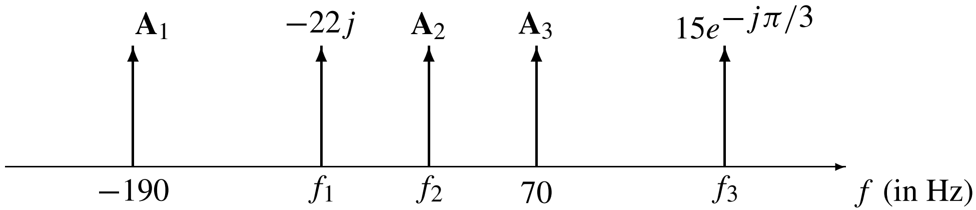
PROBLEM:

The two-sided spectrum representation of a real-valued signal $x(t)$ is shown below, but it is missing some information:



Assume that the time signal $x(t)$ for this spectrum is real-valued, and that the DC value of $x(t)$ is zero.

(a) Determine the values for the missing frequencies (in Hz):

$f_1 =$

$f_2 =$

$f_3 =$

(b) Determine the values for the missing complex amplitudes:

$\mathbf{A}_1 =$

$\mathbf{A}_2 =$

$\mathbf{A}_3 =$

(c) Write an equation for $x(t)$ using real-valued quantities only.

PROBLEM:

For the following short answer questions, write your answers in the space provided or circle the correct answer:

(a) $x(t)$ is defined by: $x(t) = \sum_{k=-10}^{10} \frac{4k}{j\pi} e^{j2.5\pi kt}$. Pick the correct response about the period:

(i) The period (T) of $x(t)$ is $T = 1.25$ sec.

(ii) The period (T) of $x(t)$ is $T = 2.5$ sec.

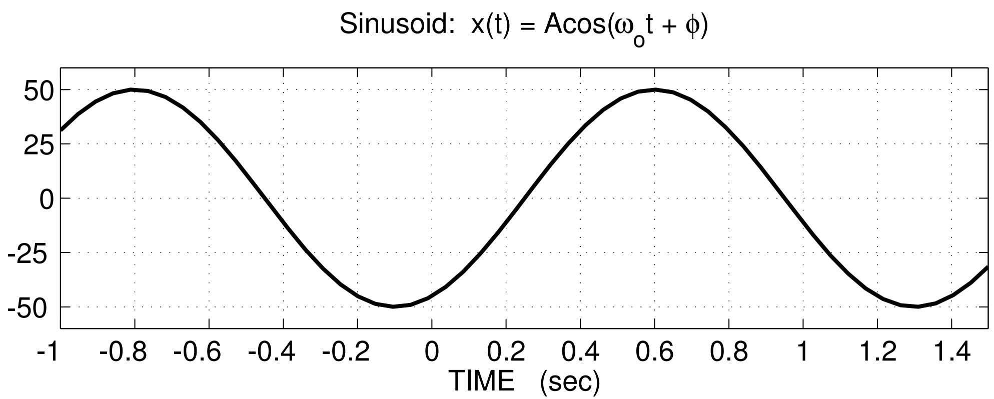
(iii) The period (T) of $x(t)$ is $T = 0.4$ sec.

(iv) The period (T) of $x(t)$ is $T = 0.8$ sec.

(v) The period (T) of $x(t)$ is $T = 2.5\pi$ sec.

(b) **TRUE** or **FALSE**: "Suppose that the signal $x(t)$ is a *single frequency* sinusoid and its spectrum has frequency components only at $f = \pm 7$ Hz. If a new signal is defined by $y(t) = x(t - \frac{1}{2})$ then $y(t)$ has frequency components at the same frequencies **and** the complex amplitudes in the spectrum are the same." EXPLAIN.

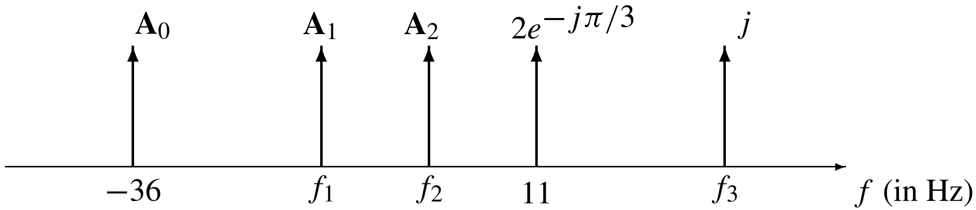
(c) In the figure below determine the phase of the sinusoid. Write your answer here: $\boxed{\phi = \quad}$



(d) In the figure above determine the frequency (ω_0) in radians/sec. $\boxed{\text{Circle}}$ the correct answer.
(A) 1.4 **(B)** $5\pi/7$ **(C)** $10\pi/7$ **(D)** $5/7$ **(E)** 2.8π

PROBLEM:

The two-sided spectrum representation of a real-valued signal $x(t)$ is shown below, but it is missing some numerical information:



Assume that the time signal $x(t)$ for this spectrum is real-valued, and that the DC value of $x(t)$ is zero.

(a) Determine the values for the missing frequencies (in Hz):

$f_1 =$

$f_2 =$

$f_3 =$

(b) Determine the values for the missing complex amplitudes:

$\mathbf{A}_0 =$

$\mathbf{A}_1 =$

$\mathbf{A}_2 =$

(c) Write an equation for $x(t)$ using real-valued quantities only.

PROBLEM:

For the following short answer questions, write your answers in the space provided or circle the correct answer:

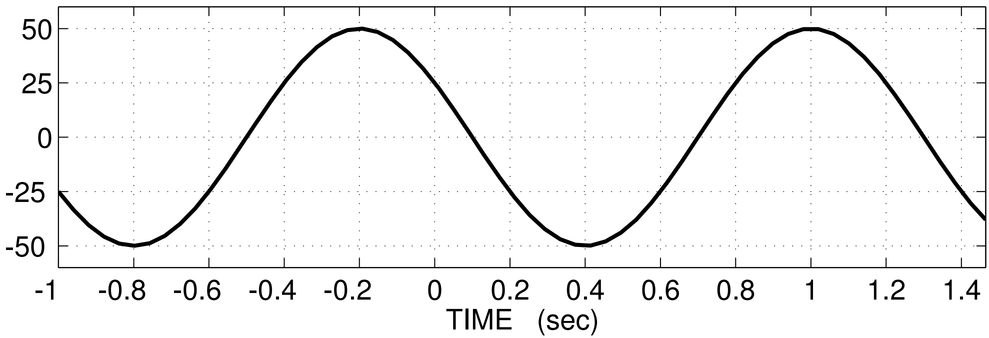
(a) The signal $x(t) = \cos(100\pi t) + \sin(101\pi t) - \cos(102\pi t)$ is

- (i) periodic with period equal to 1/100 sec.
- (ii) periodic with period equal to 1/50 sec.
- (iii) periodic with period equal to 2 sec.
- (iv) periodic with period equal to 1 sec.
- (v) periodic with period equal to $\frac{1}{2}$ sec.

(b) **TRUE** or **FALSE**: "Suppose that the signal $x(t)$ is a *single frequency* sinusoid and its spectrum has frequency components only at $f = \pm 2$ Hz. If a new signal is defined by $y(t) = x(t - \frac{1}{2})$ then $y(t)$ has frequency components at the same frequencies **but** the complex amplitudes are different." EXPLAIN.

(c) In the figure below determine the phase of the sinusoid. Write your answer here: $\boxed{\phi = \qquad}$

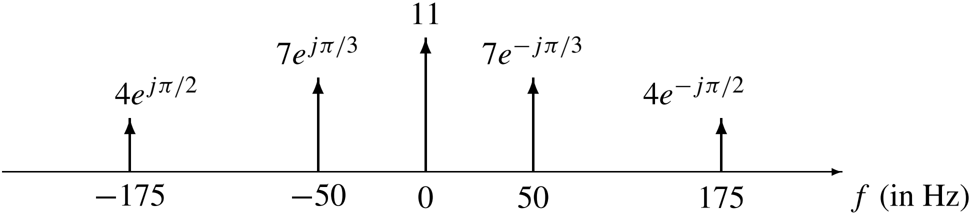
Sinusoid: $x(t) = A\cos(\omega_o t + \phi)$



(d) In the figure above determine the frequency (ω_0) in radians/sec. $\boxed{\text{Circle}}$ the correct answer.
(A) $5\pi/3$ **(B)** $5\pi/6$ **(C)** $5\pi/12$ **(D)** $5/6$ **(E)** 2.4π

PROBLEM:

A signal $x(t)$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x(t)$.

(b) Is $x(t)$ a periodic signal? If so, what is its period?

(c) Explain why "negative" frequency is needed in the spectrum.

PROBLEM:

Let $x(t) = \sin^3(27\pi t)$.

(a) Determine a formula for $x(t)$ as a sum of complex exponentials.

(b) What is the fundamental period for $x(t)$?

(c) Plot the *spectrum* for $x(t)$.

PROBLEM:

An amplitude modulated (AM) cosine wave is represented by the formula

$$x(t) = [12 + 7\sin(\pi t - \pi/3)]\cos(13\pi t)$$

(a) Use *phasors* to show that $x(t)$ can be expressed in the form:

$$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2) + A_3\cos(\omega_3 t + \phi_3)$$

where $\omega_1 < \omega_2 < \omega_3$; i.e., find $A_1,\ A_2,\ A_3,\ \phi_1,\ \phi_2,\ \phi_3,\ \omega_1,\ \omega_2,\ \omega_3$ in terms of A, ω_0, and ω_c.

(b) Sketch the two-sided spectrum of this signal on a frequency axis. Be sure to label important features of the plot. Label your plot in terms of the numerical values of the A_i's ϕ_i's and ω_i's.

PROBLEM:

A computer music student decided that he would create a new type of signal where the frequency moved as a function of time. In particular, he planned to generate a "chirp" signal—one that swept in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi$ as time goes from $t = 0$ to $t = T_2$.

It turns out that the *instantaneous frequency* of the chirp can be derived from a derivative operation. If we define $x(t)$ in the following manner:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

then we can think of this as the cosine function operating on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp.

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians} \tag{2}$$

(a) For the "chirp" in (1) determine formulas for the beginning frequency (ω_1) and ending frequency (ω_2) in terms of α, β and T_2.

(b) For the "chirp" signal

$$x(t) = \Re e\{e^{j(40t^2 + 27t + 13)}\}$$

derive a formula for the *instantaneous* frequency.

(c) Make a plot of the *instantaneous* frequency (in Hertz) versus time over the range $0 \le t \le 1$ sec.

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(c) Make a plot of the *instantaneous* frequency (in Hertz) versus time over the range $0 \le t \le 1$ sec.

PROBLEM:

It might be difficult to see why the derivative of the phase would be the instantaneous frequency. The following experiment provides a clue.

(a) Use the following parameters to define a "chirp" signal:

$$\begin{aligned} \omega_1 &= 2\pi(1) \text{ rad/sec} \\ \omega_2 &= 2\pi(9) \text{ rad/sec} \\ T_2 &= 2 \text{ sec} \end{aligned}$$

In other words, determine α and β in equation (**??**) to define $x(t)$ so that it sweeps the specified frequency range.

(b) Now make a plot of the signal synthesized in part (a). Pick a time sampling interval that is small enough so that the plot is very smooth. Put this plot in the middle panel of a 3×1 subplot, i.e., `subplot(3,1,2)`.

(c) It is difficult to verify whether or not this chirp signal will have the correct frequency content. However, the rest of this problem is devoted to an experiment that will demonstrate that the derivative of the phase is the "correct" definition of instantaneous frequency. First of all, make a plot of the instantaneous frequency $f_i(t)$ (in Hz) versus time.

(d) Now generate and plot a 4 Hz sinusoid. Put this plot in the upper panel of a 3×1 subplot, i.e., `subplot(3,1,1)`.

(e) Finally, generate and plot an 8 Hz sinusoid. Put this plot in the lower panel of a 3×1 subplot, i.e., `subplot(3,1,3)`.

(f) Compare the three signals and comment on the frequency content of the chirp. Concentrate on the frequency of the chirp in the time range $1.6 \le t \le 2$ sec. Which sinusoid matches the chirp in this time region? Compare the expected $f_i(t)$ in this region to 4 Hz and 8 Hz.

PROBLEM:

A periodic signal $x(t)$ with a period $T_0 = 10$ is described *over one period,* $0 \leq t \leq 10$, by the equation

$$x(t) = \begin{cases} 0 & 0 \leq t \leq 5 \\ 2 & 5 < t \leq 10. \end{cases}$$

This signal can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t},$$

which is valid for all time $-\infty < t < \infty$.

(a) Sketch the periodic function $x(t)$ for $-10 < t < 20$.

(b) Determine a_0, the D.C. coefficient of the Fourier Series.

(c) Use the Fourier $\boxed{\textit{analysis}}$ integral [1] (for $k \neq 0$)

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jk\omega_0 t} dt$$

to find the first ($k = 1$) Fourier series coefficient, a_1. Note: $\omega_0 = 2\pi/T_0$.

(d) If we add a constant value of one to $x(t)$, we obtain the signal $y(t) = 1 + x(t)$ with $y(t)$ given over one period by

$$y(t) = \begin{cases} 1 & 0 \leq t \leq 5 \\ 3 & 5 < t \leq 10. \end{cases}$$

This signal can also be represented by a Fourier series,

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}.$$

Explain how b_0 and b_1 are related to a_0 and a_1. (Note: You should not have to evaluate any new integrals explicitly to answer this question.)

[1] The Fourier integral can be done over any period of the signal; in this case, the most convenient choice is from -5 to 0.

PROBLEM:

It might be difficult to see why the derivative of the phase would be the instantaneous frequency. The following experiment provides a clue.

(a) Use the following parameters to define a "chirp" signal:

$$\begin{aligned} \omega_1 &= 2\pi(1) \text{ rad/sec} \\ \omega_2 &= 2\pi(9) \text{ rad/sec} \\ T_2 &= 2 \text{ sec} \end{aligned}$$

In other words, determine α and β in equation (**??**) to define $x(t)$ so that it sweeps the specified frequency range.

(b) Now make a plot of the signal synthesized in part (a). Pick a time sampling interval that is small enough so that the plot is very smooth. Put this plot in the middle panel of a 3×1 subplot, i.e., `subplot(3,1,2)`.

(c) It is difficult to verify whether or not this chirp signal will have the correct frequency content. However, the rest of this problem is devoted to an experiment that will demonstrate that the derivative of the phase is the "correct" definition of instantaneous frequency. First of all, make a plot of the instantaneous frequency $f_i(t)$ (in Hz) versus time.

(d) Now generate and plot a 4 Hz sinusoid. Put this plot in the upper panel of a 3×1 subplot, i.e., `subplot(3,1,1)`.

(e) Finally, generate and plot an 8 Hz sinusoid. Put this plot in the lower panel of a 3×1 subplot, i.e., `subplot(3,1,3)`.

(f) Compare the three signals and comment on the frequency content of the chirp. Concentrate on the frequency of the chirp in the time range $1.6 \leq t \leq 2$ sec. Which sinusoid matches the chirp in this time region? Compare the expected $f_i(t)$ in this region to 4 Hz and 8 Hz.

PROBLEM:

We have seen that a periodic signal $x(t)$ can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}. \tag{1}$$

It turns out that we can transform many operations on the signal into corresponding operations on the Fourier coefficients a_k. For example, suppose that we want to consider a new periodic signal $y(t) = \frac{dx(t)}{dt}$. What would the Fourier coefficients be for $y(t)$? To see this, we simply need to differentiate the Fourier series representation; i.e.,

$$y(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right] = \sum_{k=-\infty}^{\infty} a_k \frac{d}{dt}\left[e^{jk\omega_0 t}\right] = \sum_{k=-\infty}^{\infty} a_k \left[(jk\omega_0)e^{jk\omega_0 t}\right]. \tag{2}$$

Thus, we see that $y(t)$ is also in the Fourier series form

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}, \quad \text{where } \; b_k = (jk\omega_0)a_k$$

but in this case the Fourier series coefficients are related to the Fourier series coefficients of $x(t)$ by $b_k = (jk\omega_0)a_k$. This is a nice result because it allows us to find the Fourier coefficients *without* actually doing the differentiation of $x(t)$ and *without* doing any tedious evaluation of integrals to obtain the Fourier coefficients b_k. It is a *general* result that holds for every periodic signal and its derivative.

We can use this style of manipulation to obtain some other useful results for Fourier series. In each case below, use Equation (1) as the starting point and the given definition for $y(t)$ to express $y(t)$ as a Fourier series and then manipulate the equation so that you can pick off an expression for the Fourier coefficients b_k as a function of the original coefficients a_k.

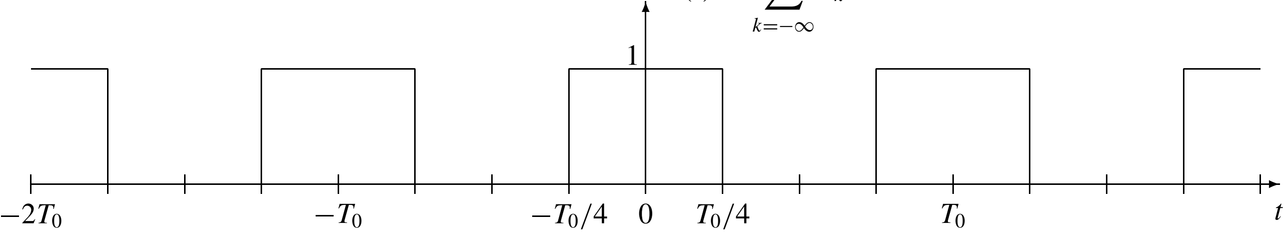
(a) Suppose that $y(t) = Ax(t)$ where A is a real number; i.e., $y(t)$ is just a scaled version of $x(t)$. Show that the Fourier coefficients for $y(t)$ are $b_k = Aa_k$.

(b) Suppose that $y(t) = x(t - t_d)$ where t_d is a real number; i.e., $y(t)$ is just a delayed version of $x(t)$. Show that the Fourier coefficients for $y(t)$ in this case are $b_k = a_k e^{-jk\omega_0 t_d}$.

PROBLEM:

Consider the periodic function $x(t)$ plotted below.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$



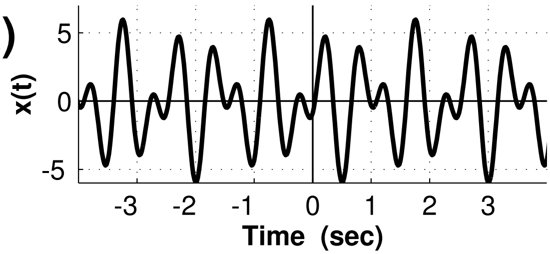
(a) Find the "DC" value a_0 and the other Fourier coefficients a_k for $k \neq 0$ in the Fourier series representation of $x(t)$.

(b) Sketch the waveform of the signal $y(t) = 2x(t - T_0/2)$ and use the results of Problem 4.4 to write down the Fourier series coefficients b_0 and b_k for $k \neq 0$ for the periodic signal $y(t)$ without evaluating any integrals. **Note: You will use this result in Section 4 of Lab #3.**

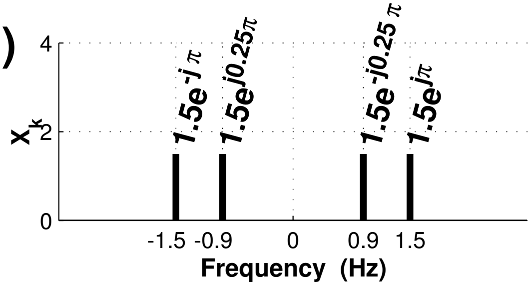
PROBLEM:

The following plots show waveforms on the left and spectra on the right. Hand in a table matching the waveform letter with its corresponding spectrum number.

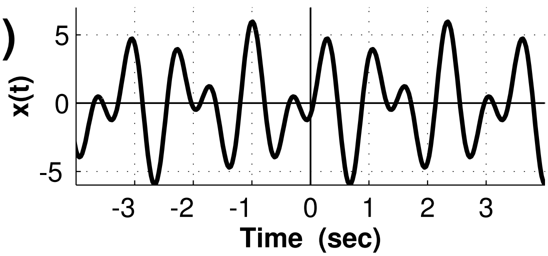
(a)



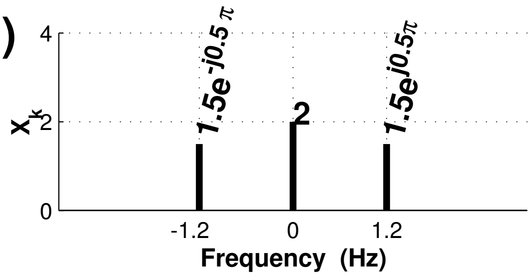
(1)



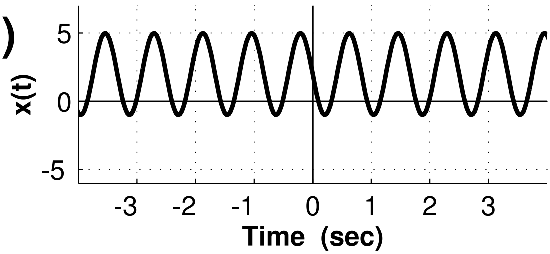
(b)



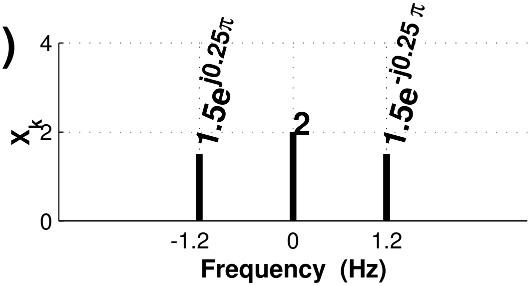
(2)



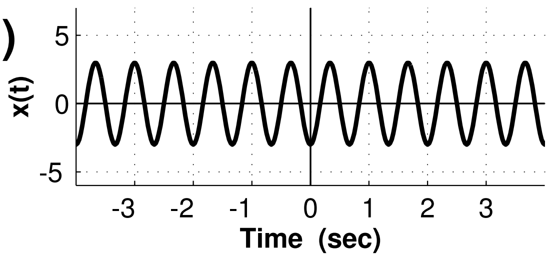
(c)



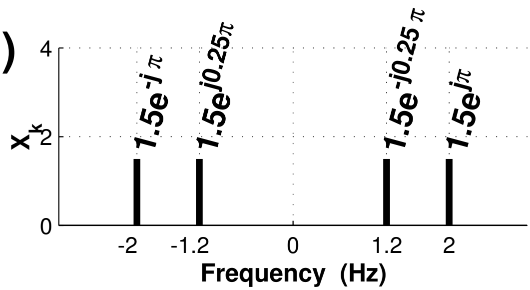
(3)



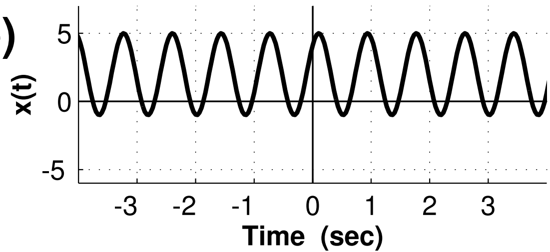
(d)



(4)



(e)



(5)