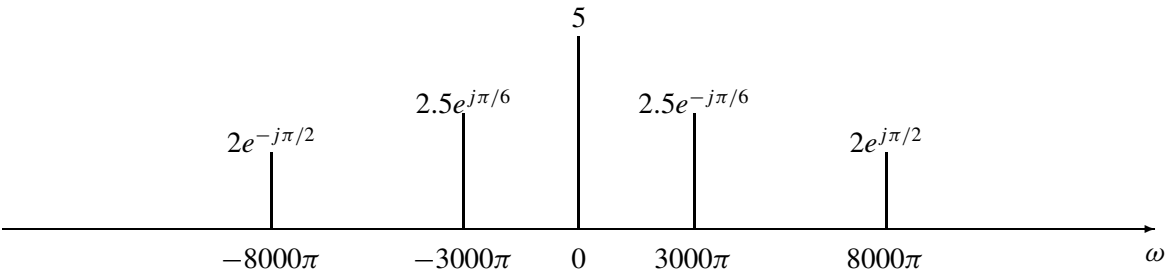


PROBLEM:

A real signal $x(t)$ has the following two-sided spectrum:



- (a) Write an equation for $x(t)$ as a sum of cosines.
- (b) Plot the spectrum of the signal $y(t) = 2x(t) - 3 \cos(5000\pi(t - 0.002))$.

PROBLEM:

In AM radio, the transmitted signal (voice or music) is modulated by a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta has a *carrier frequency* of 750 kHz. For example, if $x(t)$ is the voice/music signal, then the transmitted signal would be:

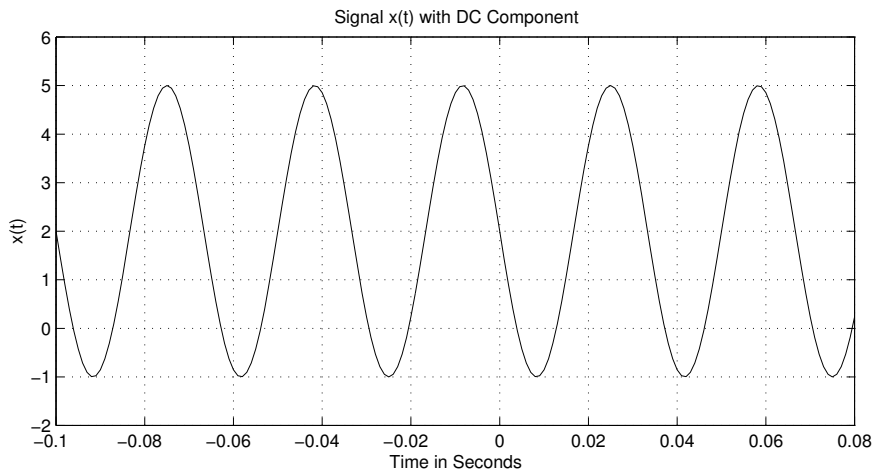
$$y(t) = [x(t) + A] \cos(2\pi(750 \times 10^3)t)$$

where A is a constant. (A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $v(t)$.) Suppose that the signal that is to be transmitted is

$$x(t) = 3 \cos(2000\pi t + \pi/4) + \cos(4000\pi t + \pi/2)$$

Draw the spectrum for $y(t)$ assuming a carrier at 750 kHz with $A = 2$. *Hint: Substitute for $x(t)$ and expand $y(t)$ into a sum of cosine terms of three different frequencies.*

PROBLEM:



The above signal $x(t)$ consists of a DC component plus a cosine signal. The terminology *DC component* means a component that is constant versus time.

- What is the frequency of the DC component? What is the frequency of the cosine component?
- Write an equation for the signal $x(t)$. You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the above graph.
- Expand the equation obtained in the previous part into a sum of positive and negative frequency complex exponential signals.
- Plot the two-sided spectrum of the signal $x(t)$. Show the complex amplitudes for each positive and negative frequency contained in $x(t)$.

PROBLEM:

A linear-FM “chirp” signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A \cos(\alpha t^2 + \beta t + \phi) \quad (1)$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly*.

$$\omega_i(t) = \frac{d}{dt} \psi(t) \quad \text{radians/sec} \quad (2)$$

There are examples on the CD-ROM in the Chapter 3 demos.

- (a) For the linear-FM “chirp” in (1), determine formulas for the beginning instantaneous frequency (ω_1) and the ending instantaneous frequency (ω_2) in terms of α , β and T_2 . For this problem, assume that the starting time of the “chirp” is $t = 0$.
- (b) For the “chirp” signal

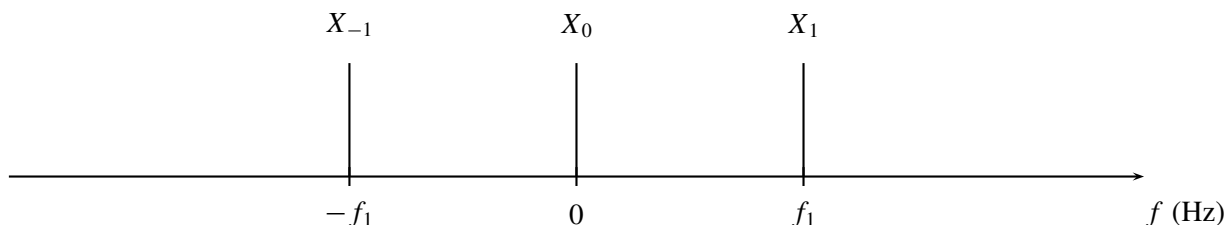
$$x(t) = \Re \left\{ e^{j2\pi(30t^2 - 30t)} \right\}$$

derive a formula for the *instantaneous frequency* versus time. Should your answer for the frequency be a positive number?

PROBLEM:

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal $x(t)$ is given by $x(t) = 3 \cos(250\pi t - \pi/6)$, and its spectrum has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

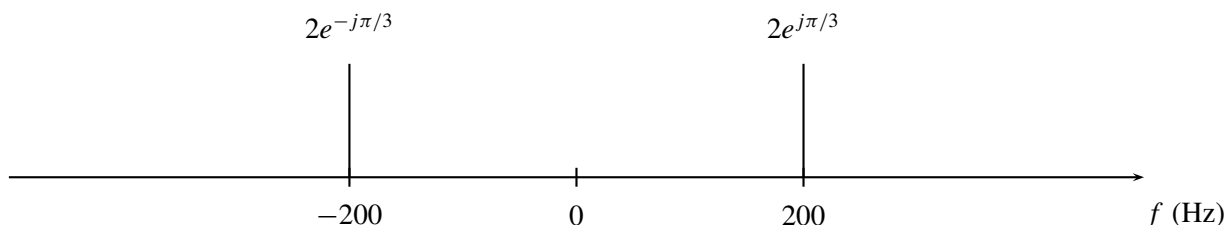
$f_1 =$

$X_0 =$

$X_1 =$

$X_{-1} =$

- (b) The spectrum of a signal $x(t)$ has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for A , f_0 , and t_0 ,

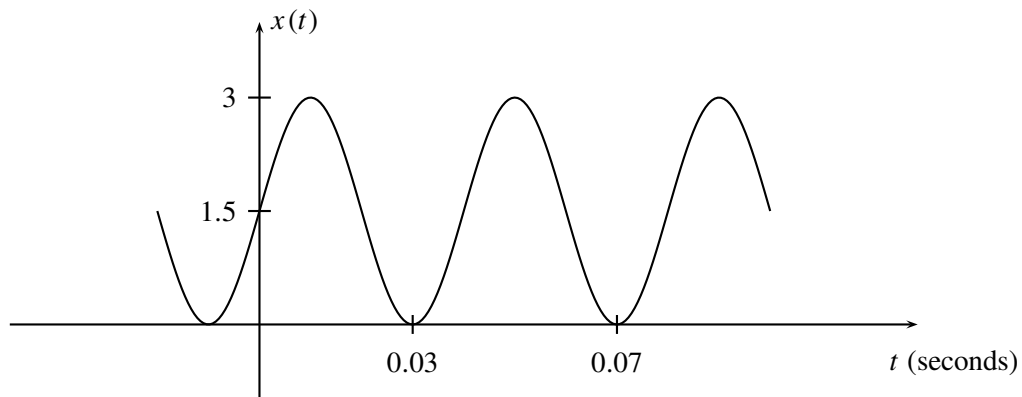
$A =$

$f_0 =$

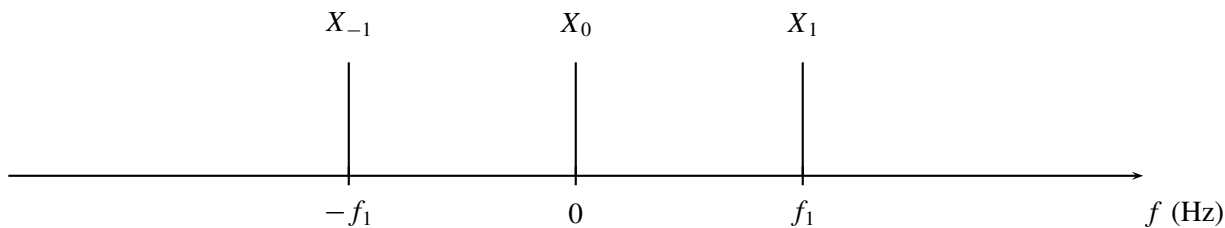
$t_0 =$

PROBLEM:

A signal $x(t) = A \cos(2\pi f_1 t + \phi)$ is shown in the figure below,



The spectrum of $x(t)$ has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

$f_1 =$

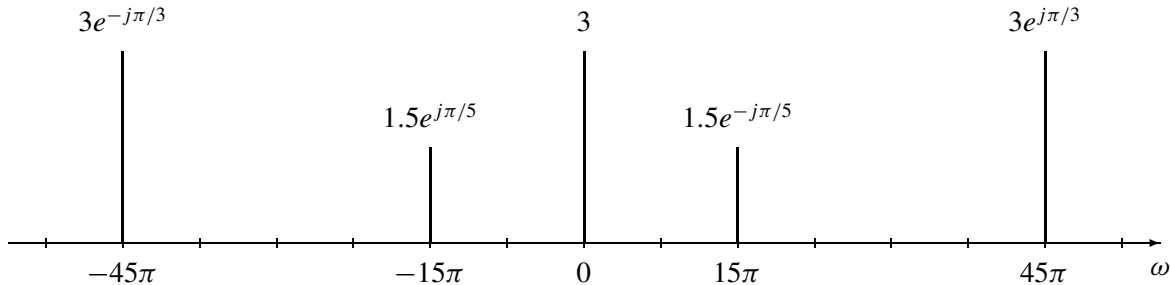
$X_0 =$

$X_1 =$

$X_{-1} =$

PROBLEM:

The spectrum of a signal $x(t)$ is shown in the following figure:



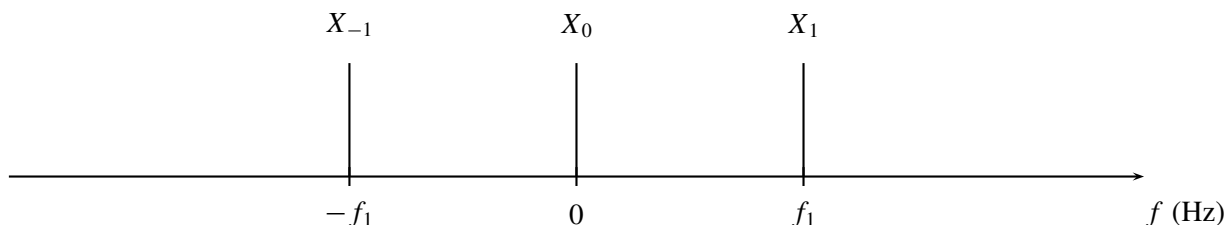
Note that the frequency axis is radian frequency (ω) *not* cyclic frequency (f).

- Write an equation for $x(t)$ in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of $x(t)$?

PROBLEM:

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal $x(t)$ is given by $x(t) = 2 \cos(200\pi t + \pi/8)$, and its spectrum has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

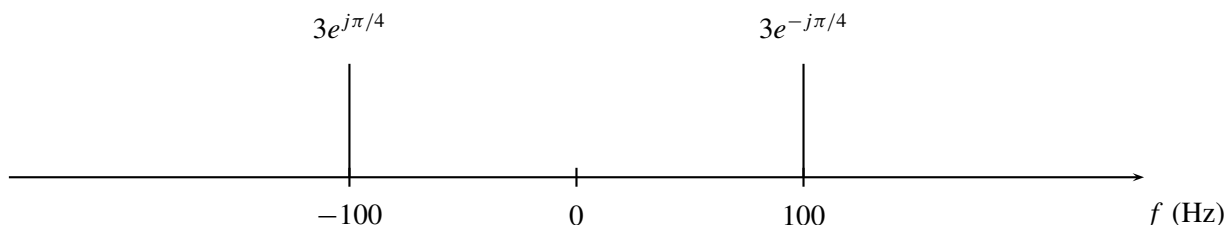
$f_1 =$

$X_0 =$

$X_1 =$

$X_{-1} =$

- (b) The spectrum of a signal $x(t)$ has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for A , f_0 , and t_0 ,

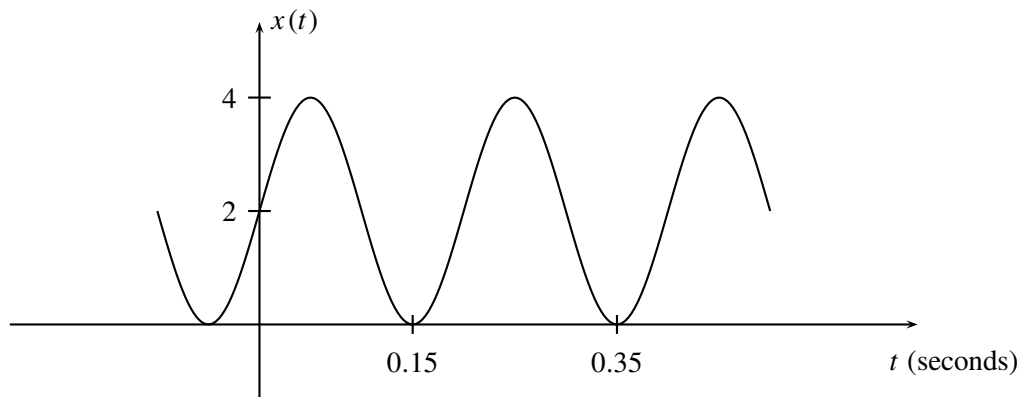
$A =$

$f_0 =$

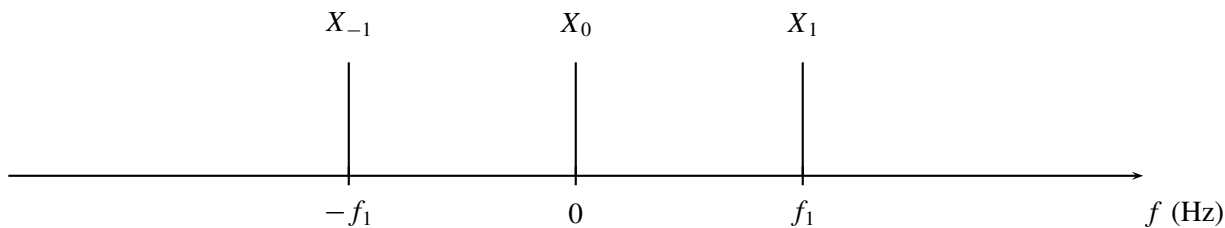
$t_0 =$

PROBLEM:

A signal $x(t) = A \cos(2\pi f_1 t + \phi)$ is shown in the figure below,



The spectrum of $x(t)$ has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

$f_1 =$

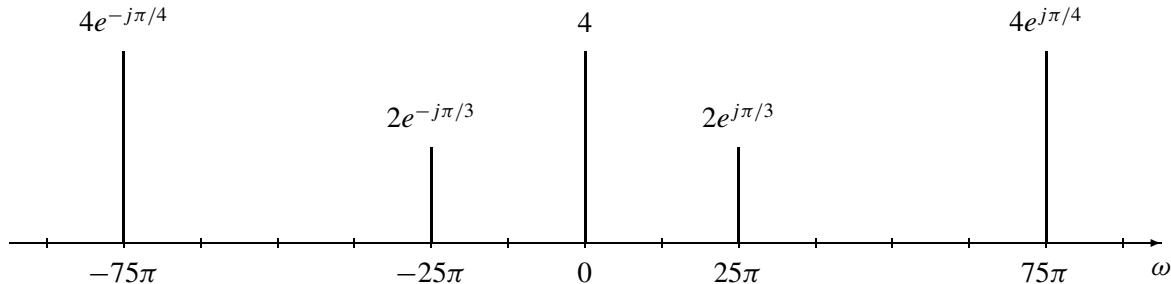
$X_0 =$

$X_1 =$

$X_{-1} =$

PROBLEM:

The spectrum of a signal $x(t)$ is shown in the following figure:



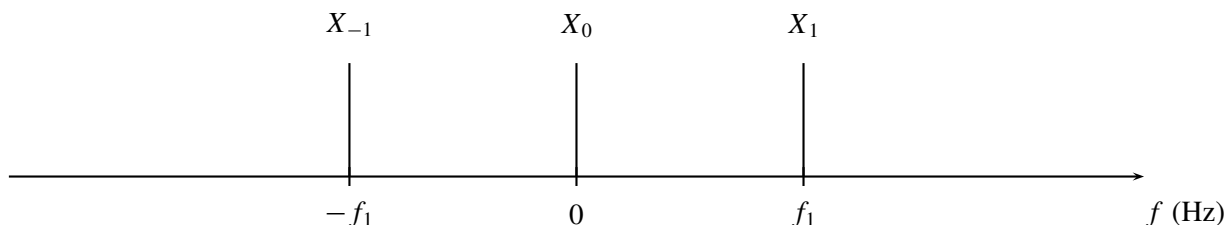
Note that the frequency axis is radian frequency (ω) *not* cyclic frequency (f).

- Write an equation for $x(t)$ in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of $x(t)$?

PROBLEM:

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal $x(t)$ is given by $x(t) = 4 \cos(300\pi t - \pi/3)$, and its spectrum has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

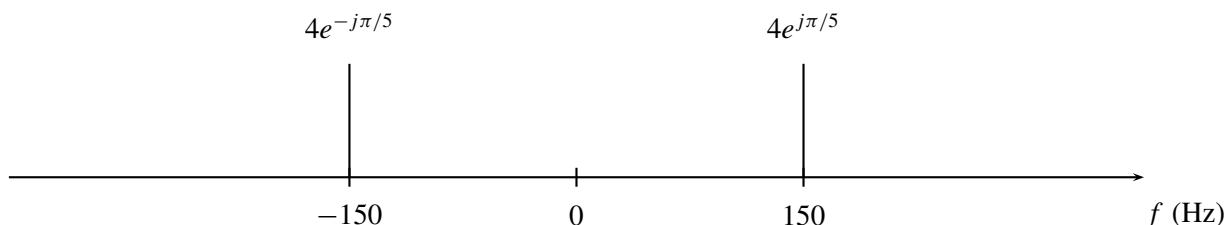
$f_1 =$

$X_0 =$

$X_1 =$

$X_{-1} =$

- (b) The spectrum of a signal $x(t)$ has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for A , f_0 , and t_0 ,

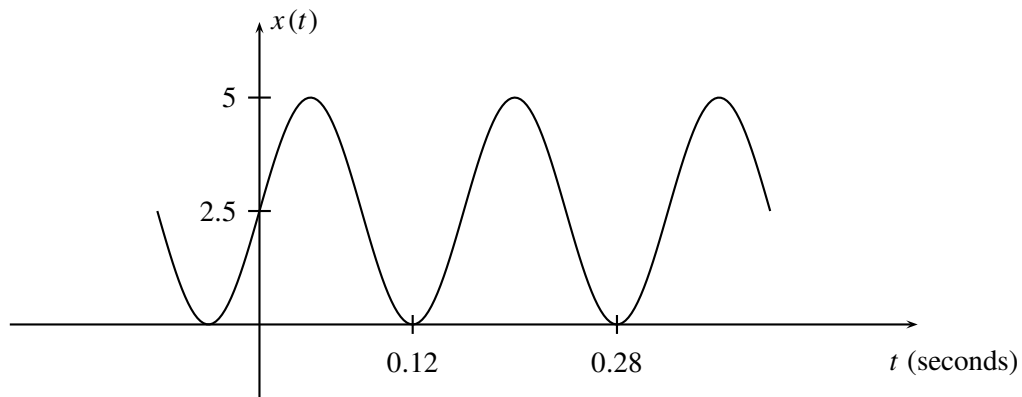
$A =$

$f_0 =$

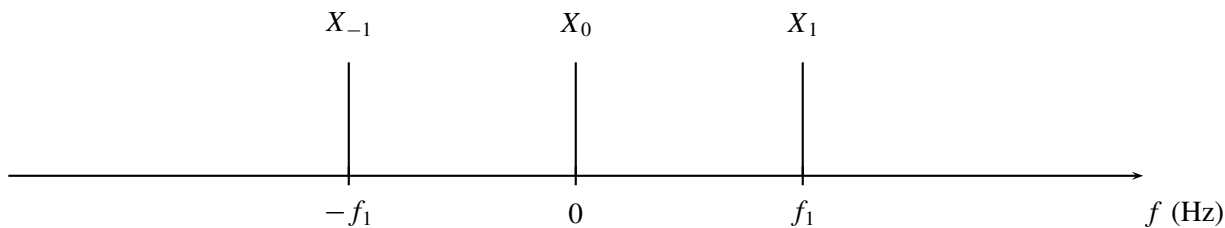
$t_0 =$

PROBLEM:

A signal $x(t) = A \cos(2\pi f_1 t + \phi)$ is shown in the figure below,



The spectrum of $x(t)$ has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

$f_1 =$

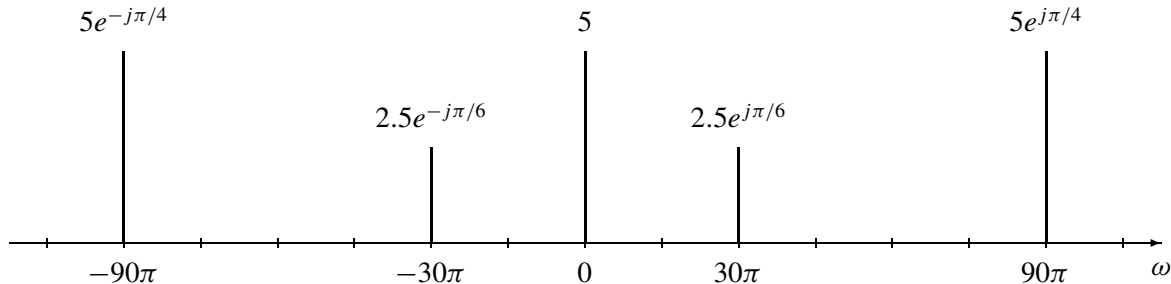
$X_0 =$

$X_1 =$

$X_{-1} =$

PROBLEM:

The spectrum of a signal $x(t)$ is shown in the following figure:



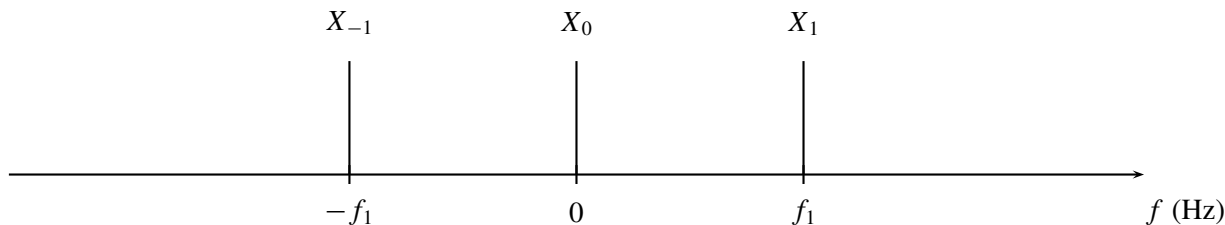
Note that the frequency axis is radian frequency (ω) *not* cyclic frequency (f).

- Write an equation for $x(t)$ in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of $x(t)$?

PROBLEM:

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal $x(t)$ is given by $x(t) = 5 \cos(350\pi t - \pi/7)$, and its spectrum has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

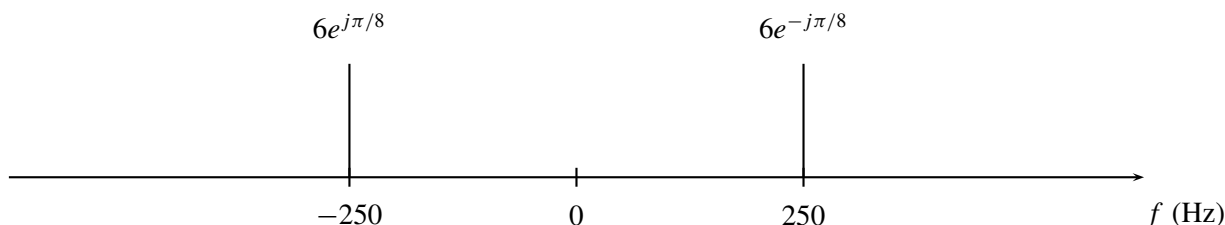
$$f_1 =$$

$$X_0 =$$

$$X_1 =$$

$$X_{-1} =$$

- (b) The spectrum of a signal $x(t)$ has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for A , f_0 , and t_0 ,

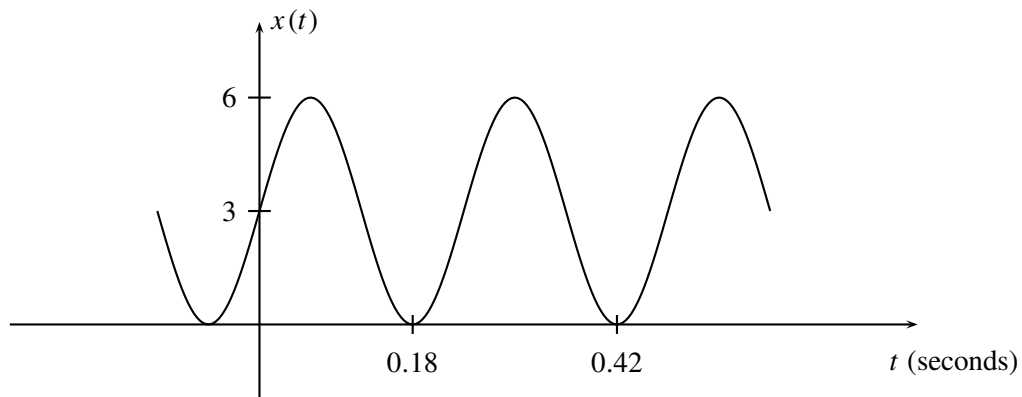
$$A =$$

$$f_0 =$$

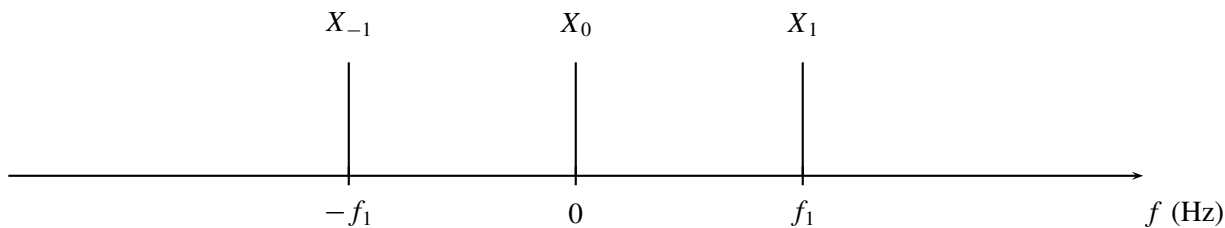
$$t_0 =$$

PROBLEM:

A signal $x(t) = A \cos(2\pi f_1 t + \phi)$ is shown in the figure below,



The spectrum of $x(t)$ has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

$f_1 =$

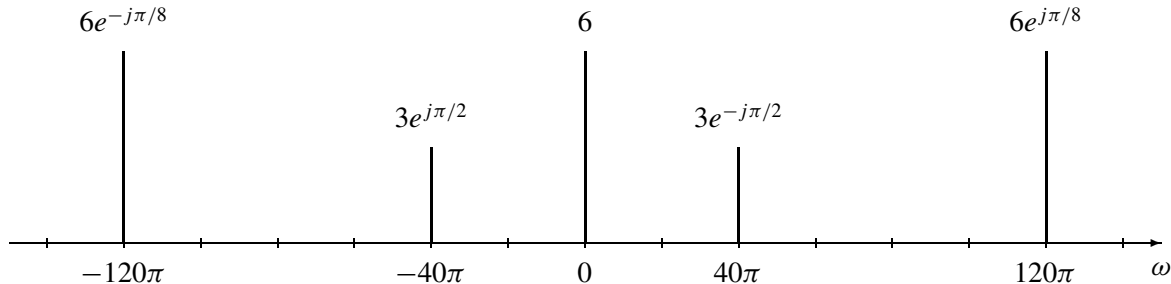
$X_0 =$

$X_1 =$

$X_{-1} =$

PROBLEM:

The spectrum of a signal $x(t)$ is shown in the following figure:



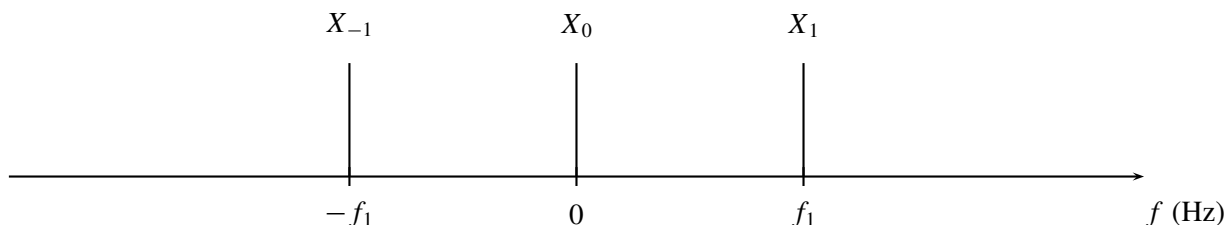
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- Write an equation for $x(t)$ in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of $x(t)$?

PROBLEM:

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal $x(t)$ is given by $x(t) = 3 \cos(400\pi t + 3\pi/16)$, and its spectrum has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

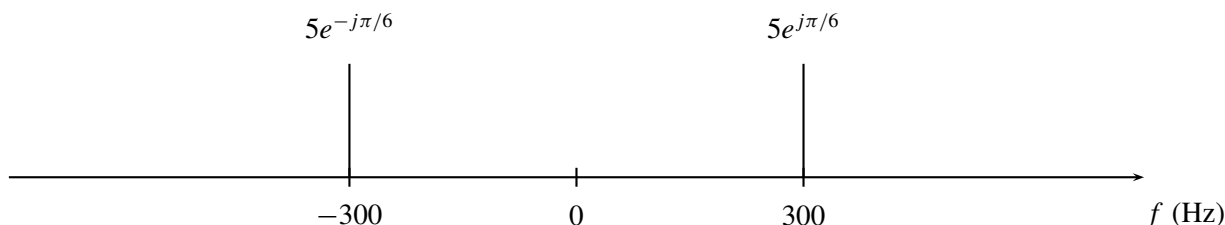
$f_1 =$

$X_0 =$

$X_1 =$

$X_{-1} =$

- (b) The spectrum of a signal $x(t)$ has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for A , f_0 , and t_0 ,

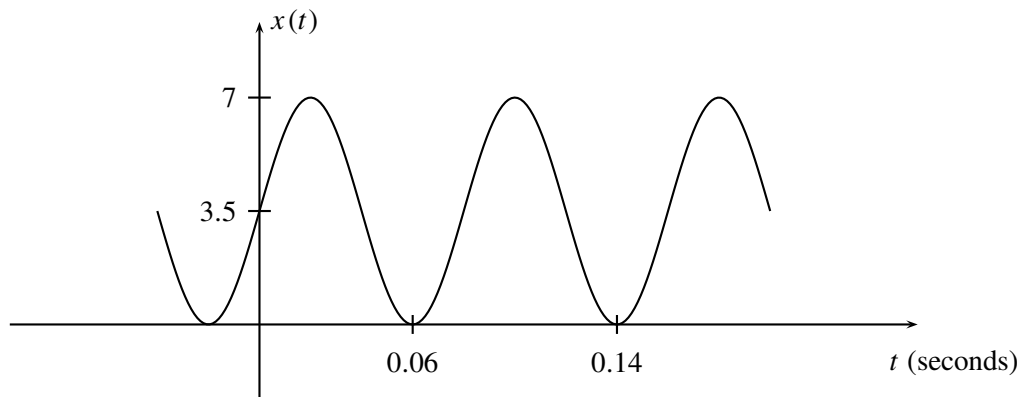
$A =$

$f_0 =$

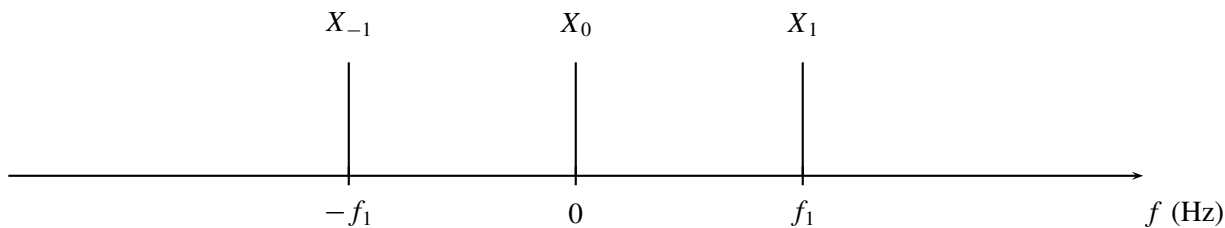
$t_0 =$

PROBLEM:

A signal $x(t) = A \cos(2\pi f_1 t + \phi)$ is shown in the figure below,



The spectrum of $x(t)$ has the form



Determine the values for f_1 , X_0 , X_1 , and X_{-1} . Note that the frequencies f are given in Hertz.

$$f_1 =$$

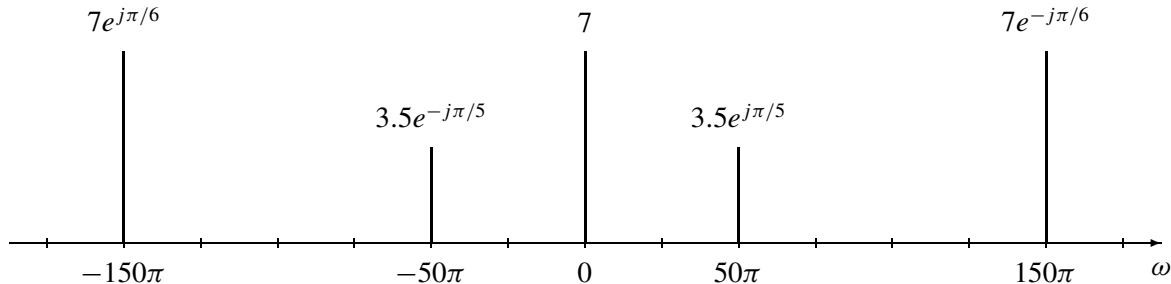
$$X_0 =$$

$$X_1 =$$

$$X_{-1} =$$

PROBLEM:

The spectrum of a signal $x(t)$ is shown in the following figure:



Note that the frequency axis is radian frequency (ω) *not* cyclic frequency (f).

- Write an equation for $x(t)$ in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of $x(t)$?

PROBLEM:

A periodic signal, $x(t)$, is given by

$$x(t) = 1 + 3 \cos(300\pi t) + 2 \sin(500\pi t - \pi/4)$$

- (a) What is the period of $x(t)$?
- (b) Find the Fourier series coefficients of $x(t)$.

PROBLEM:

A periodic signal, $x(t)$, is given by

$$x(t) = 2 + \cos(150\pi t - \pi/6) + 2 \sin(450\pi t)$$

- (a) What is the period of $x(t)$?
- (b) Find the Fourier series coefficients of $x(t)$.

PROBLEM:

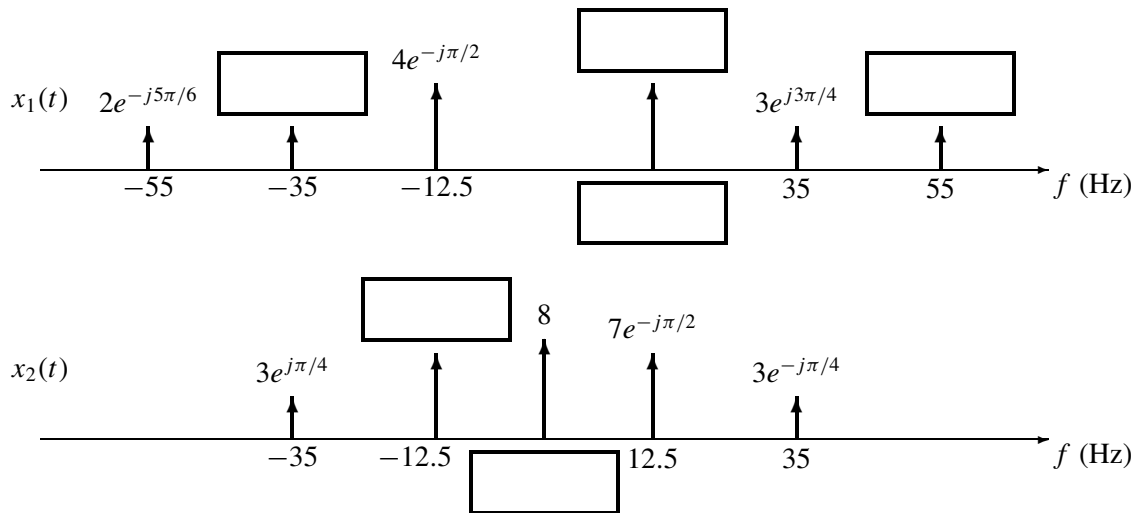
A periodic signal, $x(t)$, is given by

$$x(t) = 3 + \sin(200\pi t) + 3 \cos(600\pi t + \pi/3)$$

- (a) What is the period of $x(t)$?
- (b) Find the Fourier series coefficients of $x(t)$.

PROBLEM:

- (a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.



- (b) Write an equation for $x_2(t)$ in terms of cosine functions.

PROBLEM:

The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation. Assume that

$$v(t) = 3 + 3 \cos(5t + \pi/3)$$

and that

$$x(t) = v(t) \cos(20t).$$

- (a) Draw the spectrum for $v(t)$.
- (b) Draw the spectrum for $x(t)$.

PROBLEM:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form. Let

$$V = -\frac{1}{\sqrt{3}} - j.$$

- (a) Express jV in polar form. In addition plot jV as a vector.
- (b) Express the inverse of V in rectangular form. In addition plot $\frac{1}{V}$ as a vector.
- (c) If $Z = \frac{|V|}{V^*}$, express Z in polar form. In addition plot Z as a vector.
- (d) Express $\Re\{j^3 V e^{j15t}\}$ in the standard “cosine” form.

