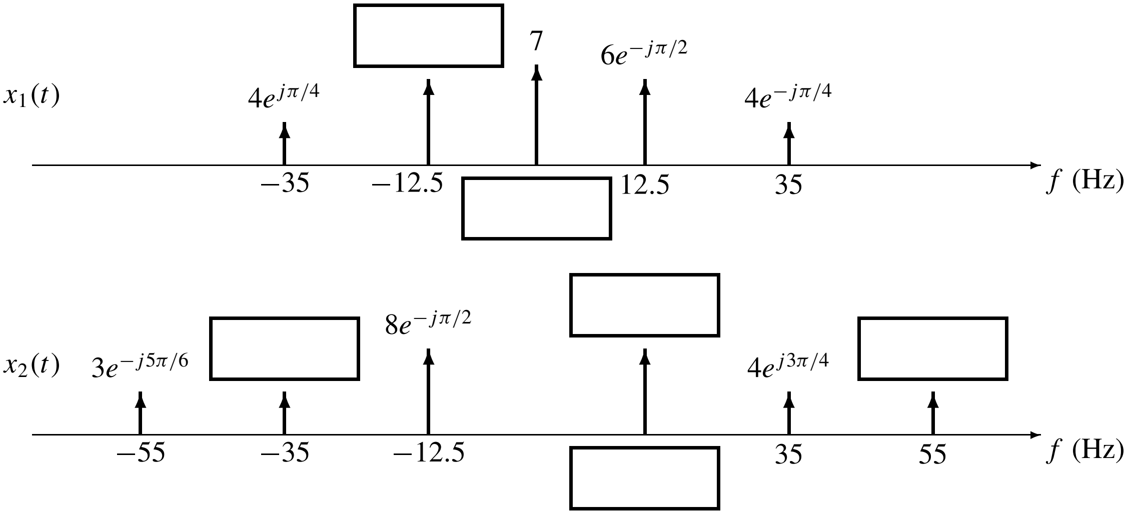
PROBLEM:

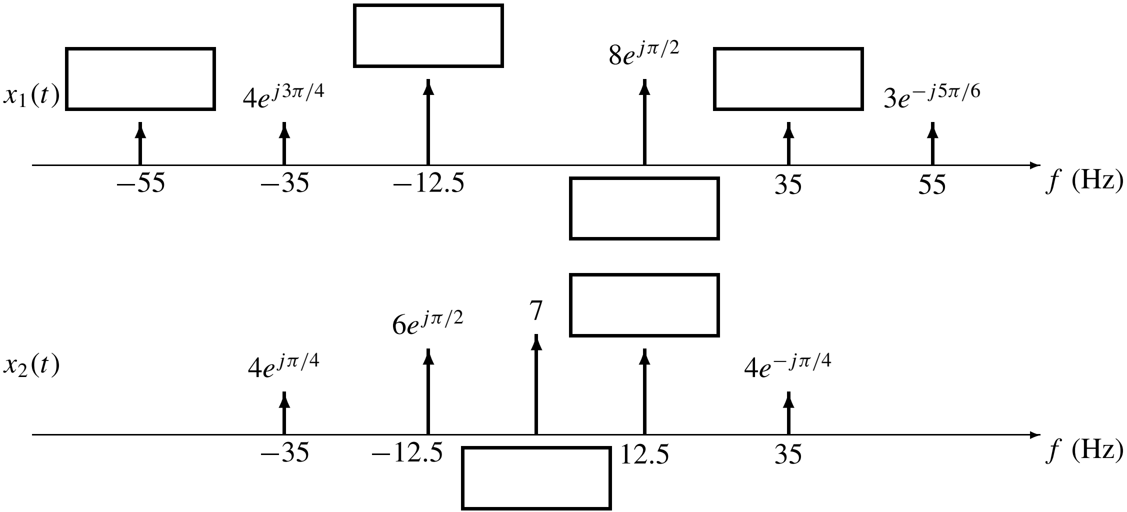
(a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.



(b) Write an equation for $x_2(t)$ in terms of cosine functions.

PROBLEM:

(a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.

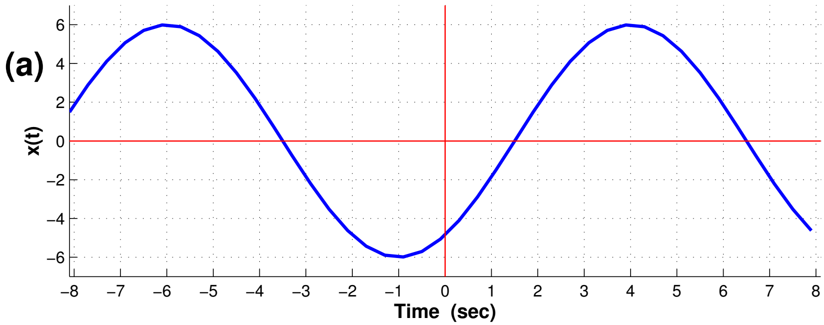


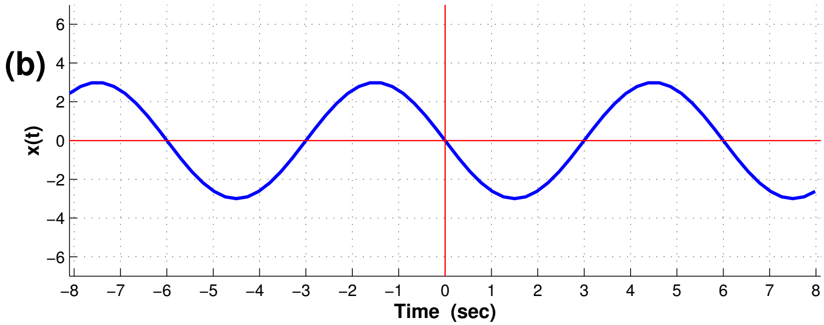
(b) Write an equation for $x_2(t)$ in terms of cosine functions.

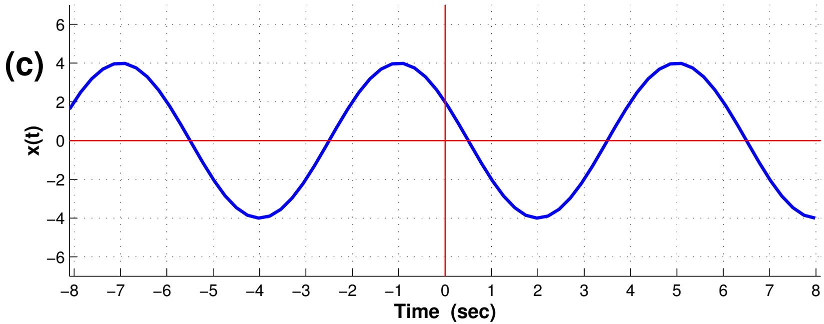
PROBLEM:

Several sinusoidal signals are plotted below. For each plot (a)–(c), determine the amplitude, phase (in radians) and frequency (in Hz). Write your answers in the following table:

PLOT	**(a)**	**(b)**	**(c)**
AMPLITUDE			
PHASE (in radians)			
FREQUENCY (in Hz)			







PROBLEM:

The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation. Assume that

$$v(t) = -2 + 2\cos(6t - \pi/4)$$

and that

$$x(t) = v(t)\cos(30t).$$

(a) Draw the spectrum for $v(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.

(b) Draw the spectrum for $x(t)$. Your sketch should be clearly labeled and all complex amplitudes should be clearly indicated.

PROBLEM:

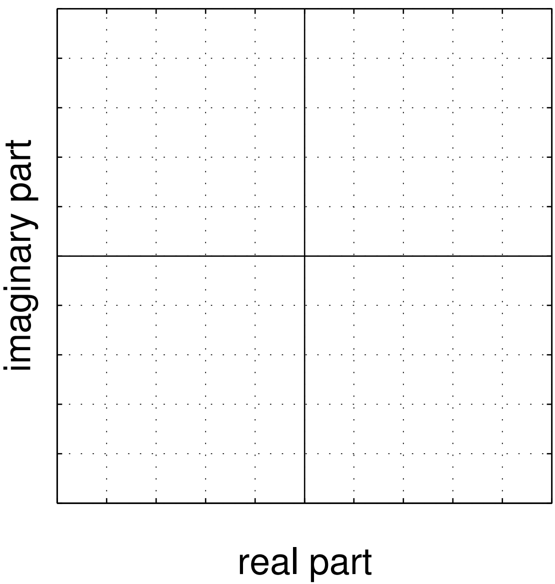
Define $x(t)$ as

$$x(t) = 2\cos(5\pi(t - .6)) + \sqrt{2}\cos(5\pi t + \pi/4)$$

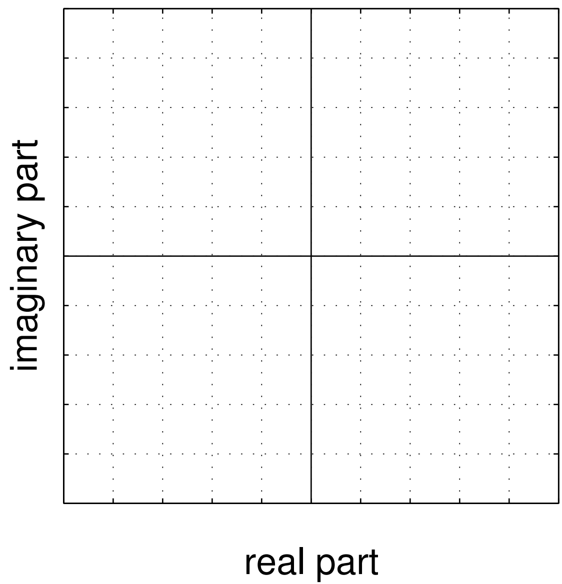
(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ, as well as ω_0.

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).

Two vectors here.



Head-to-tail plot here.



PROBLEM:

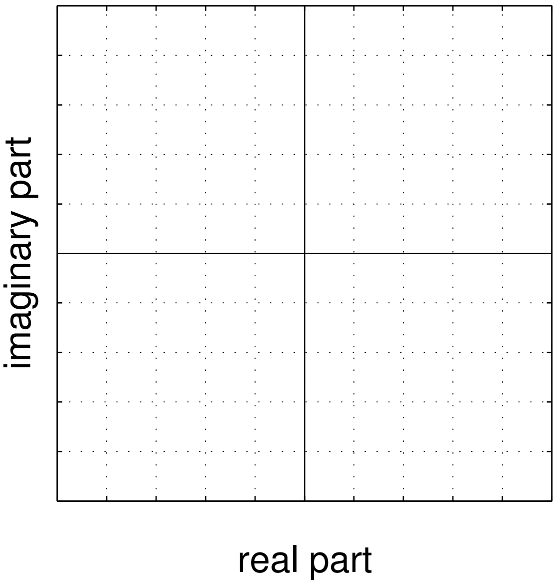
Define $x(t)$ as

$$x(t) = 4\cos(20\pi(t + .075)) + 4\sqrt{3}\cos(20\pi t + 2\pi/3)$$

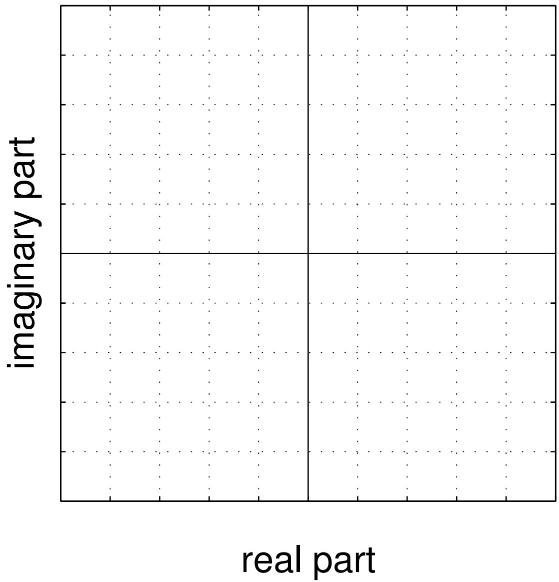
(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ, as well as ω_0.

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).

Two vectors here.

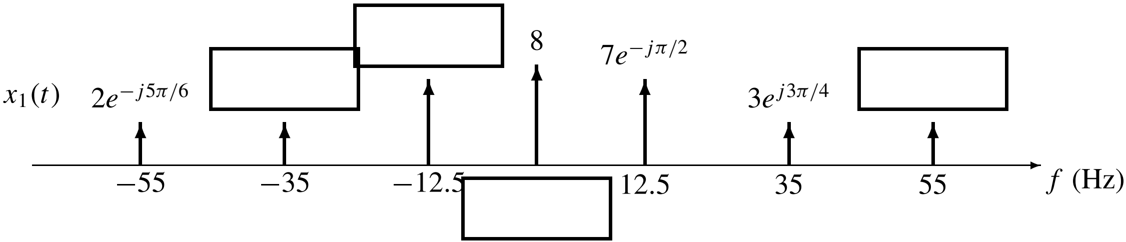


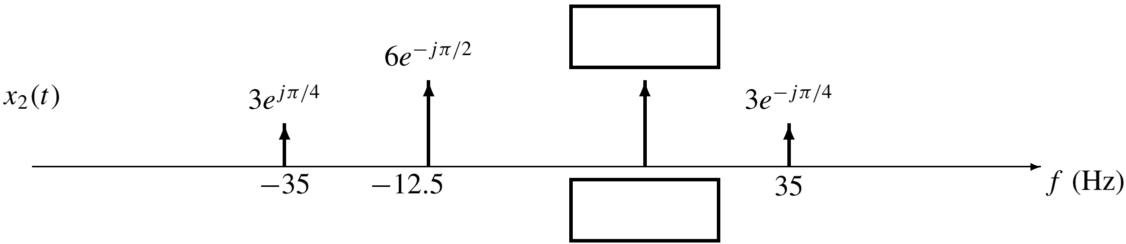
Head-to-tail plot here.



PROBLEM:

(a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.



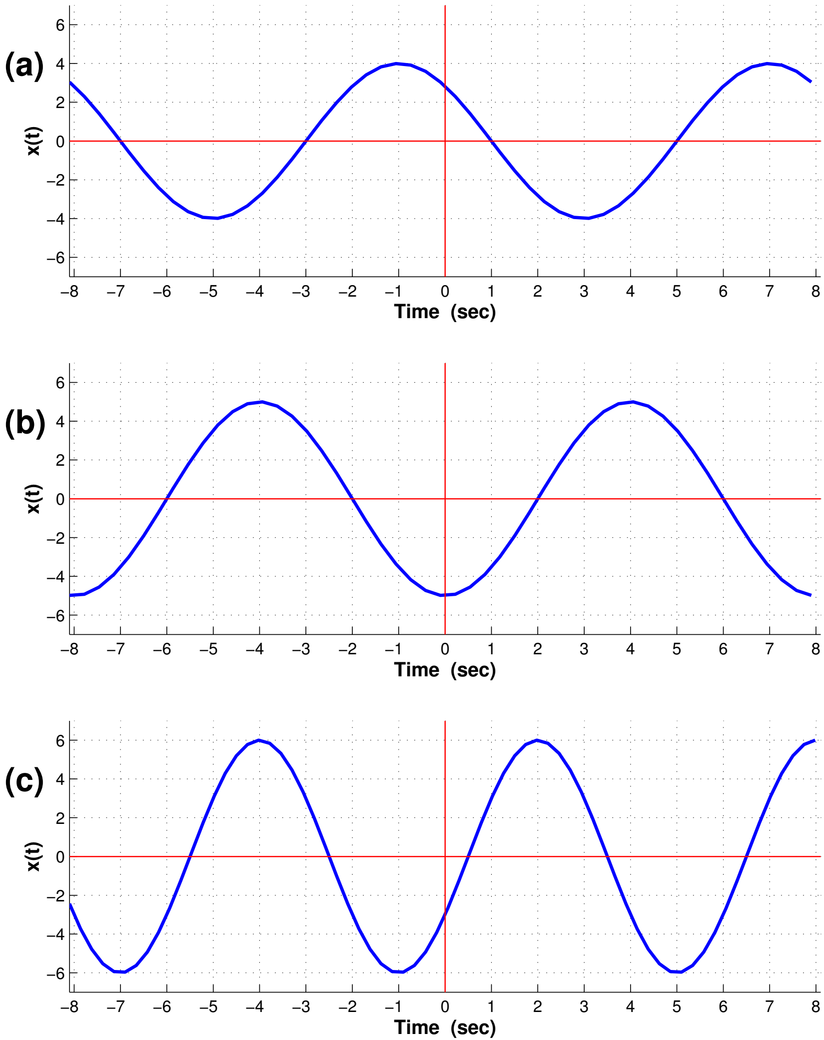


(b) Write an equation for $x_2(t)$ in terms of cosine functions.

PROBLEM:

Several sinusoidal signals are plotted below. For each plot (a)–(c), determine the amplitude, phase (in radians) and frequency (in Hz). Write your answers in the following table:

PLOT	(a)	(b)	(c)
AMPLITUDE			
PHASE (in radians)			
FREQUENCY (in Hz)			



PROBLEM:

The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation. Assume that

$$v(t) = 4 + 4\cos(6t + \pi/2)$$

and that

$$x(t) = v(t)\cos(20t).$$

(a) Draw the spectrum for $v(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.

(b) Draw the spectrum for $x(t)$. Your sketch should be clearly labeled and all complex amplitudes should be clearly indicated.

PROBLEM:

A periodic signal, $x(t)$, is given by

$$x(t) = 1 + 2\cos(300\pi t + \pi/4) + \sin(500\pi t)$$

(a) What is the period of $x(t)$?

(b) Find the Fourier series coefficients of $x(t)$ for $-6 \leq k \leq 6$.

PROBLEM:

A periodic signal, $x(t)$, is given by

$$x(t) = 2 + \cos(250\pi t - \pi) + 2\sin(750\pi t)$$

(a) What is the period of $x(t)$?

(b) Find the Fourier series coefficients of $x(t)$ for $-6 \leq k \leq 6$.

PROBLEM:

A periodic signal, $x(t)$, is given by

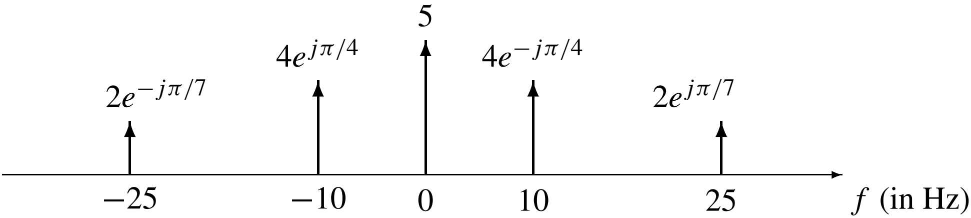
$$x(t) = 2 + \sin(300\pi t) + 3\cos(600\pi t + \pi/3)$$

(a) What is the period of $x(t)$?

(b) Find the Fourier series coefficients of $x(t)$ for $-6 \leq k \leq 6$.

PROBLEM:

A signal $x(t)$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x(t)$.

(b) Is $x(t)$ a periodic signal? If so, what is its period?

(c) Explain why "negative" frequency is needed in the spectrum.

PROBLEM:

Let $x(t) = \sin^3(27\pi t)$.

(a) Determine a formula for $x(t)$ as a sum of complex exponentials.

(b) What is the fundamental period for $x(t)$?

(c) Plot the *spectrum* for $x(t)$.

PROBLEM:

The following MATLAB program makes a plot of a "sine-times-cosine" signal:

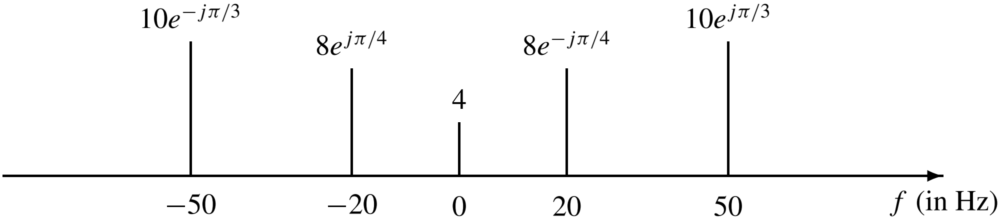
```
tt = 0:0.01:4;
xx = sin(18*pi*tt) .* cos(pi*tt);
plot(tt,xx)
```

(a) Make a sketch of the plot that will be done by MATLAB. Label the time axis carefully.

(b) The "spectrum" diagram gives the frequency content of a signal. Draw a sketch of the spectrum of the signal represented by `xx`. Label the frequencies and complex amplitudes of each component.

PROBLEM:

A signal $x(t)$ has the two-sided spectrum representation shown below.



(a) In the above plot, circle the spectral components that correspond to rotating complex phasors that rotate in the clock-wise direction.

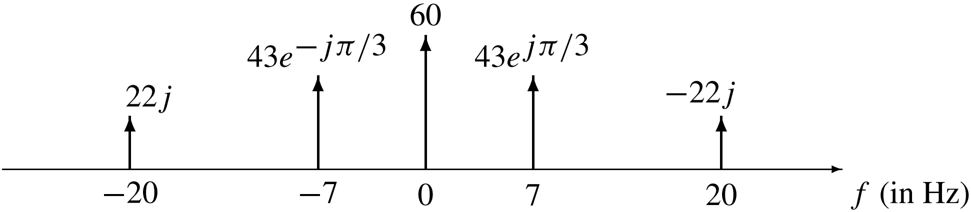
(b) Write an equation for $x(t)$ as a sum of complex rotating phasors. (These phasors may rotate clockwise, counter-clockwise or not at all.)

(c) Write an equation for $x(t)$ as a sum of real functions.

(d) Is the signal $x(t)$ periodic? If so, what is the period?

PROBLEM:

A signal $x(t)$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x(t)$.

(b) Is $x(t)$ a periodic signal? If so, what is its period?

PROBLEM:

The following MATLAB program makes a plot of the *amplitude modulated* signal that is "cosine-times-sine." (Actually the plot is of a finite time segment of the signal.)

```
tt = -1:0.01:1;
xx = cos(33*pi*tt) .* sin(pi*tt);
plot(tt,xx)
```

(a) Make a *sketch* of the plot that will be done by MATLAB. Label the time axis carefully.
Note: this can be done *without* running the MATLAB commands.

(b) The "spectrum" diagram gives the frequency content of a signal. Draw a sketch of the spectrum of the signal represented by `xx`. Label the frequencies and the complex amplitudes of each component.

PROBLEM:

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with middle-C and ending with high-C are:

note name	C	$C^{\#}$	D	$E^{\flat}$	E	F	$F^{\#}$	G	$G^{\#}$	A	$B^{\flat}$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

(d) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The D Major chord is composed of the tones of $D \quad F^{\#} \quad A$ sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the D Major chord assuming that each note is realized by a pure sinusoidal tone. (You do not have to specify the complex amplitudes precisely.)

PROBLEM:

A signal composed of sinusoids is given by the equation

$$x(t) = 44\cos(3\pi t + \pi/6) + 55\cos(6\pi t) - 33\sin(12\pi t)$$

(a) Sketch the spectrum of this signal indicating the complex size of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex amplitude value at the appropriate frequency.

(b) Is $x(t)$ periodic? If so, what is the smallest period?

(c) Now consider a new signal $y(t) = x(t) + 11\cos(5\pi t - \pi/6)$. Draw a carefully labelled sketch of the spectrum for $y(t)$. Is $y(t)$ still periodic? If so, what is the period?

(d) Finally, consider another new signal $w(t) = x(t) + 22\cos(18t + \pi/6)$. Draw a carefully labelled sketch of the spectrum for $w(t)$. Is $w(t)$ still periodic? If so, what is the period?

PROBLEM:

Consider a signal $x(t)$ such that

$$x(t) = 2\cos(\omega_1 t)\cos(\omega_2 t) = \cos[(\omega_2 + \omega_1)t] + \cos[(\omega_2 - \omega_1)t]$$

where $0 < \omega_1 < \omega_2$.

(a) What is the general condition that must be satisfied by $\omega_2 - \omega_1$ and $\omega_2 + \omega_1$ so that $x(t) = x(t + T_0)$; i.e., so that $x(t)$ is *periodic* with period T_0? In addition, determine T_0 in terms of ω_1 and ω_2.

(b) What does the result of part (a) imply about ω_1 and ω_2?

PROBLEM:

A periodic signal $x(t) = x(t + T_0)$ is described over one period $-T_0/2 \le t \le T_0/2$ by the equation

$$x(t) = \begin{cases} 1 & |t| < t_c \\ 0 & t_c < |t| \le T_0/2 \end{cases}$$

where $t_c < T_0/2$.

(a) Sketch the periodic function $x(t)$ for $-T_0 < t < 2T_0$ for the case $t_c = T_0/8$.

(b) Determine the D.C. coefficient X_0. (This answer will depend on t_c and T_0.)

(c) Use the Fourier $\boxed{\textit{analysis}}$ integral (for $k \neq 0$)

$$X_k = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t} dt$$

to determine a general formula for the Fourier coefficients X_k in the representation

$$x(t) = X_0 + \Re e \left\{ \sum_{k=1}^{\infty} X_k e^{jk\omega_0 t} \right\}$$

Your final result should depend on t_c and T_0.
Notes: the frequency ω_0 is given in radians/sec. The integral can be done over any period of the signal; in this case, the most convenient choice is from $-T_0/2$ to $T_0/2$.

(d) Sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(100)$ rad/sec and $t_c = T_0/4$ for frequencies between $-5\omega_0$ and $+5\omega_0$. Label each component with its complex amplitude (magnitude and phase).

(e) Compare your answer in part (d) to the formula for the Fourier coefficients of a 50% duty cycle square wave given in class (and in equation (3.4.5) in the book). Compare both the magnitudes and phases of X_k, as well as the trend versus k. State the similarities and also the differences.

(f) Sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(100)$ rad/sec and $t_c = T_0/8$ for frequencies between $-5\omega_0$ and $+5\omega_0$. Label each component with its complex amplitude (magnitude and phase).

PROBLEM:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp.

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

(a) For the linear-FM "chirp" in (1), determine formulas for the beginning instantaneous frequency (ω_1) and the ending instantaneous frequency (ω_2) in terms of α, β and T_2. For this problem, assume that the starting time of the "chirp" is $t = 0$.

(b) For the "chirp" signal

$$x(t) = \Re e\left\{e^{j2\pi(-33t^2 + 98t - 0.2)}\right\}$$

derive a formula for the *instantaneous* frequency versus time.

(c) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 1$ sec.

(d) *(Optional part)* What would happen if the instantaneous frequency were to become negative? Since instantaneous frequency often corresponds to what we hear, would we hear negative frequency?

PROBLEM:

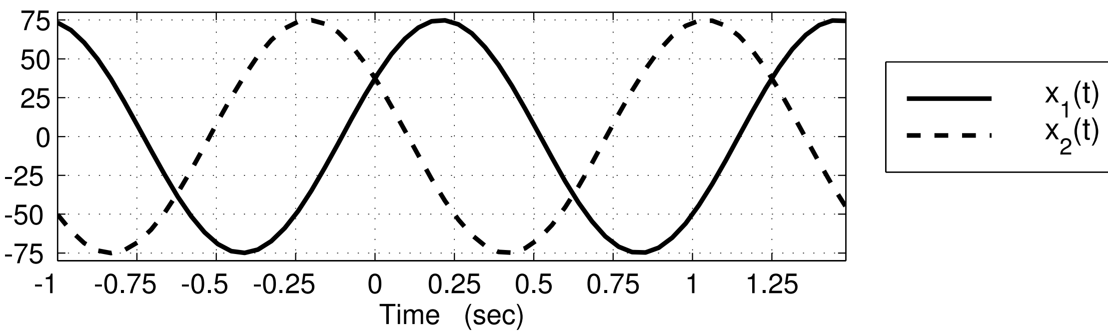
For the following short answer questions, write your answers in the space provided or circle the correct answer:

(a) The periodic signal $x(t)$ has a spectrum containing frequency components at $f = 0, \pm 2$ and ± 2.4 Hz. Determine the *fundamental period,* i.e., the shortest possible period. Make sure your answer has the correct units.

(b) **TRUE** or **FALSE**: "If the signal $x(t)$ is a sinusoid and its spectrum has frequency components at $f = \pm 55$ Hz, then the signal $y(t) = x^2(t)$ has frequency components at the same frequencies."

(c) $\boxed{\text{Circle}}$ the correct answer: When you add $2\cos(2t + 3\pi/4) + 3\cos(2t + \pi/3)$ the maximum value of the resulting signal is:
(A) equal to 0, (B) equal to 5, (C) greater than 5, (D) less than 5, but not 0.

(d) In the figure below two sinusoidal signals are shown. Which one has a phase of $+\pi/3$?
$\boxed{\text{Circle}}$ the correct answer: $\mathbf{x_1(t)}$ or $\mathbf{x_2(t)}$.



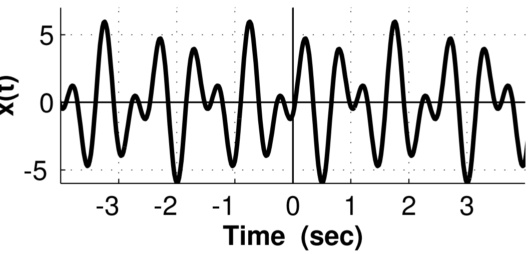
(e) In the figure above both sinusoidal signals have the same frequency. What is the frequency (ω_0) in radians/sec ? $\boxed{\text{Circle}}$ the correct answer.
(A) 2.5π **(B)** 1.25π **(C)** 1.25 **(D)** $\pi/3$ **(E)** 1.6π

PROBLEM:

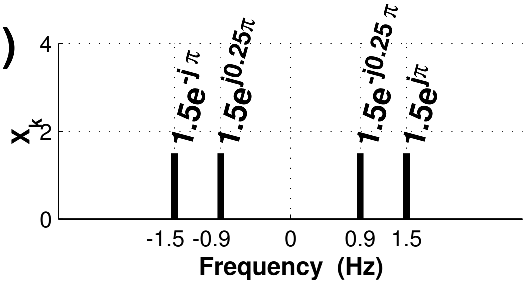
Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each of the signals (a)–(e), determine the correct spectrum (1)–(5). Write your answers in the following table:

(a)	(b)	(c)	(d)	(e)

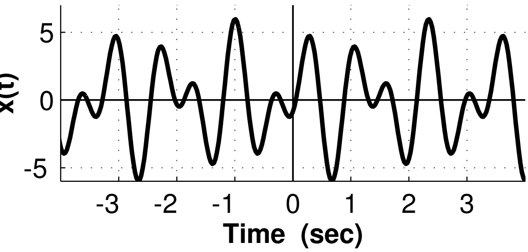
(a)



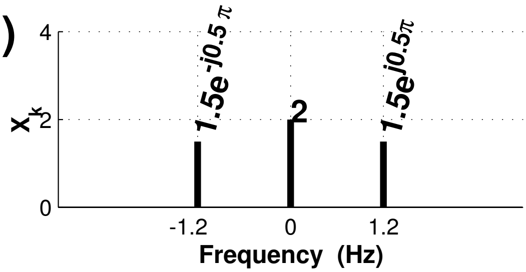
(1)



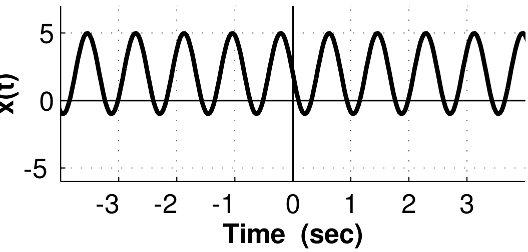
(b)



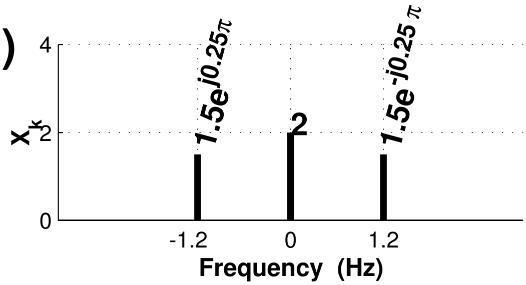
(2)



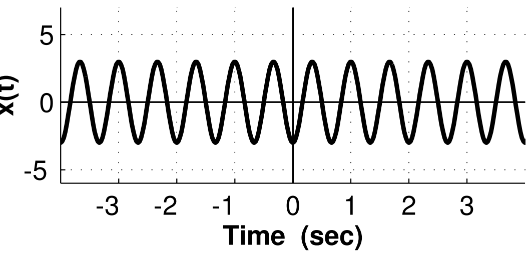
(c)



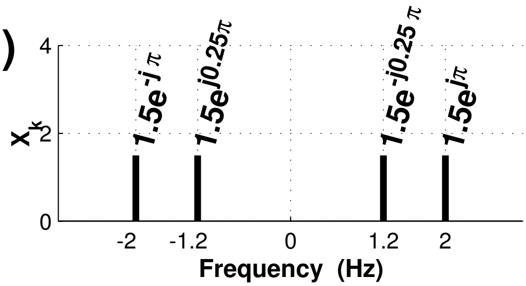
(3)



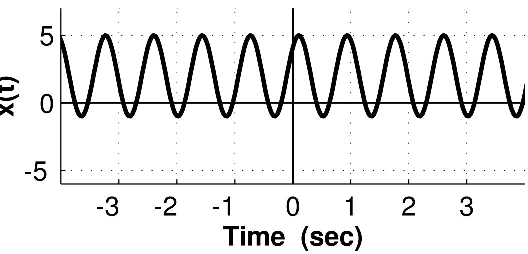
(d)



(4)



(e)



(5)