# BLM2041 Signals and Systems

## Syllabus

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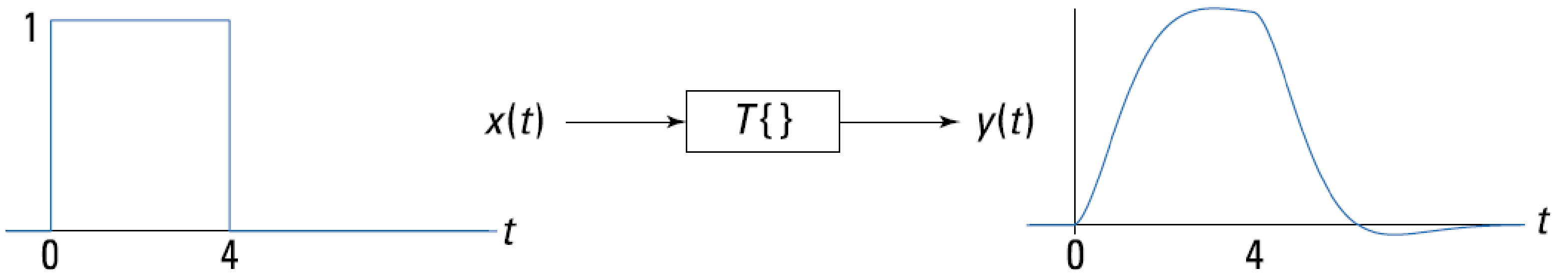
# Responses to arbitrary signals

- Although we have focused on responses to simple signals ($\delta[n]$,$\delta(t)$) we are generally interested in responses to more complicated signals.
- How do we compute responses to a more complicated input signals?



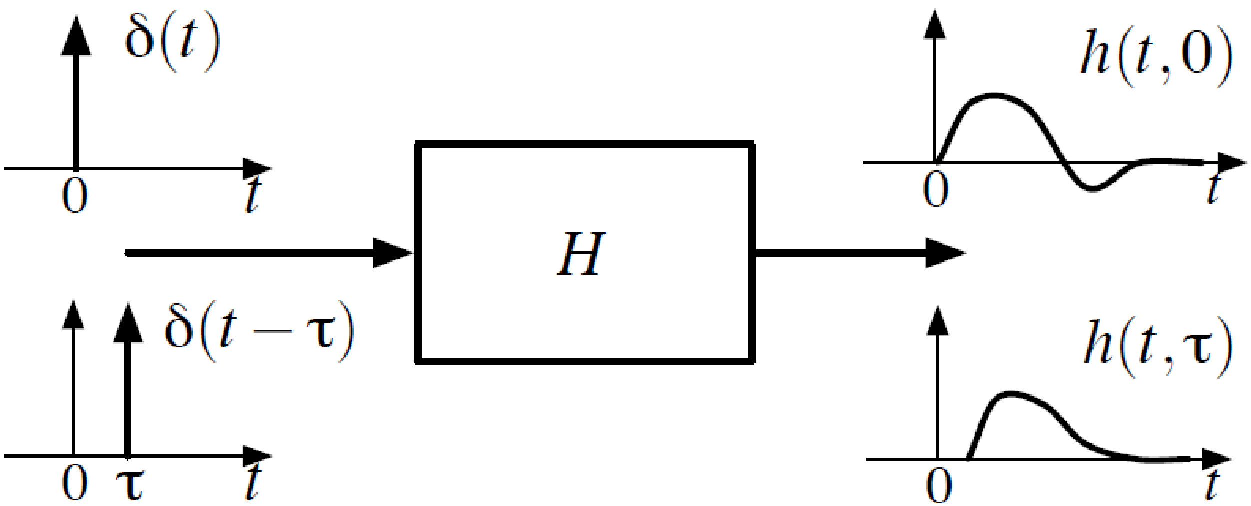

Block diagram depicting a general input/output relationship.

# Impulse Response

The *impulse response* of a linear system $h_\tau(t)$ is the output of the system at time $t$ to an impulse at time $\tau$. This can be written as

$$h_\tau = H(\delta_\tau)$$

Care is required in interpreting this expression!



**Note:** Be aware of potential confusion here:

When you write

$$h_{\tau}(t) = H(\delta_{\tau}(t))$$

the variable $t$ serves different roles on each side of the equation.

- $t$ on the left is a specific value for time, the time at which the output is being sampled.
- $t$ on the right is varying over all real numbers, it is not the same $t$ as on the left.
- The output at time specific time $t$ on the left in general depends on the input at all times $t$ on the right (the entire input waveform).

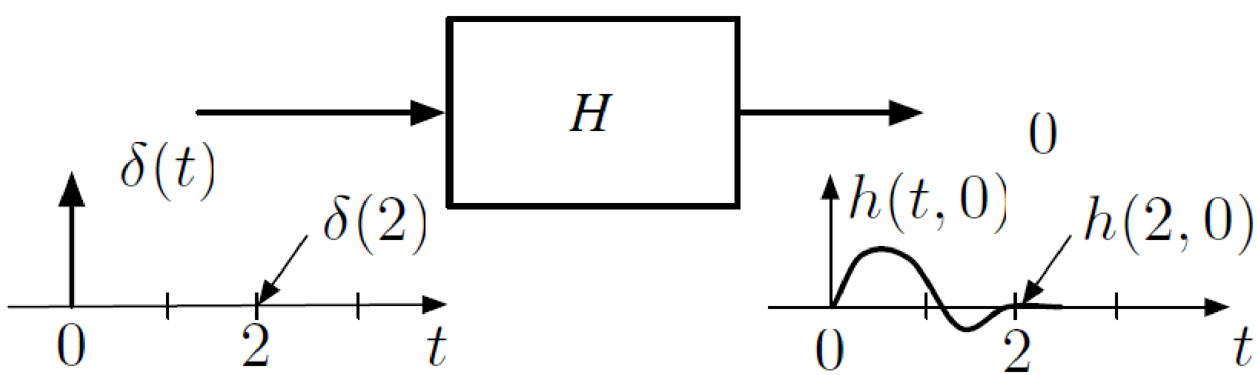
- Assume the input impulse is at $\tau = 0$,

$$h = h_0 = H(\delta_0).$$

We want to know the impulse response at time $t = 2$. It doesn't make any sense to set $t = 2$, and write

$$h(2) = H(\delta(2)) \qquad\qquad \Leftarrow \text{No!}$$

First, $\delta(2)$ is something like zero, so $H(0)$ would be zero. Second, the value of $h(2)$ depends on the entire input waveform, not just the value at $t = 2$.

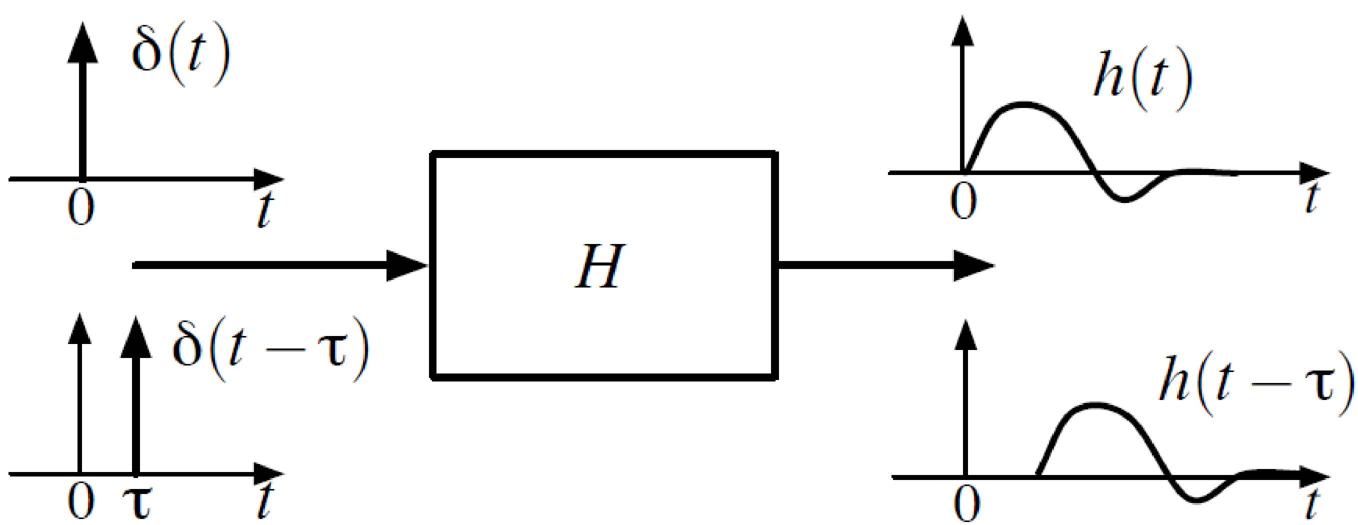


# Time-invariance

If $H$ is time invariant, delaying the input and output both by a time $\tau$ should produce the same response

$$h_\tau(t) = h(t - \tau).$$

In this case, we don't need to worry about $h_\tau$ because it is just $h$ shifted in time.



# Linearity and Extended Linearity

**Linearity:** A system $S$ is linear if it satisfies both

- *Homogeneity:* If $y = Sx$, and $a$ is a constant then
$$ay = S(ax).$$
- *Superposition:* If $y_1 = Sx_1$ and $y_2 = Sx_2$, then
$$y_1 + y_2 = S(x_1 + x_2).$$

**Combined Homogeneity and Superposition:**
If $y_1 = Sx_1$ and $y_2 = Sx_2$, and $a$ and $b$ are constants,
$$ay_1 + by_2 = S(ax_1 + bx_2)$$

## Extended Linearity

- *Summation:* If $y_n = S(x_n)$ for all $n$, an integer from $(-\infty < n < \infty)$, and $a_n$ are constants

$$\sum_n a_n y_n = S\left(\sum_n a_n x_n\right)$$

  Summation and the system operator commute, and can be interchanged.

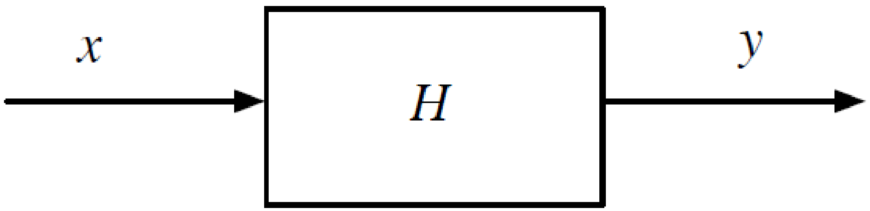
- *Integration (Simple Example)* : If $y = S(x)$,

$$\int_{-\infty}^{\infty} a(\tau)y(t-\tau)\, d\tau = S\left(\int_{-\infty}^{\infty} a(\tau)x(t-\tau)d\tau\right)$$

  Integration and the system operator commute, and can be interchanged.

# Output of an LTI System

We would like to determine an expression for the output $y(t)$ of an linear time invariant system, given an input $x(t)$




We can write a signal $x(t)$ as a sample of itself

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta_\tau(t)\ d\tau$$

This means that $x(t)$ can be written as a weighted integral of $\delta$ functions.

Applying the system $H$ to the input $x(t)$,

$$\begin{aligned} y(t) &= H\left(x(t)\right) \\ &= H\left(\int_{-\infty}^{\infty} x(\tau)\delta_\tau(t)d\tau\right) \end{aligned}$$

If the system obeys extended linearity we can interchange the order of the system operator and the integration

$$y(t) = \int_{-\infty}^{\infty} x(\tau)H\left(\delta_\tau(t)\right)d\tau.$$

The impulse response is

$$h_\tau(t) = H(\delta_\tau(t)).$$

Substituting for the impulse response gives

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h_\tau(t) d\tau.$$

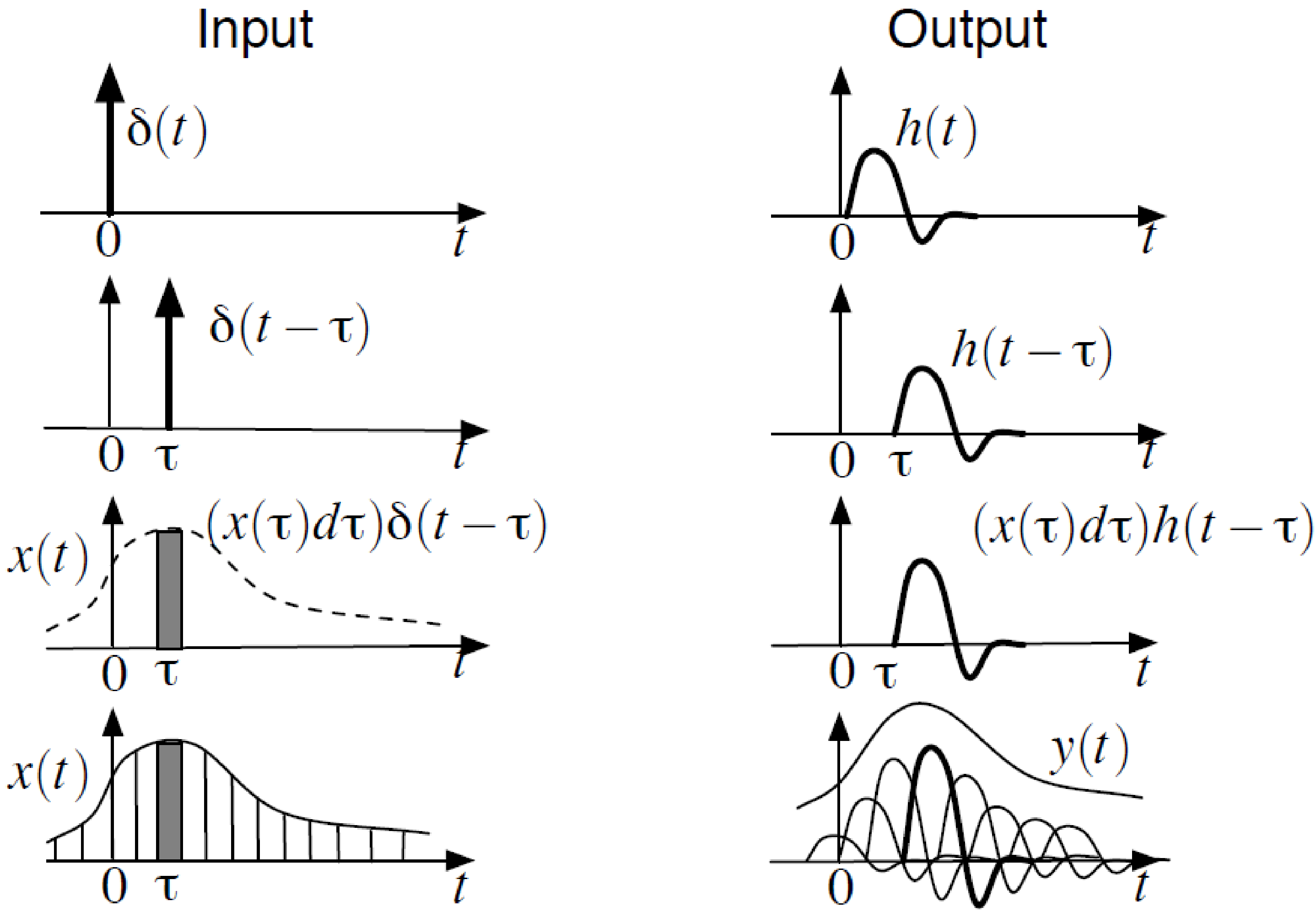
This is a *superposition integral*. The values of $x(\tau)h(t,\tau)d\tau$ are superimposed (added up) for each input time $\tau$.

If $H$ is time invariant, this written more simply as

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h_\tau(t) d\tau.$$

This is in the form of a *convolution integral*, which will be the subject of the next class.

Graphically, this can be represented as:




$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \qquad y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

If a system is linear and time-invariant (LTI) then its output is the integral of weighted and shifted unit-impulse responses.

$$\delta(t) \longrightarrow \boxed{\text{system}} \longrightarrow h(t)$$

$$\delta(t-\tau) \longrightarrow \boxed{\text{system}} \longrightarrow h(t-\tau)$$

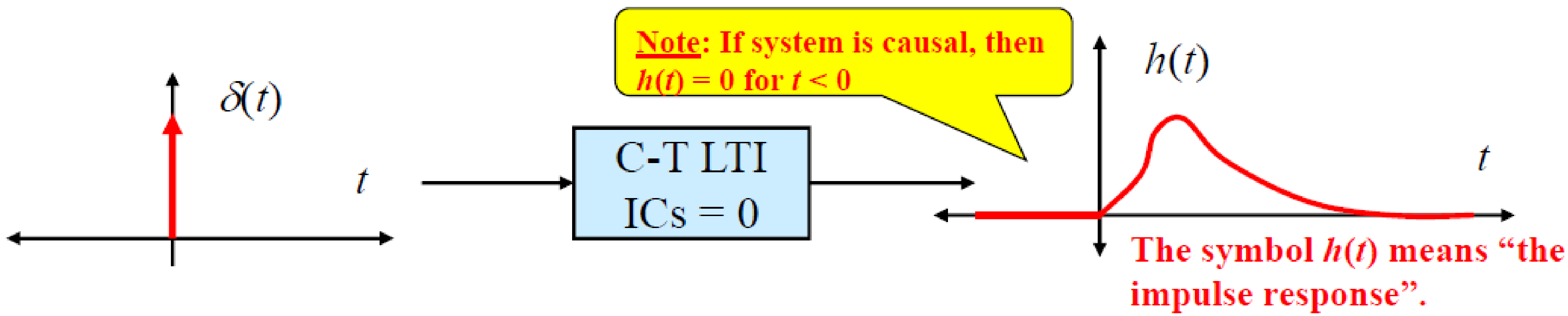
$$x(\tau)\delta(t-\tau) \longrightarrow \boxed{\text{system}} \longrightarrow x(\tau)h(t-\tau)$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \longrightarrow \boxed{\text{system}} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

# Recall: Impulse Response

Earlier we introduced the concept of impulse response…

…what comes out of a system when the input is an impulse (delta function)




Noting that the LT of $\delta(t) = 1$ and using the properties of the transfer function and the Z transform we said that

$$h(t) = \mathscr{L}^{-1}\left\{H(s)\mathscr{L}\left\{\delta(t)\right\}\right\} \qquad \boxed{h(t) = \mathscr{L}^{-1}\left\{H(s)\right\}} \qquad \boxed{h(t) = \mathscr{F}^{-1}\left\{H(\omega)\right\}}$$

So…once we have either $H(s)$ or $H(\omega)$ we can get the impulse response $h(t)$

# Convolution Property and System Output

Let $x(t)$ be a signal with CTFT $X(\omega)$ and LT of $X(s)$

$$x(t) \leftrightarrow X(\omega)$$
$$x(t) \leftrightarrow X(s)$$

Consider a system w/ freq resp $H(\omega)$ & trans func $H(s)$

$$h(t) \leftrightarrow H(\omega)$$
$$h(t) \leftrightarrow H(s)$$

We've spent much time using these tools to analyze system outputs this way:

$$Y(\omega) = H(\omega)X(\omega) \quad \leftrightarrow \quad y(t) = \mathscr{F}^{-1}\{H(\omega)X(\omega)\}$$

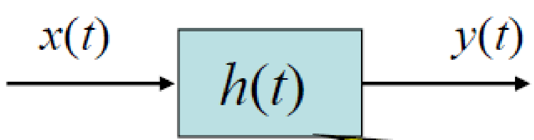
$$Y(s) = H(s)X(s) \quad \leftrightarrow \quad y[n] = \mathscr{L}^{-1}\{H(s)X(s)\}$$

**The convolution property of the CTFT and LT gives an alternate way to find $y(t)$:**

$$\boxed{\mathscr{F}^{-1}\{X(\omega)H(\omega)\} = x(t) * h(t)}$$

$$\boxed{\mathscr{L}^{-1}\{X(s)H(s)\} = x(t) * h(t)}$$

$$\boxed{x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau}$$




$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau}$$

**"Convolving" input $x(t)$ with the impulse response $h(t)$ gives the output $y(t)$!**

**LTI System with impulse response $h(t)$**

## Convolution for Causal System & with Causal Input

An arbitrary LTI system's output can be found using the general convolution form:

$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau}$$

**General LTI System**

If the system is causal then $h(t) = 0$ for $t < 0$.... Thus $h(t - \tau) = 0$ for $t > \tau$ … so:

$$\boxed{y(t) = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau}$$

**Causal LTI System**

If the input is causal then $x(t) = 0$ for $t < 0$.... so:

$$\boxed{y(t) = \int_{0}^{\infty} x(\tau)h(t-\tau)d\tau}$$

**Causal Input & General LTI System**

If the system & signal are both causal then

$$\boxed{y(t) = \int_{0}^{t} x(\tau)h(t-\tau)d\tau}$$

**Causal Input & Causal LTI System**

## Convolution Properties

1. **Commutativity** $\boxed{x(t)*h(t)=h(t)*x(t)}$

2. **Associativity** $\boxed{[x(t)*h_1(t)]*h_2(t)=x(t)*[h_1(t)*h_2(t)]}$

Associativity together with commutativity says we **can interchange the order of two cascaded systems**.

3. **Distributivity** $\boxed{x(t)*[h_1(t)+h_2(t)]=x(t)*h_1(t)+x(t)*h_2(t)}$

4. **Derivative Property:**

derivative

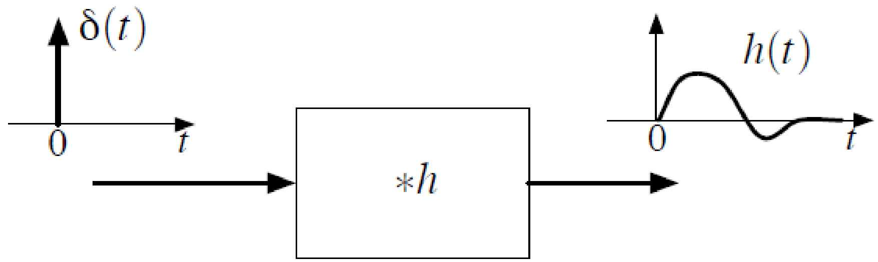
$$\frac{d}{dt}[x(t)*v(t)]=\dot{x}(t)*v(t)$$
$$=x(t)*\dot{v}(t)$$

5. **Integration Property** Let $y(t)=x(t)*h(t)$, then

$$\int_{-\infty}^{t}y(\lambda)d\lambda=\left[\int_{-\infty}^{t}x(\lambda)d\lambda\right]*h(t)=x(t)*\left[\int_{-\infty}^{t}h(\lambda)d\lambda\right]$$

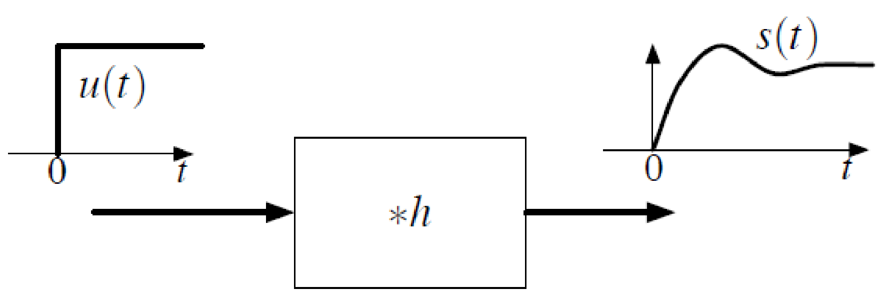
**Example:** Measuring the impulse response of an LTI system.

We would like to measure the impulse response of an LTI system, described by the impulse response $h(t)$

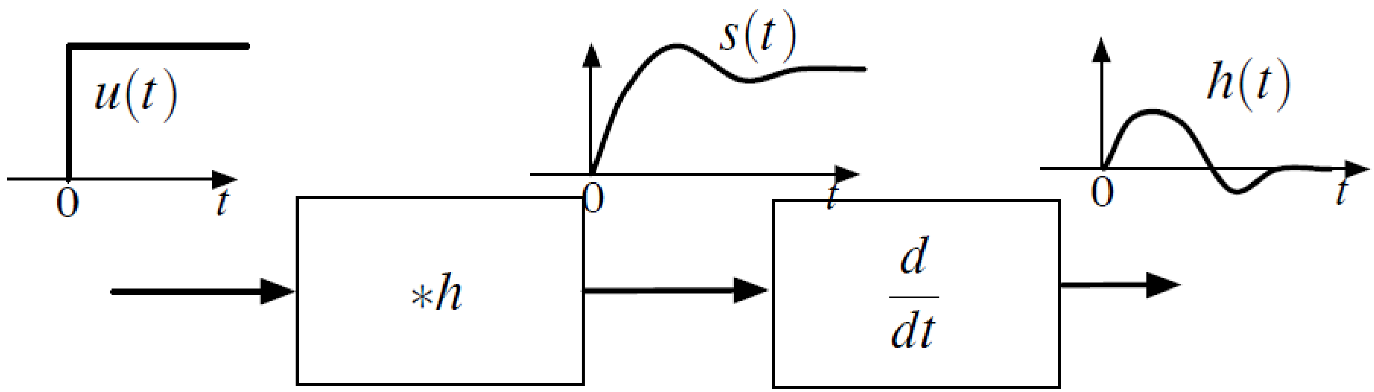



This can be practically difficult because input amplitude is often limited. A very short pulse then has very little energy.

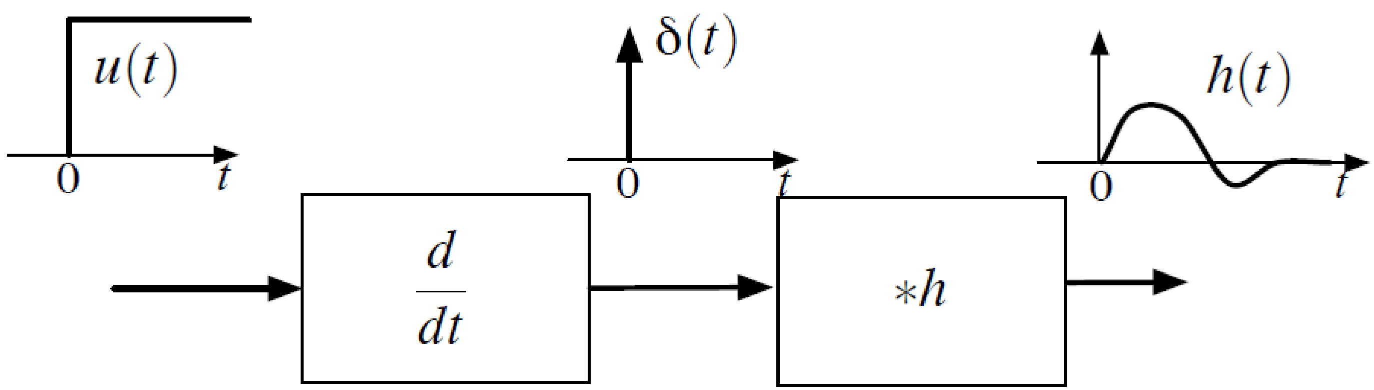
A common alternative is to measure the *step response* $s(t)$, the response to a unit step input $u(t)$



The impulse response is determined by differentiating the step response,




To show this, commute the convolution system and the differentiator to produce a system with the same overall impulse response



# Steps for Graphical Convolution $x(t)*h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

1. **Re-Write the signals as functions of τ**: $x(\tau)$ and $h(\tau)$

2. **Flip** just one of the signals around $t = 0$ to get either $x(-\tau)$ or $h(-\tau)$
   a. It is usually best to flip the signal with shorter duration
   b. For notational purposes here: we'll flip $h(\tau)$ to get $h(-\tau)$

3. **Find Edges** of the flipped signal
   a. Find the left-hand-edge τ-value of $h(-\tau)$: **call it $\tau_{L,0}$**
   b. Find the right-hand-edge τ-value of $h(-\tau)$: **call it $\tau_{R,0}$**

4. **Shift** $h(-\tau)$ by an arbitrary value of $t$ to get $h(t - \tau)$ and **get its edges**
   a. Find the left-hand-edge τ-value of $h(t - \tau)$ as a function of $t$: **call it $\tau_{L,t}$**
      - **Important:** It will *always* be… $\tau_{L,t} = t + \tau_{L,0}$

   b. Find the right-hand-edge τ-value of $h(t - \tau)$ as a function of $t$: **call it $\tau_{R,t}$**
      - **Important:** It will *always* be… $\tau_{R,t} = t + \tau_{R,0}$

   Note: If the signal you flipped is NOT finite duration,
   one or both of $\tau_{L,t}$ and $\tau_{R,t}$ will be infinite ($\tau_{L,t} = -\infty$ and/or $\tau_{R,t} = \infty$)

# Steps Continued

**5. <u>Find Regions of τ-Overlap</u>**

a. What you are trying to do here is find intervals of $t$ over which the product $x(\tau)\, h(t - \tau)$ has a single mathematical form in terms of $\tau$

b. In each region find: Interval of $t$ that makes the identified overlap happen

c. Working examples is the best way to learn how this is done

**Tips**: Regions should be contiguous with no gaps!!!
Don't worry about $<$ vs. $\leq$ etc.

6. For Each Region: **Form the Product $x(\tau)\, h(t - \tau)$ and Integrate**

a. Form product $x(\tau)\, h(t - \tau)$

b. Find the Limits of Integration by finding the interval of $\tau$ over which the product is nonzero

  i. Found by seeing where the edges of $x(\tau)$ and $h(t - \tau)$ lie

  ii. Recall that the edges of $h(t - \tau)$ are $\tau_{L,t}$ and $\tau_{R,t}$, which often depend on the value of $t$

   • So… the limits of integration may depend on $t$

c. Integrate the product $x(\tau)\, h(t - \tau)$ over the limits found in 6b

  i. The result is generally a function of $t$, but is only valid for the interval of t found for the current region

  ii. Think of the result as a "time-section" of the output $y(t)$

# Steps Continued

7. **“Assemble” the output** from the output time-sections for all the regions
   a. Note: you do NOT add the sections together
   b. You define the output “piecewise”
   c. Finally, if possible, look for a way to write the output in a simpler form

# Example: Graphically Convolve Two Signals

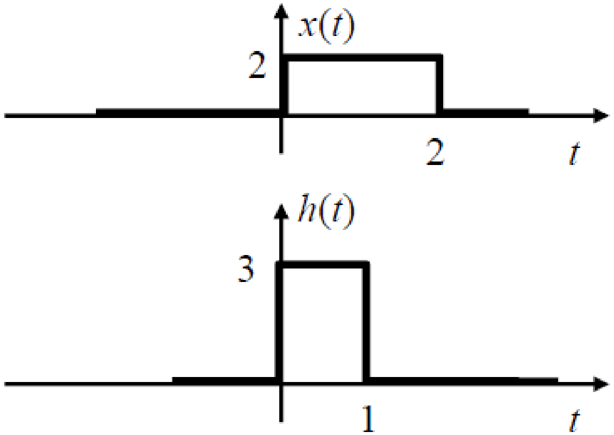
$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

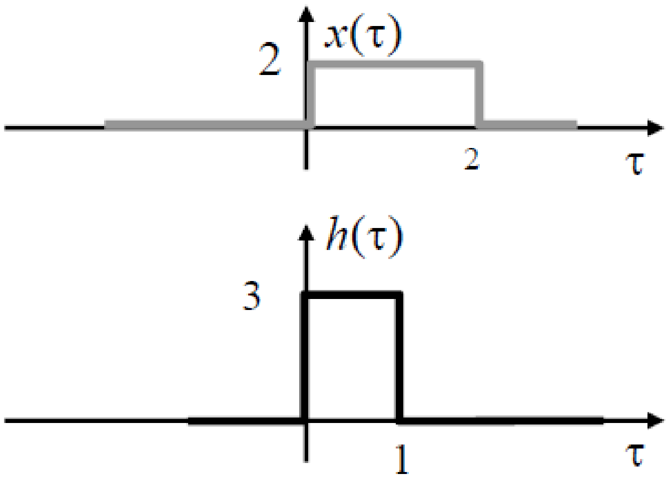
**By "Properties of Convolution"… these two forms are Equal**

**This is why we can flip either signal**

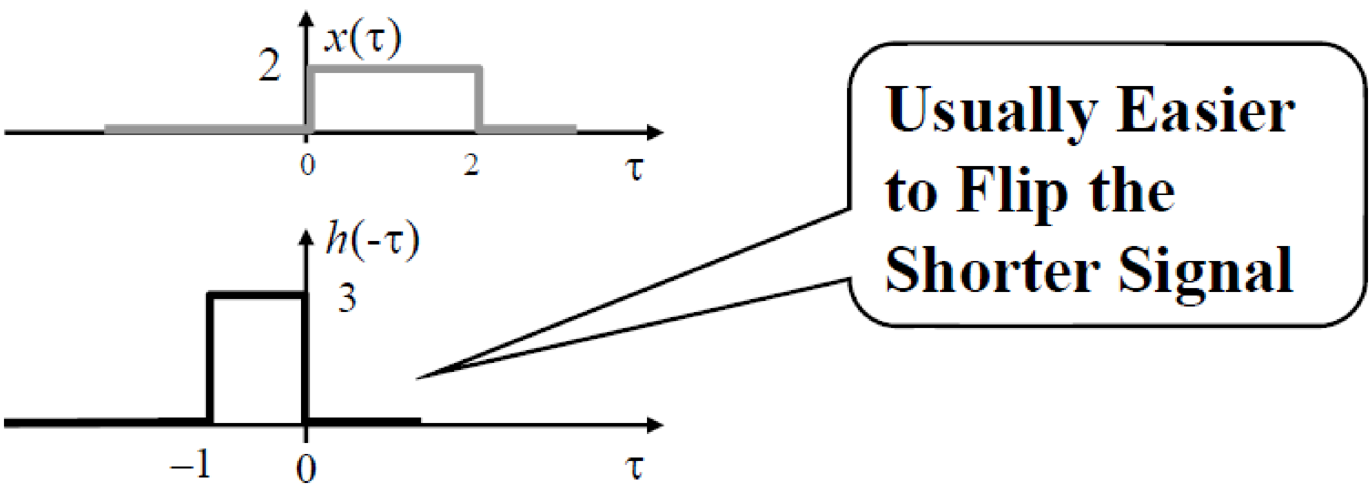
**Convolve these two signals:**



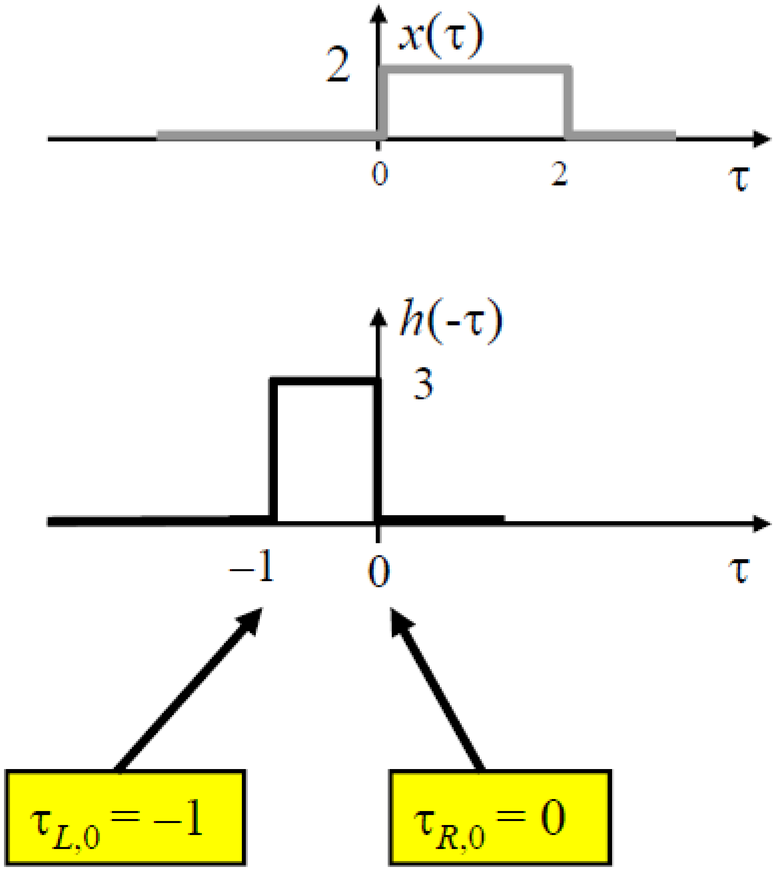
## Step #1: Write as Function of τ




## Step #2: Flip $h(\tau)$ to get $h(-\tau)$



# Step #3: Find Edges of Flipped Signal



## *Motivating* Step #4: Shift by $t$ to get $h(t\text{-}\tau)$ & Its Edges

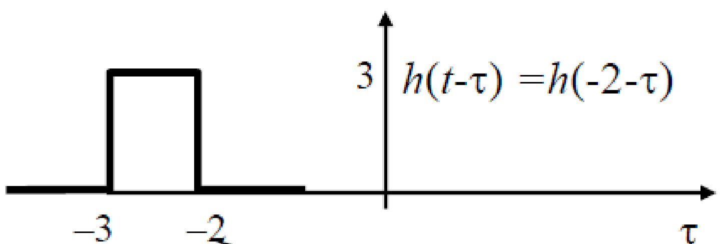
Just looking at 2 "arbitrary" $t$ values

In Each Case We Get

$$\tau_{L,t} = t + \tau_{L,0}$$

$$\tau_{R,t} = t + \tau_{R,0}$$

**For $t = -2$**




$\tau_{L,t} = t + \tau_{L,0}$

$\tau_{L,t} = t - 1$

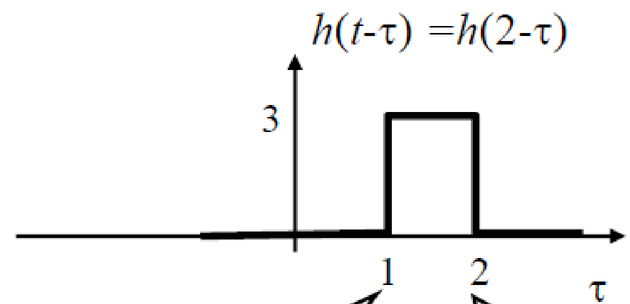
$\tau_{L,-2} = -2 - 1$

$\tau_{R,t} = t + \tau_{R,0}$

$\tau_{R,t} = t + 0$

$\tau_{R,-2} = -2 + 0$

**For $t = 2$**




$\tau_{L,t} = t + \tau_{L,0}$

$\tau_{L,t} = t - 1$

$\tau_{L,2} = 2 - 1$

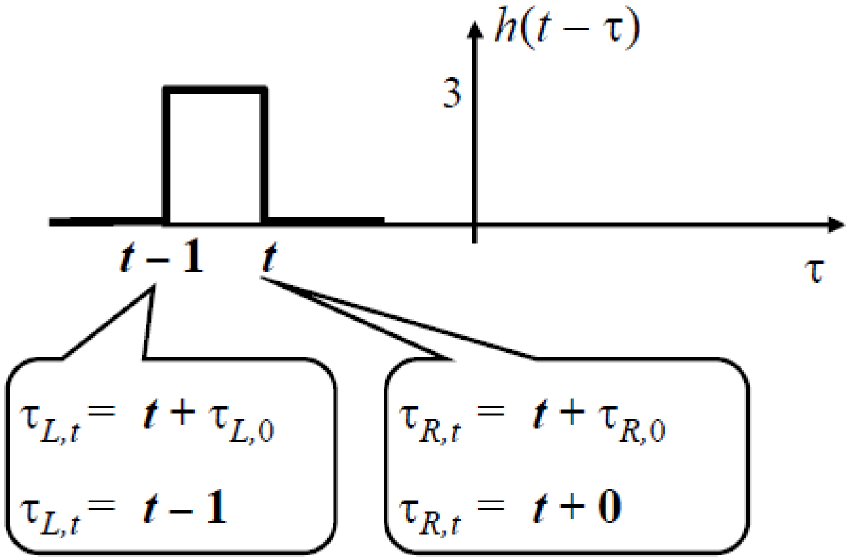
$\tau_{R,t} = t + \tau_{R,0}$

$\tau_{R,t} = t + 0$

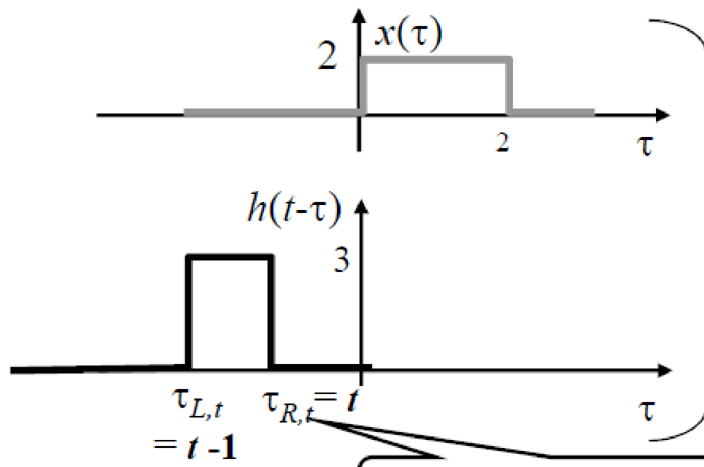
$\tau_{R,2} = 2 + 0$

# *Doing* Step #4: Shift by $t$ to get $h(t\text{-}\tau)$ & Its Edges

**For *Arbitrary* Shift by $t$**



# Step #5: Find Regions of τ-Overlap

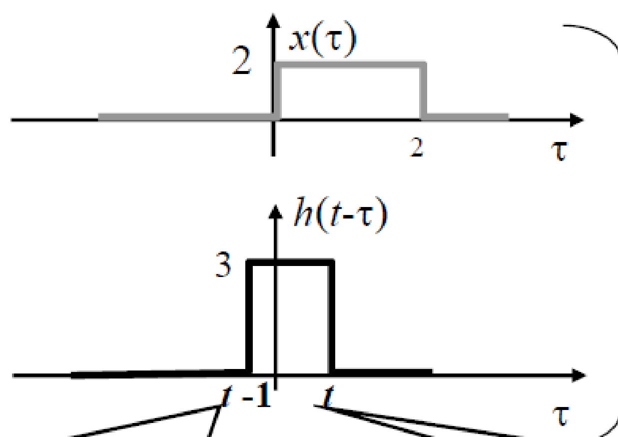



Want $\tau_{R,t} < 0$ ➔ $t < 0$

**Region I**

**No τ-Overlap**

**$t < 0$**




Want $\tau_{L,t} \leq 0$ ➔ $t\text{-}1 \leq 0$ ➔ $t \leq 1$

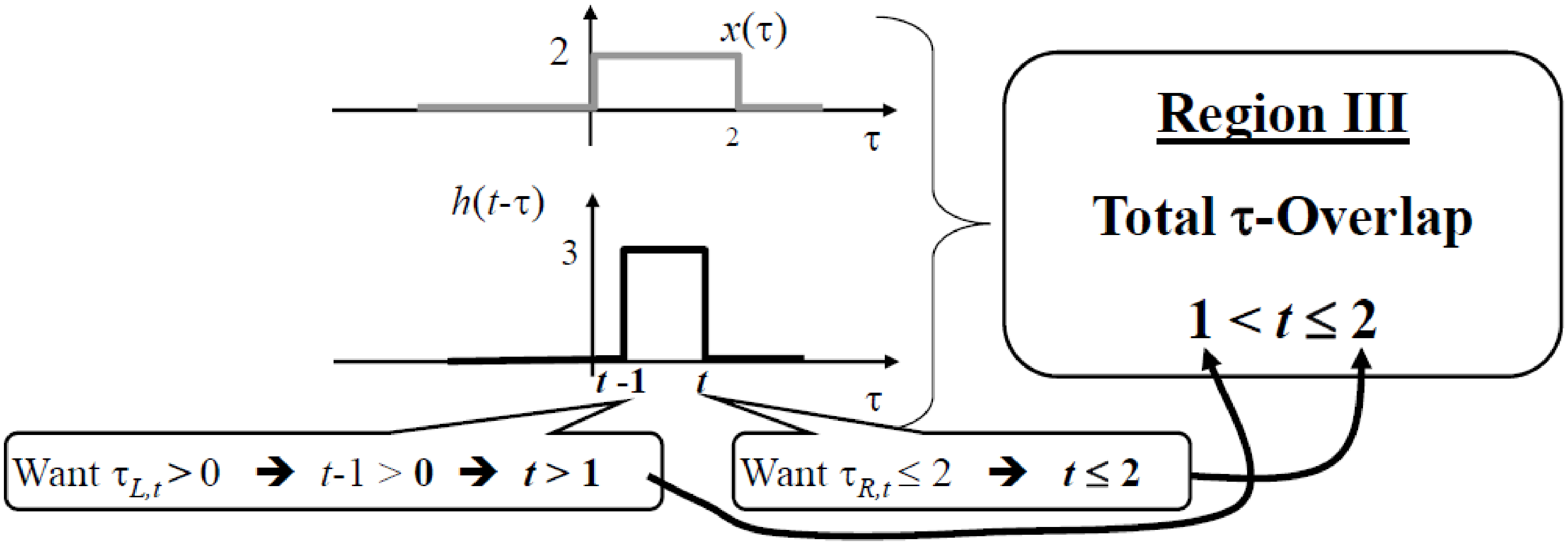
Want $\tau_{R,t} \geq 0$ ➔ $t \geq 0$

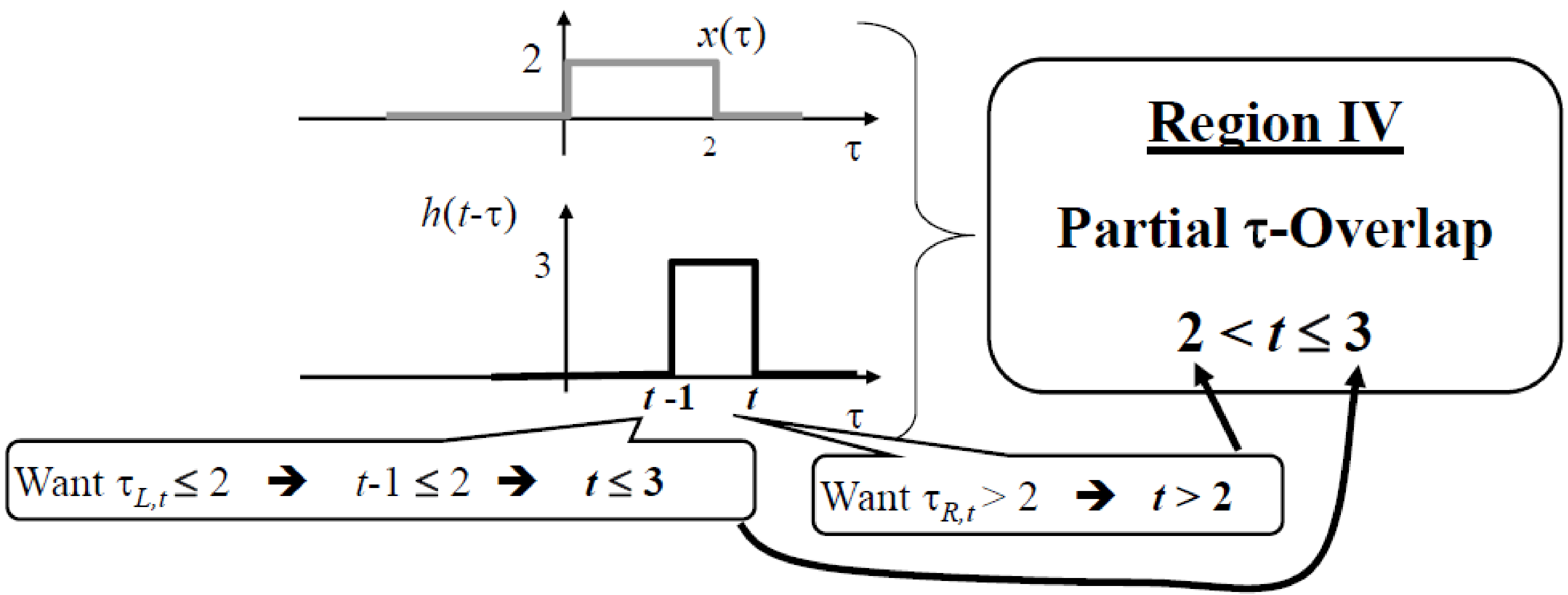
**Region II**

**Partial τ-Overlap**

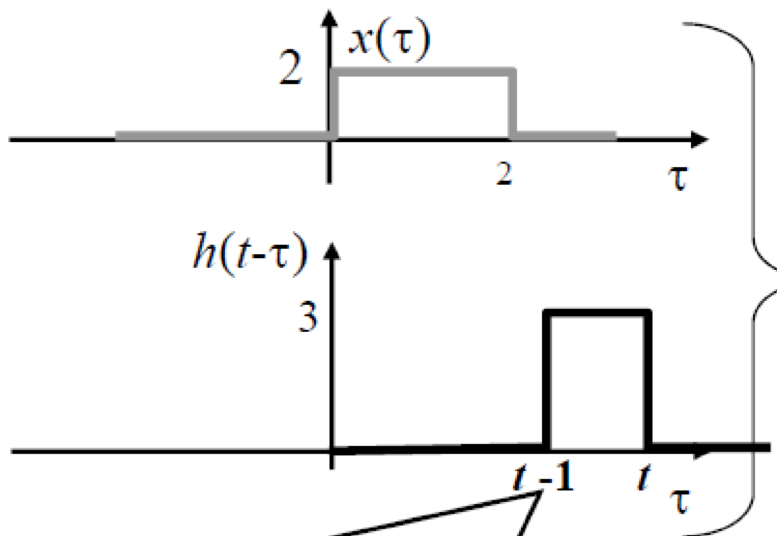
**$0 \leq t \leq 1$**

# Step #5 (Continued): Find Regions of τ-Overlap



# Step #5 (Continued): Find Regions of τ-Overlap



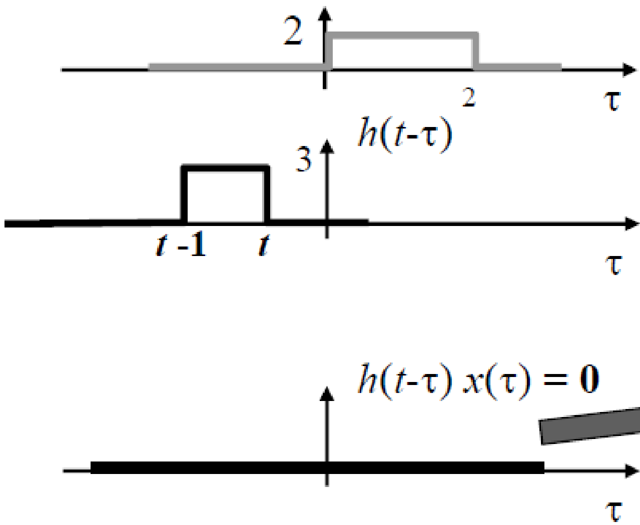

Want $\tau_{L,t} > 2$ ➔ $t$-1 > 2 ➔ $t > 3$

**Region V**

**No τ-Overlap**

$t > 3$

# Step #6: Form Product & Integrate For Each Region



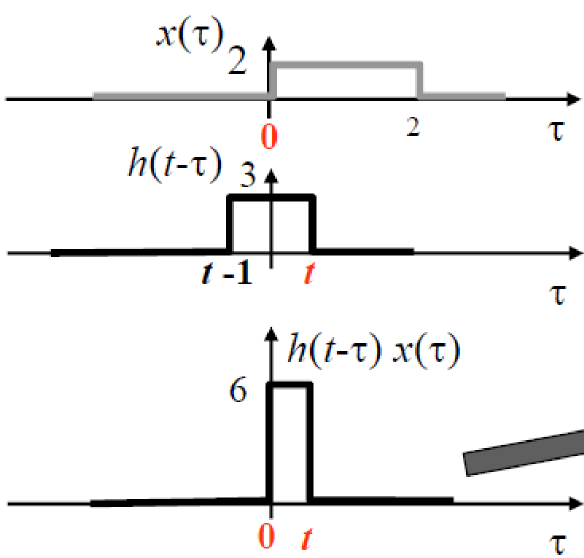

## Region I: $t < 0$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} 0 d\tau = 0$$

$$y(t) = 0 \quad \text{for all} \quad t < 0$$

With 0 integrand the limits don't matter!!!



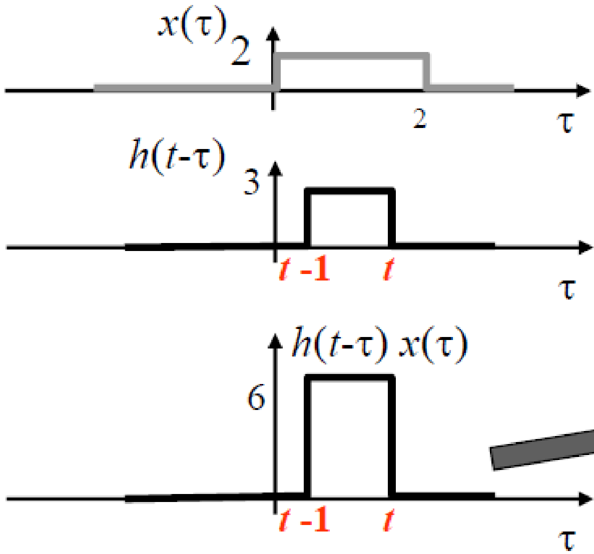

## Region II: $0 \le t \le 1$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_{0}^{t} 6 d\tau = [6\tau]_0^t = 6t - 6\times 0 = 6t$$

$$y(t) = 6t \quad \text{for} \quad 0 \le t \le 1$$

# Step #6 (Continued): Form Product & Integrate For Each Region

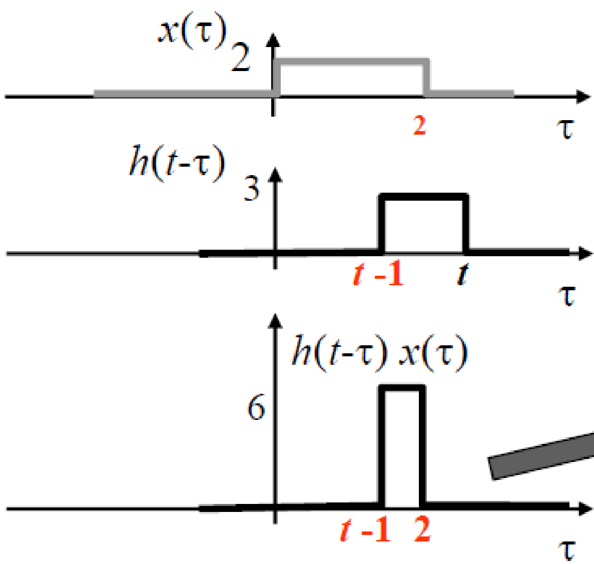



**Region III: $1 < t \le 2$**

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_{t-1}^{t} 6d\tau = [6\tau]_{t-1}^{t} = 6t - 6(t-1) = 6$$

$y(t) = 6 \quad$ for all $t$ such that : $\; 1 < t \le 2$




**Region IV: $2 < t \le 3$**

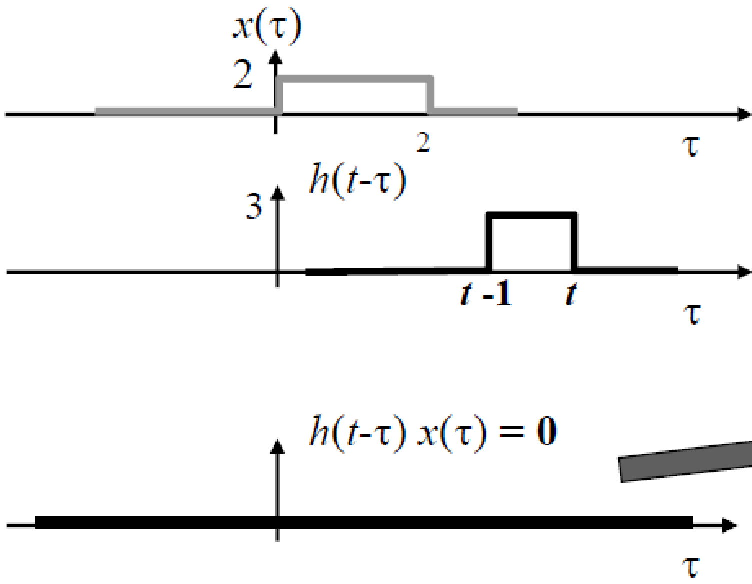
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_{t-1}^{2} 6d\tau = [6\tau]_{t-1}^{2} = 6 \times 2 - 6(t-1) = -6t + 18$$

$y(t) = -6t + 18 \quad$ for $\; 2 < t \le 3$

## Step #6 (Continued): Form Product & Integrate For Each Region

**Region V: $t > 3$**

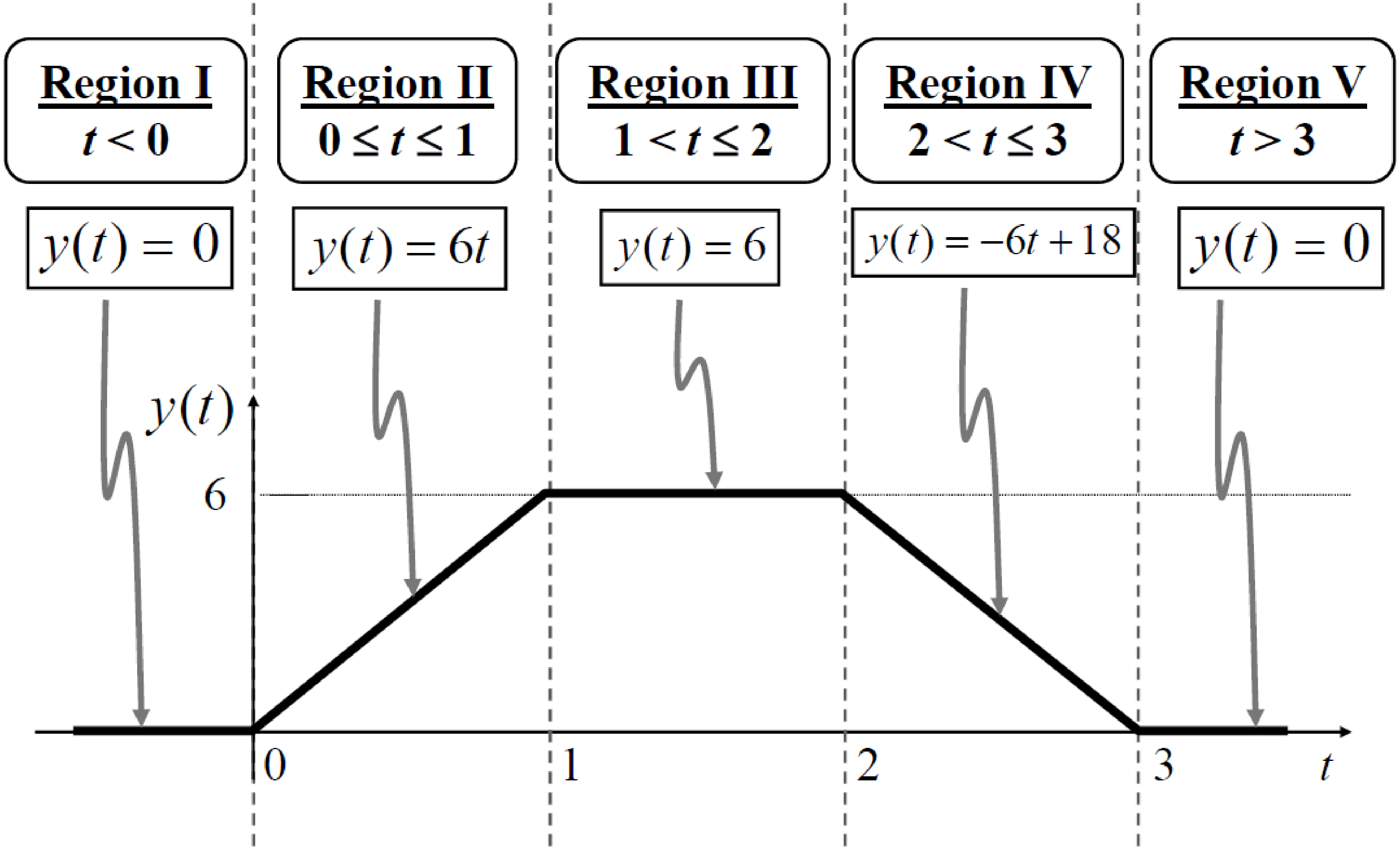



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} 0 d\tau = 0$$

$$y(t) = 0 \quad \text{for all} \quad t > 3$$

# Step #7: "Assemble" Output Signal



# Discrete Convolution

If a system is linear and time-invariant (LTI) then its output is the sum of weighted and shifted unit-sample responses.

$$\delta[n] \longrightarrow \boxed{\text{system}} \longrightarrow h[n]$$

$$\delta[n-k] \longrightarrow \boxed{\text{system}} \longrightarrow h[n-k]$$

$$x[k]\delta[n-k] \longrightarrow \boxed{\text{system}} \longrightarrow x[k]h[n-k]$$

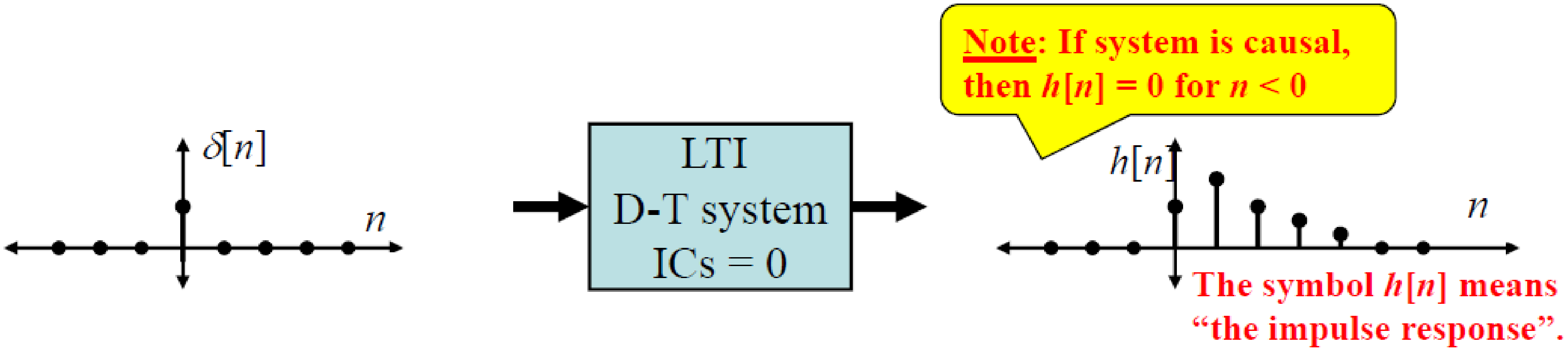
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \longrightarrow \boxed{\text{system}} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

# Discrete Convolution

## Recall: Impulse Response

Earlier we introduced the concept of impulse response…

…what comes out of a system when the input is an impulse (delta sequence)




Noting that the ZT of $\delta[n] = 1$ and using the properties of the transfer function and the Z transform we said that

$$h[n] = Z^{-1}\{H(z)Z\{\delta[n]\}\} \qquad \boxed{h[n] = Z^{-1}\{H(z)\}} \qquad \boxed{h[n] = IDTFT\{H(\Omega)\}}$$

So…once we have either $H$(z) or $H(\Omega)$ we can get the impulse response $h[n]$

# Convolution Property and System Output

Let $x[n]$ be a signal with DTFT $X(\Omega)$ and ZT of $X(z)$

$$x[n] \leftrightarrow X(\Omega)$$
$$x[n] \leftrightarrow X(z)$$

Consider a system w/ freq resp $H(\Omega)$ & trans func $H(z)$

$$h[n] \leftrightarrow H(\Omega)$$
$$h[n] \leftrightarrow H(z)$$

We've spent much time using these tools to analyze system outputs this way:

$$Y(\Omega) = H(\Omega)X(\Omega) \quad \leftrightarrow \quad y[n] = DTFT^{-1}\{H(\Omega)X(\Omega)\}$$

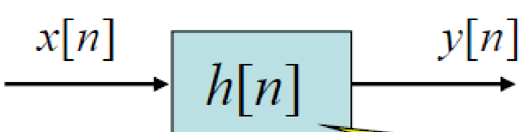
$$Y(z) = H(z)X(z) \quad \leftrightarrow \quad y[n] = Z^{-1}\{H(z)X(z)\}$$

**The convolution property of the DTFT and ZT gives an alternate way to find $y[n]$:**

$$\boxed{DTFT^{-1}\{X(\Omega)H(\Omega)\} = x[n] * h[n]}$$

$$\boxed{Z^{-1}\{X(z)H(z)\} = x[n] * h[n]}$$

$$\boxed{x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]}$$




$$\boxed{y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]}$$

**"Convolving" input $x[n]$ with the impulse response $h[n]$ gives the output $y[n]$!**

**LTI System with impulse response $h[n]$**

# Convolution for *Causal* System & with *Causal* Input

An arbitrary LTI system's output can be found using the general convolution form:

$$\boxed{y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]}$$

**General LTI System**

If the system is causal then $h[n] = 0$ for $n < 0$… Thus $h[n-m] = 0$ for $m > n$ … so:

$$\boxed{y[n] = \sum_{m=-\infty}^{n} x[m]h[n-m]}$$

**Causal LTI System**

If the input is causal then $x[n] = 0$ for $n < 0$…. so:

$$\boxed{y[n] = \sum_{m=0}^{\infty} x[m]h[n-m]}$$

**Causal Input & General LTI System**

If the system & signal are both causal then

$$\boxed{y[n] = \sum_{m=0}^{n} x[m]h[n-m]}$$

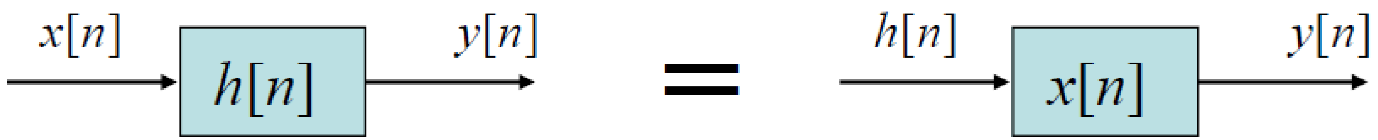
**Causal Input & Causal LTI System**

## Convolution Properties (can sometimes exploit to make things easier)

### 1. Commutativity

$$\boxed{x[n]*h[n]=h[n]*x[n]}$$

$$\sum_{m=-\infty}^{\infty} x[m]h[n-m]=\sum_{m=-\infty}^{\infty} h[m]x[n-m]$$



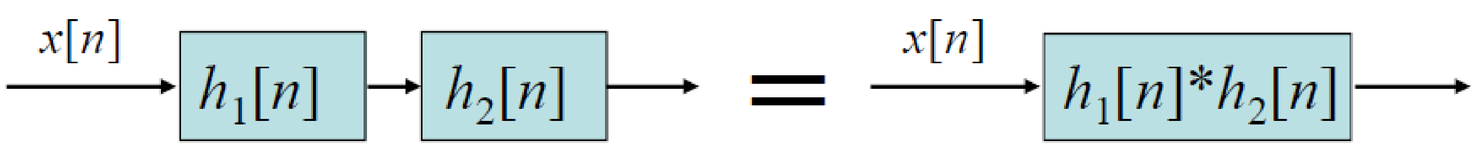

This is obvious from the frequency domain (or z domain) viewpoint:

$$\boxed{x[n]*h[n]=h[n]*x[n]} \Rightarrow \boxed{X(\Omega)H(\Omega)=H(\Omega)X(\Omega)}$$

### 2. Associativity

$$\boxed{(x[n]*h_1[n])*h_2[n]=x[n]*(h_1[n]*h_2[n])}$$

$\Rightarrow$ Can combine cascade into single equivalent system

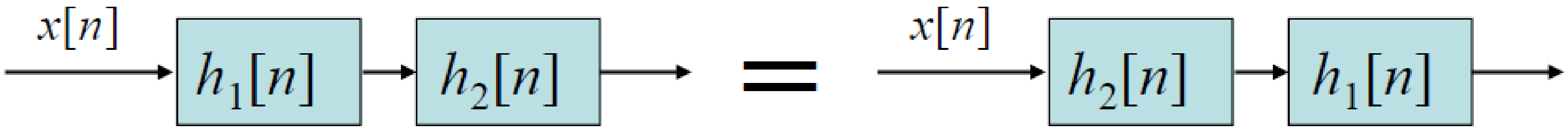



This is obvious from the frequency domain (or z domain) viewpoint:

$$\boxed{\left[X(\Omega)H_1(\Omega)\right]H_2(\Omega)=X(\Omega)\underbrace{\left[H_1(\Omega)H_2(\Omega)\right]}}$$

**Tells us what the Freq Resp is for a cascade**

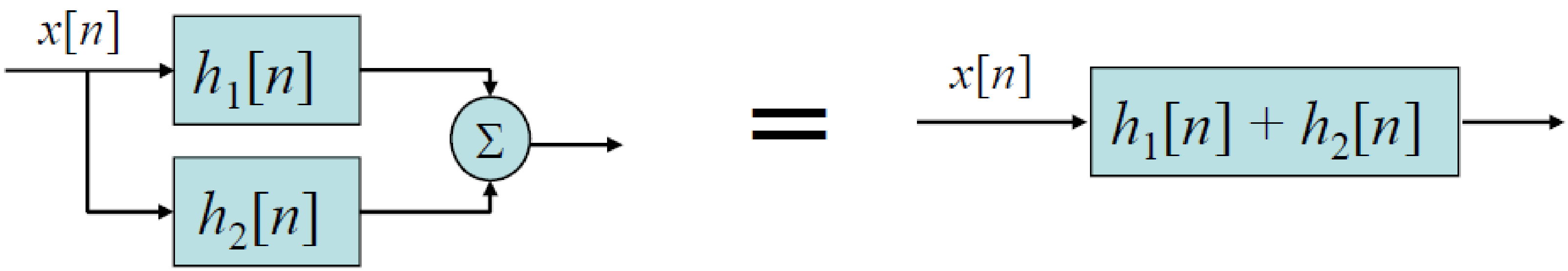
Associativity together with commutativity says we **can interchange the order of two cascaded systems**:




**Warning**: This holds in theory but in practice there may be physical issues that prevent this!!!

**3. Distributivity** $\boxed{x[n]*(h_1[n]+h_2[n]) = x[n]*h_1[n]+x[n]*h_2[n]}$

$\Rightarrow$ can combine sum of two outputs into a single system (or vice versa)




With commutativity this says we can split a complicated input into sum of simple ones… which is nothing more than "linearity"!!

## Graphical Convolution – To Visualize & Test Real Systems

***Can do convolution this way when signals are know numerically or by equation***

- Convolution involves the sum of a product of two signals: $x[i]h[n-i]$

- At each output index $n$, the product changes

"Commutativity" says we can flip either $x[i]$ or $h[i]$ and get the same answer

**Step 1**: Write both as functions of $i$: $x[i]$ & $h[i]$

**Step 2**: Flip $h[i]$ to get $h[-i]$ (The book calls this "fold")

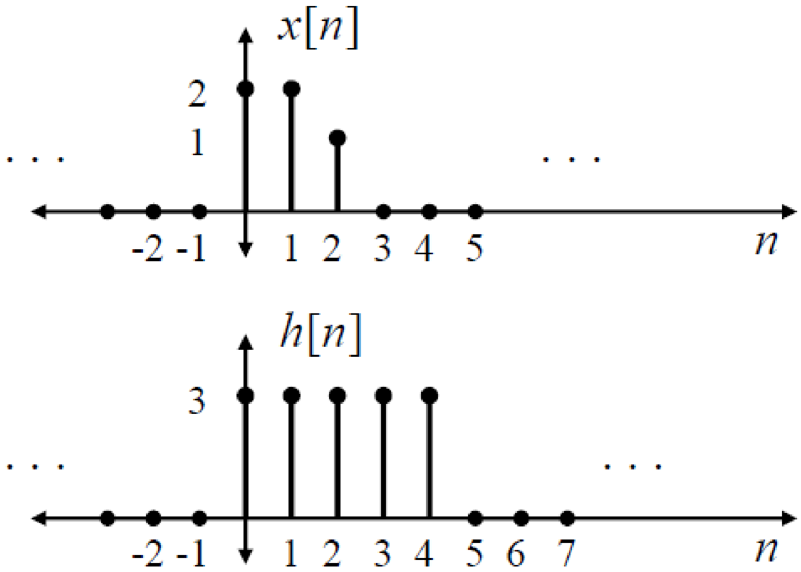
**Repeat for each $n$**

**Step 3**: For each output index $n$ value of interest, shift by $n$ to get $h[n - i]$

(Note: positive $n$ gives right shift!!!!)

**Step 4**: Form product $x[i]h[\mathrm{n} - i]$ and sum its elements to get the number $y[n]$

# Example of Graphical Convolution



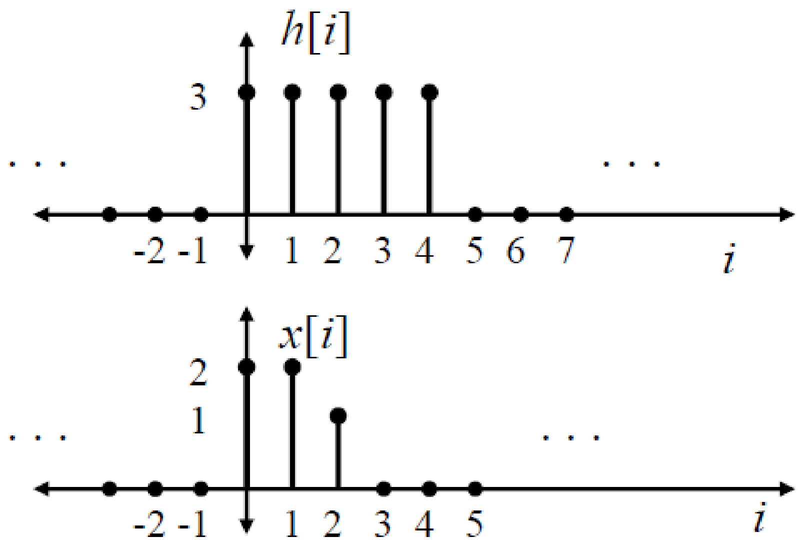

Find $y[n]=x[n]*h[n]$ for all integer values of $n$

## Solution

For this problem I choose to flip $x[n]$

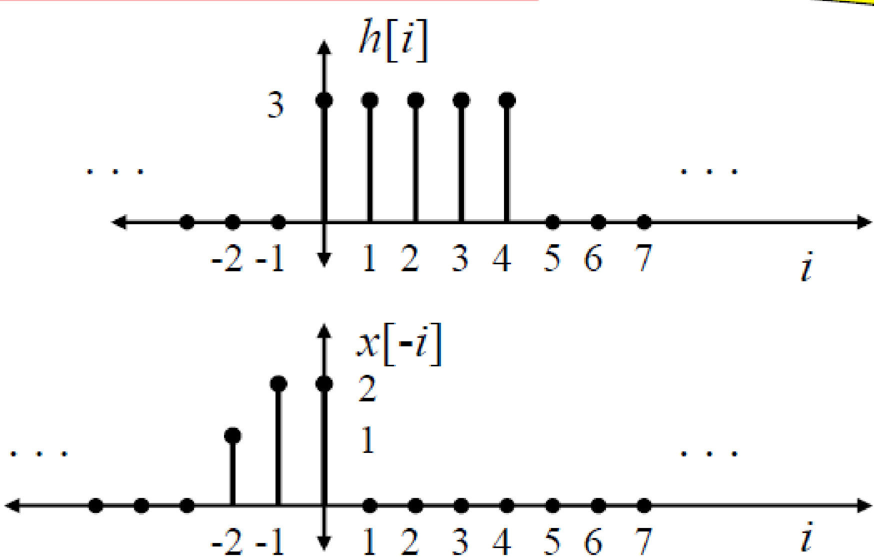
My personal preference is to flip the shorter signal although I sometimes don't follow that "rule"… only through lots of practice can you learn how to best choose which one to flip.

**Step 1**: Write both as functions of $i$: $x[i]$ & $h[i]$




**Step 2**: Flip $x[i]$ to get $x[-i]$

**"Commutativity"** says we can flip either $x[i]$ or $h[i]$ and get the same answer…
**Here I flipped $x[i]$**

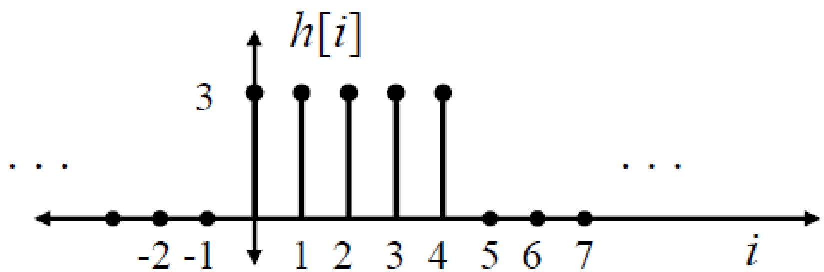


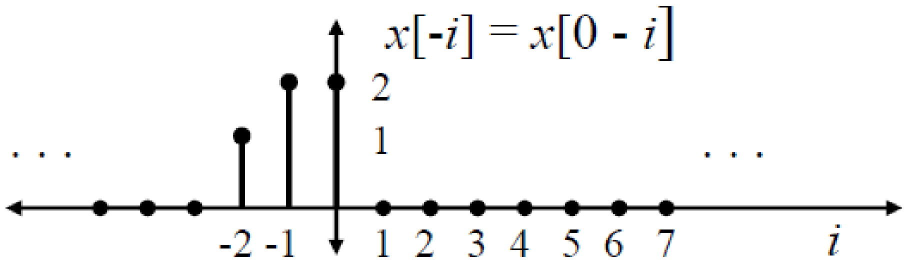
We want a solution for $n = \ldots -2, -1, 0, 1, 2, \ldots$ so must do Steps 3&4 for all $n$.

But… let's first do: **Steps 3&4 for $n = 0$** and then proceed from there.

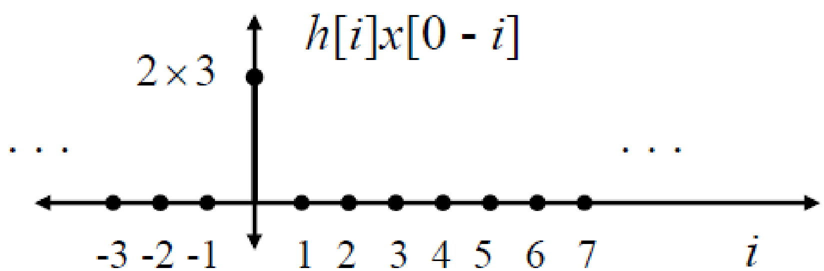
**Step 3**: For $n = 0$, shift by $n$ to get $x[n - i]$

For $n = 0$ case there is no shift!
$x[0 - i] = x[-i]$




**Step 4**: For $n = 0$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

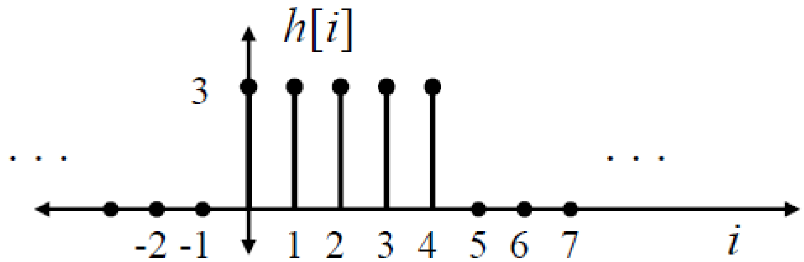


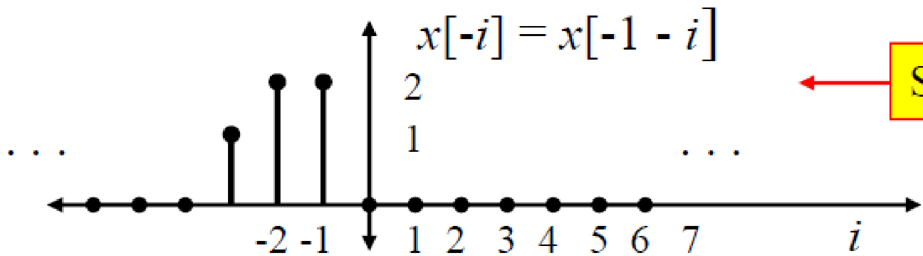

**Sum over $i \Rightarrow$** $y[0] = 6$

## Steps 3&4 for all $n < 0$

**Step 3**: For $n < 0$, shift by $n$ to get $x[n - i]$

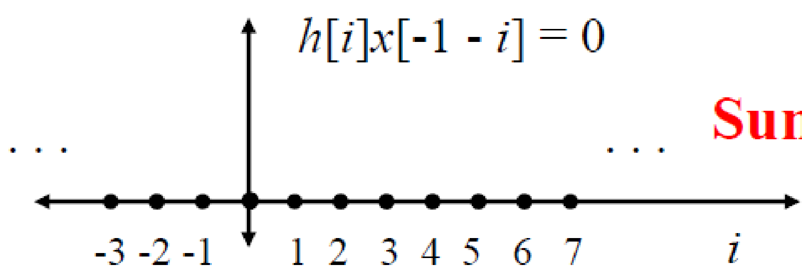
Negative $n$ gives a left-shift



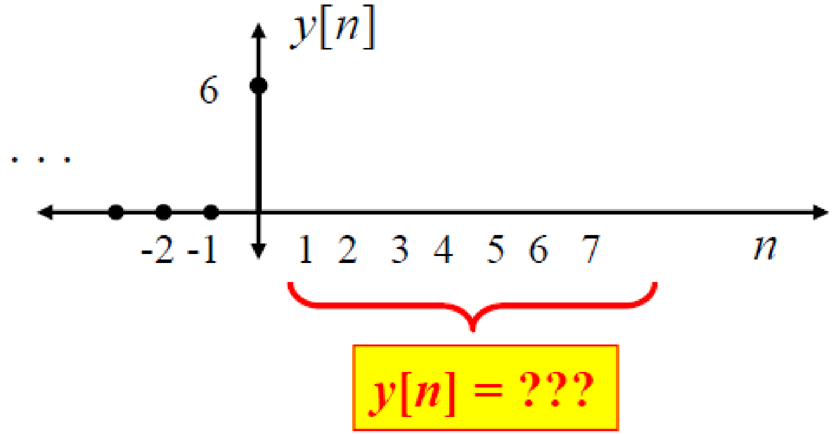

Shown here for $n$ = -1

**Step 4**: For $n < 0$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$




**Sum over $i$** $\Rightarrow$ $\boxed{y[n] = 0 \quad \forall n < 0}$

So… what we know so far is that: $\boxed{y[n] = \begin{cases} 0, & \forall n < 0 \\ 6, & n = 0 \end{cases}}$

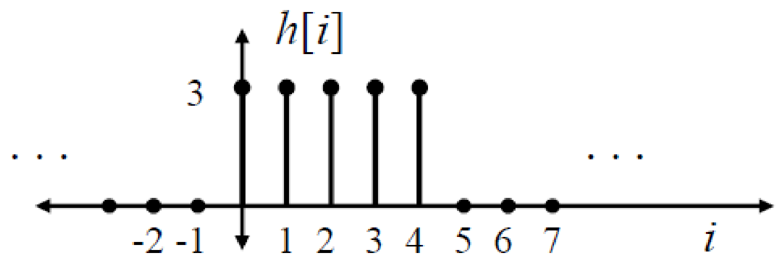


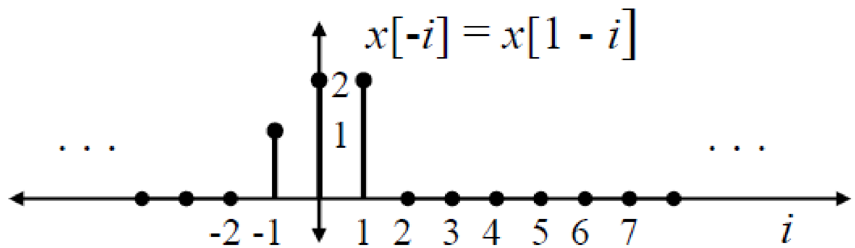

So now we have to do Steps 3&4 for $n > 0$…

## Steps 3&4 for $n = 1$

**Step 3**: For $n = 1$, shift by $n$ to get $x[n - i]$

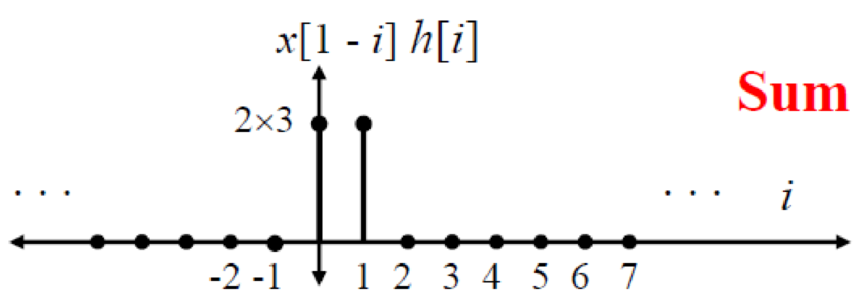
Positive $n$ gives a Right-shift




shifted to the right by one

**Step 4**: For $n = 1$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

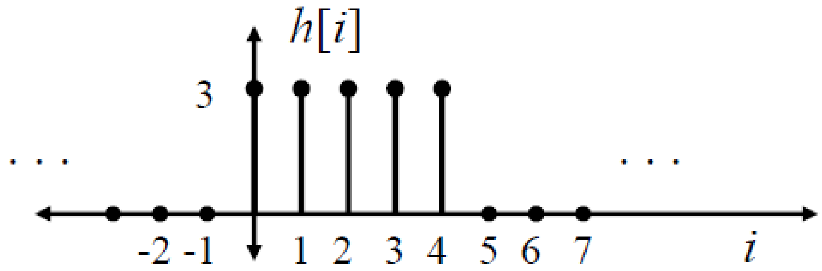


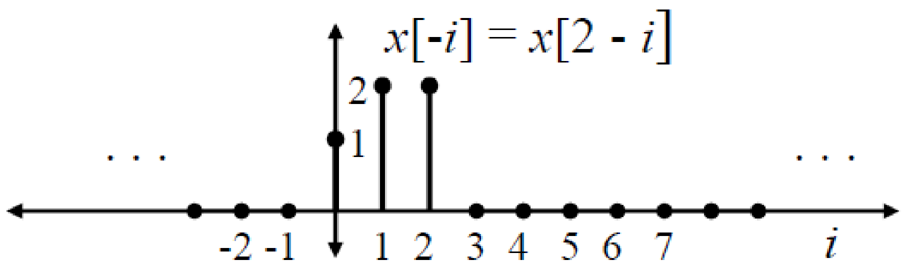

**Sum over $i$** $\Rightarrow$ $y[1] = 6 + 6 = 12$

## Steps 3&4 for $n = 2$

**Step 3**: For $n = 2$, shift by $n$ to get $x[n - i]$

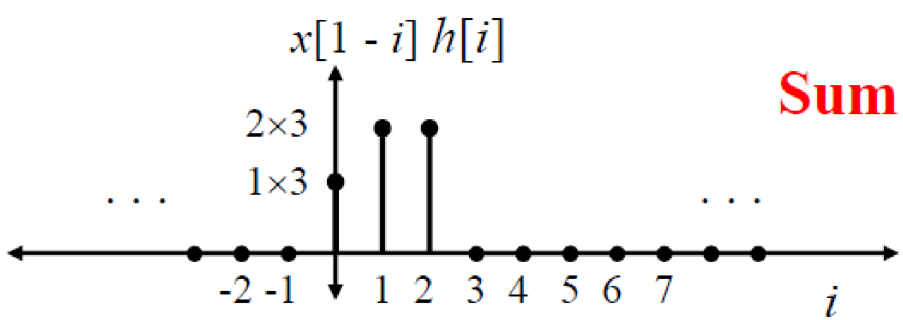
Positive $n$ gives a Right-shift




shifted to the right by two

**Step 4**: For $n = 2$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

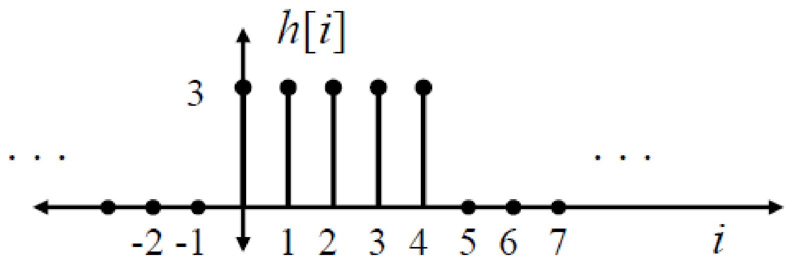


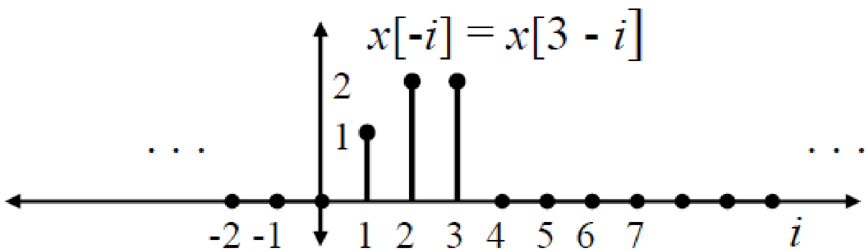

**Sum over $i$ ⇒** $y[2] = 3 + 6 + 6 = 15$

## Steps 3&4 for $n = 3$

**Step 3**: For $n = 3$, shift by $n$ to get $x[n - i]$

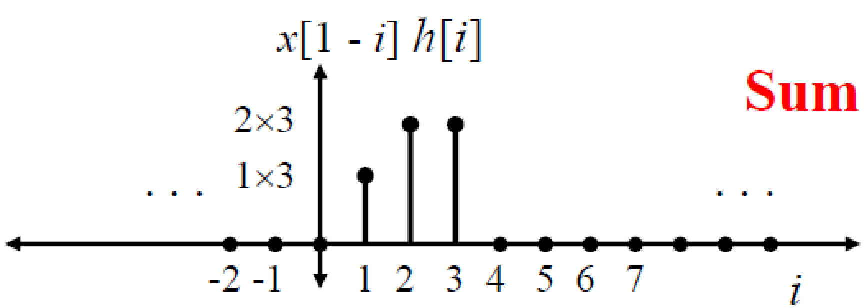
Positive $n$ gives a Right-shift




shifted to the right by three

**Step 4**: For $n = 3$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

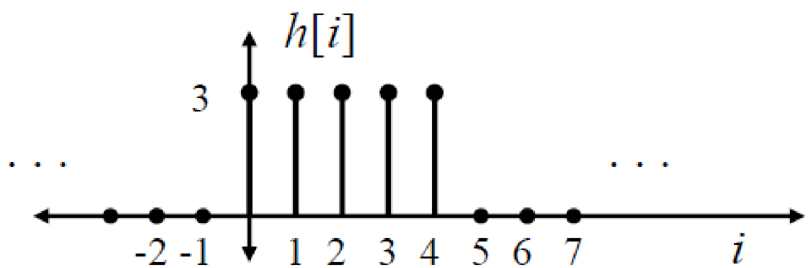


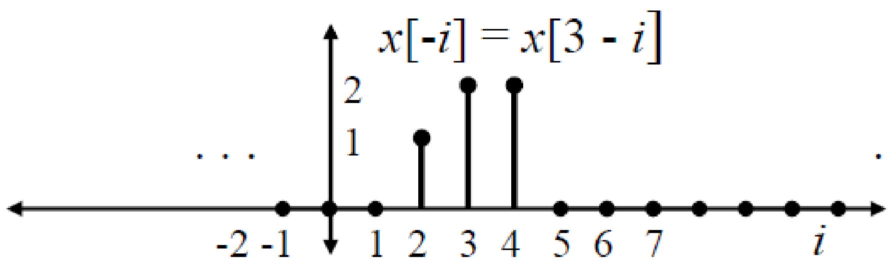

**Sum over $i$ ⇒** $y[3] = 3 + 6 + 6 = 15$

# Steps 3&4 for $n = 4$

**Step 3**: For $n = 4$, shift by $n$ to get $x[n - i]$

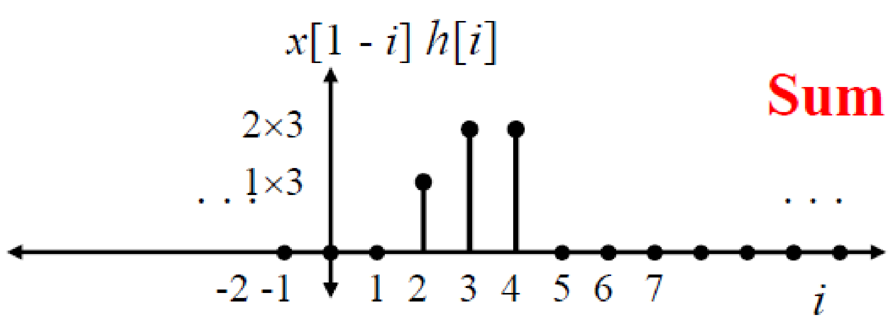
Positive $n$ gives a Right-shift




shifted to the right by four

**Step 4**: For $n = 4$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

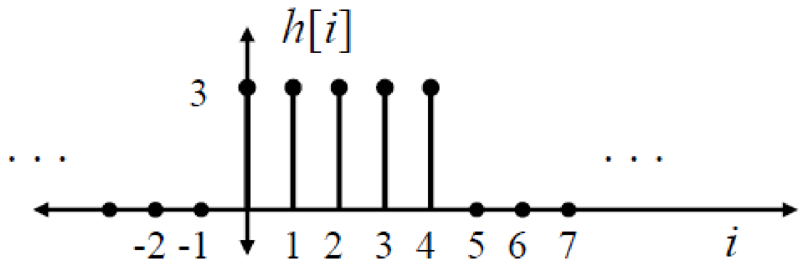


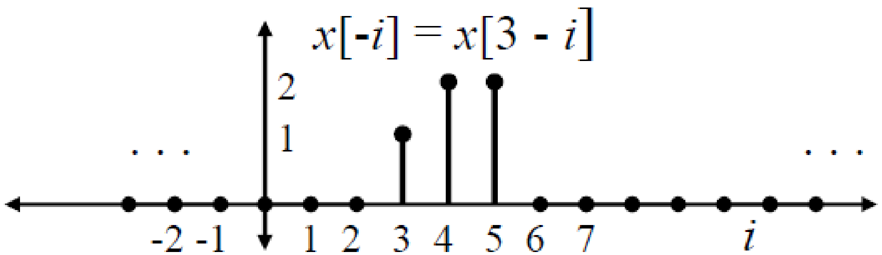

**Sum over $i$ ⇒** $y[4] = 3 + 6 + 6 = 15$

## Steps 3&4 for $n = 5$

**Step 3**: For $n = 5$, shift by $n$ to get $x[n - i]$

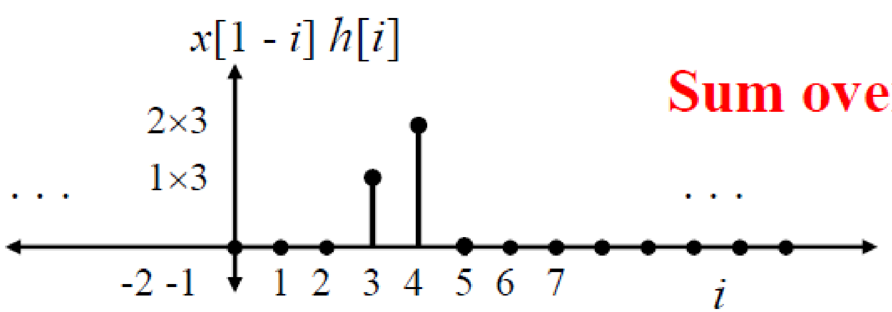
Positive $n$ gives a Right-shift




shifted to the right by five

**Step 4**: For $n = 5$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

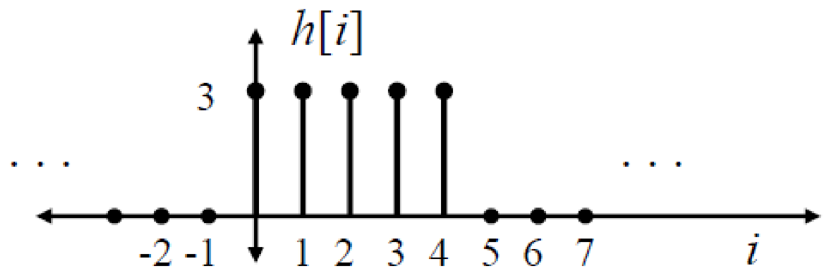


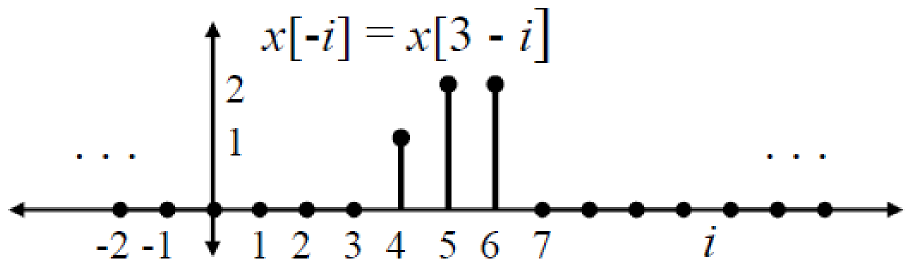

**Sum over $i$ ⇒** $y[5] = 3 + 6 = 9$

## Steps 3&4 for $n = 6$

**Step 3**: For $n = 6$, shift by $n$ to get $x[n - i]$

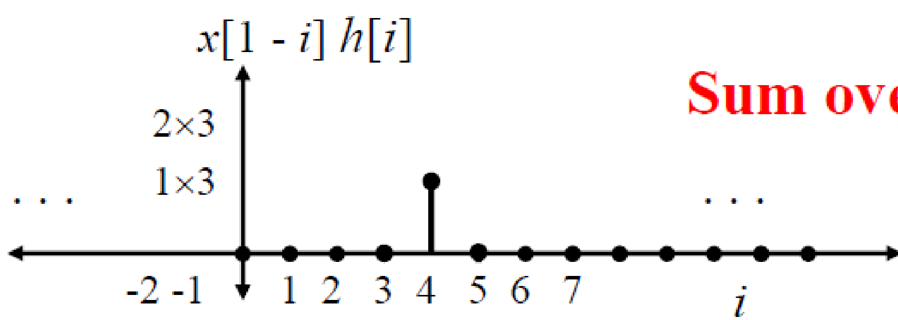
Positive $n$ gives a Right-shift




shifted to the right by six

**Step 4**: For $n = 6$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

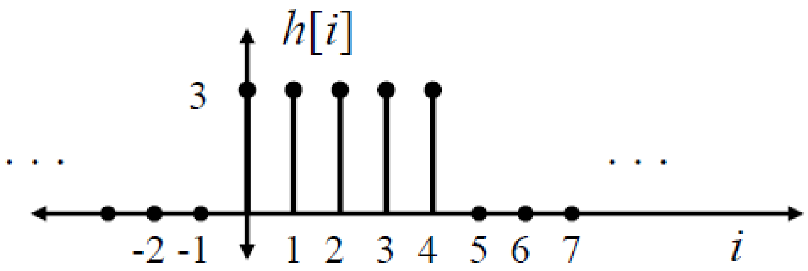


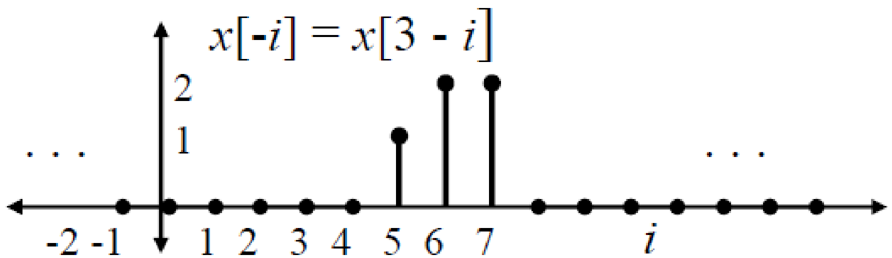

**Sum over $i$** $\Rightarrow$ $y[6] = 3$

## Steps 3&4 for all $n > 6$

**Step 3**: For $n > 6$, shift by $n$ to get $x[n - i]$

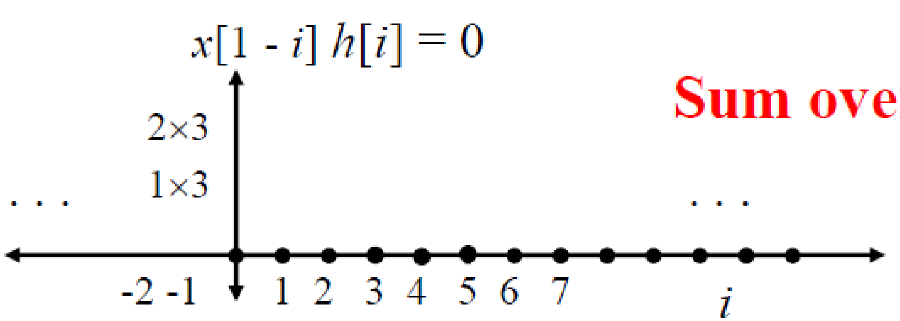
Positive $n$ gives a Right-shift




shifted to the right by seven

**Step 4**: For $n > 6$, Form the product $x[i]h[n-i]$ and sum its elements to give $y[n]$

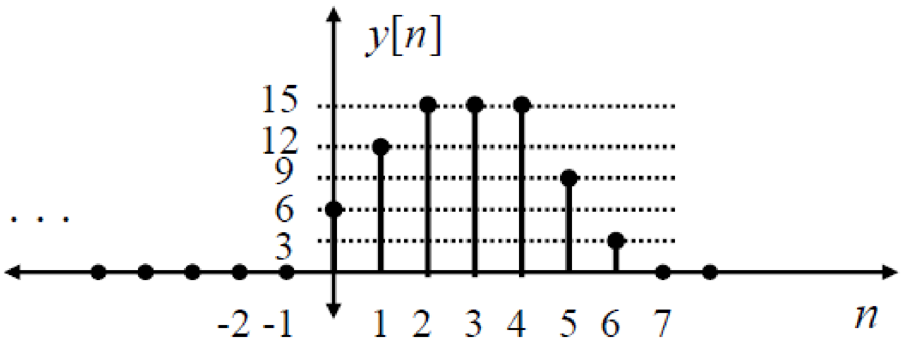



**Sum over $i$ ⇒** $\boxed{y[n] = 0 \quad \forall n > 6}$

So… now we know the values of $y[n]$ for all values of $n$

We just need to put it all together as a function…

Here it is easiest to just plot it… you could also list it as a table.




Note that convolving these kinds of signals gives a "ramp-up" at the beginning and a "ramp-down" at the end.

Various kinds of "transients" at the beginning and end of a convolution are common.