

- Conjunction AND,  $\wedge$
- Inclusive Disjunction, OR  $\vee$
- Exclusive disjunction, XOR  $\vee \wedge$ ,  $\oplus$
- Negation  $\sim$ ,  $\neg$
- Implication  $\rightarrow$
- Double Implication  $\leftrightarrow$

**şafak bilici**

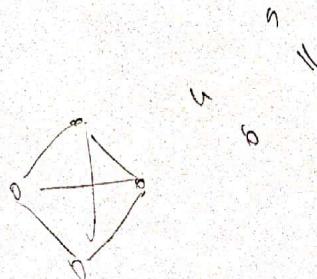
Logical Equivalence,  $\rightarrow (p \leftrightarrow q) \rightarrow ((p \rightarrow q) \wedge (q \rightarrow p))$

Logical Equivalence,  $\rightarrow (p \rightarrow q) \rightarrow \neg p \vee q$

Contrapositive,  $\rightarrow (p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$

Autology,  $\rightarrow$  Her zaman true  $\rightarrow (p \rightarrow (p \vee q))$

Contradiction,  $\rightarrow$  Her zaman false  $\rightarrow (p \wedge \neg p)$



4 Degişkenli

E)  $F(A, B, C, D) = \sum(0, 5, 7, 9, 10, 13, 15)$

		$\bar{A}\bar{B}\bar{C}\bar{D}$			
		00	01	11	10
$AB$		00	1	0	0
$CD$		00	1	1	1
01		1	0	1	1
11		0	1	1	0
10		0	0	0	1

$\rightarrow A\bar{C}D$        $\begin{smallmatrix} 2^1 & 2^2 & 2^3 & 2^4 \\ \bar{A} & \bar{B} & C & D \end{smallmatrix}$

Grup  
 $2^4 = 16 X$   
 $2^3 = 8 X$   
 $2^2 = 4 \checkmark$   
 $2^1 = 2 \checkmark$

ONTAKOLAN  $B \vee D$        $\bar{A}\bar{B}\bar{C}\bar{D}$

↓  
 $BD$

$F_3(A, B, C, D) = A\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + BD + A\bar{B}C\bar{D}$

		$\bar{B}\bar{C}\bar{D}$			
		00	01	11	10
$AB$		00	1	0	1
$CD$		00	1	1	1
01		0	1	1	0
11		0	1	1	0
10		0	0	1	1

$\rightarrow A\bar{D}$

$\rightarrow BD$

$\rightarrow \bar{B}D + A\bar{D} + \bar{B}\bar{C}\bar{D}$

		$\bar{A}\bar{C}$			
		00	01	11	10
$AB$		00	1	1	1
$CD$		00	1	1	1
01		1	1	0	0
10		0	0	0	0
11		1	1	1	1

$\rightarrow \bar{D}$

$\rightarrow \bar{A}\bar{C} + \bar{D}$

		$D$			
		00	01	11	10
$AB$		00	1	1	1
$CD$		00	1	1	1
01		1	0	0	1
11		1	0	0	1
10		1	1	1	1

$\rightarrow \bar{B}$

$\rightarrow \bar{D} + \bar{B}$

E/ Bir öğrenci bir dönemde  $\rightarrow^{\text{max}}$  3 ders alabiliyor. Bir üst dönemde yeni ders secebilmesinin şartı en az 2'sinde geçmelidir. Bu durumun matematiksel modelini çizelim.

indeks-1

	A	B	C	F
1	0	0	0	0
2	0	0	1	0
3	0	1	0	0
4	0	1	1	1
5	1	0	0	0
6	1	0	1	1
7	1	1	0	1
8	1	1	1	1

$$F(A, B, C) = \sum' (3, 5, 6, 7)$$

	AB	00	01	11	10
C	00	0	0	1	0
	10	1	1	0	1
		1	1	0	1

$$F_S(A, B, C) = AB + BC + AC = AB + C(A + B)$$

String

$(ab)^* c$	$a^* c^+$	$\rightarrow$ Kleene Operator
c	c	
abc	cc	
ababc	ccc	
abababc	acc	
	caacc	
	acaac	

Bağıntı

$$a = \{x, y\} \quad b = \{1, 2\}$$

$$axb = \{(x,1), (x,2), (y,1), (y,2)\}$$

Bağıntı Özellikleri

\* A relation  $R$  on a set  $A$  is called Reflexive (yansıma)

if  $(x,x) \in R$  for every element  $x \in A$

$$A \times A \rightarrow R : \{(1,1), (2,2), (3,3)\}$$

	1	2	3
1	1	0	0
2	0	1	0
3	0	0	1

\* A relation  $R$  on a set  $A$  is called nonreflexive if  $(x,x) \notin R$  for some element  $x \in A$ .

	1	2	3
1	0	0	1
2	1	0	0
3	1	1	1

\* A relation  $R$  on a set  $A$  is called irreflexive if  $(x,x) \notin R$  for every element  $x \in A$ .

	1	2	3
1	0	1	1
2	1	0	1
3	0	0	0

\* A relation  $R$  on a set  $A$  is called symmetric (transpose)

if  $[(x,y) \in R \text{ whenever } (x,y) \in R] \text{ or } [(y,x) \in R \text{ whenever } (x,y) \in R] \text{ or } (x=y)$ ;

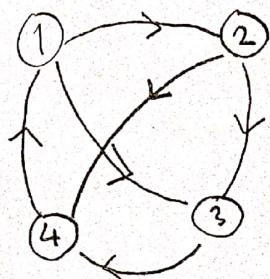
for  $x, y \in A$

\* A relation  $R$  on a set  $A$  such that  $(x,y) \in R$  and  $(y,x) \in R$  only if  $x=y$ ; for  $x,y \in A$  is called antisymmetric.  $(x,y)$  varken  $(y,x)$  olmeceklidir.

\* A relation  $R$  on a set  $A$  is called transitive (gecislik) if whenever  $(x,y) \in R$  and  $(y,z) \in R$  then  $(x,z) \in R$  for  $x,y,z \in A$

$$X = \{1, 2, 3, 4\} \quad (x,y) \in R \quad R: x \leq y \quad x, y \in X$$

$$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4)\}$$



$$\{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$$

$$\{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$$

$$\{(2,4), (4,2)\}$$

$$\{(1,2), (2,3), (3,4)\}$$

$$\{(1,1), (2,2), (3,3), (4,4)\}$$

$$\{(1,3), (1,4), (2,3), (2,4), (3,1), (3,4)\}$$

Ref	IrRef	NRef	Sym	Asym	Trans
X	X	✓	X	X	✓
✓	X	X	✓	X	✓
X	✓	X	✓	X	X
X	✓	X	X	✓	X
✓	X	X	✓	✓	✓
X	✓	X	X	X	X

$$\text{if } a=b^2, (a,b) \in R \quad A = \{1, 2, 3, 4\}$$

$$R = \{(1,1), (4,2)\}$$

Ref X

NRef ✓

IrRef X

Sym X

Asym ✓

Trans ✓

## Bağıntının Bileşkesi (Composition)

$$R^1 = R$$

$$R^2 = R \circ R$$

$$R^3 = R^2 \circ R$$

$$\dots$$
  
$$R^n = R^{n-1} \circ R$$

$R = \{(1,1), (2,1), (3,2), (4,3)\}$  iken  $R^2$  ve  $R^3$  bul.

$$R^2 = R \circ R = \{(1,1)(2,1)(3,1)(1,2)\}$$

$$R^3 = R^2 \circ R = \{(1,1)(2,1)(3,1)(4,2)\}$$

$$R = \{(1,1), (2,1), (3,2), (4,3)\} \quad \text{reflexive, symmetric}$$
  
$$R = \{(1,1), (2,1), (3,2), (4,3)\}$$

$$R^2 = \{(1,1)(2,1)(3,1)(1,2)\}$$

$$R = \{(1,1)(2,1)(3,2)(4,3)\}$$

## Denklik Bağıntısı (Equivalence Relation)

$X$  bir kume,  $R$ 'de  $X$  üzerinde bir bağıntı olsun.

$R$  bağıntısı üzerinde reflexive, symmetric, transitive özellikleri mercut ise bu bir denklik bağıntısı olup  $X \leftrightarrow R$  şeklinde gösterilir.

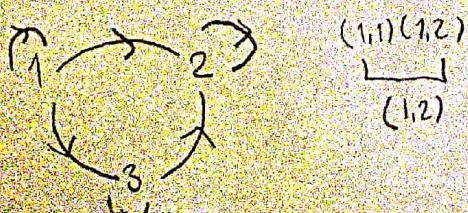
$X = \{\text{integers}\}$  ve  $X$  kumesi üzerinde tamsayı olsun  $R$  bağıntısı da  $xRy \Leftrightarrow x-y=5$  olarak verilsin.  $R$ 'nin Equivalence Relation olmadığını göster.

$$X = \{1, 6\}, R = \{(6, 1)\} \rightarrow \text{IRaf}$$
  
Asym  
Trans

$$X = \{1, 2, 3, 4, 5, 6\}$$

$$R = \{(1,1)(1,3)(1,5)(3,1)(3,3)(3,5)(5,1)(5,3)(5,5)(2,2)(2,6)(6,2)(6,6)(4,4)\}$$

↓  
Sym Trans  
Ref



$$(1,1)(1,2) \\ \underbrace{(1,2)}_{1} \\ (1,2)$$

## Sıralama Bağıntısı (Partial Order Relation)

$y/x$

$R$ : if  $x$  divides  $y$   $(x, y) \in R$   $A = \{1, 2, 3, 4\}$   $x, y \in A$

$$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (1,3), (1,4), (2,4)\}$$

Ref ✓  
 Asym ✓  
 Trans ✓

} Partial Order ✓

## Hasse Diagramları

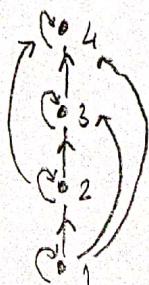
Sıralama bağıntısının özellikleri sağlanır

$$R: \{(a, b) \mid a \leq b, a \in X, b \in X\}, X = \{1, 2, 3, 4\}$$

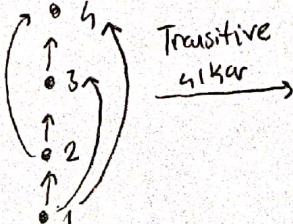
$$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$$

Reflexive ✓  
 Asym ✓  
 Transitive ✓

} Partial Order



Reflexive  
ulkar



Transitive  
ulkar



Hasse Diagram

Bundan  
geride  
git

## Kapalılık (Closure)

reflexive yok  
symmetric yok  
transitive yok

bağlantının bu  
özelliklere sahip  
olabilirliğini sağlama  
ıstemidi.

\* Transitive Closure

\* Warshall Algorithm (by Stephen Warshall)

$$\rightarrow W = \cap R$$

for  $k=1, n$

    for  $i=1, n$

        for  $j=1, n$

$$W_{ij}^k = W_{ij}^0 \vee (W_{ik}^0 \wedge W_{kj}^0)$$

Elle göz

    end

end

end

	1	2	3
1	1	1	0
2	0	1	0
3	1	0	0

$$R = \{(1,1) (1,2) (2,2) (3,1)\}$$

## Chromatic Polinoms

$\overset{x}{\circ}$  noktasal graf

$\overset{x}{\circ} - \overset{x-1}{\circ}$  çizgisel graf

### U Grafi

$$U_n = x \cdot (x-1)^{n-1} \longrightarrow U_2 = x \cdot (x-1) \longrightarrow \overset{x}{\circ} - \overset{x-1}{\circ}$$

$$\longrightarrow U_3 = x \cdot (x-1)^2 \longrightarrow \overset{x}{\circ} - \overset{x-1}{\circ} - \overset{x-1}{\circ}$$

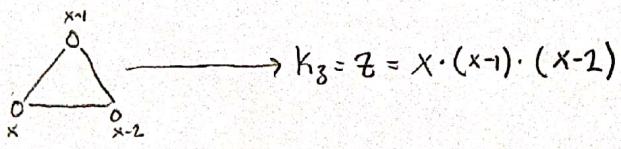
$$\longrightarrow U_5 = x \cdot (x-1)^4 \longrightarrow \begin{array}{c} x-1 \\ \overset{x}{\circ} \end{array} - \begin{array}{c} x-1 \\ \overset{x-1}{\circ} \end{array} - \begin{array}{c} x-1 \\ \overset{x-1}{\circ} \end{array} - \begin{array}{c} x-1 \\ \overset{x-1}{\circ} \end{array}$$

### K- grafi

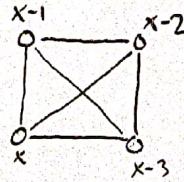
- her düğüm kendisi dışındaki doğrularla bağlıdır

$K_3 \rightarrow Z$  grafi (özel durum)

// polinomun 2. teriminin katsayısı grafin  
kenar sayısını verir.



$$K_n = \prod_{k=0}^{n-1} (x-k) \longrightarrow x \cdot (x-1) \cdot (x-2) \cdot (x-3) \longrightarrow$$



- polinomun en yüksek dereceli terimin katsayıısı 1'dir

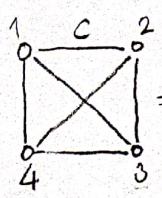
$$K_4 = (x^2-x) \cdot (x^2-5x+6) = x^4 - 6x^3 + 11x^2 - 6x$$

- polinomun derecesi graftaki doğrular sayısını verir.

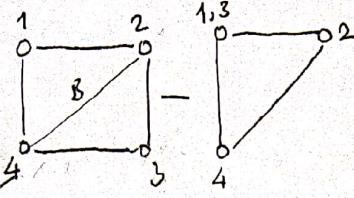
- polinomun katsayıları toplamı sıfırdır.

- polinomda kesinlikle sabit sayı olmaz.

## Graf'in Sadeleştirilmesi



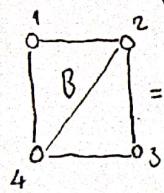
1,3 arasındaki  
path'i aksar



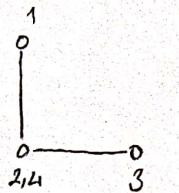
1,3 üstüste  
gelecek şekilde  
Katla

$$C = x \cdot (x-1) \cdot (x^2 - 4x + 4) - x \cdot (x-1) \cdot (x-2)$$

$$C = x^4 - 6x^3 + 11x^2 - 6x$$



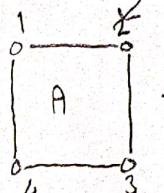
2,4 arası  
path'i aksar



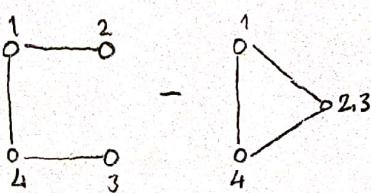
2,4 üstüste  
gelecek şekilde  
Katla

$$B = x \cdot (x-1) \cdot (x^2 - 3x + 3) - x \cdot (x-1)^2$$

$$B = x \cdot (x-1) \cdot (x^2 - 4x + 4)$$



2,3 arası  
path'i kopar

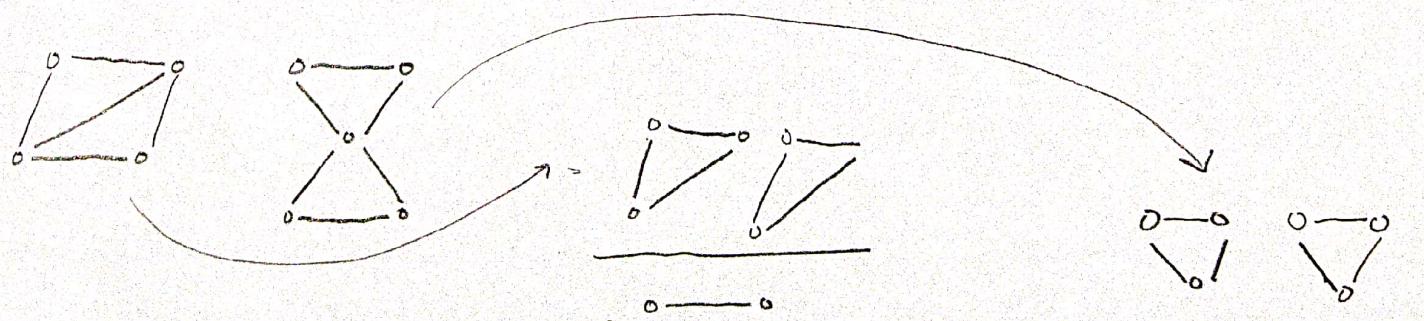


2,3 üst  
üste Katla

$$A = x \cdot (x-1)^3 - x \cdot (x-1) \cdot (x-2)$$

$$A = x \cdot (x-1) \cdot (x^2 - 3x + 3)$$

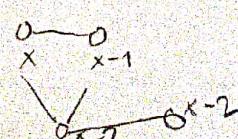
Eğer 2 graf birbirinden noktasal veya çizgisel bir grafta ayırlıysa



$$\frac{x^2 \cdot (x-1)^2 \cdot (x-2)^2}{x \cdot (x-1)}$$

$$x \cdot (x-1) \cdot (x-2)$$

$$\frac{x^2 \cdot (x-1)^2 \cdot (x-2)^2}{x}$$



## Trees

Dairesel yapı igermez

Bir node'dan diğerine tek yol var

$$h = \max \{ I(v) \}$$

En az 1 çocuğu olsa doğmalar internal

Hıç çocuğu olmayanlar terminal

### Ağacların Karakteristiği

$T \rightarrow n$  doğmalar bir graf

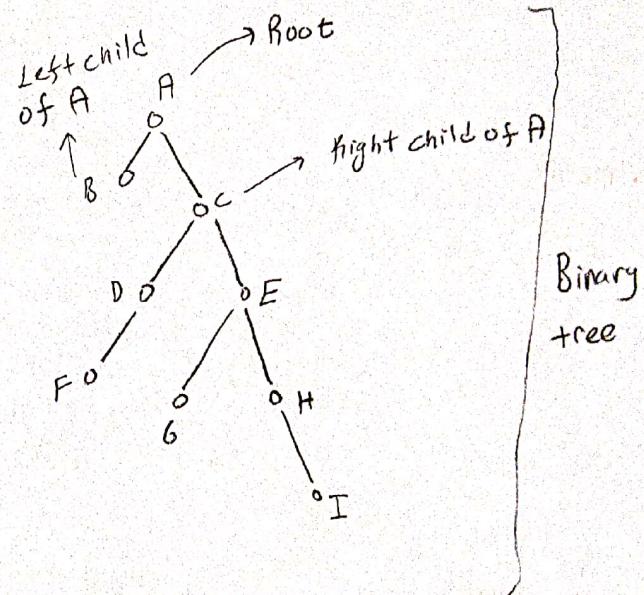
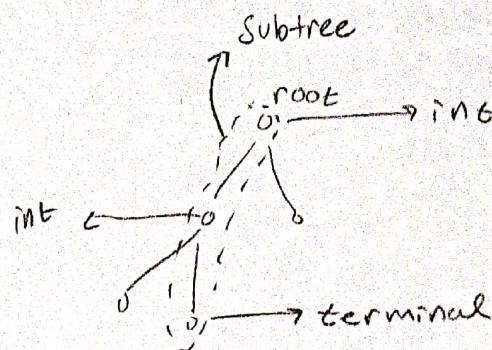
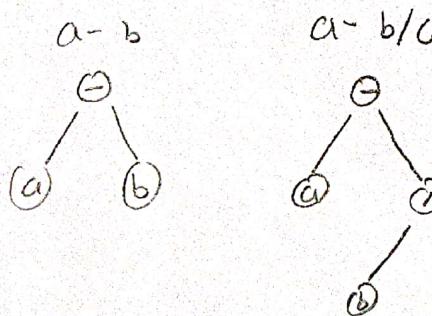
a)  $T$  bir graf'tır

b)  $T$  bağılantılı ve acyclic

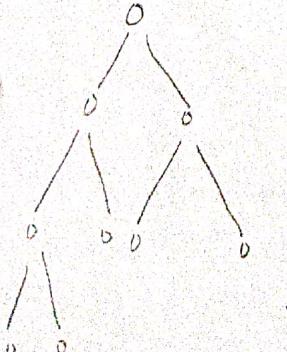
c)  $T$  bağılantılı ve  $n-1$  kenara sahip

d)  $T$  acyclic ve  $n-1$  kenara =

### Binary Tree Matematiksel ifadelerin gösterimi



### Tam ikili Ağacı (Full Binary Tree)



Root node dışı her node 2 çocuğu sahip

$t = \text{Üç doğm sayısı} , l = \text{ara doğm sayısı}$

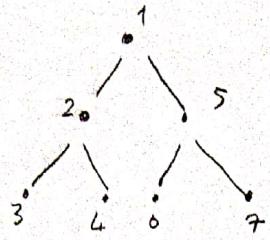
$$- \lg(t) \leq h$$

$$- l+1 = \text{Üç doğm}$$

$$- 2^l + 1 = \text{toplam doğm}$$

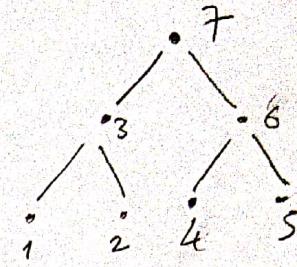
## Tree Traversals

Pre-order



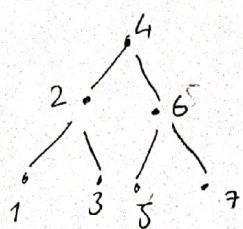
Post Order →

Post modernist  
sağ

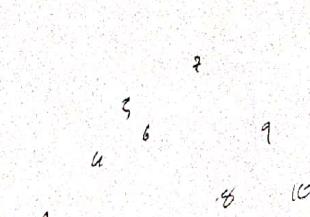


In-Order

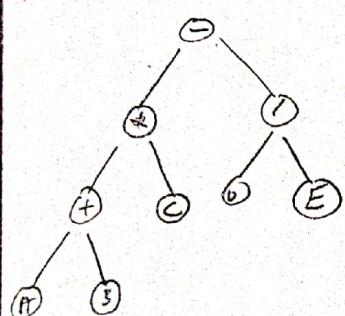
14



reverse Post-Order →



## Aritmetik



$(A+B)^*C - D/E$  infix

$AB+C^*DE/-$  postfix  
 $-+ABC/DE$  prefix  
 $-(A+B)^*C - D/E$

Isomorphism:

Düğüm sayıları aynı

Kenar sayıları aynı

Karşılıklı düğüm derecesi aynı

## Spanning Tree

Grafında bir T spanning tree

Grafının bir subtree'si

Grafının bütün düğümlerini içeren

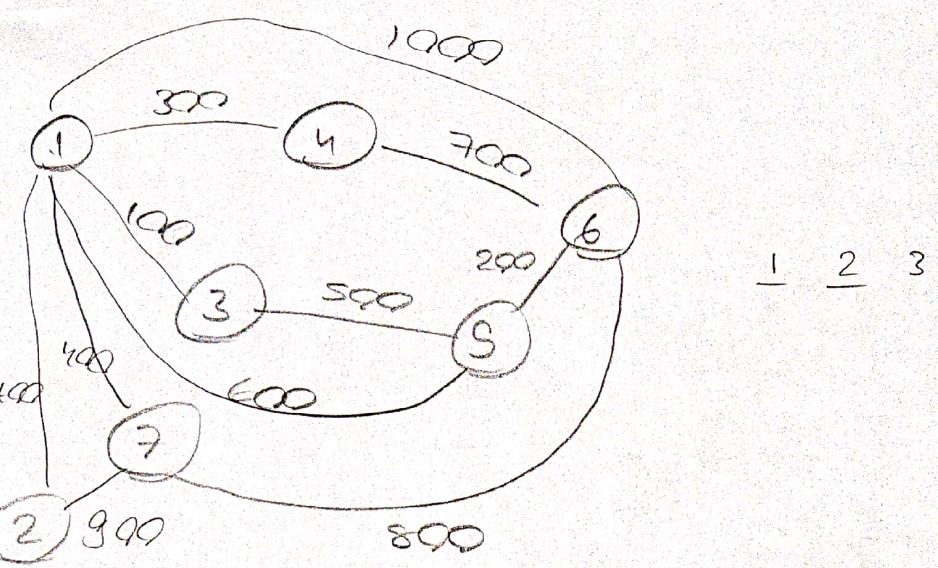
## Kruskal's Algorithm

Adım 1: En küçük ağırlıklı kenarı bul

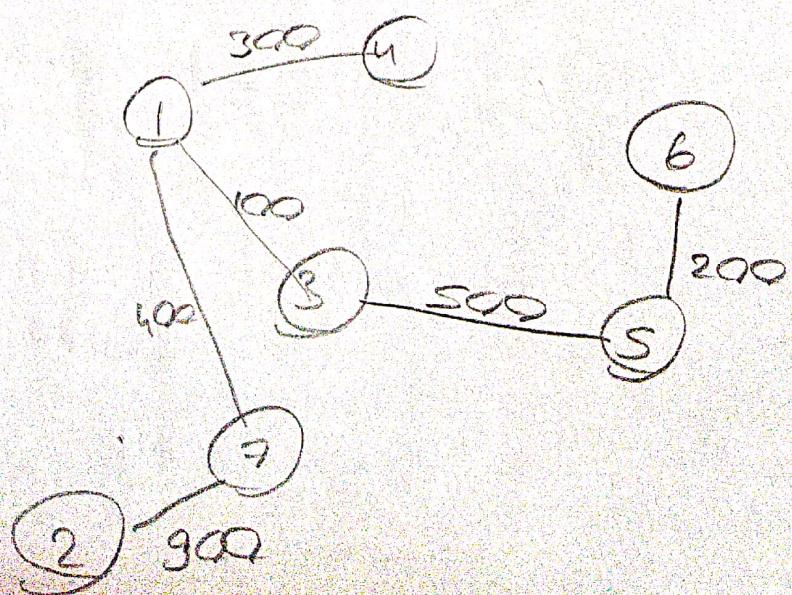
Adım 2: En küçük değere sahip

bir sonraki kenarı bul  
ancak kapalı döngü  
olusturmamasın

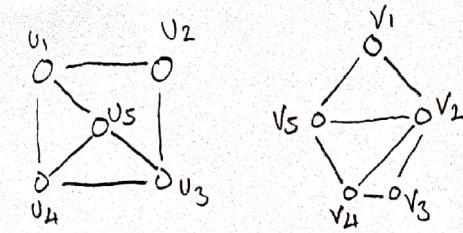
Adım 8: Grafındaki her bir düğümü  
dolasma kadar adım 2 yap



	1	2	3	4	5	6	7
X 1	m	100	100	300	600	1000	400
X 2	1000	m	m	m	m	m	900
X 3	100	m	m	m	500	m	m
X 4	300	m	m	m	m	700	m
X 5	600	M	500	M	M	200	M
X 6	1000	m	m	200	200	m	800
X 7	400	900	m	m	m	800	M



# Graf isomorfizmi

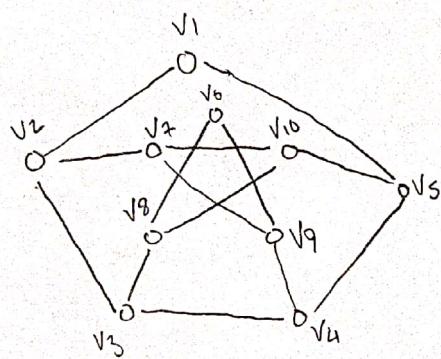


Degüm = 5

Kenar = 7

$$\begin{cases} \text{1 tane deg=2} \\ \text{4 tane deg=3} \end{cases} \xleftarrow{\neq} \begin{cases} 2 \text{ tane deg=2} \\ 2 \text{ tane deg=3} \\ 1 \text{ tane deg=4} \end{cases}$$

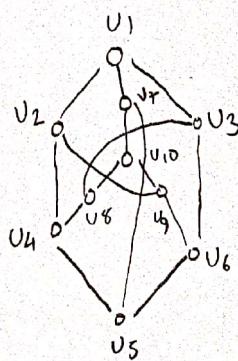
isomorfik degiller



Degüm = 10

Kenar = 15

10 tane deg=3

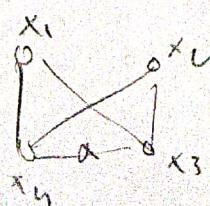
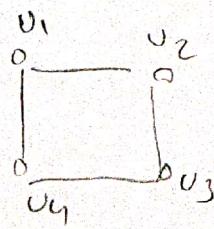


Degüm = 10

Kenar = 15

10 tane deg=3

kendisinin  
ve komşularının  
degree'si aynı mı?



dereceleri a. ft olmasi kural

E graf  $\rightarrow$  başladığın yere

E path  $\rightarrow$  == == torunlu değil

H cycle  $\rightarrow$  her node'1 kuz  
Kenar torunlu değil

	$x_1$	$x_2$	$x_3$	$x_n$		$u_1$	$u_2$	$u_3$	$u_4$
$x_1$	0	0	1	1	$u_1$	0	1	0	1
$x_2$	0	0	1	1	$u_2$	1	0	1	0
$x_3$	1	1	0	0	$u_3$	0	1	0	1
$x_n$	1	1	0	0	$u_4$	1	0	1	0

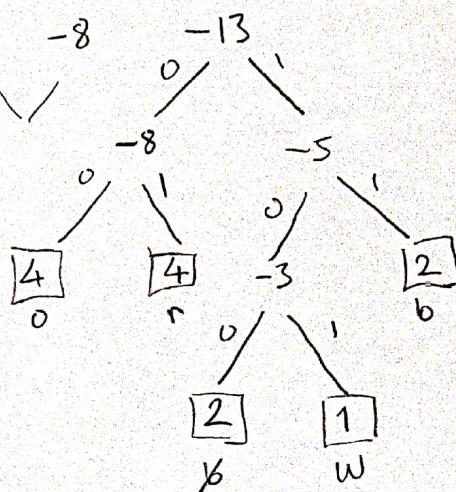
# Sınav Soruları

1) "borrow & or & rob"

$$\begin{cases} b=2 \\ o=4 \\ r=4 \\ w=1 \\ y=2 \end{cases}$$

$$13 * 8 = 104 \text{ bit}$$

1 2 2 -3 4 4 -5 -8



$$\frac{29}{104} \rightarrow \%28$$

$$\hookrightarrow 100 - 28 = \%72 \text{ katang}$$

$$\hookrightarrow o = 00 \rightarrow 2^4 \cdot 4 = 48$$

$$r = 01 \rightarrow 2^4 \cdot 4 = 48$$

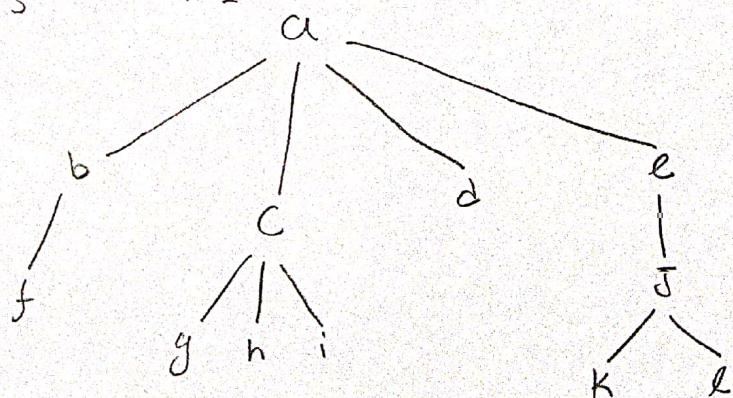
$$b = 11 \rightarrow 2^2 \cdot 2 = 4$$

$$y = 100 \rightarrow 3^2 \cdot 2 = 6$$

$$w = 101 \rightarrow 3^2 \cdot 1 = 3$$

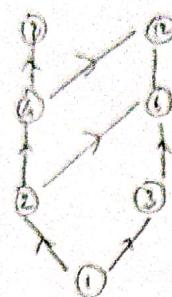
29 bit

2) a b f c g h i d e j k l

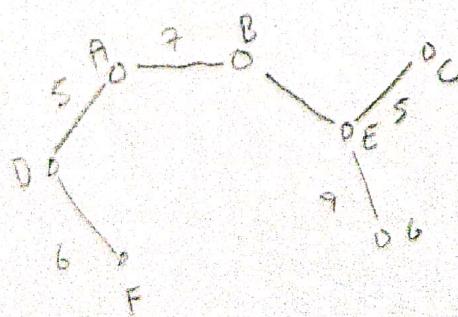
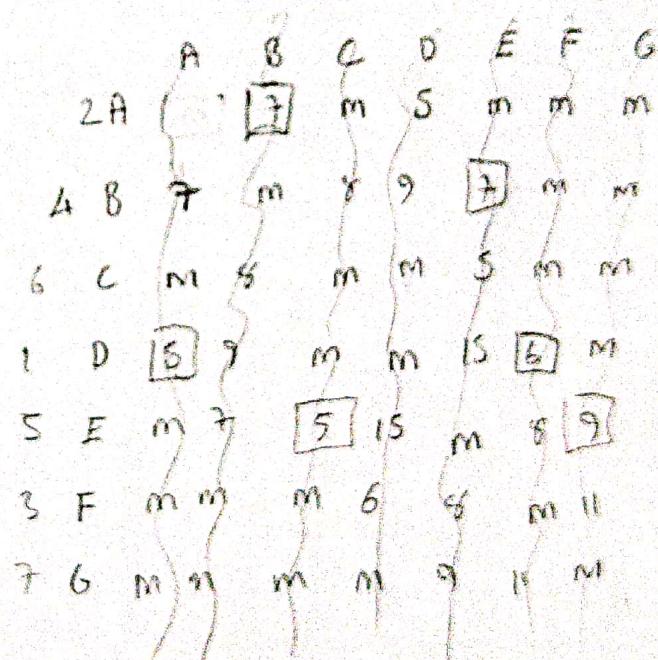
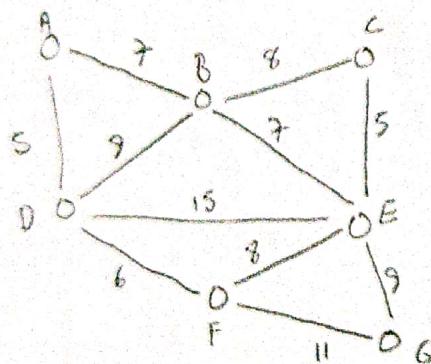


3)  $A = \{1, 2, 3, 4, 6, 8, 12\}$  a b'yi böwü.  $(a, b)$

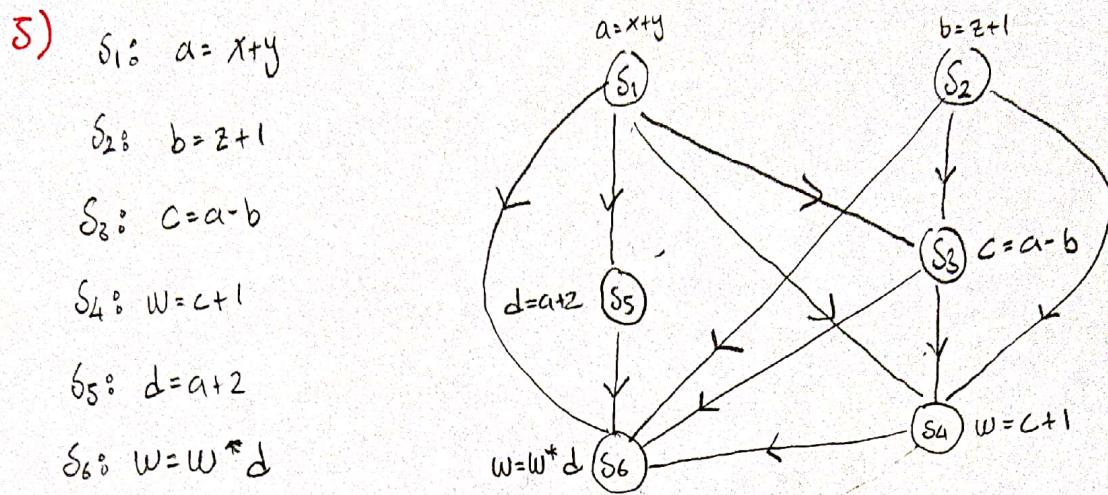
$$B = \{(1,2)(1,6)(1,4)(1,8)(1,12)(2,6)(2,8)(2,12)(3,6)(3,12)(4,8)(4,12)(6,12) \\ (1,1)(2,2)(3,3)(4,4)(6,6)(7,7)(12,12)\}$$

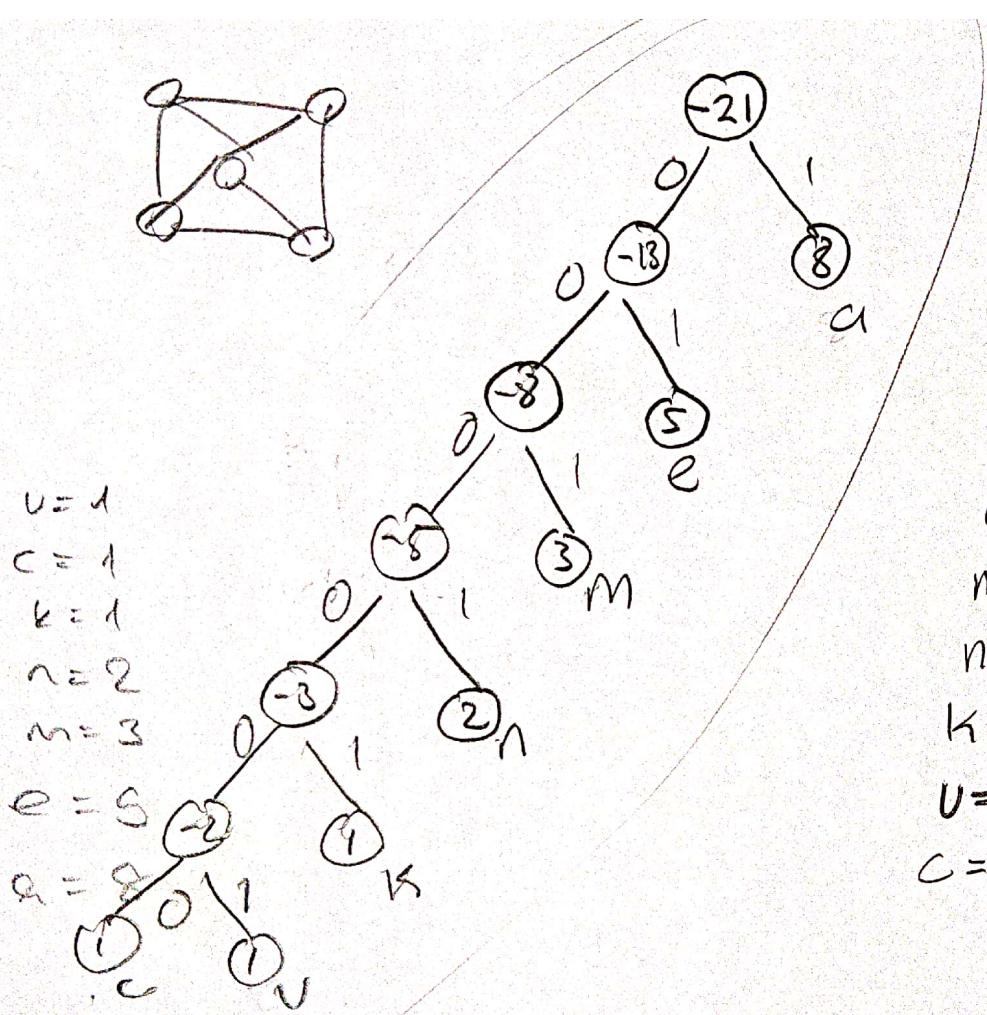


4)



$AB + 2A$



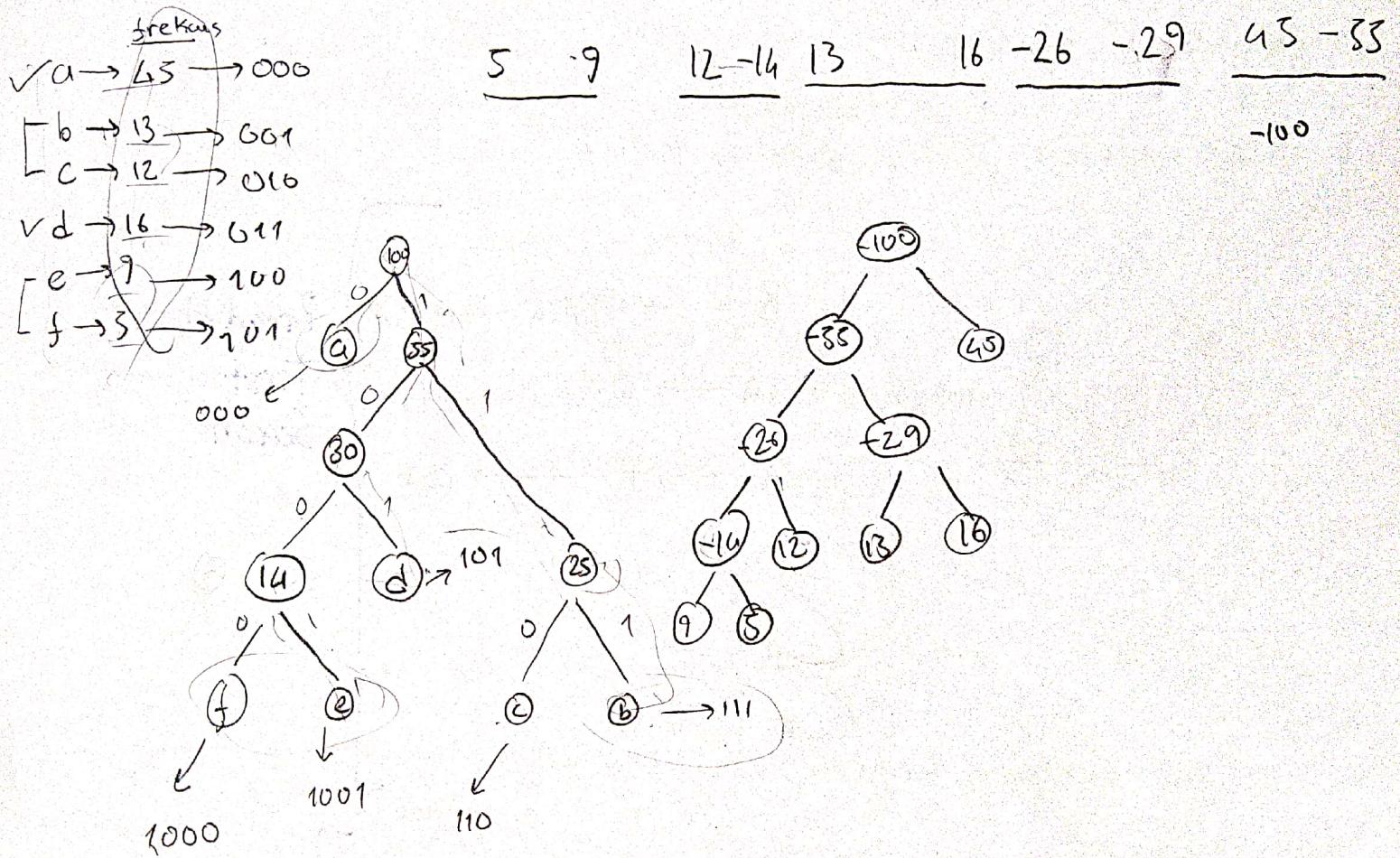


$a = 1 \rightarrow 8$   
 $e = 01 \rightarrow 10$   
 $m = 001 \rightarrow 9$   
 $n = 0001 \rightarrow 8$   
 $K = 000001 \rightarrow 5$   
 $V = 0000001 \rightarrow 6$   
 $C = 0000000 \rightarrow 6$

$$\frac{1}{1} \quad \frac{1}{1} \quad \frac{1}{2} \quad \underline{-2} \quad \underline{3} \quad \underline{-3} \quad \underline{5} \quad \underline{-5} \quad \underline{8} \quad \underline{-8} \quad \underline{-13} \quad \underline{-21}$$

$$\frac{1}{1} \quad \frac{1}{-2} \quad \underline{2} \quad \underline{-3} \quad \underline{3} \quad \underline{-5} \quad \underline{5} \quad \underline{-8} \quad \underline{8} \quad \underline{-13} \quad \underline{-21}$$

## Huffman Trees

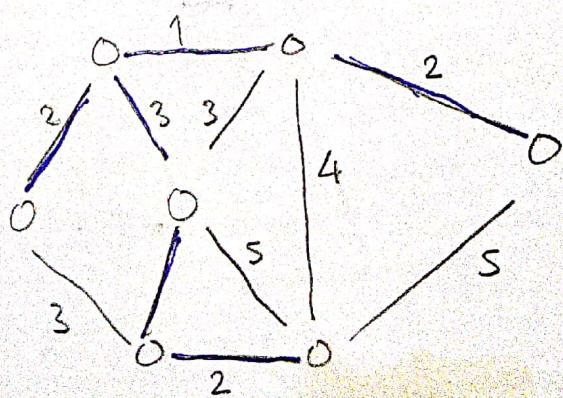


## Kruskal's Algorithm

Adım 1 → En küçük ağırlığa sahip kenarı bul

Adım 2 → " " " " " 2. Kenarı bul ancak kapalı döngü olmasın

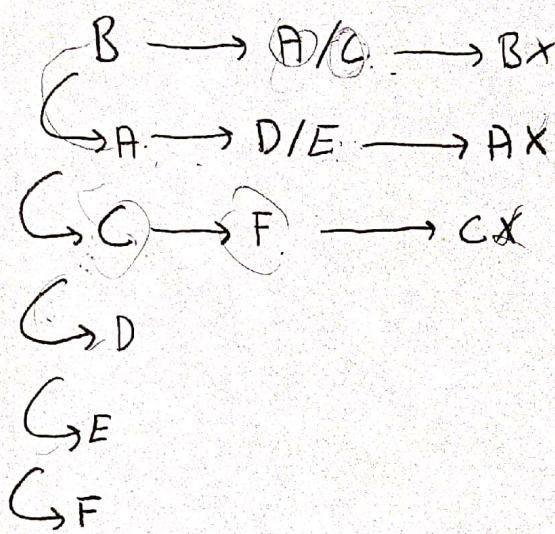
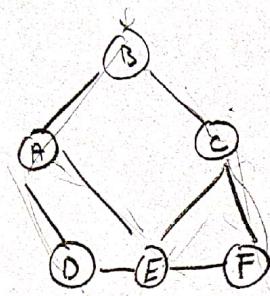
Adım 3 → Iter doğru dolaşana kadar adım 2'yi yap



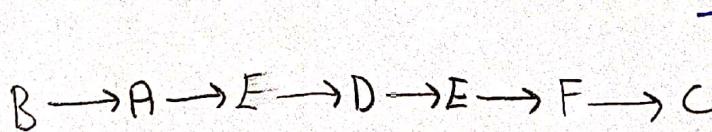
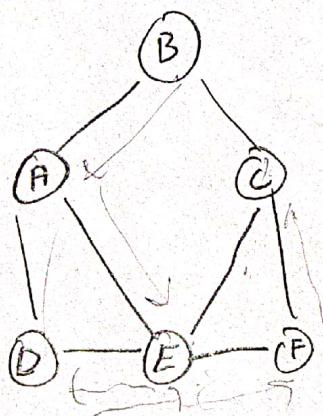
## Spanning Trees

G grafinda T spanning tree

T G' nin subtreesi ve tüm nodeleri kesis.

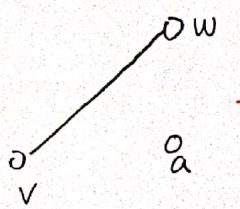


Breadth  
First  
Search

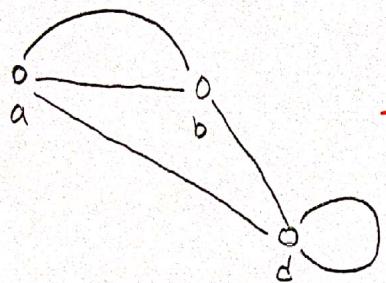


Depth  
First  
Search

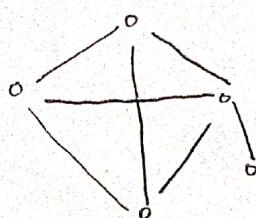
# Graflar



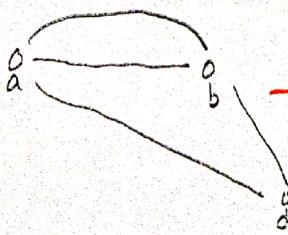
→ Ayrik düğüm (Isolated Vertex) = a



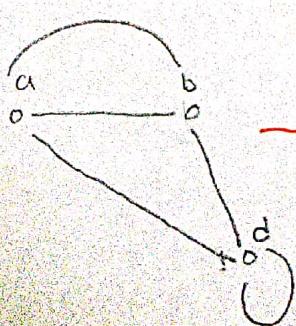
- a ve b iki paralel Kenar ile birleştirilmiştir.
- d düğümünde döngü var



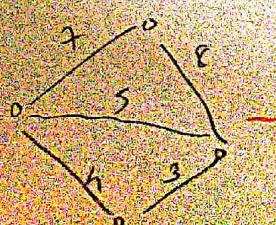
→ Basit (Simple) Graf yonsuz, paralelkenar olmayan ve döngü içermeyen graflarıdır.



→ Çoklu (Multi) graph, yonsuz, döngüsüz ama paralelkenar var.



→ Pseudo graph, yonsuz, döngülü, paralelkenar var.



→ Ağırlıklı (Weighted) graf

## Graflarda Benzerlik

$$V(G) \longrightarrow \{v_1, v_2, v_3, v_4, v_5\};$$

Her node  $v_i$ , bir vektör ile gösterilir  $\longrightarrow (p_1, p_2, p_3)$

$$v_1 = (66, 20, 1)$$

$$v_2 = (41, 10, 2)$$

$$v_3 = (68, 5, 8)$$

$$v_4 = (90, 34, 5)$$

$$v_5 = (78, 12, 14)$$

$$V = (p_1, p_2, p_3), \quad W = (q_1, q_2, q_3)$$

$$\bullet \quad S(V, W) = \sum_{i=1}^3 |p_i - q_i|$$

$\bullet N$  sekiller sabit sayı seçilsin

$\bullet S(V, W) < N$  ise  $V$  ve  $W$  arasında kenar eklenir.

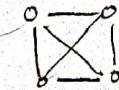
Eğer  $V=W$  veya  $V$   $W$  arasında yol varsa  $V$   $W$ 'nın aynı sınıfta olduğunu söyleyebiliriz.

## Tam (Complete) Graf $K_n$

- $n \geq 3$
- $K_n$ :  $n$  adet düğüm, her düğüm diğer düğümlerle bir kenar ile baglantıstır.

$$n \rightarrow \text{düğüm}$$

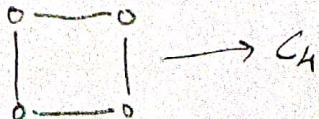
$$n \cdot (n-1)/2 \rightarrow \text{kenar}$$



## Cycles (açıksız) Graf $C_n$

$$- n \geq 3$$

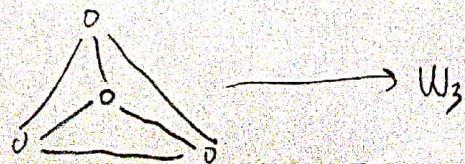
- $\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \dots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$  şeklindedir



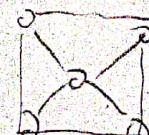
$$\begin{matrix} n \rightarrow \text{düğüm} \\ n \rightarrow \text{kenar} \end{matrix}$$

## Wheel (Tekerkelik) Graf $W_n$

- $C_n$  grafına diğer bütün düğümlere bağlanacak şekilde düğüm eklemesi.



$$\begin{matrix} n+1 \rightarrow \text{düğüm} \\ 2n \rightarrow \text{kenar} \end{matrix}$$

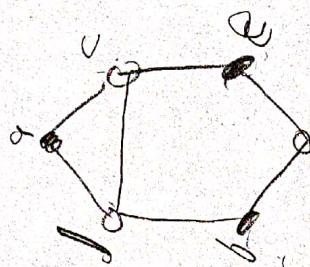
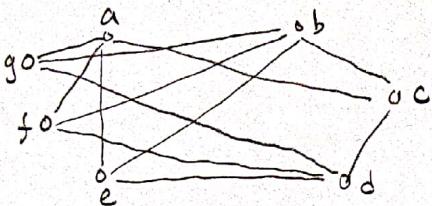
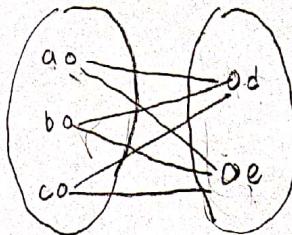


## İki Parçalı (Bipartite) Graf

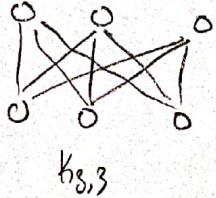
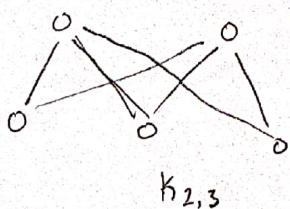
$$V(G) = V(G_1) \cup V(G_2)$$

$$|V(G_1)| = m, |V(G_2)| = n \rightarrow$$

$$V(G_1) \cap V(G_2) = \emptyset$$

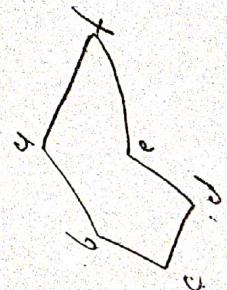


## Tam (Complete) Bipartite Graf

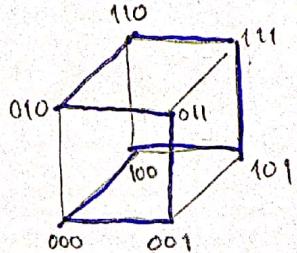


$m+n \rightarrow \text{dogru}$

$m \times n \rightarrow \text{kenar}$

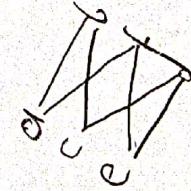


## N-Cube Graf $\mathcal{Q}_n$



$2^n \rightarrow \text{dogru}$

$n \cdot 2^{n-1} \rightarrow \text{kenar}$



a	b
c	d
e	f

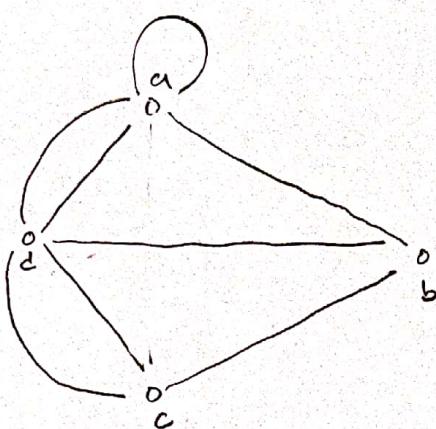
## Euler Döngüsü (Euler Cycles)

Grafı içerdikti her kenardan 1 kez geçerek başlangıç noktasına dönmemek.

### Düğüm Derecesi

$$\delta(a) = 4$$

$$\delta(d) = 5$$



## Euler Grafi

Grafı Euler Cycle'ına sahipse Euler grafı denir

Tüm düğümlerin derecesi çift olmalıdır

### Teorem

$e \rightarrow$  Kenar

$n \rightarrow$  Düğüm

$$\sum_{i=1}^n \delta(v_i) = 2e$$

$\delta(1) \rightarrow$  pendant

$\delta(0) \rightarrow$  isolated

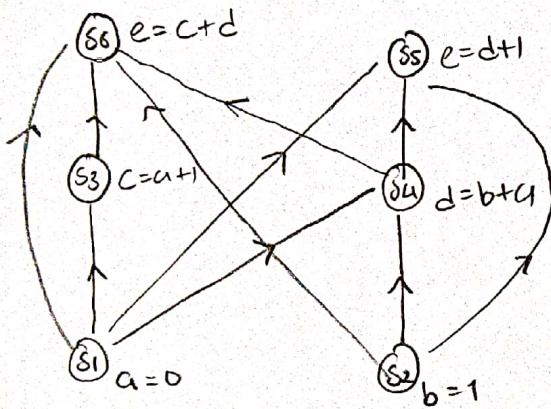
## Hamiltonian Cycle

grafin üzerinde her node'dan 1 kez geçerets kapalı yol

G grafı şahipse hamiltonic graf

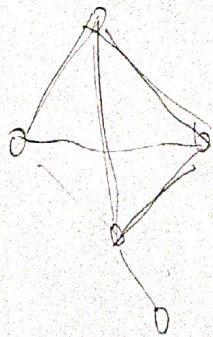
## Precedence Graf

$$S_1 : a=0 \quad S_2 : b=1 \quad S_3 : c=a+1 \quad S_4 : d=b+c+1 \quad S_5 : e=d+1 \quad S_6 : e=c+d$$

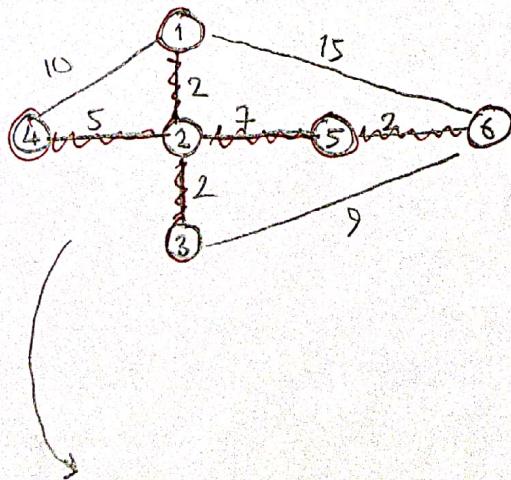
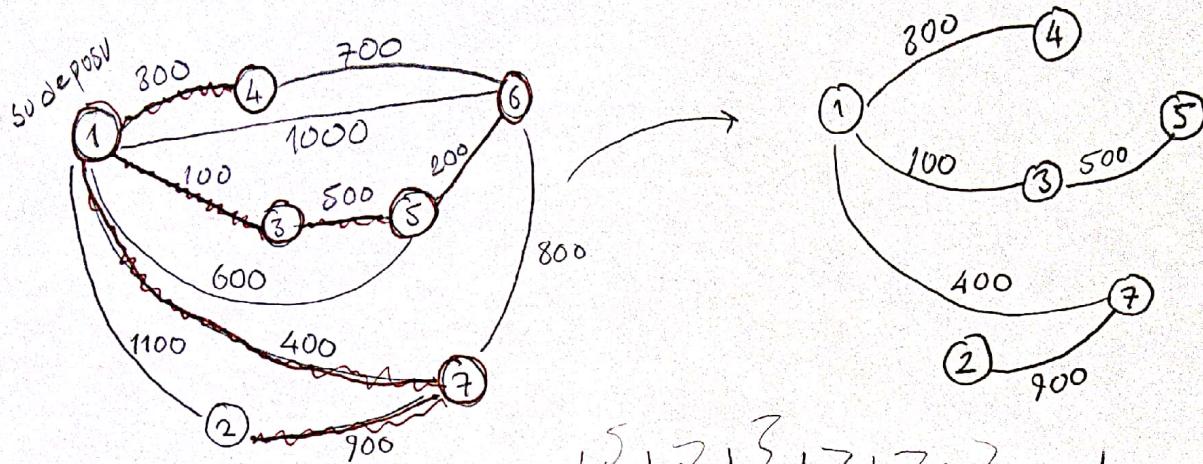


## Planar Graf

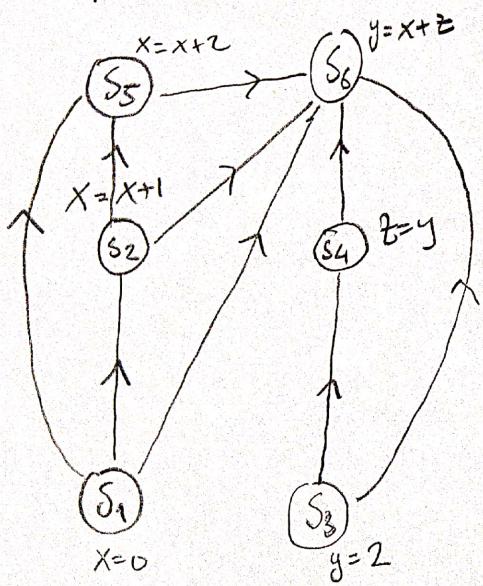
Kenarlar birbirini kesmiyorsa planar



## Prim Algoritması (Minimum Spanning Tree)



$$S_1: x=0 \quad S_2: x=x+1 \quad S_3: y=2 \quad S_4: z=y \quad S_5: x=x+2 \quad S_6: y=x+z \quad S_7: z=4$$

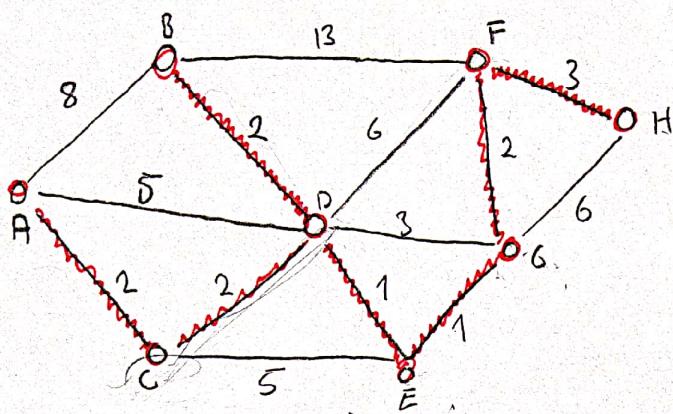


gemishe back

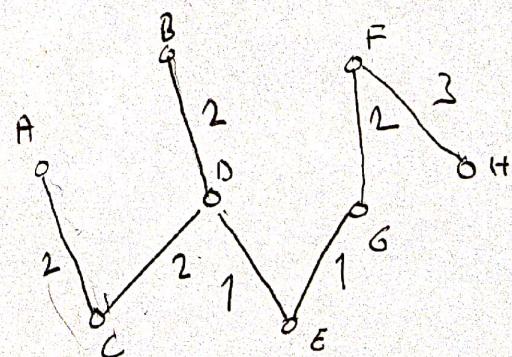
precedence  
graph

29.11.2019

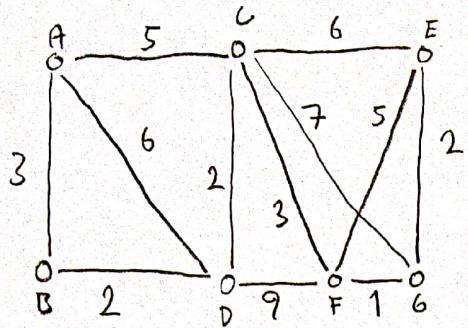
## Dijkstra Algoritması



	A	B	C	D	E	F	G	H
A	0_A	8_A	2_A*	5_A	$\infty$	$\infty$	$\infty$	$\infty$
C		8_A	2_A	4_C*	7_C	$\infty$	$\infty$	$\infty$
D			6_D	4_C	5_D*	10_D	7_D	$\infty$
E				6_D*	5_E	10_D	6_E	$\infty$
B		6_D			10_D	6_E*	$\infty$	
G					8_G*	6_E	12_G	
F					8_G	11_F*		
H						11_F		



	A	B	C	D	E	F	G	H
A	0_A	8_A	2_A*	5_A	$\infty$	$\infty$	$\infty$	$\infty$
C		8_A	2_A	4_C*	7_C	$\infty$	$\infty$	$\infty$
D			6_D	4_C	5_D*	10_D	7_D	$\infty$
E				6_D*	5_E	10_D	6_E	$\infty$
B		6_D			10_D	6_E*	$\infty$	
G					8_G*	6_E	12_G	
F					8_G	11_F*		
H						11_F		



	A	B	C	D	E	F	6
A	$A_0$	$3_A^*$	$5_A$	$6_A$	$\infty$	$\infty$	$\infty$
B		$3_A$	$5_A$	$5_B^*$	$\infty$	$\infty$	$\infty$
D			$5_A^*$	$5_B$	$\infty$	$14_D$	$\infty$
C			$5_A$	$11_C$	$8_C^*$	$12_C$	
F				$11_C$	$8_F$	$9_F^*$	
G						$9_F$	

# Finite State Automata

Initial State ( $S_0$ )

Durumlar Listesi ( $S$ )

Her durum geçisi için olmasız gerekeli Koşullar Listesi (conditions) f, g---

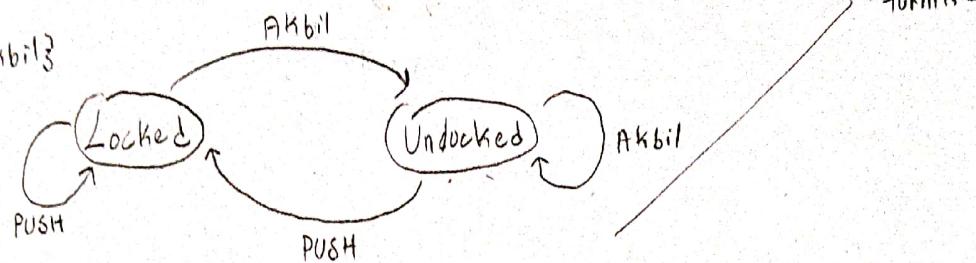
- I, O

FSM' nin hafızası : State #

örnek:

$S = \{\text{Locked}, \text{Unlocked}\}$

I = {Push, Aktion}



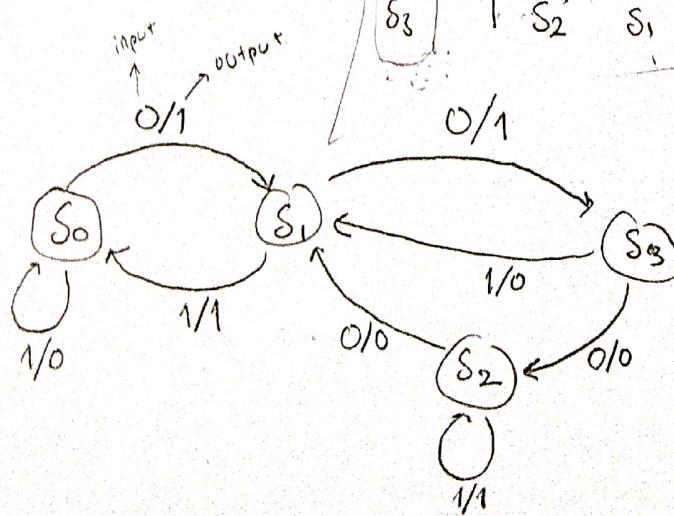
örnek:

$S = \{S_0, S_1, S_2, S_3\}$

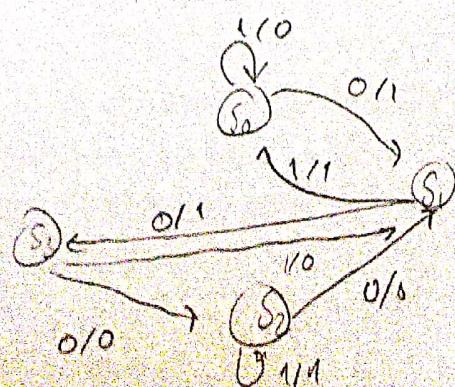
I = {0, 1}

O = {0, 1}

State	Input		Output
	0	1	
$S_0$	$S_1$	$S_0$	1 0
$S_1$	$S_3$	$S_0$	1 1
$S_2$	$S_1$	$S_2$	0 1
$S_3$	$S_2$	$S_1$	0 0



radc (%) 4



50.30

190

35.30

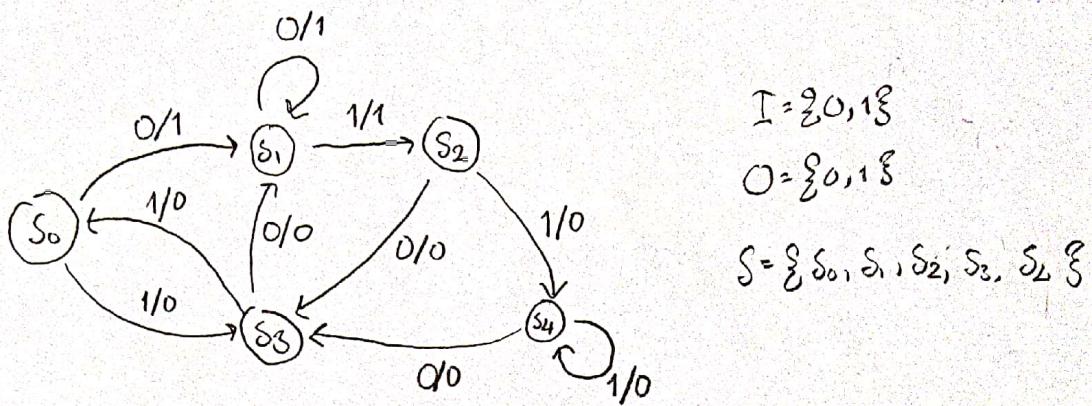
18+

10.5

105

$$\begin{array}{r}
 68.5 \\
 + 28.5 \\
 \hline
 97.00
 \end{array}
 \quad
 \begin{array}{r}
 32.10 \\
 \hline
 41.00
 \end{array}
 \quad
 \begin{array}{r}
 102.40 \\
 \hline
 100
 \end{array}$$

Örnek:



$$I = \{0, 1\}$$

$$O = \{0, 1\}$$

$$S = \{S_0, S_1, S_2, S_3, S_4\}$$

State	Input		Output
	0	1	
$S_0$	$s_1$	$s_3$	1 0
$S_1$	$s_1$	$s_2$	1 1
$S_2$	$s_3$	$s_4$	0 0
$S_3$	$s_1$	$s_0$	0 0
$S_4$	$s_3$	$s_4$	0 0

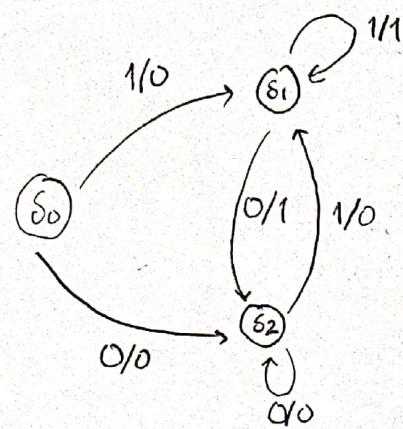
Örnek 2

Yukarıdakine göre

Input: 101011  
Output: 001000

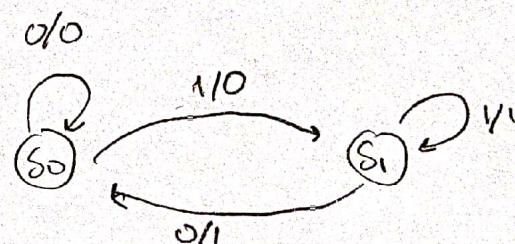
Output?

## Örnek:



<u>Inputs</u>	<u>Output</u>
10101	01010
1111	0111
0000	0000
010100	001010

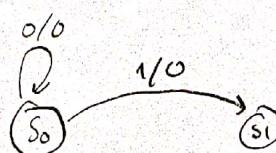
Saya 1 bit shift



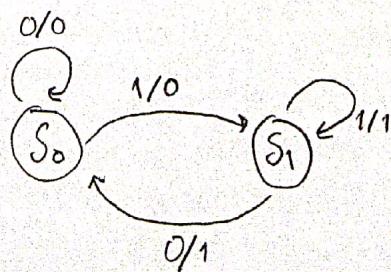
## Örnek

Sola shift eden FGM

<u>Input</u>	<u>Output</u>
0110	1100
1100	1000
0001	0010



0110  
1100



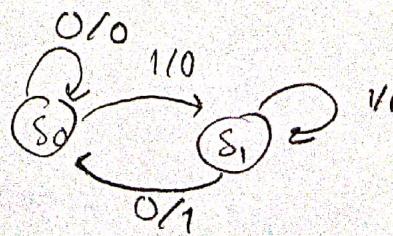
1100 1000

0

1111

1110

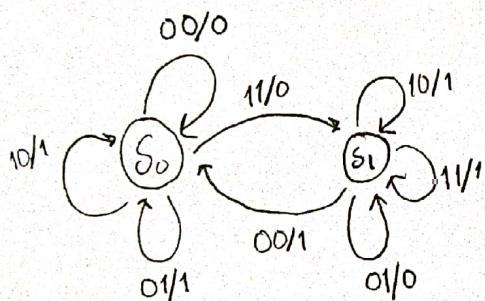
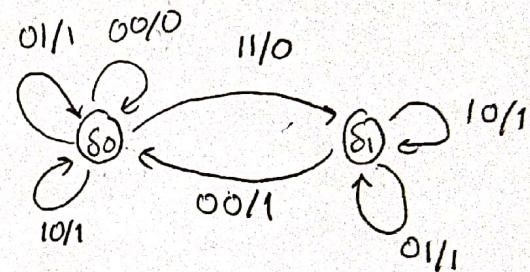
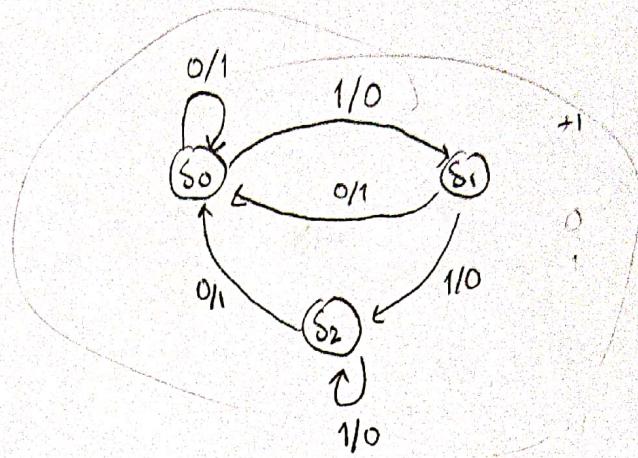
1010



### Örnek

Binary toplama

<u>I<sub>1</sub></u>	<u>I<sub>2</sub></u>	<u>Output</u>
0	0	0
0	1	1
1	0	1
1	1	0      Eg = 1

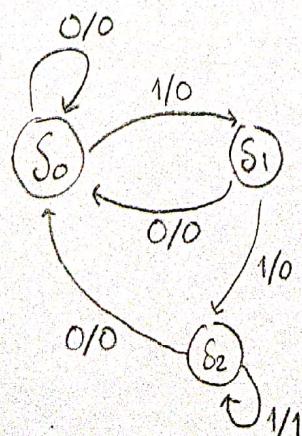


$$\begin{array}{r} 00111 \\ +00110 \\ \hline 01101 \end{array}$$

### Örnek

gönderiler inputta art arda 3 tane 1 varsa hatta mesajı gönderilsin.

$$\begin{array}{l} I: 1\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 1\ 1 \\ \downarrow \quad \downarrow \\ O: 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1 \end{array}$$



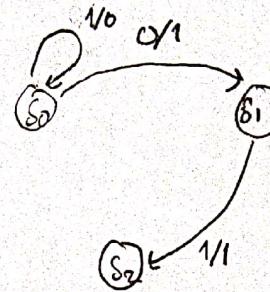
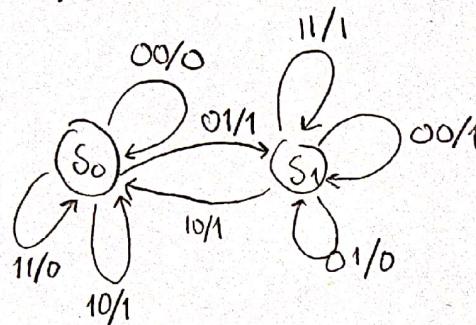
### Örnek

likarmanın 2 bit

I	O
0 0	0
0 1	1 (bora)
1 0	1
1 1	0

$$\begin{array}{r} 1101 \ 0 \ 1 \\ 0110 \ 0 \ 1 \\ \hline 0111000 \end{array}$$

$$= \begin{array}{r} 0 \ 0 \\ 1 \ 1 \end{array}$$



### Örnek

Degişken ismi 4 char uzunluğunda

ilk karakter A-Z <= Z aralığında olmalı

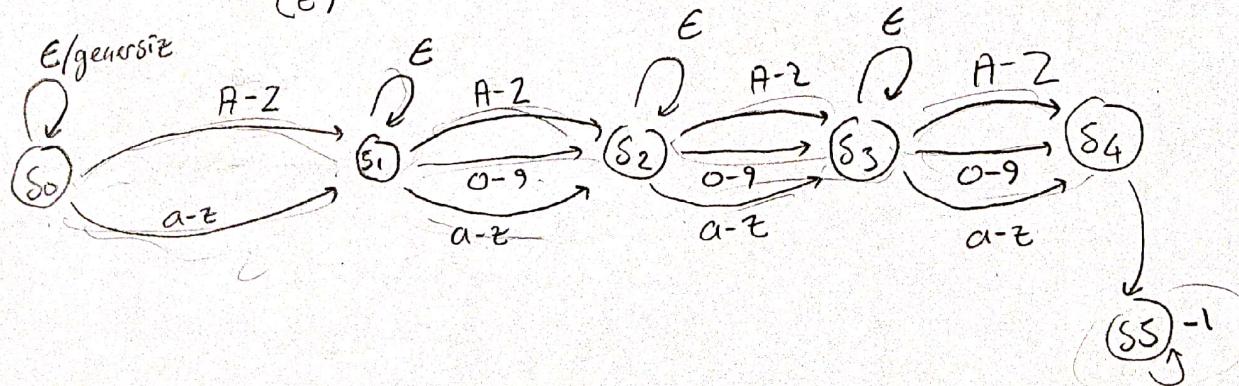
Sonrası

a-z

A-Z

0-9

Baska karakter girilirse geçersiz sayılacak ve yeni bir karakter isteneceks (E)



## Örnek

login

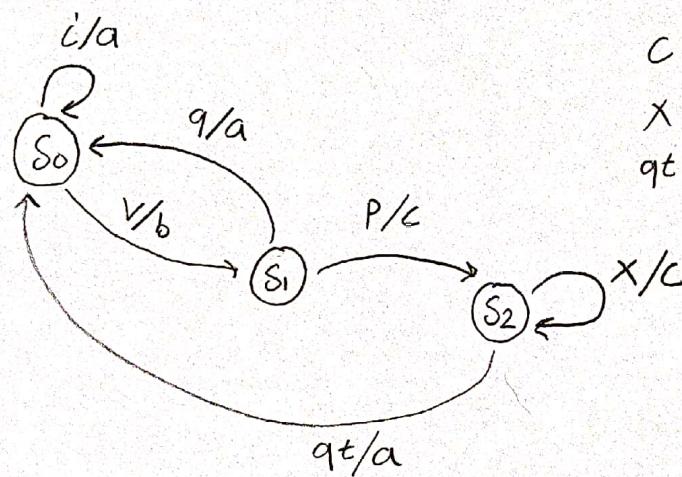
Kullanıcı ID'si isteniyor

ID geçersizse geleneksel ID

ID geçerliyse password

password güclü ise ID girişini tekrarlar

"dogru" ✓



v → valid ID

i → invalid ID

p → valid password

q → invalid password

a → "enter ID"

b → "enter password"

c → prompt

x → exit input

qt → quit