

BLM2041 Signals and Systems

Syllabus

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BLM2041 Signals and Systems

FIR Filtering and Frequency Response

LECTURE OBJECTIVES

- INTRODUCE FILTERING IDEA
 - Weighted Average
 - Running Average
- FINITE IMPULSE RESPONSE FILTERS
 - FIR Filters
 - Show how to compute the output $y[n]$ from the input signal, $x[n]$

LECTURE OBJECTIVES

- SINUSOIDAL INPUT SIGNAL
 - DETERMINE the FIR FILTER OUTPUT

- FREQUENCY RESPONSE of FIR

- PLOTTING vs. Frequency

- MAGNITUDE vs. Freq

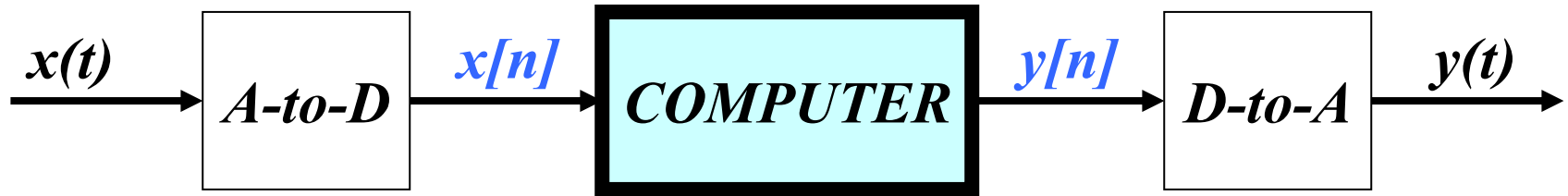
- PHASE vs. Freq

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})|e^{j\angle H(e^{j\hat{\omega}})}$$

MAG

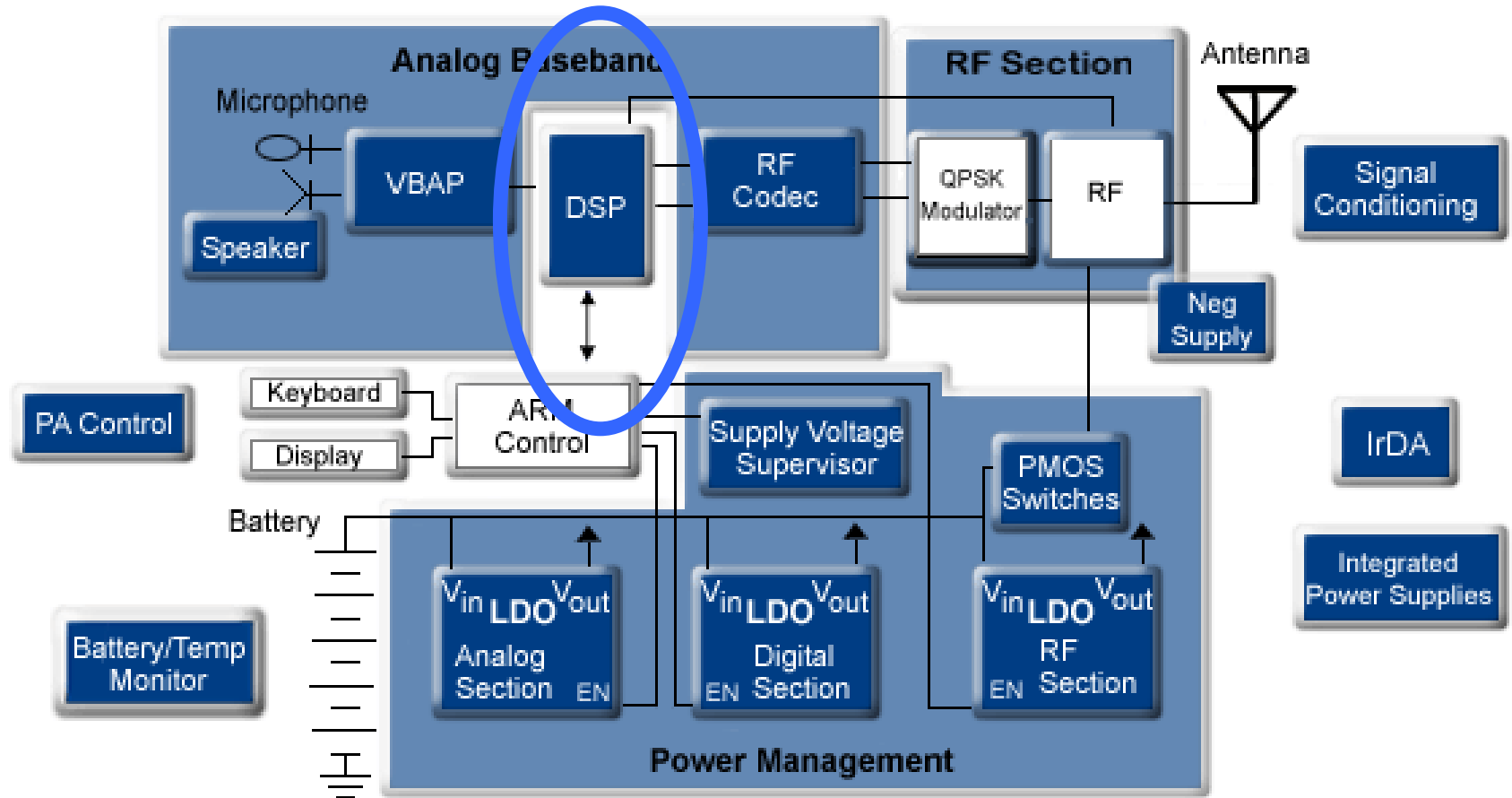
PHASE

DIGITAL FILTERING



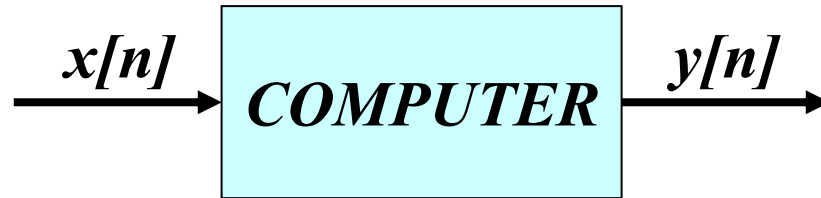
- CONCENTRATE on the COMPUTER
 - PROCESSING ALGORITHMS
 - SOFTWARE (MATLAB)
 - HARDWARE: DSP chips, VLSI
- **DSP**: DIGITAL SIGNAL PROCESSING

Digital Cell Phone



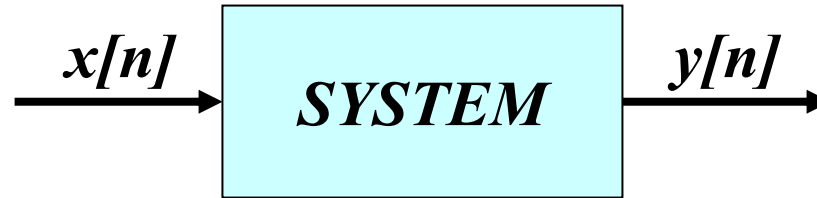
Now it plays video

DISCRETE-TIME SYSTEM



- OPERATE on $x[n]$ to get $y[n]$
- WANT a **GENERAL** CLASS of SYSTEMS
 - **ANALYZE** the **SYSTEM**
 - **TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN**
 - **SYNTHESIZE** the **SYSTEM**

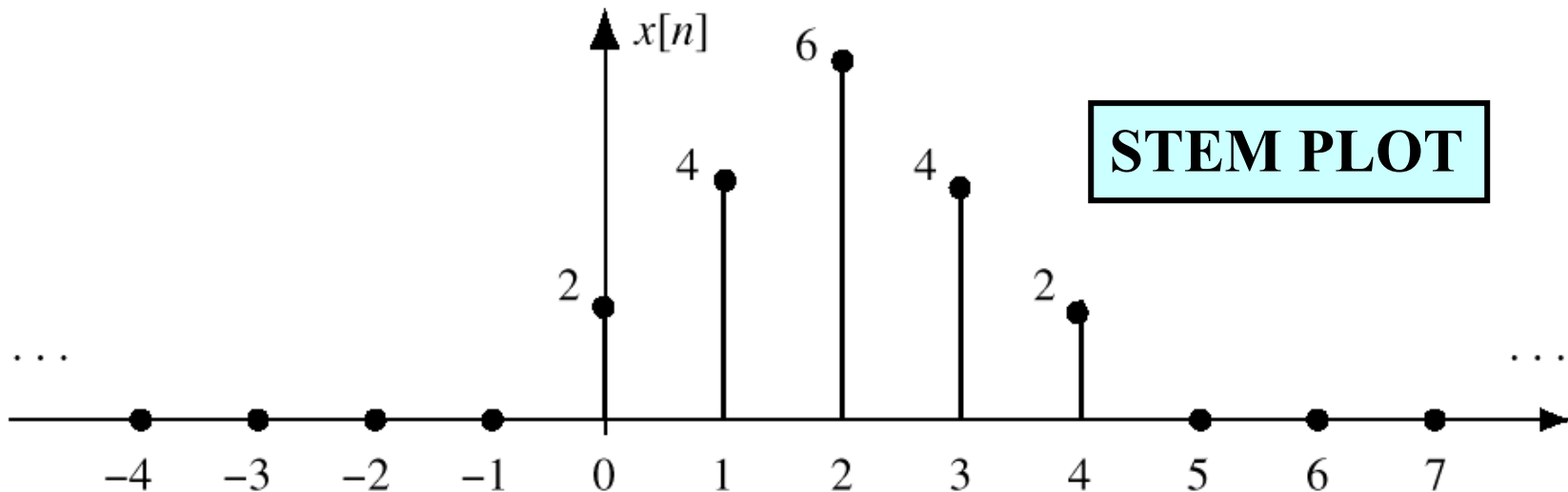
D-T SYSTEM EXAMPLES



- EXAMPLES:
 - POINTWISE OPERATORS
 - SQUARING: $y[n] = (x[n])^2$
 - RUNNING AVERAGE
 - RULE: “the output at time n is the average of three consecutive input values”

DISCRETE-TIME SIGNAL

- $x[n]$ is a LIST of NUMBERS
 - INDEXED by “ n ”



3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
 - Do this for each “ n ”

the following input–output equation

Make a TABLE

$$y[n] = \frac{1}{3} (x[n] + x[n+1] + x[n+2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	6	4	2	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

$n=0$ $y[0] = \frac{1}{3}(x[0] + x[1] + x[2])$

$n=1$ $y[1] = \frac{1}{3}(x[1] + x[2] + x[3])$

3-PT AVERAGE SYSTEM

INPUT SIGNAL

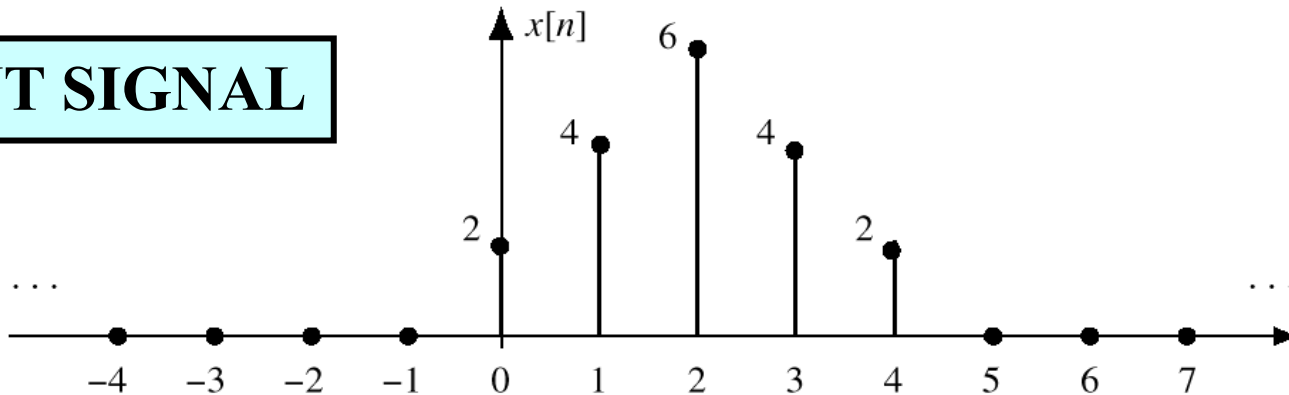


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \frac{1}{3} (x[n] + x[n+1] + x[n+2])$$

OUTPUT SIGNAL

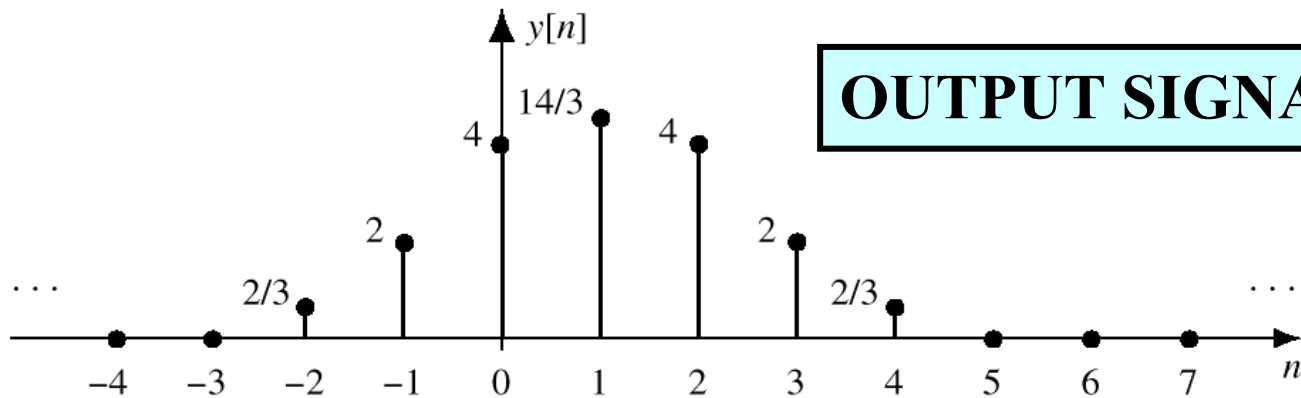


Figure 5.3 Output of running average, $y[n]$.

PAST, PRESENT, FUTURE

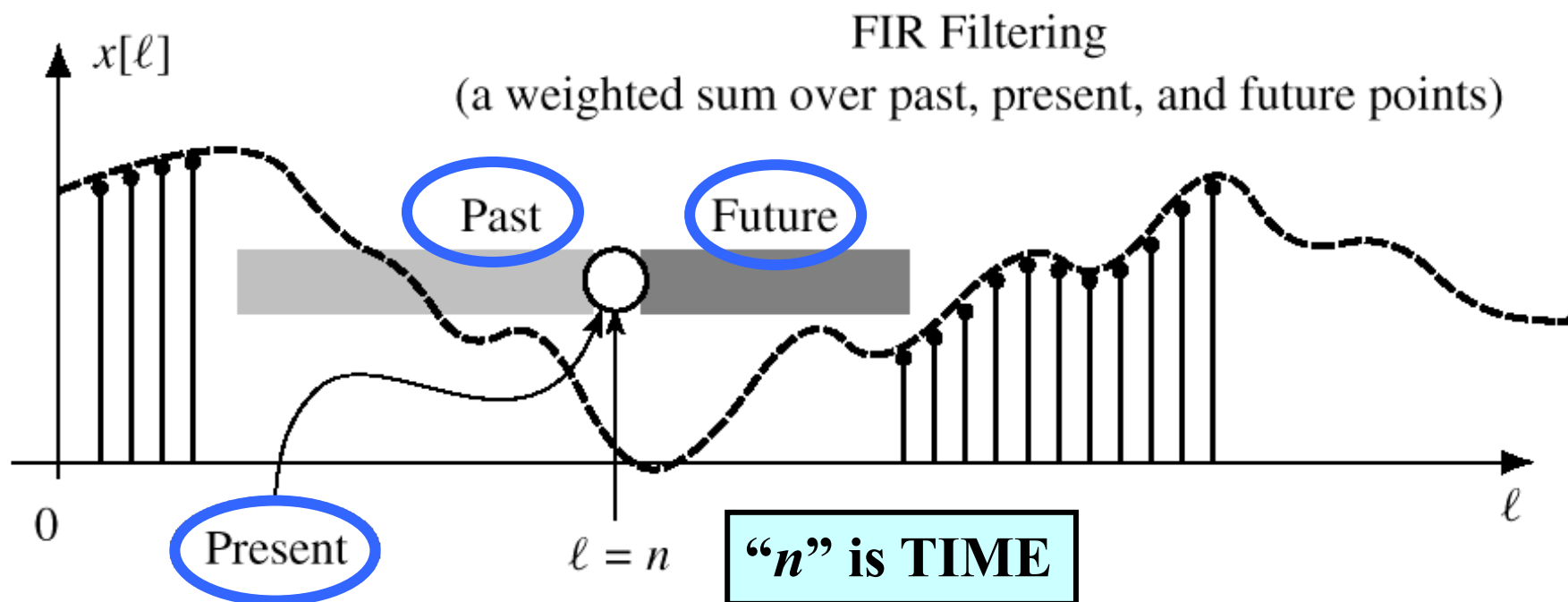


Figure 5.4 The running-average filter calculation at time index n uses values within a sliding window (shaded). Dark shading indicates the future ($l > n$); light shading, the past ($l < n$).

ANOTHER 3-pt AVERAGER

- Uses “PAST” VALUES of $x[n]$
 - IMPORTANT IF “ n ” represents REAL TIME
 - WHEN $x[n]$ & $y[n]$ ARE STREAMS

$$y[n] = \frac{1}{3} (x[n] + x[n-1] + x[n-2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	6	4	2	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

– DEFINE THE FILTER

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

– For example,

$$b_k = \{3, -1, 2, 1\}$$

$$\begin{aligned} y[n] &= \sum_{k=0}^3 b_k x[n-k] \\ &= 3x[n] - x[n-1] + 2x[n-2] + x[n-3] \end{aligned}$$

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

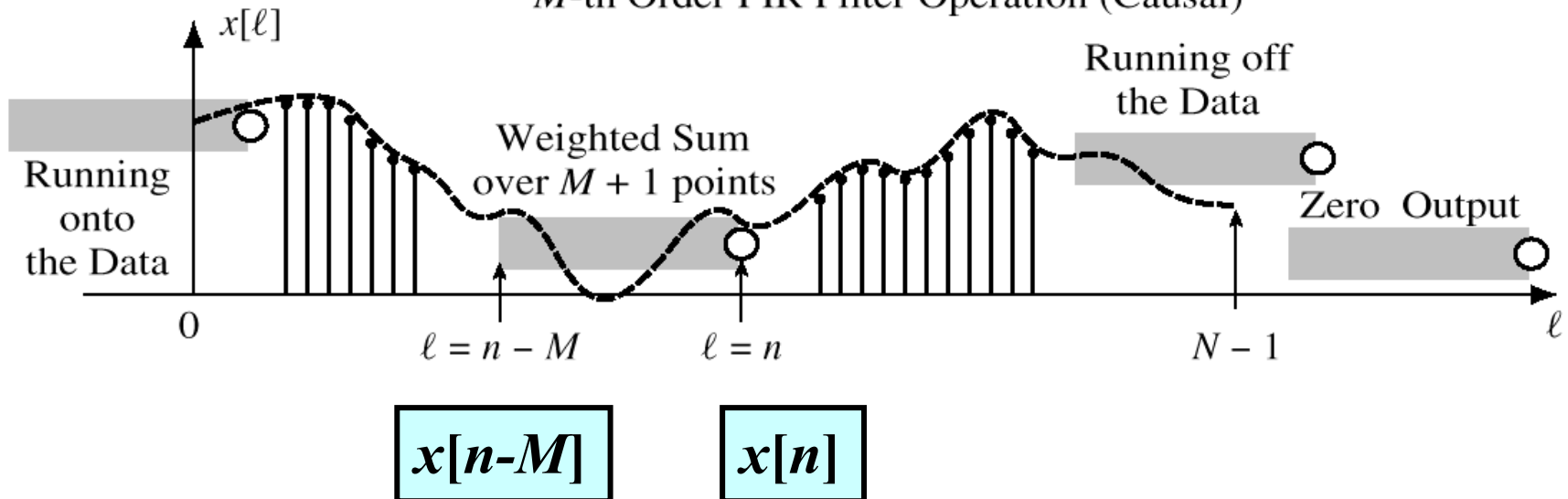
- FILTER ORDER is M
- FILTER LENGTH is $L = M+1$
 - NUMBER of FILTER COEFFS is L

GENERAL FIR FILTER

- SLIDE a WINDOW across $x[n]$

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

M -th Order FIR Filter Operation (Causal)



FILTERED STOCK SIGNAL

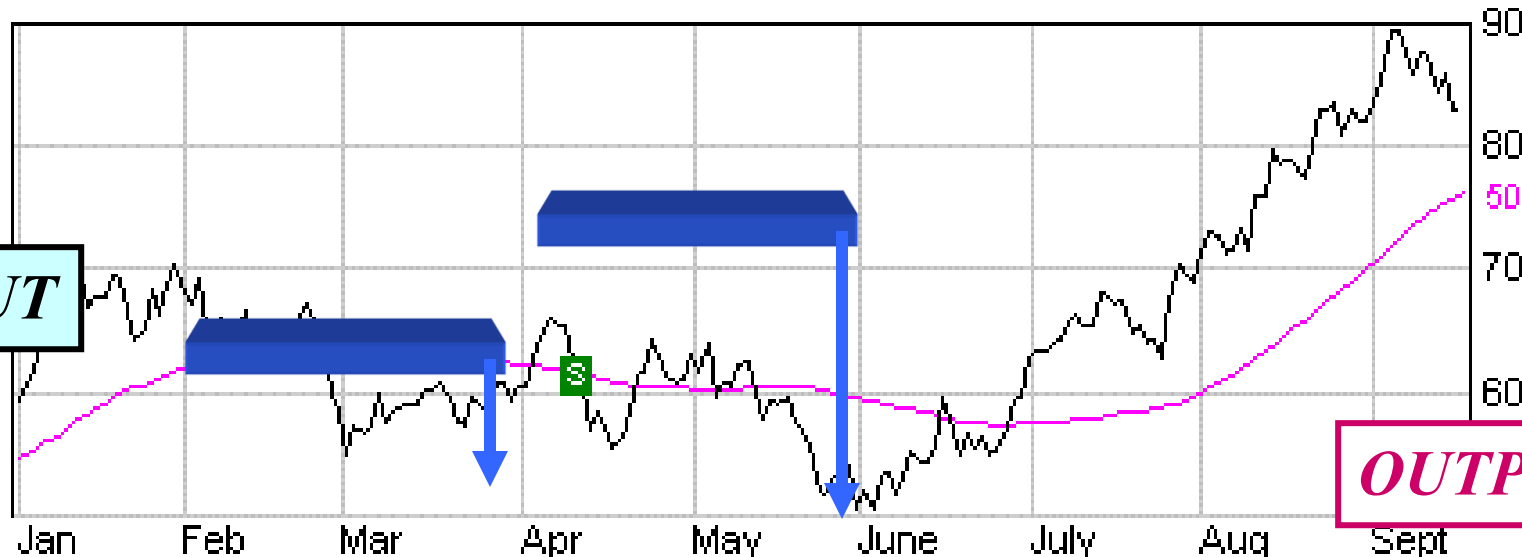
Period: **YTD**

Chart Type: **Closing Prices**

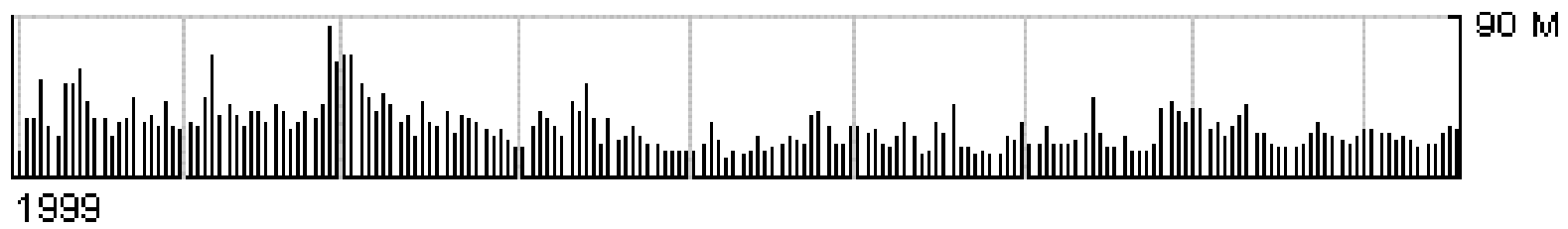
INTC **B4 3/4** **+ 1/8**

[S] = Stock split

INPUT



OUTPUT



Moving Averages: ☐ None ☐ 25 ☒ 50 ☐ 100 ☐ 200

50-pt Averager

SPECIAL INPUT SIGNALS

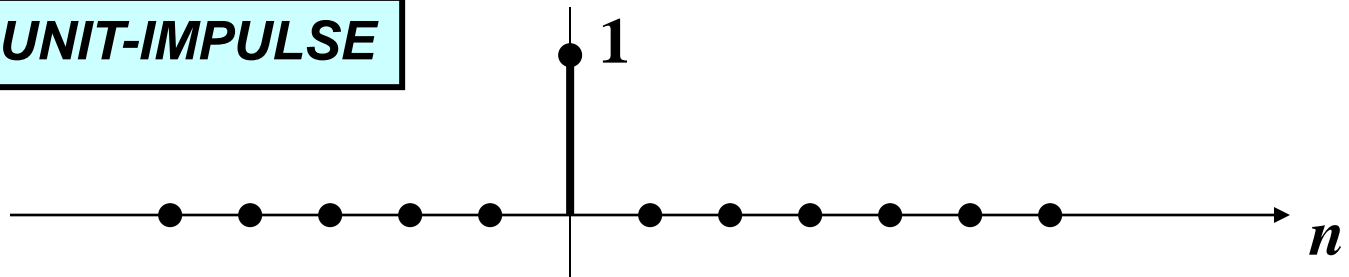
- $x[n] = \text{SINUSOID}$

FREQUENCY RESPONSE (LATER)

- $x[n]$ has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



UNIT IMPULSE SIGNAL $\delta[n]$

n	...	-2	-1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n - 3]$	0	0	0	0	0	0	1	0	0	0	0

$\delta[n]$ is NON-ZERO
When its argument
is equal to ZERO

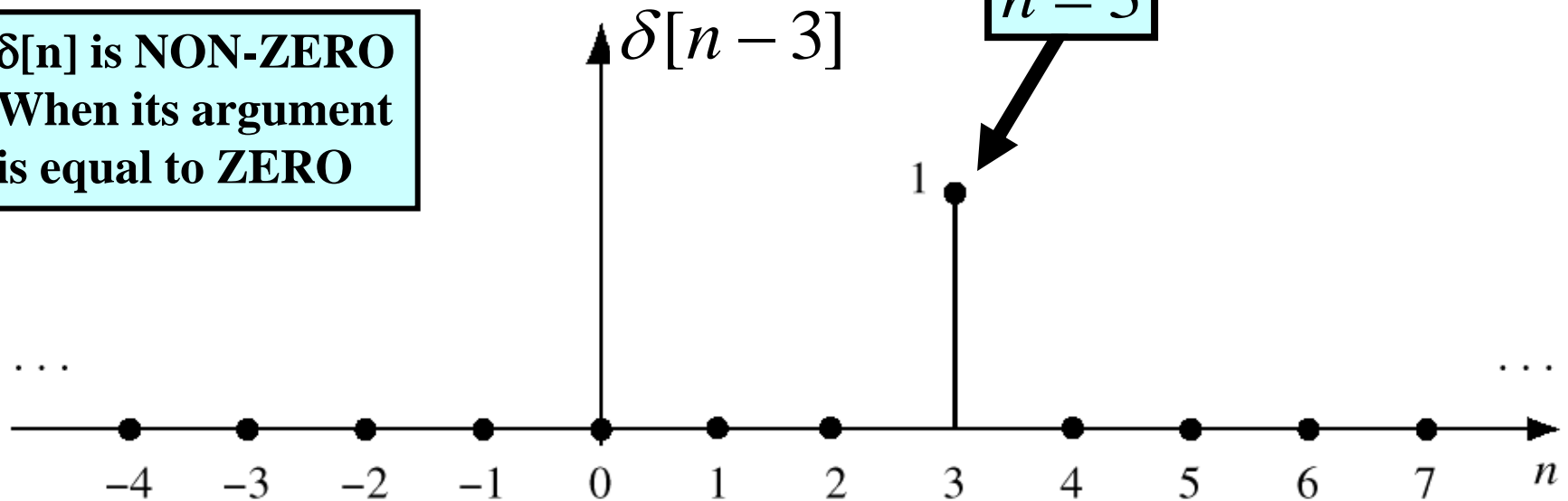
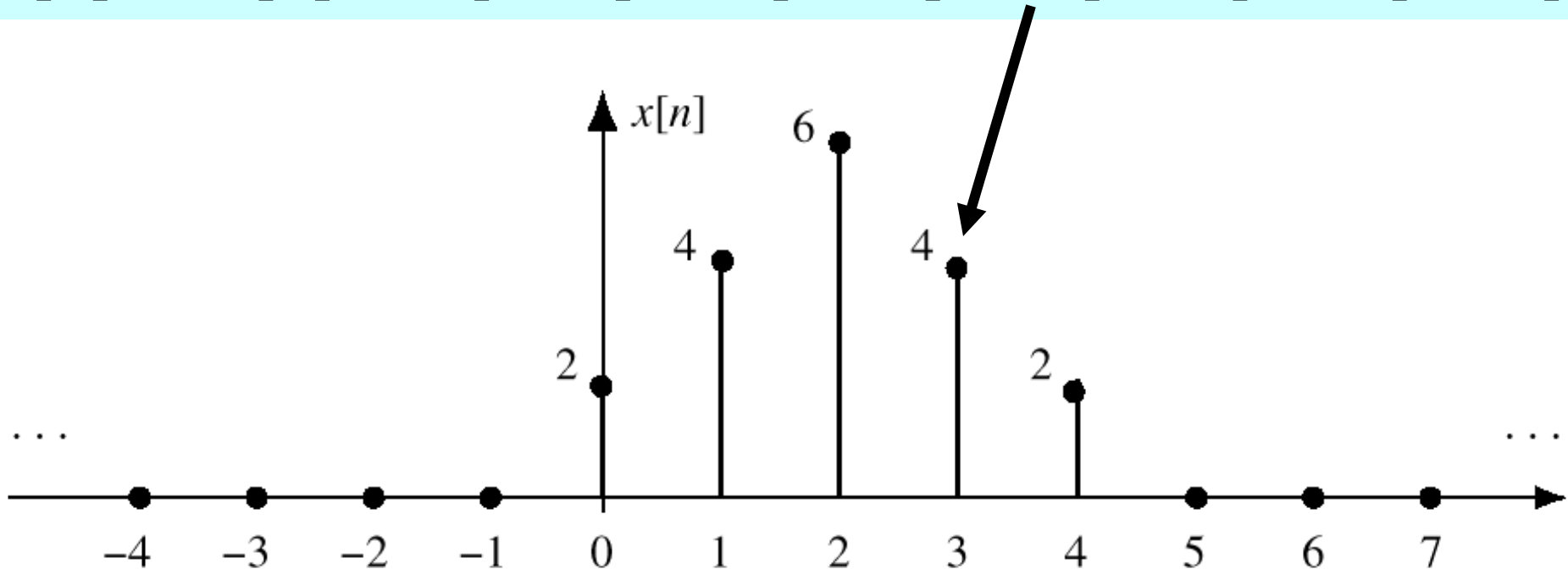


Figure 5.7 Shifted impulse sequence, $\delta[n - 3]$.

MATH FORMULA for $x[n]$

- Use **SHIFTED** IMPULSES to write $x[n]$

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$



SUM of **SHIFTED** IMPULSES

n	...	-2	-1	0	1	2	3	4	5	6	...
$2\delta[n]$	0	0	0	2	0	0	0	0	0	0	0
$4\delta[n - 1]$	0	0	0	0	4	0	0	0	0	0	0
$6\delta[n - 2]$	0	0	0	0	0	6	0	0	0	0	0
$4\delta[n - 3]$	0	0	0	0	0	0	4	0	0	0	0
$2\delta[n - 4]$	0	0	0	0	0	0	0	2	0	0	0
$x[n]$	0	0	0	2	4	6	4	2	0	0	0

$$x[n] = \sum_k x[k]\delta[n - k]$$

This formula ALWAYS works

$$= \dots + x[-1]\delta[n + 1] + x[0]\delta[n] + x[1]\delta[n - 1] + \dots \quad (5.3.6)$$

4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES

$$y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL = $\delta[n]$

$$x[n] = \delta[n]$$

$$y[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$$

- OUTPUT is called “IMPULSE RESPONSE”

$$h[n] = \{\dots, 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, \dots\}$$

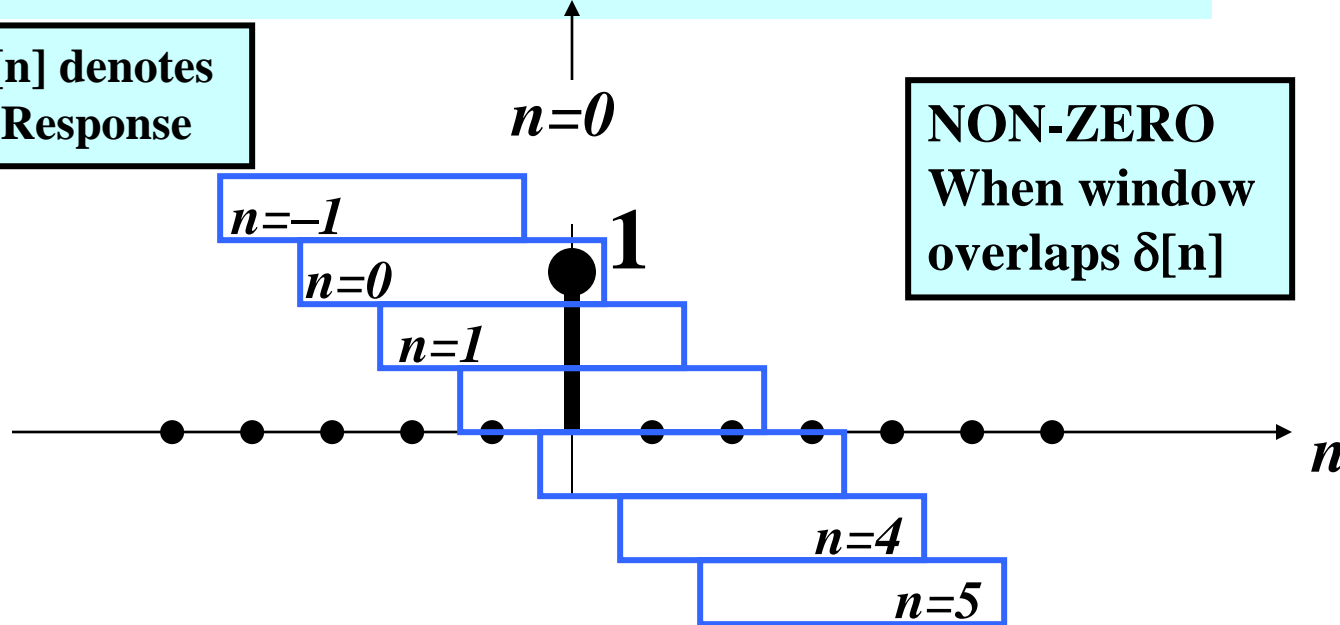
4-pt Avg Impulse Response

$$y[n] = \frac{1}{4} (x[n] + x[n-1] + x[n-2] + x[n-3])$$

$\delta[n]$ “READS OUT” the FILTER COEFFICIENTS

$$h[n] = \{\dots, 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, \dots\}$$

“h” in $h[n]$ denotes
Impulse Response



FIR IMPULSE RESPONSE

- Convolution = Filter Definition
 - Filter Coeffs = Impulse Response

n	$n < 0$	0	1	2	3	\dots	M	$M + 1$	$n > M + 1$
$x[n] = \delta[n]$	0	1	0	0	0	0	0	0	0
$y[n] = h[n]$	0	b_0	b_1	b_2	b_3	\dots	b_M	0	0

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

$$y[n] = \sum_{k=0}^M h[k] x[n-k]$$

CONVOLUTION

FILTERING EXAMPLE

- 7-point AVERAGER

- Removes cosine

- By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^6 \left(\frac{1}{7}\right) x[n-k]$$

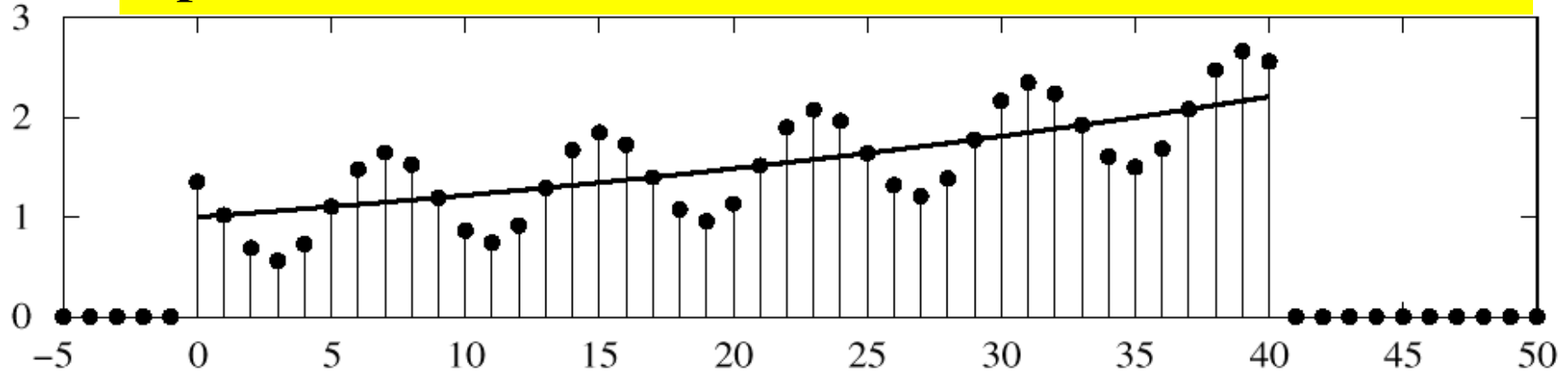
- 3-point AVERAGER

- Changes A slightly

$$y_3[n] = \sum_{k=0}^2 \left(\frac{1}{3}\right) x[n-k]$$

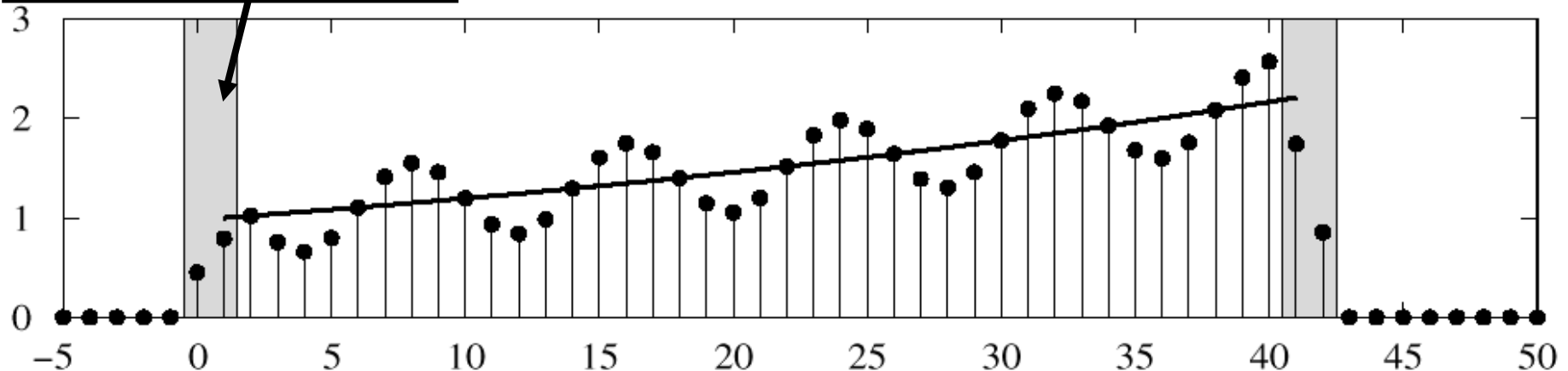
3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$



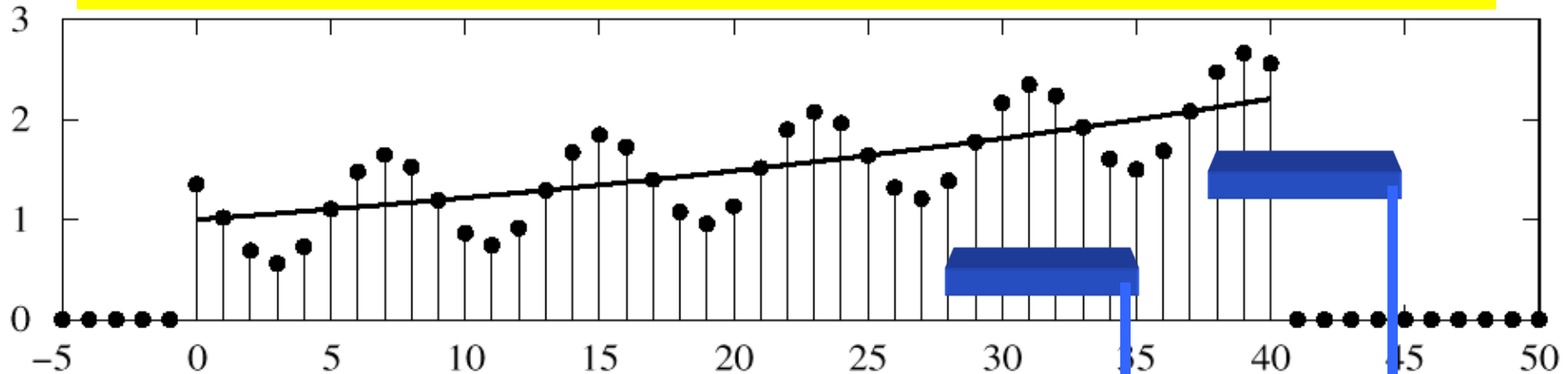
USE PAST VALUES

Output of 3-Point Running-Average Filter



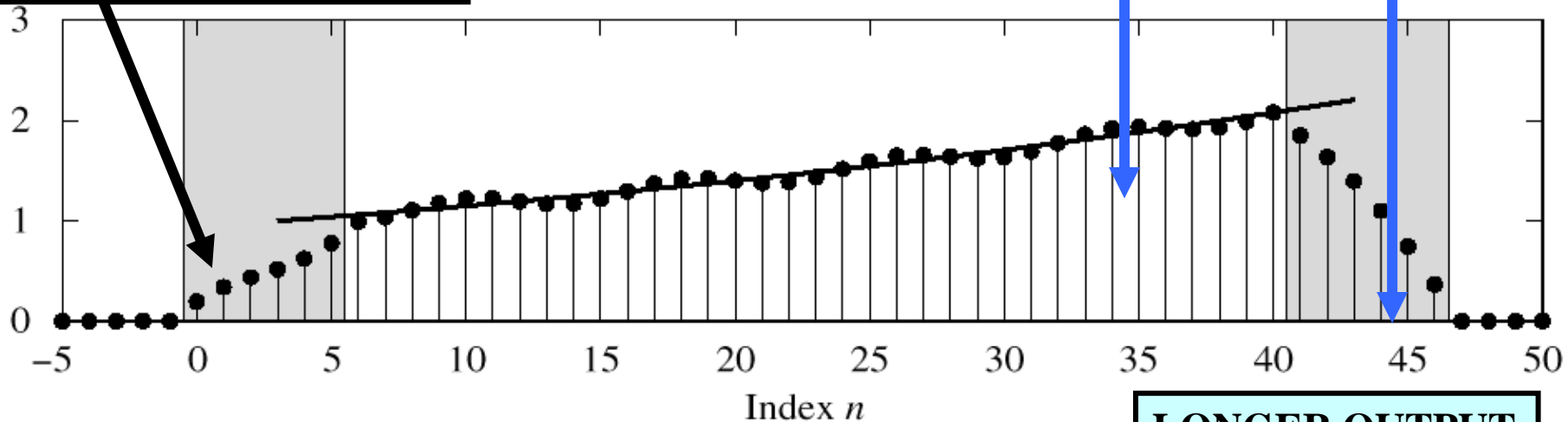
7-pt FIR EXAMPLE (AVG)

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$



CAUSAL: Use Previous

Output of 7-Point Running-Average Filter



LONGER OUTPUT

Linearity & Time-Invariance, Convolution

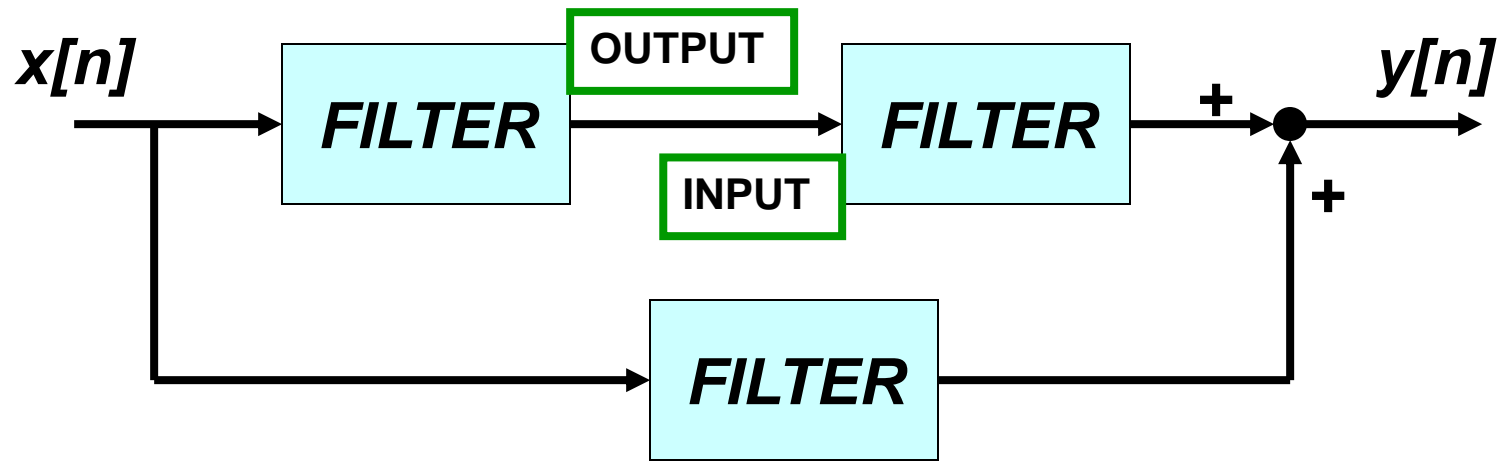
- IMPULSE RESPONSE, $h[n]$
 - FIR case: same as $\{b_k\}$
- CONVOLUTION
 - GENERAL: $y[n] = h[n] * x[n]$
 - GENERAL CLASS of SYSTEMS
 - LINEAR and TIME-INVARIANT
- ALL LTI systems have $h[n]$ & use convolution

DIGITAL FILTERING



- CONCENTRATE on the FILTER (DSP)
- DISCRETE-TIME SIGNALS
 - FUNCTIONS of n , the “time index”
 - INPUT $x[n]$
 - OUTPUT $y[n]$

BUILDING BLOCKS

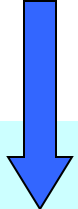


- BUILD UP COMPLICATED FILTERS
 - FROM SIMPLE MODULES
 - Ex: FILTER MODULE MIGHT BE 3-pt FIR

GENERAL FIR FILTER


- FILTER COEFFICIENTS $\{b_k\}$

– DEFINE THE FILTER


$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

– For example,

$$b_k = \{3, -1, 2, 1\}$$


$$y[n] = \sum_{k=0}^3 b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

MATLAB for FIR FILTER

- **yy = conv(bb, xx)**
 - VECTOR **bb** contains Filter Coefficients
 - <https://www.mathworks.com/help/matlab/ref/conv.html>
- FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^M b_k x[n - k]$$

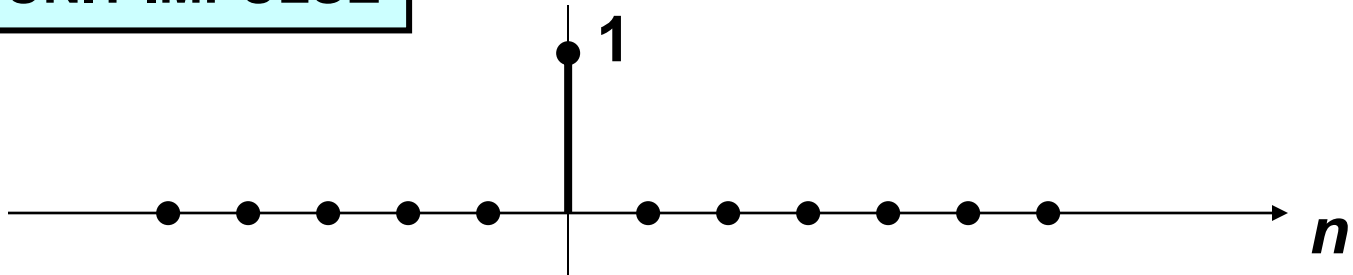
conv2 ()
for images

SPECIAL INPUT SIGNALS

- $x[n] = \text{SINUSOID}$ **FREQUENCY RESPONSE**
- $x[n]$ has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



FIR IMPULSE RESPONSE

- Convolution = Filter Definition
 - Filter Coeffs = Impulse Response

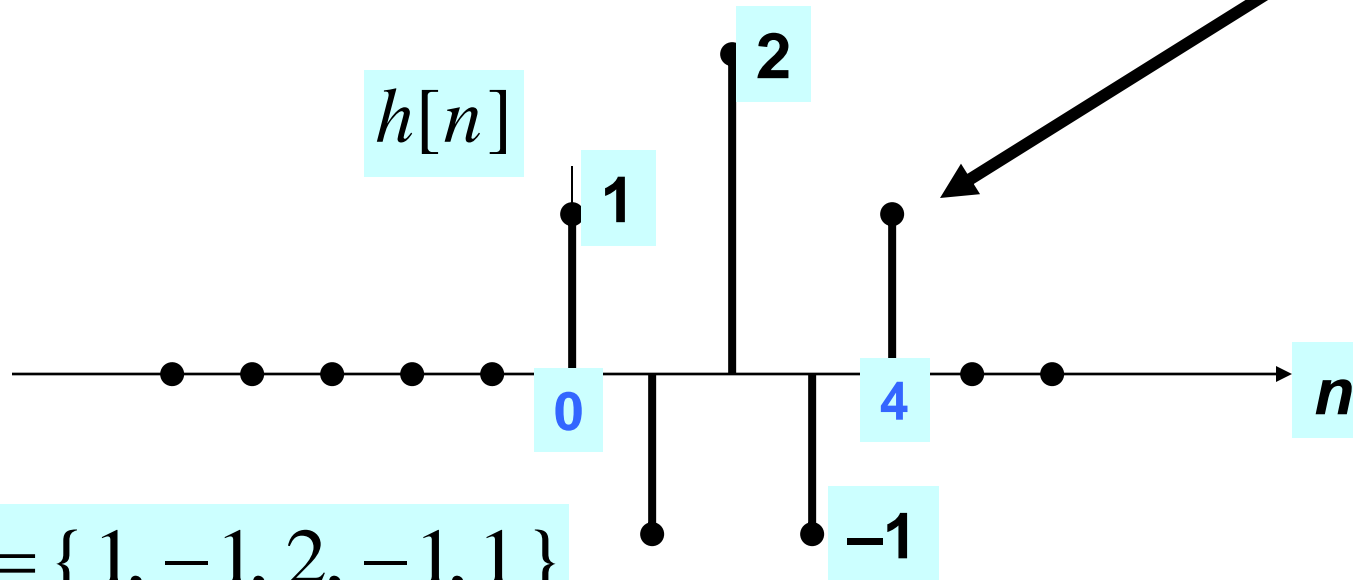
n	$n < 0$	0	1	2	3	...	M	$M + 1$	$n > M + 1$
$x[n] = \delta[n]$	0	1	0	0	0	0	0	0	0
$y[n] = h[n]$	0	b_0	b_1	b_2	b_3	...	b_M	0	0

$$h[n] = \sum_{k=0}^M b_k \delta[n - k]$$

MATH FORMULA for $h[n]$

- Use **SHIFTED** IMPULSES to write $h[n]$

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$



$$b_k = \{ 1, -1, 2, -1, 1 \}$$

LTI: Convolution Sum

- **Output = Convolution of $x[n]$ & $h[n]$**
 - **NOTATION:** $y[n] = h[n] * x[n]$
 - Here is the FIR case:

The diagram shows the FIR convolution sum equation $y[n] = \sum_{k=0}^M h[k]x[n-k]$ centered on a light blue background. Three callout boxes with black borders and light blue backgrounds point to specific parts of the equation: 'FINITE LIMITS' points to the upper limit M , another 'FINITE LIMITS' points to the lower limit $k=0$, and 'Same as b_k ' points to the coefficient $h[k]$. A blue curved arrow connects the 'Same as b_k ' box to the $k=0$ limit.

$$y[n] = \sum_{k=0}^M h[k]x[n-k]$$

FINITE LIMITS

FINITE LIMITS

Same as b_k

CONVOLUTION Example

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$

$$x[n] = u[n]$$

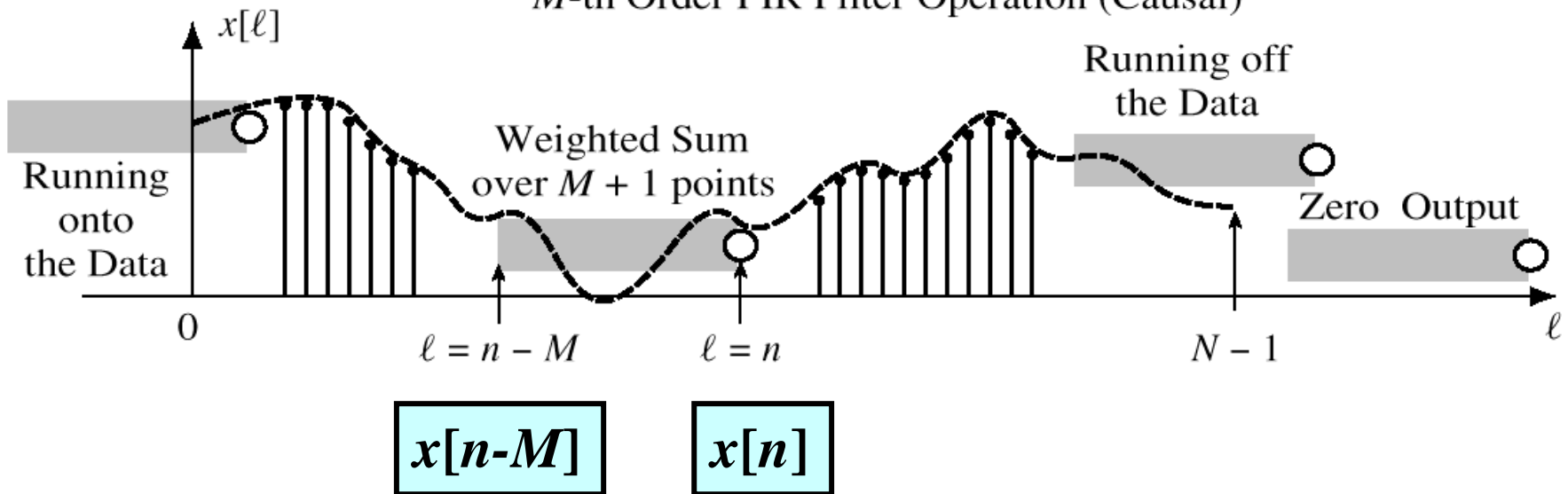
n	-1	0	1	2	3	4	5	6	7
$x[n]$	0	1	1	1	1	1	1	1	...
$h[n]$	0	1	-1	2	-1	1	0	0	0
$h[0]x[n]$	0	1	1	1	1	1	1	1	1
$h[1]x[n-1]$	0	0	-1	-1	-1	-1	-1	-1	-1
$h[2]x[n-2]$	0	0	0	2	2	2	2	2	2
$h[3]x[n-3]$	0	0	0	0	-1	-1	-1	-1	-1
$h[4]x[n-4]$	0	0	0	0	0	1	1	1	1
$y[n]$	0	1	0	2	1	2	2	2	...

GENERAL FIR FILTER

- SLIDE a Length-L WINDOW over $x[n]$

$$y[n] = \sum_{k=0}^M b_k x[n - k]$$

M -th Order FIR Filter Operation (Causal)



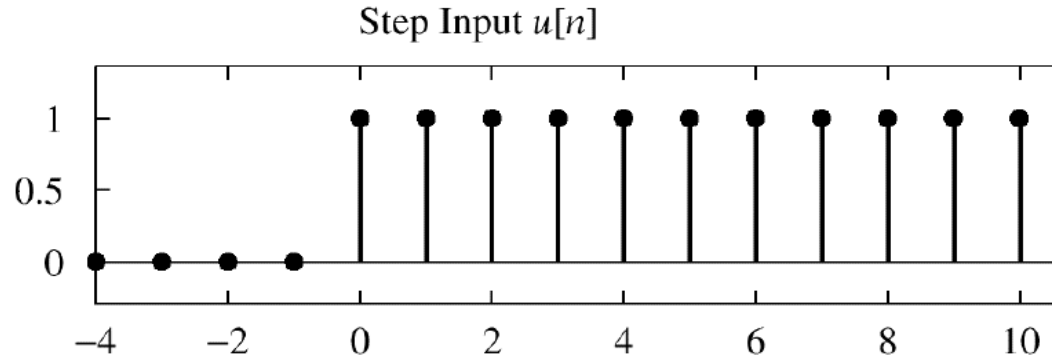
POP QUIZ

- FIR Filter is “FIRST DIFFERENCE”

$$y[n] = x[n] - x[n-1]$$

- INPUT is “UNIT STEP”

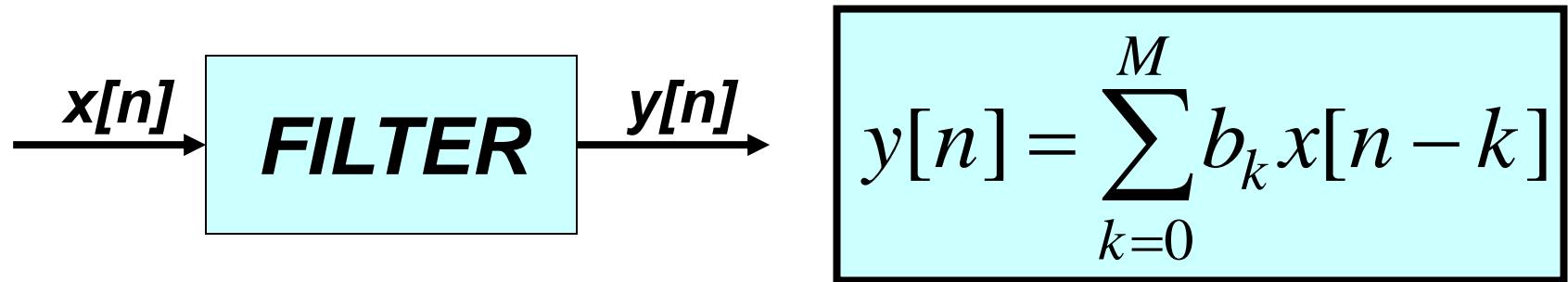
$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



- Find $y[n]$

$$y[n] = u[n] - u[n-1] = \delta[n]$$

HARDWARE STRUCTURES

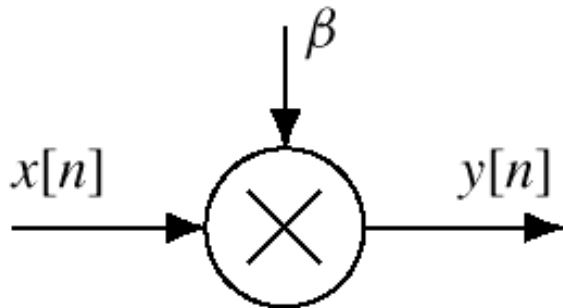


- INTERNAL STRUCTURE of “FILTER”
 - WHAT COMPONENTS ARE NEEDED?
 - HOW DO WE “HOOK” THEM TOGETHER?
- SIGNAL FLOW GRAPH NOTATION

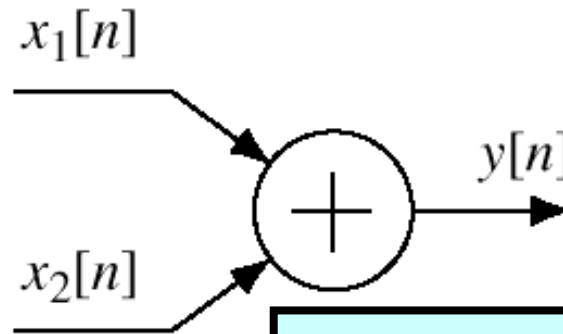
HARDWARE ATOMS

- Add, Multiply & Store

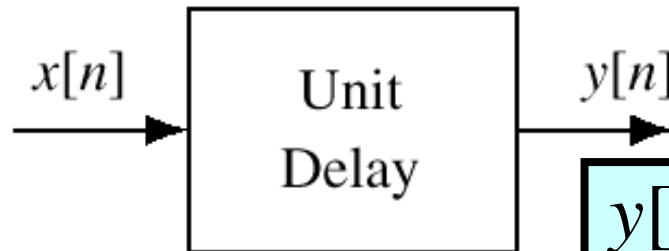
$$y[n] = \sum_{k=0}^M b_k x[n-k]$$



$$y[n] = \beta x[n]$$



$$y[n] = x_1[n] + x_2[n]$$



$$y[n] = x[n-1]$$

FIR STRUCTURE

- Direct Form

**SIGNAL
FLOW GRAPH**

$$y[n] = \sum_{k=0}^M b_k x[n - k]$$

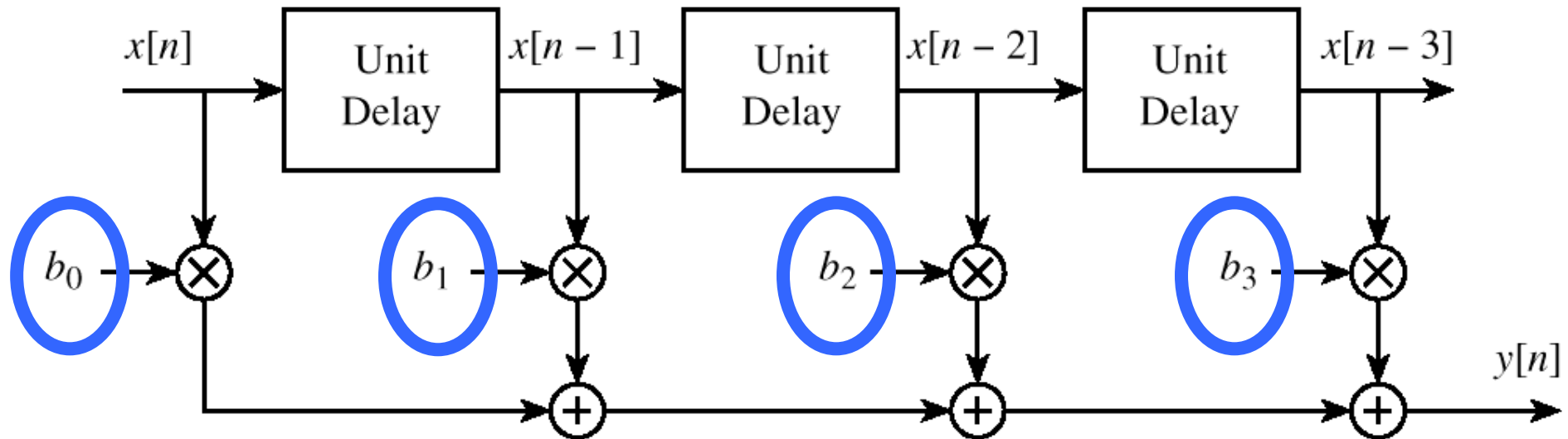
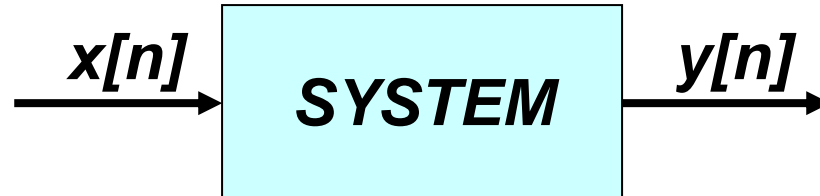


Figure 5.13 Block-diagram structure for the M th order FIR filter.

SYSTEM PROPERTIES



- MATHEMATICAL DESCRIPTION
- TIME-INVARIANCE
- LINEARITY
- CAUSALITY
 - “No output prior to input”

TIME-INVARIANCE

- IDEA:
 - “Time-Shifting the input will cause the same time-shift in the output”
- EQUIVALENTLY,
 - We can prove that
 - The time origin ($n=0$) is picked arbitrary

TESTING Time-Invariance

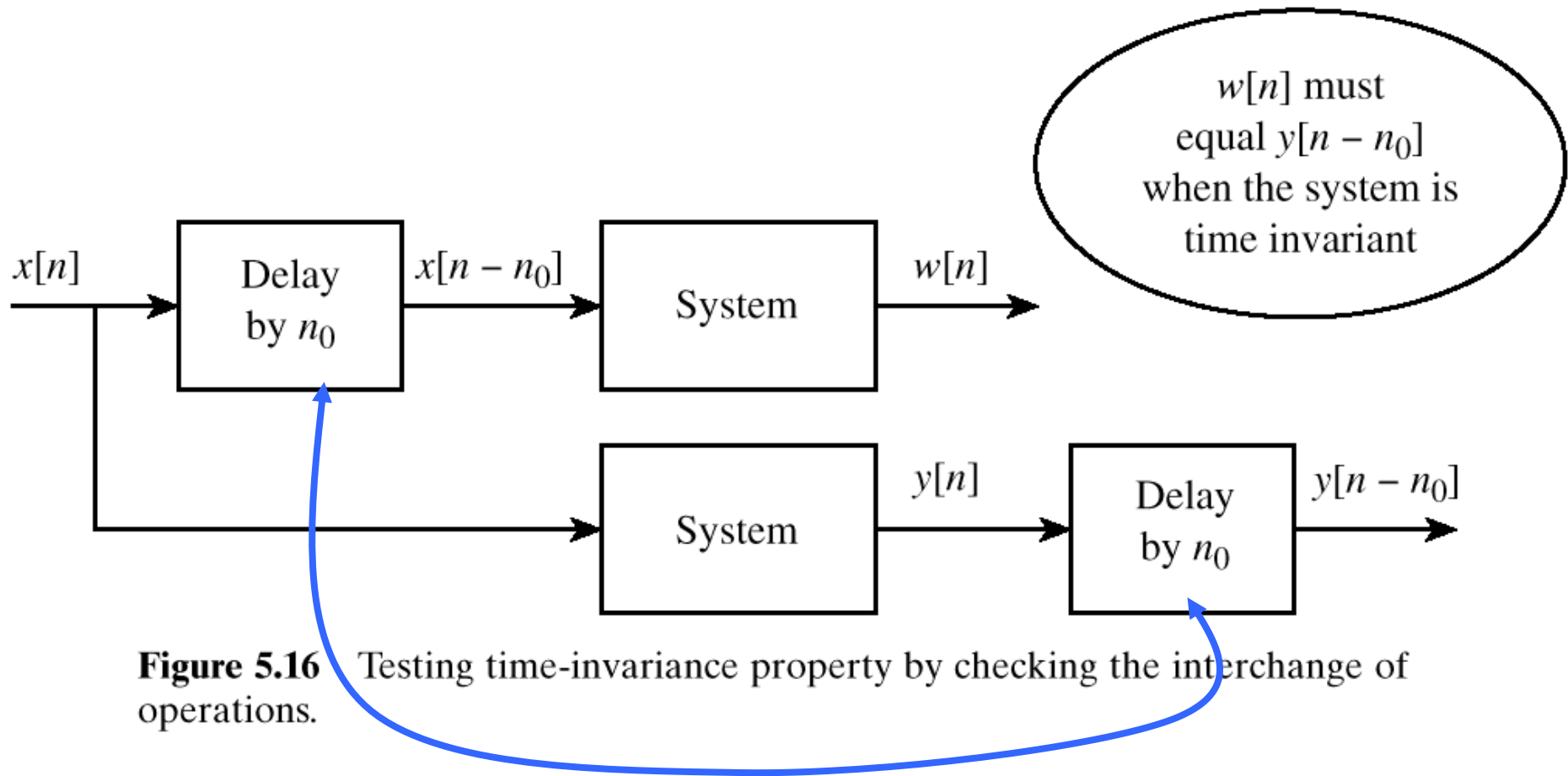


Figure 5.16 Testing time-invariance property by checking the interchange of operations.

LINEAR SYSTEM

- LINEARITY = Two Properties
- SCALING
 - “Doubling $x[n]$ will double $y[n]$ ”
- SUPERPOSITION:
 - “Adding two inputs gives an output that is the sum of the individual outputs”

TESTING LINEARITY

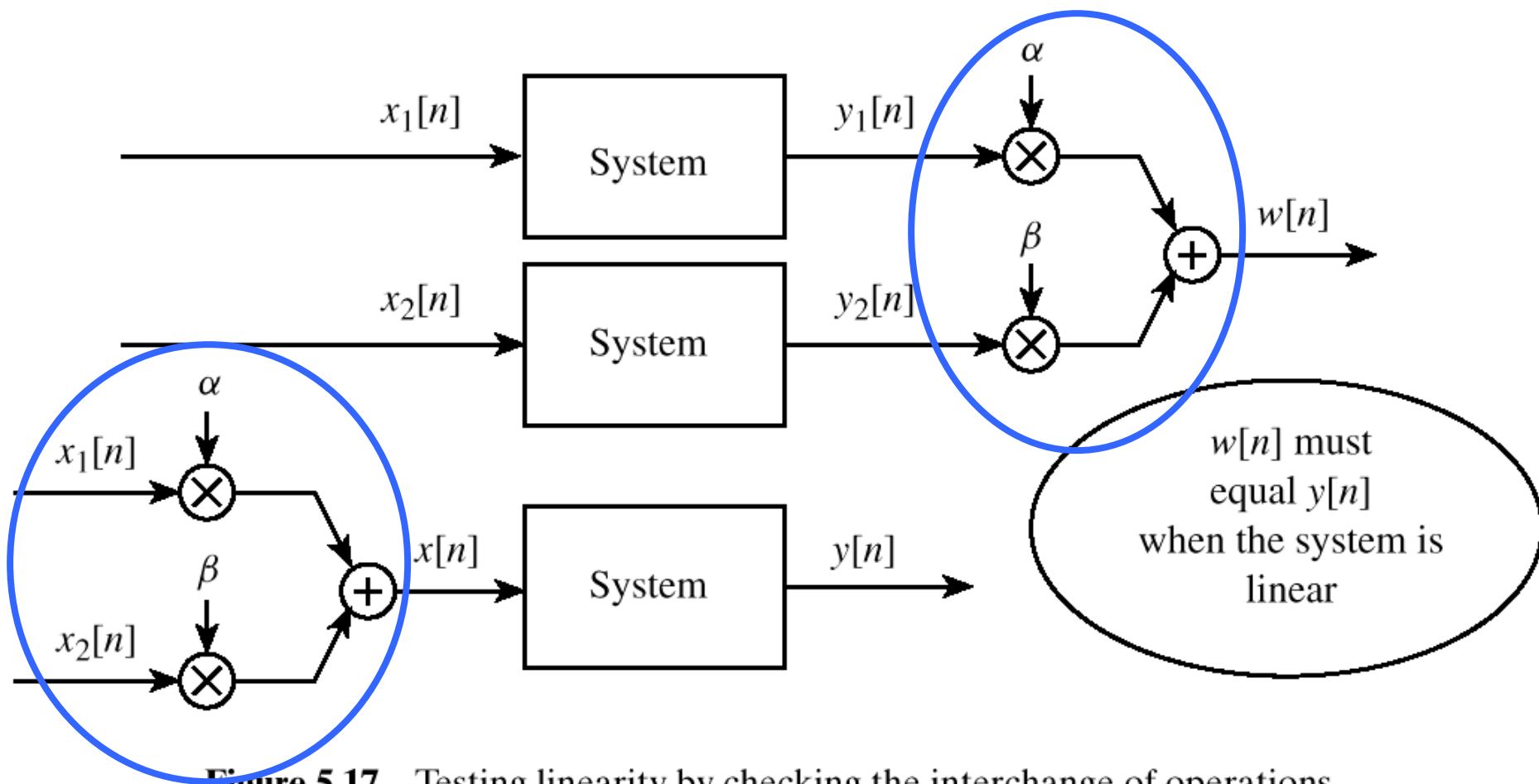


Figure 5.17 Testing linearity by checking the interchange of operations.

LTI SYSTEMS

- LTI: **L**inear & **T**ime-**I**nvariant
- COMPLETELY CHARACTERIZED by:
 - IMPULSE RESPONSE $h[n]$
 - CONVOLUTION: $y[n] = x[n] * h[n]$
 - The “rule” defining the system can ALWAYS be re-written as convolution
- FIR Example: $h[n]$ is same as b_k

POP QUIZ

- FIR Filter is “FIRST DIFFERENCE”
 - $y[n] = x[n] - x[n - 1]$
- Write output as a convolution
 - Need impulse response

$$h[n] = \delta[n] - \delta[n - 1]$$

- Then, another way to compute the output:

$$y[n] = (\delta[n] - \delta[n - 1]) * x[n]$$

CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
 - NO, LTI SYSTEMS can be rearranged !!!
 - WHAT ARE THE FILTER COEFFS? $\{b_k\}$

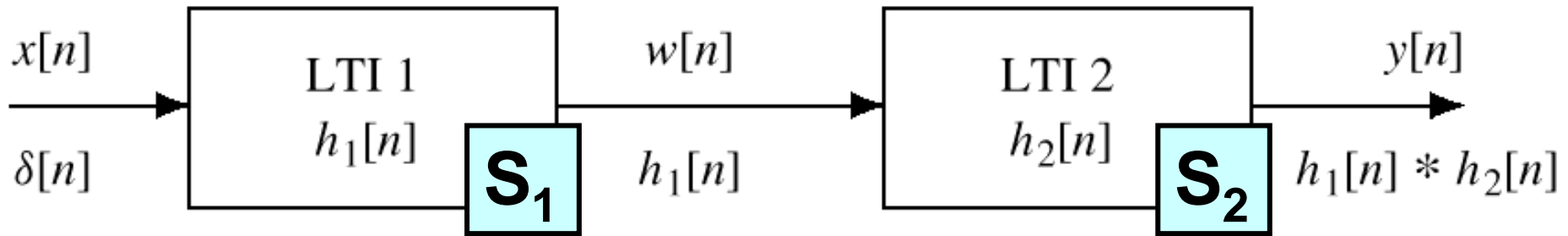


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

– Find “overall” $h[n]$ for a cascade ?

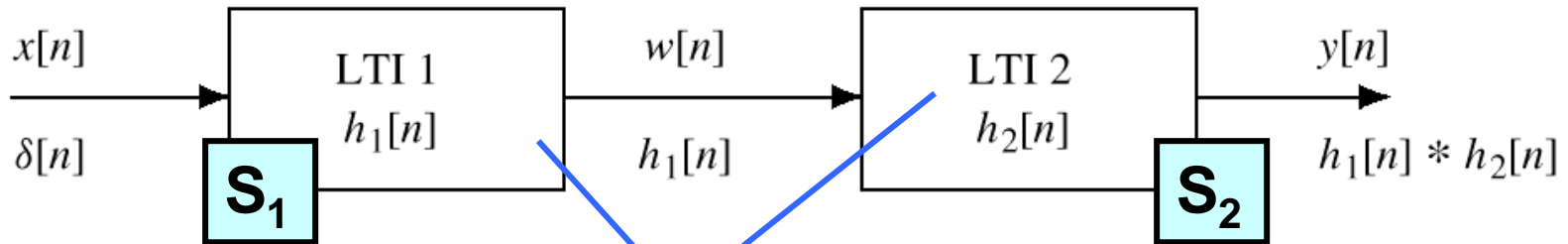


Figure 5.19 A Cascade of Two LTI Systems.

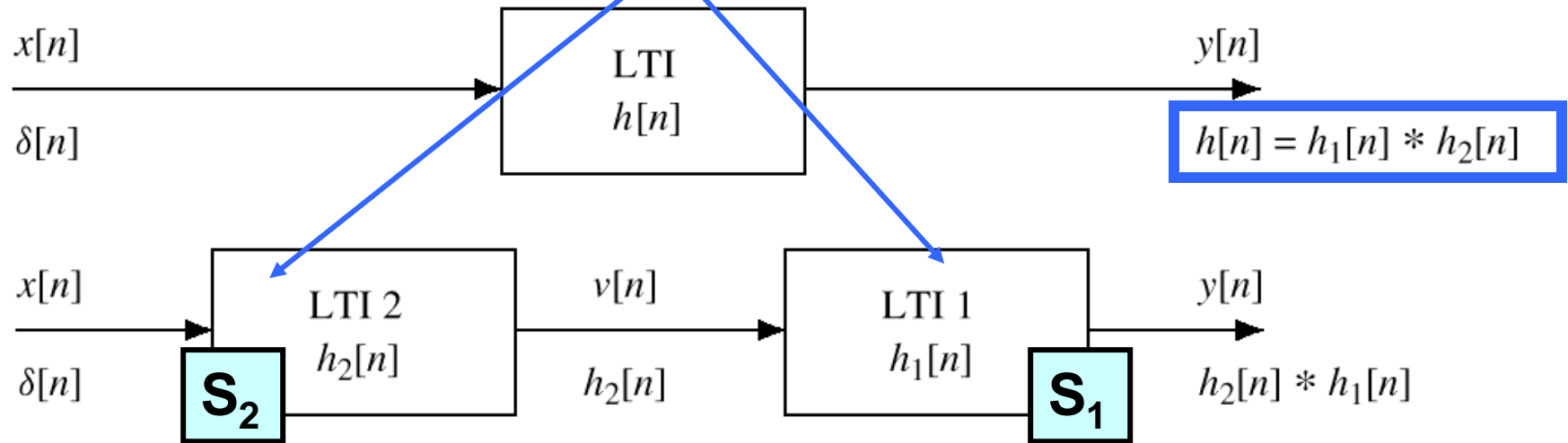


Figure 5.20 Switching the order of cascaded LTI systems.

DOMAINS: Time & Frequency

- **Time-Domain: “ n ” = time**
 - $x[n]$ discrete-time signal
 - $x(t)$ continuous-time signal
- **Frequency Domain (sum of sinusoids)**
 - Spectrum vs. f (Hz)
 - ANALOG vs. DIGITAL
 - Spectrum vs. ω -hat
- Move back and forth QUICKLY

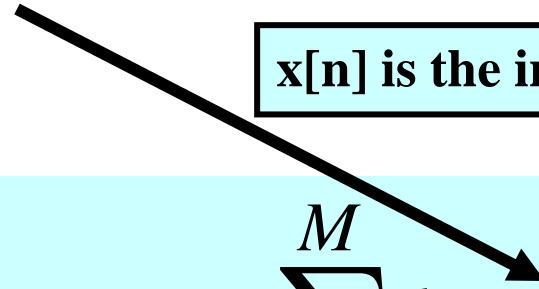
FREQUENCY RESPONSE

- INPUT: $x[n] = \text{SINUSOID}$
- OUTPUT: $y[n]$ will also be a SINUSOID
 - Different Amplitude and Phase
 - SAME Frequency
- AMPLITUDE & PHASE CHANGE
 - Called the FREQUENCY RESPONSE

COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\varphi} e^{j\hat{\omega}n} \quad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential


$$y[n] = \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M h[k] x[n-k]$$

FIR DIFFERENCE EQUATION

COMPLEX EXP OUTPUT

- Use the FIR “Difference Equation”

$$y[n] = \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M b_k A e^{j\varphi} e^{j\hat{\omega}(n-k)}$$

$$= \left(\sum_{k=0}^M b_k e^{j\hat{\omega}(-k)} \right) A e^{j\varphi} e^{j\hat{\omega}n}$$

$$= H(\hat{\omega}) A e^{j\varphi} e^{j\hat{\omega}n}$$

FREQUENCY RESPONSE

- At each frequency, we can **DEFINE**

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

FREQUENCY RESPONSE

- Complex-valued formula
 - Has **MAGNITUDE** vs. frequency
 - And **PHASE** vs. frequency
- Notation: $H(e^{j\hat{\omega}})$ in place of $H(\hat{\omega})$

EXAMPLE 6.1

$$\{b_k\} = \{1, 2, 1\}$$

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\ &= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) \\ &= e^{-j\hat{\omega}}(2 + 2\cos \hat{\omega}) \end{aligned}$$

**EXPLOIT
SYMMETRY**



Since $(2 + 2\cos \hat{\omega}) \geq 0$

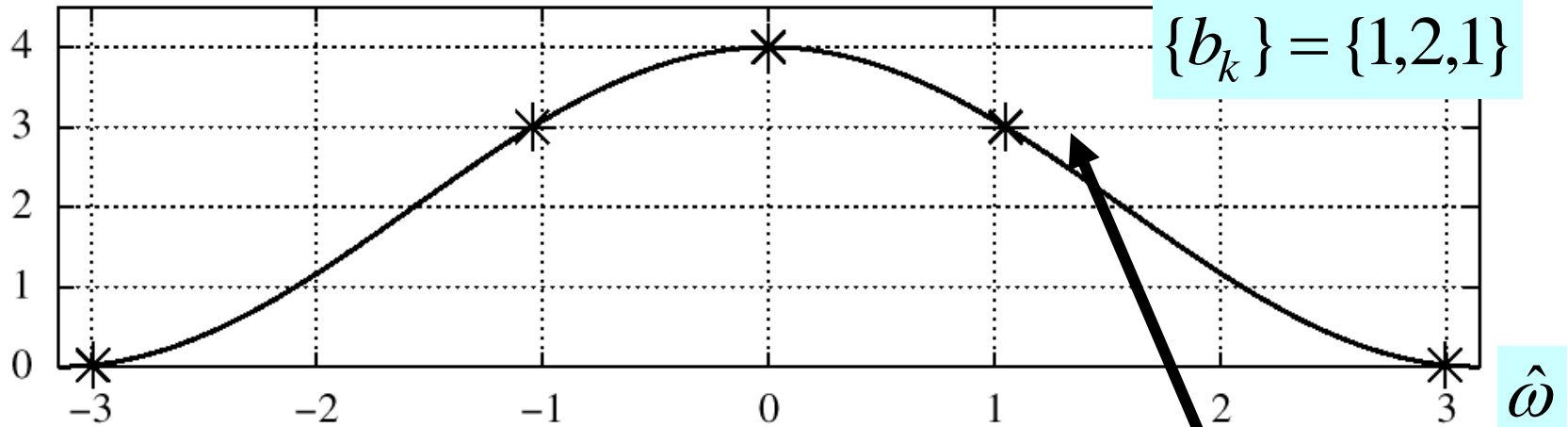
Magnitude is $|H(e^{j\hat{\omega}})| = (2 + 2\cos \hat{\omega})$

and Phase is $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$

PLOT of FREQ RESPONSE

Magnitude of Frequency Response of FIR Filter with Coefficients [1, 2, 1]

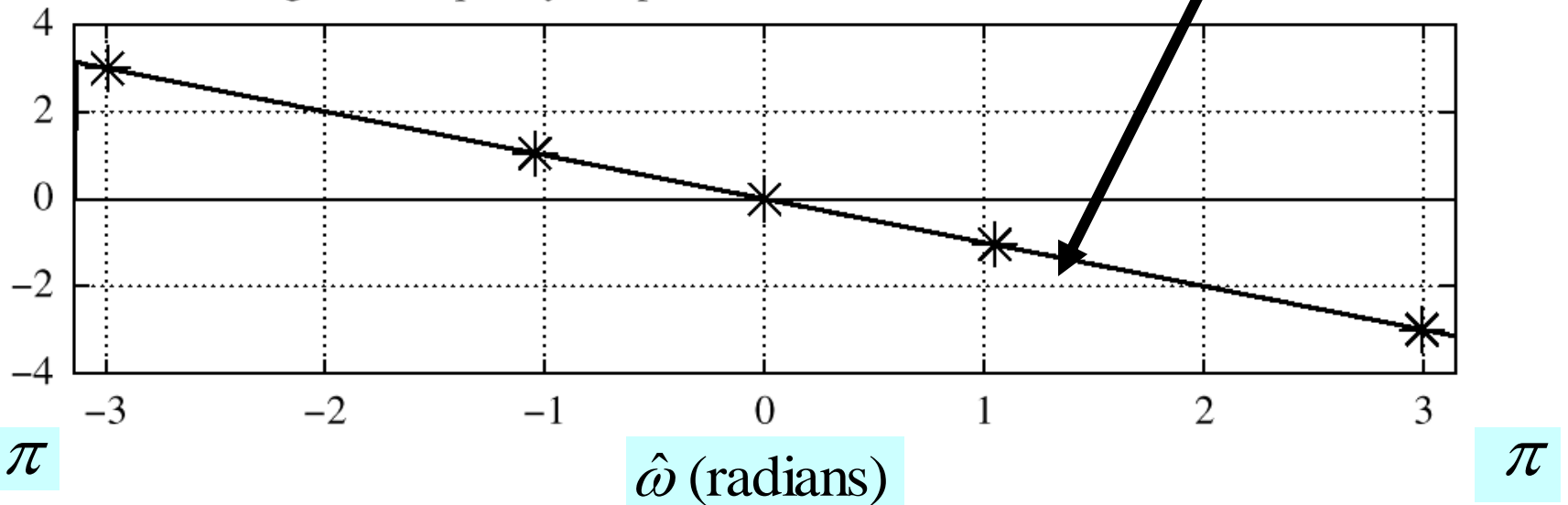
$$\{b_k\} = \{1, 2, 1\}$$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos \hat{\omega})e^{-j\hat{\omega}}$$

RESPONSE at $\pi/3$

Phase Angle of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



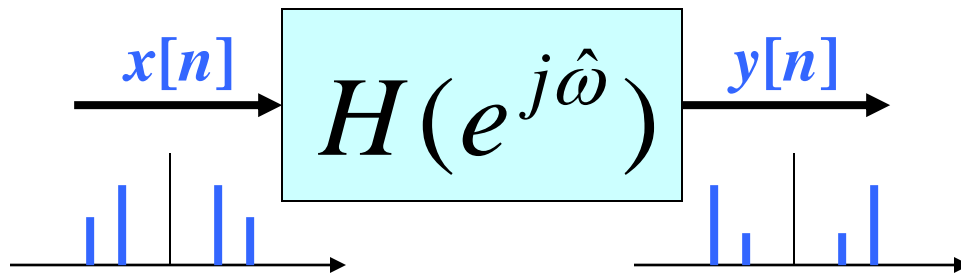
$-\pi$

$\hat{\omega}$ (radians)

π

EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known
and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos \hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

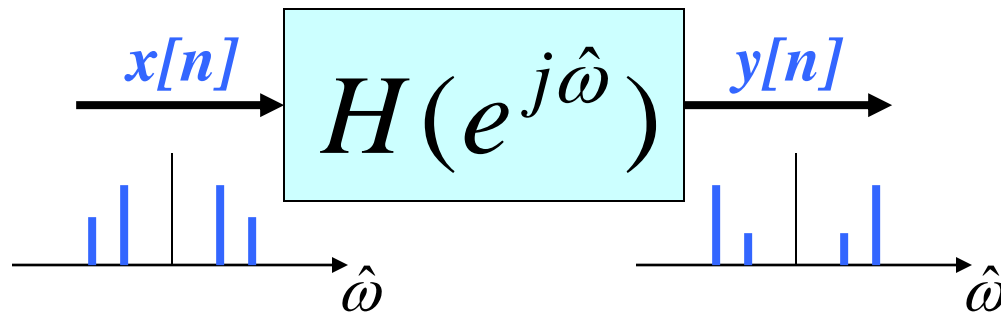
$$H(e^{j\hat{\omega}}) = (2 + 2\cos \hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @ \hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4} e^{j(\pi/3)n} = 6e^{-j\pi/12} e^{j(\pi/3)n}$$

EXAMPLE: COSINE INPUT

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known
and $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

EX: COSINE INPUT

Find $y[n]$ when $x[n] = 2 \cos(\frac{\pi}{3} n + \frac{\pi}{4})$

$$2 \cos(\frac{\pi}{3} n + \frac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

Use
Linearity

$$y_1[n] = H(e^{j\pi/3}) e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3}) e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

EX: COSINE INPUT (ans-2)

Find $y[n]$ when $x[n] = 2 \cos(\frac{\pi}{3} n + \frac{\pi}{4})$

$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega}) e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3}) e^{j(\pi n/3 + \pi/4)} = 3e^{-j(\pi/3)} e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3}) e^{-j(\pi n/3 + \pi/4)} = 3e^{j(\pi/3)} e^{-j(\pi n/3 + \pi/4)}$$

$$y[n] = 3e^{j(\pi n/3 - \pi/12)} + 3e^{-j(\pi n/3 - \pi/12)}$$

$$\Rightarrow y[n] = 6 \cos(\frac{\pi}{3} n - \frac{\pi}{12})$$

MATLAB: FREQUENCY RESPONSE

- `HH = freqz(bb, 1, ww)`

- VECTOR `bb` contains Filter Coefficients

- <https://www.mathworks.com/help/signal/ref/freqz.html>

- FILTER COEFFICIENTS $\{b_k\}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

Time & Frequency Relation

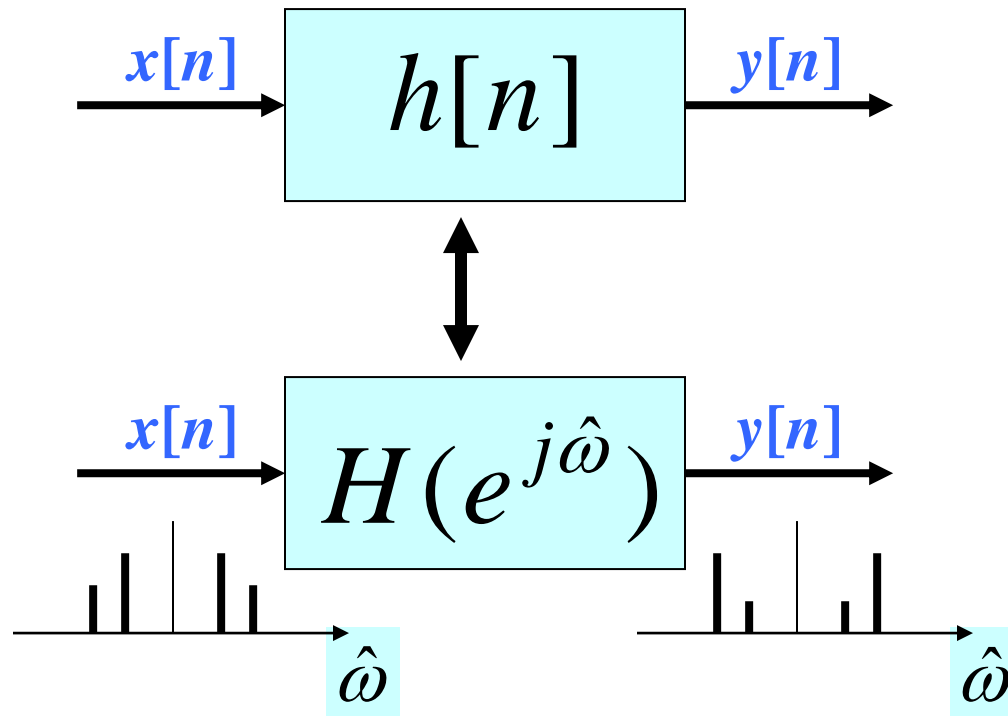
- Get Frequency Response from $h[n]$
 - Here is the FIR case:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k} = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k}$$

IMPULSE RESPONSE

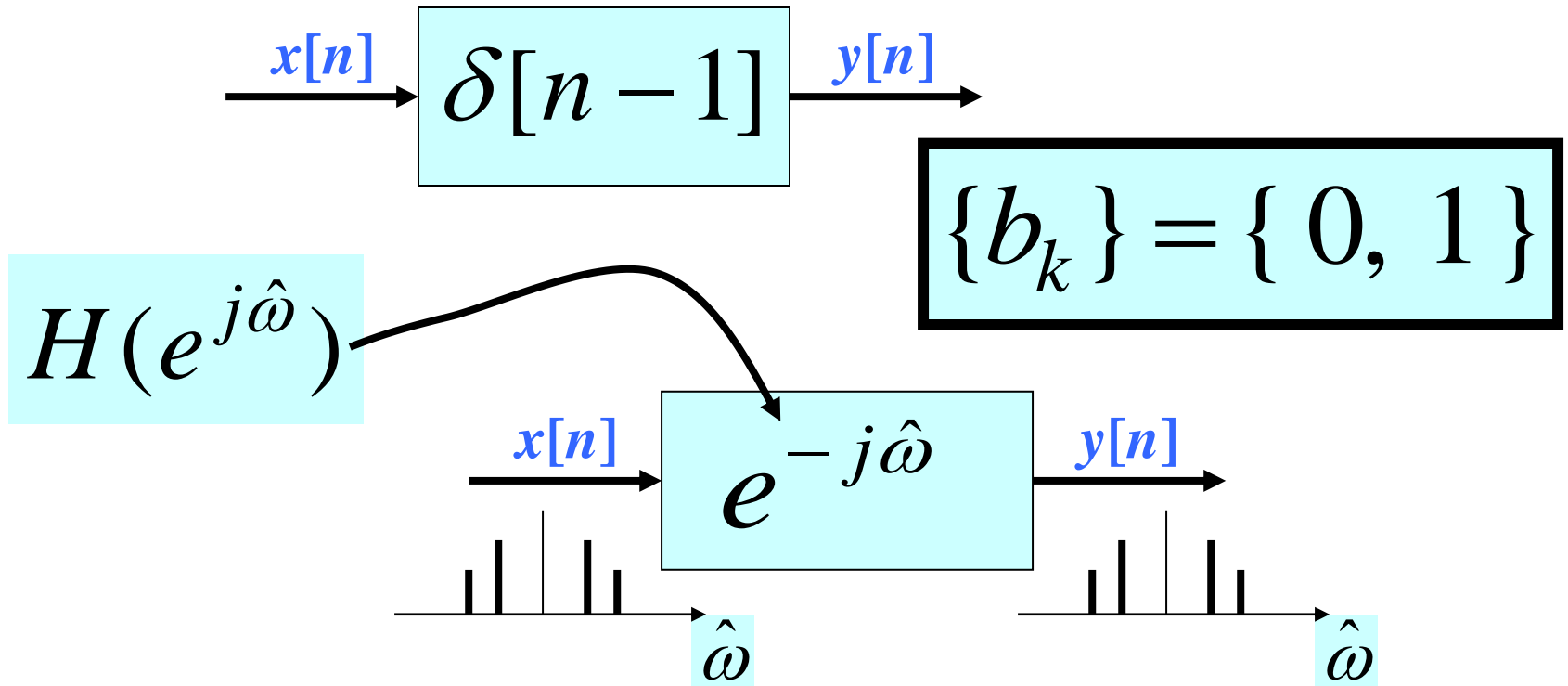
BLOCK DIAGRAMS

- Equivalent Representations



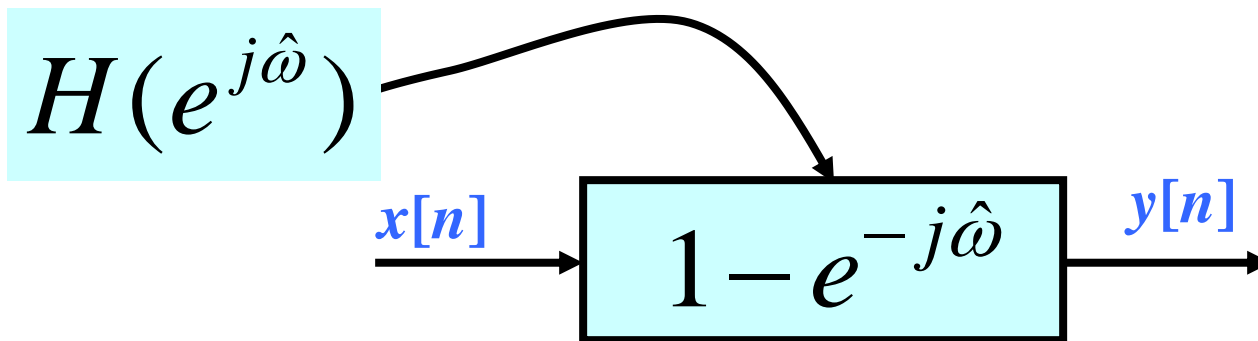
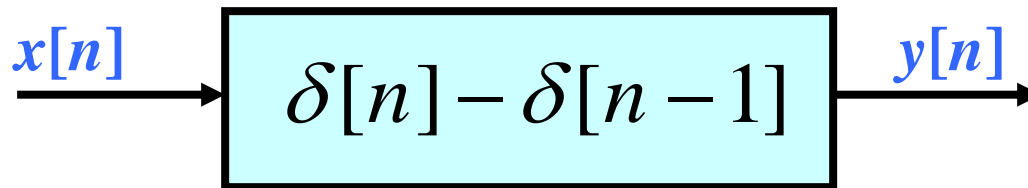
UNIT-DELAY SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n - 1]$



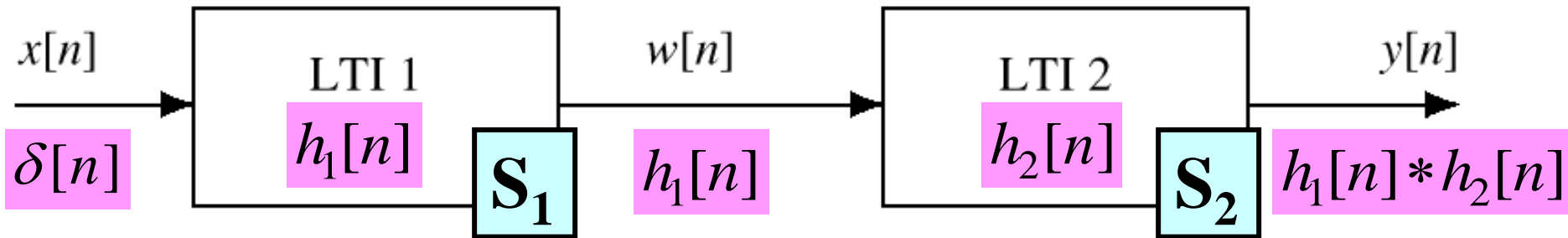
FIRST DIFFERENCE SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for the Difference Equation : $y[n] = x[n] - x[n - 1]$



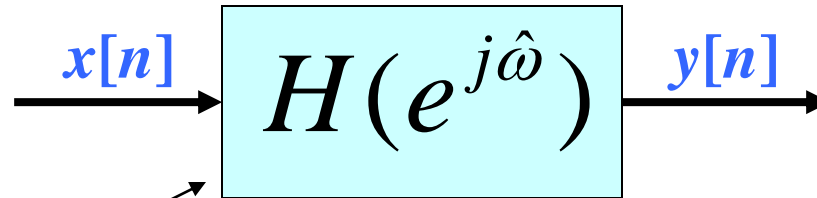
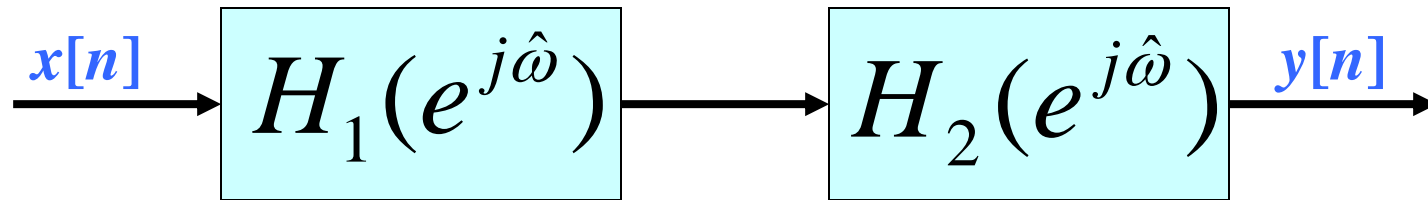
CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
 - NO, LTI SYSTEMS can be rearranged !!!
 - WHAT ARE THE FILTER COEFFS? $\{b_k\}$
 - WHAT is the overall FREQUENCY RESPONSE ?



CASCADE EQUIVALENT

- MULTIPLY the Frequency Responses



**EQUIVALENT
SYSTEM**

$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$$