BLM3620 Digital Signal Processing*

Erkan Uslu

euslu@yildiz.edu.tr

Yıldız Technical University – Computer Engineering

*Based on lecture notes from Ali Can Karaca & Ahmet Elbir

Lecture #7 – Convolution and FIR Filters

- Convolution Example
- Graphical Convolution
- MATLAB demo
- FIR Filter
- FIR Filter Application

Remember: Classification of Impulse Response h[n]

FIR – Finite Impulse Response:

- Number of impulses are limited.
- Always stable.

For example: $h[n] = \delta[n-1] + 5\delta[n-5]$

IIR – Infinite Impulse Response:

- Number of impulses are infinite.
- Sometimes these systems are not stable.

For example: $h[n] = u[n-1] + 5u[n-5]$

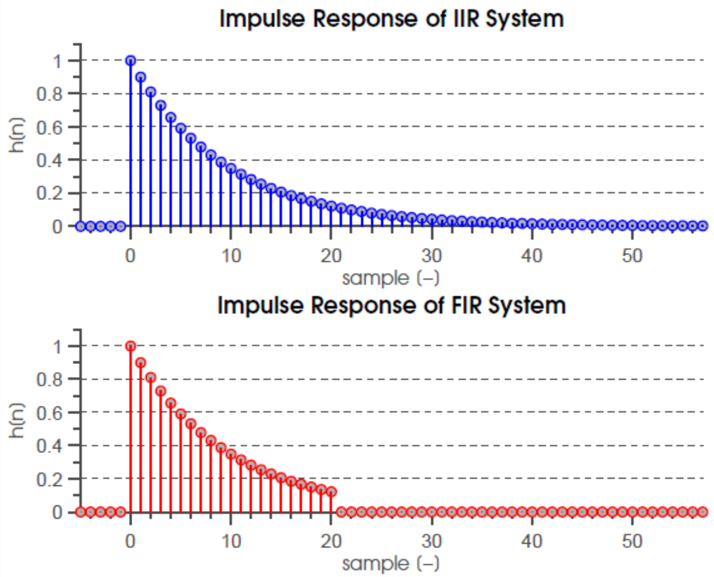
Finite impulse response (FIR):

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

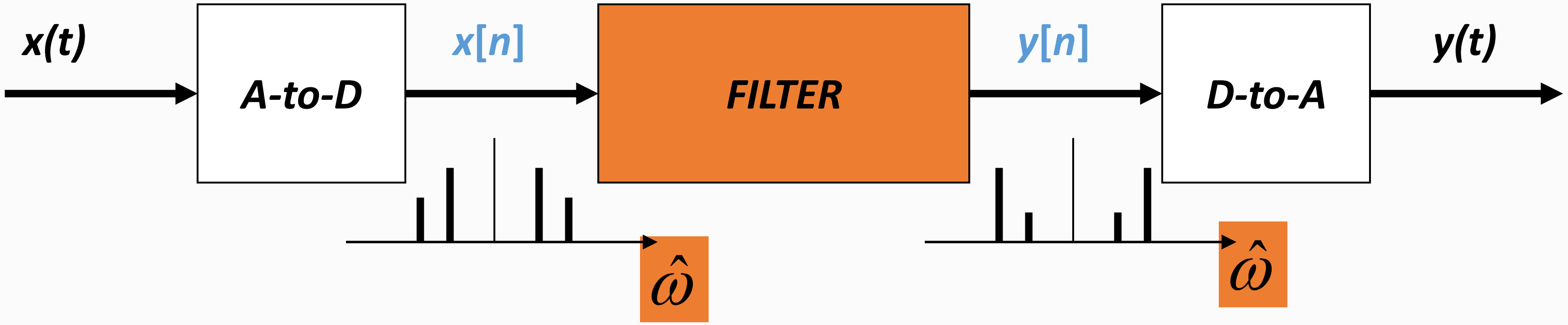
Infinite impulse response (IIR):

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

Another example:



DOMAINS: Time & Frequency



- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT x[n] = SUM of SINUSOIDS
  - Then, OUTPUT y[n] = SUM of SINUSOIDS

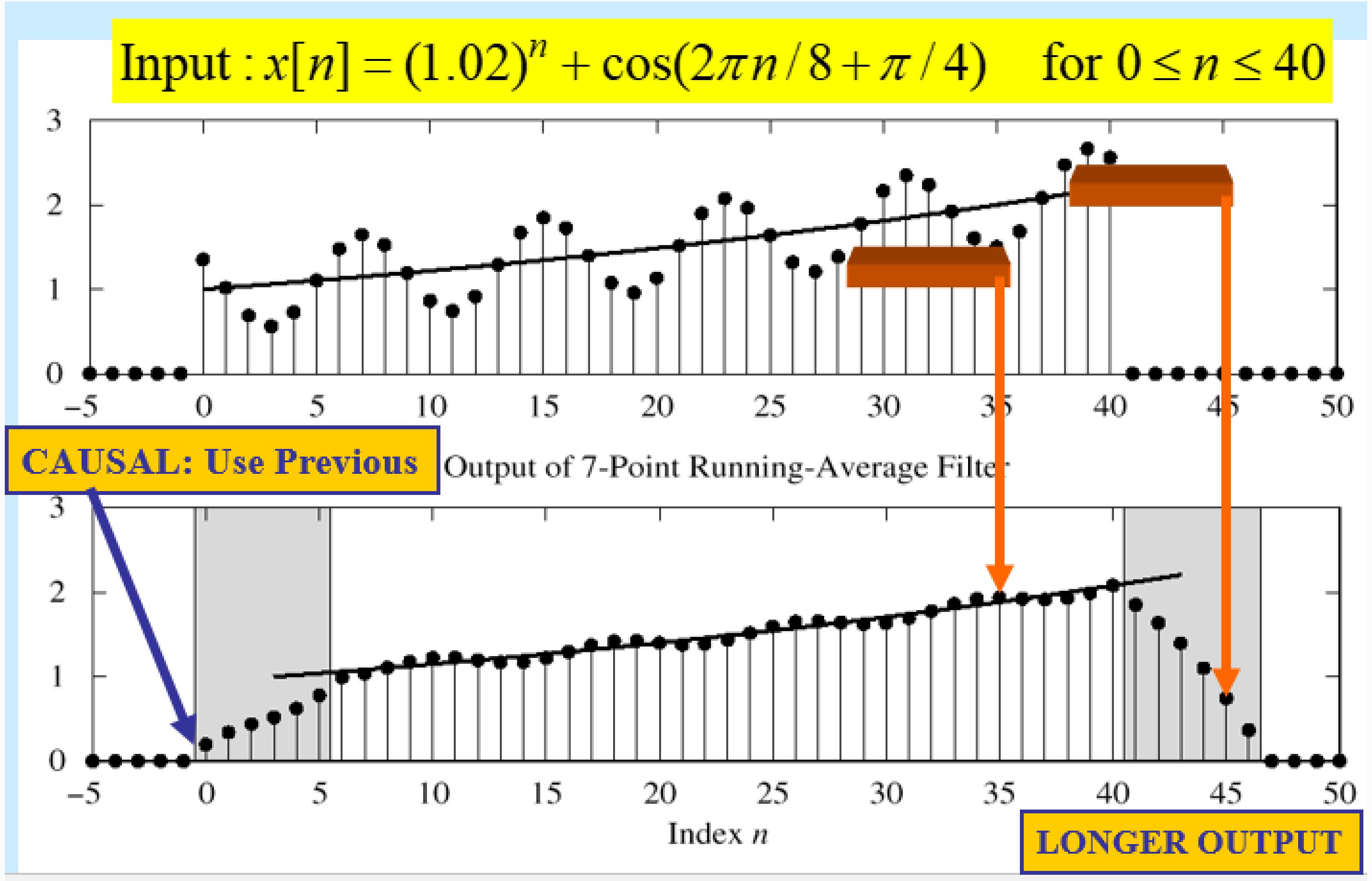
- **Time-Domain: "n" = time**
  - x[n] discrete-time signal
  - x(t) continuous-time signal

- 3-point AVERAGER

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

- 7-point AVERAGER

$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$



But…

How can I calculate the effects of this filter on digital frequency?

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$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

But…

How can I calculate the effects of this filter on digital frequency?

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FIR DIFFERENCE EQUATION

- Use the FIR "Difference Equation"

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} b_k Ae^{j\varphi}e^{j\hat{\omega}(n-k)}$$

$$= \left(\sum_{k=0}^{M} b_k e^{j\hat{\omega}(-k)}\right) Ae^{j\varphi}e^{j\hat{\omega}n}$$

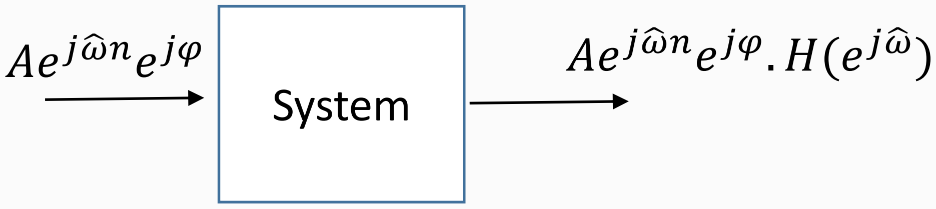
$$= H(\hat{\omega})Ae^{j\varphi}e^{j\hat{\omega}n}$$

New Term: Frequency Response $H(e^{j\hat{\omega}})$

- At each frequency, we can DEFINE

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \longleftarrow \text{FREQUENCY RESPONSE}$$

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency
- Notation: $H(e^{j\hat{\omega}})$ in place of $H(\hat{\omega})$



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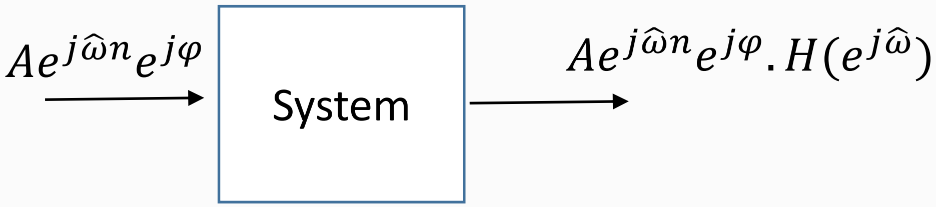
FREQUENCY RESPONSE



- Complex-valued formula
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- Notation: $H(e^{j\hat{\omega}})$ in place of $H(\hat{\omega})$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \cdots$$
$$= \left|H(e^{j\hat{\omega}})\right| e^{j\angle H(e^{j\hat{\omega}})}$$

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$$A e^{j\hat{\omega}n} e^{j\varphi} \longrightarrow \boxed{\text{System}} \longrightarrow A e^{j\hat{\omega}n} e^{j\varphi} . H(e^{j\hat{\omega}})$$

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  - And PHASE vs. frequency
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Complex Number:

1- A Phase Component

2- A Magnitude Component

Example

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

$$\{b_k\} = \{1,\ 2,\ 1\}$$

$$H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

$$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}})$$

$$= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$$

EXPLOIT SYMMETRY

Since $(2 + 2\cos\hat{\omega}) \geq 0$

Magnitude is $\left|H(e^{j\hat{\omega}})\right| = (2 + 2\cos\hat{\omega})$

and Phase is $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$

Example

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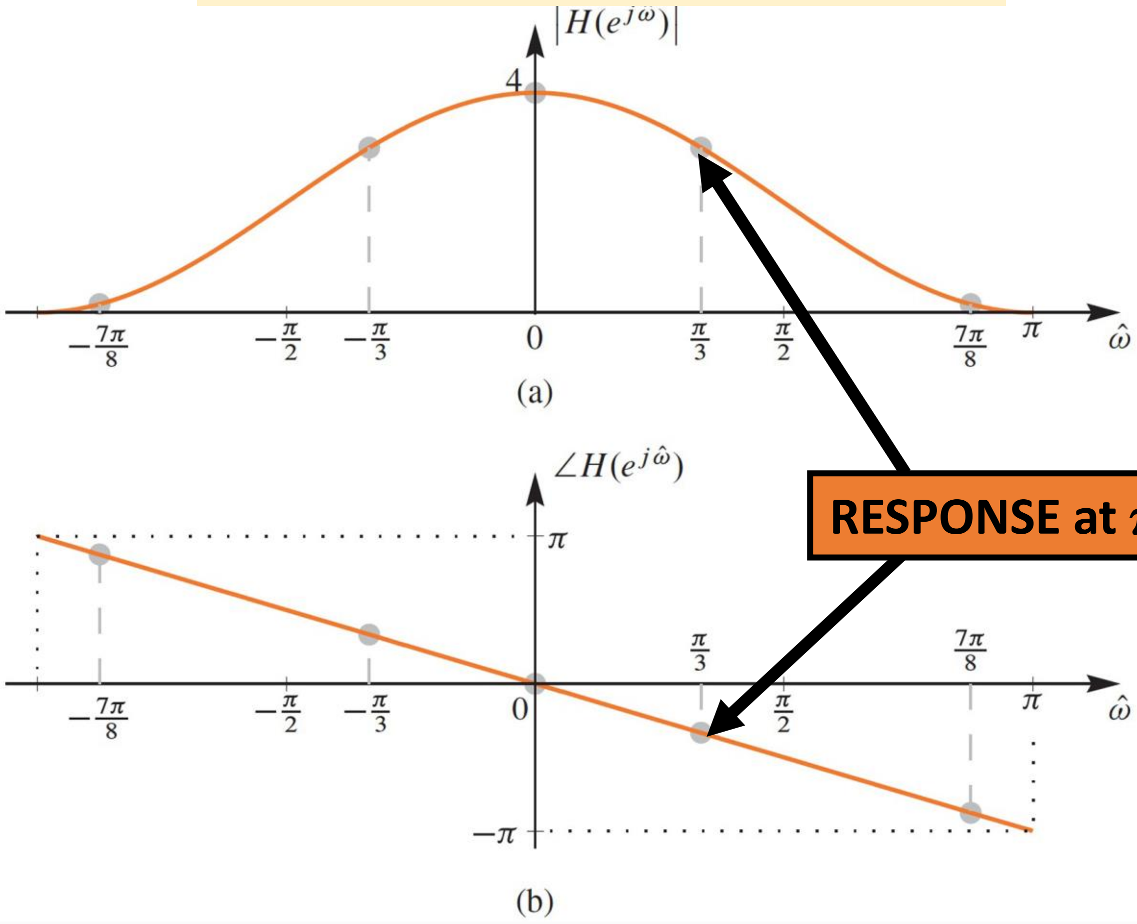
$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$



Example

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$\{b_k\} = \{1,\ 2,\ 1\}$$

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\ &= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) \\ &= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega}) \end{aligned}$$

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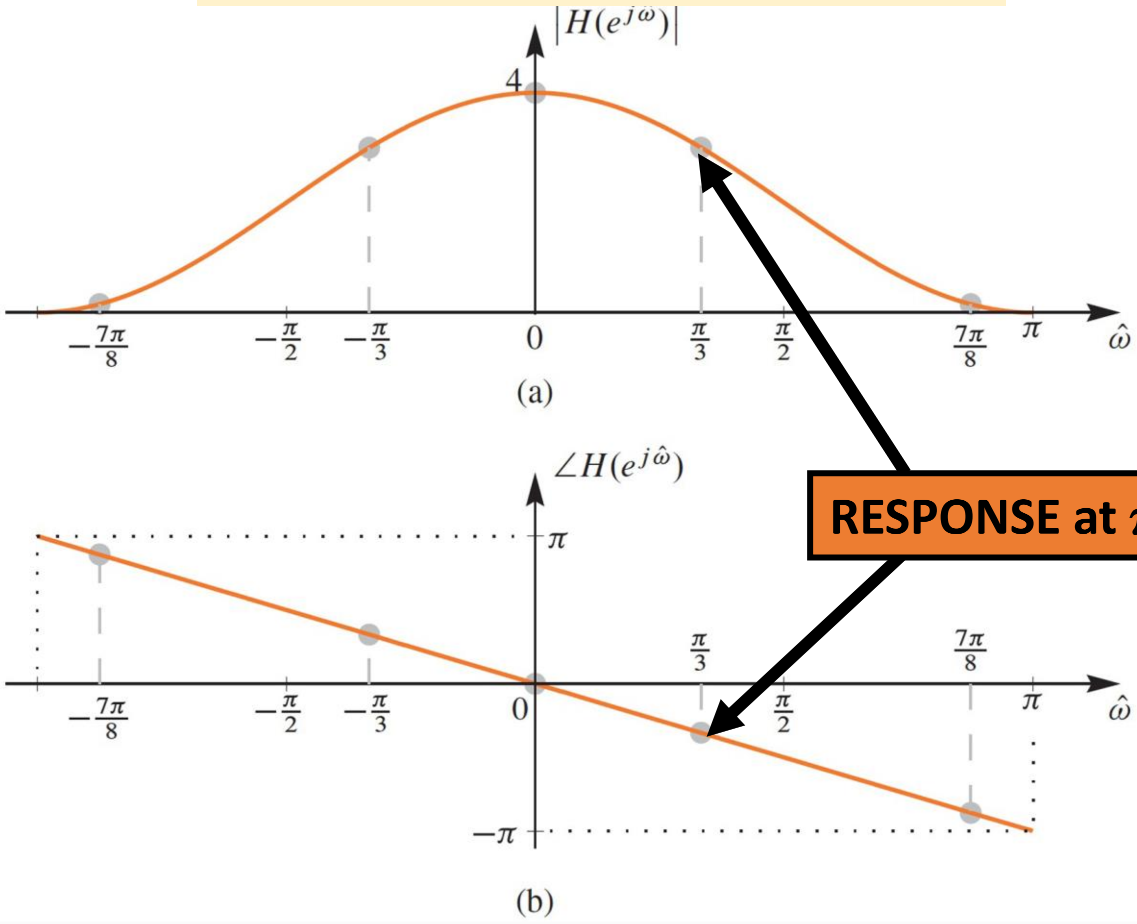

What does this filter do in frequency domain?

Example – 2 : For the previous system…

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

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Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\,\hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6\,e^{-j\pi/12}e^{j(\pi/3)n}$$

Example - 3 : For the previous system...

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = e^{j(\pi n/3+\pi/4)} + e^{-j(\pi n/3+\pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

Use Linearity

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3+\pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3+\pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

Example - 3 : For the previous system...

$$\text{Find } y[n] \text{ when } x[n] = 2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4})$$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

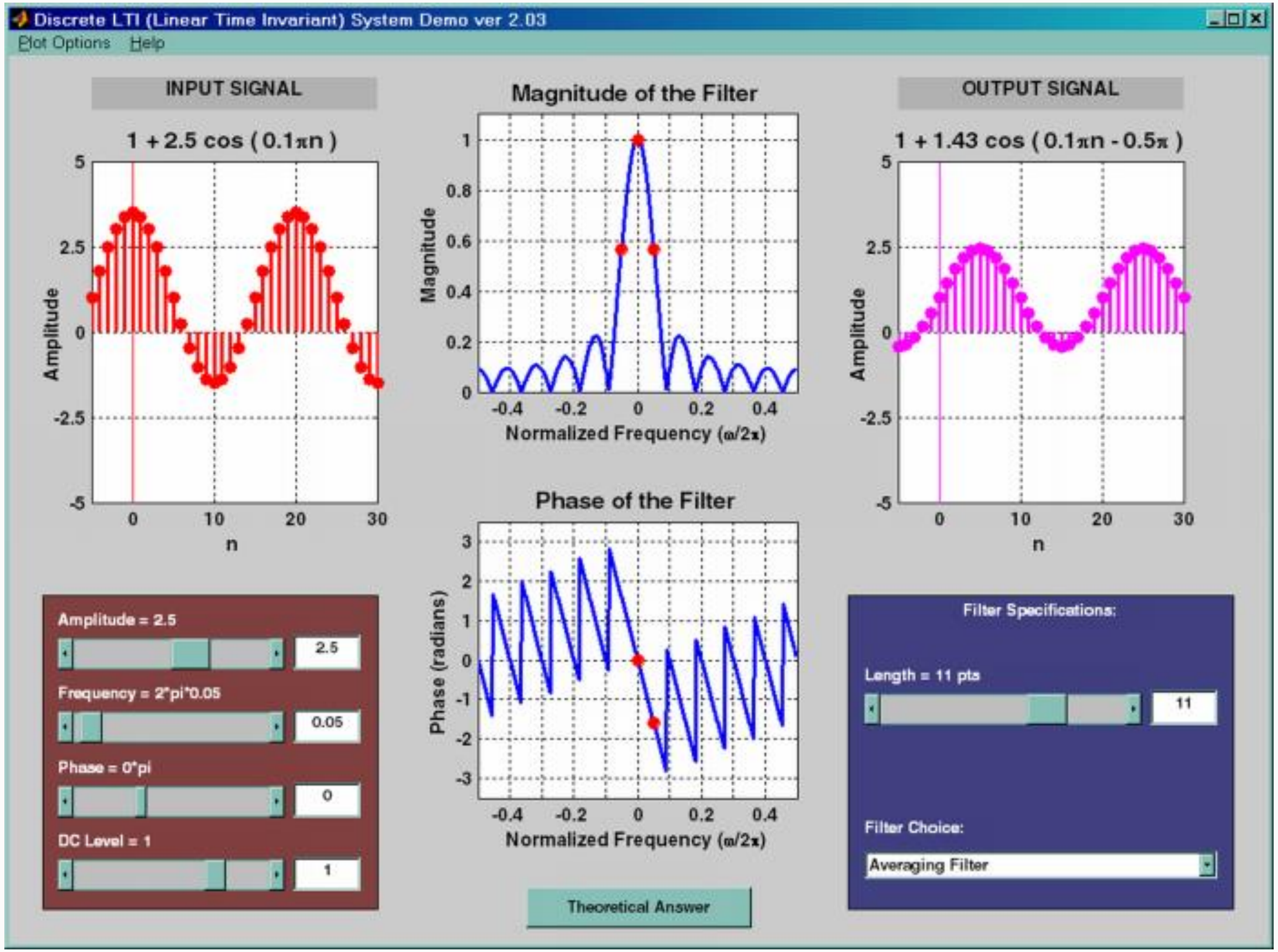
$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3+\pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3+\pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3+\pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3+\pi/4)}$$

$$y[n] = 3e^{j(\pi n/3-\pi/12)} + 3e^{-j(\pi n/3-\pi/12)}$$

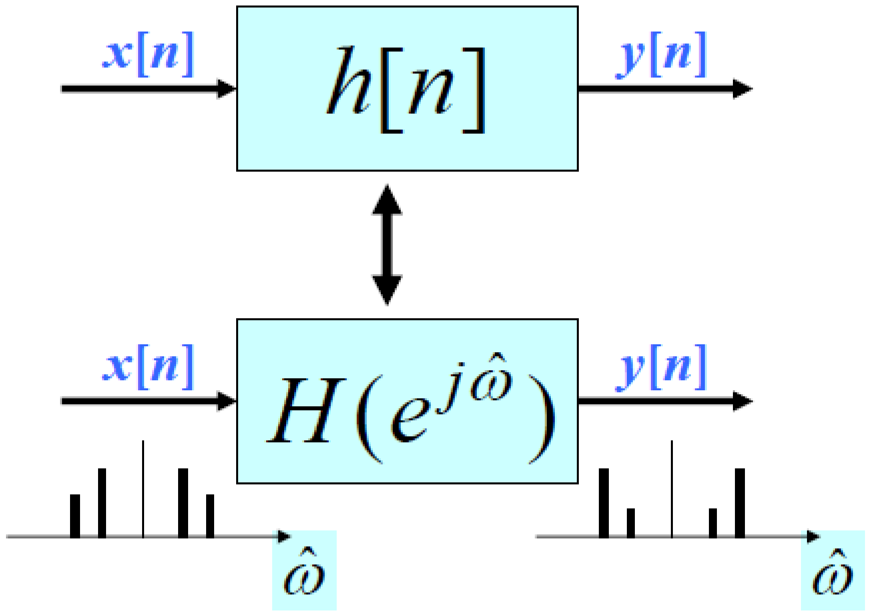
$$\Rightarrow y[n] = 6\cos(\tfrac{\pi}{3}n - \tfrac{\pi}{12})$$

DLTI Demo with Sinuzoids

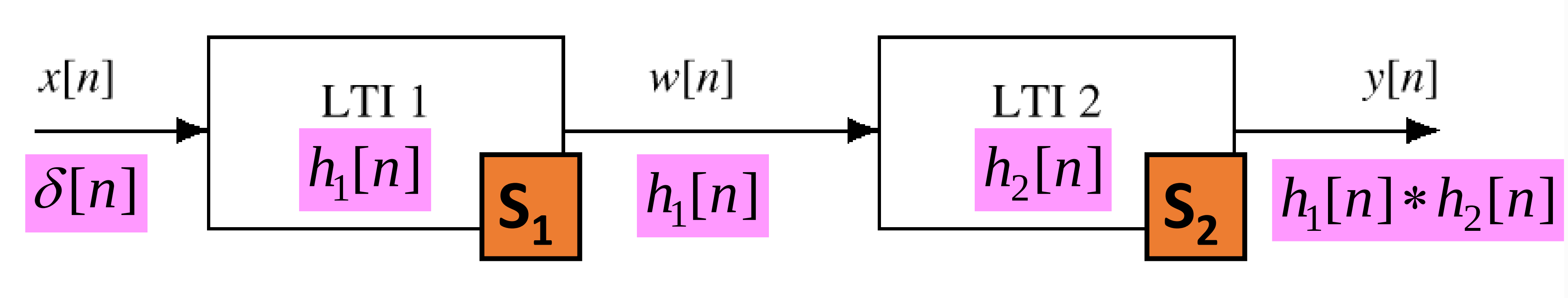


https://dspfirst.gatech.edu/matlab/#dltidemo

- Equivalent Representations



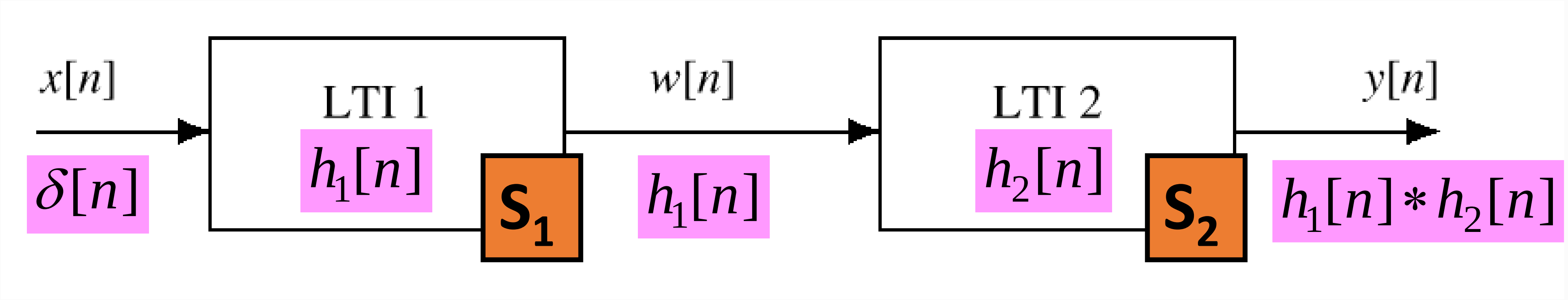
Cascaded LTI Systems



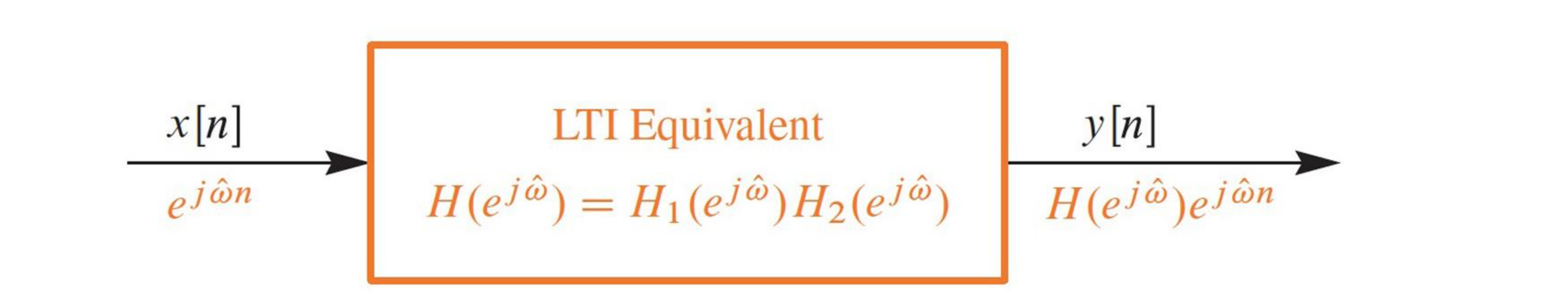
WHAT is the overall FREQUENCY RESPONSE ?

https://dspfirst.gatech.edu/chapters/06firfreq/demos/blockd/index.html

Cascaded LTI Systems



WHAT is the overall FREQUENCY RESPONSE ?

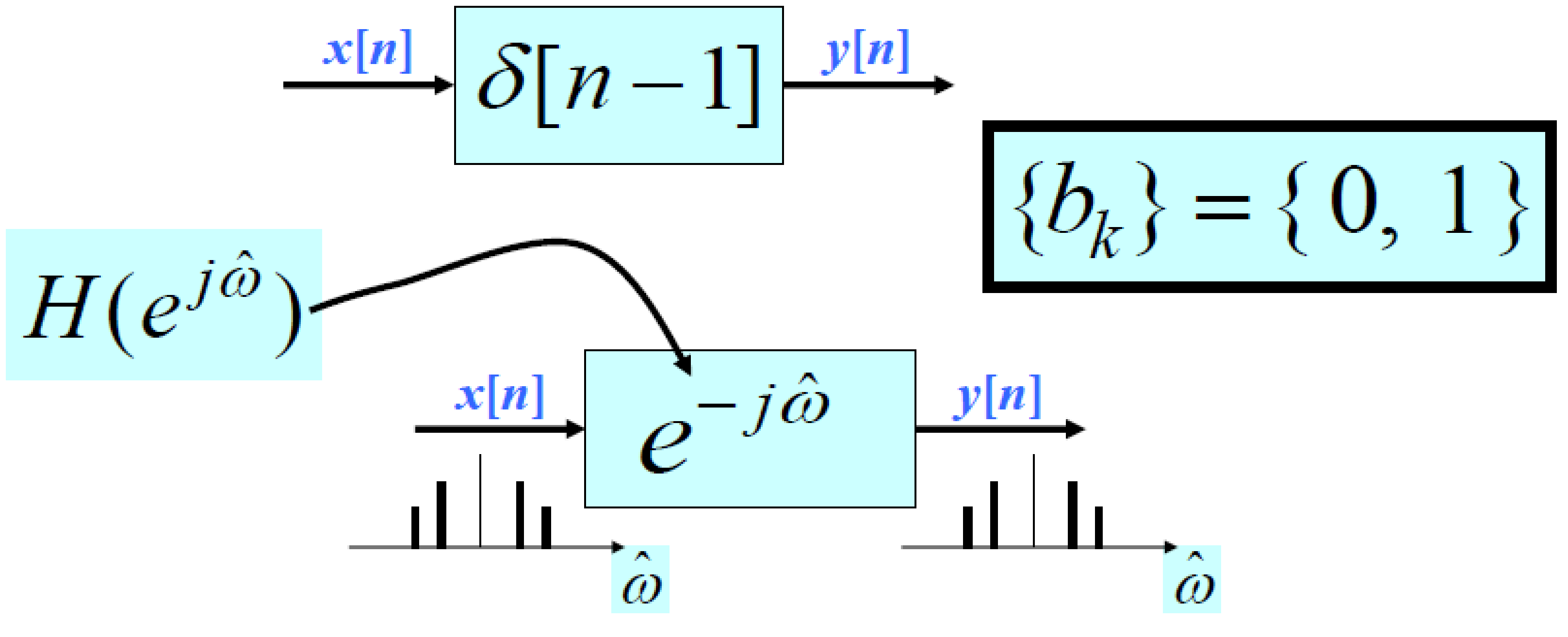


$$\boxed{H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})}$$

https://dspfirst.gatech.edu/chapters/06firfreq/demos/blockd/index.html

Example – 4: Unit Delay System

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-1]$



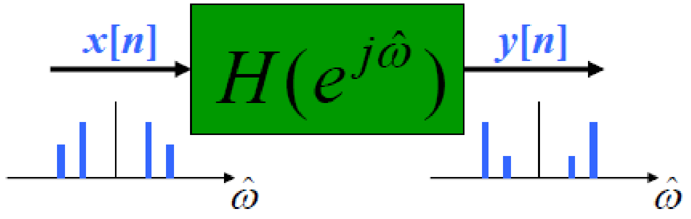
Example – 5: Freq. Domain to Time

$H(e^{j\hat{\omega}})$ and find $h[n]$ or b_k

$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

$$h[n] = ?$$

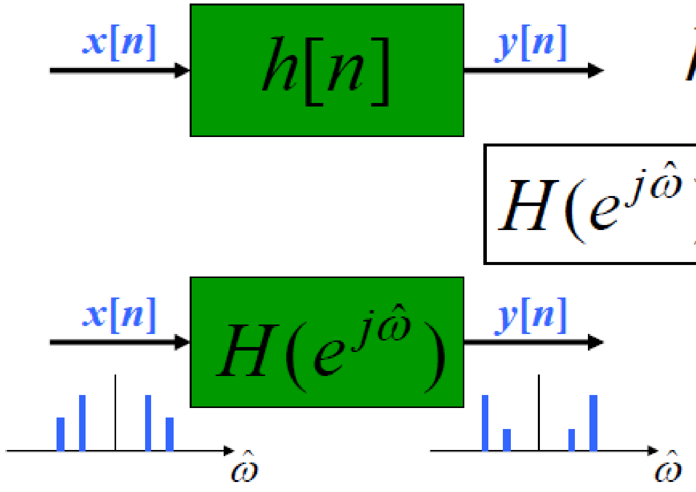
$$\boxed{H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})}$$



$$\boxed{H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}}$$

Example – 5: Freq. Domain to Time

$H(e^{j\hat{\omega}})$ and find $h[n]$ or b_k



$$h[n] = ?$$

$$\boxed{H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})}$$

$$\boxed{H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}}$$

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$

EULER's Formula

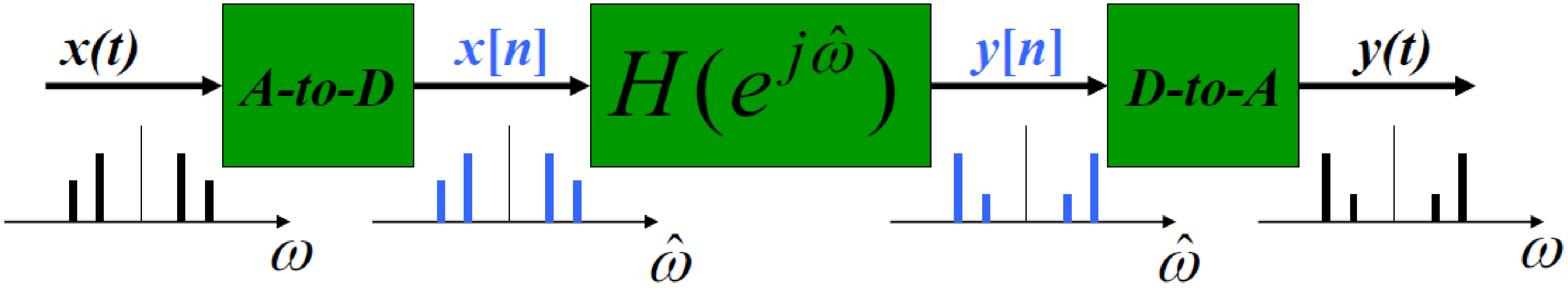
$$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{0, 3.5, 0, 3.5\}$$

Digital Filtering



$\boxed{\omega}$ – SPECTRUM of x(t) (SUM of SINUSOIDS)

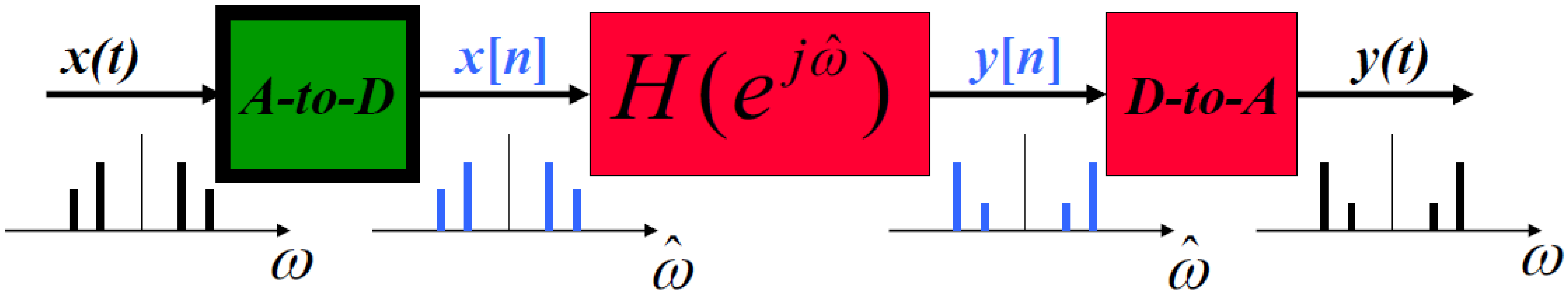
$\boxed{\hat{\omega}}$ – SPECTRUM of x[n]

- Is ALIASING a PROBLEM ?

– SPECTRUM y[n] (FIR Gain or Nulls)

$\boxed{\omega}$ – Then, OUTPUT y(t) = SUM of SINUSOIDS

Frequency Scaling

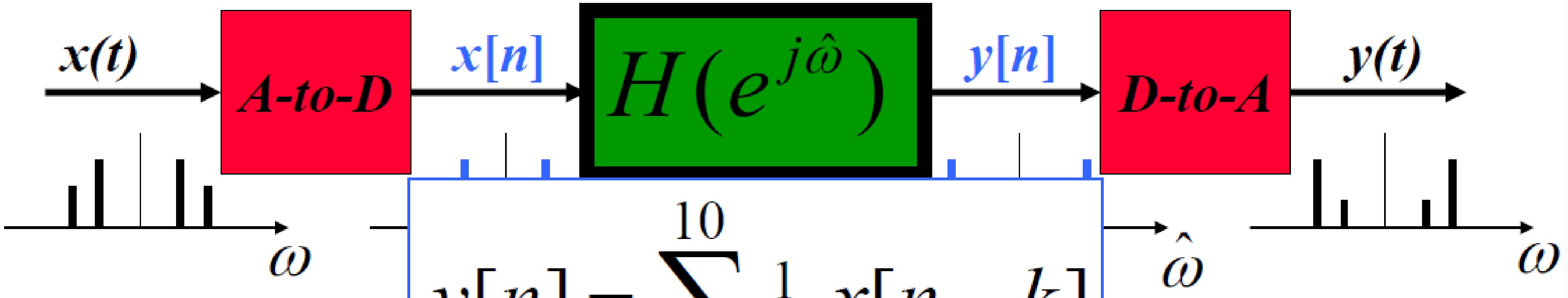


- TIME SAMPLING:
  - IF NO ALIASING:
  - FREQUENCY SCALING

$$\boxed{t = nT_s}$$

$$\boxed{\hat{\omega} = \omega T_s = \frac{\omega}{f_s}}$$

11-pt Averager



$$y[n] = \sum_{k=0}^{10} \frac{1}{11} x[n-k]$$

250 Hz

25 Hz

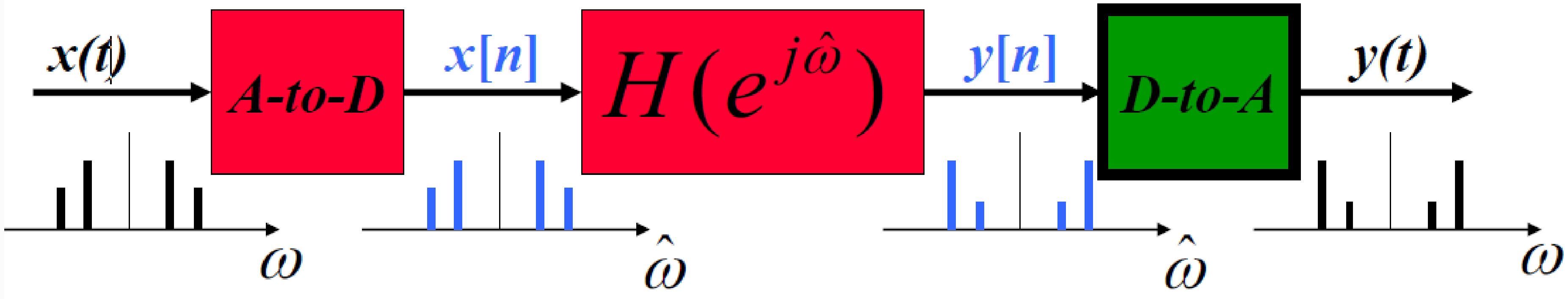
$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

?

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \tfrac{1}{2}\pi)$$

24

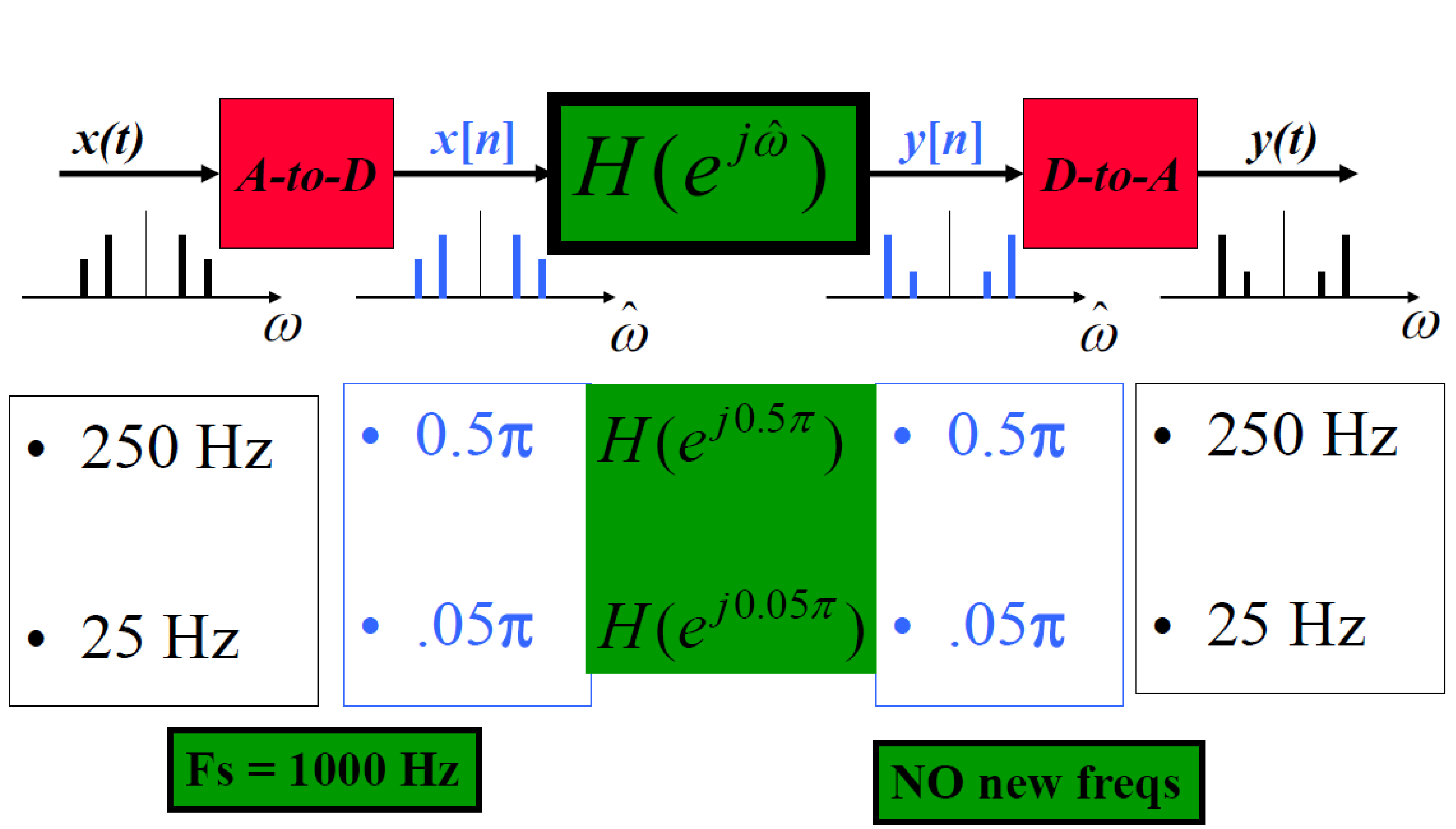
D-A Frequency Scaling



- TIME SAMPLING: $\boxed{t = nT_s \Rightarrow n \leftarrow t f_s}$
- RECONSTRUCT up to $0.5f_s$
  - FREQUENCY SCALING

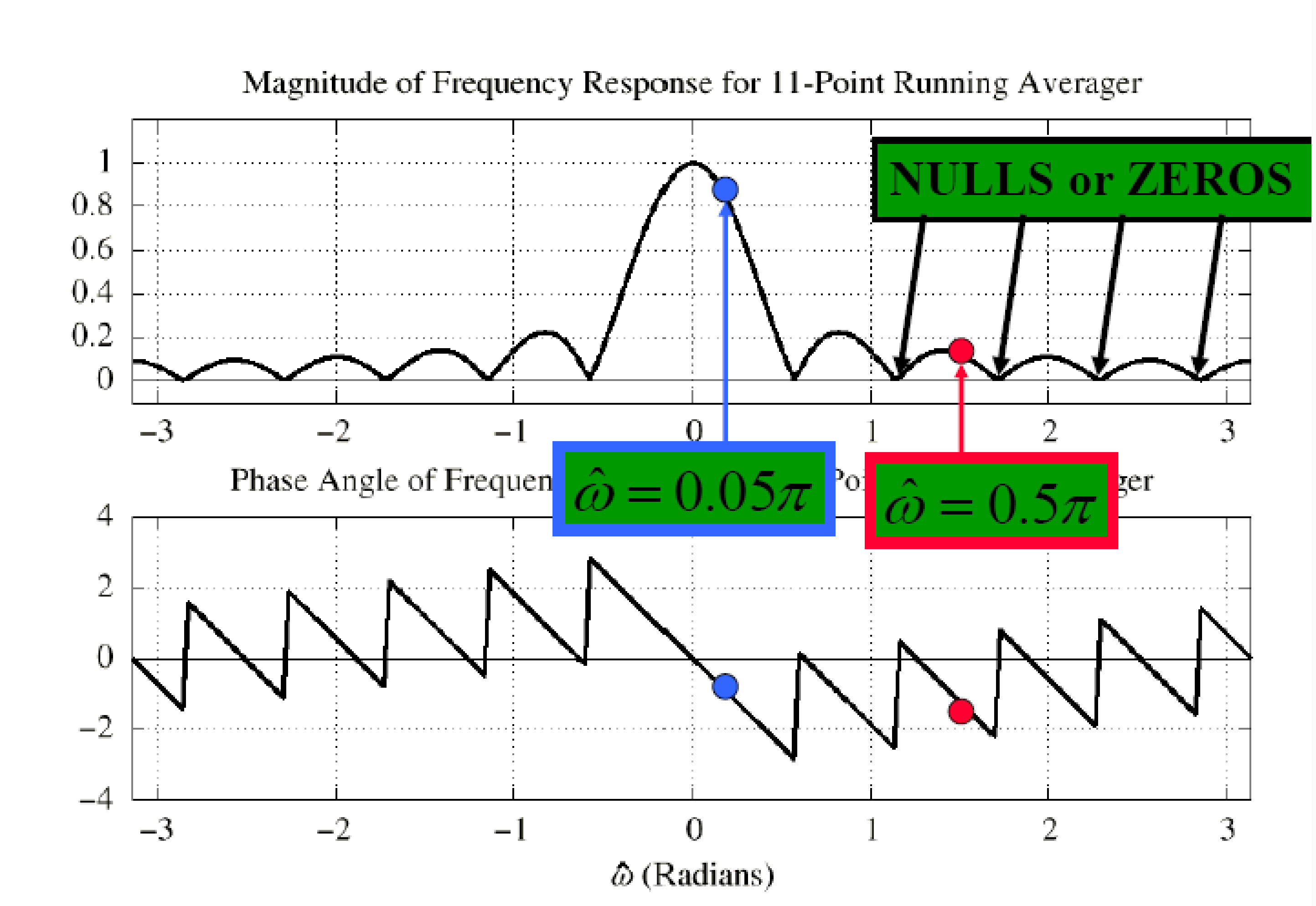
$$\boxed{\omega = \hat{\omega} f_s}$$

Summary



$t = nTs = \ n/1000$ $\quad$ $cos\ (2pi250t) \rightarrow \ cos\left(2.pi.250.\frac{n}{100}\right) = cos(0.5pi)$

Magnitude of Frequency Response



$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

At $\hat{\omega} = 0.5\pi$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}(0.5\pi))}{11\sin(\frac{1}{2}(0.5\pi))} e^{-j5(0.5\pi)}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

Magnitude of Frequency Response

$$\boxed{H(e^{j2\pi(25)/1000})} = \frac{\sin(\pi(25)(11)/1000)}{11\sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

MAG SCALE

$\boxed{f_s = 1000}$

$$= 0.8811 e^{-j\pi/4}$$

PHASE CHANGE

$$\boxed{H(e^{j2\pi(250)/1000})} = \frac{\sin(\pi(250)(11)/1000)}{11\sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

$$= 0.0909 e^{-j\pi/2}$$

$$y(t) = \underline{0.8811}\cos(2\pi(25)t \underline{- \pi/4}) + \underline{0.0909}\sin(2\pi(250)t \underline{- \pi/2})$$

Remember: 17-pt Centralized Average filter to Noisy Audio

```
clc; clear all;

%% Load Sound
load ('piano2.mat');
x = x(1:16000);
soundsc(x,Fs);

%% Add noise
K = awgn(x,40);
soundsc(K,Fs);

%%  Filter
N = 17;
h = 1/N*ones(1,N);

%% Apply Convolution
y = conv(K,h,'same');
soundsc(y,Fs);
%%
plot(x,'r'); hold on; plot(y,'b');
```

LOAD THE SIGNAL

ADD A NOISE TO SIGNAL

FILTER THE SIGNAL

Remember: 17-pt Centralized Average filter to Noisy Audio

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LOAD THE SIGNAL

ADD A NOISE TO SIGNAL

FILTER THE SIGNAL

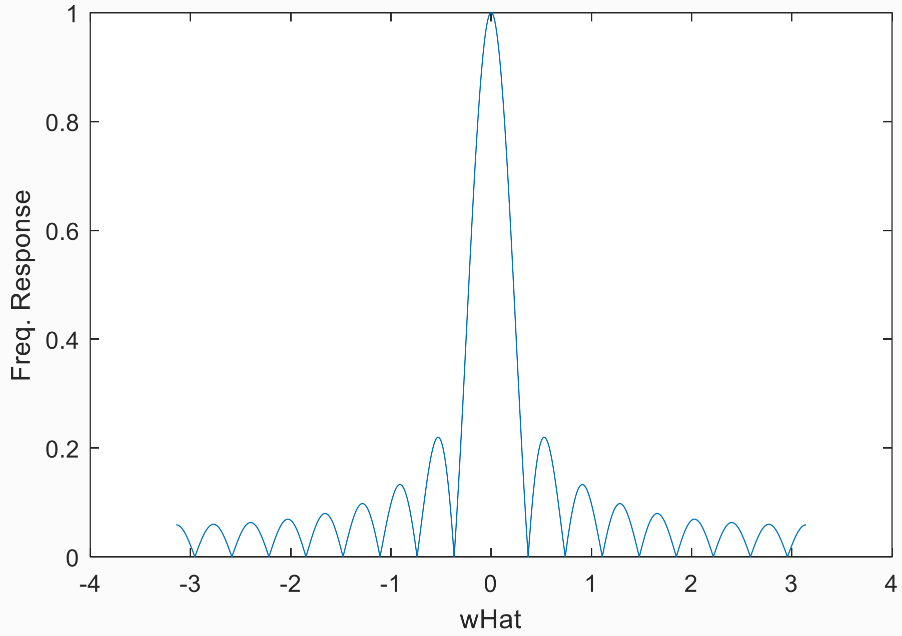
But How it works? What is the frequency response?

17-pt Averager

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

```
wHat = linspace(-pi,pi,Fs);
b = (1/17)*ones(1,17);
%%
H = zeros(1,Fs);
for k = 1:17
    H = H + b(k)*exp(-1j*wHat*k);
end

plot (wHat, abs(H));
xlabel('wHat');
ylabel('Freq. Response');
```



17-pt Averager

$$y(n) = \frac{1}{17}\sum_{k=0}^{16} x(n-k)$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

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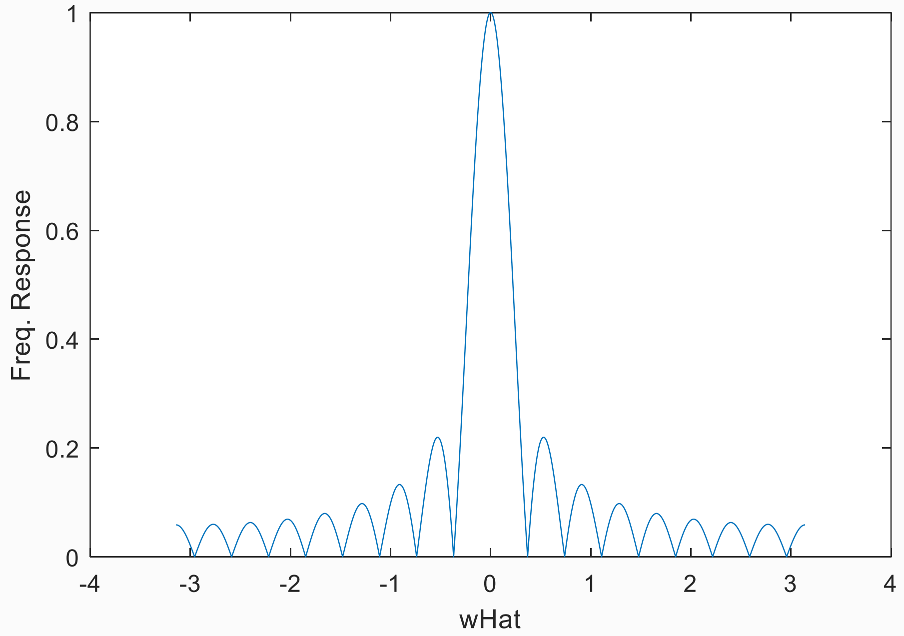

17-pt Averager

$$y(n) = \frac{1}{17}\sum_{k=0}^{16} x(n-k)$$

$$h(n) = \frac{1}{17}\sum_{k=0}^{16} \delta(n-k)$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

```
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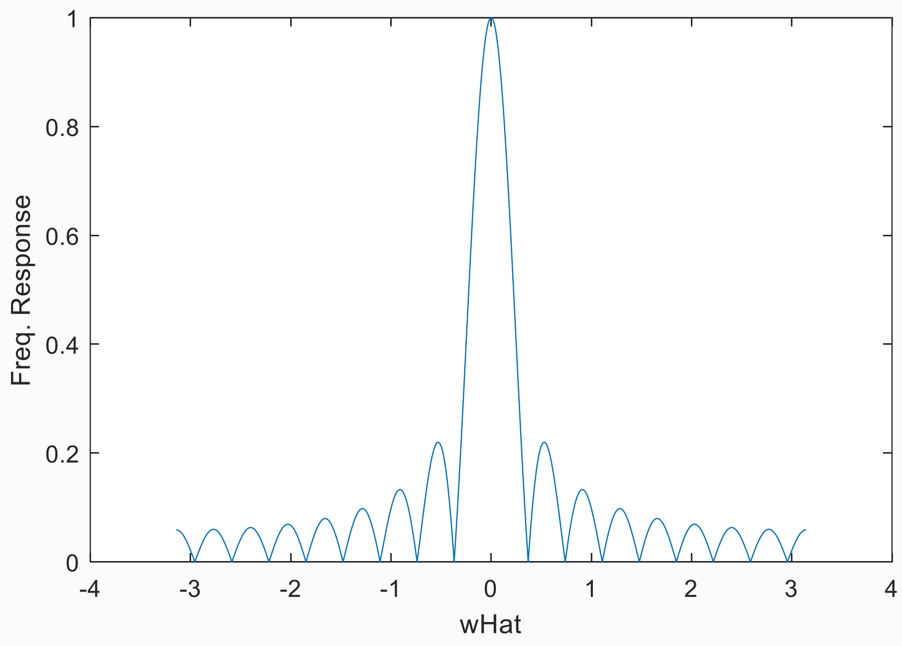


17-pt Averager

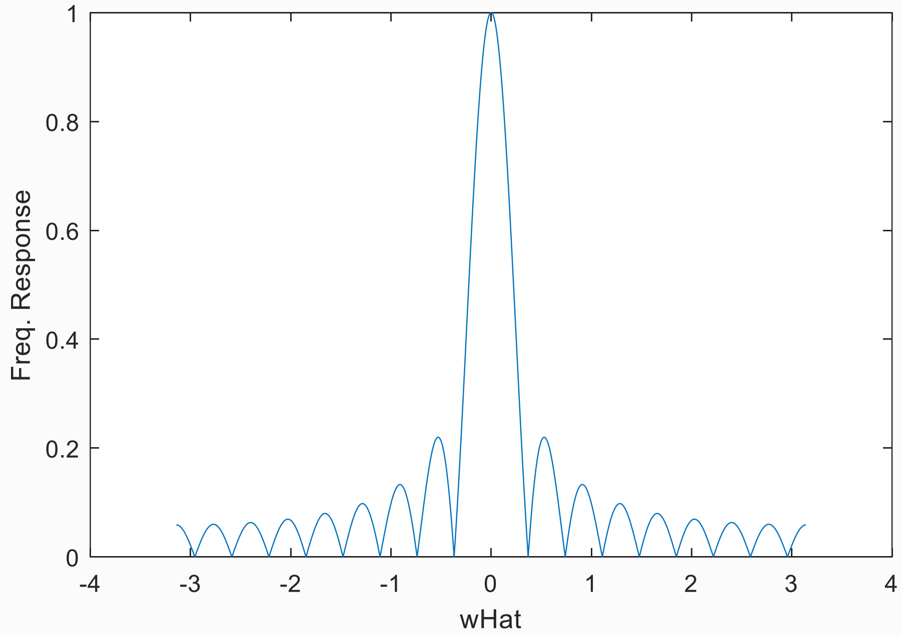
$$y(n) = \frac{1}{17}\sum_{k=0}^{16} x(n-k)$$

$$h(n) = \frac{1}{17}\sum_{k=0}^{16} \delta(n-k) = \frac{1}{17}\delta(n) + \cdots + \frac{1}{17}\delta(n-16)$$

$$\boxed{H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}}$$

```matlab
wHat = linspace(-pi,pi,Fs);
b = (1/17)*ones(1,17);
%%
H = zeros(1,Fs);
for k = 1:17
    H = H + b(k)*exp(-1j*wHat*k);
end

plot (wHat, abs(H));
xlabel('wHat');
ylabel('Freq. Response');
```

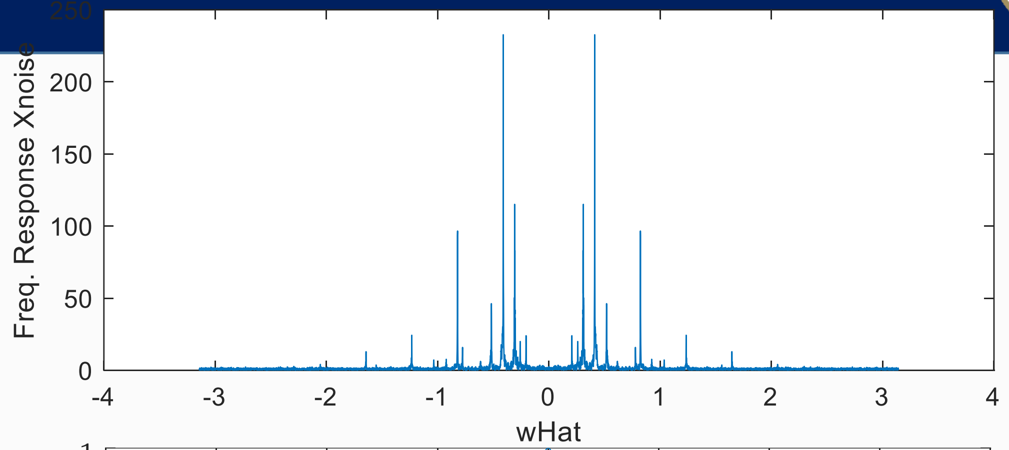


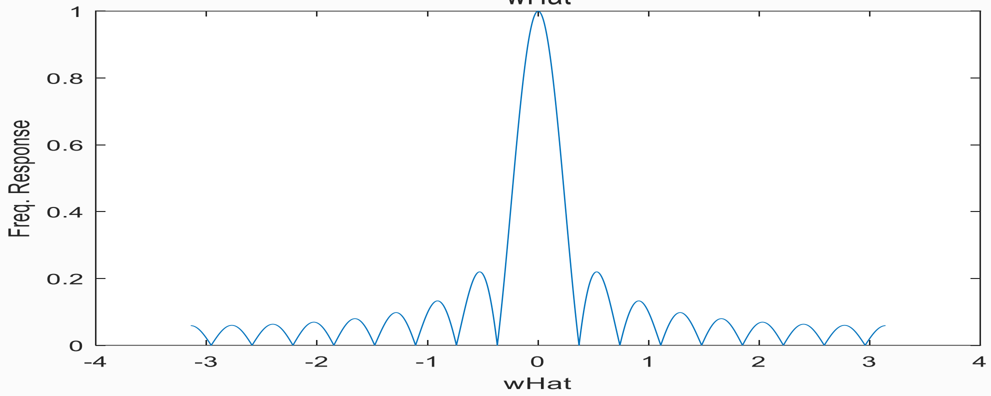
Let’s make a deep analysis

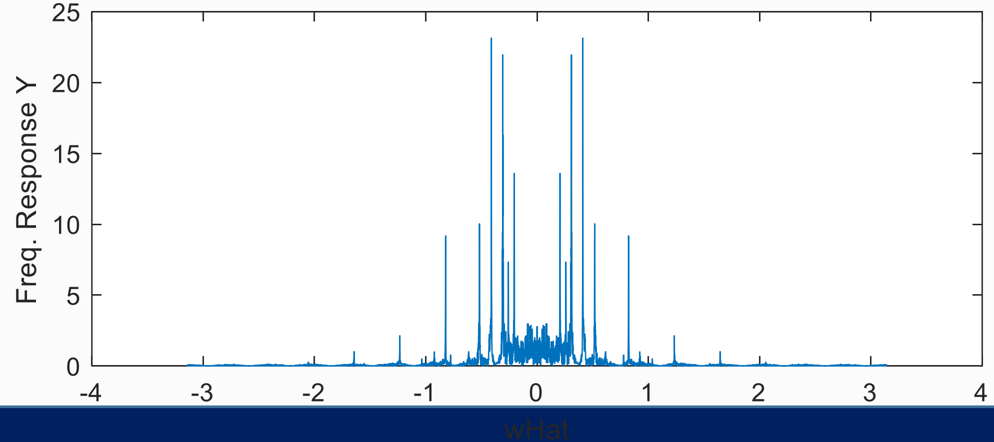
```
clc; clear all;
%% Load Sound
load ('piano2.mat');
x = x(1:16000);
X = fftshift(fft(x,Fs));
wHat = linspace(-pi,pi,Fs);
plot(wHat,abs(X));
xlabel('wHat');
ylabel('Freq. Response X');

%% Add Noise
Xnoise = awgn(x,40);
Xnoisef = fftshift(fft(Xnoise,Fs));
figure(2);
plot(wHat,abs(Xnoisef));
xlabel('wHat');
ylabel('Freq. Response Xnoise');

%%  Filter
N = 17; h = 1/N*ones(1,N);
y = conv(Xnoise,h,'same');
yf = fftshift(fft(y,Fs));
figure(3);
plot(wHat,abs(yf));
xlabel('wHat');
ylabel('Freq. Response Y');
```







Let’s make a deep analysis

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X = fftshift(fft(x,Fs));
wHat = linspace(-pi,pi,Fs);
plot(wHat,abs(X));
xlabel('wHat');
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Xnoise = awgn(x,40);
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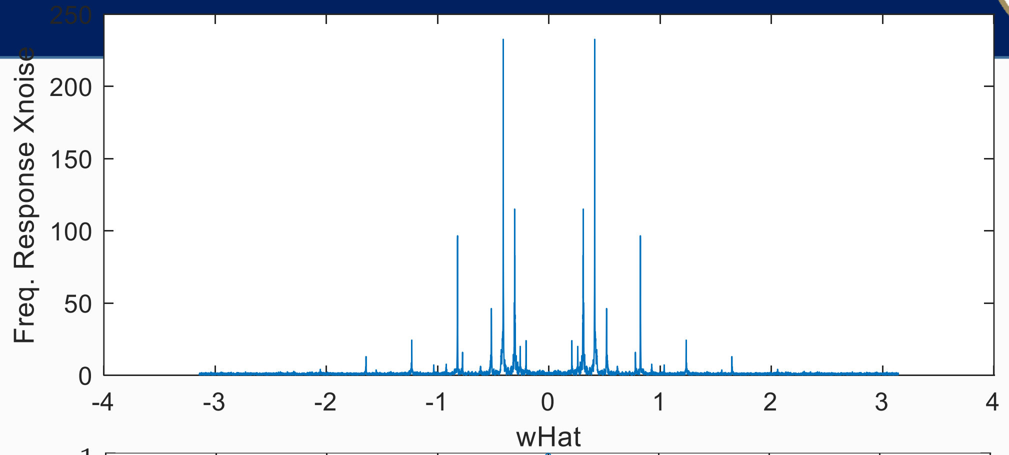
figure(3);
plot(wHat,abs(yf));
xlabel('wHat');
ylabel('Freq. Response Y');
```

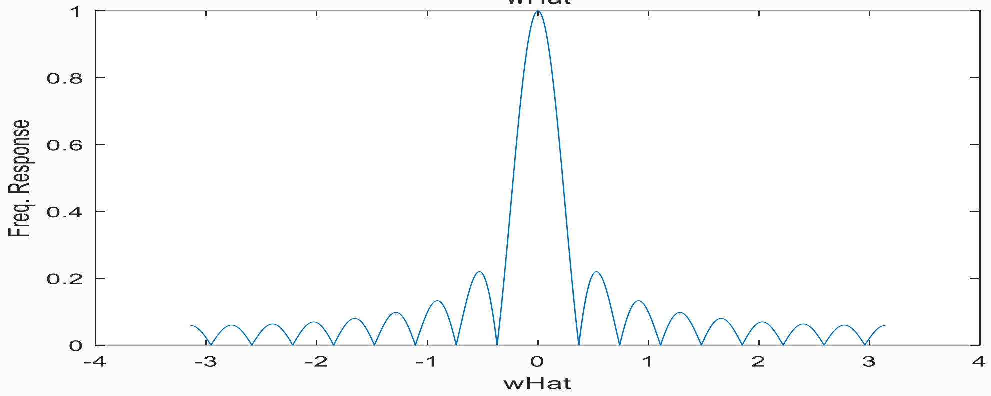
Input

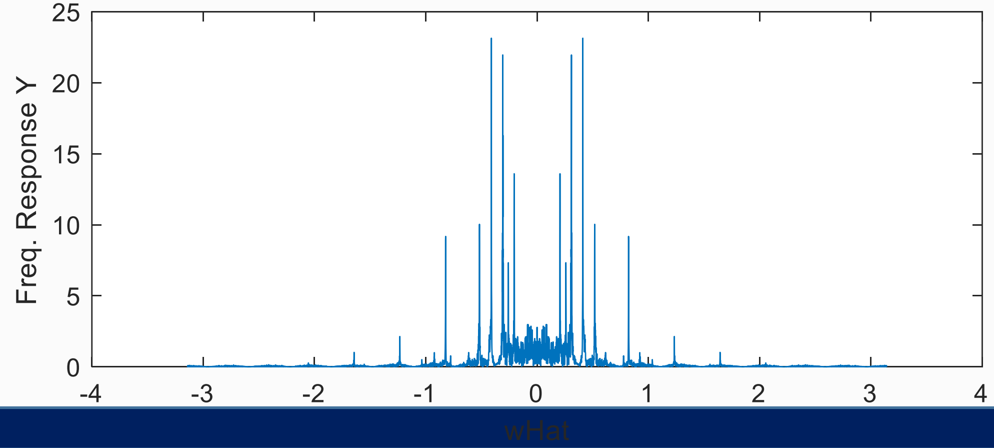
x

Filter

Output







Homework : Hearing Test – Audiometry Test

Conduct a test of your hearing, and present the results as a frequency response plot.

https://dspfirst.gatech.edu/chapters/06firfreq/labs/HearingTestFreqResponse/HearingTestFreqResponse.pdf

http://hyperphysics.phy-astr.gsu.edu/hbase/Sound/maxsens.html

Define a sampling frequency (Fs)

From 20 Hz to 22000 Hz with 100 Hz step do:

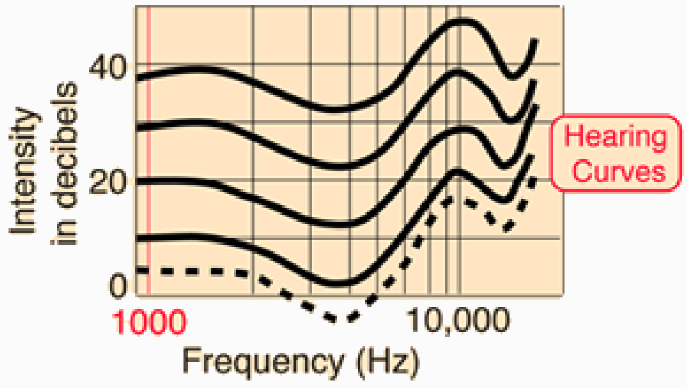
Play a tone with the selected frequency

Did you hear it: Give a score from 0-100.

Save this value for the last plot

Continue loop.

Use the hearing test to determine the frequency where your hearing sensitivity starts to drop significantly.



- Plot analog frequency vs. |H|. (Freq in logspace)
- Plot digital frequency vs. |H|. (Freq in logspace)

Exercise - 1

PROBLEM:

A linear time-invariant system is described by the FIR difference equation

$$y[n] = x[n] - 3x[n-1] + 9x[n-2] - 3x[n-3] + x[n-4]$$

(a) Write a simple formula for the magnitude of the frequency response $|H(e^{j\hat{\omega}})|$. Express your answer in terms of real-valued functions only.

(b) Derive a simple formula for the phase of the frequency response $\angle H(e^{j\hat{\omega}})$.

$$H(e^{j\hat{\omega}}) = 1 - 3e^{-j\hat{\omega}} + 9e^{-j2\hat{\omega}} - 3e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$= e^{-j2\hat{\omega}}\left(e^{j2\hat{\omega}} - 3e^{j\hat{\omega}} + 9 - 3e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right)$$

$$= e^{-j2\hat{\omega}}\left(2\cos 2\hat{\omega} - 6\cos\hat{\omega} + 9\right)$$

$\Rightarrow$

$$|H(e^{j\hat{\omega}})| = 2\cos 2\hat{\omega} - 6\cos\hat{\omega} + 9$$

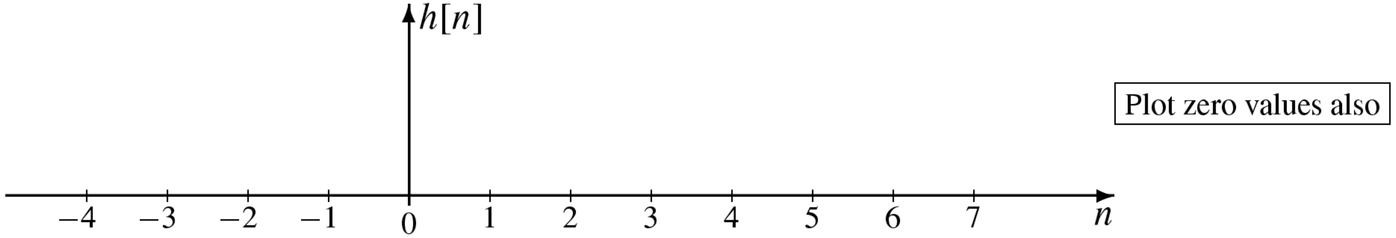
$$\angle H(e^{j\hat{\omega}}) = -2\hat{\omega}$$

Exercise - 2

PROBLEM:

The following FIR filter is specified by the filter coefficients $\{b_k\} = \{2, 0, -4, 0, 2\}$

(a) Determine the impulse response: give your answer as a $\boxed{\text{plot}}$ of $h[n]$ vs. n.

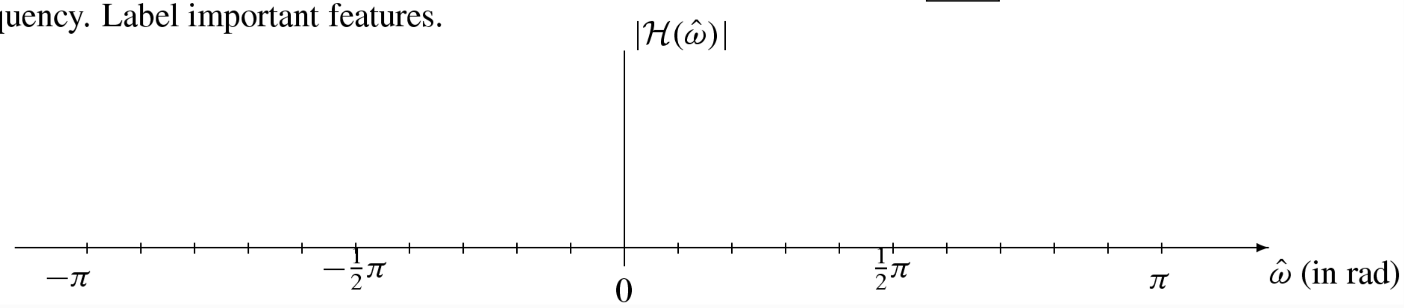


(b) Determine the frequency response, $\mathcal{H}(\hat{\omega})$, and select one of the following as the correct answer:

(A) $(4 - 4\cos(2\hat{\omega}))e^{-j(2\hat{\omega}-\pi)}$ **(B)** $2\cos\hat{\omega} + 4e^{-j(2\hat{\omega}+\pi)}$ **(C)** $(4\cos(2\hat{\omega}) - 4)e^{-j\hat{\omega}}$

(D) $2\cos(2\hat{\omega}) - 4$

(c) Determine the magnitude of $\mathcal{H}(\hat{\omega})$ and present your answer as a a $\boxed{\text{plot}}$ of the magnitude vs. frequency. Label important features.



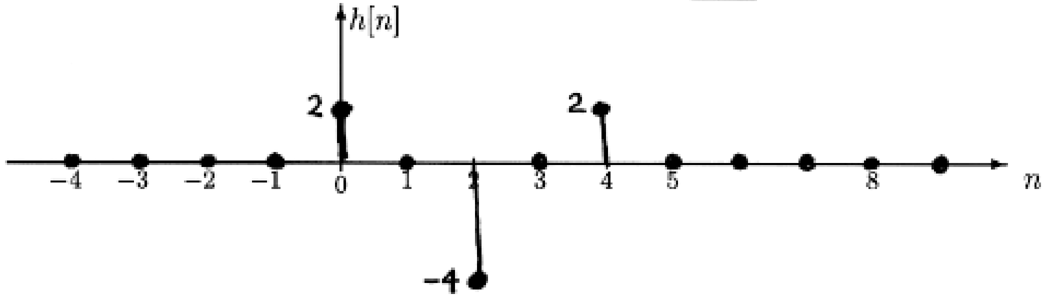
Solution

The following FIR filter is specified by the filter coefficients $\{b_k\} = \{2, 0, -4, 0, 2\}$

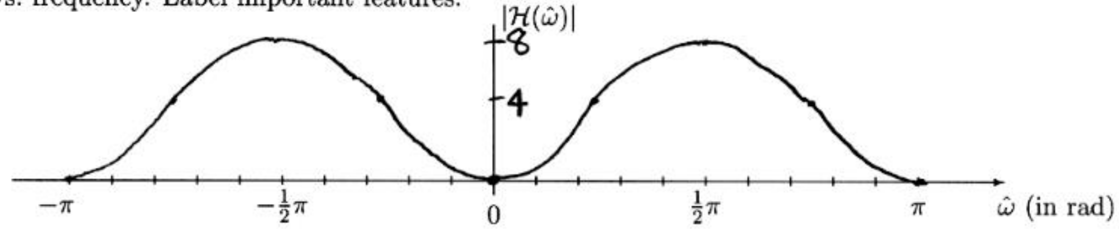
$y[n] = 2x[n] - 4x[n-2] + 2x[n-4]$

$h[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-4]$.

(a) Determine the impulse response: give your answer as a $\boxed{\text{plot}}$ of $h[n]$ vs. n.



(c) Determine the magnitude of $\mathcal{H}(\hat{\omega})$ and present your answer as a a $\boxed{\text{plot}}$ of the magnitude vs. frequency. Label important features.



$$|\mathcal{H}(\hat{\omega})| = \underbrace{|4 - 4\cos 2\hat{\omega}|}_{\text{This is non-negative}} = 4 - 4\cos 2\hat{\omega}$$

$\hat{\omega} = 0 \Rightarrow 4 - 4 = 0$

$\hat{\omega} = \pi \Rightarrow 4 - 4 = 0$

$\hat{\omega} = \pi/2 \Rightarrow 4 - 4(-1) = 8$

$\hat{\omega} = \pi/4 \Rightarrow 4 - 4(0) = 4$

(b) Determine the frequency response, $\mathcal{H}(\hat{\omega})$, and select one of the following as the correct answer:

(A) $(4 - 4\cos(2\hat{\omega}))e^{-j(2\hat{\omega}-\pi)}$ **(B)** $2\cos\hat{\omega} + 4e^{-j(2\hat{\omega}+\pi)}$ **(C)** $(4\cos(2\hat{\omega}) - 4)e^{-j\hat{\omega}}$

(D) $2\cos(2\hat{\omega}) - 4$

$$\mathcal{H}(\hat{\omega}) = 2 - 4e^{-j2\hat{\omega}} + 2e^{-j4\hat{\omega}}$$

$$= e^{-j2\hat{\omega}}(2e^{+j2\hat{\omega}} - 4 + 2e^{-j2\hat{\omega}})$$

$$= e^{-j2\hat{\omega}}(4\cos 2\hat{\omega} - 4)$$

$$= e^{-j2\hat{\omega}} e^{j\pi}(4 - 4\cos 2\hat{\omega})$$

A

Exercise - 3

PROBLEM:

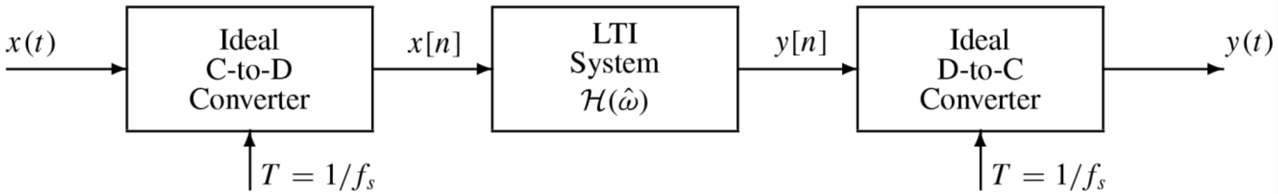
The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 2\cos(6000\pi t - \pi/4) + 11\cos(12000\pi t - \pi/3)$$

The frequency response for the digital filter (LTI system) is

$$\mathcal{H}(\hat{\omega}) = \frac{\sin(5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

If $f_s = 10000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.



Exercise - 3

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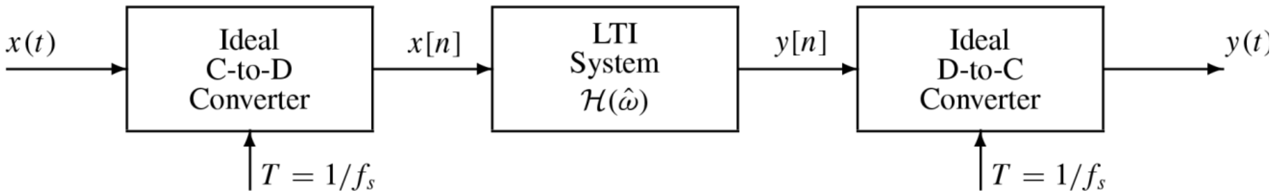
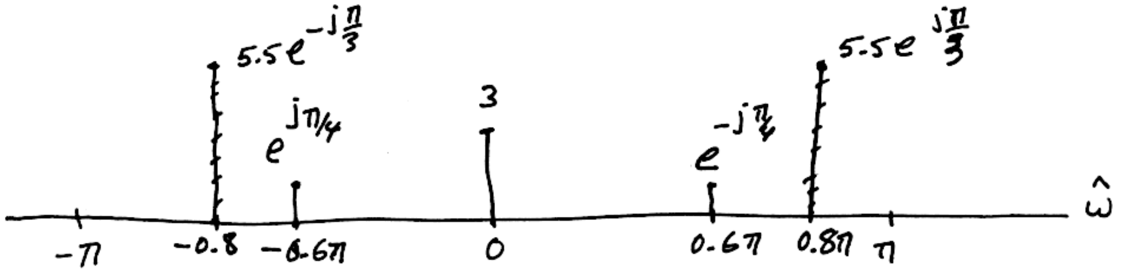


$$x[n] = 3 + 2\cos(0.6\pi n - \pi/4) + 11\cos(1.2\pi n - \pi/3)$$



$$H(\hat{\omega}) = \frac{\sin(5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}e^{-j5\hat{\omega}}$$

$$H(0) = 10$$

$$H(0.6\pi) = \frac{\sin 3\pi}{\sin 0.3\pi}e^{-j3\pi} = 0$$

$$H(0.8\pi) = \frac{\sin 4\pi}{\sin 0.4\pi}e^{-j4\pi} = 0$$

$$y[n] = 10 \times 3 = 30, \quad y(t) = 30$$