BLM3620 Digital Signal Processing*

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*Based on lecture notes from Ali Can Karaca & Ahmet Elbir

Lecture #6 – Convolution and FIR Filters

- Convolution Example
- Graphical Convolution
- MATLAB demo
- FIR Filter
- FIR Filter Application

Remember: DT Convolution

There are three approaches to calculate convolution:

1) Mathematical Approach
2) Table Approach (Polynomial Multiplication)
3) Graphical Approach

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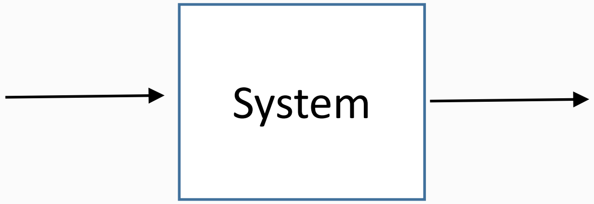

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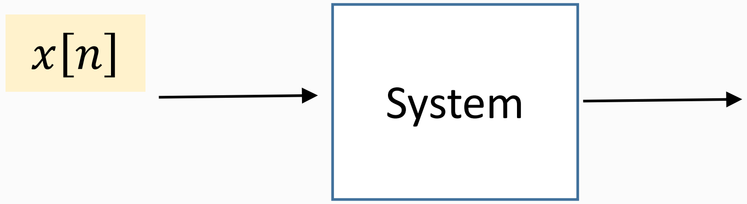

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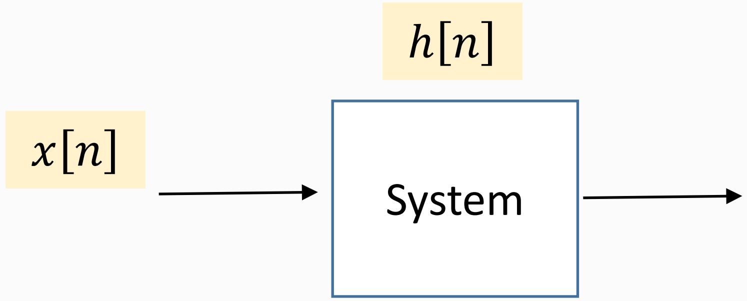

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Geometric Serial Sum

$$\sum_{n=M}^{N} r^k = \frac{r^{N+1} - r^M}{r - 1}$$

Convolution Method – 3: Graphical Approach

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

For n=0,

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[-k]$$

For n=-5,

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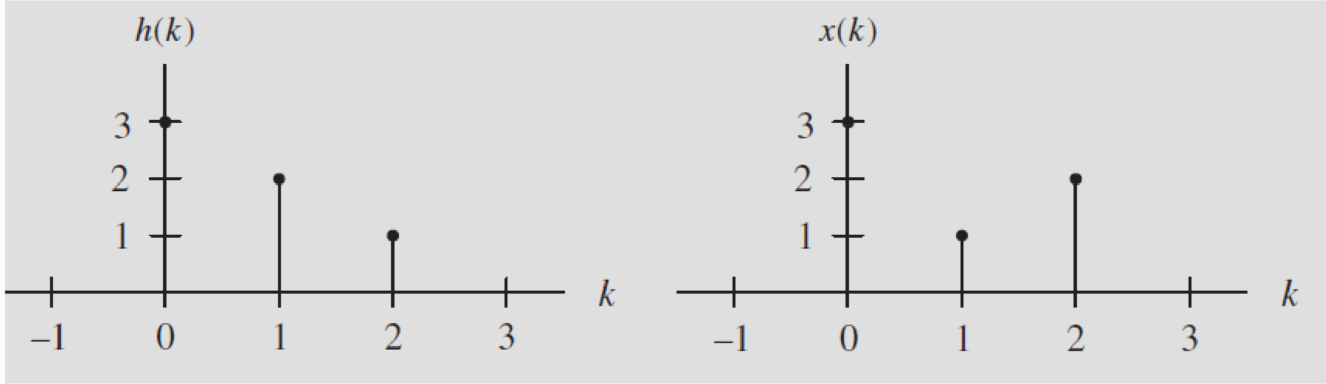
Step 1. Obtain the reversed sequence $h(-k)$.

Step 2. Shift $h(-k)$ by $|n|$ samples to get $h(n-k)$. If $n \geq 0$, $h(-k)$ will be shifted to right by n samples; but if $n < 0$, $h(-k)$ will be shifted to the left by $|n|$ samples.

Step 3. Perform the convolution sum that is the sum of products of two sequences $x(k)$ and $h(n-k)$ to get $y(n)$.

Step 4. Repeat steps (1)–(3) for the next convolution value $y(n)$.

Example for Graphical Approach



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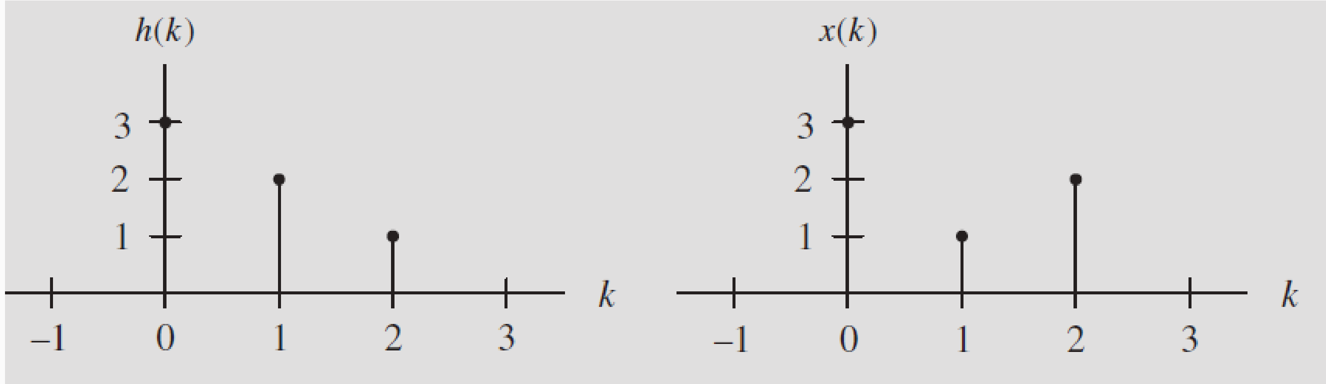

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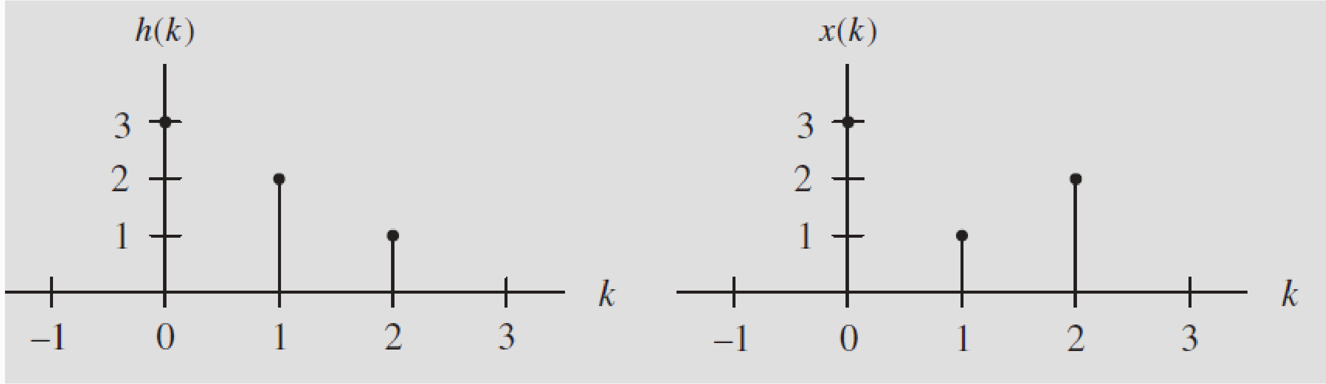

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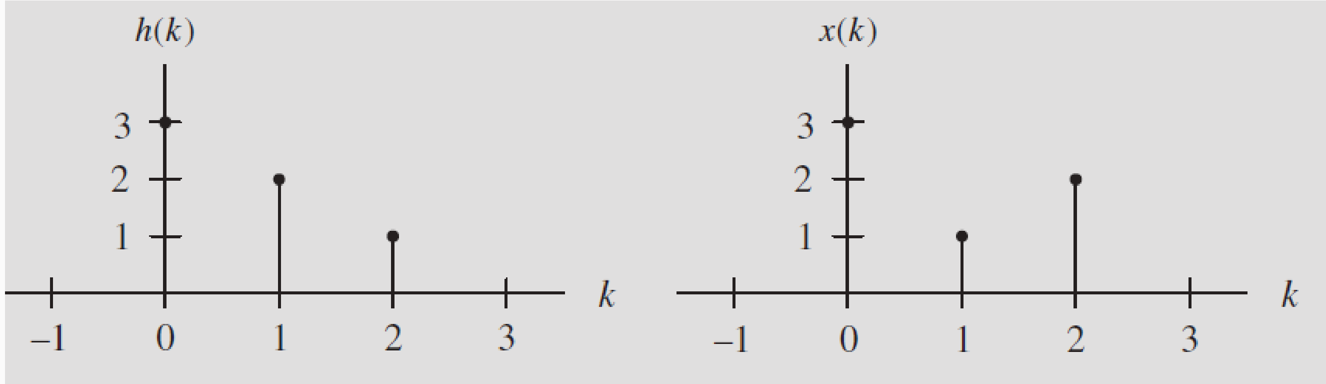

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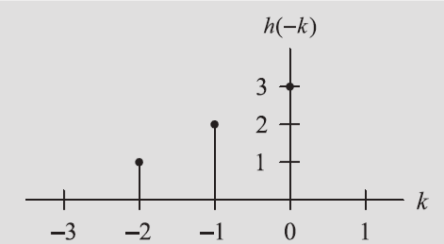

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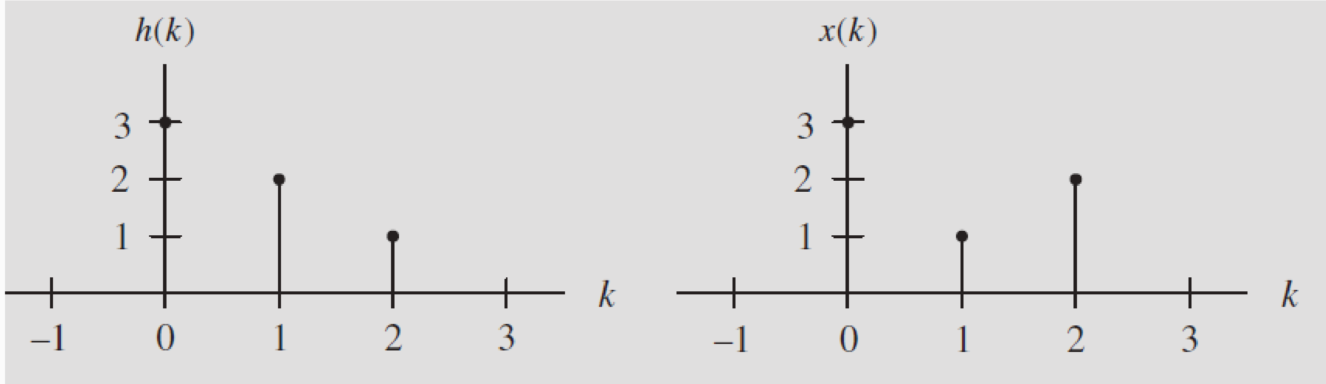

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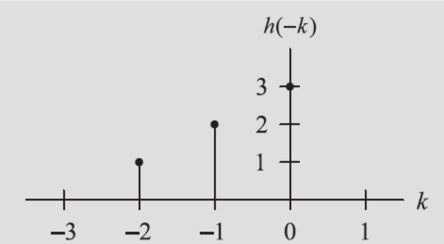

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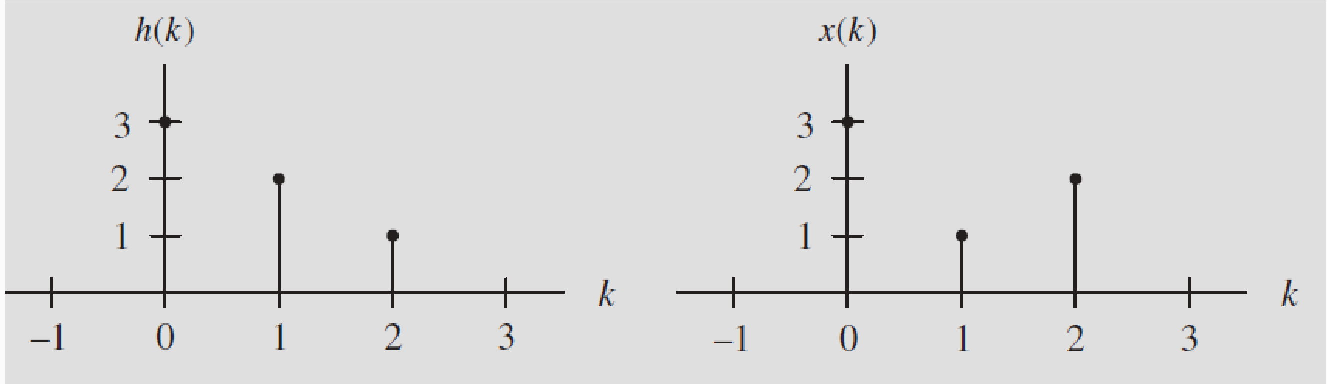

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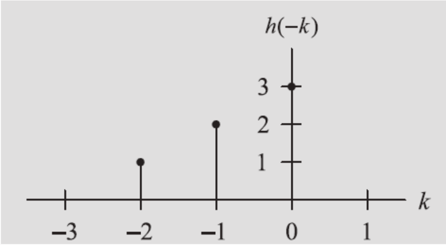

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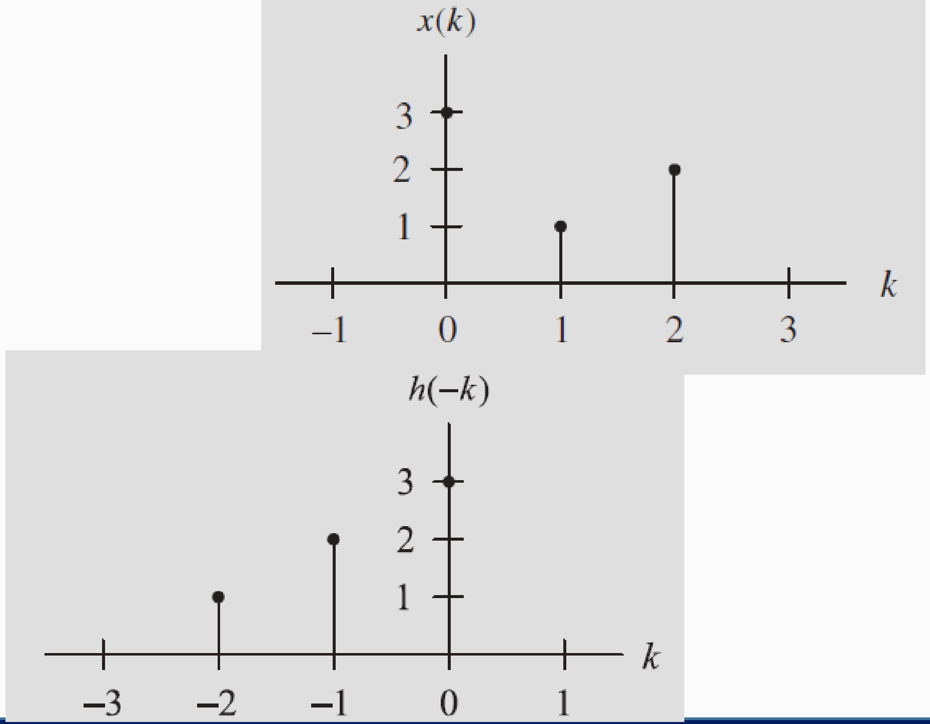

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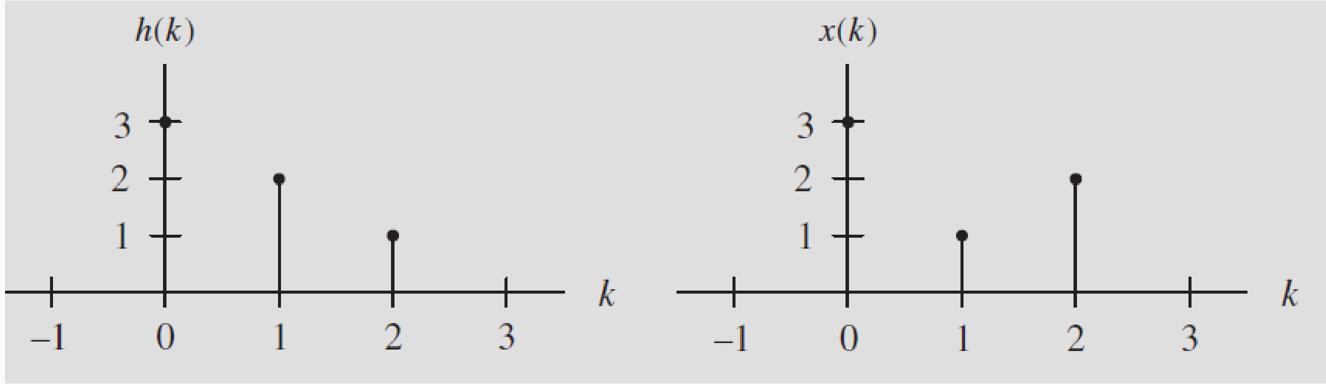

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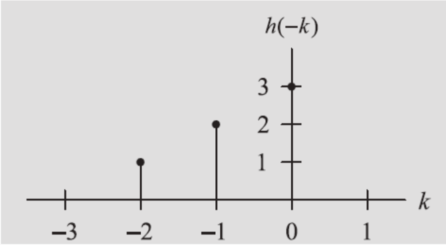

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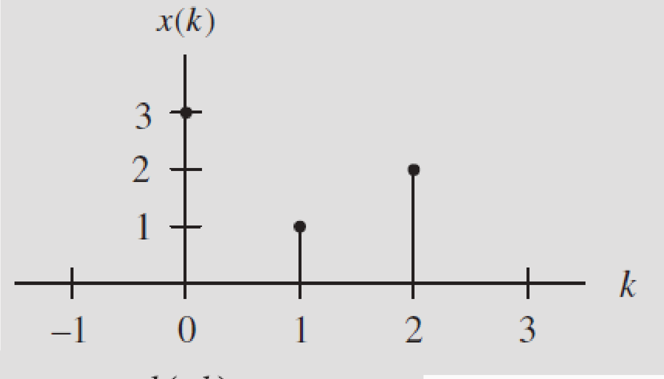

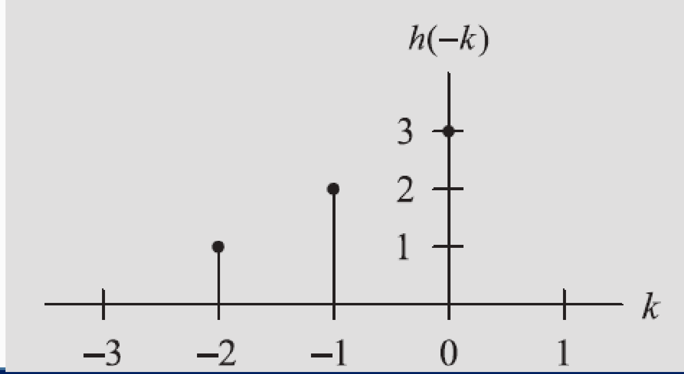

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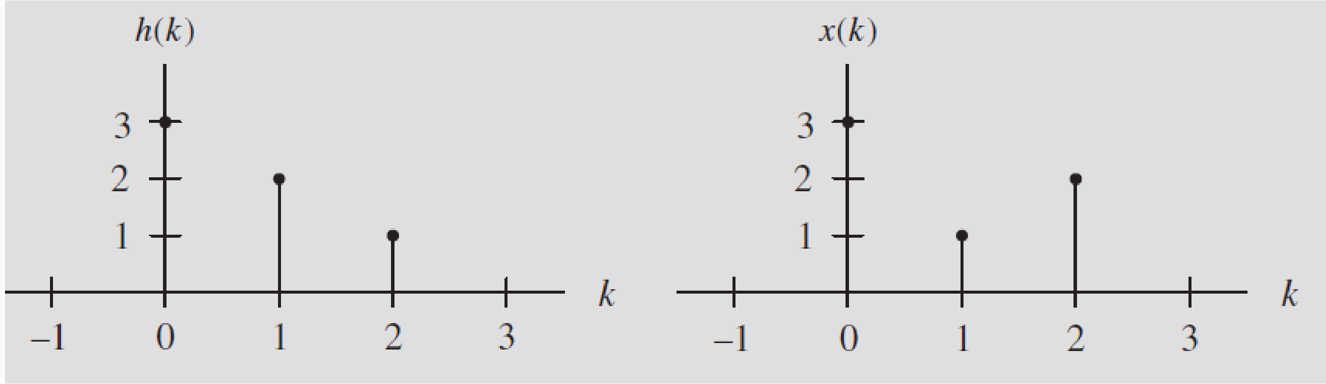

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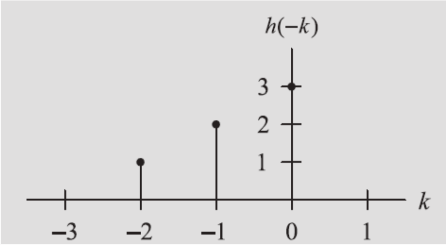

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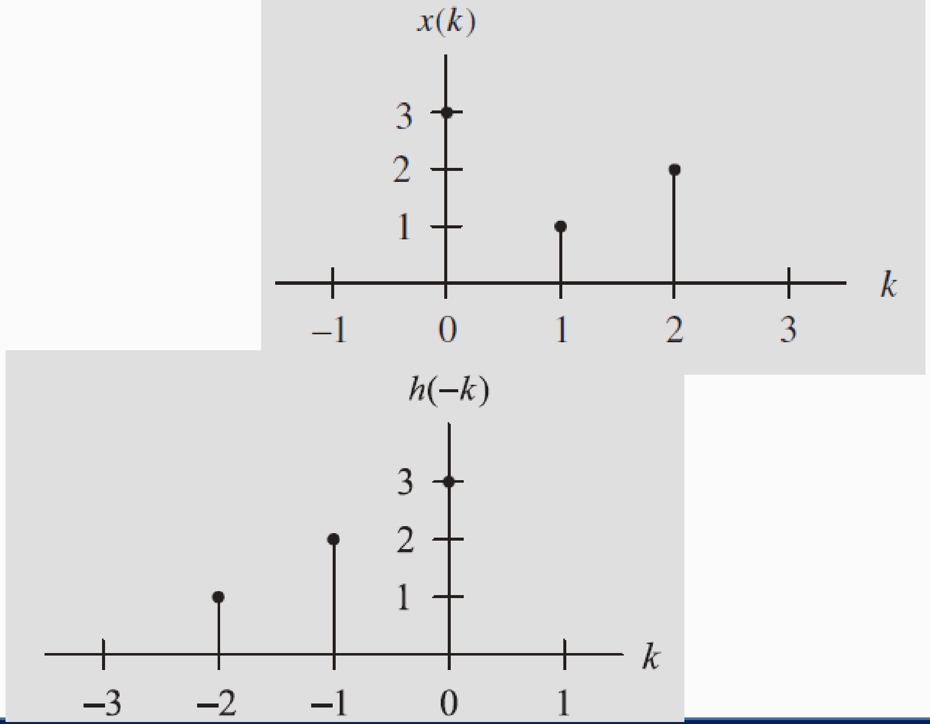

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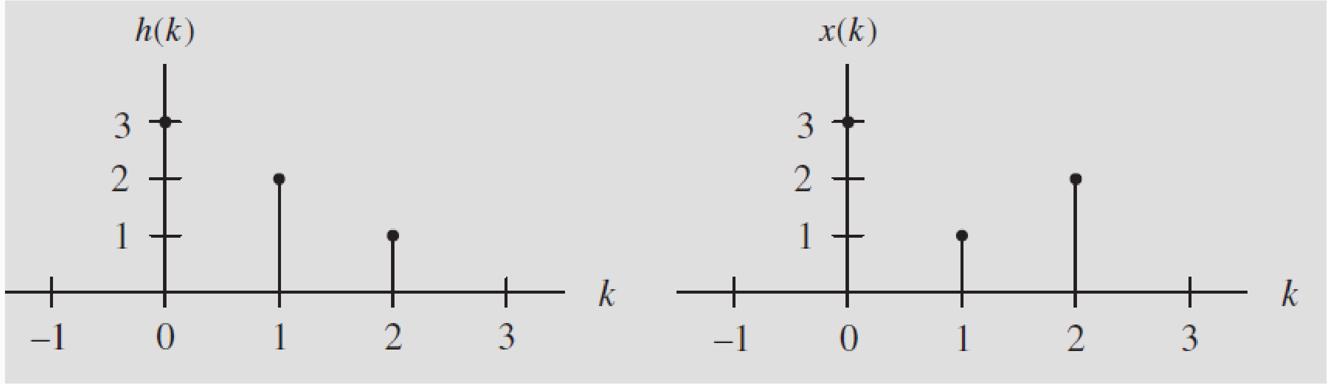

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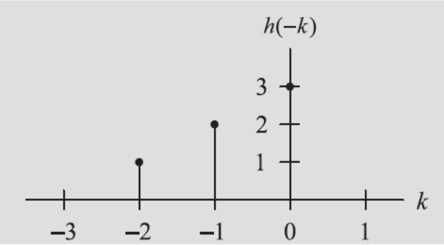

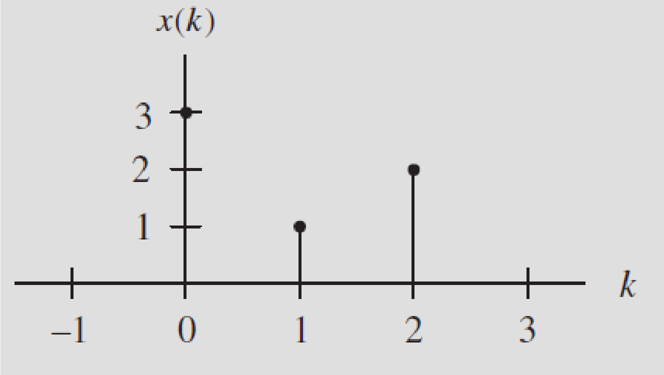

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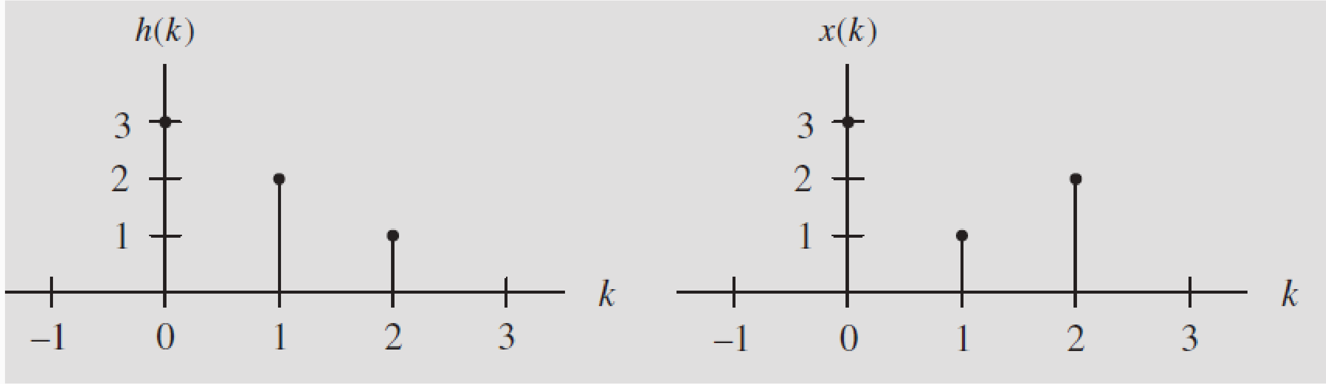

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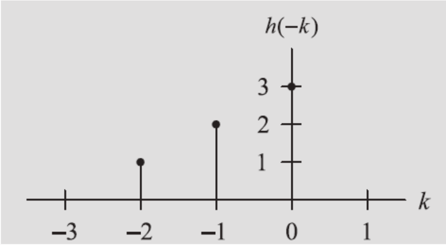

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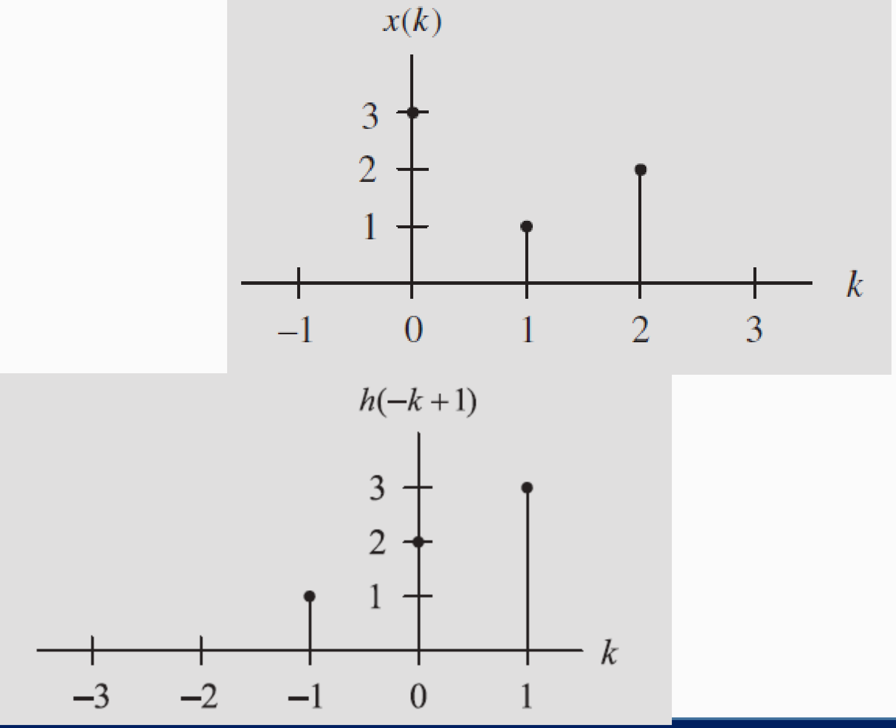

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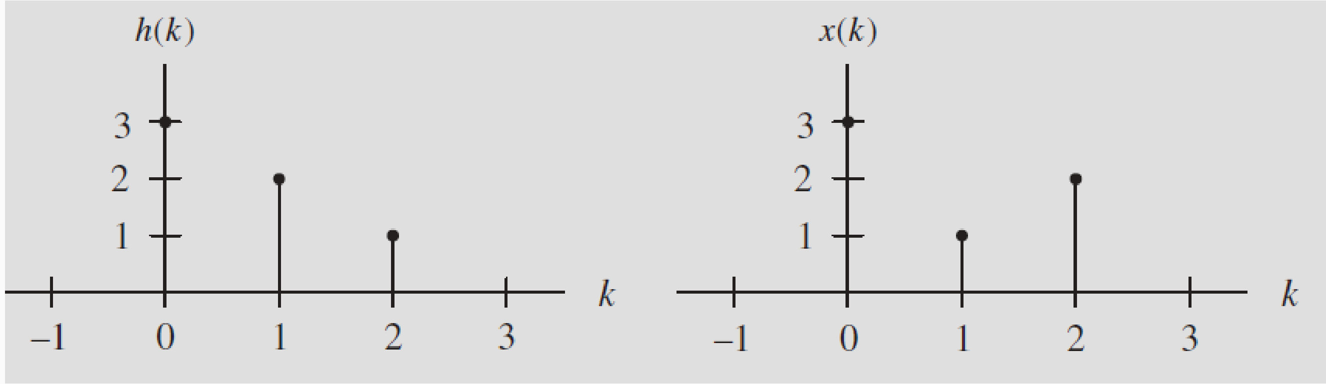

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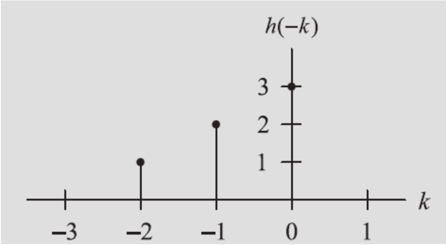

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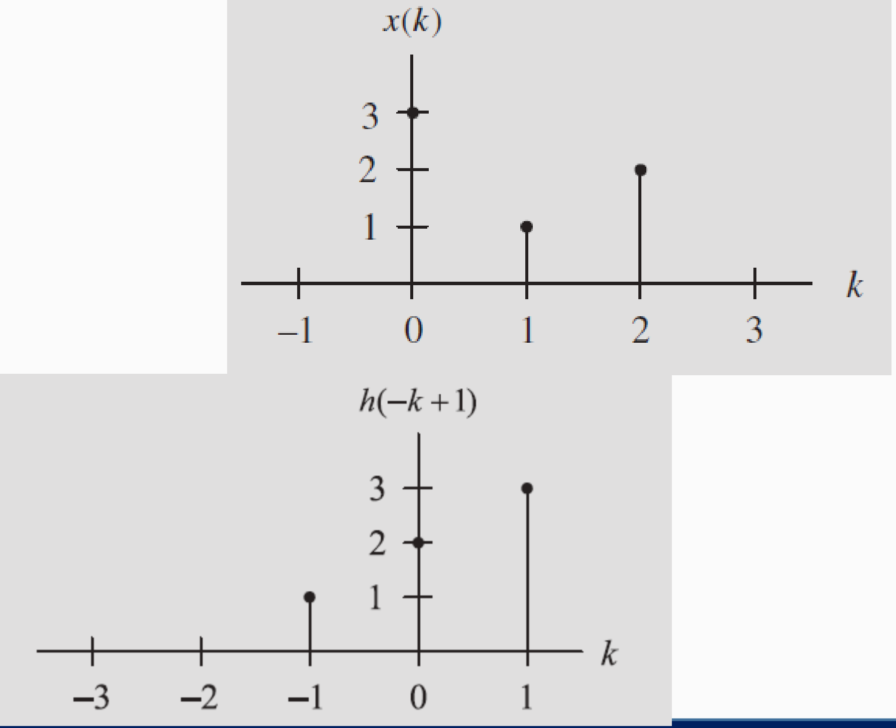

Equal to:

$$y[1] = \sum_{k=0}^{3} x[k]h[1-k]$$

y[1]=2*3+1*3=9

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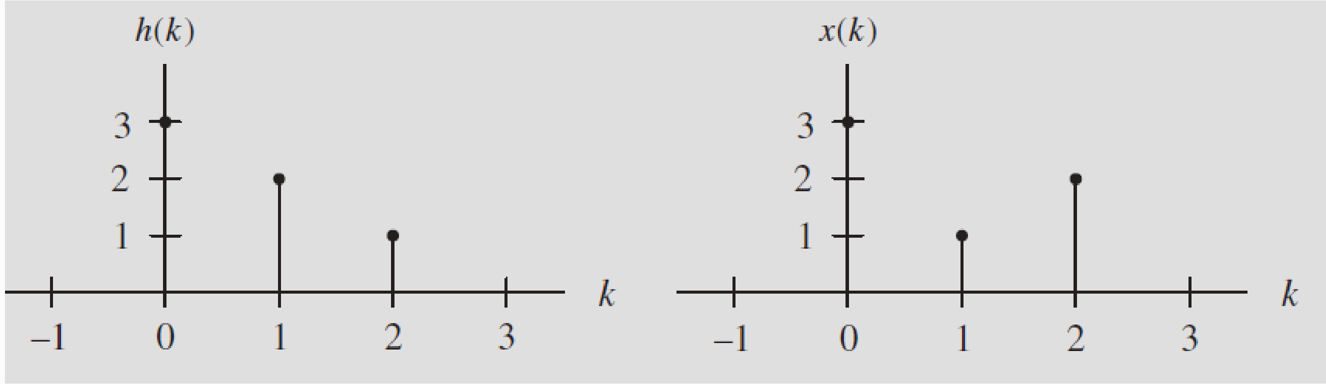

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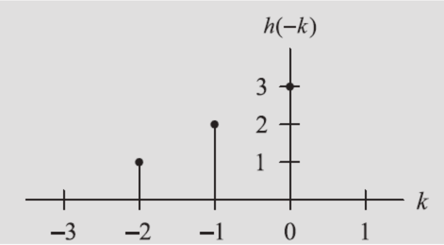

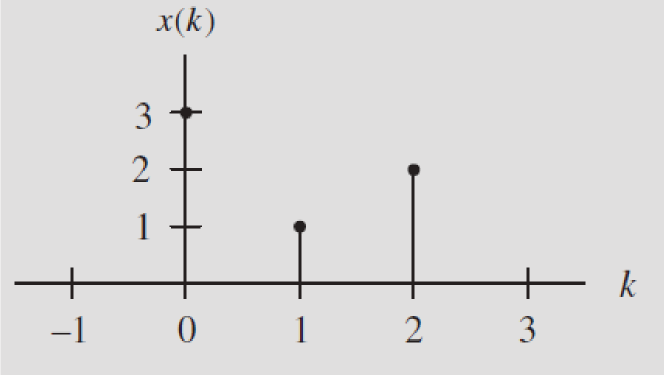

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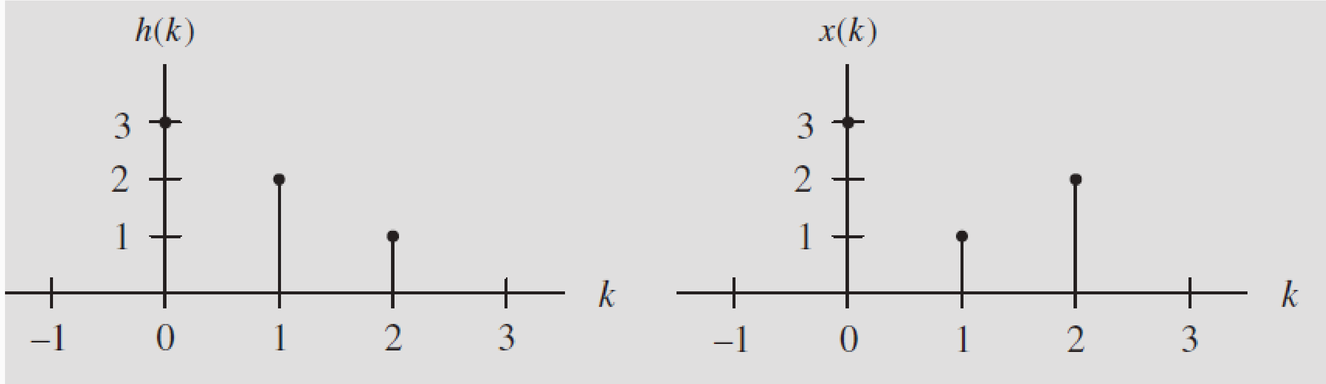

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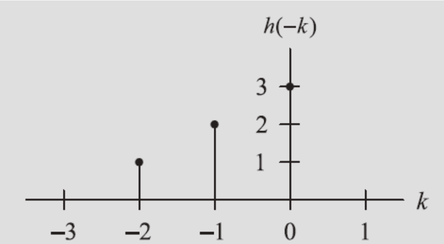

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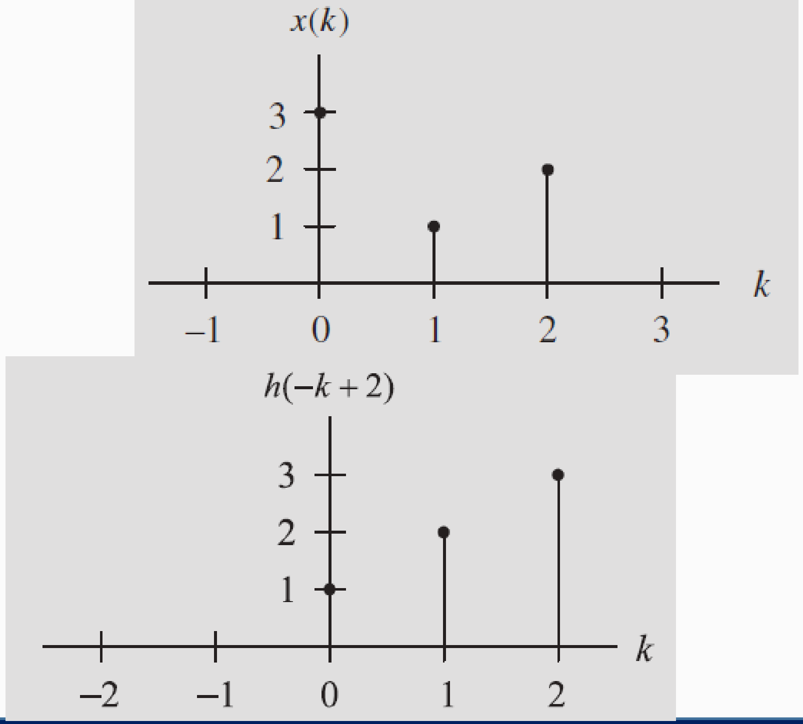

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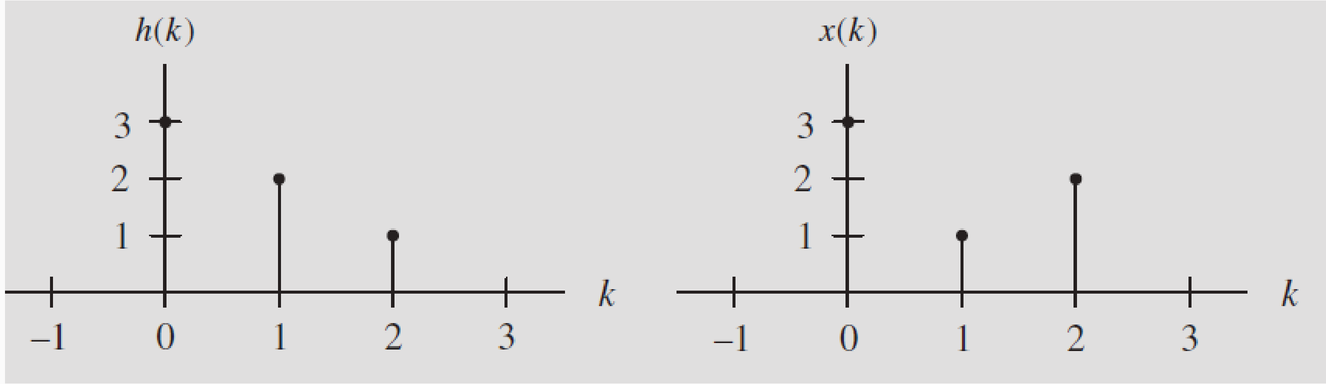

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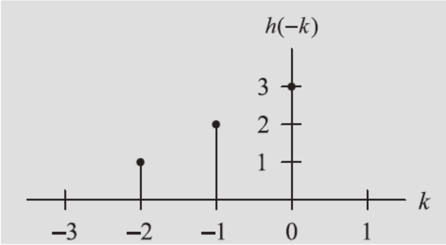
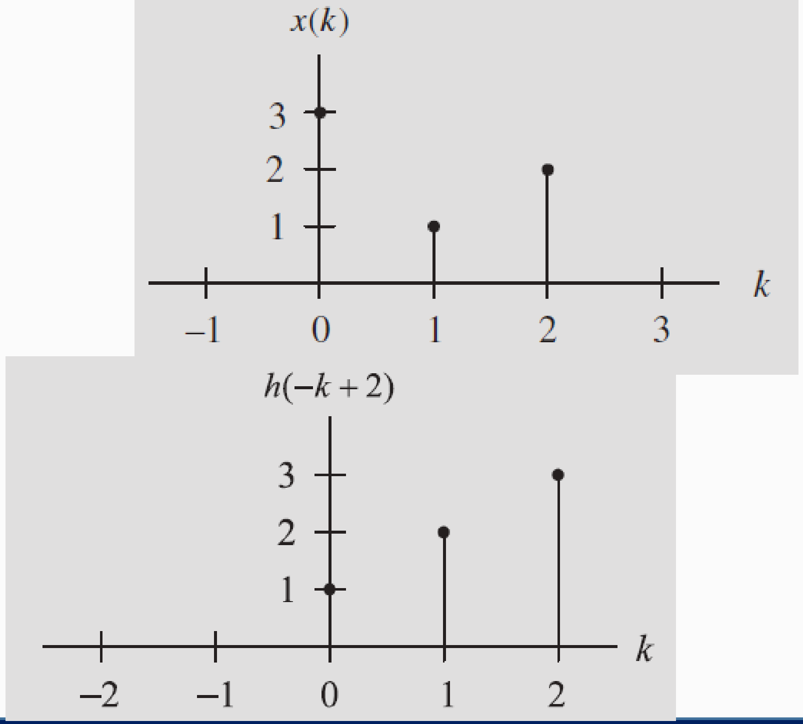


2) Shift it by *2* and get h(2-k)

3) Perform conv. sum

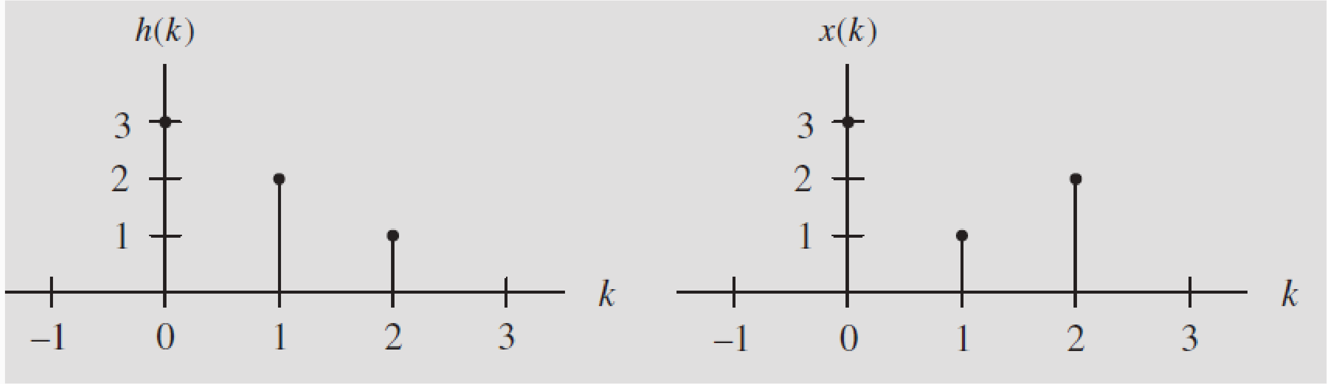


Equal to:

$$y[2] = \sum_{k=0}^{3} x[k]h[2-k]$$

y[2]=1*3+2*1+3*2=10

Example for Graphical Approach



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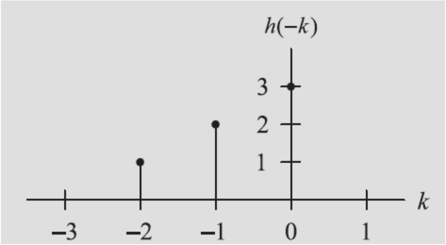
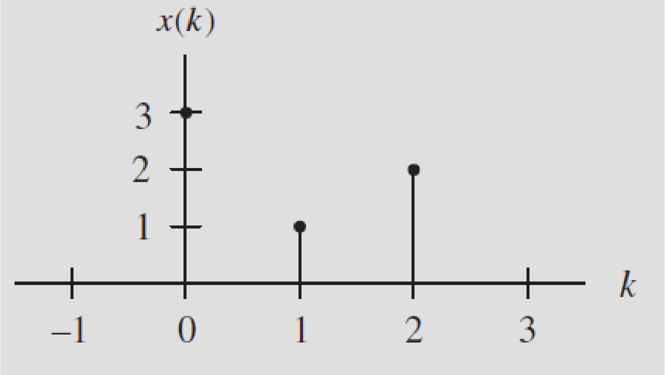
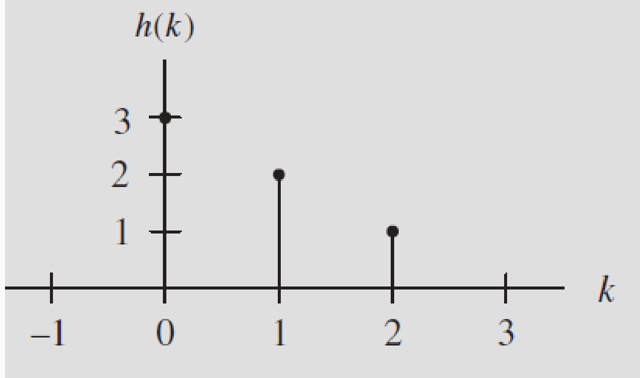


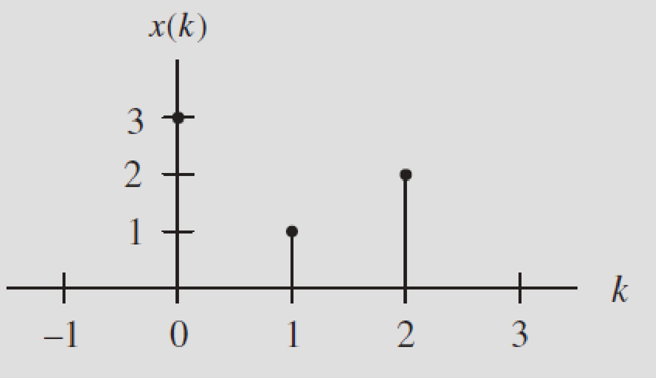


2) Shift it by *3* and get h(3-k)

3) Perform conv. sum

Example for Graphical Approach





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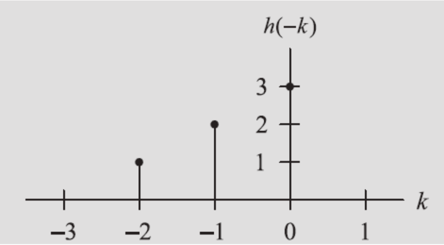

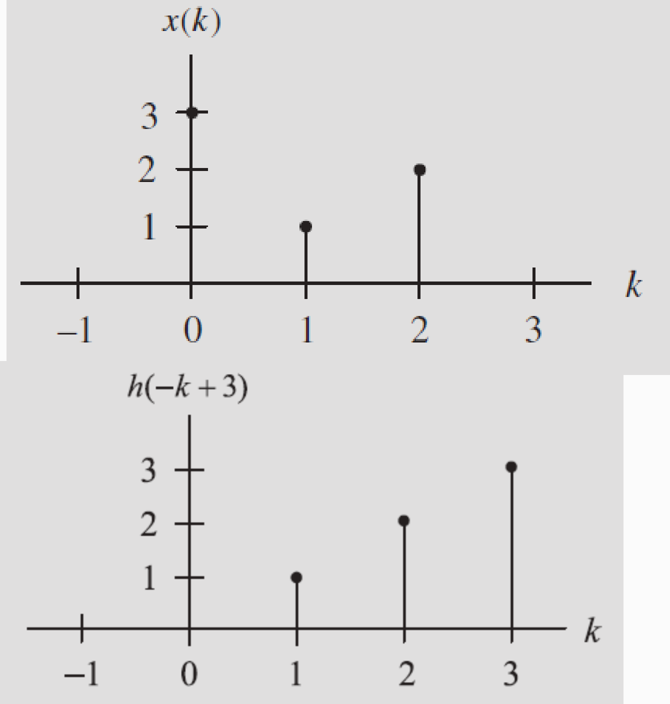

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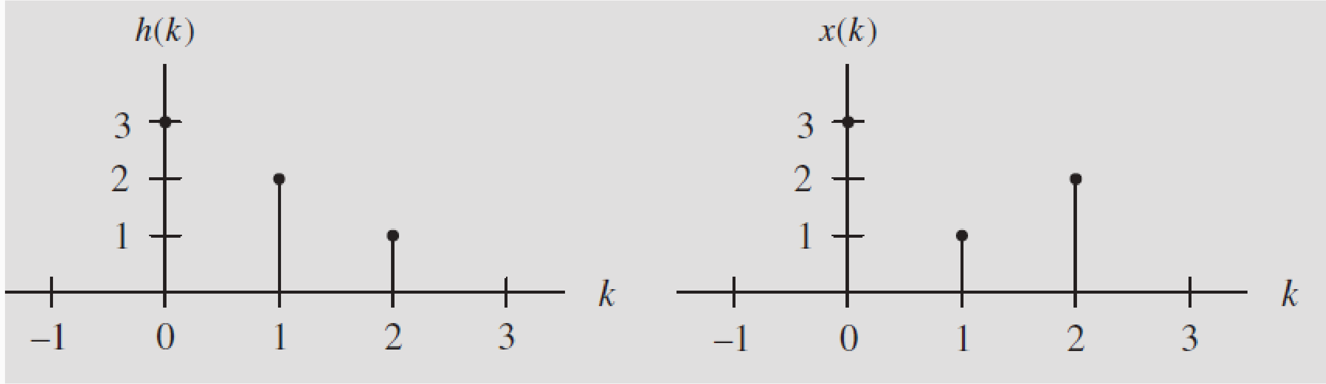

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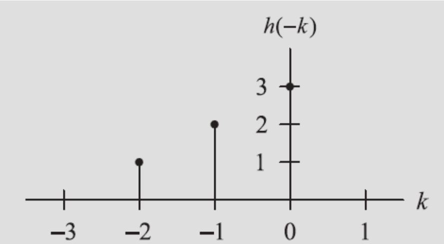

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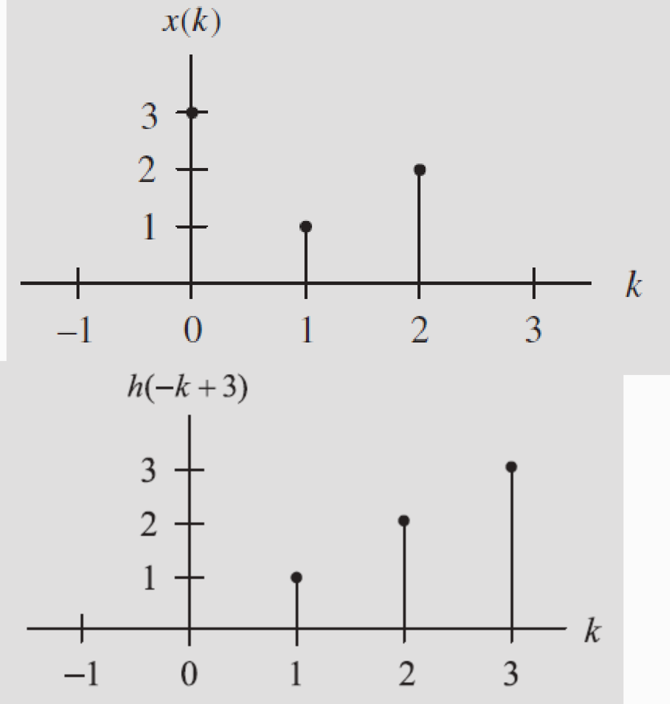
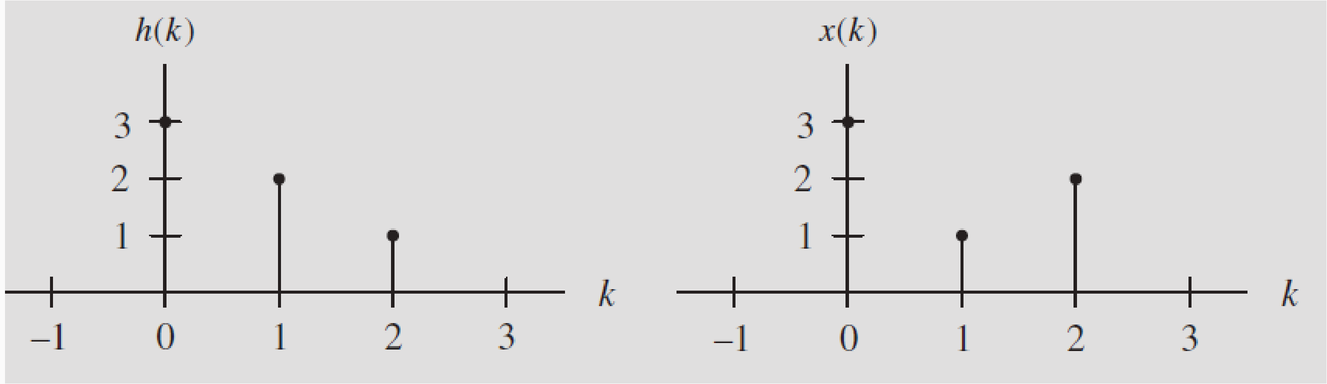


Equal to:

$$y[3] = \sum_{k=0}^{3} x[k]h[3-k]$$

y[3]=1*1+2*2=5

Example for Graphical Approach



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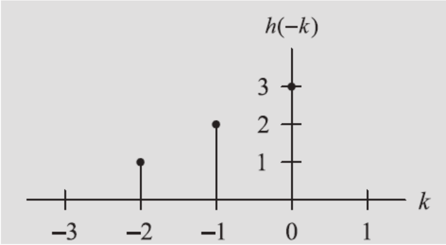
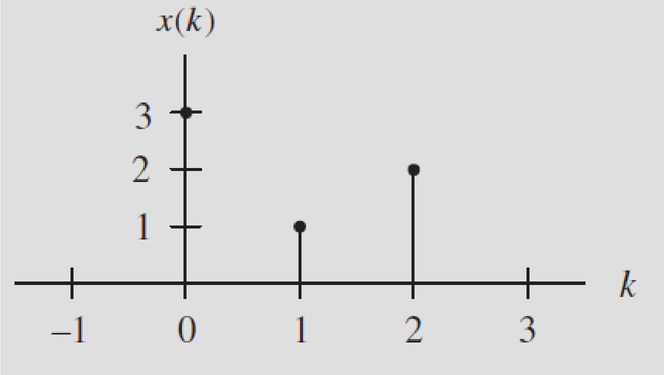
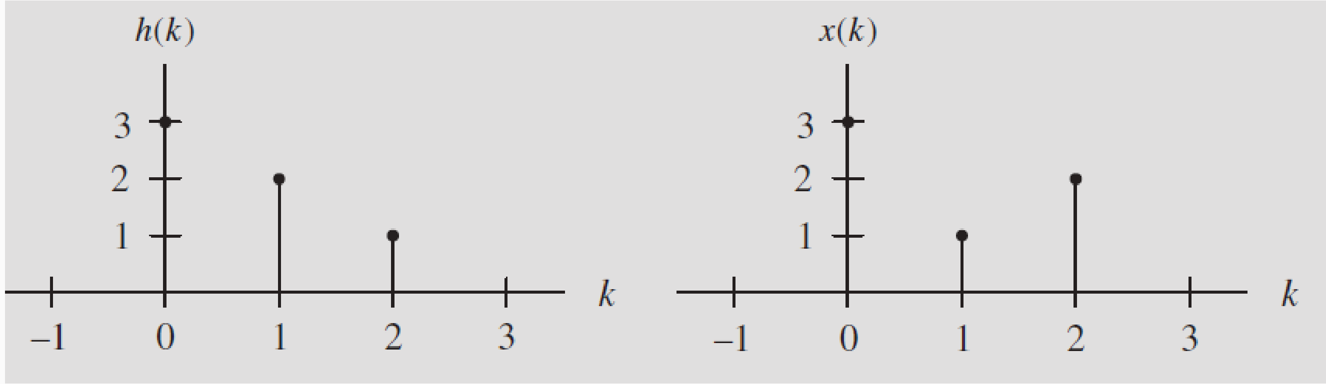




2) Shift it by *4* and get h(4-k)

3) Perform conv. sum

Example for Graphical Approach



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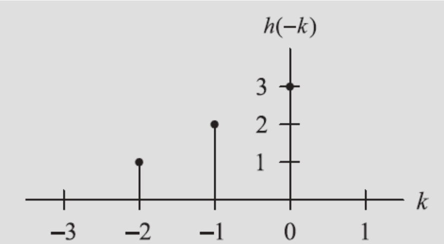

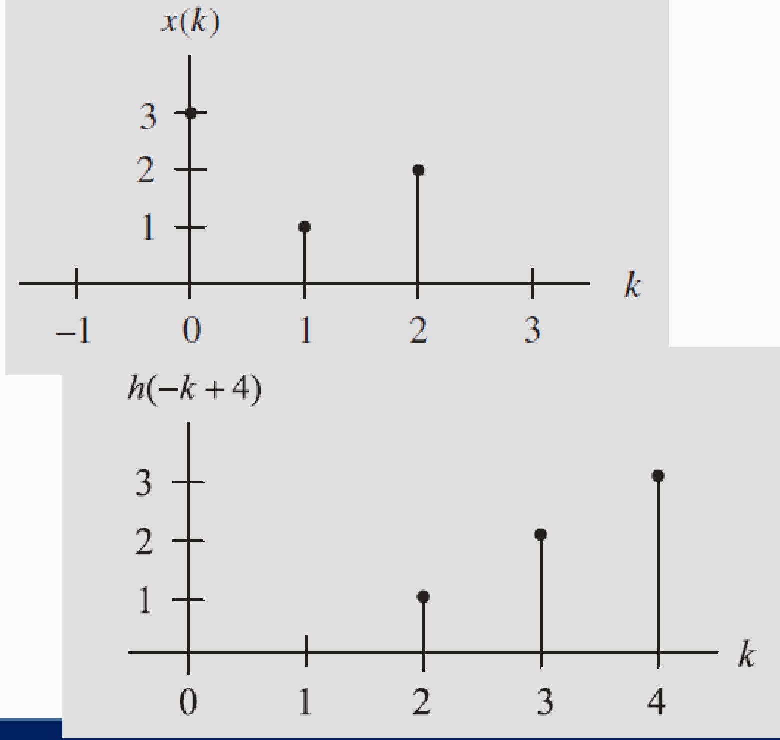

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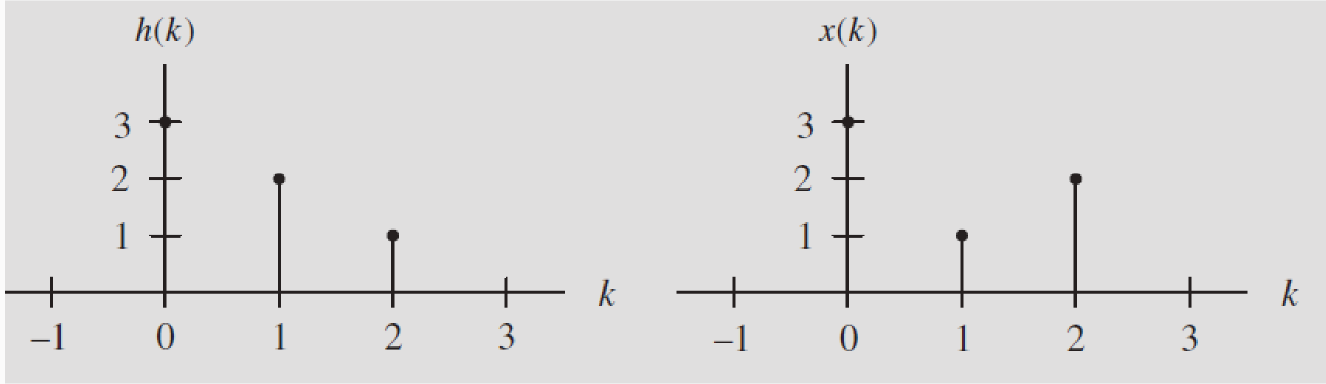
Example for Graphical Approach



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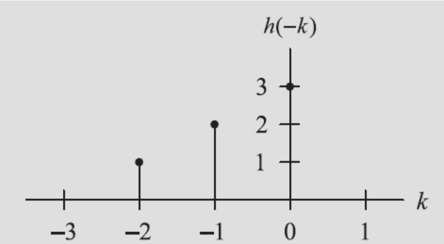

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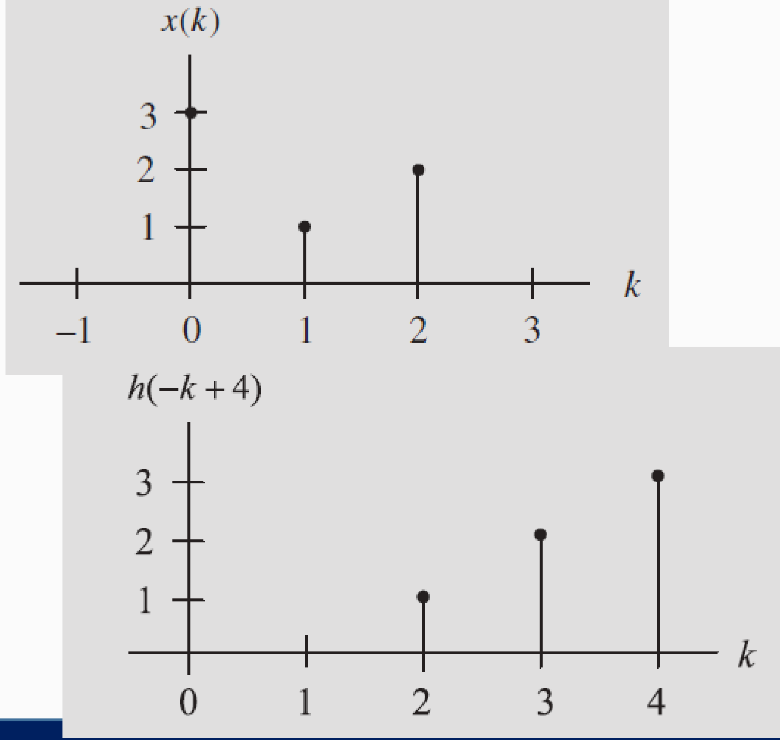
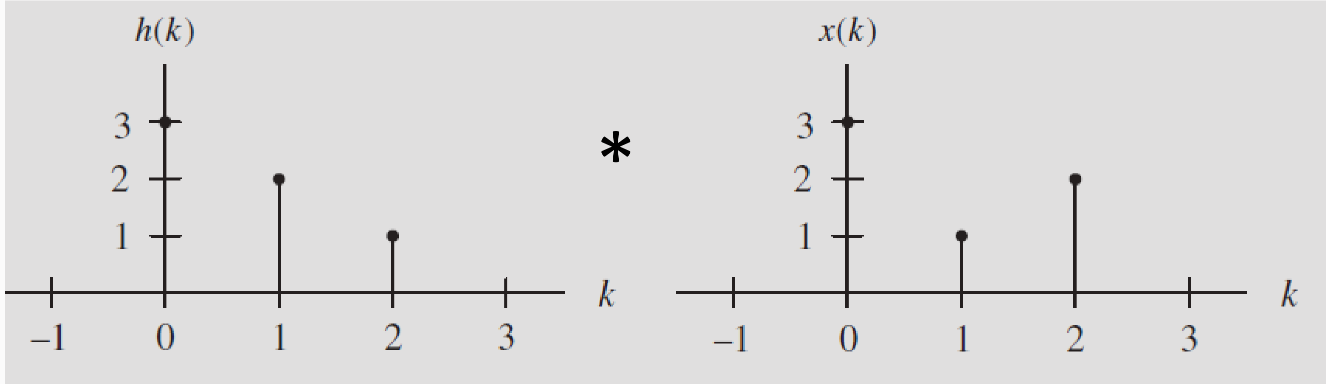


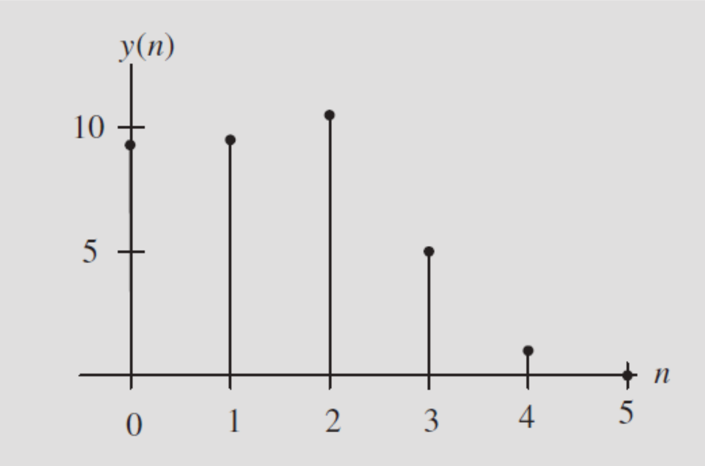
Equal to:

$$y[4] = \sum_{k=0}^{3} x[k]h[4-k]$$

$\boxed{y[4]=2}$

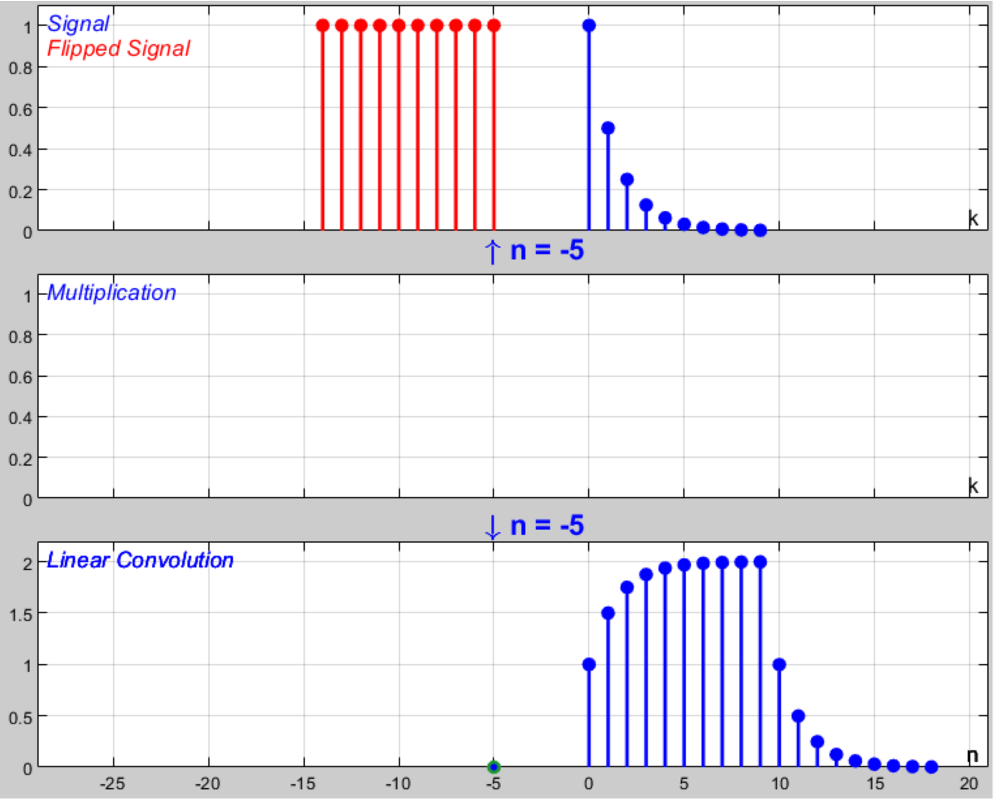
Example for Graphical Approach

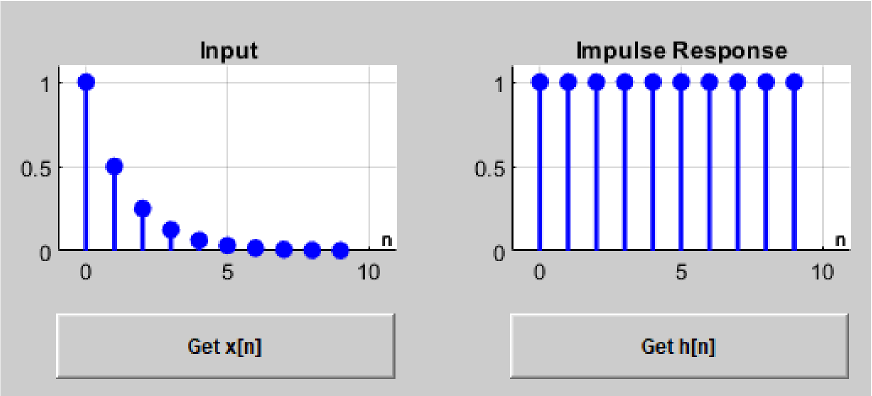




MATLAB dconv Demo

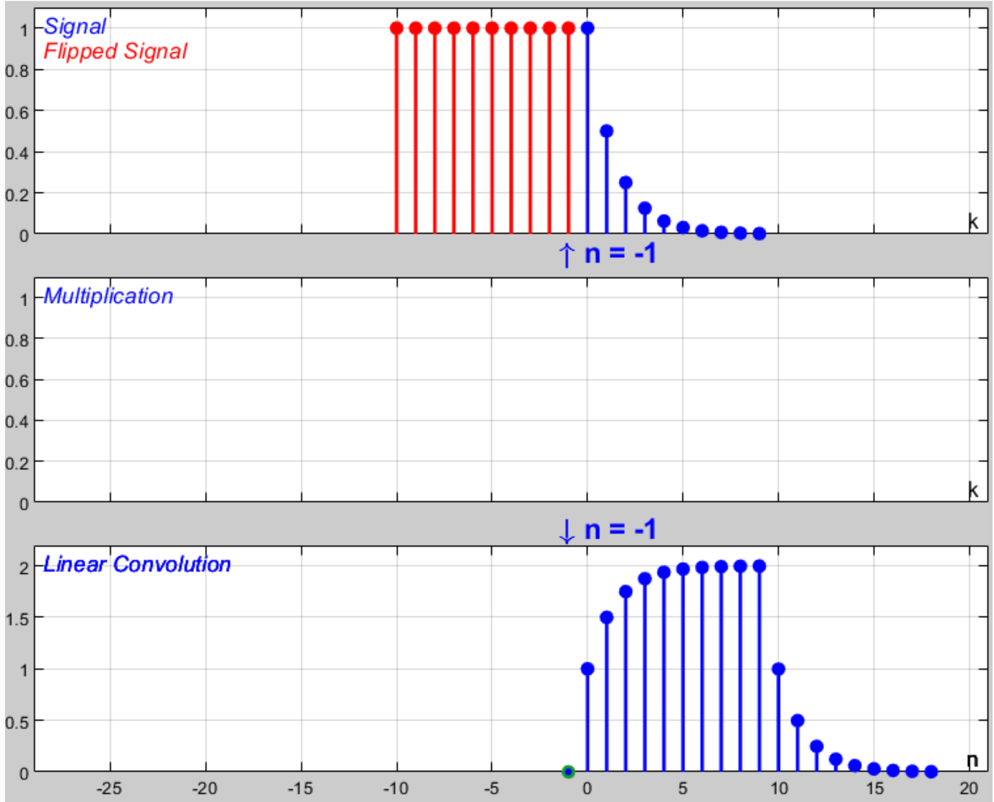
https://dspfirst.gatech.edu/matlab/#dconvdemo

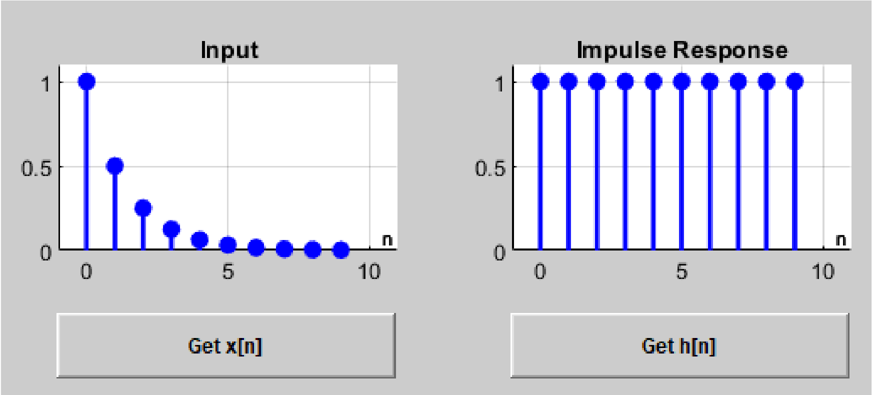




Ref. Discrete Conv. Demo v 3.15 @GitHub

MATLAB dconv Demo





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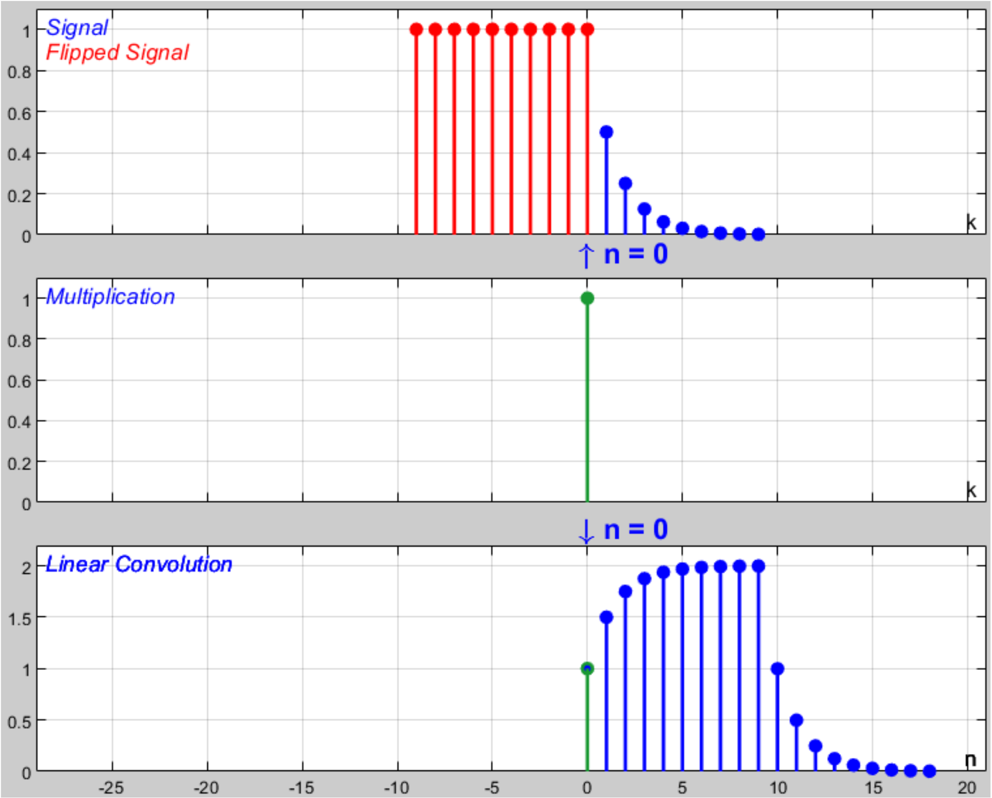

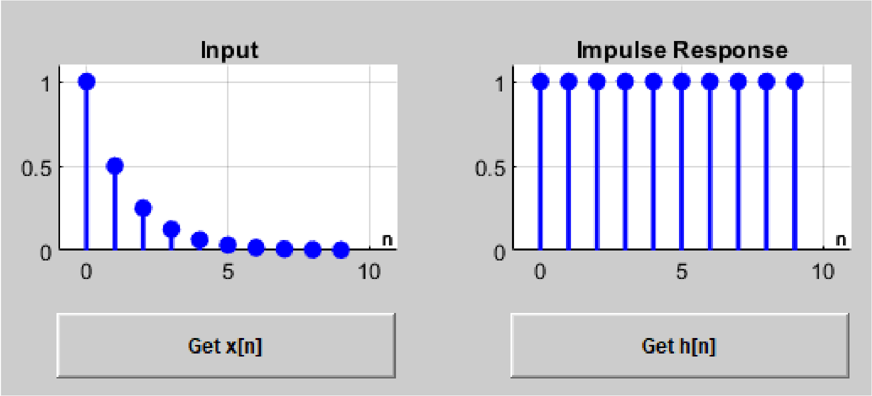

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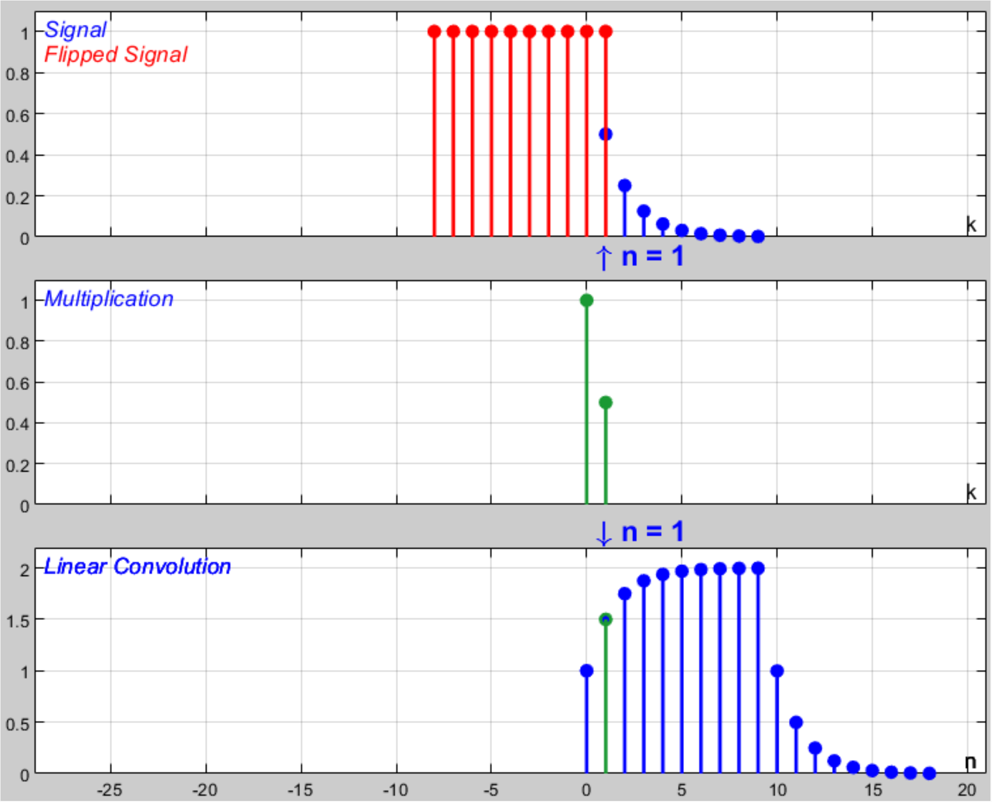

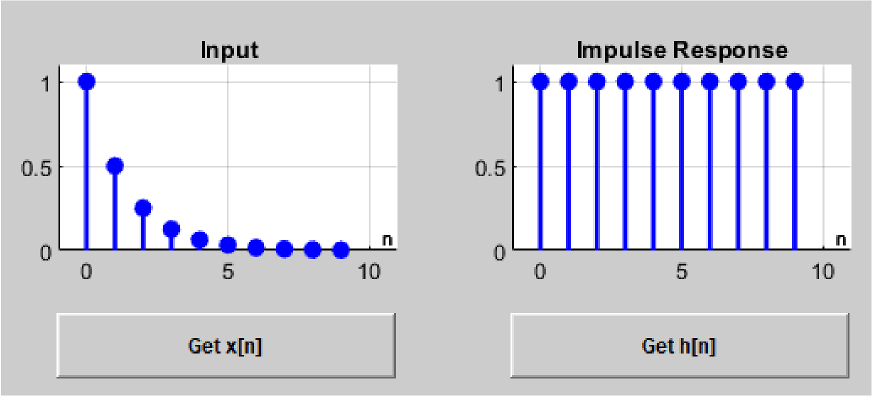

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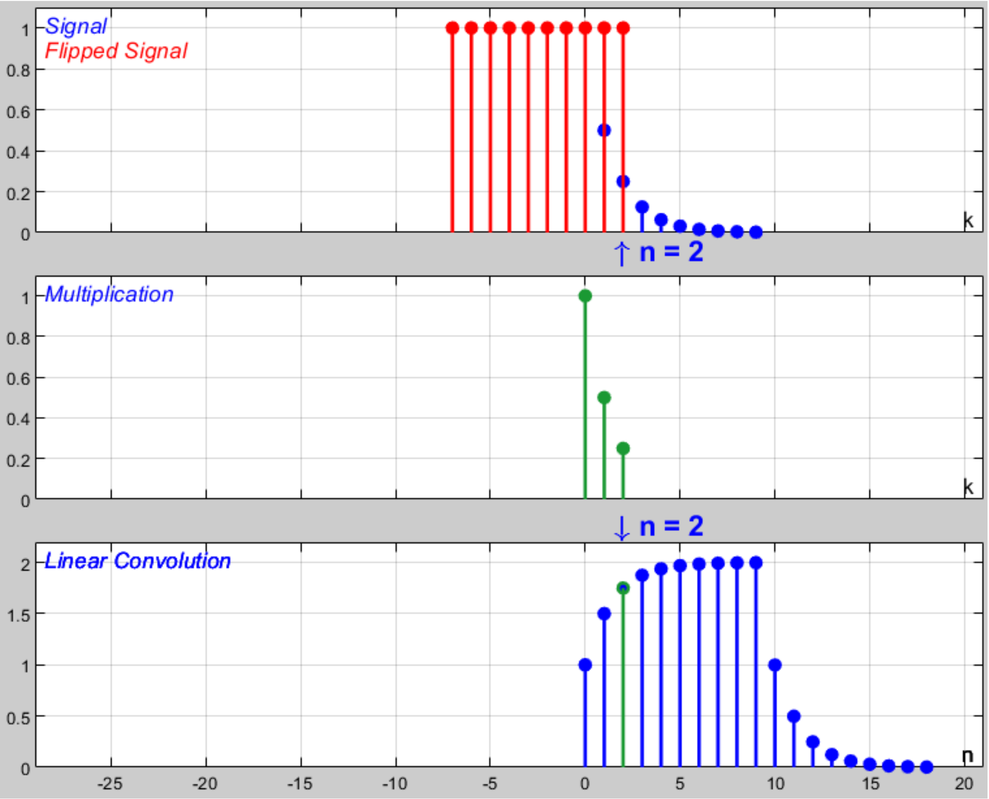

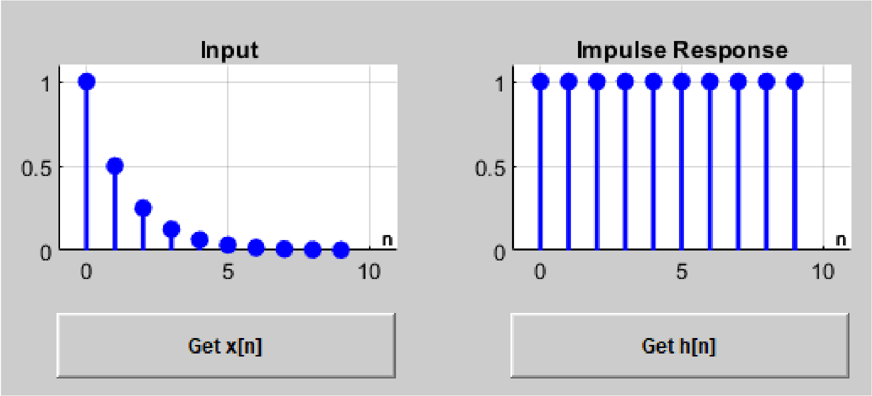

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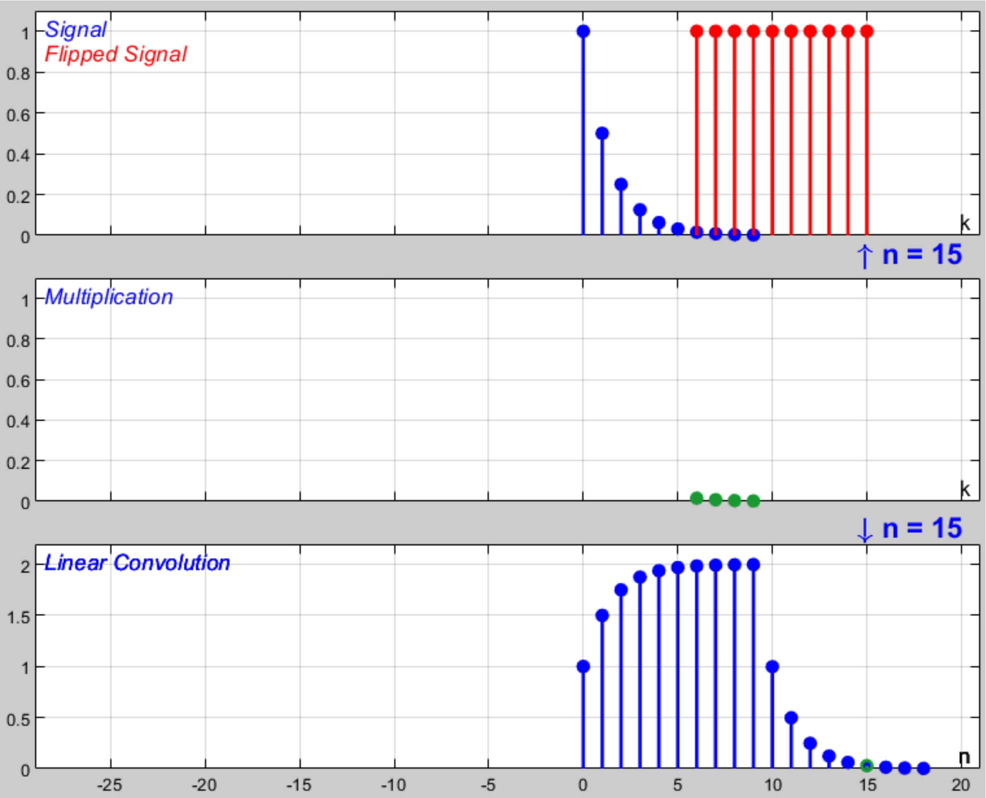

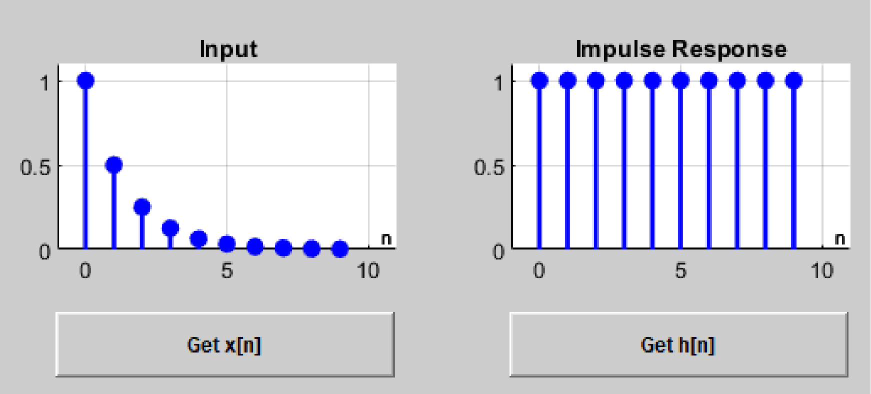

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Generalization of Discrete Time Systems

A linear, time-invariant system can be described by a difference equation having the following general form:

$$y(n)+a_1y(n-1)+\cdots+a_Ny(n-N)=b_0x(n)+b_1x(n-1)+\cdots+b_Mx(n-M)$$

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$$y(n)=-a_1y(n-1)-\cdots-a_Ny(n-N)+b_0x(n)+b_1x(n-1)+\cdots+b_Mx(n-M)$$

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$$y(n) = -a_1 y(n-1) - \cdots - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \cdots + b_M x(n-M)$$

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Linear Constant Coefficient Difference Equation

Block Diagram Representation of LCCDE

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

It is easy to implement the filters to hardware using block diagrams!

Block Diagram Representation of LCCDE

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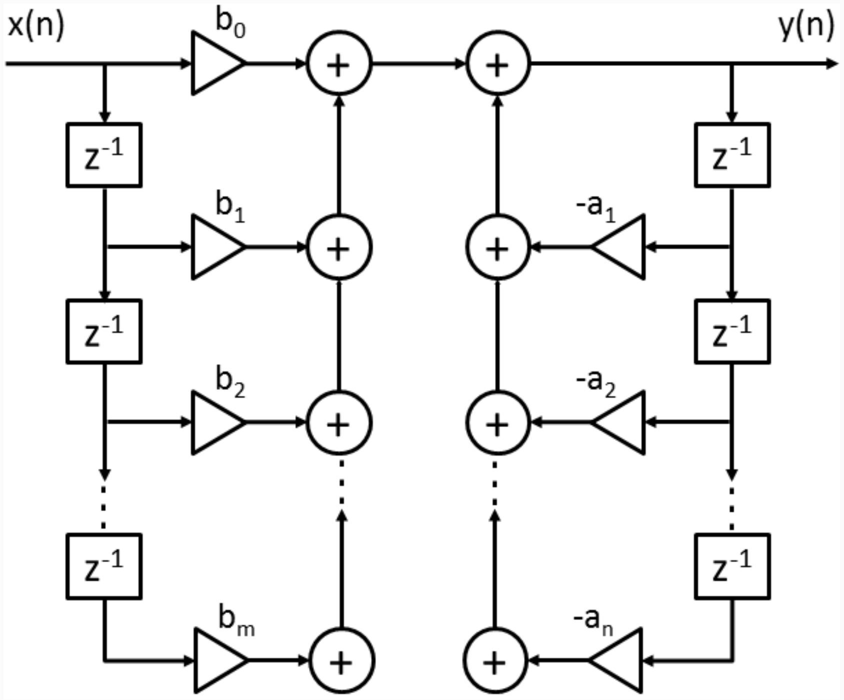

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Example

Given the following difference equation:

$$y(n) = 0.25y(n-1) + x(n),$$

identify the nonzero system coefficients.

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identify the nonzero system coefficients.

$$b_0 = 1$$

$$-a_1 = 0.25$$

If N = 0, the system has FIR

$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{j=0}^{M} b_j x(n-j)$$

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M th order FIR filter

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$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{j=0}^{M} b_j x(n-j)$$

$$y(n) = \sum_{k=0}^{M} b_k x(n-k)$$

M th order FIR filter

Finite impulse response (FIR):

$$y(n) = \sum_{k=0}^{M} h(k) x(n-k)$$

Block Diagram Representation of FIR

- Direct Form

SIGNAL FLOW GRAPH

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

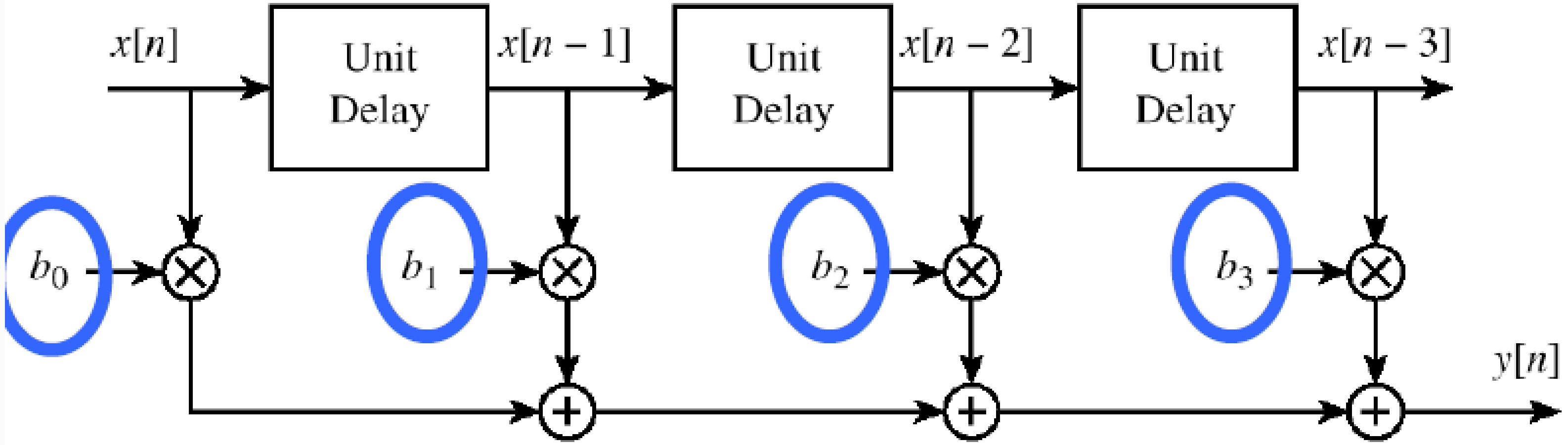


Figure 5.13 Block-diagram structure for the Mth order FIR filter.

Classification of Impulse Response h[n]

FIR – Finite Impulse Response:

- Number of impulses are limited.
- Always stable.

For example: $h[n] = \delta[n-1] + 5\delta[n-5]$

IIR – Infinite Impulse Response:

- Number of impulses are infinite.
- Sometimes these systems are not stable.

For example: $h[n] = u[n-1] + 5u[n-5]$

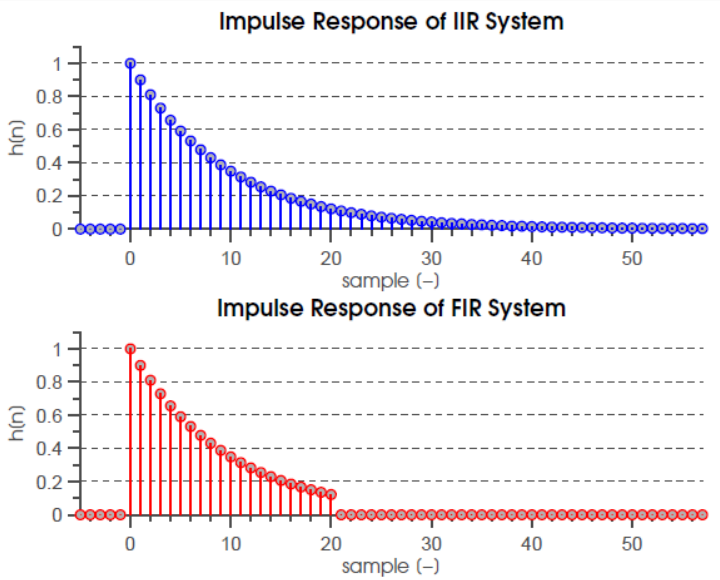
Finite impulse response (FIR):

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

Infinite impulse response (IIR):

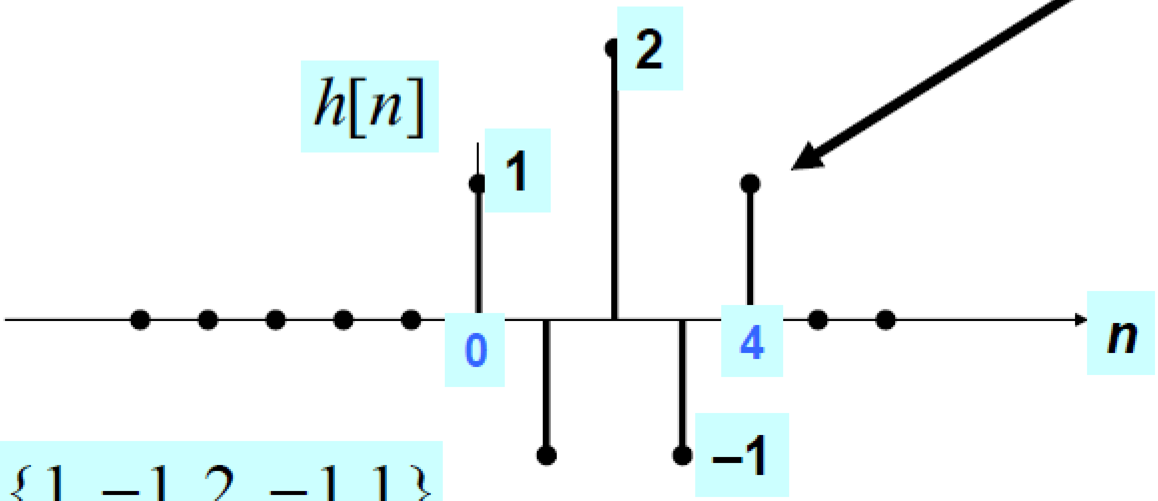
$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

Another example:



Math Formula of h[n] : FIR example

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$



$$b_k = \{1, -1, 2, -1, 1\}$$

FINITE LIMITS

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

Same as b_k

FINITE LIMITS

FIR Filter conv. - Table Method (Study it at home)

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$
$$x[n] = u[n]$$

n	-1	0	1	2	3	4	5	6	7
$x[n]$	0	1	1	1	1	1	1	1	...
$h[n]$	0	1	-1	2	-1	1	0	0	0
$h[0]x[n]$	0	1	1	1	1	1	1	1	1
$h[1]x[n-1]$	0	0	-1	-1	-1	-1	-1	-1	-1
$h[2]x[n-2]$	0	0	0	2	2	2	2	2	2
$h[3]x[n-3]$	0	0	0	0	-1	-1	-1	-1	-1
$h[4]x[n-4]$	0	0	0	0	0	1	1	1	1
$y[n]$	0	1	0	2	1	2	2	2	...

INPUT SIGNAL

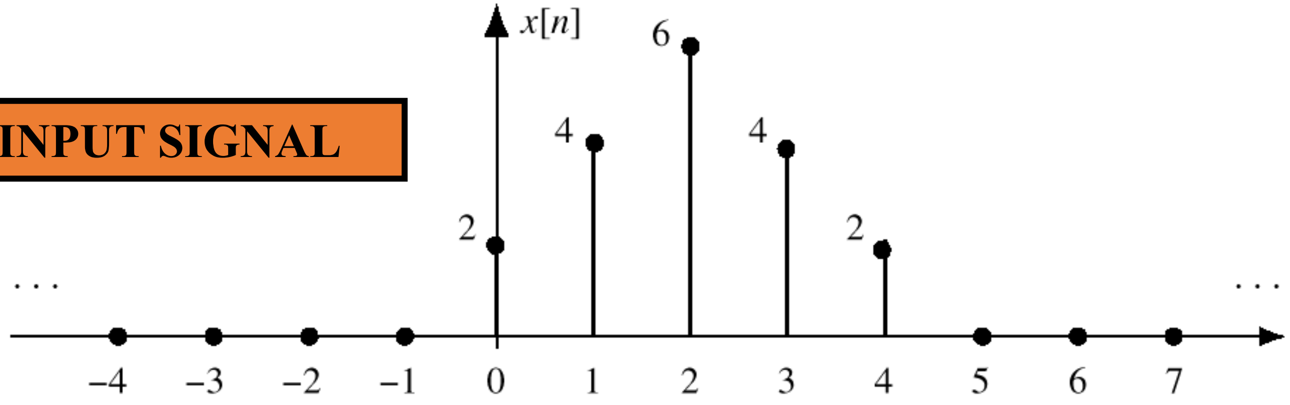


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n+2])$$

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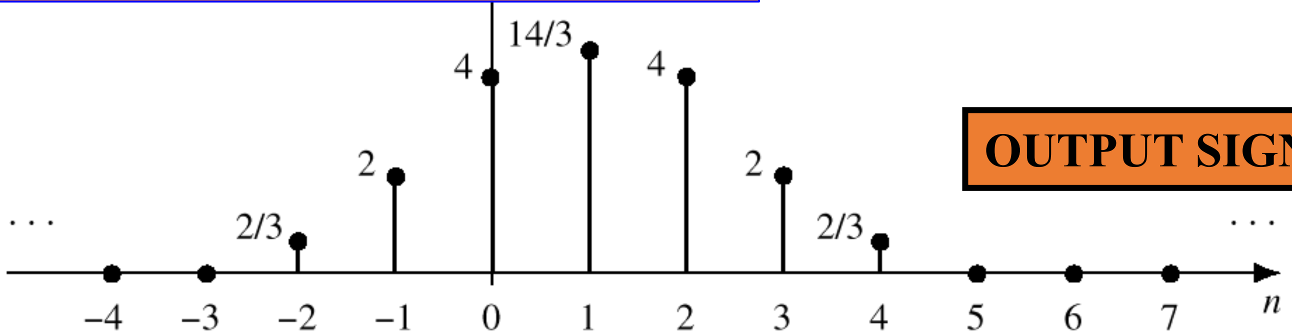


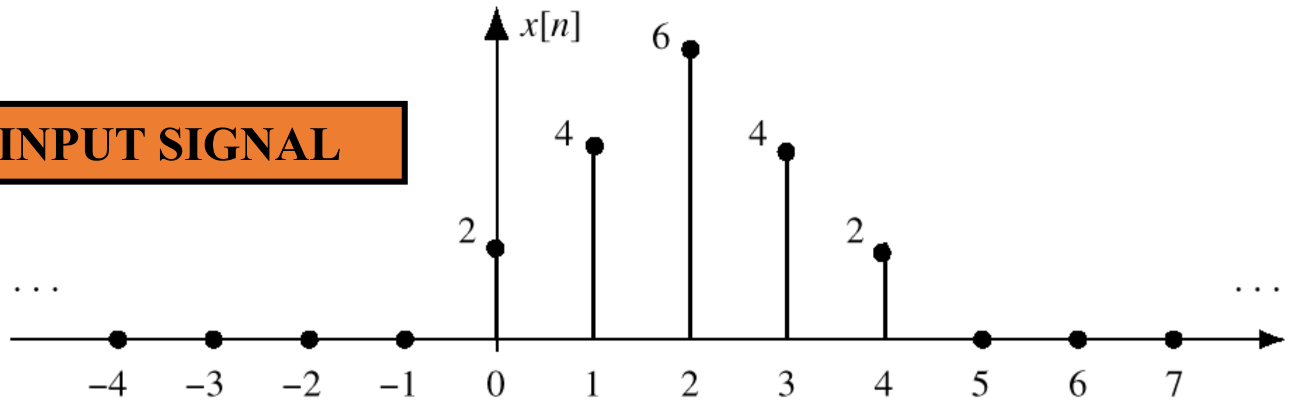
OUTPUT SIGNAL

Figure 5.3 Output of running average, $y[n]$.

Is this system causal?

3-point Average Filter

INPUT SIGNAL



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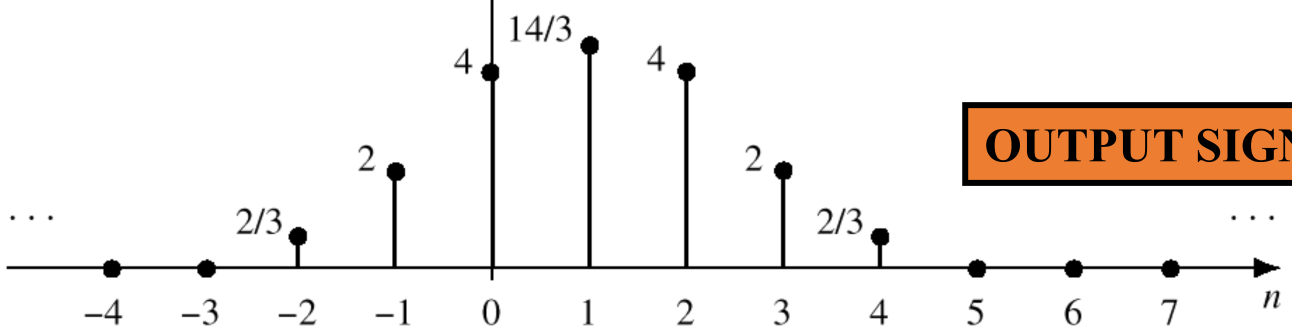

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Figure 5.3 Output of running average, $y[n]$.

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Do this system has FIR ?

3-point Average Filter

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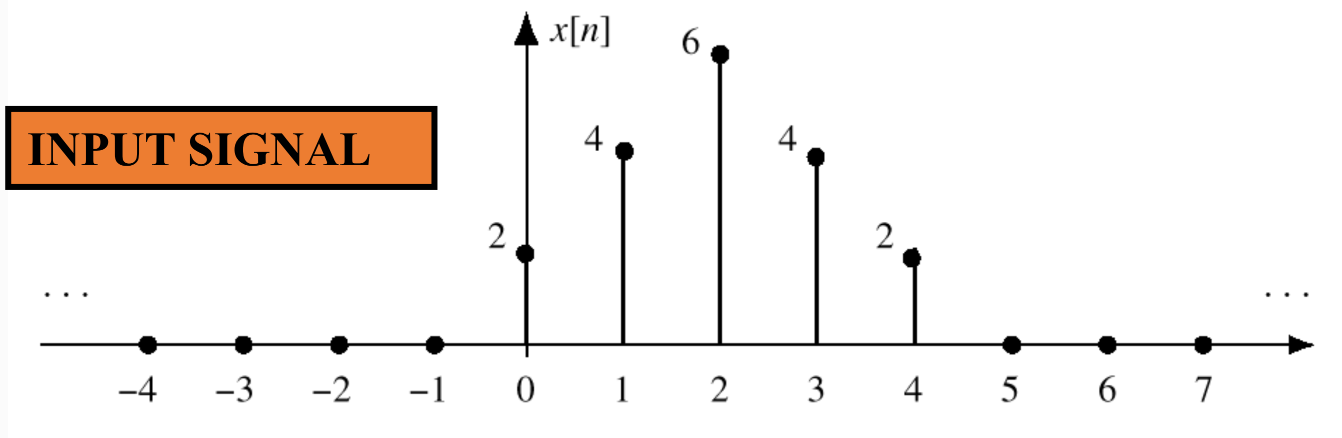

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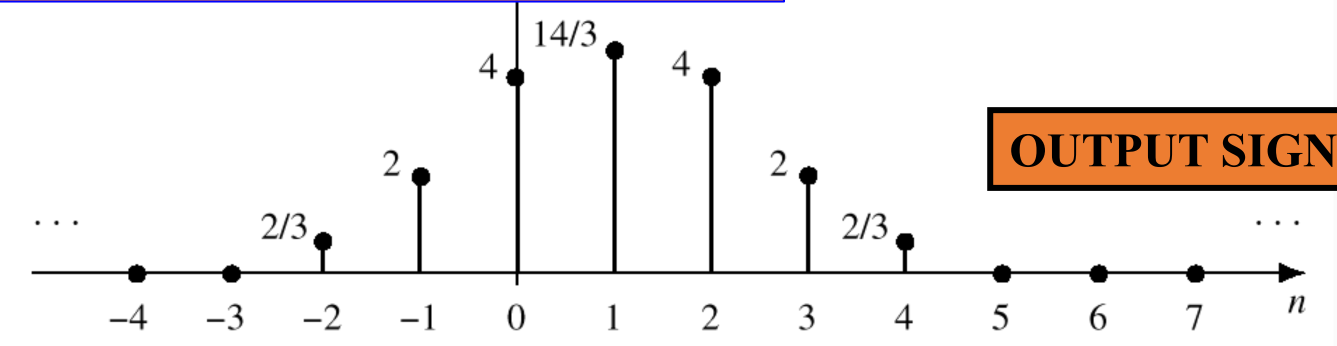

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Is this system causal?

Do this system has FIR ?

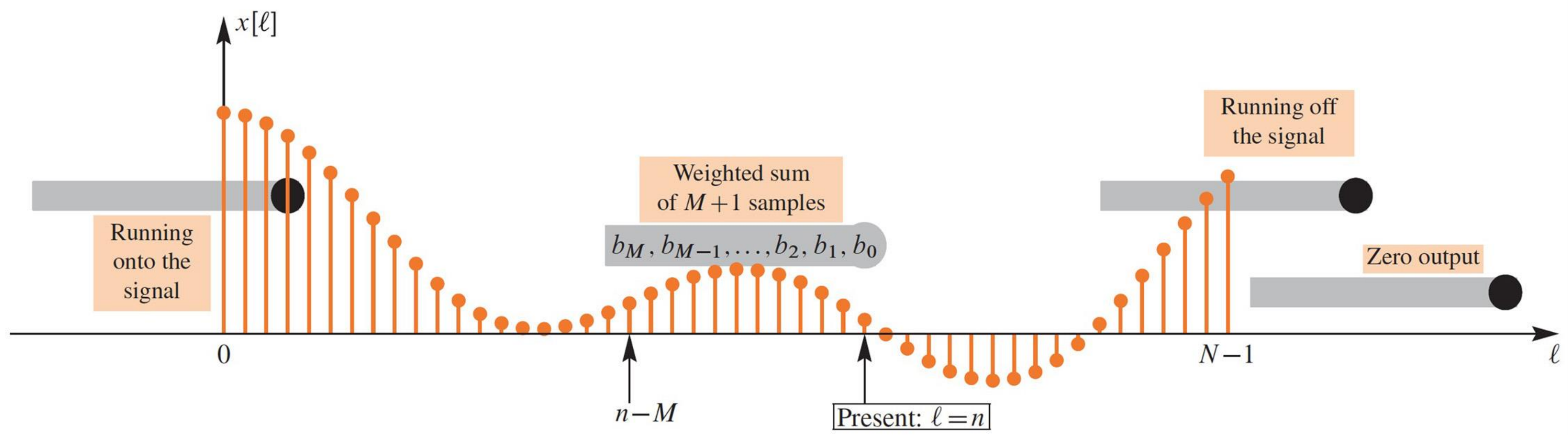
$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

FINITE LIMITS

FINITE LIMITS

Same as b_k

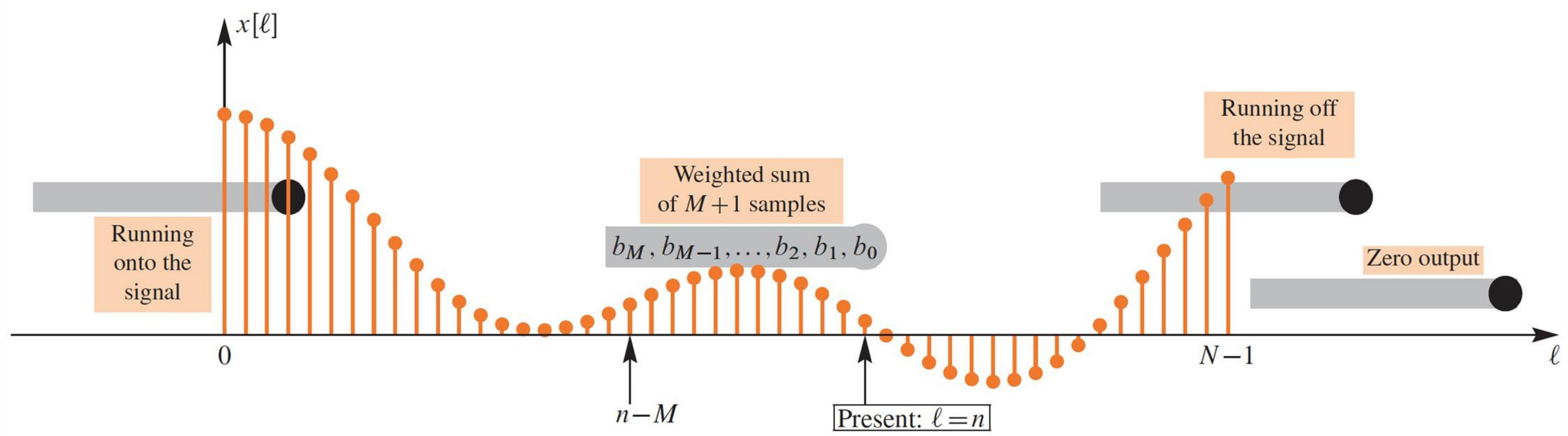
Recall: FIR Filter is always a causal system



x[n-M]

x[n]

Recall: FIR Filter is always a causal system



$x[n\text{-}M]$ $x[n]$

FINITE LIMITS

$$y[n]=\sum_{k=0}^{M} h[k]x[n-k]$$

Same as b_k

FINITE LIMITS

4-point Average FIR Filter

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

$$x[n] = \delta[n]$$
$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

4-point Average FIR Filter

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

<u>Find impulse response:</u>

$$x[n] = \delta[n]$$

$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

4-point Average FIR Filter

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

Find impulse response:

$$x[n] = \delta[n]$$
$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

n	-3	-2	-1	0	1	2	3	4	5
$x[n]$	0	0	0	1	0	0	0	0	0
$y[n]$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0

4-point Average FIR Filter

$$y[n]=\tfrac{1}{4}(x[n]+x[n-1]+x[n-2]+x[n-3])$$

Find impulse response:

$$x[n]=\delta[n]$$
$$y[n]=\tfrac{1}{4}\delta[n]+\tfrac{1}{4}\delta[n-1]+\tfrac{1}{4}\delta[n-2]+\tfrac{1}{4}\delta[n-3]$$

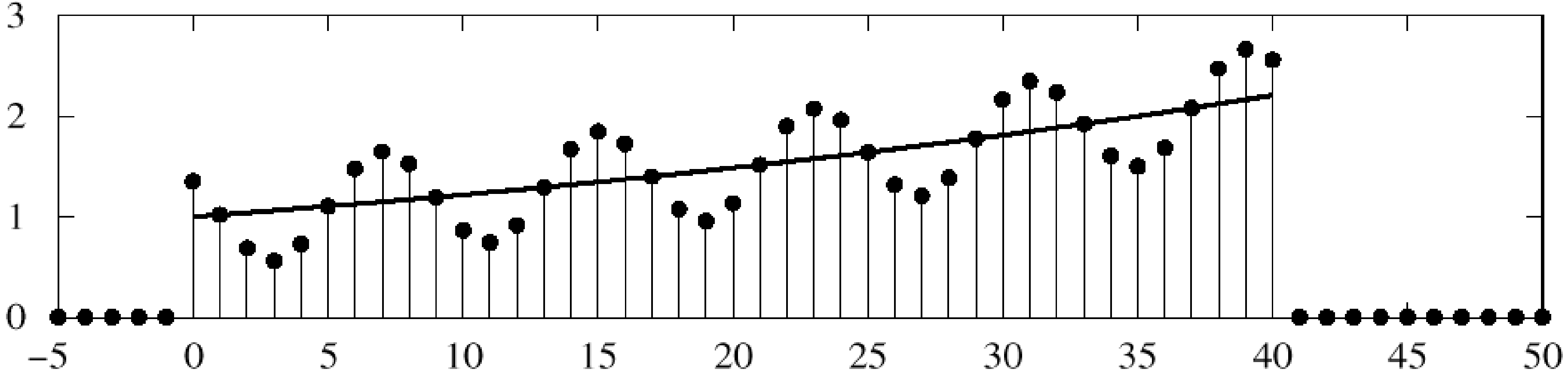
n	-3	-2	-1	0	1	2	3	4	5
$x[n]$	0	0	0	1	0	0	0	0	0
$y[n]$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0

$$h[n]=\{\ldots,0,0,\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4},0,0,\ldots\}$$

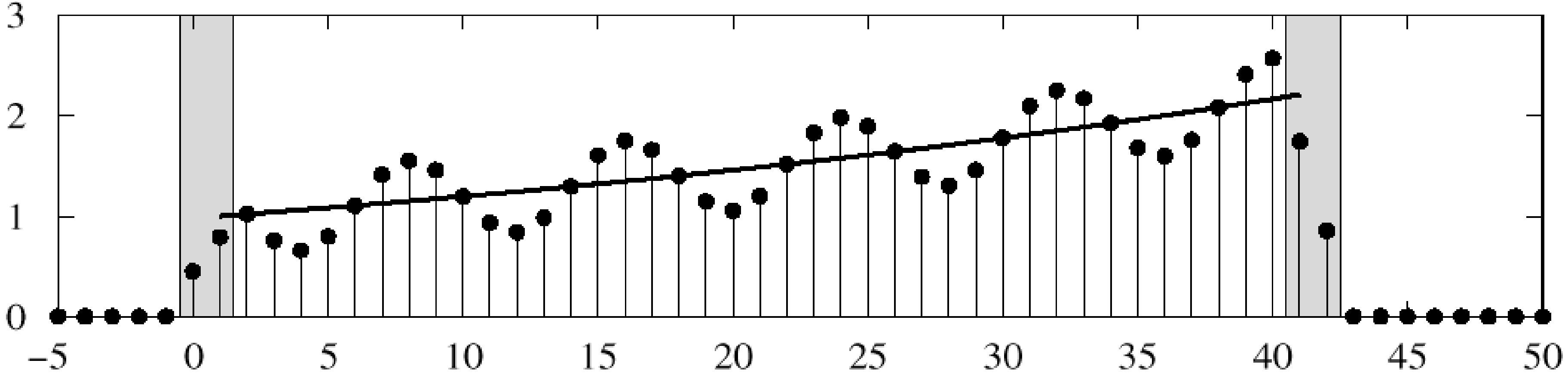
n=0

3-pt AVG EXAMPLE

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$

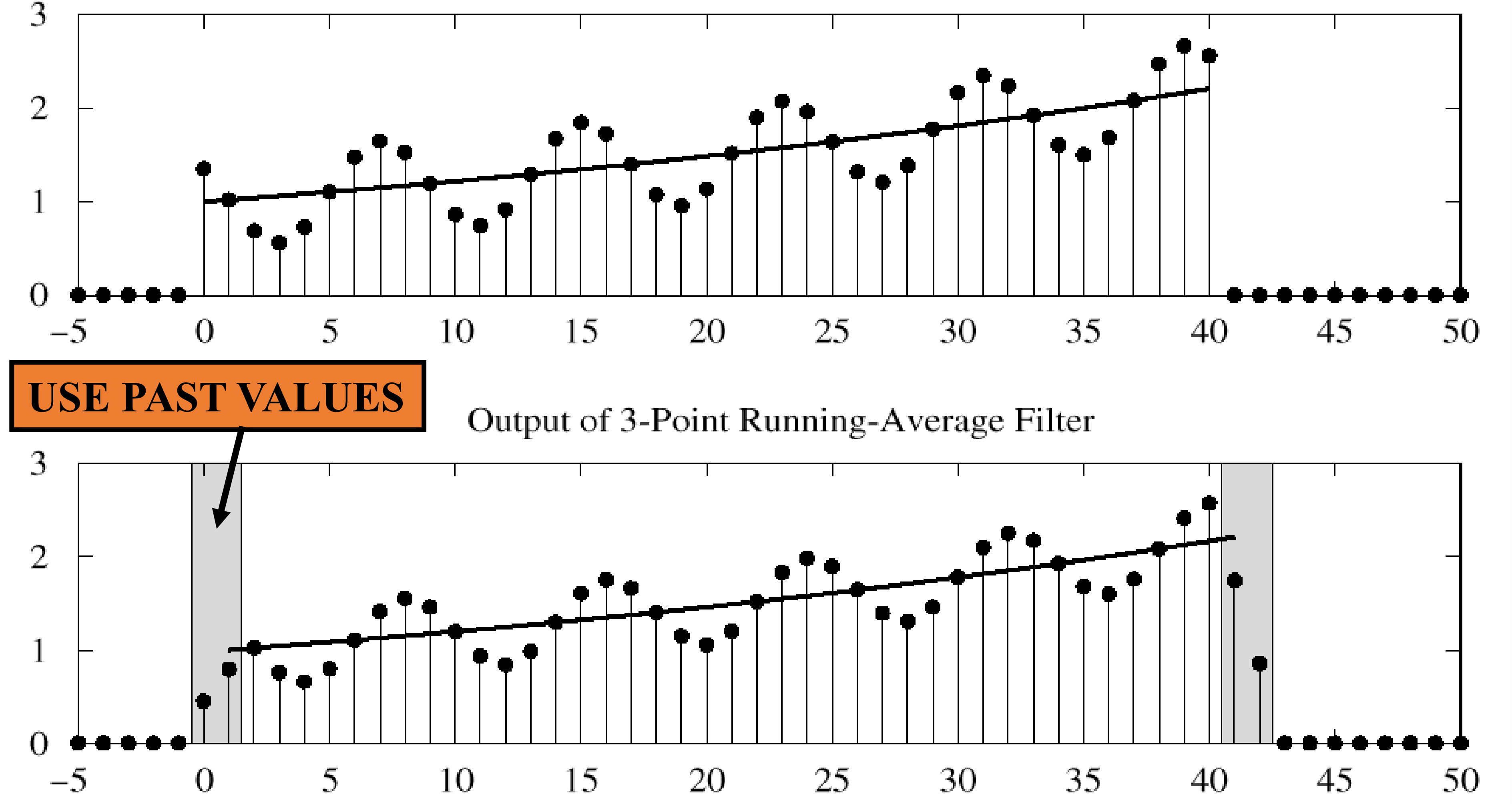


Output of 3-Point Running-Average Filter

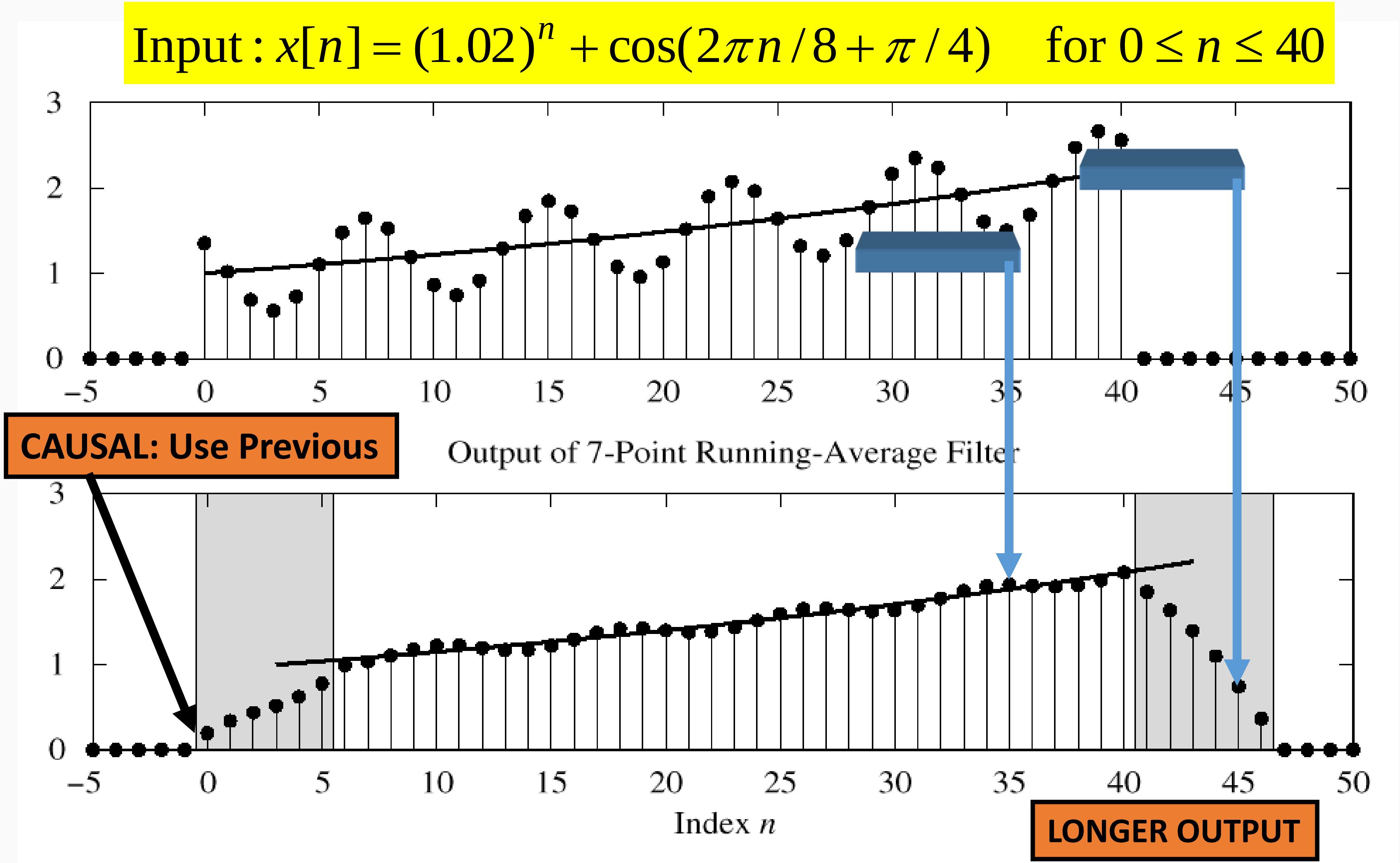


3-pt AVG EXAMPLE

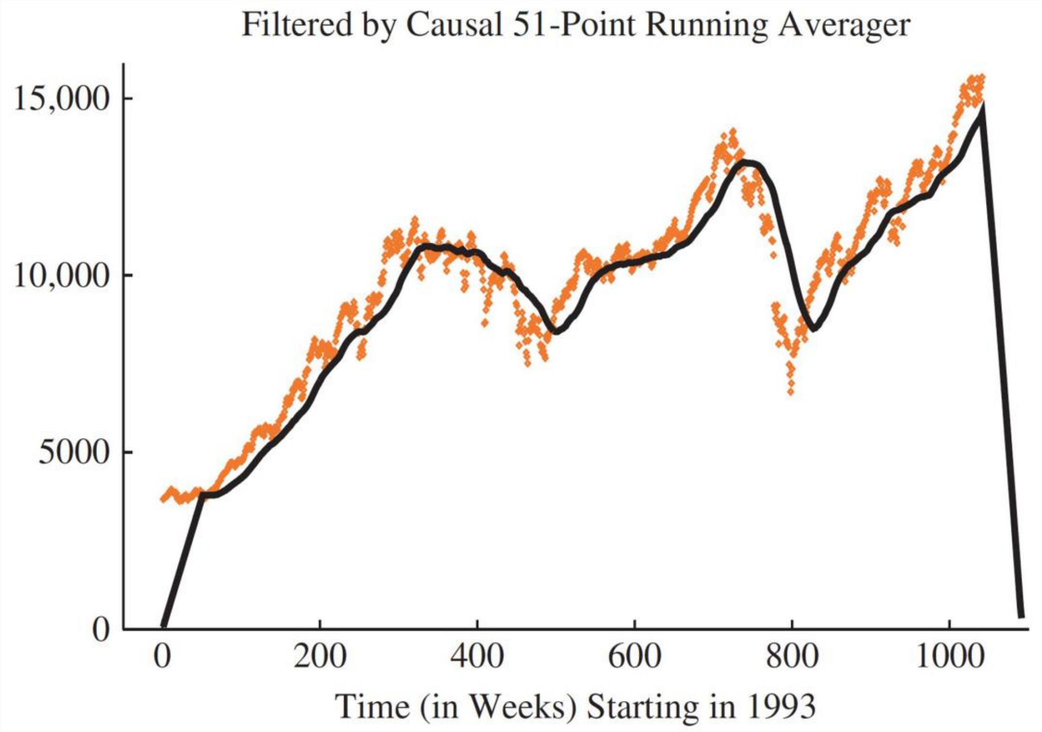
$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



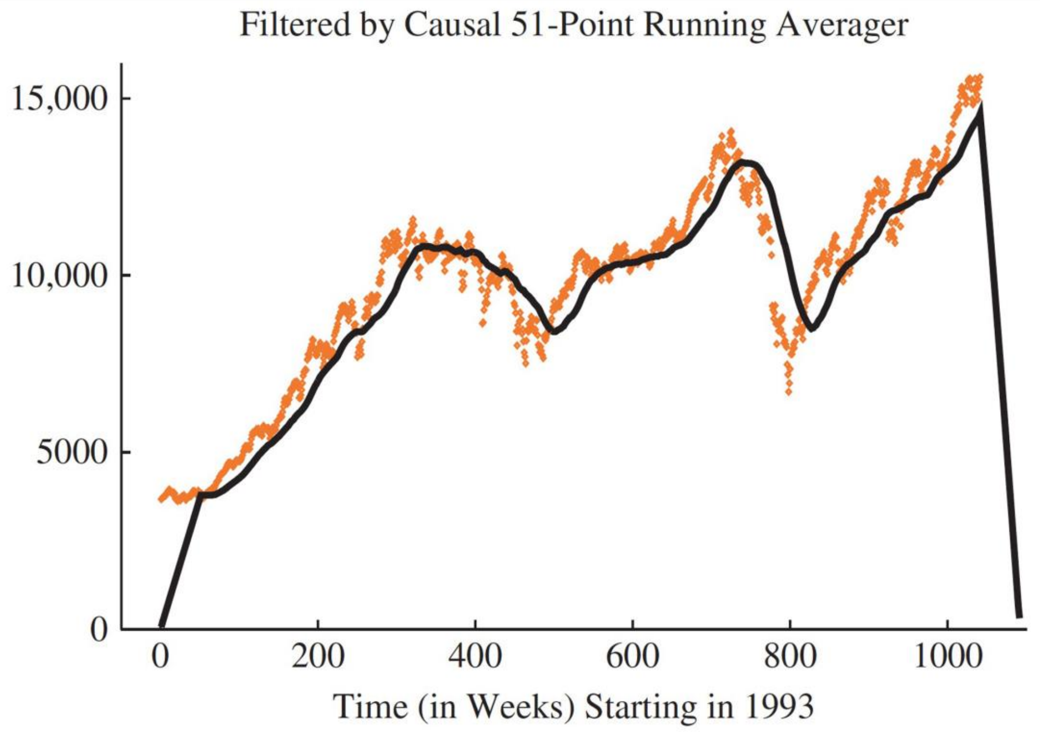
7-pt FIR EXAMPLE (AVG)

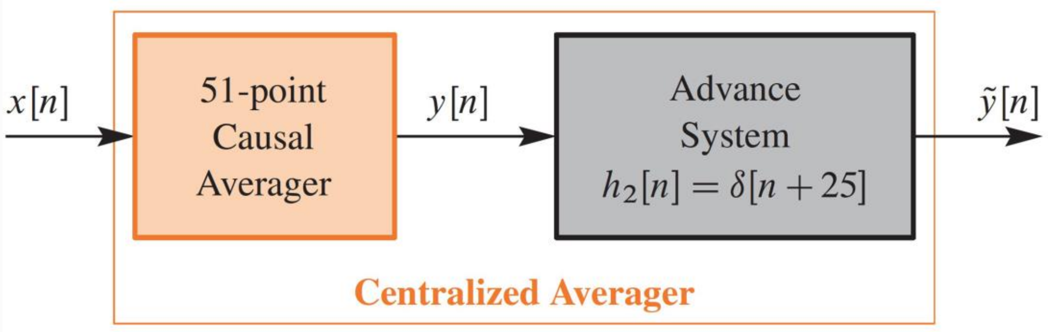


FILTER STOCK PRICES - CAUSAL VS ANTICAUSAL

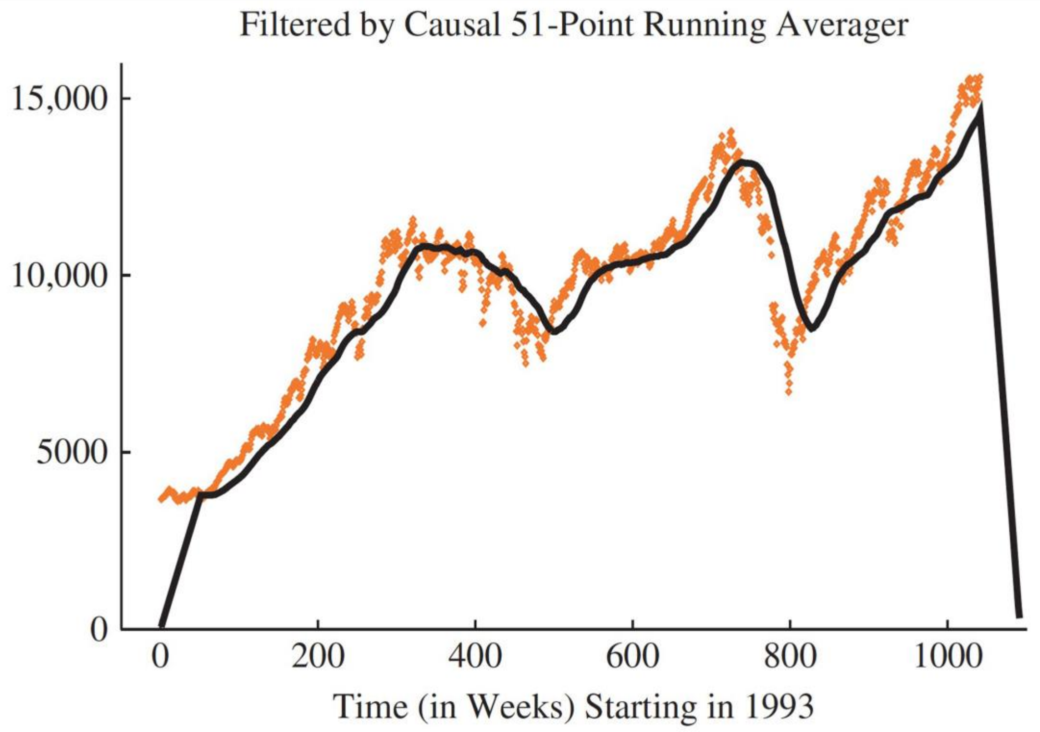


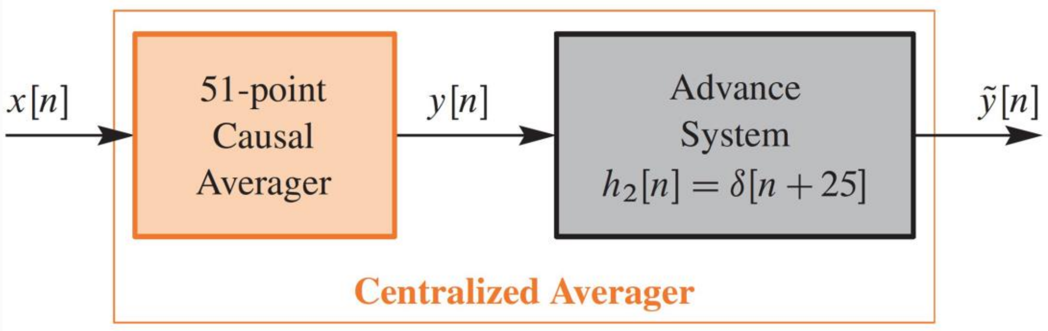
FILTER STOCK PRICES - CAUSAL VS ANTICAUSAL

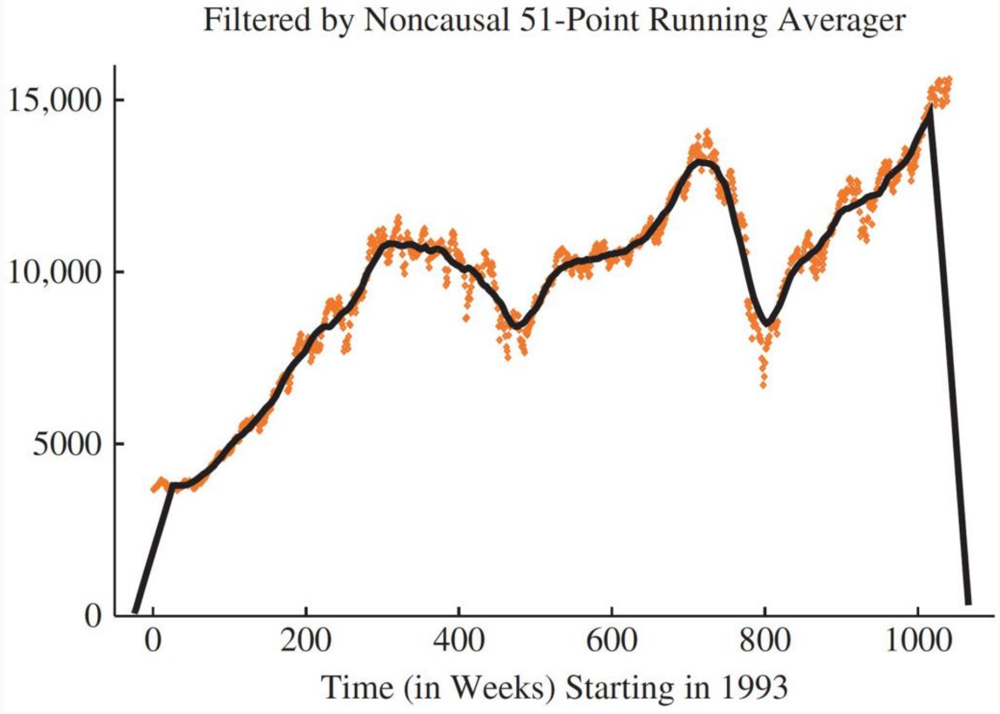




FILTER STOCK PRICES - CAUSAL VS ANTICAUSAL

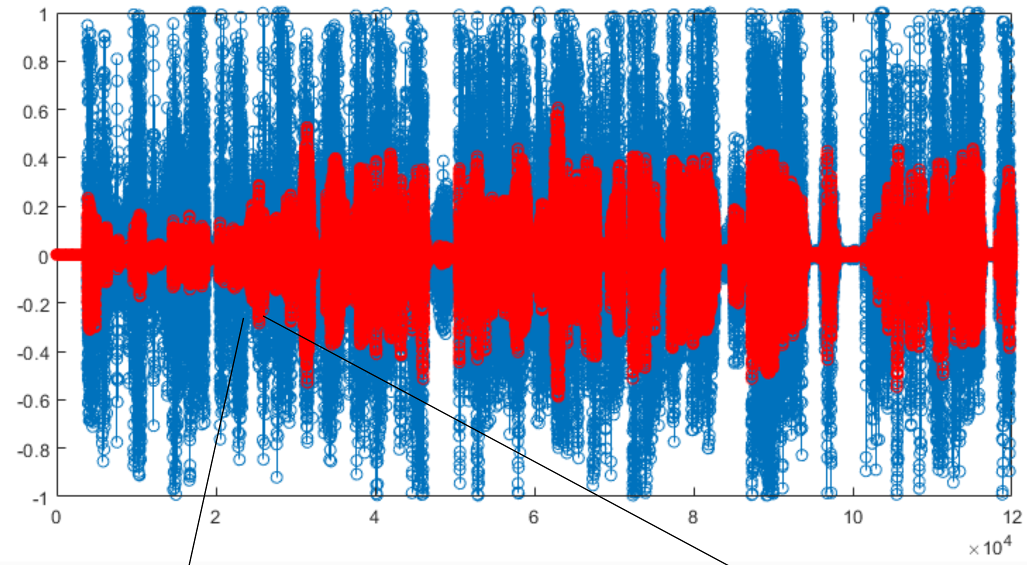


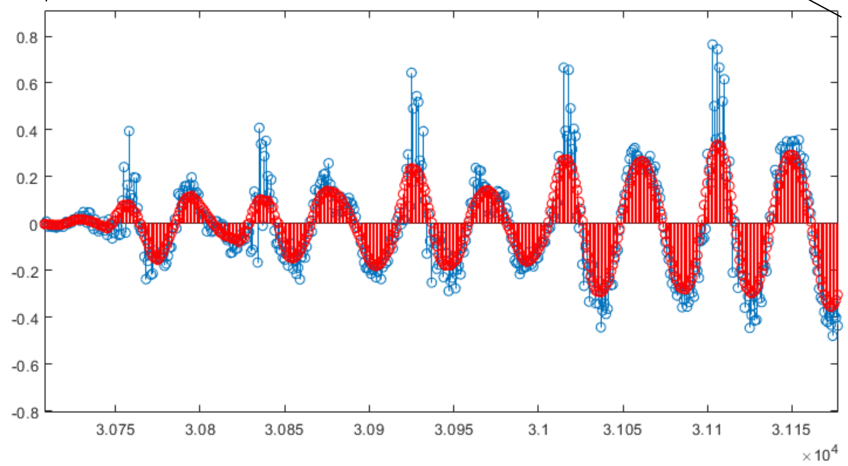




Let's apply 17-pt Centralized Average filter to Noisy Audio

```
clc; clear all;

%% Load Sound
load ('piano2.mat');
x = x(1:16000);
soundsc(x,Fs);

%% Add noise
K = awgn(x,40);
soundsc(K,Fs);

%%  Filter
N = 17;
h = 1/N*ones(1,N);

%% Apply Convolution
y = conv(K,h,'same');
soundsc(y,Fs);
%%
plot(x,'r'); hold on; plot(y,'b');
```





Apply Average Filter to An Image

```
clc; clear all;
I = imread('eight.tif');
I_noise =
imnoise(I,'gaussian',0,0.001);
%%
H = (1/9)*ones(3,3);
ortalamaSonucu =
conv2(I_noise,H,'same');

%%
figure(1), imshow(I_noise,[]);
figure(2), imshow(ortalamaSonucu,[]);
```

Exercise -1

PROBLEM:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] - 2x[n-1] + 3x[n-2] - 4x[n-3] + 2x[n-4].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders as in Figure 5.13 in the *SP First.*

(b) Determine the impulse response $h[n]$ for this system.

(c) Use convolution to determine the output due to the input

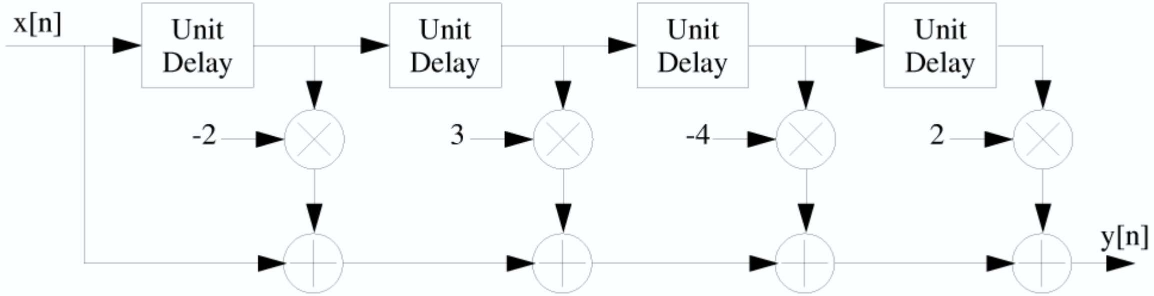
$$x[n] = \delta[n] - \delta[n-1] + \delta[n-2] = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Plot the output sequence $y[n]$ for $-3 \le n \le 10$.

Exercise -1

$$y[n] = x[n] - 2x[n-1] + 3x[n-2] - 4x[n-3] + 2x[n-4]$$

a) The block diagram for $y[n]$ is as follows.



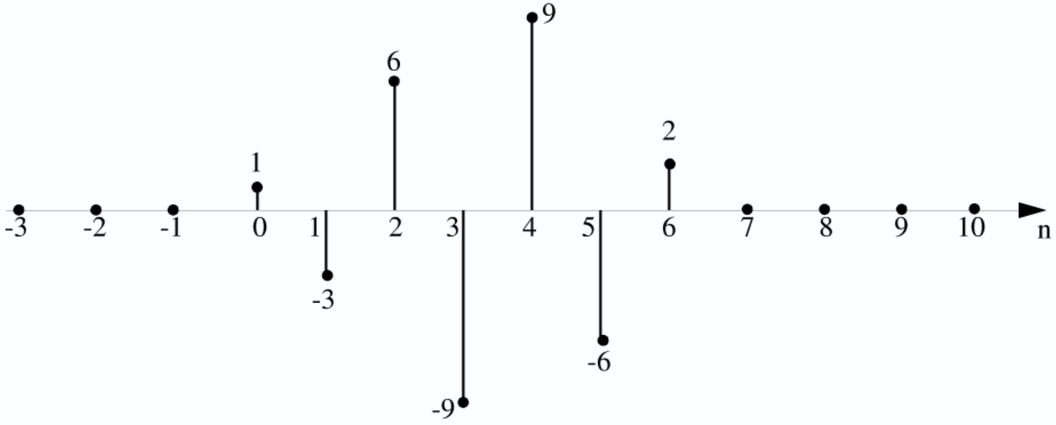
b) The impulse response for $y[n]$ can be found by using $x[n] = \delta[n]$ which results in

$$y[n] = h[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-2] - 4\delta[n-3] + 2\delta[n-4]$$

c) $y[n]$ can be tabulated as follows.

n	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
$h[n]$				1	-2	3	-4	2						
$x[n]$				1	-1	1								
				1	-2	3	-4	2						
					-1	2	-3	4	-2					
						1	-2	3	-4	2				
$y[n]$				1	-3	6	-9	9	-6	2				

Plotting $y[n]$ gives



Exercise -2

PROBLEM:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{4} (k+1)x[n-k]$$

The input to this system is *unit step* signal, denoted by $u[n]$, i.e., $x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Determine the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Use convolution to compute $y[n]$, over the range $-5 \leq n \leq \infty$, when the input is $u[n]$. Make a plot of $y[n]$ vs. n. (Hint: you might find it useful to check your results with MATLAB's `conv()` function.)

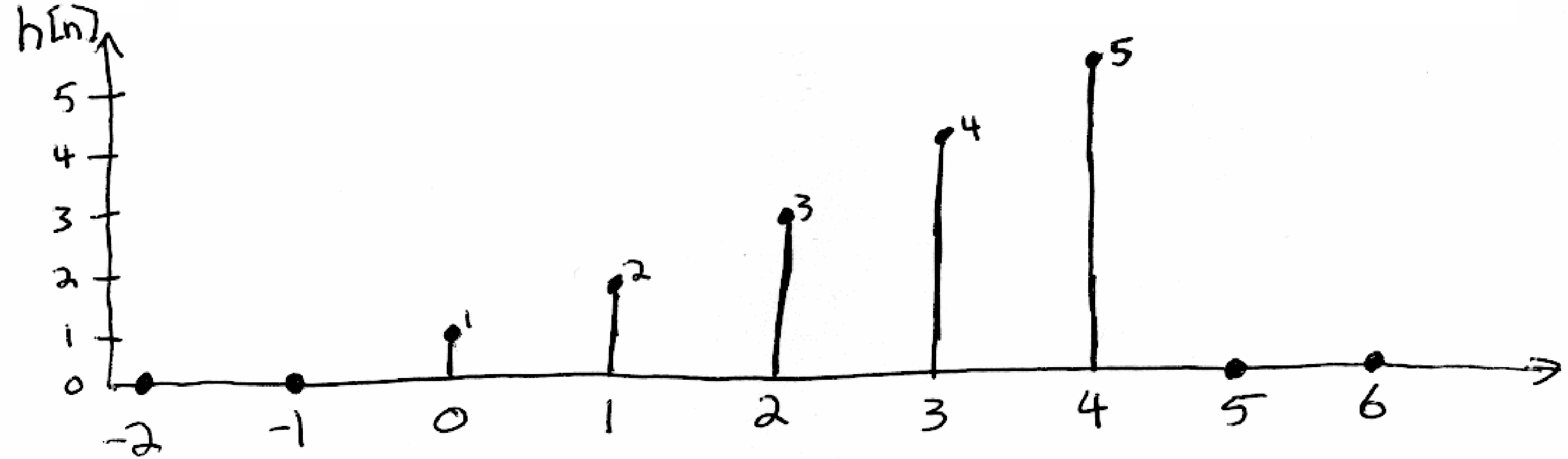
Exercise -2

a) $y[n] = 1x[n] + 2x[n-1] + 3x[n-2] + 4x[n-3] + 5x[n-4]$

Filter coefficients $\boxed{b_0 = 1 \quad b_1 = 2 \quad b_2 = 3 \quad b_3 = 4 \quad b_4 = 5}$

$(b_n = 0$ for $n < 0$ and $n > 4)$

b) $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 5\delta[n-4]$

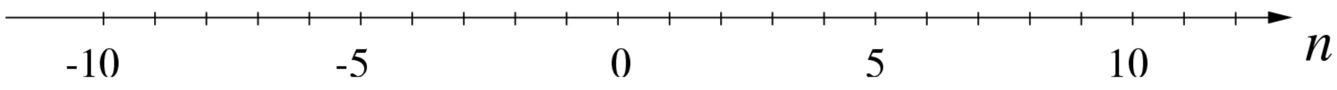


Exercise - 3

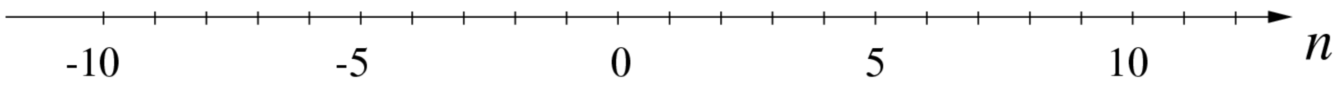
PROBLEM:

Let $x[n] = u[n] - u[n-7]$ and $h[n] = \begin{cases} (\frac{1}{2})^n & 0 \leq n \leq 3 \\ 0 & \text{otherwise.} \end{cases}$

(a) Plot $x[n]$.

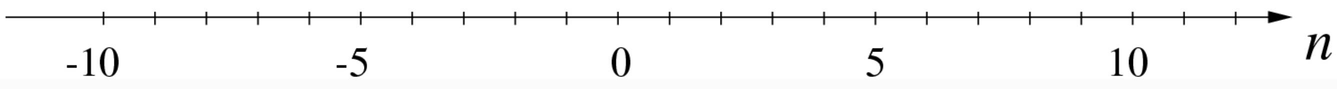


Plot $h[n]$.



Label the amplitudes for each sample.

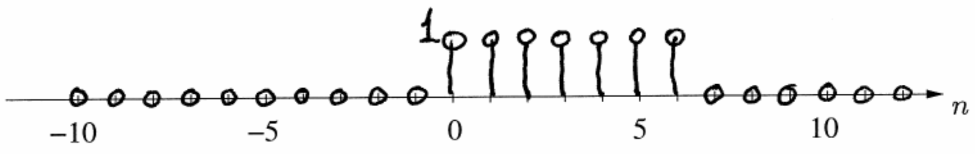
(b) If we now assume $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ and $y[n] = x[n] * h[n]$, where $h[n]$ is as defined above, plot $y[n]$ on the axis below.



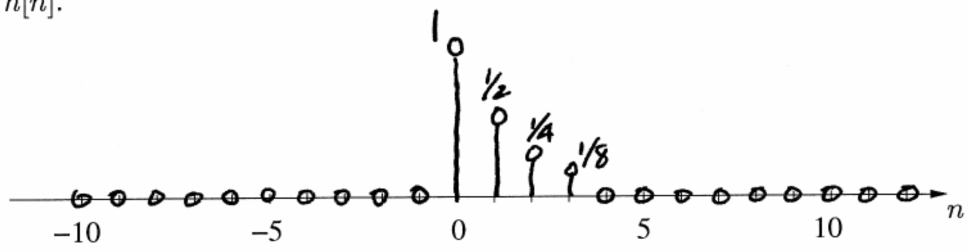
Exercise – 3

Let $x[n] = u[n] - u[n-7]$ and $h[n] = \begin{cases} (\frac{1}{2})^n & 0 \le n \le 3 \\ 0 & \text{otherwise.} \end{cases}$

(a) Plot $x[n]$.



Plot $h[n]$.



(b) If we now assume $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ and $y[n] = x[n] * h[n]$, where $h[n]$ is as defined above, plot $y[n]$ on the axis below.

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$

$1 \quad 1 \quad 1$

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$

$1 \quad \frac{3}{2} \quad \frac{7}{4} \quad \frac{7}{8} \quad \frac{3}{8} \quad \frac{1}{8}$