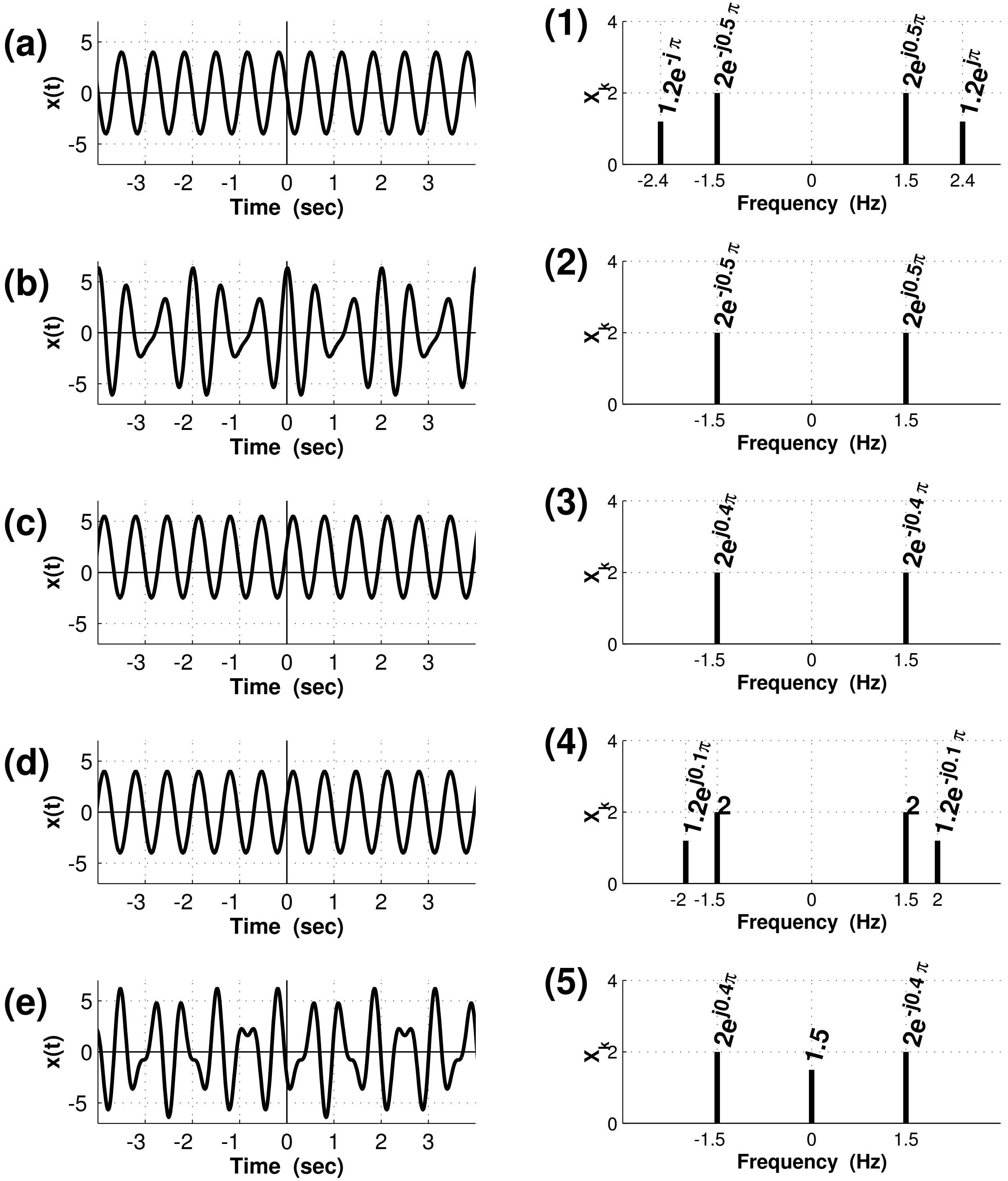
PROBLEM:

The following plots show waveforms on the left and spectra on the right. Hand in a table matching the waveform letter with its corresponding spectrum number.



PROBLEM:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly.*

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

There are examples on the CD-ROM in the Chapter 3 demos.

(a) For the linear-FM "chirp" in (1), determine formulas for the beginning instantaneous frequency (ω_1) and the ending instantaneous frequency (ω_2) in terms of α, β and T_2. For this problem, assume that the starting time of the "chirp" is $t = 0$.

(b) For the "chirp" signal

$$x(t) = \Re e\left\{e^{j2\pi(30t^2 - 30t)}\right\}$$

derive a formula for the *instantaneous* frequency versus time. Should your answer for the frequency be a positive number?

(c) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 1$ sec.

PROBLEM:

A signal $x(t)$ is periodic with period $T_0 = 8$. Therefore it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/8)kt}.$$

It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{8} \int_{-4}^{0} (4+t) e^{-j(2\pi/8)kt} dt. \tag{1}$$

(a) In the expression for a_k in Equation (1) above, the integral and its limits define the signal $x(t)$. Determine an equation for $x(t)$ that is valid over one period.

(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-10 \le t \le 10$ seconds. Label it carefully.

(c) Determine a_0, the DC value of $x(t)$.

PROBLEM:

We have seen that a periodic signal $x(t)$ can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \tag{1}$$

where $\omega_0 = 2\pi/T_0 = 2\pi f_0$. It turns out that we can transform many operations on the signal into corresponding operations on the Fourier coefficients a_k. For example, suppose that we want to consider a new periodic signal $y(t) = \frac{dx(t)}{dt}$. What would the Fourier coefficients be for $y(t)$? To see this, we simply need to differentiate the Fourier series representation; i.e.,

$$y(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right] = \sum_{k=-\infty}^{\infty} a_k \frac{d}{dt}\left[e^{jk\omega_0 t}\right] = \sum_{k=-\infty}^{\infty} a_k \left[(jk\omega_0)e^{jk\omega_0 t}\right]. \tag{2}$$

Thus, we see that $y(t)$ is also in the Fourier series form

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}, \quad \text{where } b_k = (jk\omega_0)a_k$$

but in this case the Fourier series coefficients are related to the Fourier series coefficients of $x(t)$ by $b_k = (jk\omega_0)a_k$. This is a nice result because it allows us to find the Fourier coefficients *without* actually doing the differentiation of $x(t)$ and *without* doing any tedious evaluation of integrals to obtain the Fourier coefficients b_k. It is a *general* result that holds for every periodic signal and its derivative.

We can use this style of manipulation to obtain some other useful results for Fourier series. In each case below, use Equation (2) as the starting point and the given definition for $y(t)$ to express $y(t)$ as a Fourier series and then manipulate the equation so that you can pick off an expression for the Fourier coefficients b_k as a function of the original coefficients a_k.

(a) Suppose that $y(t) = Ax(t)$ where A is a real number; i.e., $y(t)$ is just a scaled version of $x(t)$. Show that the Fourier coefficients for $y(t)$ are $b_k = Aa_k$.

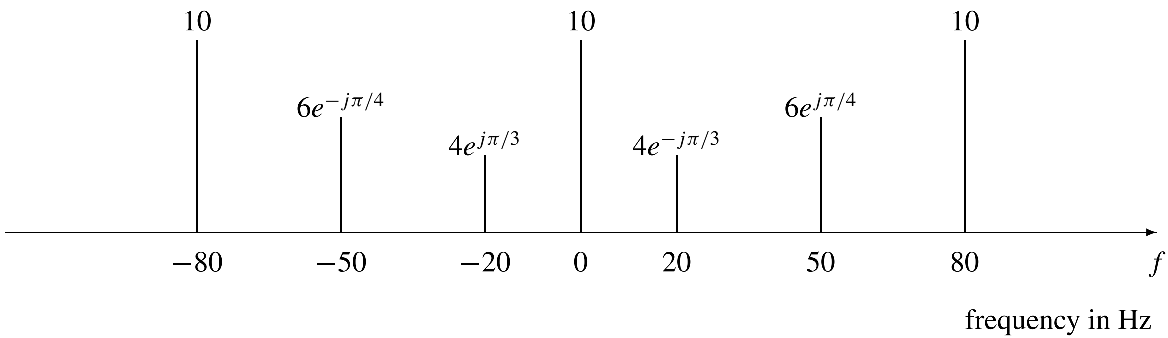
(b) Suppose that $y(t) = x(t - t_d)$ where t_d is a real number; i.e., $y(t)$ is just a delayed version of $x(t)$. Show that the Fourier coefficients for $y(t)$ in this case are $b_k = a_k e^{-jk\omega_0 t_d}$.

PROBLEM:

A real signal

$$x(t) = A\cos(40\pi t + \phi) + B\cos(\omega_1(t - \tau)) + C\cos(\omega_2 t) + D$$

has the following two-sided spectrum:



(a) Determine A, B, C, D, ω_1, ω_2, ϕ, and τ the signal $x(t)$ with the above spectrum.

$A =$___________

$B =$___________

$C =$___________

$D =$___________

$\phi =$___________

$\omega_1 =$___________

$\omega_2 =$___________

$\tau =$___________

(b) The signal $x(t)$ is periodic. Determine the fundamental frequency f_0, of the signal $x(t)$.

$f_0 =$___________

PROBLEM:

A signal $x(t)$ is given by the equation

$$x(t) = [A + \cos(40\pi t)]\cos(200\pi t - \pi/2).$$

The signal $x(t)$, which is given above as a *product*, can also be expressed as a *sum* of sinusoids of the form

$$x(t) = \sum_{k=1}^{N} D_k \cos(\omega_k t + \phi_k), \tag{1}$$

where the ω_k's are different freqeuncies.

(a) Determine the number of cosine terms in $x(t)$, i.e. the value of N in Equation (1).

$N =$ ________________

(b) What are the lowest and highest frequencies of all the sinusoids in the sum form [Eq. (1)] of $x(t)$?

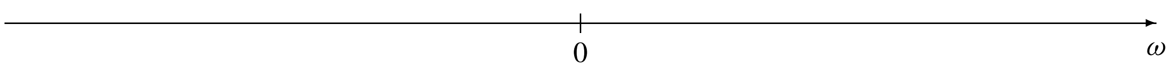
lowest $\omega_k =$ ________________

highest $\omega_k =$ ________________

(c) The spectrum of $x(t)$ contains a component at frequency 200π rad/sec with complex amplitude $-2j$. What is the numerical value of A?

$A =$________________

(d) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency (in radians/sec) and their complex amplitude. You may need to use your result from part (c) to label the plot properly.



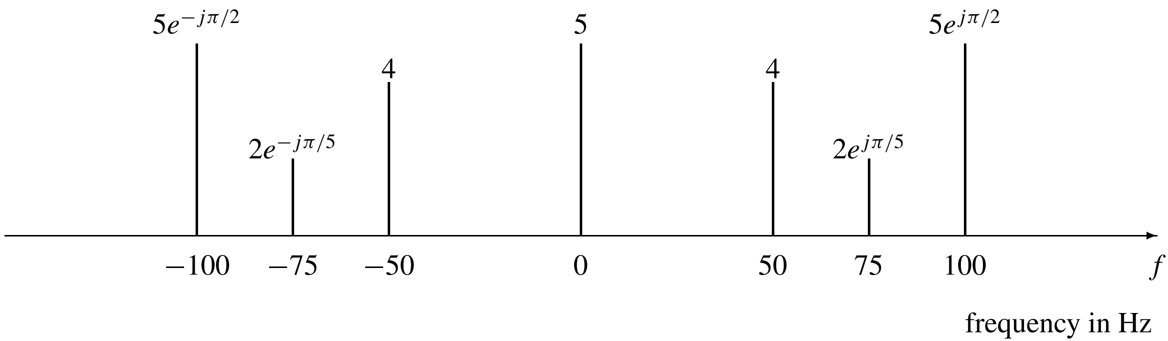
frequency in rad/sec

PROBLEM:

A real signal

$$x(t) = A\cos(200\pi t + \phi) + B\cos(\omega_1(t - \tau)) + C\cos(\omega_2 t) + D$$

has the following two-sided spectrum:



(a) Determine A, B, C, D, ω_1, ω_2, ϕ, and τ the signal $x(t)$ with the above spectrum.

$A =$____________

$B =$____________

$C =$____________

$D =$____________

$\phi =$____________

$\omega_1 =$____________

$\omega_2 =$____________

$\tau =$____________

(b) The signal $x(t)$ is periodic. Determine the fundamental frequency f_0, of the signal $x(t)$.

$f_0 =$____________

PROBLEM:

A signal $x(t)$ is given by the equation

$$x(t) = 4[A + \cos(300\pi t)]\cos(1000\pi t + \pi/2).$$

The signal $x(t)$, which is given above as a *product*, can also be expressed as a *sum* of sinusoids of the form

$$x(t) = \sum_{k=1}^{N} D_k \cos(\omega_k t + \phi_k), \tag{1}$$

where the ω_k's are different freqeuncies.

(a) Determine the number of cosine terms in $x(t)$, i.e. the value of N in Equation (1).

$N =$ _______________

(b) What are the lowest and highest frequencies of all the sinusoids in the sum form [Eq. (1)] of $x(t)$?

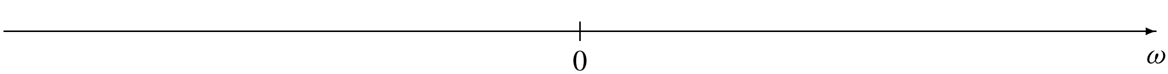
lowest $\omega_k =$ _______________

highest $\omega_k =$ _______________

(c) The spectrum of $x(t)$ contains a component at frequency 1000π rad/sec with complex amplitude $4j$. What is the numerical value of A?

$A =$_______________

(d) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency (in radians/sec) and their complex amplitude. You may need to use your result from part (c) to label the plot properly.



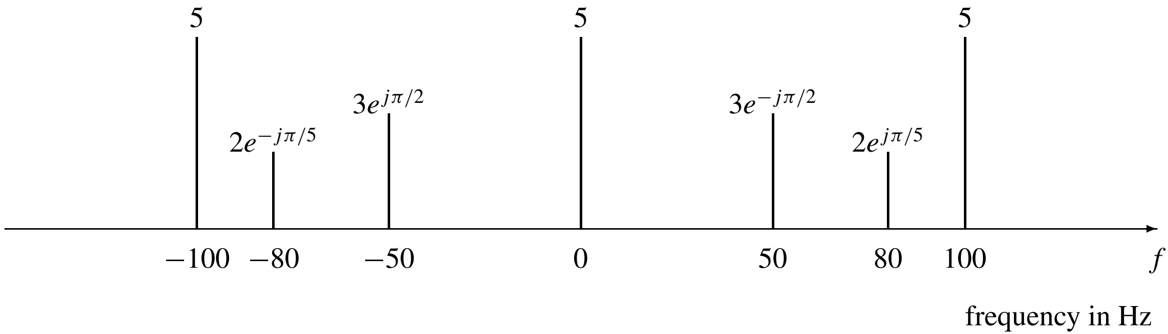
frequency in rad/sec

PROBLEM:

A real signal

$$x(t) = A\cos(160\pi t + \phi) + B\cos(\omega_1(t - \tau)) + C\cos(\omega_2 t) + D$$

has the following two-sided spectrum:



(a) Determine A, B, C, D, ω_1, ω_2, ϕ, and τ the signal $x(t)$ with the above spectrum.

$A =$____________

$B =$____________

$C =$____________

$D =$____________

$\phi =$____________

$\omega_1 =$____________

$\omega_2 =$____________

$\tau =$____________

(b) The signal $x(t)$ is periodic. Determine the fundamental frequency f_0, of the signal $x(t)$.

$f_0 =$____________

PROBLEM:

A signal $x(t)$ is given by the equation

$$x(t) = 2[A + \cos(200\pi t)]\cos(2000\pi t + \pi/2).$$

The signal $x(t)$, which is given above as a *product*, can also be expressed as a *sum* of sinusoids of the form

$$x(t) = \sum_{k=1}^{N} D_k \cos(\omega_k t + \phi_k), \tag{1}$$

where the ω_k's are different freqeuncies.

(a) Determine the number of cosine terms in $x(t)$, i.e. the value of N in Equation (1).

$N =$ _______________

(b) What are the lowest and highest frequencies of all the sinusoids in the sum form [Eq. (1)] of $x(t)$?

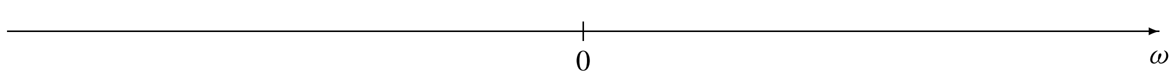
lowest $\omega_k =$ _______________

highest $\omega_k =$ _______________

(c) The spectrum of $x(t)$ contains a component at frequency 2000π rad/sec with complex amplitude $6j$. What is the numerical value of A?

$A =$_______________

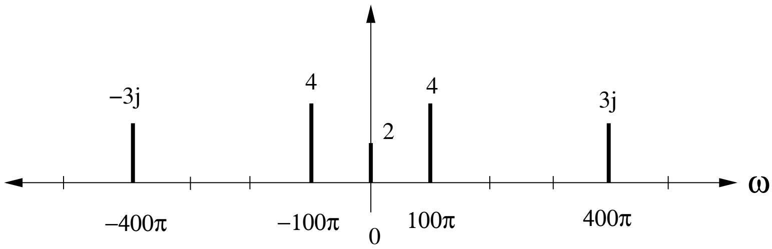
(d) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency (in radians/sec) and their complex amplitude. You may need to use your result from part (c) to label the plot properly.



frequency in rad/sec

PROBLEM:

Shown in the figure is a spectrum plot for the periodic signal $x(t)$.



(a) Determine the period T_0 of $x(t)$.

$T_0 =$

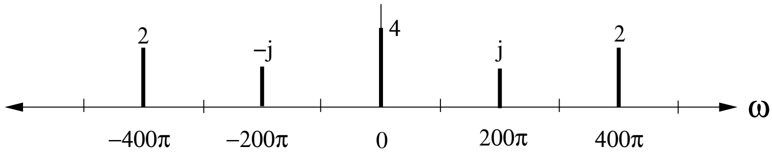
(b) A periodic signal of this type can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}.$$

If the Fourier series coefficients of $x(t)$ are denoted by a_k, $k = 0, \pm 1, \pm 2, \pm 3, \ldots$, determine which coefficients have non-zero value. List these Fourier series coefficients and their values below.

PROBLEM:

Shown in the figure is a spectrum plot for the periodic signal $x(t)$.



(a) Determine the period T_0 of $x(t)$.

$T_0 =$

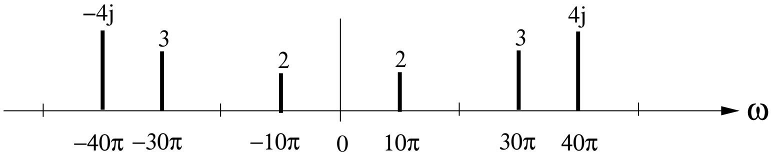
(b) A periodic signal of this type can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}.$$

If the Fourier series coefficients of $x(t)$ are denoted by a_k, $k = 0, \pm 1, \pm 2, \pm 3, \ldots$, determine which coefficients have non-zero value. List these Fourier series coefficients and their values below.

PROBLEM:

Shown in the figure is a spectrum plot for the periodic signal $x(t)$.



(a) Determine the period T_0 of $x(t)$.

$T_0 =$

(b) A periodic signal of this type can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}.$$

If the Fourier series coefficients of $x(t)$ are denoted by a_k, $k = 0, \pm 1, \pm 2, \pm 3, \ldots$, determine which coefficients have non-zero value. List these Fourier series coefficients and their values below.

PROBLEM:

The following MATLAB program makes a plot of a "sine-times-sine" signal:

```
tt = 0:0.01:3;
xx = sin(16*pi*tt) .* sin(pi*tt);
plot(tt,xx)
```

(a) Make a sketch of the plot that will be done by MATLAB. Label the time axis carefully.

(b) The "spectrum" diagram gives the frequency content of a signal. Draw a sketch of the spectrum of the signal represented by `xx`. Label the frequencies and complex amplitudes of each component.

PROBLEM:

The following MATLAB program makes a plot of a "sum-of-sines" signal:

```
ttt = 0:(1/1000):0.5;
xxx = sin(100*pi*ttt) +  sin(108*pi*ttt);
plot(ttt, xxx)
```

(a) Make a sketch of the plot that will be done by MATLAB. Label the time axis carefully.

(b) The "spectrum" diagram gives the frequency content of a signal. Draw a sketch of the spectrum of the signal represented by `xxx`. Label the frequencies and complex amplitudes of each component.

PROBLEM:

Express the following sinusoidal signal as the sum of four complex exponential terms–each a rotating phasor.

$$x(t) = 4\cos(33\pi t - \pi/4) + 2\sin(44\pi t)$$

In addition, determine the period of $x(t)$.

PROBLEM:

The following MATLAB program makes a plot of a "cosine-times-sine" signal:

```
tt = 0:0.01:2;
xc = cos(21*pi*tt);
xs = sin(3pi*tt);
xx = xc .* xs;
plot(tt,xx)
```

(a) Make a sketch of the plot that will be done by MATLAB. Label the time axis carefully.

(b) The "spectrum" diagram gives the frequency content of a signal. Draw a sketch of the spectrum for each of the three signals represented by `xc`, `xs` and `xx`. Label the frequencies and complex amplitudes of each component.

PROBLEM:

A chirp signal is synthesized according to the following formula:

$$x(t) = \Re e\{e^{j2\pi(500t^2 + 700t + 900)}\} \qquad \text{for } 0 \leq t \leq 3$$

(a) Derive the sinusoidal formula for $x(t)$.

(b) Determine the formula for the instantaneous frequency of the chirp.

(c) Make a plot of the instantaneous frequency versus time.

(d) Derive a formula similar to $x(t)$ for a chirp signal whose instantaneous frequency starts at 7 kHz and falls linearly to 3 kHz in 3 seconds.

PROBLEM:

A chirp signal is synthesized according to the following formula:

$$x(t) = \Re e\{e^{j600\pi t^2} \cos(1600\pi t)\} \qquad \text{for } 0 \leq t \leq 5$$

(a) Determine the formula for the instantaneous frequency of the chirp. Make a plot of the instantaneous frequency versus time.

(b) Derive a formula for a chirp signal whose instantaneous frequency starts at 3 kHz and falls linearly to 0 kHz in 2 seconds.

PROBLEM:

Circle the correct answer to each of these short answer questions:

1. A signal $x(t)$ is defined by: $x(t) = \Re e\{e^{j12\pi t} + e^{j21\pi t}\}$. Its fundamental frequency is:
   (a) $f_0 = 1.5$ Hz
   (b) $f_0 = 3\pi$ Hz
   (c) $f_0 = 6$ Hz
   (d) $f_0 = 21$ Hz
   (e) none of the above

2. A sinusoidal signal $x(t)$ is defined by: $x(t) = \Re e\{(1 + j)e^{j\pi t}\}$. When plotted versus time (t), a maximum value of $x(t)$ will be located at:
   (a) $t = 0$ sec.
   (b) $t = 1/4$ sec.
   (c) $t = 1$ sec.
   (d) $t = 7/4$ sec.
   (e) none of the above

3. Determine the amplitude (A) and phase (ϕ) of the sinusoid that is the sum of the following three sinusoids: $\cos(\pi t + \pi/2) + \cos(\pi t + \pi/4) + \cos(\pi t + 3\pi/4)$.
   (a) $A = 0$ and $\phi = 0$.
   (b) $A = 1$ and $\phi = \pi/2$.
   (c) $A = 1 + \sqrt{2}$ and $\phi = 0$.
   (d) $A = 1 + \sqrt{2}$ and $\phi = \pi/2$.
   (e) $A = 3$ and $\phi = \pi/2$.

4. Evaluate the complex number: $z = \sum_{k=0}^{4} e^{-j\pi k/2}$
   (a) $z = 0$
   (b) $z = j$
   (c) $z = -j$
   (d) $z = 1$
   (e) $z = -1$

PROBLEM:

A signal composed of sinusoids is given by the equation

$$x(t) = 2\cos(15t) + 3\cos(25t - \pi/4)$$

(a) Sketch the spectrum of this signal indicating the complex amplitude of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex amplitude value at the appropriate frequency.

(b) Define a new signal $w(t) = x(t - 0.1)$. Draw a carefully labelled sketch of the spectrum for $w(t)$.

PROBLEM:

In AM radio, the transmitted signal is voice (or music) mixed with a *carrier signal.* The carrier is a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta is 750 kHz. If we use the notation $v(t)$ to denote the voice/music signal, then the actual transmitted signal for WSB might be:

$$x(t) = (v(t) + A)\cos(2\pi(750 \times 10^3)t)$$

where A is a constant. (A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $v(t)$.)

(a) Voice-band signals tend to contain frequencies less than 4000 Hz. Suppose that $v(t)$ is a sinusoid, $v(t) = \sin(2\pi(1000)t)$. Draw the spectrum for $v(t)$.

(b) Now draw the spectrum for $x(t)$, assuming a carrier at 750 kHz. Use $v(t)$ from part (a).

(c) Music signals might contain frequencies as high as 20 kHz. Suppose that $v(t)$ is a higher frequency sinusoid, $v(t) = \sin(2\pi(13000)t)$. Draw the spectrum for $x(t)$, assuming a carrier at 750 kHz.

(d) If the carrier frequency is changed to 680 kHz (WCNN), describe in words how the spectrum will change for both of the cases above. In particular, describe the relationship between the spectrum for $v(t)$ and the carrier frequency. Use the spectra that you sketched in the previous parts as examples to support your explanation.

PROBLEM:

A signal composed of sinusoids is defined by an equation that uses the Fourier Series *synthesis* notation:

$$x(t) = \sum_{k=-3}^{3} jke^{j\pi kt}$$

(a) Sketch the spectrum of this signal indicating the complex amplitude of each frequency component. Label the complex amplitude value at the appropriate frequencies.

(b) Is $x(t)$ periodic? If so, what is the fundamental period?

(c) Determine the DC value of $x(t)$, i.e., the average value over one period: $\frac{1}{T_0}\int_0^{T_0} x(t)dt$.

PROBLEM:

A periodic signal $x(t) = x(t + T_0)$ is described over one period $-T_0/2 \le t \le T_0/2$ by the equation

$$x(t) = \begin{cases} 10 & |t| < t_c \\ -2 & t_c < |t| \le T_0/2 \end{cases}$$

where $t_c < T_0/2$. In this problem assume that $T_0 = 5$ and $t_c = 1$.

(a) Sketch the periodic function $x(t)$ for t in the range $-T_0 < t < 2T_0$.

(b) Determine the D.C. coefficient X_0 using the parameters $T_0 = 5$ and $t_c = 1$.

(c) Determine the *fundamental frequency* ω_0 in the Fourier Series representation (rad/sec).

(d) Use the Fourier $\boxed{\textit{analysis}}$ integral(for $k \neq 0$)

$$X_k = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t} dt$$

to determine a general formula for the Fourier coefficients X_k in the representation

$$x(t) = X_0 + \Re e \left\{ \sum_{k=1}^{\infty} X_k e^{jk\omega_0 t} \right\}$$

Your final result could depend on t_c and T_0, but use $t_c = 1$ and $T_0 = 5$.
Note: The integral can be done over any period of the signal; in this case, the most convenient choice is from $-T_0/2$ to $T_0/2$.

(e) Sketch the spectrum of $x(t)$ for the case $t_c = 1$ and $T_0 = 5$. Include the DC component and also the first 2 non-zero frequency components in both positive and negative frequency. Label each component with its complex amplitude (magnitude and phase). Check your work by verifying that the conjugate property, $\frac{1}{2}X_{-k} = \frac{1}{2}X_k^*$, holds.
Note: When converting from $\Re e\{X\}$ to the spectrum, remember that $\Re e\{X\} = \frac{1}{2}X + \frac{1}{2}X^*$.

PROBLEM:

Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each of the signals (a)–(e), determine the correct spectrum (1)–(5). Explain your answers by deriving the formula for a time signal from each of the spectrum plots.