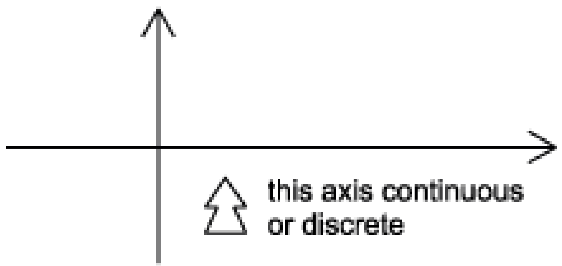


This can easily do something you don't want or expect, and oscillate out of control.

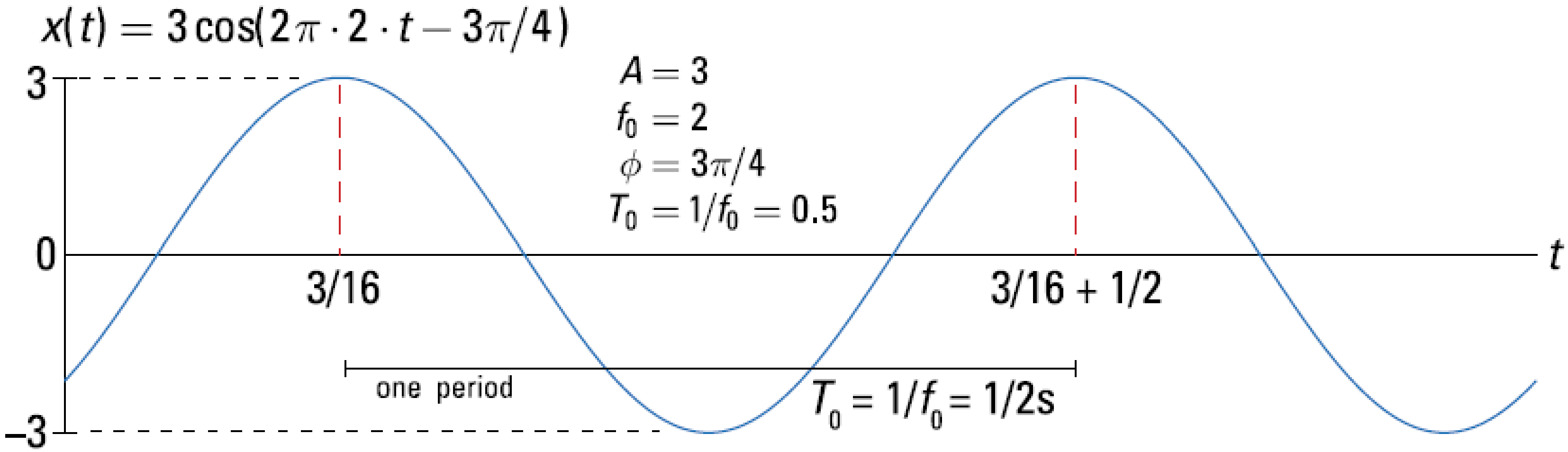
Frequency domain analysis explains why, and tells you how to design the system to do what you want.

Classifications of Signals

Continuous-Time vs. Discrete-Time

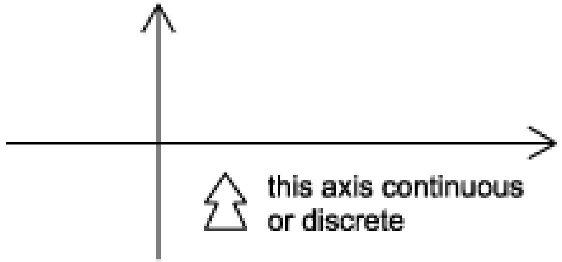


- As the names suggest, this classification is determined by whether or not the time axis is discrete (countable) or continuous.
- A continuous-time signal will contain a value for all real numbers along the time axis.
-In contrast to this, a discrete-time signal, often created by sampling a continuous signal, will only have values at equally spaced intervals along the time axis.



Classifications of Signals

Continuous-Time vs. Discrete-Time



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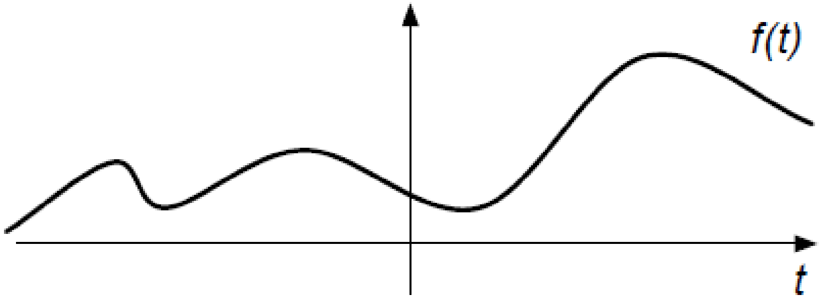
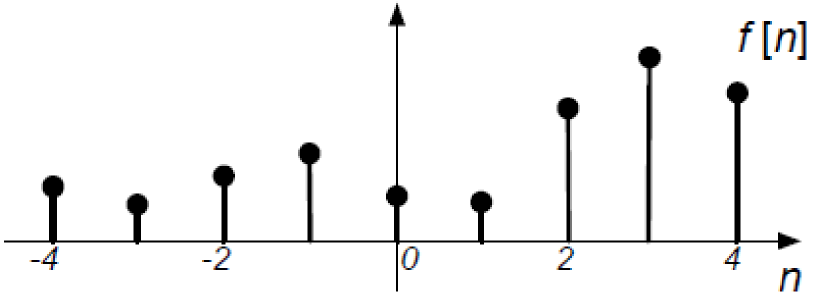


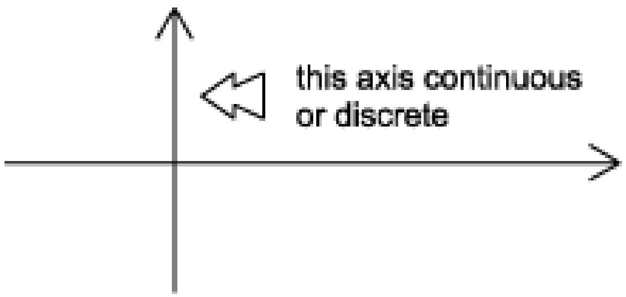
Continuous



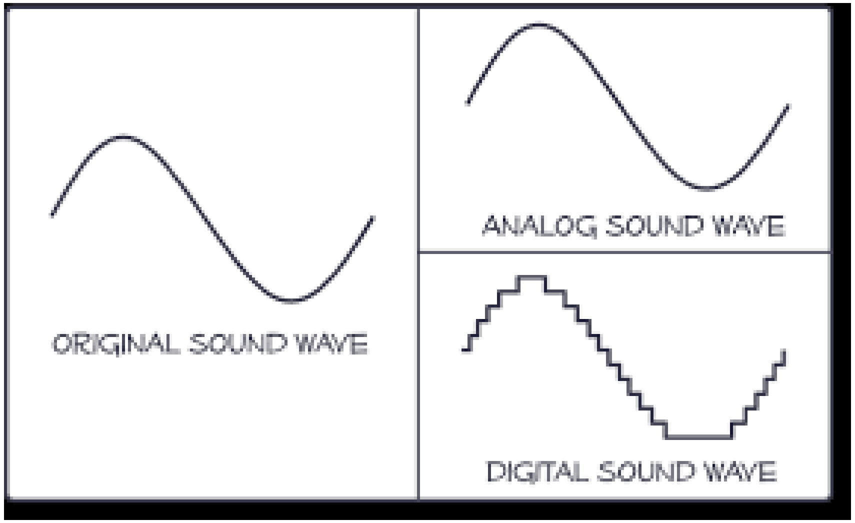
Discrete

Classifications of Signals

Analog vs. Digital



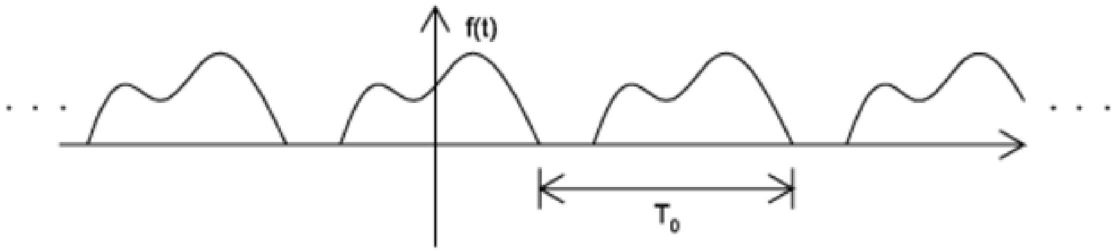
- The difference between **analog** and **digital** is similar to the difference between continuous-time and discrete-time.
- However, in this case the difference involves the values of the function. Analog corresponds to a continuous set of possible function values, while digital corresponds to a discrete set of possible function values.
- An common example of a digital signal is a binary sequence, where the values of the function can only be one or zero.



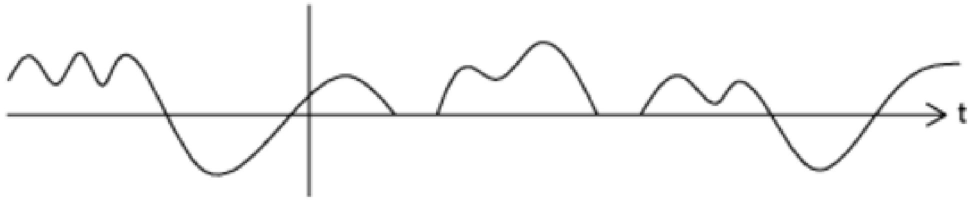
Analog – Digital Signal Example

Classifications of Signals

Periodic vs. Aperiodic



3a: A periodic signal with period T_0



3b: An aperiodic signal

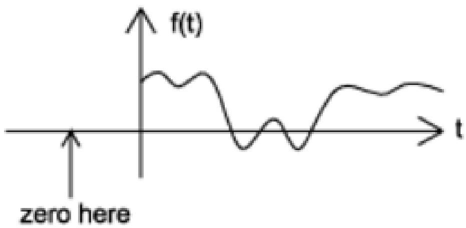
Periodic signals repeat with some **period** T, while aperiodic, or nonperiodic, signals do not. We can define a periodic function through the following mathematical expression, where t can be any number and T is a positive constant

$$f(t) = f(t+T)$$

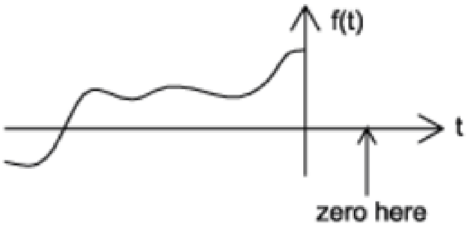
fundamental period of our function, f(t), is the smallest value of T that the still allows Equation to be true.

Classifications of Signals

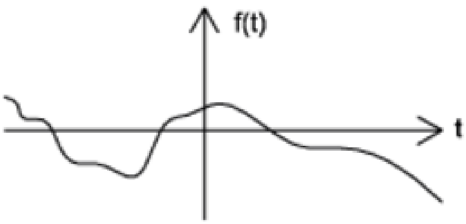
Causal vs. Anticausal vs. Noncausal



4a: A causal signal



4b: An anticausal signal



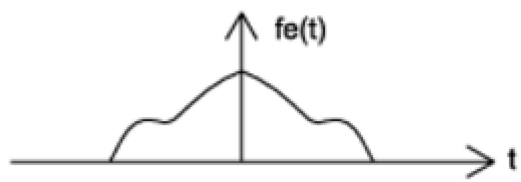
4c: A noncausal signal

- **Causal** signals are signals that are zero for all negative time, while **anticausal** are signals that are zero for all positive time.
- **Noncausal** signals are signals that have nonzero values in both positive and negative time

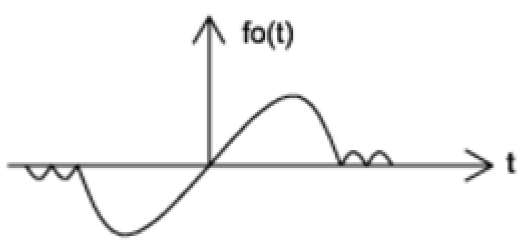
Classifications of Signals

Even vs. Odd

- An **even signal** is any signal f such that $f(t)=f(-t)$.
Even signals can be easily spotted as they are **symmetric** around the vertical axis.
- An **odd signal**, on the other hand, is a signal f such that $f(t)=-f(-t)$



5a: An even signal



5b: An odd signal

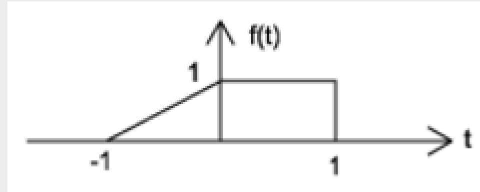
Even – Odd Decomposition

- Using the definitions of even and odd signals, we can show that any signal can be written as a combination of an even and odd signal. That is, every signal has an odd-even decomposition.
- To demonstrate this, we have to look no further than a single equation

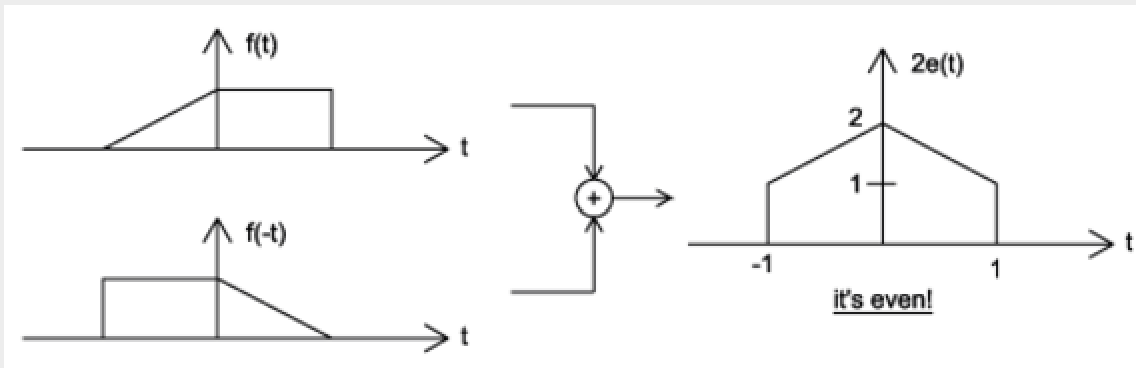
$$f(t) = \frac{1}{2}\left(f(t) + f(-t)\right) + \frac{1}{2}\left(f(t) - f(-t)\right)$$

By multiplying and adding this expression out, it can be shown to be true.
Also, it can be shown that $f(t)+f(-t)$ fulfills the requirement of an even function, while $f(t)-f(-t)$ fulfills the requirement of an odd function.

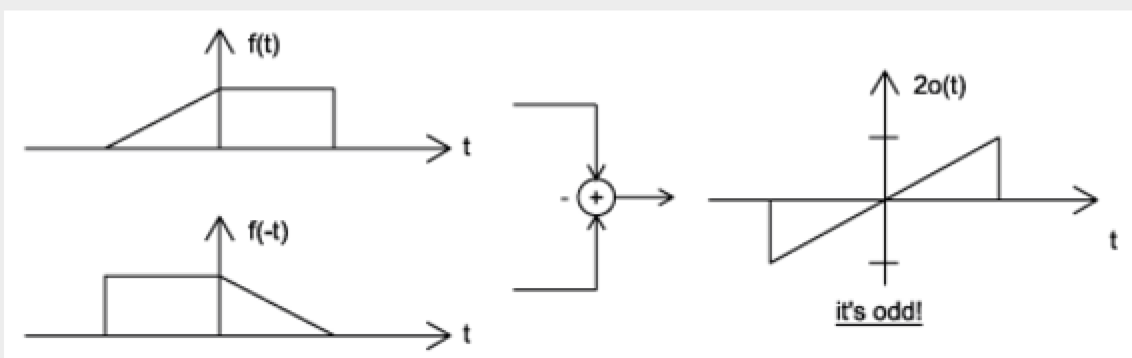
Even – Odd Decomposition Example



The signal we will decompose using odd-even decomposition

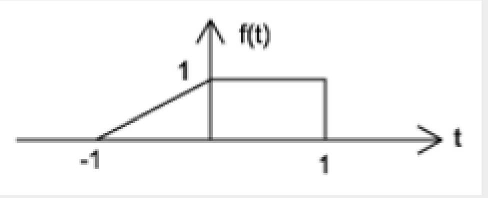


6b: Even part: $e(t) = \frac{1}{2}\left(f(t) + f(-t)\right)$

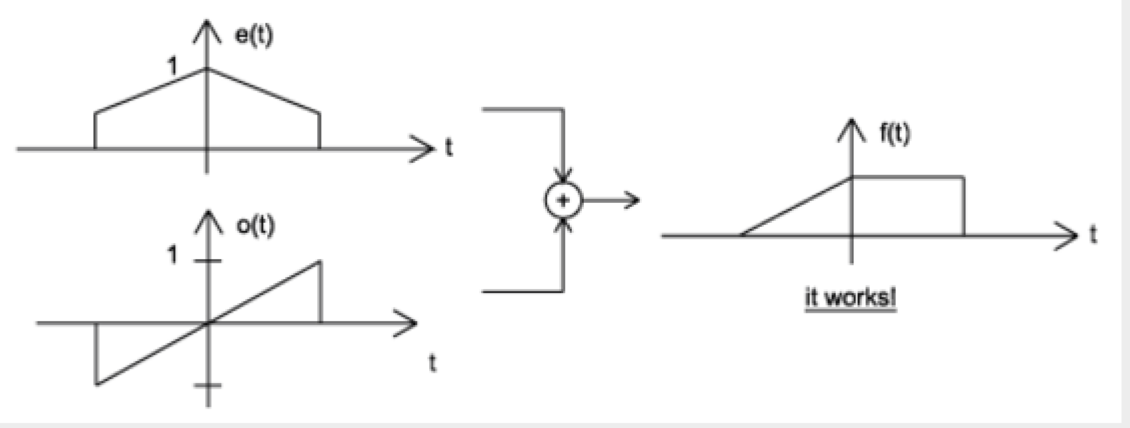


6c: Odd part: $o(t) = \frac{1}{2}\left(f(t) - f(-t)\right)$

Even – Odd Decomposition Example



The signal we will decompose using odd-even decomposition

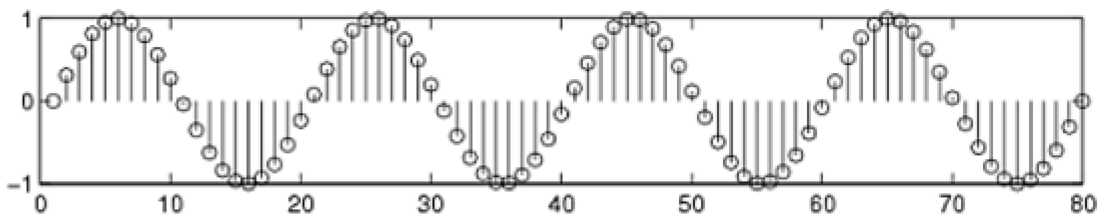


6d: Check: $e(t) + o(t) = f(t)$

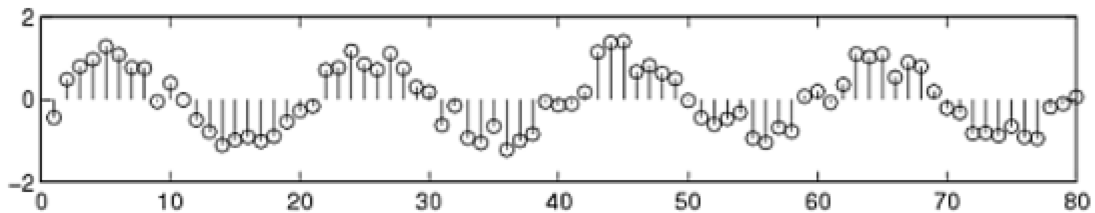
Classifications of Signals

Deterministic vs. Random

A **deterministic signal** is a signal in which each value of the signal is fixed, being determined by a mathematical expression, rule, or table. On the other hand, the values of a **random signal** are not strictly defined, but are subject to some amount of variability.



7a: Deterministic Signal



7b: Random Signal

Example

Consider the signal defined for all real t described by

$$f(t) = \begin{cases} \sin(2\pi t)/t & t \geq 1 \\ 0 & t < 1 \end{cases}$$

Write down the properties of this signal

This signal is continuous time, analog, aperiodic, infinite length, causal, neither even nor odd, and, by definition, deterministic.

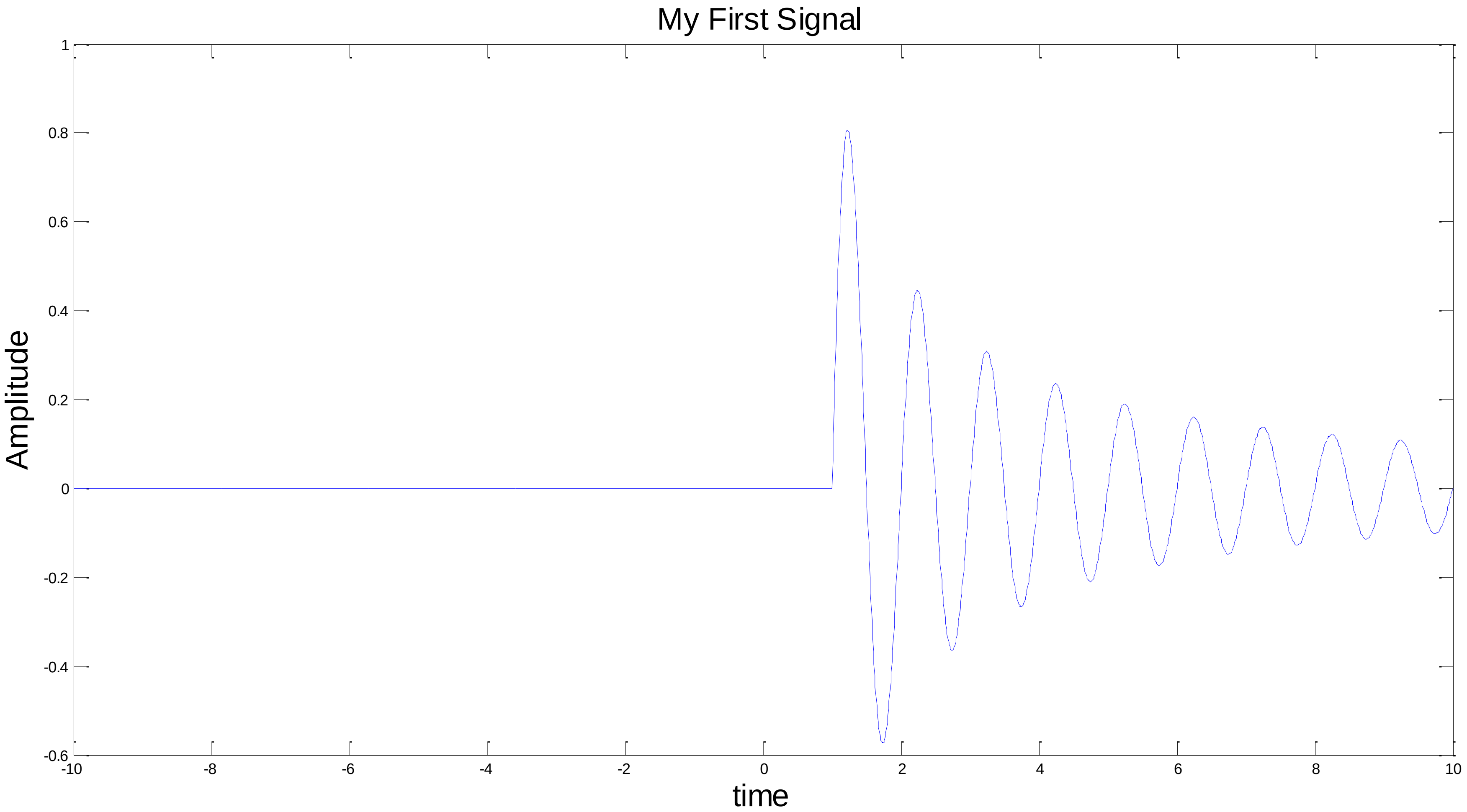
Example

```
% Code written for Last Example in Lecture1

clc
clear all
close all

t1 = 1:0.01:10;
t2 = -10:0.01:1-0.01;
timeAxis = [t2 t1];
MySignal = [zeros(1,length(t2))
sin(2*pi*t1)./t1];
plot(timeAxis,MySignal)
ylabel('Amplitude', 'fontsize', 20)
xlabel('time', 'fontsize', 20)
title('My First Signal','fontsize', 20)
```

Example



My First Signal
Amplitude
time
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-10
-8
-6
-4
-2
0
2
4
6
8
10