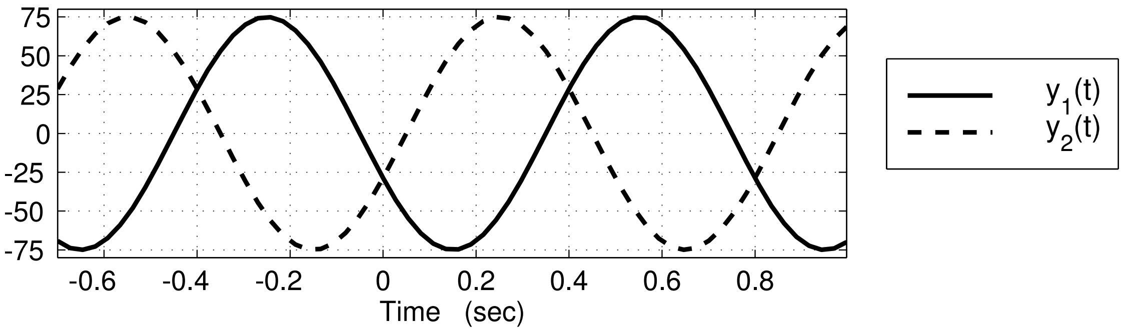
PROBLEM:

For the following short answer questions, write your answers in the space provided or circle the correct answer:

(a) In the figure below two sinusoidal signals are shown. Which one has a phase of $+5\pi/8$?
$\boxed{\text{Circle}}$ the correct answer: $\mathbf{y_1(t)}$ or $\mathbf{y_2(t)}$.



(b) In the figure above both sinusoidal signals have the same frequency. What is the frequency (ω_0) in radians/sec ? $\boxed{\text{Circle}}$ the correct answer.
(A) $5\pi/8$ **(B)** 1.6π **(C)** 2.5π **(D)** 1.25π **(E)** 0.8

(c) **TRUE** or **FALSE**: "If the signal $x(t)$ is a sinusoid and its spectrum has frequency components at $f = \pm 2$ Hz, then a new signal defined by $y(t) = x(t)\cos(200\pi t)$ has frequency components at $f = \pm 102$ Hz and $f = \pm 98$ Hz."

(d) The signal $x(t)$ has a spectrum containing frequency components at $f = 0, \pm 0.6$, and ± 2 Hz. Determine the *fundamental period,* i.e., the shortest possible period.

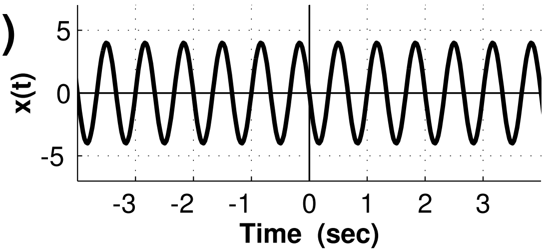
(e) $\boxed{\text{Circle}}$ the correct answer: When you add $4\cos(16\pi t + 3\pi/4) + 4\cos(16\pi t - \pi/4)$ the maximum value of the resulting signal is:
(A) equal to 0, (B) equal to 8, (C) greater than 8, (D) less than 8, but not 0.

PROBLEM:

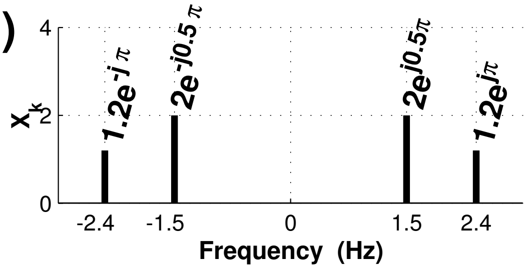
Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each of the signals (a)–(e), determine the correct spectrum (1)–(5). Write your answers in the following table:

(a)	(b)	(c)	(d)	(e)

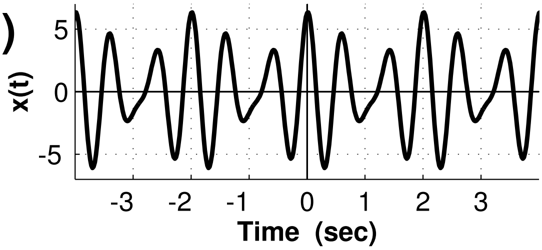
(a)



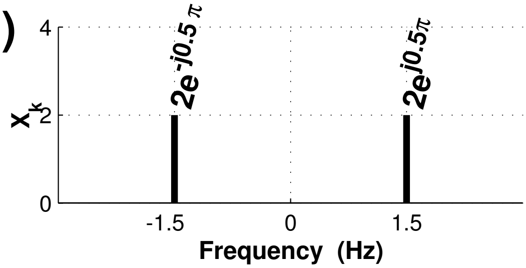
(1)



(b)



(2)



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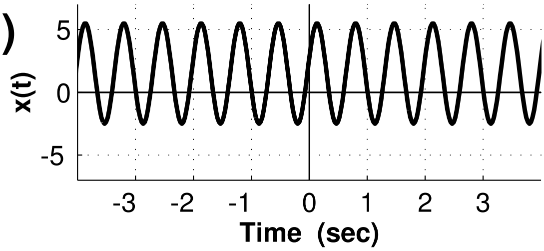
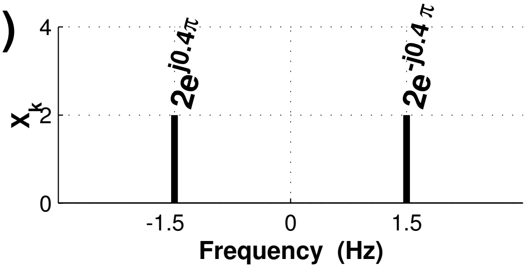
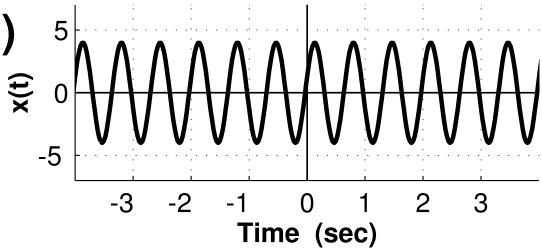


(3)



(d)



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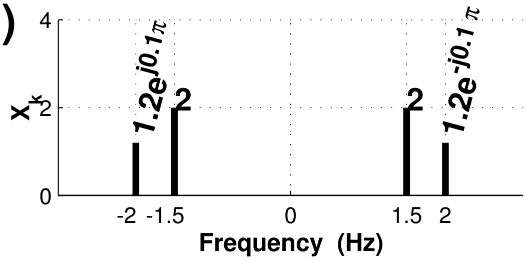
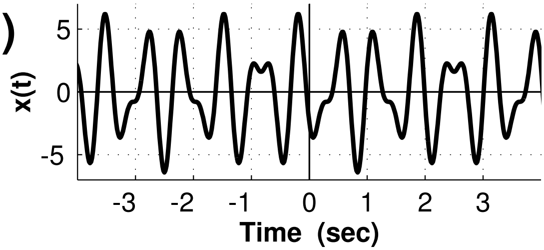
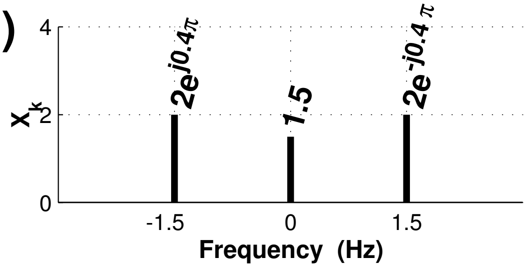


(e)



(5)



PROBLEM:

The two-sided spectrum of a signal $x(t)$ is given in the following table:

frequency (ω)	complex phasor
-150π	X_{-2}
-90π	$3e^{j\pi/4}$
0	5
ω_1	X_1
150π	$1+\sqrt{3}j$

(a) If $x(t)$ is a real signal, what are X_1, X_{-2}, and ω_1?

(b) Write an expression for $x(t)$ involving only real numbers and cosine functions.

PROBLEM:

In AM radio, the transmitted signal is voice (or music) mixed with a *carrier signal.* The carrier is a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta has a *carrier frequency* of 750 kHz. If we use the notation $v(t)$ to denote the voice/music signal, then the actual transmitted signal for WSB might be:

$$x(t) = (v(t) + A)\cos(2\pi(750 \times 10^3)t)$$

where A is a constant. (A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $v(t)$.)

(a) Voice-band signals tend to contain frequencies less than 4000 Hz (4 kHz). Suppose that $v(t)$ is a 1 kHz sinusoid, $v(t) = \cos(2\pi(1000)t)$. Draw the spectrum for $v(t)$.

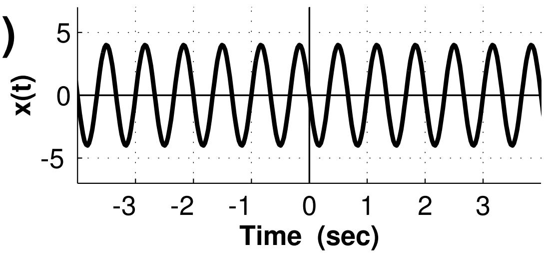
(b) Now draw the spectrum for $x(t)$, assuming a carrier at 750 kHz. Use $v(t)$ from part (a) and assume that $A = 2$. *Hint: Substitute for* $v(t)$ *and expand* $x(t)$ *into a sum of cosine terms of three different frequencies.*

(c) How would the spectrum of the AM radio signal change if the carrier frequency is changed to 680 kHz (WCNN) and $v(t)$ and A are the same as defined in parts (a) and (b).

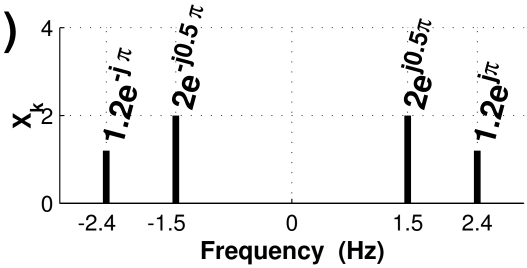
PROBLEM:

Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each of the signals (a)–(e), determine the correct spectrum (1)–(5). Explain your answers by deriving the formula for a time signal from each of the spectrum plots.

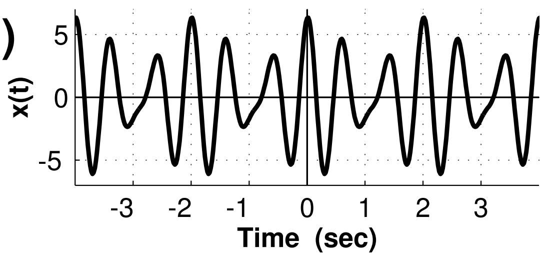
(a)



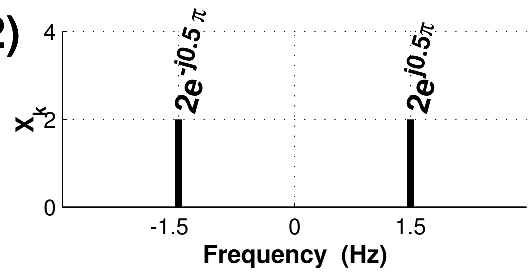
(1)



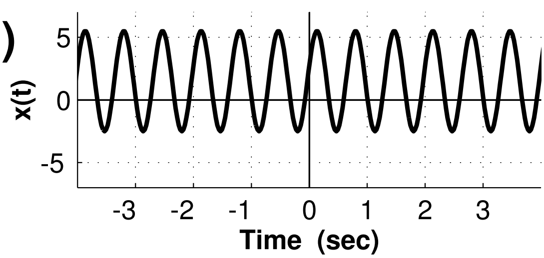
(b)



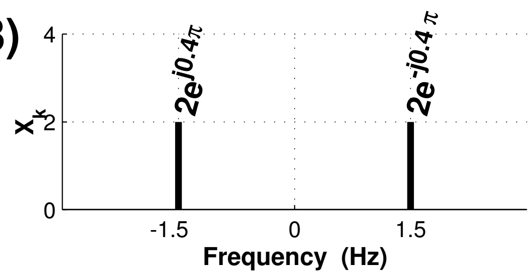
(2)



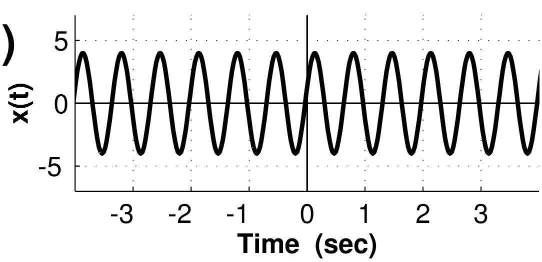
(c)



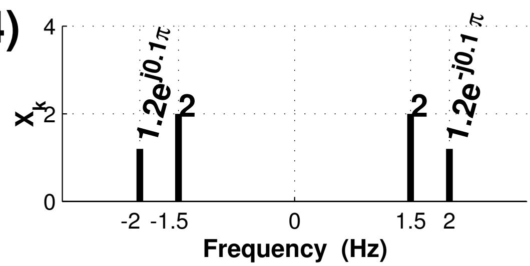
(3)



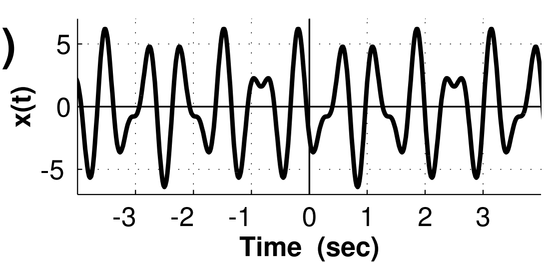
(d)



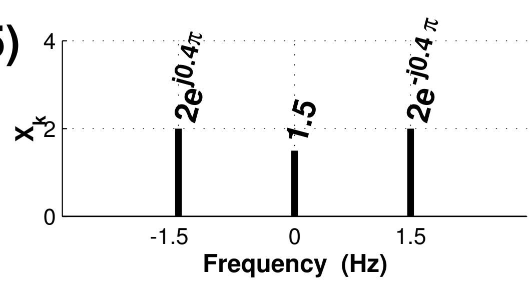
(4)



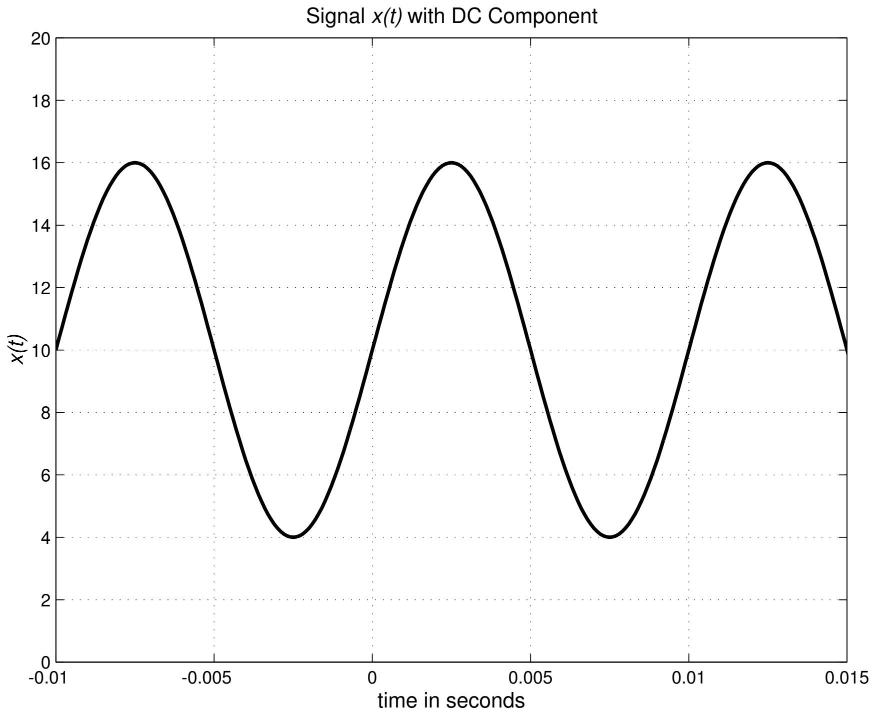
(e)



(5)



PROBLEM:



The above signal $x(t)$ consists of a DC (or constant) component plus a cosine signal.

(a) What is the frequency of the constant component? What is the frequency of the cosine component?

(b) Write an equation for the signal $x(t)$. You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the above graph.

(c) Expand the equation obtained in part (a) into a sum of positive and negative frequency complex exponential signals and plot the two-sided spectrum of the signal $x(t)$. Show the complex amplitudes for each positive and negative frequency contained in $x(t)$.

PROBLEM:

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with middle-C and ending with high-C are:

note name	C	$C^{\#}$	D	$E^{\flat}$	E	F	$F^{\#}$	G	$G^{\#}$	A	$B^{\flat}$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C (note #49) is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

(d) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The E Minor chord is composed of the tones of E, G, B sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the E-Minor chord assuming that each note is realized by a pure sinusoidal tone and that each note is equally loud. (You do not have to specify the complex amplitudes precisely.)

PROBLEM:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly.*

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

There are examples on the CD-ROM in the Chapter 3 demos.

(a) For the linear-FM "chirp" in (1), determine formulas for the beginning instantaneous frequency (ω_1) and the ending instantaneous frequency (ω_2) in terms of α, β and T_2. For this problem, assume that the starting time of the "chirp" is $t = 0$.

(b) For the "chirp" signal

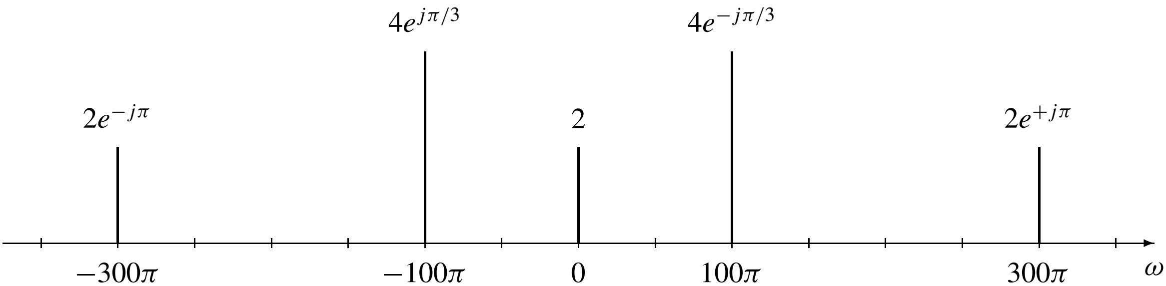
$$x(t) = \Re e\left\{e^{j2\pi(25t^2 - 25t)}\right\}$$

derive a formula for the *instantaneous* frequency versus time. Should your answer for the frequency be a positive number?

(c) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 1$ sec.

PROBLEM:

The spectrum of a signal $x(t)$ is shown in the following figure:



Note carefully that the frequency axis is radian frequency (ω) ***not*** **cyclic frequency** (f).

(a) Write an equation for $x(t)$ in terms of cosine functions.

(b) Is $x(t)$ periodic? **You must explain this answer. Why or why not?**
If it is periodic, what is the fundamental frequency and corresponding period of $x(t)$?

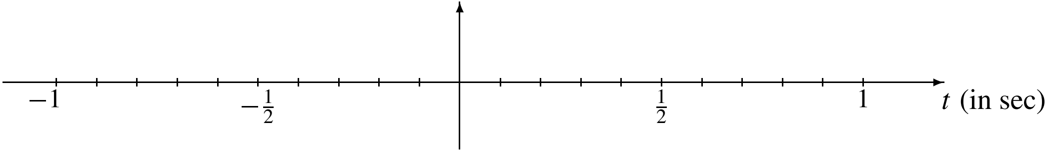
(c) A new signal is defined as $y(t) = \cos(\alpha t + \pi) + x(t)$. It is known that $y(t)$ is periodic with period $T_0 = 0.04$ sec. Determine **two** positive values for the frequency α that will satisfy this condition.

(d) Using either of the frequencies α found in (c), modify the spectrum plot above so that it becomes the spectrum of $y(t)$.

PROBLEM:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 1 & \text{for } 0.7 < t < 0.8 \\ -1 & \text{for } 0 < t < 0.7 \end{cases}$

(a) Assume that the period of $x(t)$ is 0.8 s. Sketch $x(t)$ over the ENTIRE range $-1 \le t \le 1$ s.



(b) Write the general Fourier integral expression for the coefficient a_k in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integrals (e.g., limits of integration), but do not evaluate the integrals. All parameters in the integrals should have numeric values.*

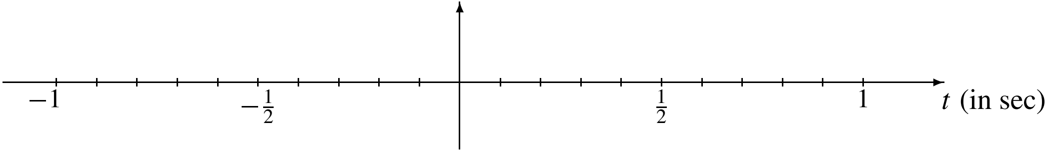
(c) Evaluate the Fourier integral below. Simplify your answer and express it in **polar form.**

$$\frac{1}{4}\int_{-0.5}^{0.5} \cos(\pi t)e^{-j2\pi(2)t/4}dt$$

PROBLEM:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 1 & \text{for } 0.2 < t < 0.5 \\ -1 & \text{for } 0 < t < 0.2 \end{cases}$

(a) Assume that the period of $x(t)$ is 0.5 s. Sketch $x(t)$ over the ENTIRE range $-1 \leq t \leq 1$ s.



(b) Write the general Fourier integral expression for the coefficient a_k in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integrals (e.g., limits of integration), but do not evaluate the integrals. All parameters in the integrals should have numeric values.*

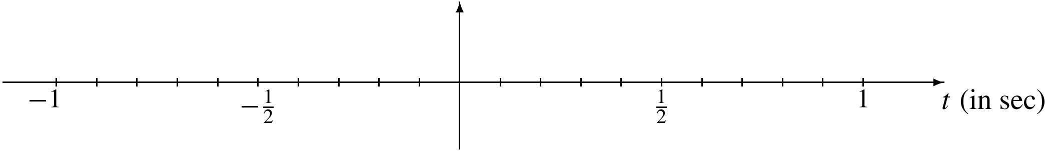
(c) Evaluate the Fourier integral below. Simplify your answer and express it in **polar form.**

$$\frac{1}{2}\int_0^1 \sin(\pi t)e^{-j2\pi(1)t/2}dt$$

PROBLEM:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 1 & \text{for } 0.0 < t < 0.3 \\ -1 & \text{for } 0.3 < t < 0.6 \end{cases}$

(a) Assume that the period of $x(t)$ is 0.6 s. Sketch $x(t)$ over the ENTIRE range $-1 \leq t \leq 1$ s.

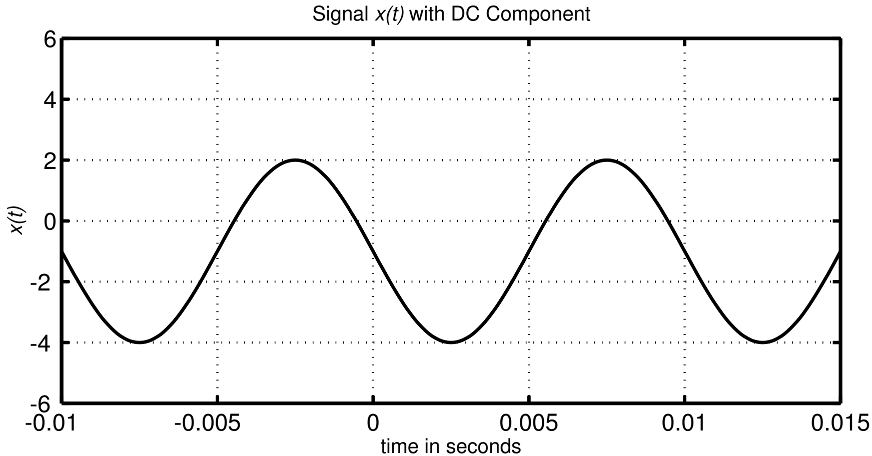


(b) Write the general Fourier integral expression for the coefficient a_k in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integrals (e.g., limits of integration), but do not evaluate the integrals. All parameters in the integrals should have numeric values.*

(c) Evaluate the Fourier integral below. Simplify your answer and express it in **polar form.**

$$\frac{1}{4}\int_{-1}^{1} \cos\left(\tfrac{1}{2}\pi t\right) e^{-j2\pi(1)t/4}\, dt$$

PROBLEM:



The above signal $x(t)$ consists of a DC component plus a cosine signal. The terminology *DC component* means a component that is constant versus time.

(a) What is the frequency of the DC component? What is the frequency of the cosine component?

(b) Write an equation for the signal $x(t)$. You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the above graph.

(c) Expand the equation obtained in the previous part into a sum of positive and negative frequency complex exponential signals.

(d) Then plot the two-sided spectrum of the signal $x(t)$. Show the complex amplitudes for each positive and negative frequency contained in $x(t)$.

PROBLEM:

In this problem you will consider the general case of the "beating" phenomenon. When you multiply two sinusoids:

$$x(t) = \cos(2\pi(40)t - \pi/3)\cos(2\pi(600)t + \pi/4)$$

the signal can still be expressed as a "spectrum." In order to do this, you need an *additive* combination of sinusoids.

(a) Use the inverse Euler formula to obtain a set of complex exponential signals that sum together to make $x(t)$.

(b) Plot the spectrum of $x(t)$.

(c) Find a complex signal $z(t)$ such that $x(t) = \Re e\{z(t)\}$.

(d) Use the spectrum to write an alternate formula for $x(t)$ as:

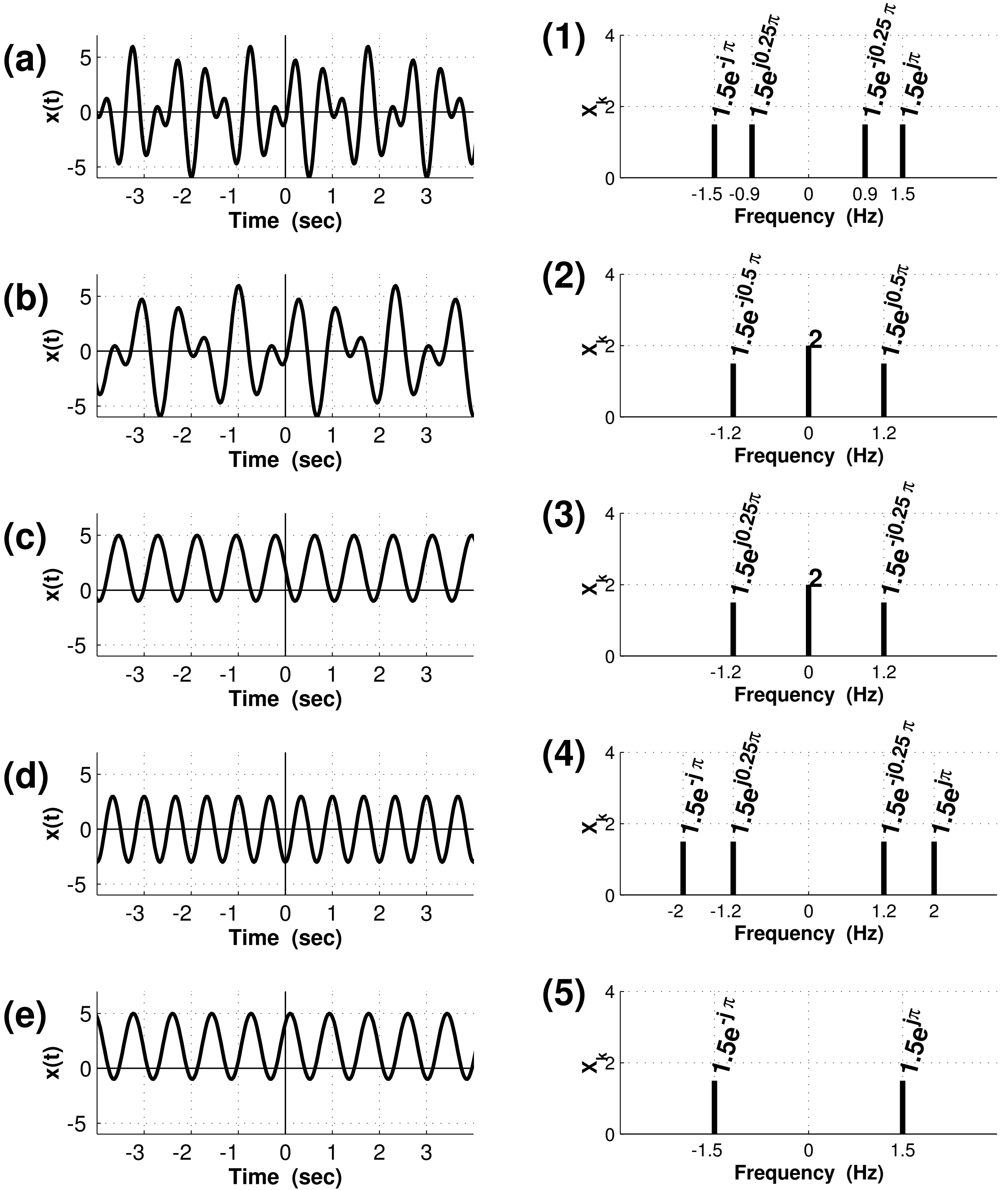
$$x(t) = A\cos[2\pi(f_c - \Delta)t + \phi_1] + B\cos[2\pi(f_c + \Delta)t + \phi_2]$$

Find the numerical values for all the parameters: A, B, f_c, Δ, ϕ_1, and ϕ_2.

(e) This signal is periodic; determine its fundamental period.

PROBLEM:

Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each of the signals (a)–(e), determine the correct spectrum (1)–(5). Explain your answers by deriving the mathematical formula for a time signal from each of the spectrum plots.



PROBLEM:

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with A-440 and ending with A-880 are:

note name	A	$B^\flat$	B	C	$C^\#$	D	$E^\flat$	E	F	$F^\#$	G	$G^\#$	A
note number	49	50	51	52	53	54	55	56	57	58	59	60	61
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with A-440 and ending at A-880. Recall that A-440 is the A above middle C (note #49) which is tuned to 440 Hz.

(c) The notes (from part (b)) on a piano ketboard are numbered 49 through 61. If n denotes the note number, and f denotes the frequency of the corresponding tone in hertz, give a formula for the frequency of the tone as a function of the note number.

(d) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The A Minor chord is composed of the tones of A, C, E sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the A-Minor chord assuming that each note is realized by a pure sinusoidal tone and that each note is equally loud. (You do not have to specify the complex amplitudes precisely.)

PROBLEM:

A periodic signal is represented by the Fourier Series synthesis formula:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j30\pi kt} \qquad \text{where} \quad a_k = \begin{cases} \dfrac{1}{4 + j2k} & \text{for } k = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{for } |k| > 3 \end{cases}$$

(a) Sketch the two-sided spectrum of this signal. Label all complex amplitudes in **polar form.**

(b) Determine the fundamental frequency (in Hz) and the fundamental period (in secs.) of this signal.

PROBLEM:

A periodic signal $x(t) = x(t + T_0)$ is described *over one period*, $0 \leq t \leq T_0$, by the equation

$$x(t) = \begin{cases} t & 0 \leq t \leq t_c \\ 0 & t_c < t \leq T_0 \end{cases}$$

where $0 < t_c < T_0$.

(a) Sketch the periodic function $x(t)$ for $-T_0 < t < 2T_0$ for the specific case $t_c = \frac{1}{2}T_0$.

(b) Determine the D.C. coefficient of the Fourier Series, a_0. Once again, use the specific case of $t_c = \frac{1}{2}T_0$.

PROBLEM:

Use the signal $x(t)$ defined by the equation

$$x(t) = \begin{cases} t & 0 \le t \le t_c \\ 0 & t_c < t \le T_0 \end{cases}$$

where $t_c = \frac{1}{2}T_0$.

(a) Use the Fourier $\boxed{\textit{analysis}}$ integral (for $k \neq 0$)

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

to determine a general formula for the Fourier Series coefficients a_k. Your final result for a_k should depend on k.

Notes: This Fourier integral requires integration by parts; in addition, the Fourier integral can be done over any period of the signal; in this case, the most convenient choice is from 0 to T_0.

Note: the frequency ω_0 would be given in rads/sec, but it does not have a specific value. However, you can simplify your formulas by using the identity $\omega_0 T_0 = 2\pi$.

(b) Use the Fourier Series coefficients to sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(\frac{1}{4})$ rad/sec and $t_c = \frac{1}{2}T_0$. Include *only* those frequency components corresponding to $k = 0, \pm 1, \pm 2, \pm 3$. Label each component with its frequency and its complex amplitude (i.e., numerical values of magnitude and phase).

PROBLEM:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the "angle" of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying angle argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the angle argument $\psi(t)$ is the *instantaneous frequency*, which is also the audible frequency heard from the chirp. (The instantaneous frequency is the frequency heard by the human ear when the chirp rate is relatively slow. There are cases of FM where the audible signal is quite different, but these happen when the chirp rate is very high.)

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

(a) For the "chirp" signal

$$x(t) = \Re e\left\{e^{j2\pi(-75t^2 + 900t + 33)}\right\}$$

derive a formula for the *instantaneous* frequency versus time.

(b) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 2$ sec.

PROBLEM:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$.

(a) Determine the formula for a signal $x(t)$ that sweeps from $f_1 = 5000$ Hz at $T_1 = 0$ secs. to $f_2 = 1000$Hz at $T_2 = 2$ secs.

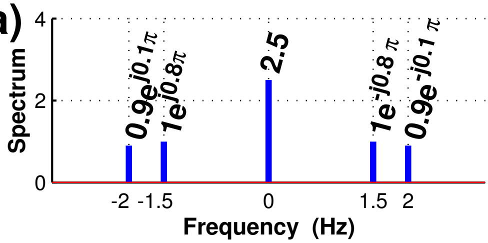
(b) Sketch the time-frequency diagram showing the instantaneous frequency versus time for the signal in part (a).

PROBLEM:

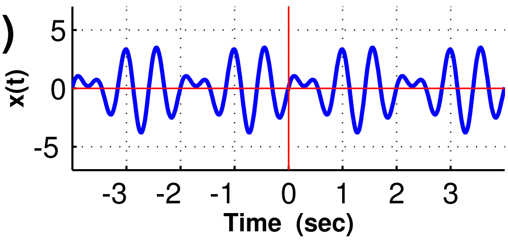
Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each spectrum plot (a)–(e), determine the fundamental frequency in Hz, and also determine the correct signal (1)–(5). Write your answers in the following table:

SPECTRUM	(a)	(b)	(c)	(d)	(e)
FUNDAMENTAL FREQUENCY (HZ)					
SIGNAL					

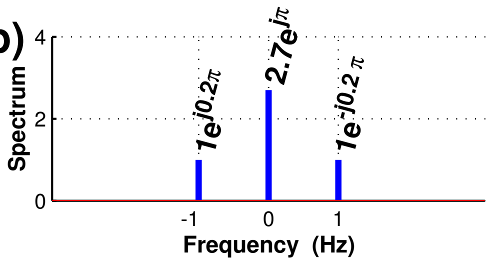
(a)



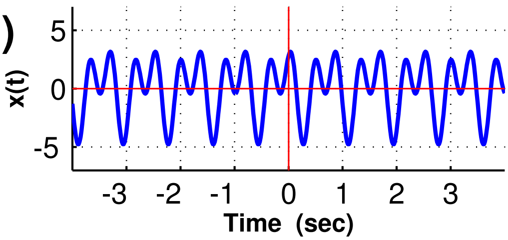
(1)



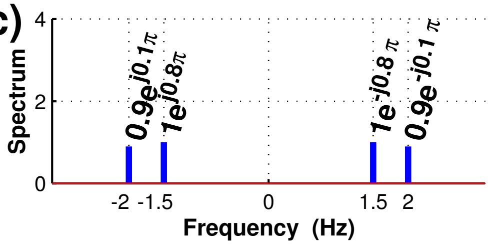
(b)



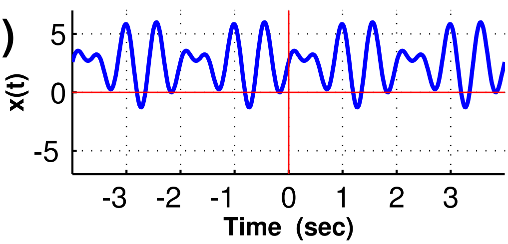
(2)



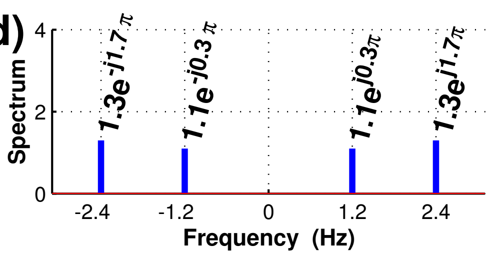
(c)



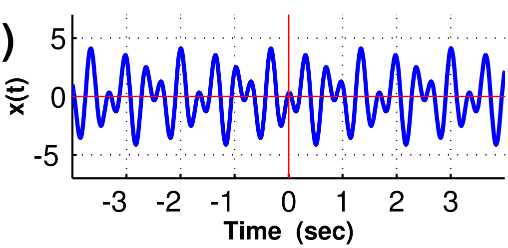
(3)



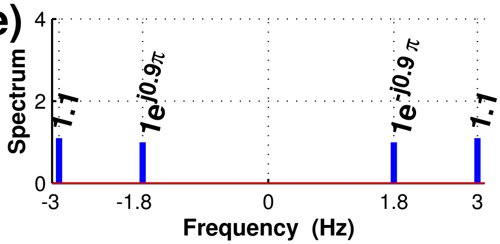
(d)



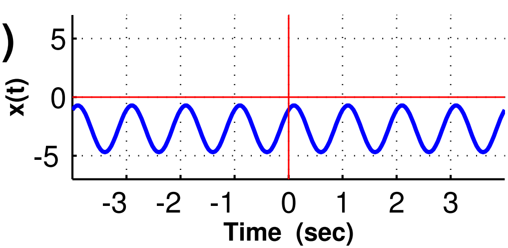
(4)



(e)



(5)



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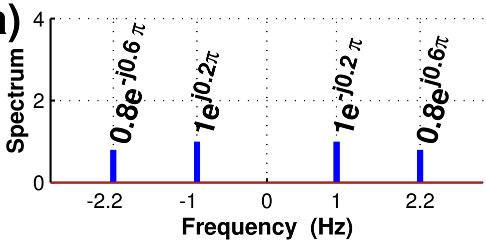
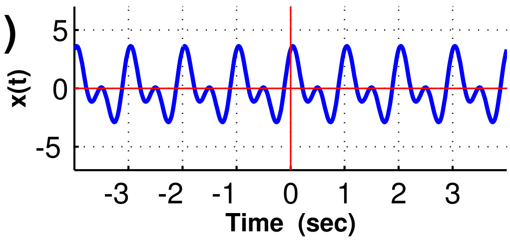
FUNDAMENTAL FREQUENCY (HZ)					
SIGNAL					

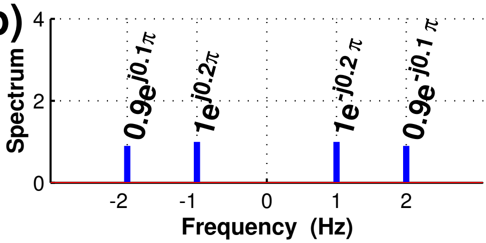
(a)



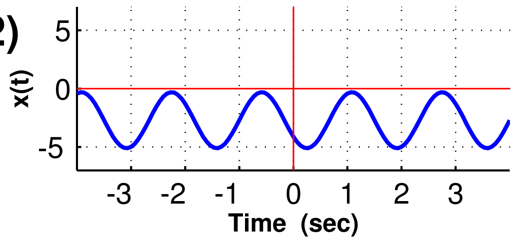
(1)



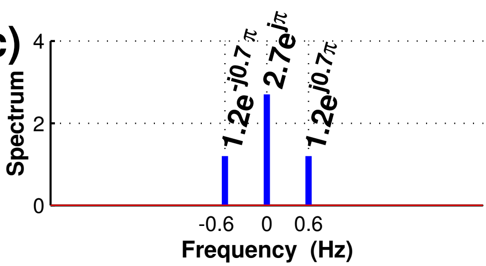
(b)



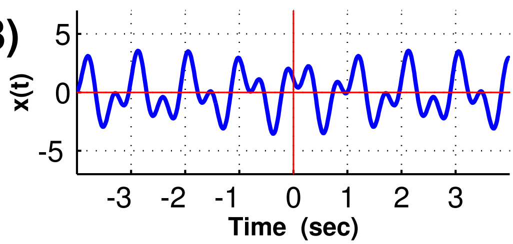
(2)



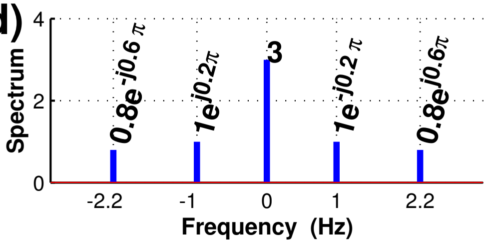
(c)



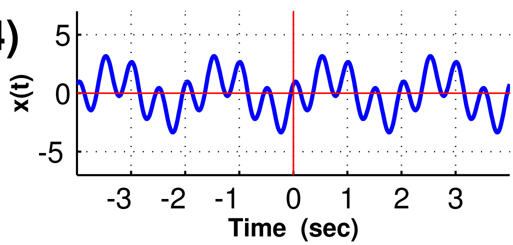
(3)



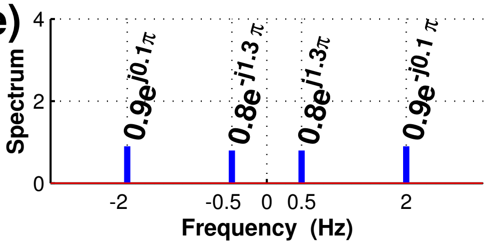
(d)



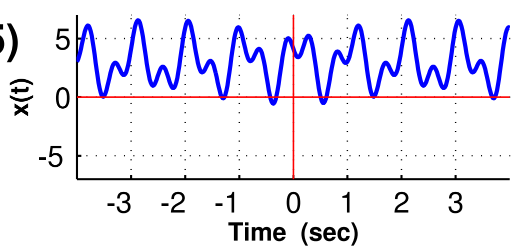
(4)



(e)



(5)



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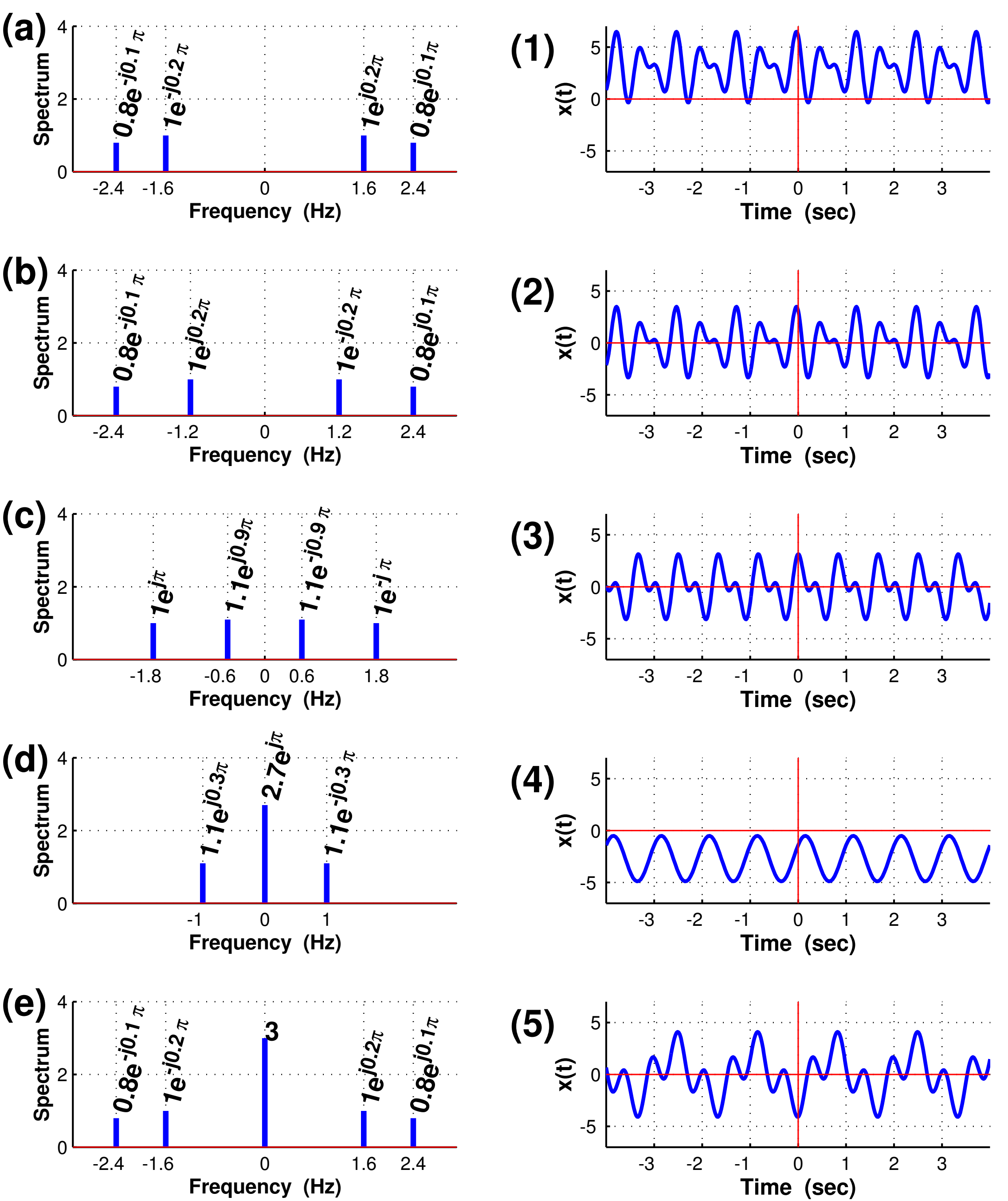
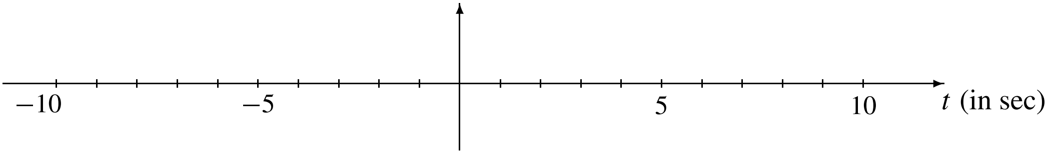
SPECTRUM	(a)	(b)	(c)	(d)	(e)
FUNDAMENTAL FREQUENCY (HZ)					
SIGNAL					



PROBLEM:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 1 & \text{for } |t| \leq 1 \\ 0 & \text{for } 1 < |t| < 4 \end{cases}$

(a) Assume that the period of $x(t)$ is 8 s. Draw a plot of $x(t)$ over the range $-10 \leq t \leq 10$ s.



(b) Determine the DC value of $x(t)$.

(c) Write the Fourier integral expression for the coefficient a_7 in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integral (e.g., limits of integration), but do not evaluate the integral. All parameters in the integral should have numeric values.*

(d) Evaluate the following integral: $\displaystyle\int_{-1}^{0} e^{-j2\pi(5)t/10}\,dt$ Simplify your answer and express it in **polar form.**