BLM3620 Digital Signal Processing*

Erkan Uslu

euslu@yildiz.edu.tr

Yıldız Technical University – Computer Engineering

*Based on lecture notes from Ali Can Karaca & Ahmet Elbir

Lecture #5 – Discrete Time Signals and Convolution

- Basic discrete time signals
- Linear, Time-Invariant, Causal and Stable Systems
- Impulse Response
- Discrete Convolution
- MATLAB Application

Before we begin…

Is convolution topic important for AI?

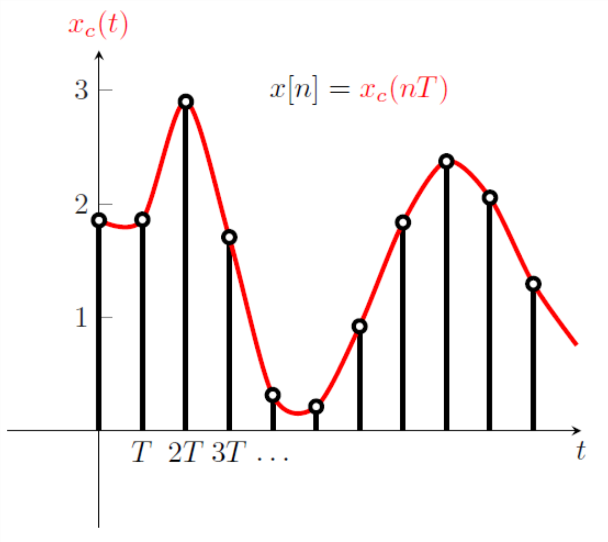
Before we begin…

Is convolution topic important for AI?

Remember: Last Lecture

After sampling, we have discrete time signal.

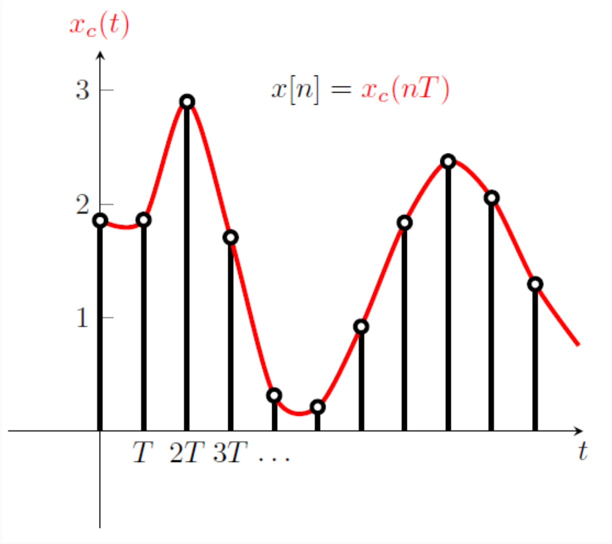
We should learn how to process this digital signal.



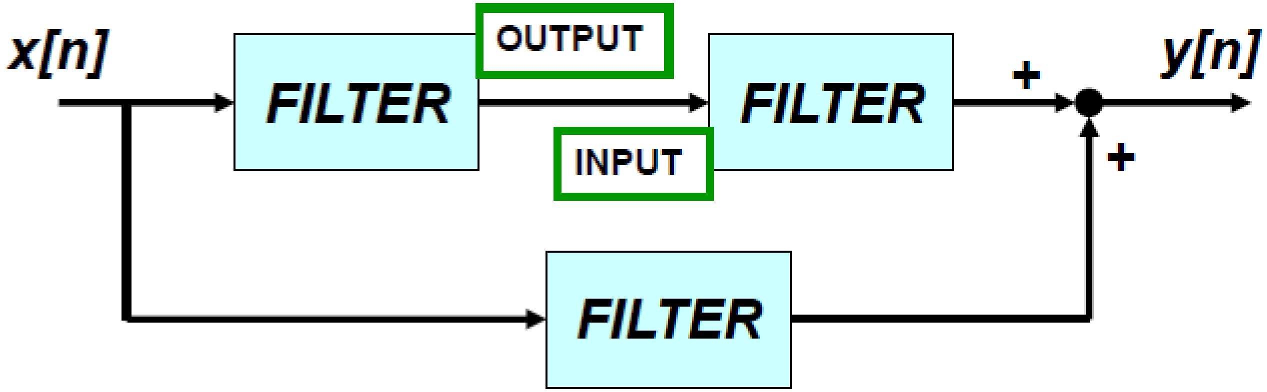
Concentrate on the Filtering theory

After sampling, we have discrete time signal.

We should learn how to process this digital signal.

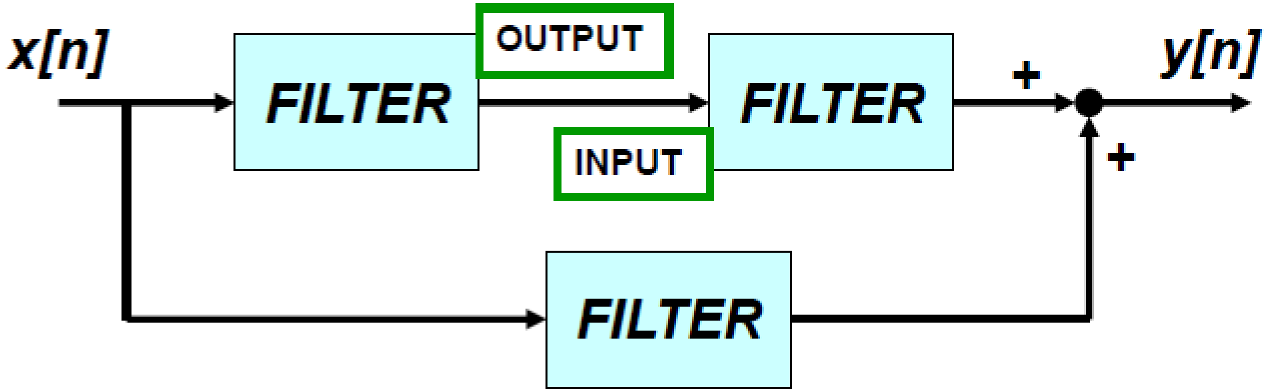


Filtering - Block Diagram Representation

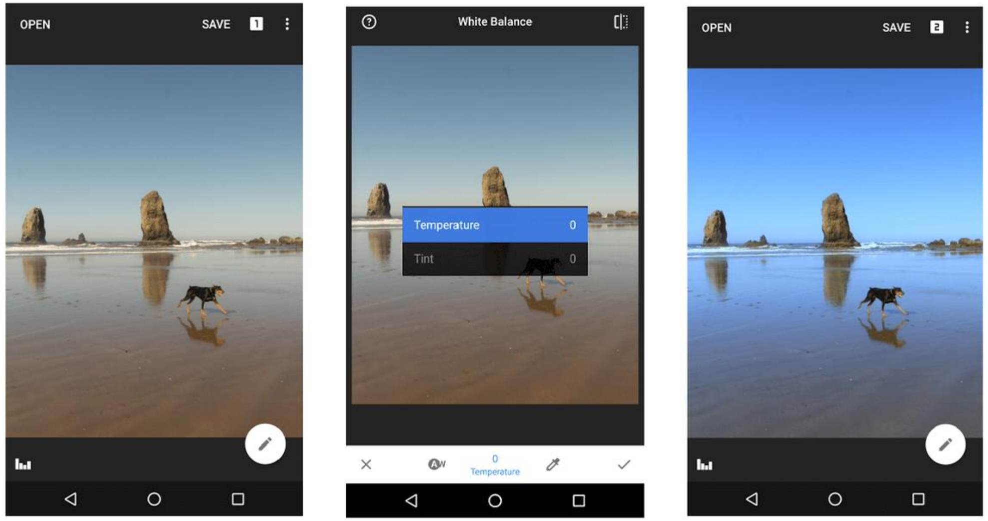


- BUILD UP COMPLICATED FILTERS
  - FROM SIMPLE MODULES
  - Ex: FILTER MODULE MIGHT BE 3-pt FIR

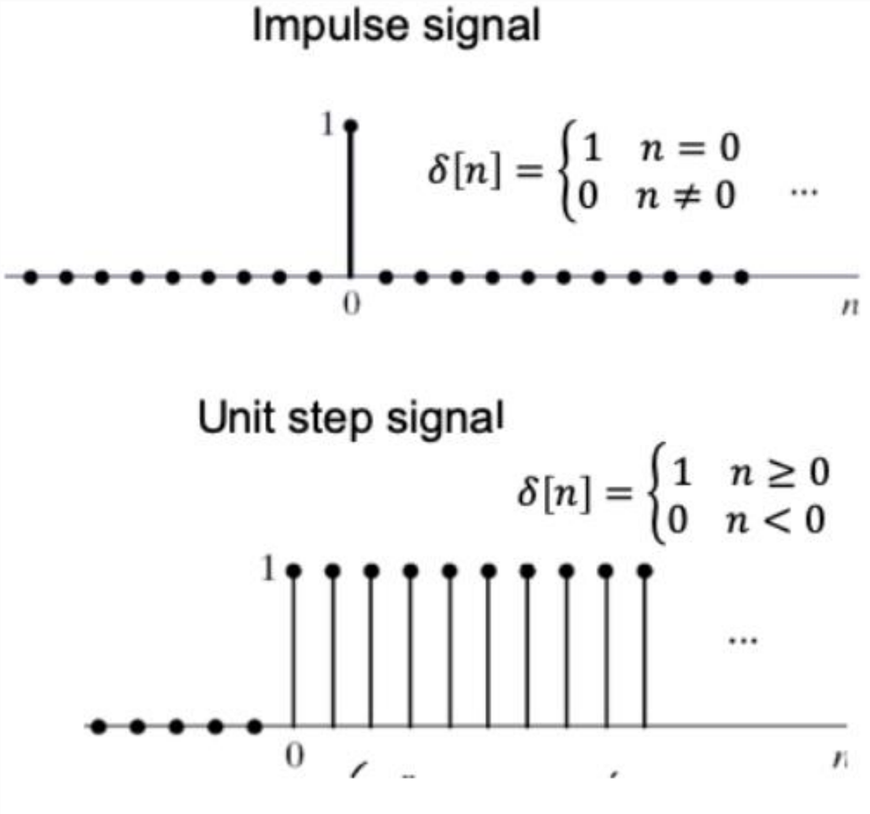
Filtering - Block Diagram Representation



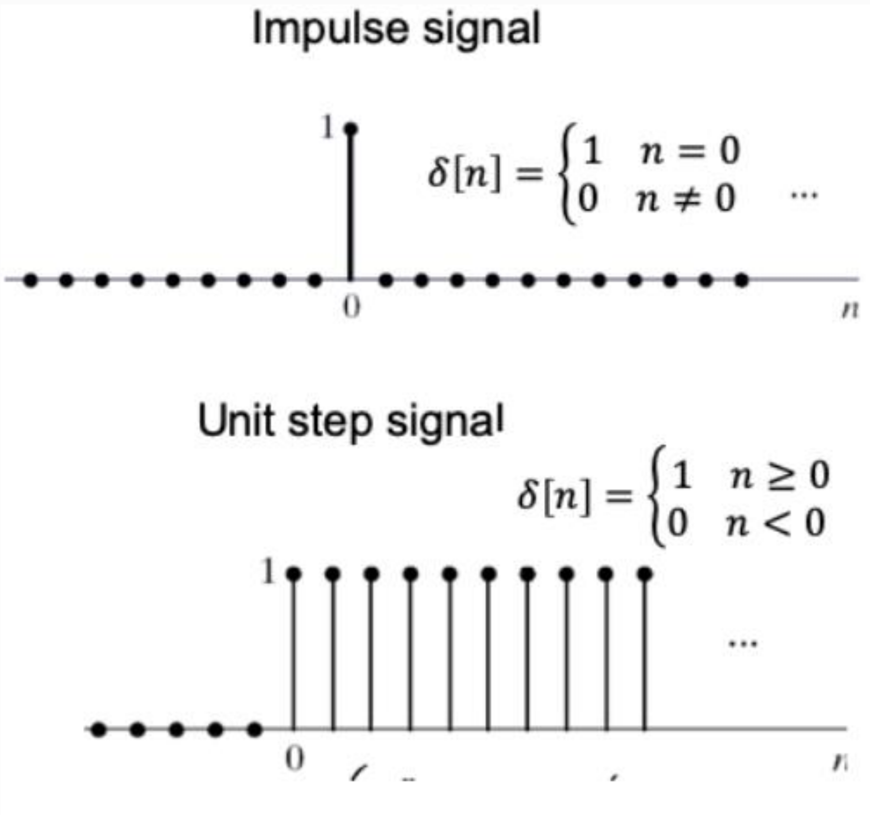
- BUILD UP COMPLICATED FILTERS
  - FROM SIMPLE MODULES
  - Ex: FILTER MODULE MIGHT BE 3-pt FIR

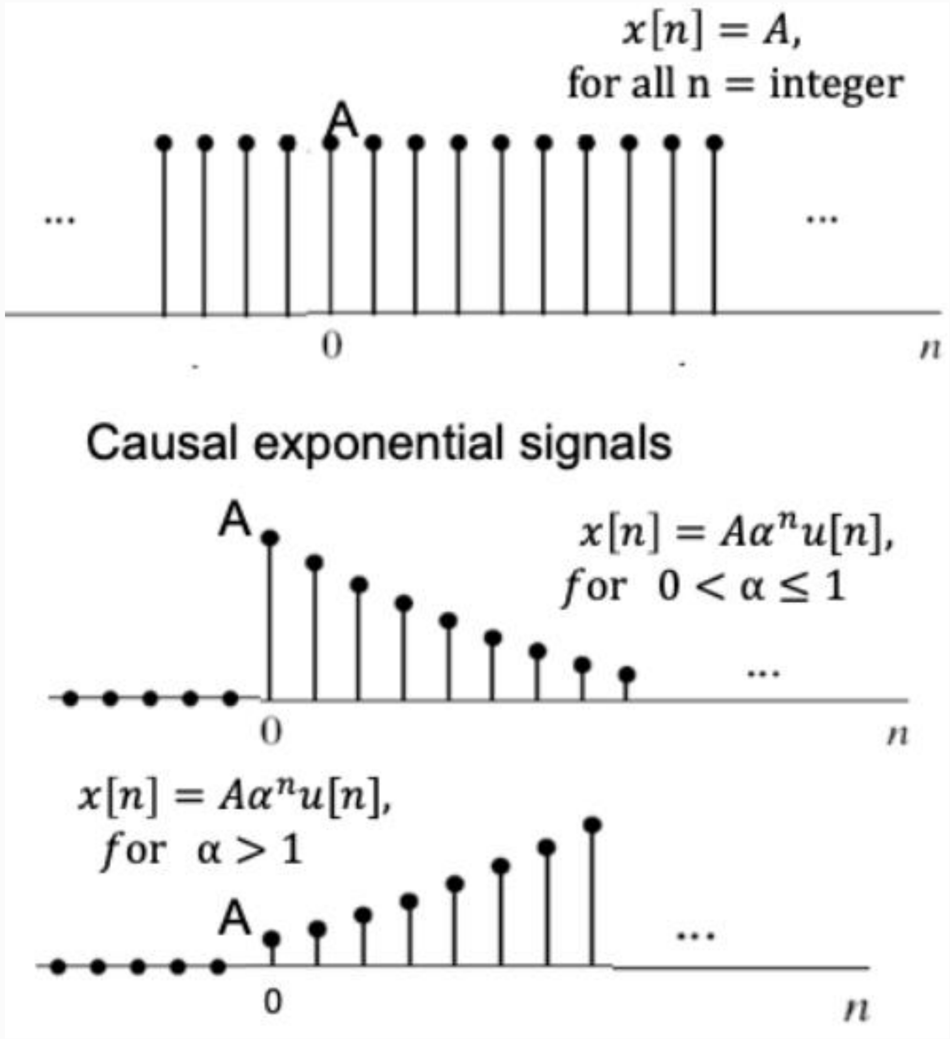


Basic Discrete Signals

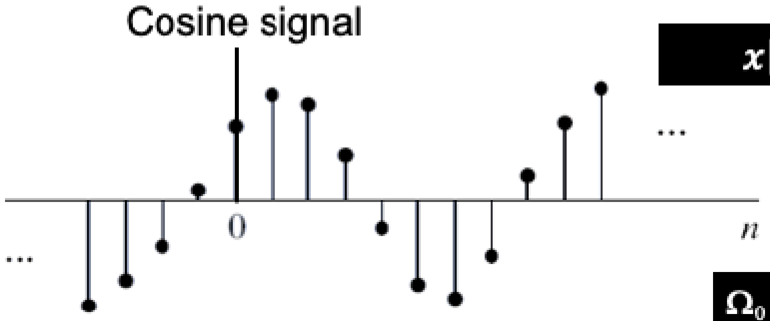


Basic Discrete Signals





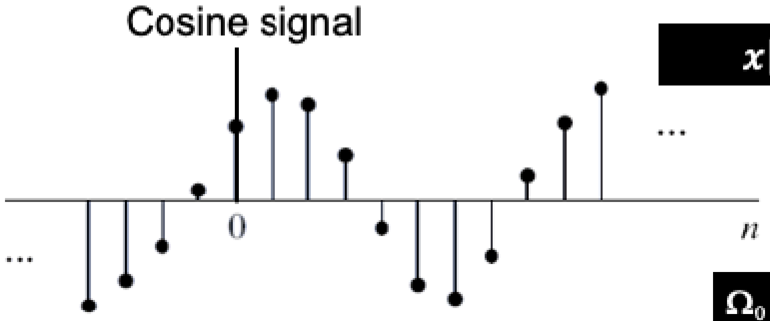
Basic Discrete Signals - 2



$$x[n] = A\cos(\Omega_0 n + \phi) \quad \text{for n = integer}$$

Ω_0 is angle increment between samples
In radian/sample

Basic Discrete Signals - 2

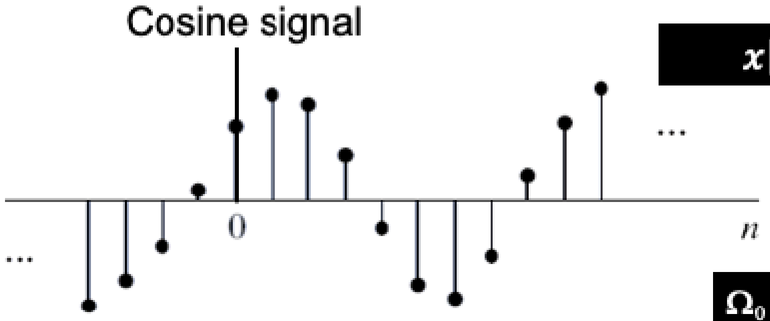


$x[n] = A\cos(\Omega_0 n + \phi) \quad \text{for n = integer}$

Ω_0 is angle increment between samples
In radian/sample

$x[n] = \cos\left(\frac{2\pi n}{12}\right)$ -> Plot this signal in MATLAB.

Basic Discrete Signals - 2



$x[n] = A\cos(\Omega_0 n + \phi)$ for n = integer

Ω_0 is angle increment between samples
In radian/sample

$x[n] = \cos\left(\frac{2\pi n}{12}\right)$ -> Plot this signal in MATLAB.

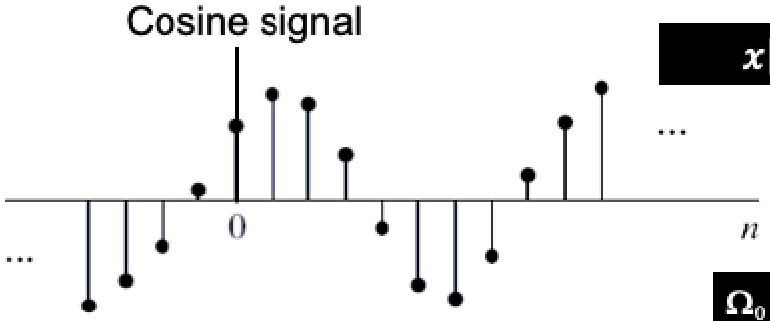
MATLAB kodu:

```
clc; clear all;
%%

n=-20:20;
x = 0.*n;
for i=1:length(n)
   x(i)=cos(2*pi*n(i)/12);
End

stem(n,x,'filled');
```

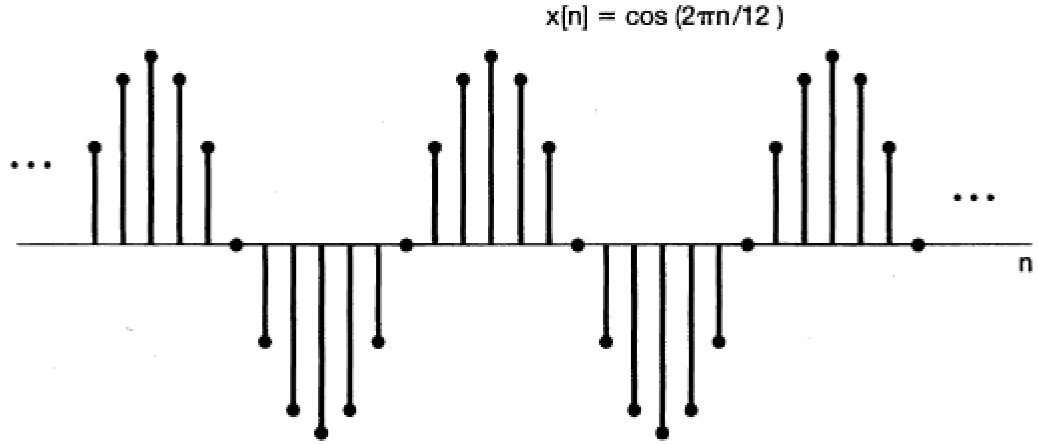
Basic Discrete Signals - 2



$x[n] = A\cos(\Omega_0 n + \phi)$ for n = integer

Ω_0 is angle increment between samples
In radian/sample

$x[n] = \cos\left(\frac{2\pi n}{12}\right)$ -> Plot this signal in MATLAB.



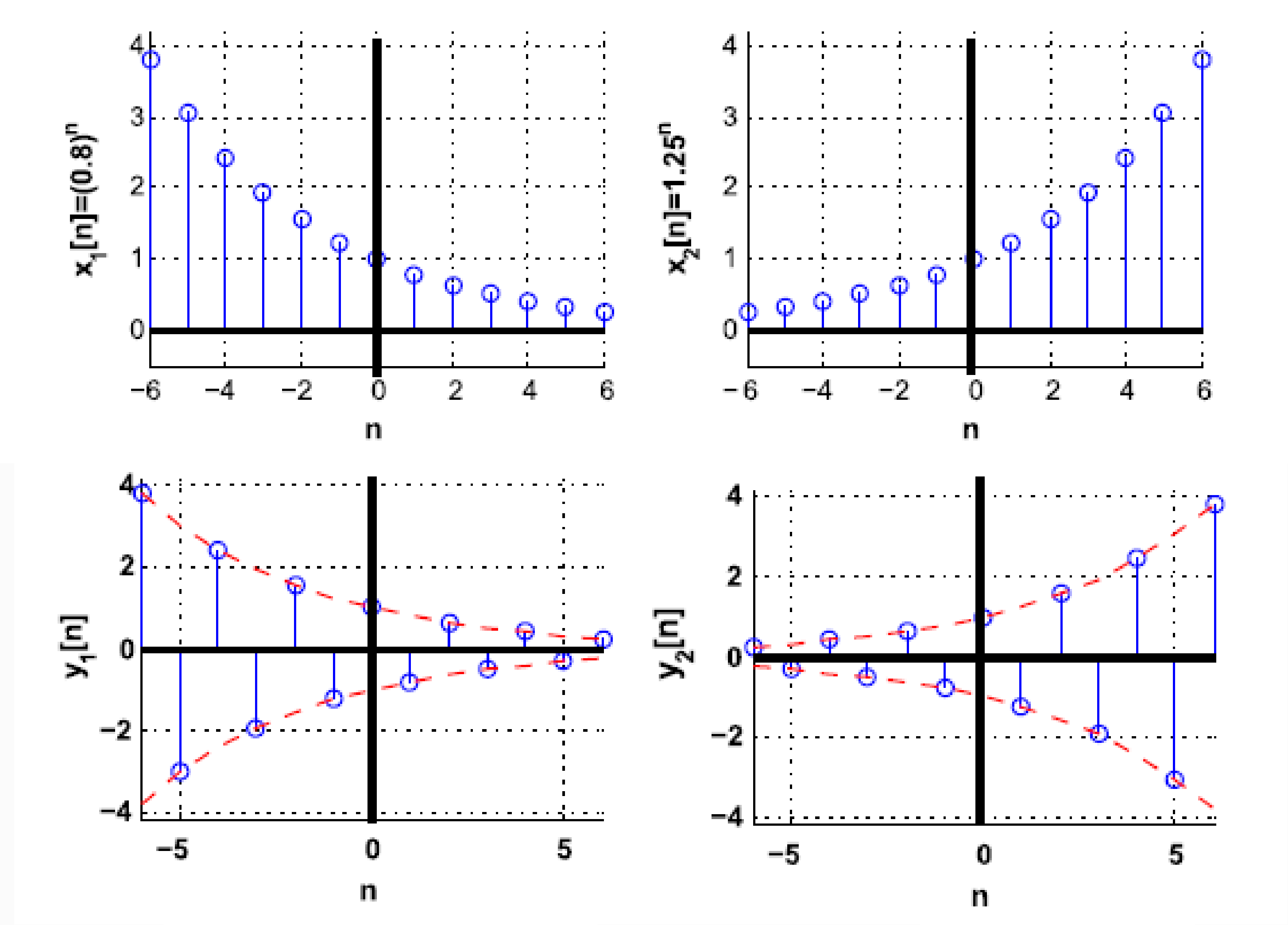
MATLAB kodu:

```
clc; clear all;
%%

n=-20:20;
x = 0.*n;
for i=1:length(n)
   x(i)=cos(2*pi*n(i)/12);
End

stem(n,x,'filled');
```

Basic Discrete Signals - 3



$$y_1[n] = x_1[n]\cos(\pi n) \qquad x_2[n]\cos(\pi n)$$

MATLAB kodu:

```
clc; clear all;
%%

n=-20:20;
x = 0.*n;

for i=1:length(n)
   x(i)=(0.8^n(i))*cos(pi*n(i));
end

stem(n,x,'filled');
```

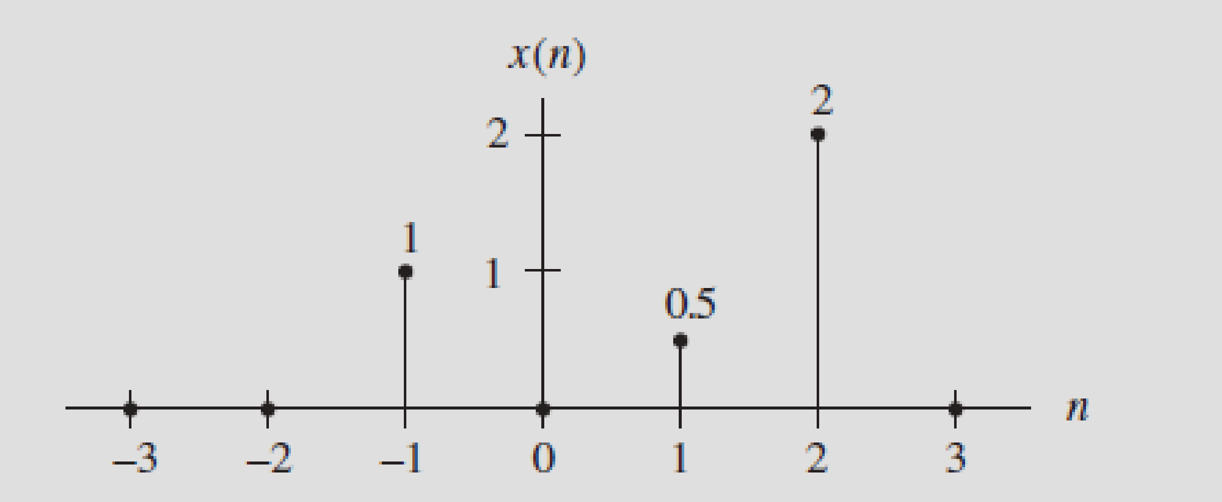
Exponential Sinuzoids

Example

Given the following,

$$x(n) = \delta(n+1) + 0.5\delta(n-1) + 2\delta(n-2),$$

Sketch this sequence.



Example with Sampling

Assuming a DSP system with a sampling time interval of 125 μs,

(a) Convert each of following analog signal $x(t)$ to the digital signal $x(n)$.

1. $x(t) = 10e^{-5000t}u(t)$
2. $x(t) = 10\sin(2000\pi t)u(t)$

(b) Determine and plot the sample values from each obtained digital function.

Example with Sampling

Assuming a DSP system with a sampling time interval of 125 μs,

(a) Convert each of following analog signal $x(t)$ to the digital signal $x(n)$.

1. $x(t) = 10e^{-5000t}u(t)$
2. $x(t) = 10\sin(2000\pi t)u(t)$

(b) Determine and plot the sample values from each obtained digital function.

(a) Since $T = 0.000125$ s in Eq. (3.3), substituting $t = nT = n \times 0.000125 = 0.000125n$ into the analog signal $x(t)$ expressed in (1) leads to the digital sequence

1. $x(n) = x(nT) = 10e^{-5000 \times 0.000125n}u(nT) = 10e^{-0.625n}u(n)$.

   Similarly, the digital sequence for (2) is achieved as follows:

2. $x(n) = x(nT) = 10\sin(2000\pi \times 0.000125n)u(nT) = 10\sin(0.25\pi n)u(n)$

Writing a Sampled Signal in terms of impulse functions

Assuming a DSP system with a sampling time interval of 125 μs,

(a) Convert each of following analog signal $x(t)$ to the digital signal $x(n)$.

1. $x(t) = 10e^{-5000t}u(t)$
2. $x(t) = 10\sin(2000\pi t)u(t)$

(b) Determine and plot the sample values from each obtained digital function.

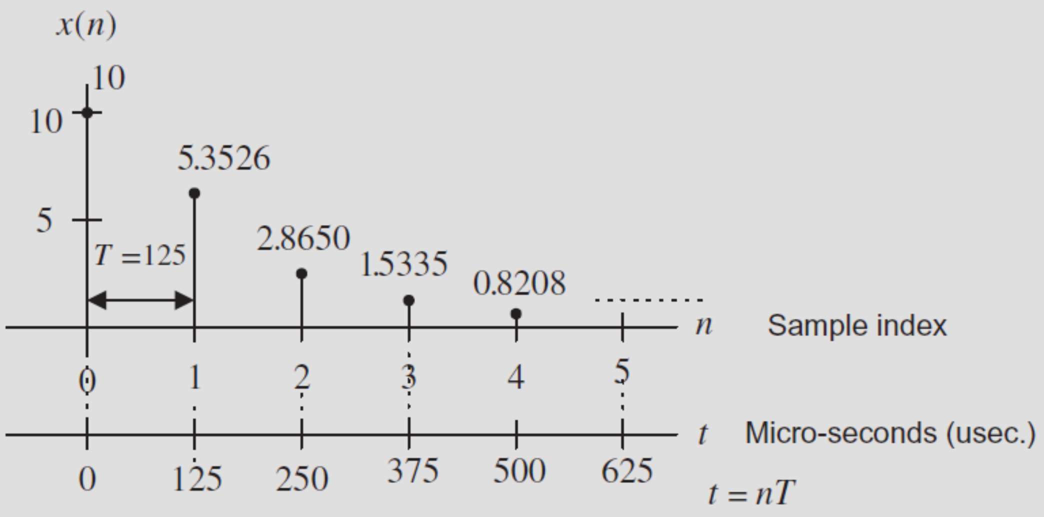
Writing a Sampled Signal in terms of impulse functions

Assuming a DSP system with a sampling time interval of 125 μs,

(a) Convert each of following analog signal $x(t)$ to the digital signal $x(n)$.

1. $x(t) = 10e^{-5000t}u(t)$
2. $x(t) = 10\sin(2000\pi t)u(t)$

(b) Determine and plot the sample values from each obtained digital function.



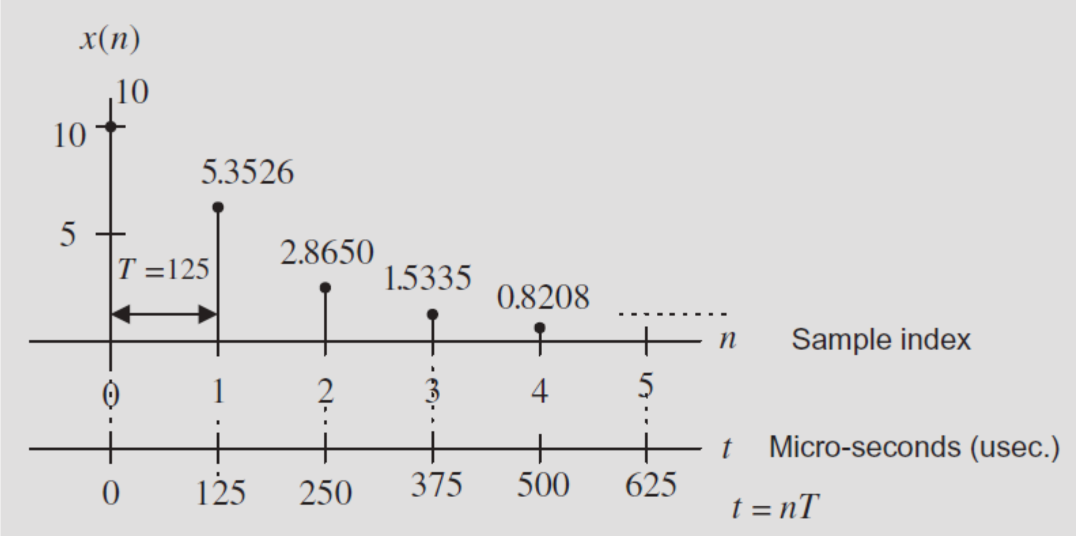
Writing a Sampled Signal in terms of impulse functions

Assuming a DSP system with a sampling time interval of 125 μs,

(a) Convert each of following analog signal $x(t)$ to the digital signal $x(n)$.

1. $x(t) = 10e^{-5000t}u(t)$
2. $x(t) = 10\sin(2000\pi t)u(t)$

(b) Determine and plot the sample values from each obtained digital function.



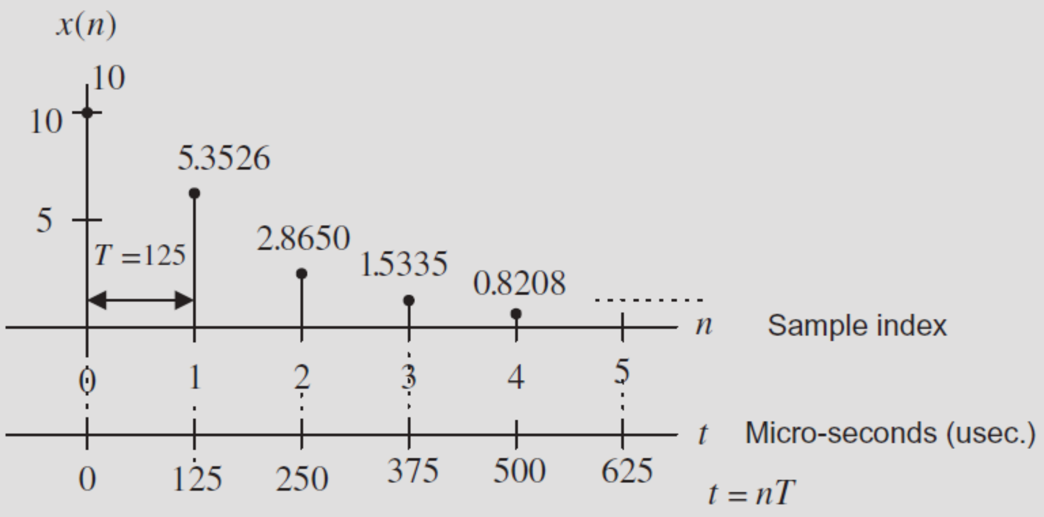
Writing a Sampled Signal in terms of impulse functions

Assuming a DSP system with a sampling time interval of 125 μs,

(a) Convert each of following analog signal $x(t)$ to the digital signal $x(n)$.

1. $x(t) = 10e^{-5000t}u(t)$
2. $x(t) = 10\sin(2000\pi t)u(t)$

(b) Determine and plot the sample values from each obtained digital function.



$$x(0) = 10e^{-0.625\times 0}u(0) = 10.0$$

$$x(1) = 10e^{-0.625\times 1}u(1) = 5.3526$$

$$x(2) = 10e^{-0.625\times 2}u(2) = 2.8650$$

$$x(3) = 10e^{-0.625\times 3}u(3) = 1.5335$$

$$x(4) = 10e^{-0.625\times 4}u(4) = 0.8208$$

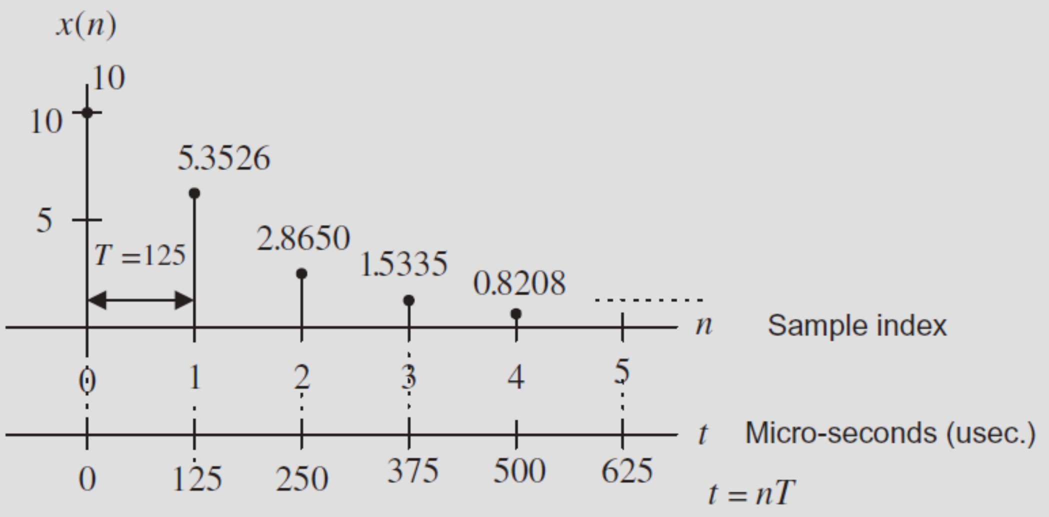
Writing a Sampled Signal in terms of impulse functions

Assuming a DSP system with a sampling time interval of 125 μs,

(a) Convert each of following analog signal $x(t)$ to the digital signal $x(n)$.

1. $x(t) = 10e^{-5000t}u(t)$
2. $x(t) = 10\sin(2000\pi t)u(t)$

(b) Determine and plot the sample values from each obtained digital function.



$$x(0) = 10e^{-0.625\times 0}u(0) = 10.0$$

$$x(1) = 10e^{-0.625\times 1}u(1) = 5.3526$$

$$x(2) = 10e^{-0.625\times 2}u(2) = 2.8650$$

$$x(3) = 10e^{-0.625\times 3}u(3) = 1.5335$$

$$x(4) = 10e^{-0.625\times 4}u(4) = 0.8208$$

$$x[n] = 10\delta[n] + 5.3526\delta[n-1] + 2.865\delta[n-2] + 1.535\delta[n-3] + 0.821\delta[n-4] + \ldots$$

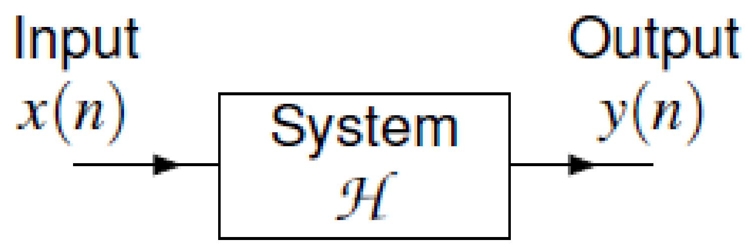
Discrete Time Systems

- A system with input x and output y can be described by the equation

$$y = \mathcal{H}\{x\},$$

where $\mathcal{H}$ denotes an operator (i.e., transformation).

- Note that the operator $\mathcal{H}$ ***maps a function to a function*** (not a number to a number).

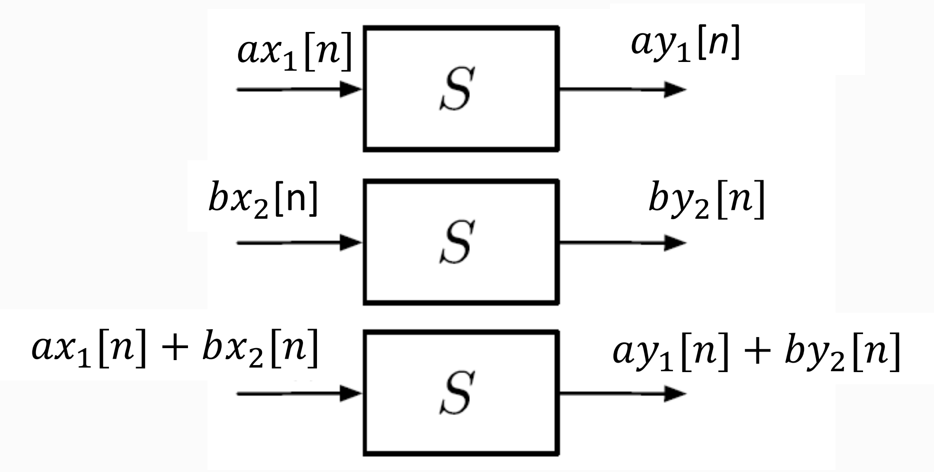


Classification of Systems: (1) Linearity

A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties.

Classification of Systems: (1) Linearity

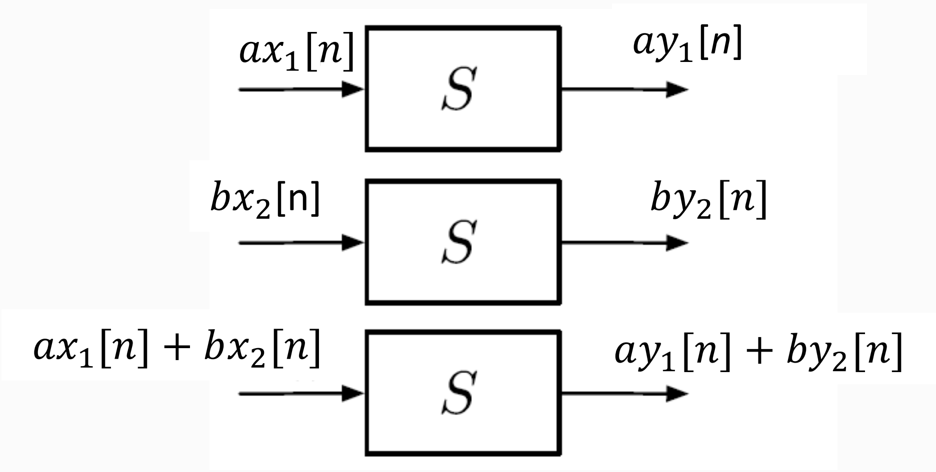
A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties.



Additivity + Homogeneity Properties

Classification of Systems: (1) Linearity

A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties.

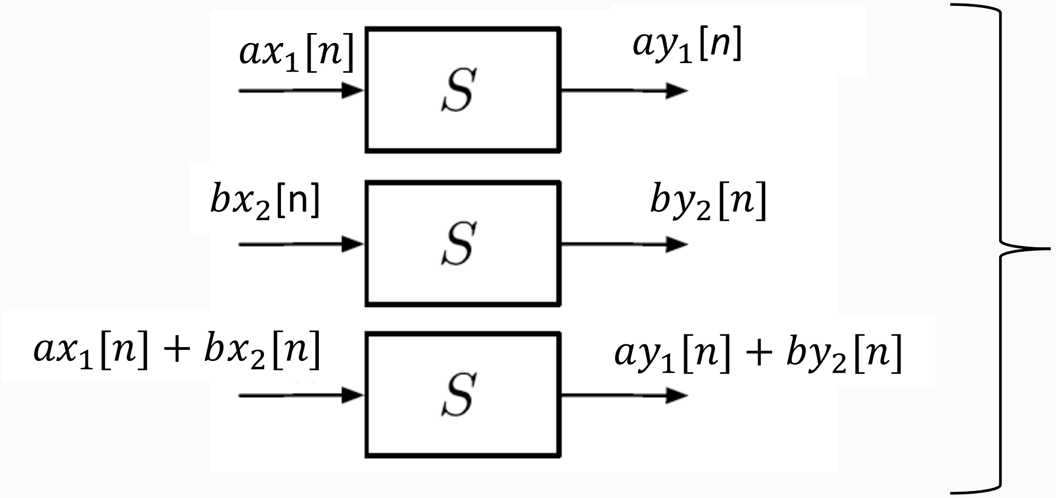


Additivity + Homogeneity Properties

For example, is y(n)=10x(n) system linear?

Classification of Systems: (1) Linearity

A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties.



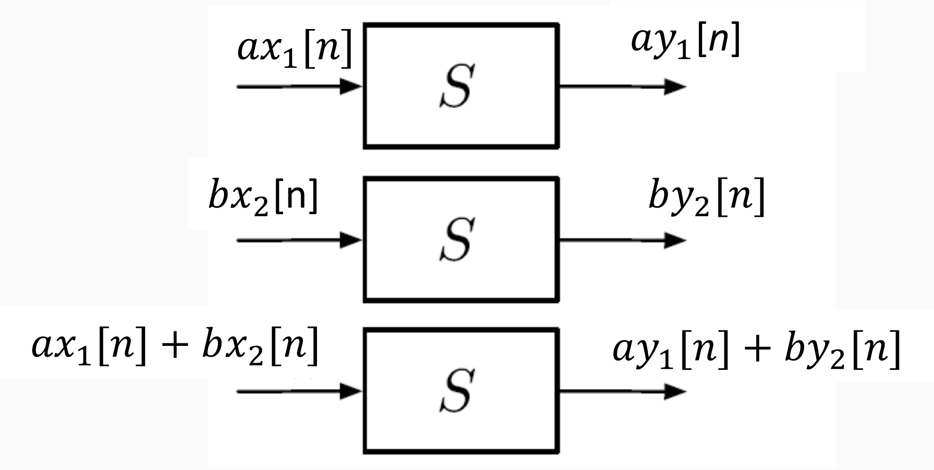
Additivity + Homogeneity Properties

For example, is y(n)=10x(n) system linear?

$y_1(n) = 10x_1(n)$ and $y_2(n) = 10x_2(n)$

Classification of Systems: (1) Linearity

A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties.



Additivity + Homogeneity Properties

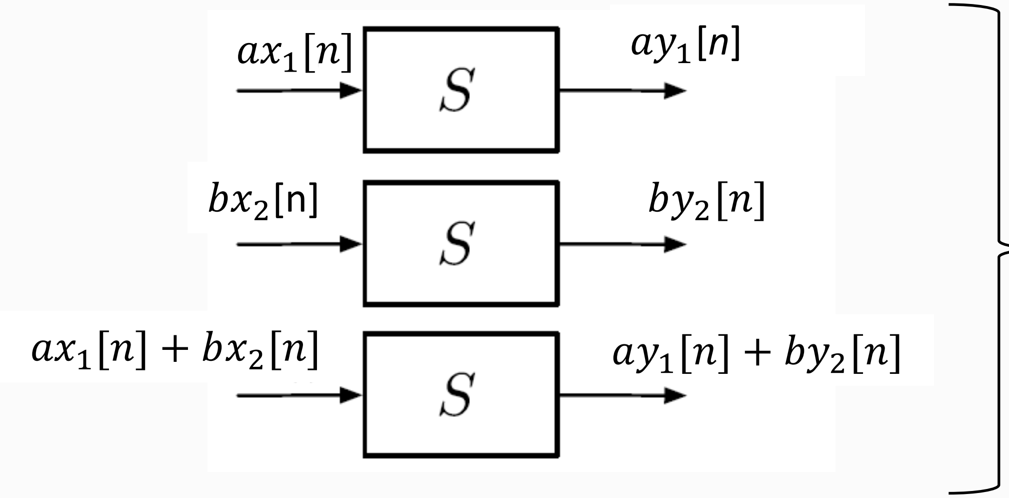
For example, is y(n)=10x(n) system linear?

$$y_1(n) = 10x_1(n) \text{ and } y_2(n) = 10x_2(n)$$

$$\alpha y_1(n) + \beta y_2(n) = \alpha[10x_1(n)] + \beta[10x_2(n)]$$

Classification of Systems: (1) Linearity

A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties.



Additivity + Homogeneity Properties

For example, is y(n)=10x(n) system linear?

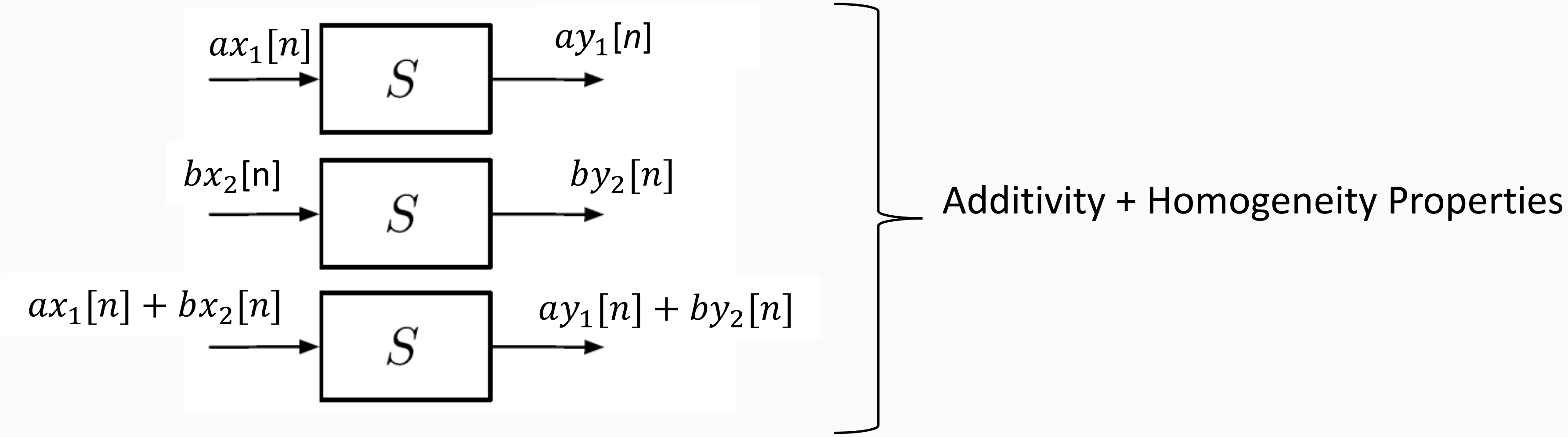
$$x(n) = \alpha x_1(n) + \beta x_2(n)$$

$$y_1(n) = 10x_1(n) \text{ and } y_2(n) = 10x_2(n)$$

$$\alpha y_1(n) + \beta y_2(n) = \alpha[10x_1(n)] + \beta[10x_2(n)]$$

Classification of Systems: (1) Linearity

A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties.



For example, is y(n)=10x(n) system linear?

$$x(n) = \alpha x_1(n) + \beta x_2(n)$$

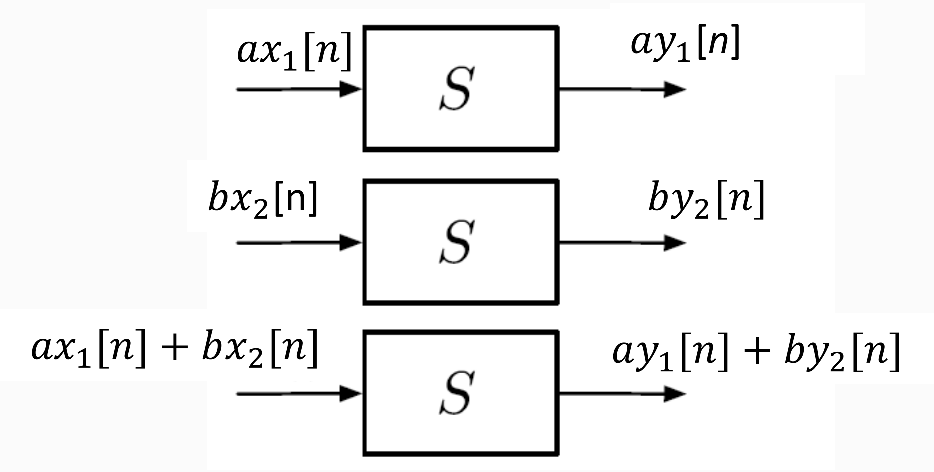
$$y(n) = 10x(n) = 10(\alpha x_1(n) + \beta x_2(n)) = 10\alpha x_1(n) + 10\beta x_2(n)$$

$y_1(n) = 10x_1(n)$ and $y_2(n) = 10x_2(n)$

$$\alpha y_1(n) + \beta y_2(n) = \alpha[10x_1(n)] + \beta[10x_2(n)]$$

Classification of Systems: (1) Linearity

A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties.



Additivity + Homogeneity Properties

For example, is y(n)=10x(n) system linear?

$$x(n) = \alpha x_1(n) + \beta x_2(n)$$

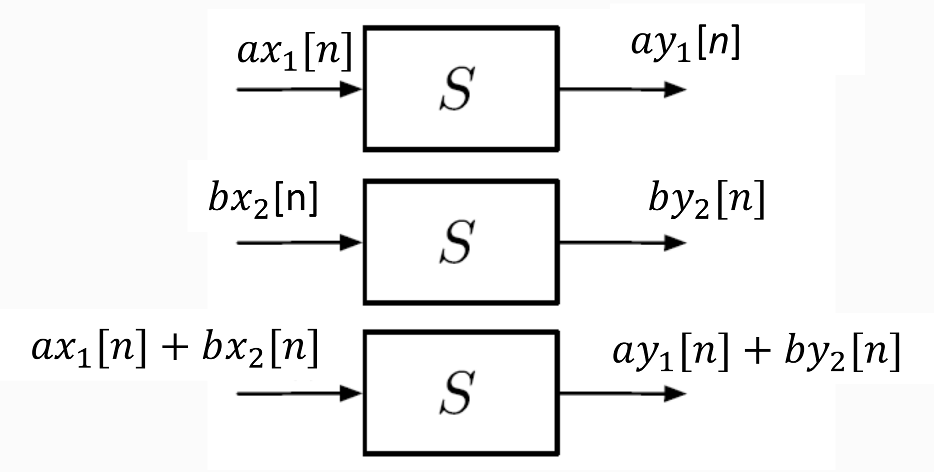
$$y(n) = 10x(n) = 10(\alpha x_1(n) + \beta x_2(n)) = 10\alpha x_1(n) + 10\beta x_2(n)$$

$$y_1(n) = 10x_1(n) \text{ and } y_2(n) = 10x_2(n)$$

$$\alpha y_1(n) + \beta y_2(n) = \alpha[10x_1(n)] + \beta[10x_2(n)]$$

Classification of Systems: (1) Linearity

A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties.



Additivity + Homogeneity Properties

For example, is y(n)=10x(n) system linear?

$$x(n) = \alpha x_1(n) + \beta x_2(n)$$

$$y(n) = 10x(n) = 10(\alpha x_1(n) + \beta x_2(n)) = 10\alpha x_1(n) + 10\beta x_2(n)$$

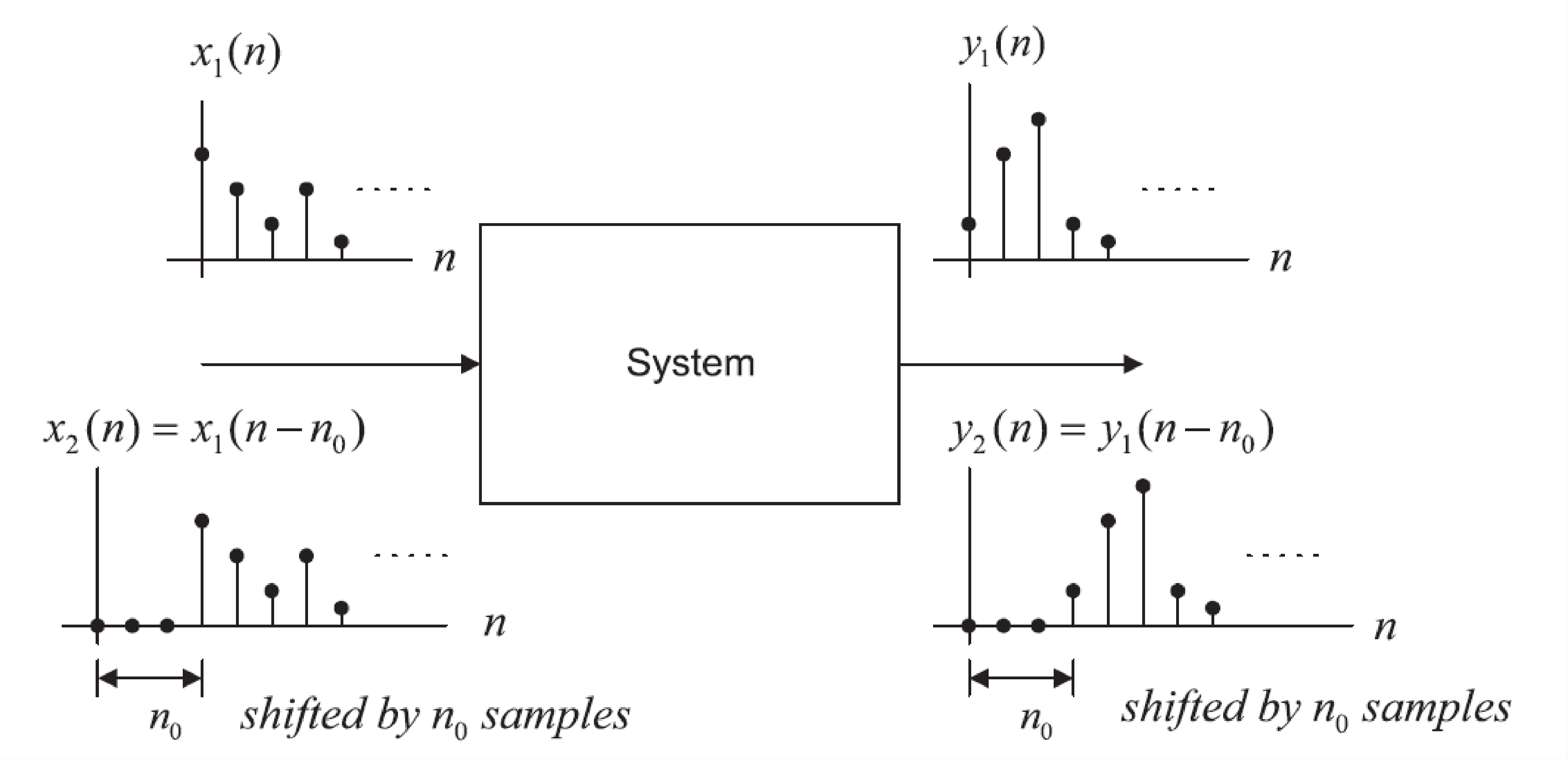
$$y_1(n) = 10x_1(n) \text{ and } y_2(n) = 10x_2(n)$$

$$\alpha y_1(n) + \beta y_2(n) = \alpha[10x_1(n)] + \beta[10x_2(n)]$$

$$y(n) = \alpha y_1(n) + \beta y_2(n)$$

Classification of Systems: (2) Time-Invariance

If the system is time invariant and $y_1(n)$ is the system output due to the input $x_1(n)$, then the shifted system input $x_1(n_0)$ will produce a shifted system output $y_1(nn_0)$ by the same amount of time n_0.



Example

$y[n] = nx[n]$ -> Is this signal time-invariant or not?

Example

$y[n] = nx[n]$ -> Is this signal time-invariant or not?

1) First, apply an input x1 and find the output y1 -> $x_1[n] \rightarrow y_1[n] = nx_1[n]$

Example

$y[n] = nx[n]$ -> Is this signal time-invariant or not?

1) First, apply an input x1 and find the output y1 -> $x_1[n] \rightarrow y_1[n] = nx_1[n]$

2) Second, shift the input x1 by n0 and find the output y2 ->

$$x_2[n] = x_1[n - n_0] \longrightarrow y_2[n] = nx_1[n - n_0]$$

Example

$y[n] = nx[n]$ -> Is this signal time-invariant or not?

1) First, apply an input x1 and find the output y1 -> $x_1[n] \rightarrow y_1[n] = nx_1[n]$

2) Second, shift the input x1 by n0 and find the output y2 ->

$$x_2[n] = x_1[n-n_0] \longrightarrow y_2[n] = nx_1[n-n_0]$$

3) Does it match with the shifted output of y1 ?

$$y_1[n-n_0] = (n-n_0)x_1[n-n_0] \neq y_2[n]$$

Example

$y[n] = nx[n]$ -> Is this signal time-invariant or not?

1) First, apply an input x1 and find the output y1 -> $x_1[n] \rightarrow y_1[n] = nx_1[n]$

2) Second, shift the input x1 by n0 and find the output y2 ->

$$x_2[n] = x_1[n-n_0] \longrightarrow y_2[n] = nx_1[n-n_0]$$

3) Does it match with the shifted output of y1 ?

$$y_1[n-n_0] = (n-n_0)x_1[n-n_0] \neq y_2[n]$$

Therefore, it is not a time-invariant system!!

Classification of Systems: (3) Causal

A causal system is the one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, and its past input sample values such as $x(n-1)$, $x(n-2)$,…. Otherwise, if a system output depends on the future input values such as $x(n+1)$, $x(n+2)$,…, the system is noncausal. The noncausal system cannot be realized in real time.

Classification of Systems: (3) Causal

A causal system is the one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, and its past input sample values such as $x(n-1)$, $x(n-2)$,…. Otherwise, if a system output depends on the future input values such as $x(n+1)$, $x(n+2)$,…, the system is noncausal. The noncausal system cannot be realized in real time.

EXAMPLE 3.4

Given the following linear systems

(a) $y(n) = 0.5x(n) + 2.5x(n-2)$, for $n \geq 0$,

Classification of Systems: (3) Causal

A causal system is the one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, and its past input sample values such as $x(n-1)$, $x(n-2)$,…. Otherwise, if a system output depends on the future input values such as $x(n+1)$, $x(n+2)$,…, the system is noncausal. The noncausal system cannot be realized in real time.

EXAMPLE 3.4

Given the following linear systems

(a) $y(n) = 0.5x(n) + 2.5x(n-2)$, for $n \geq 0$,

(a) Since for $n \geq 0$, the output $y(n)$ depends on the current input $x(n)$ and its past value $x(n-2)$, the system is causal.

Classification of Systems: (3) Causal

A causal system is the one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, and its past input sample values such as $x(n-1)$, $x(n-2)$,…. Otherwise, if a system output depends on the future input values such as $x(n+1)$, $x(n+2)$,…, the system is noncausal. The noncausal system cannot be realized in real time.

EXAMPLE 3.4

Given the following linear systems

(a) $y(n) = 0.5x(n) + 2.5x(n-2)$, for $n \geq 0$,

(a) Since for $n \geq 0$, the output $y(n)$ depends on the current input $x(n)$ and its past value $x(n-2)$, the system is causal.

$y(n) = 0.25x(n-1) + 0.5x(n+1) - 0.4y(n-1)$, for $n \geq 0$,

Classification of Systems: (3) Causal

A causal system is the one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, and its past input sample values such as $x(n-1)$, $x(n-2)$,…. Otherwise, if a system output depends on the future input values such as $x(n+1)$, $x(n+2)$,…, the system is noncausal. The noncausal system cannot be realized in real time.

EXAMPLE 3.4

Given the following linear systems

(a) $y(n) = 0.5x(n) + 2.5x(n-2)$, for $n \geq 0$,

(a) Since for $n \geq 0$, the output $y(n)$ depends on the current input $x(n)$ and its past value $x(n-2)$, the system is causal.

$y(n) = 0.25x(n-1) + 0.5x(n+1) - 0.4y(n-1)$, for $n \geq 0$,

Classification of Systems: (3) Causal

A causal system is the one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, and its past input sample values such as $x(n-1)$, $x(n-2)$,…. Otherwise, if a system output depends on the future input values such as $x(n+1)$, $x(n+2)$,…, the system is noncausal. The noncausal system cannot be realized in real time.

EXAMPLE 3.4

Given the following linear systems

(a) $y(n) = 0.5x(n) + 2.5x(n-2)$, for $n \geq 0$,

(a) Since for $n \geq 0$, the output $y(n)$ depends on the current input $x(n)$ and its past value $x(n-2)$, the system is causal.

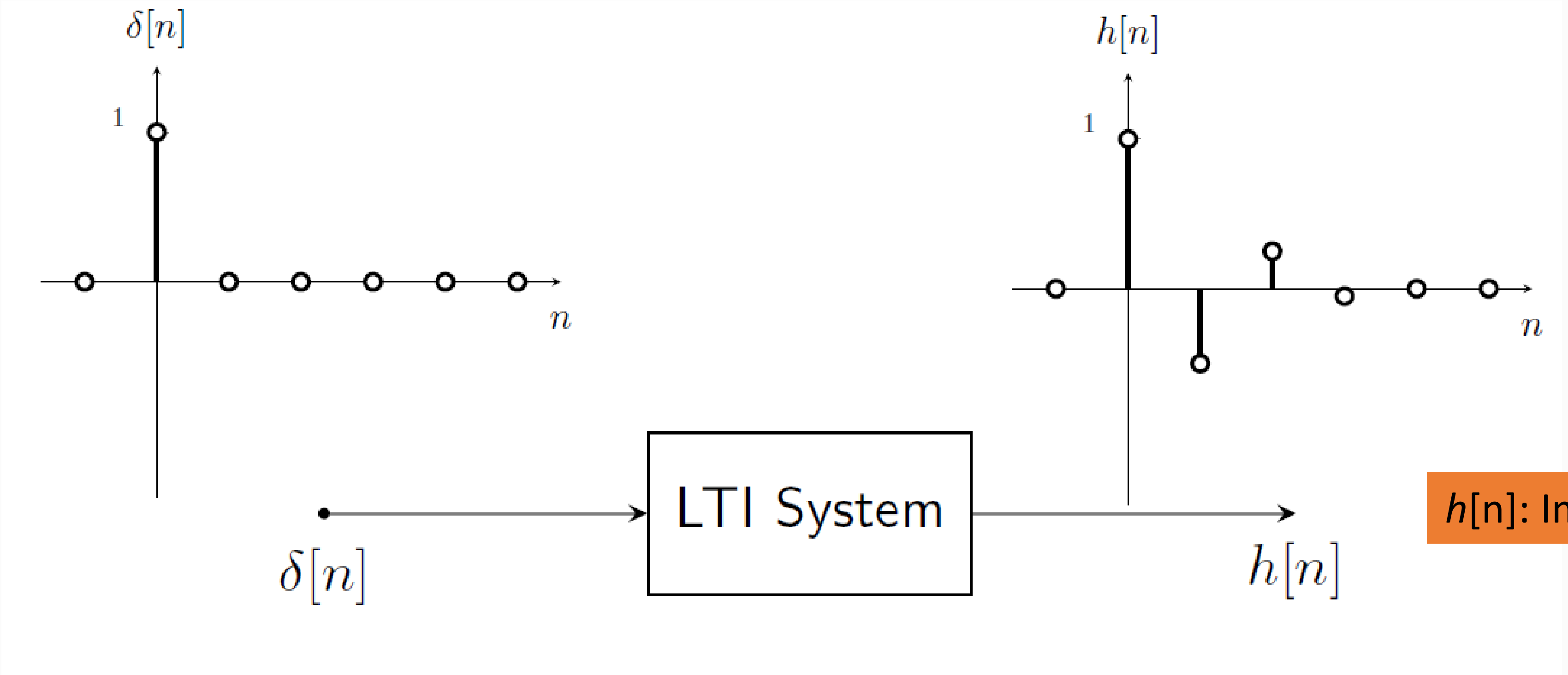
$y(n) = 0.25x(n-1) + 0.5x(n+1) - 0.4y(n-1)$, for $n \geq 0$,

-> Is this causal?

Check This Example at Home!

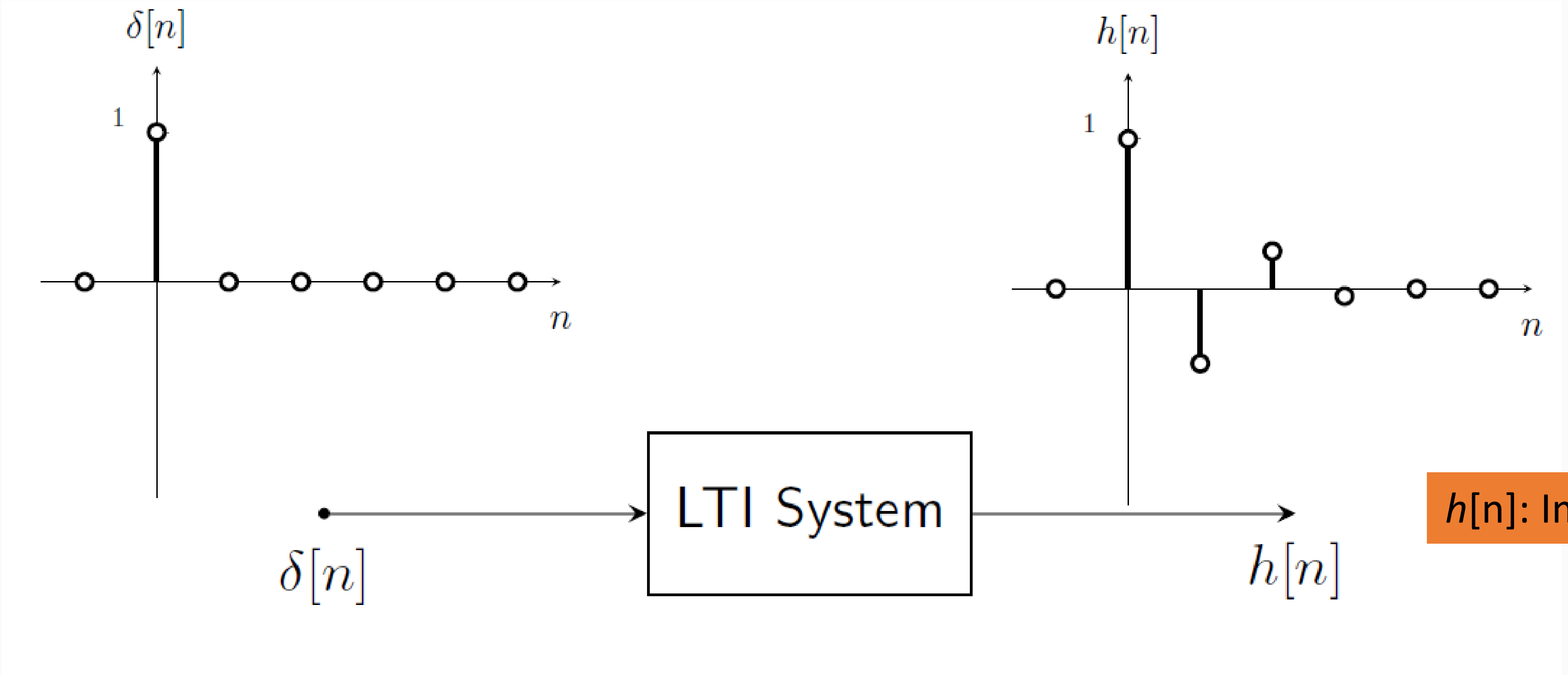
System	Linear	Time invariant	Causal
Constant offset $y[n] = x[n] + C, C \neq 0$	N	Y	Y
Time shift $y[n] = x[n - n_d]$	Y	Y	Y, if $n_d > 0$
Squaring $y[n] = x^2[n]$	N	Y	Y
Accumulator $y[n] = \sum_{k=-\infty}^{n} x[k]$	Y	Y	Y
Compressor $y[n] = x[Mn], M > 1$	Y	N	N
Differentiator $y[n] = x[n] - x[n-1]$	Y	Y	Y
A difference equation $y[n] = x[n] + y[n-1]$	Y	Y	Y

Linear and Time-Invariant (LTI) Systems (Very Important!!)



h[n]: Impulse Response

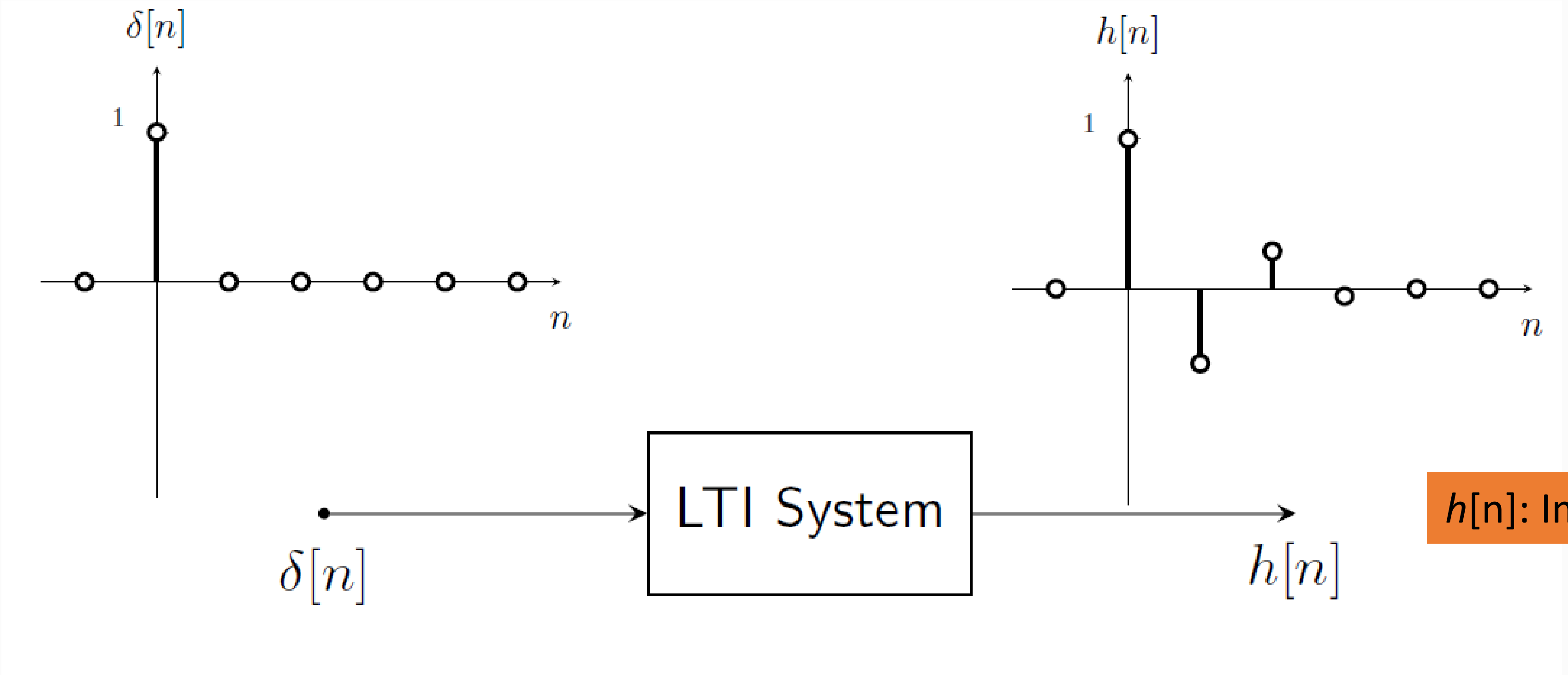
Linear and Time-Invariant (LTI) Systems (Very Important!!)



h[n]: Impulse Response

If we write $\delta[n]$ instead of x[n] in system equation, then we can easliy find impulse response h[n]

Linear and Time-Invariant (LTI) Systems (Very Important!!)

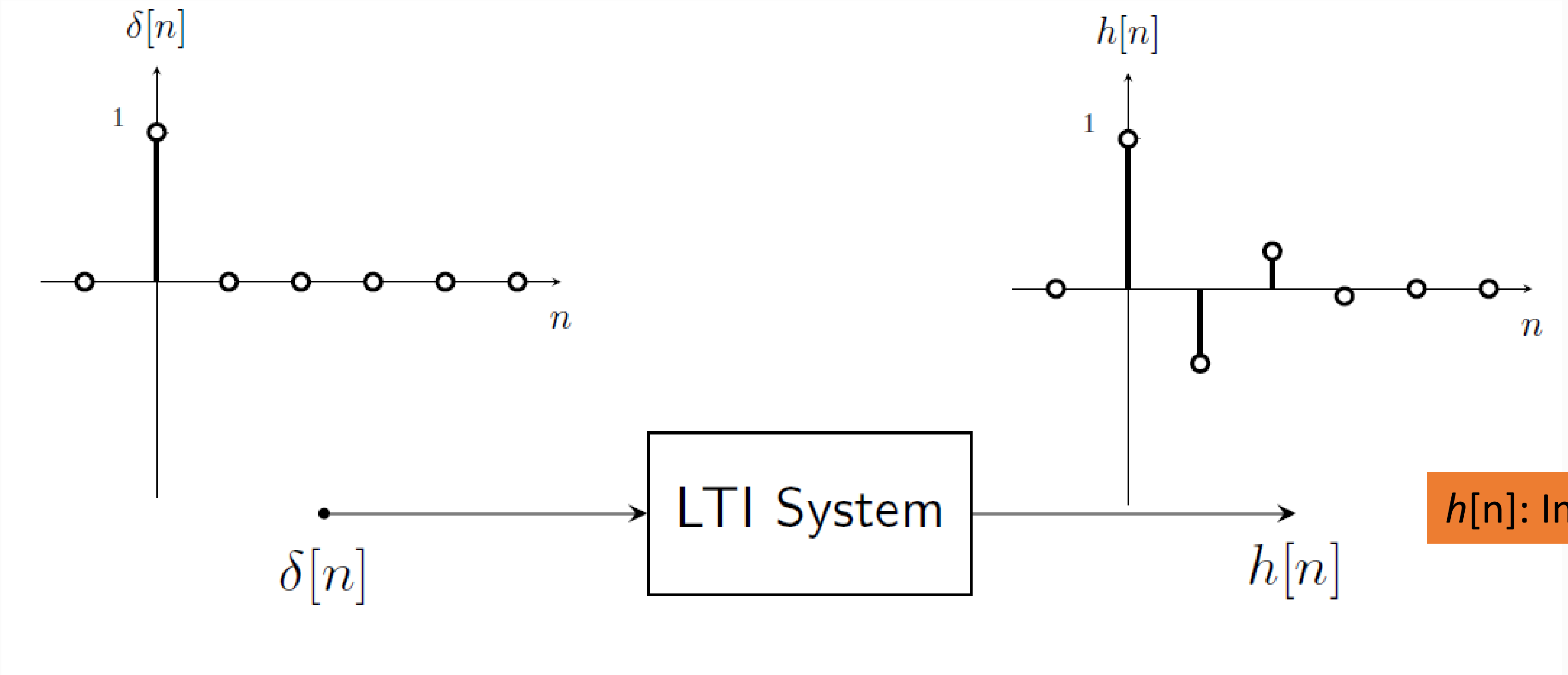


h[n]: Impulse Response

If we write $\delta[n]$ instead of x[n] in system equation, then we can easliy find impulse response h[n]

$y[n] = x[n] + kx[n-d]$

Linear and Time-Invariant (LTI) Systems (Very Important!!)



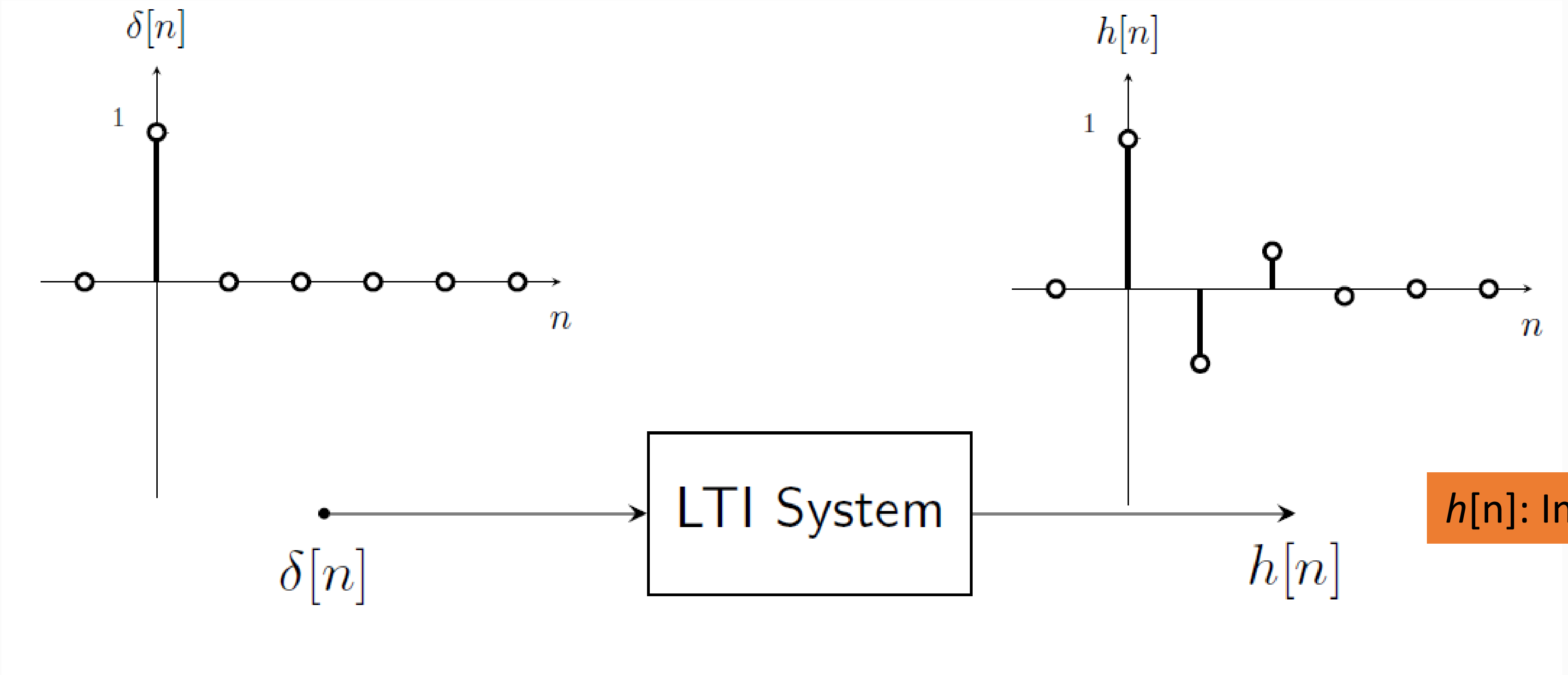
h[n]: Impulse Response

If we write $\delta[n]$ instead of x[n] in system equation, then we can easliy find impulse response h[n]

$y[n] = x[n] + kx[n-d]$

h$[n] = \delta[n] + k\delta[n-d]$

Linear and Time-Invariant (LTI) Systems (Very Important!!)

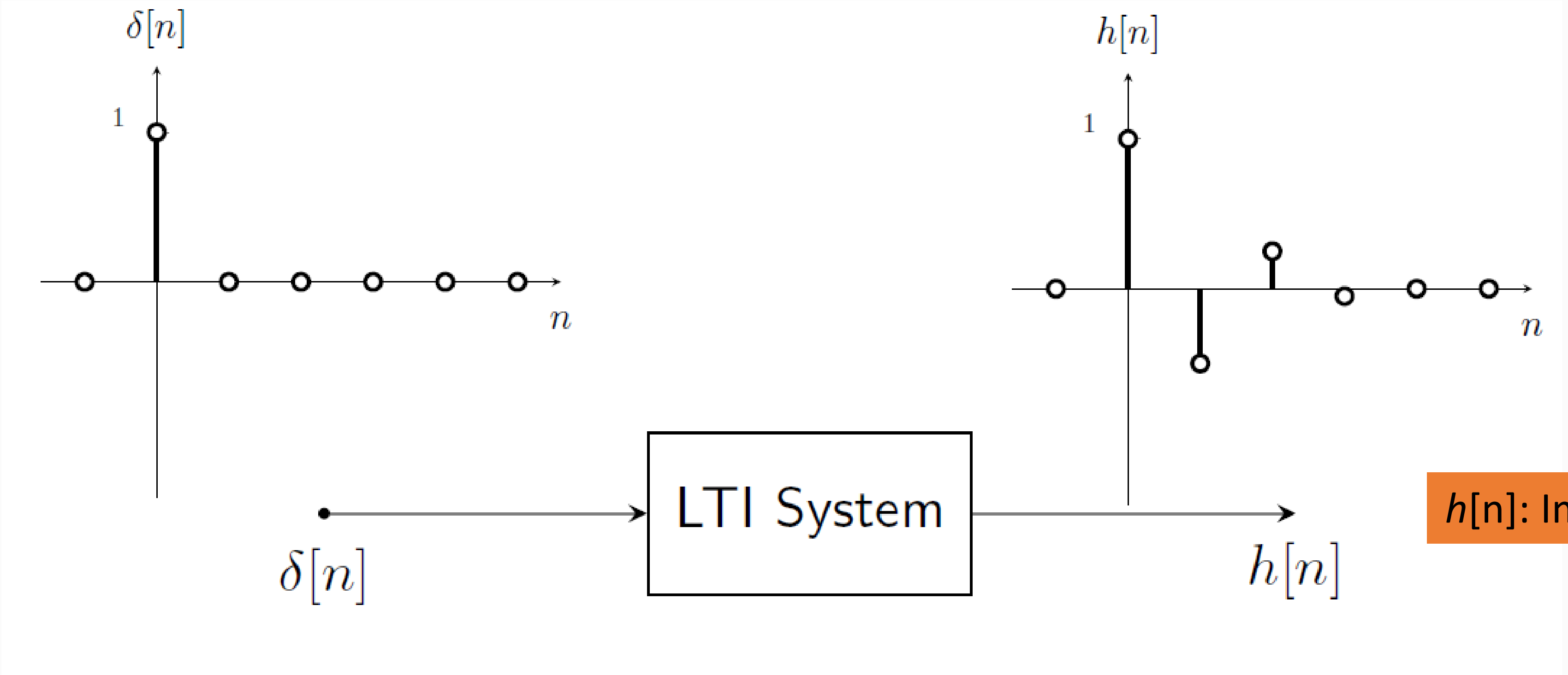


h[n]: Impulse Response

If we write $\delta[n]$ instead of x[n] in system equation, then we can easliy find impulse response h[n]

$y[n] = x[n] + kx[n-d]$ → h$[n] = \delta[n] + k\delta[n-d]$

Linear and Time-Invariant (LTI) Systems (Very Important!!)

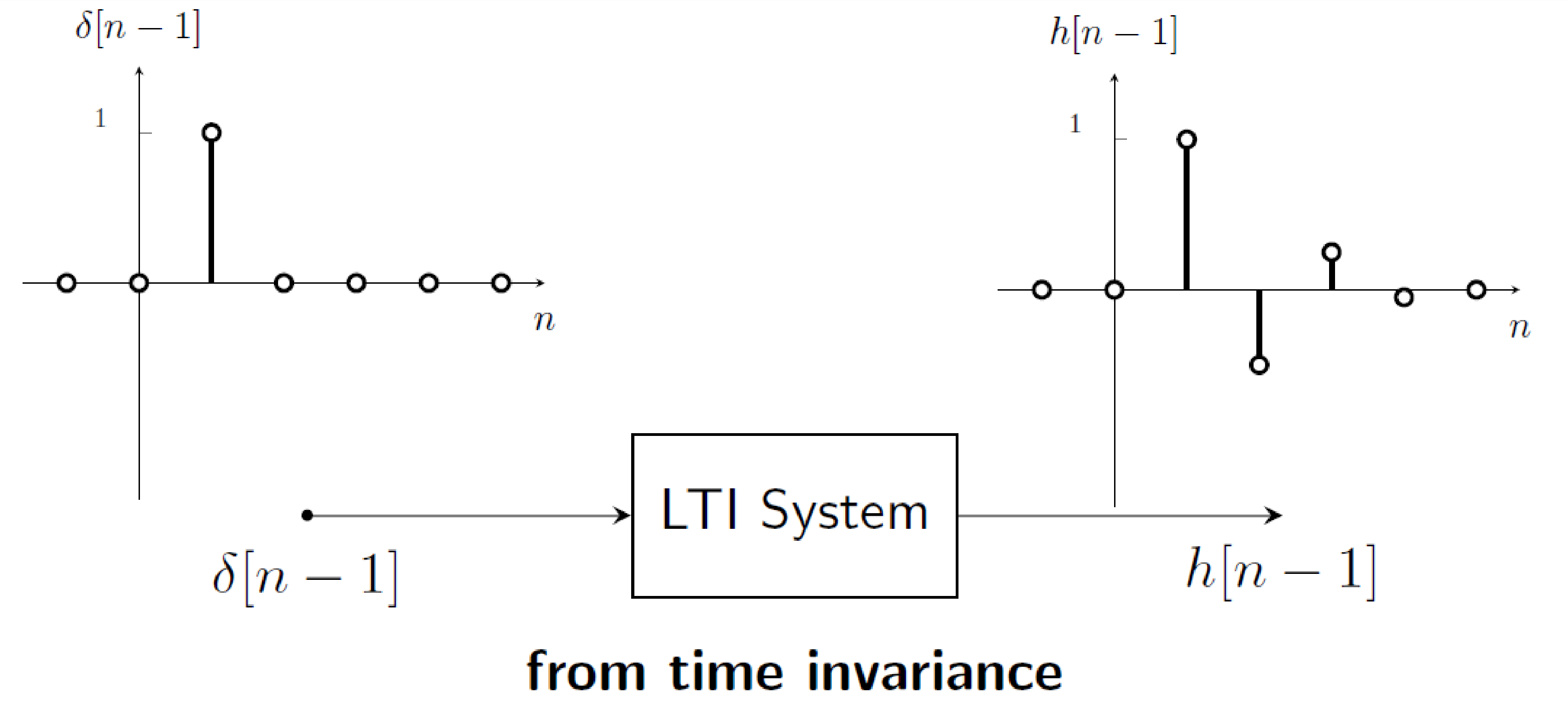


h[n]: Impulse Response

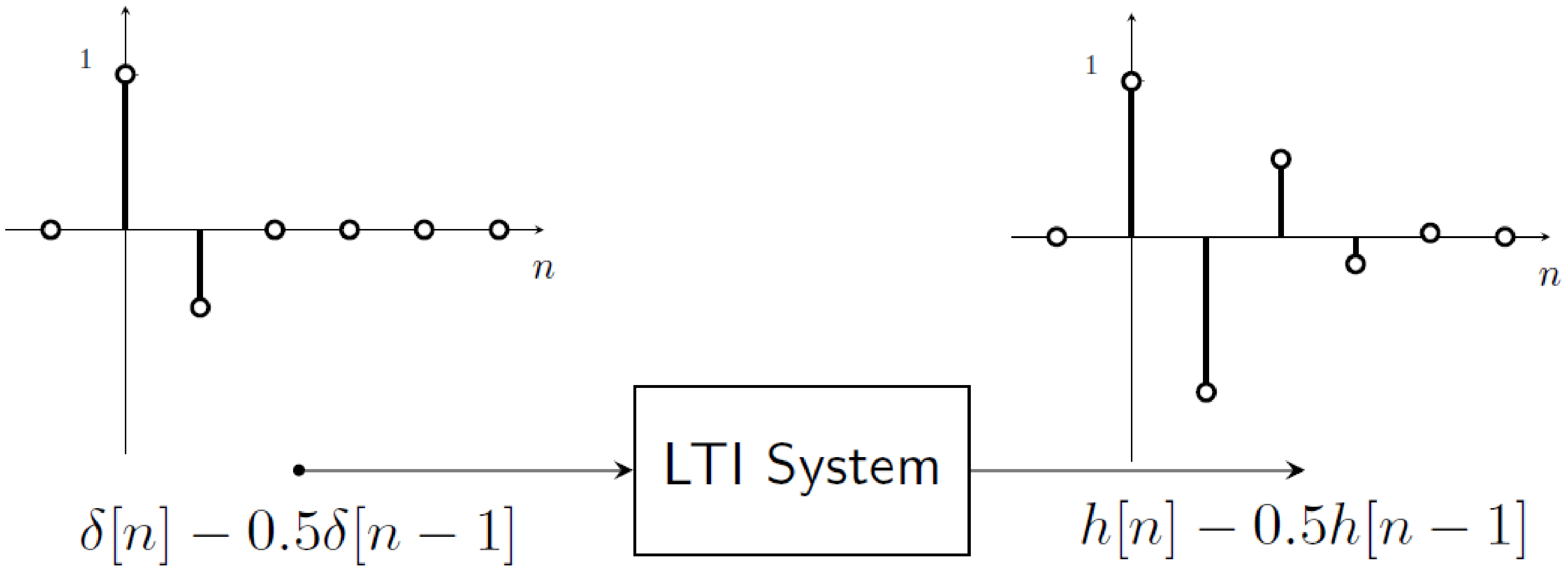
If we write $\delta[n]$ instead of x[n] in system equation, then we can easliy find impulse response h[n]

$y[n] = x[n] + kx[n-d]$ ⟶ h$[n] = \delta[n] + k\delta[n-d]$

Linear and Time-Invariant (LTI) Systems (Very Important!!)



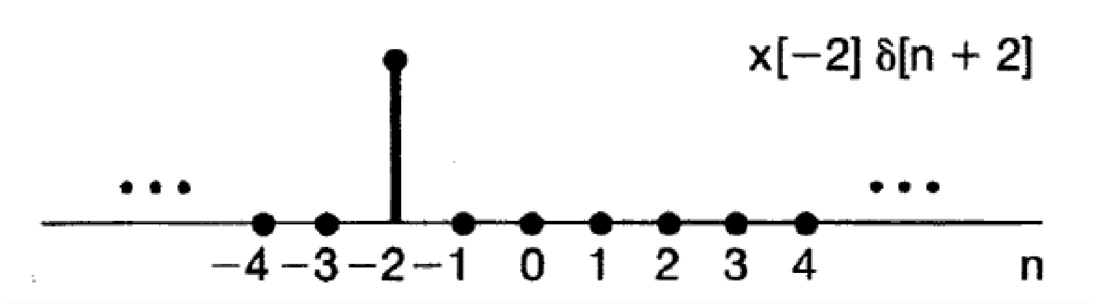
Linear and Time-Invariant (LTI) Systems (Very Important!!)

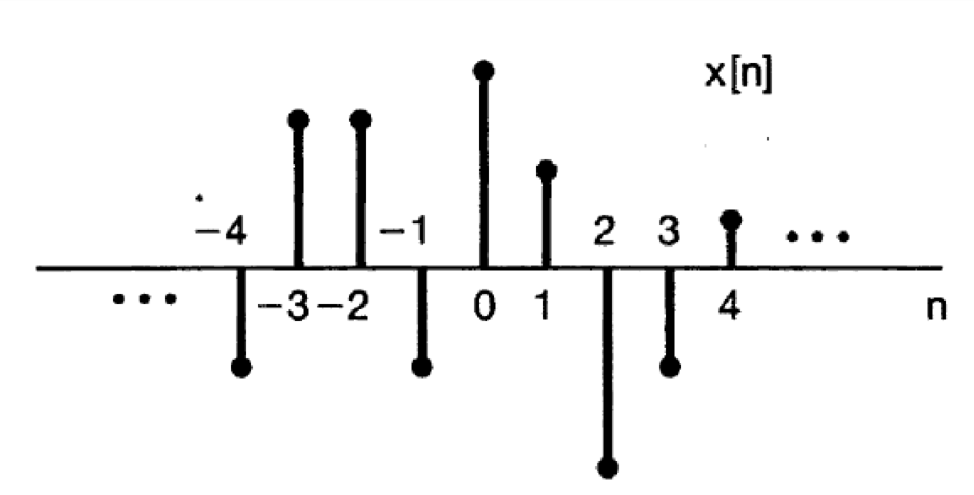


from linearity

Remember…

- We can write any sampled signal in terms of impulse functions:

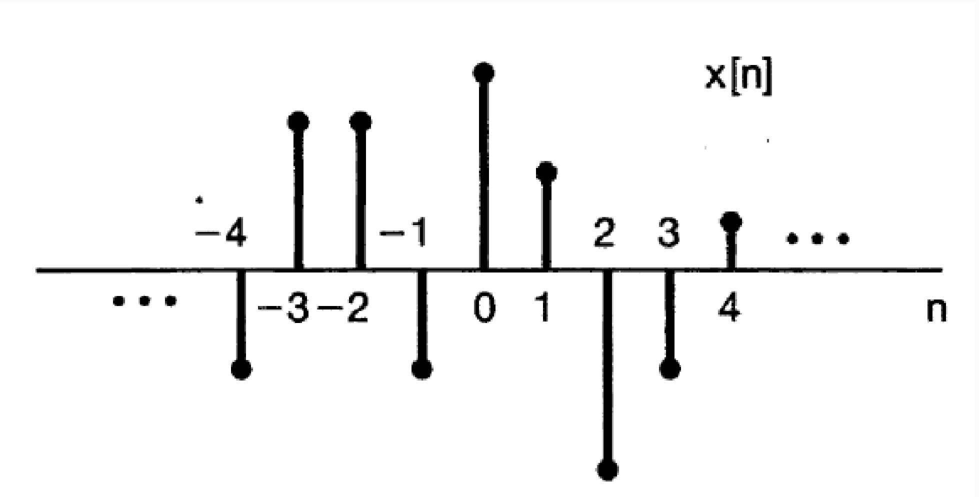




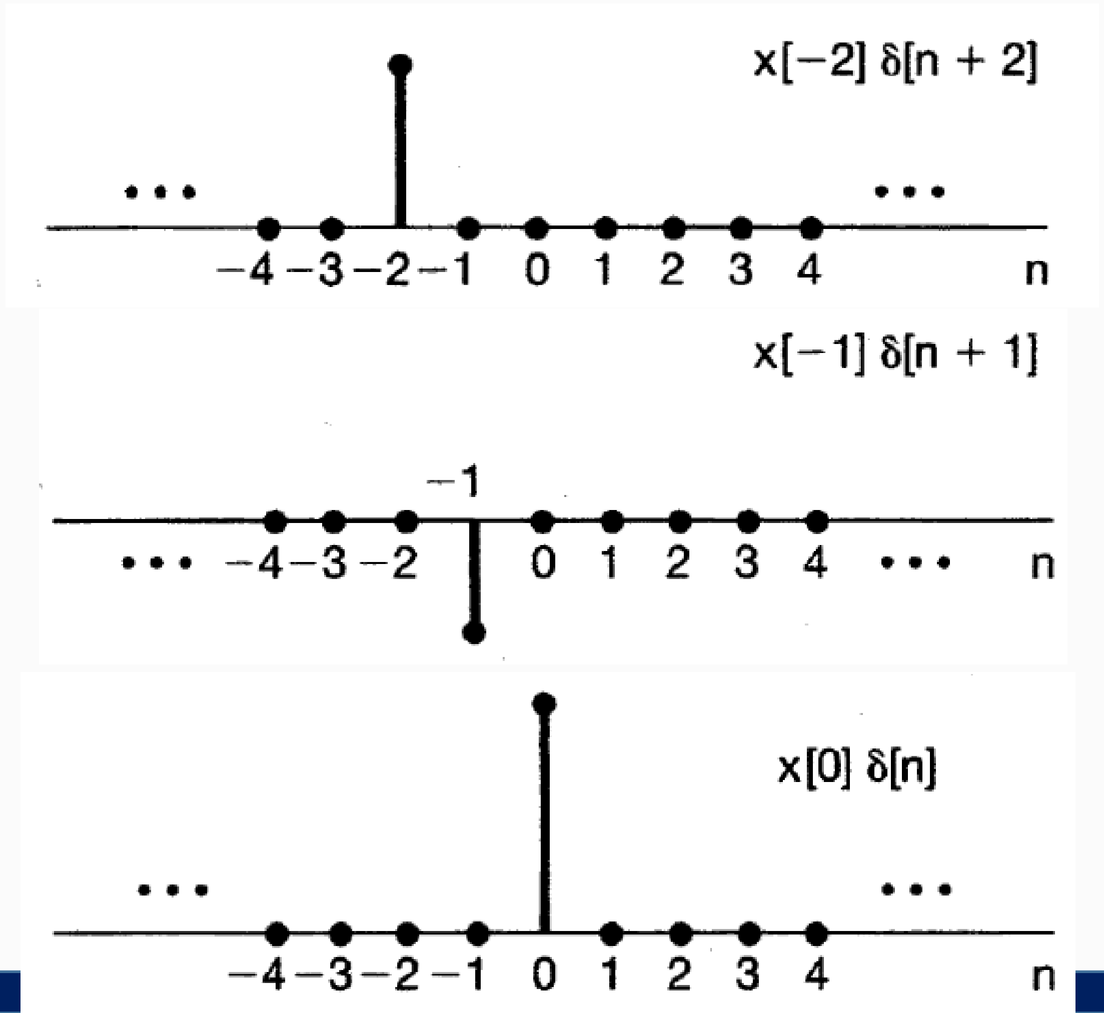
=

Remember…

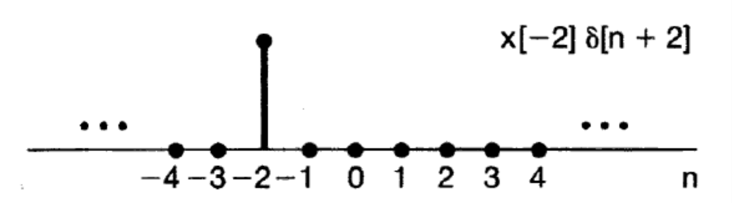
- We can write any sampled signal in terms of impulse functions:

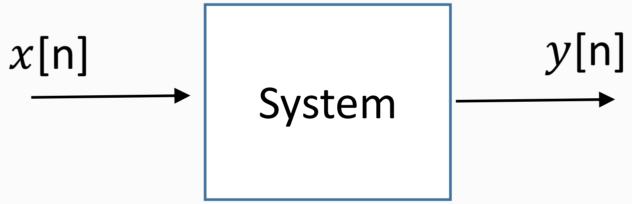


=

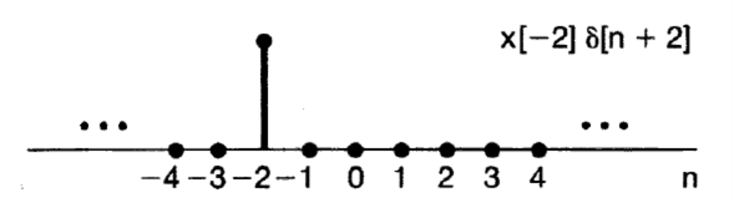


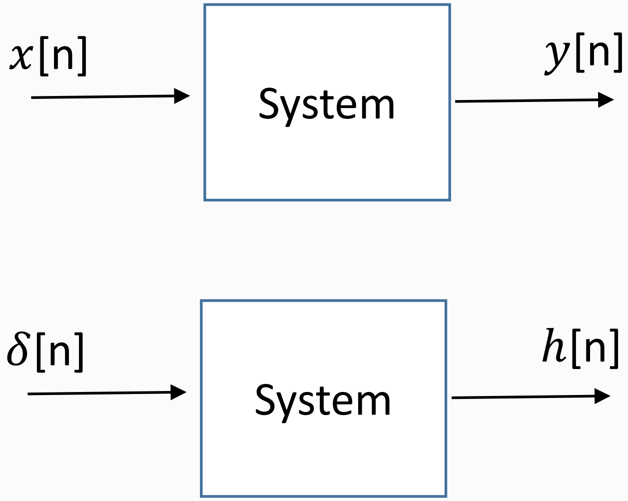
Convolution Sum !!!





Convolution Sum !!!





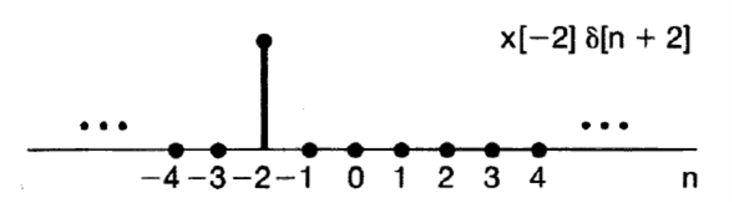
Convolution Sum !!!

$x[n] \longrightarrow$ System $\longrightarrow y[n]$

$\delta[n] \longrightarrow$ System $\longrightarrow h[n]$

$x[k]\delta[n-k] \longrightarrow$ System $\longrightarrow x[k]h[n-k]$

Convolution Sum !!!



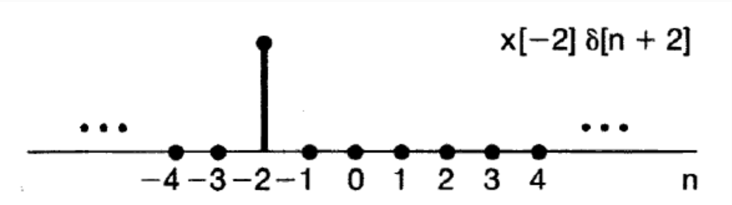
$x[n] \rightarrow$ System $\rightarrow y[n]$

$\delta[n] \rightarrow$ System $\rightarrow h[n]$

$x[k]\delta[n-k] \rightarrow$ System $\rightarrow x[k]h[n-k]$

$\rightarrow$ System $\rightarrow$

Convolution Sum !!!



$$x[n] \longrightarrow \boxed{\text{System}} \longrightarrow y[n]$$

$$\delta[n] \longrightarrow \boxed{\text{System}} \longrightarrow h[n]$$

$$x[k]\delta[n-k] \longrightarrow \boxed{\text{System}} \longrightarrow x[k]h[n-k]$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$\longrightarrow \boxed{\text{System}} \longrightarrow$$

Convolution Sum !!!

$x[\mathrm{n}] \rightarrow$ System $\rightarrow y[\mathrm{n}]$

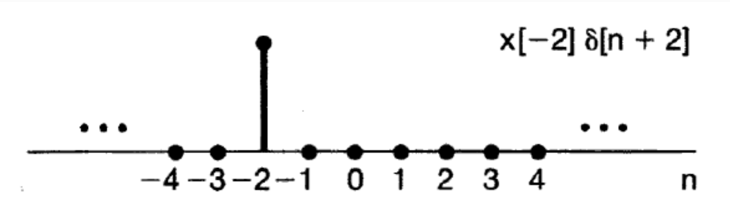
$\delta[\mathrm{n}] \rightarrow$ System $\rightarrow h[\mathrm{n}]$

$x[k]\delta[n-k] \rightarrow$ System $\rightarrow x[k]h[n-k]$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \rightarrow \text{System} \rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

CONVOLUTION SUM

Convolution Sum !!!



$$x[n] \rightarrow \boxed{\text{System}} \rightarrow y[n]$$

$$\delta[n] \rightarrow \boxed{\text{System}} \rightarrow h[n]$$

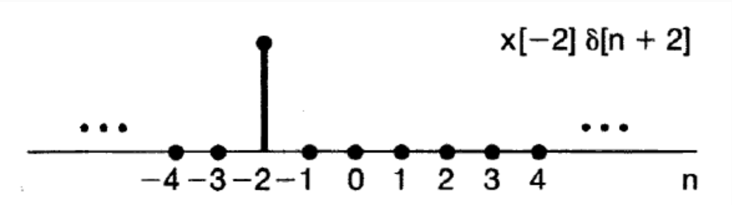
$$x[k]\delta[n-k] \rightarrow \boxed{\text{System}} \rightarrow x[k]h[n-k]$$

$$y[n] = x[n] * h[n]$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \rightarrow \boxed{\text{System}} \rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

CONVOLUTION SUM

Convolution Sum !!!



$x[n] \longrightarrow$ System $\longrightarrow y[n]$

$\delta[n] \longrightarrow$ System $\longrightarrow h[n]$

$x[k]\delta[n-k] \longrightarrow$ System $\longrightarrow x[k]h[n-k]$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$ $\longrightarrow$ System $\longrightarrow$ $$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

CONVOLUTION SUM

$$y[n] = x[n] * h[n]$$

Or

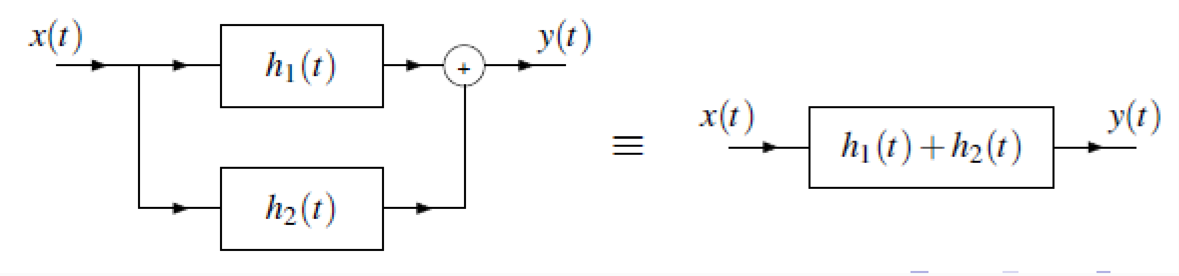
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

Properties of LTI Systems

1) Distributive Property:

$$y[n] = x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

Block Diagram:



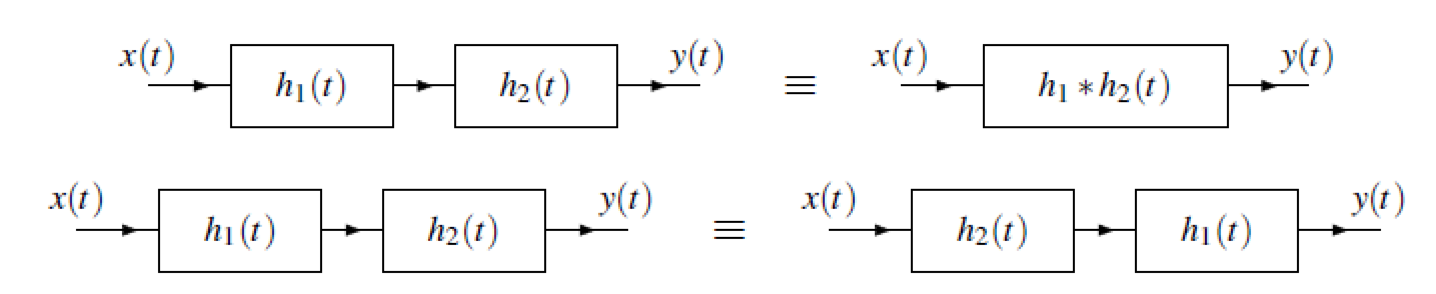
It can be used with the systems connected parallel.

Properties of LTI Systems

2) Associativity Property:

$$y[n] = x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

Blok diyagram :

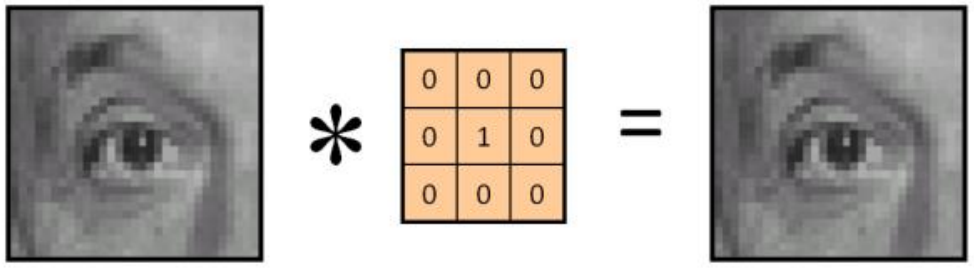


It can be used with the systems that are cascaded-connected .

Properties of LTI Systems

3) Identity Element of Convolution Property:

$$x[n] * \delta[n] = x[n]$$



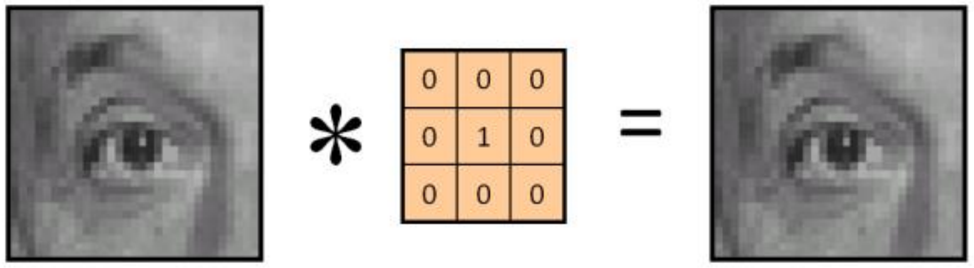
Original

Identical image

Properties of LTI Systems

3) Identity Element of Convolution Property:

$$x[n] * \delta[n] = x[n]$$



Original

Identical image

Convolution with an Impulse

$$x[n] * \delta[n - n_0] = x[n - n_0]$$

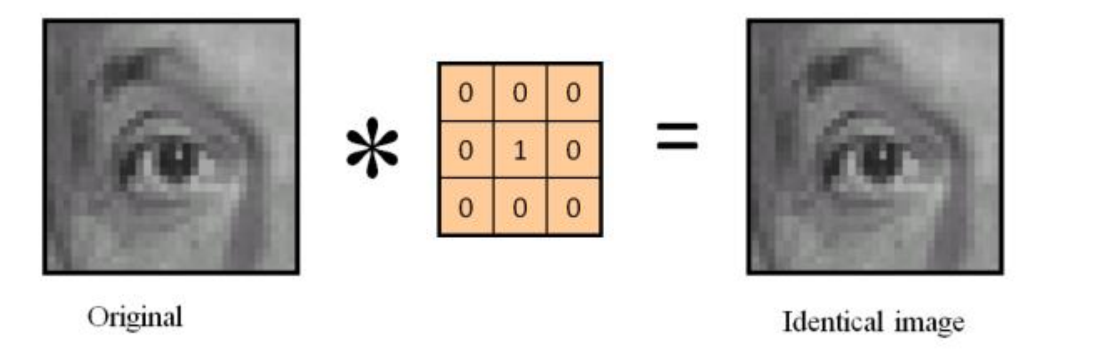
Properties of LTI Systems

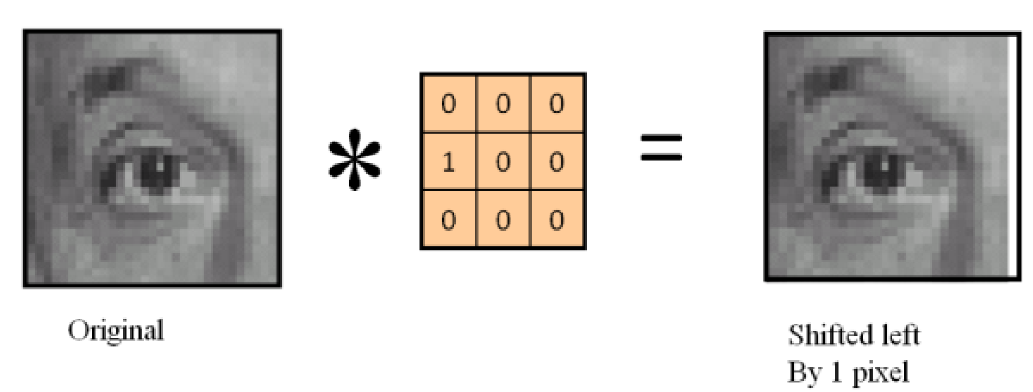
3) Identity Element of Convolution Property:

$$x[n]*\delta[n] = x[n]$$

Convolution with an Impulse

$$x[n] * \delta[n-n_0] = x[n-n_0]$$





Convolution in Discrete-Time Systems

- There are three approaches to calculate convolution:

1) Mathematical Approach

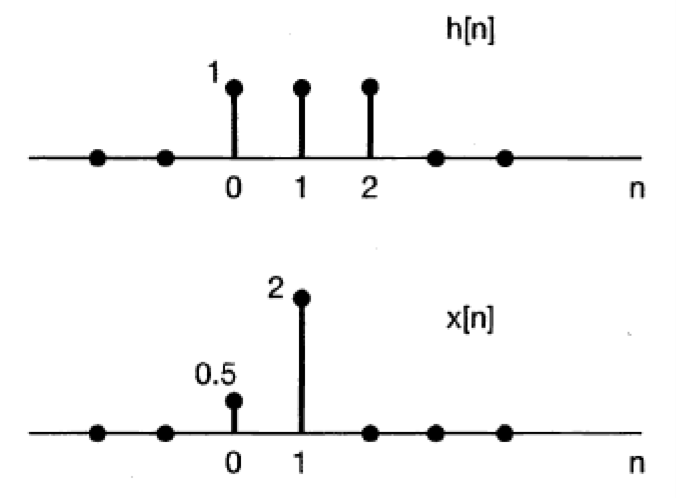
2) Table Approach (Polynomial Multiplication)

3) Graphical Approach

Example 1 (Mathematical Approach)

Impulse response of a LTI system and the input signal are given right.

Find the output y[n]!



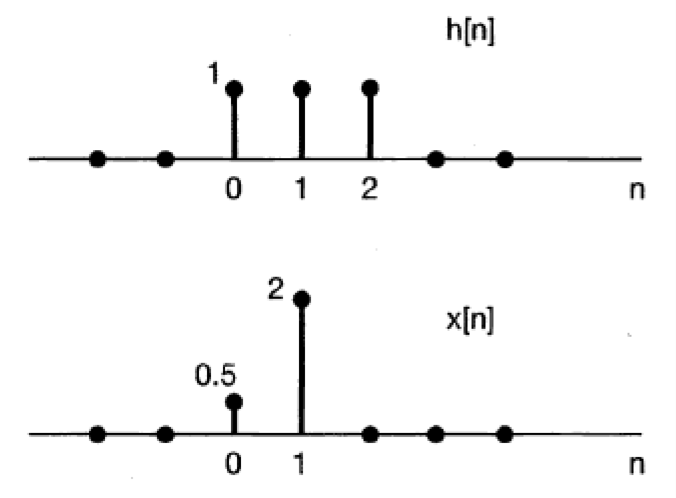
$n = 0$:

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k] = \sum_{k=0}^{1} x[k]h[-k] = x[0]h[0] + x[1]h[-1] = 0.5$$

Example 1 (Mathematical Approach)

Impulse response of a LTI system and the input signal are given right.

Find the output y[n]!



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

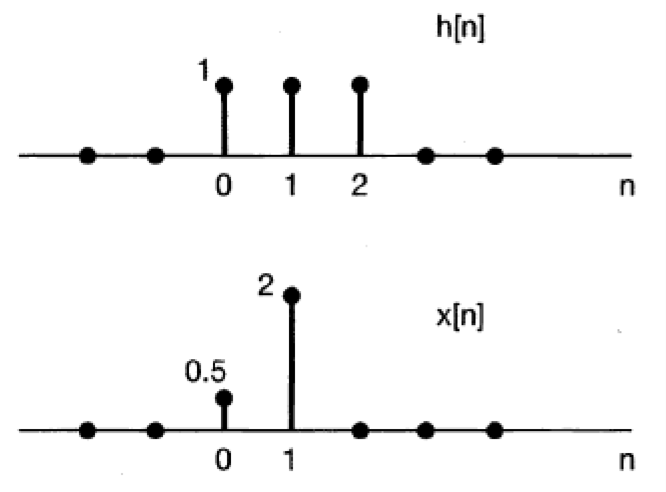
$n = 0$:

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k] = \sum_{k=0}^{1} x[k]h[-k] = x[0]h[0] + x[1]h[-1] = 0.5$$

Example 1 (Mathematical Approach)

Impulse response of a LTI system and the input signal are given right.

Find the output y[n]!



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

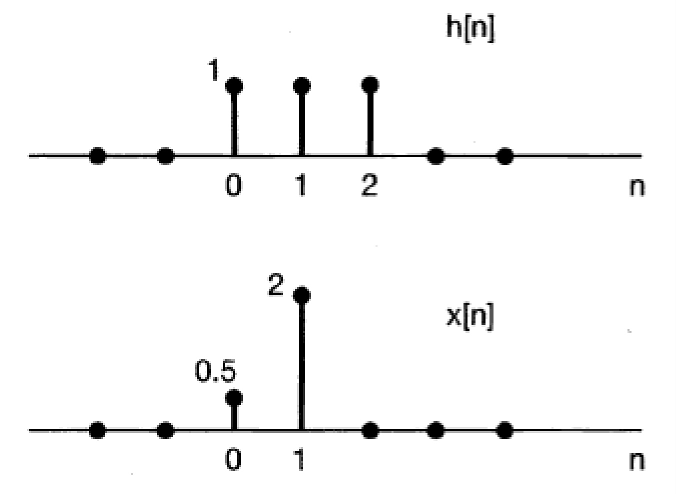
n = -1 :

n = 0 :

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k] = \sum_{k=0}^{1} x[k]h[-k] = x[0]h[0] + x[1]h[-1] = 0.5$$

Example 1 (Mathematical Approach)

Impulse response of a LTI system and the input signal are given right.



Find the output y[n]!

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

n = -1 :

$$y[-1] = \sum_{k=-\infty}^{\infty} x[k]h[-1-k] = \sum_{k=0}^{1} x[k]h[-1-k] = x[0]h[-1] + x[1]h[-2] = 0$$

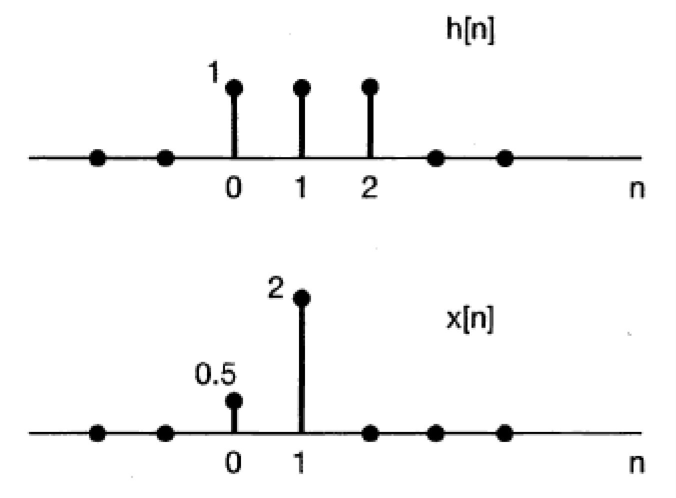
n = 0 :

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k] = \sum_{k=0}^{1} x[k]h[-k] = x[0]h[0] + x[1]h[-1] = 0.5$$

Example 1

Impulse response of a LTI system and the input signal are given right.

Find the output y[n]!



$n = 1$:

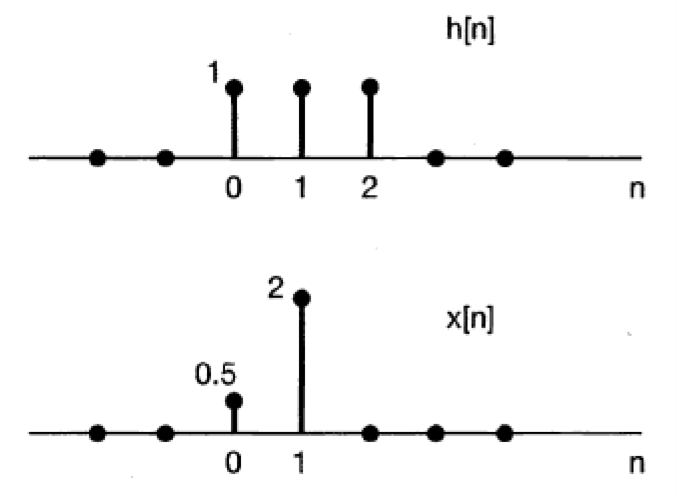
$$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k] = \sum_{k=0}^{1} x[k]h[1-k] = x[0]h[1] + x[1]h[0] = 0.5 * 1 + 2 * 1 = 2.5$$

$n = 2$:

$$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k] = \sum_{k=0}^{1} x[k]h[2-k] = x[0]h[2] + x[1]h[1] = 0.5 * 1 + 2 * 1 = 2.5$$

Example 1

Impulse response of a LTI system and the input signal are given right.



Find the output y[n]!

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

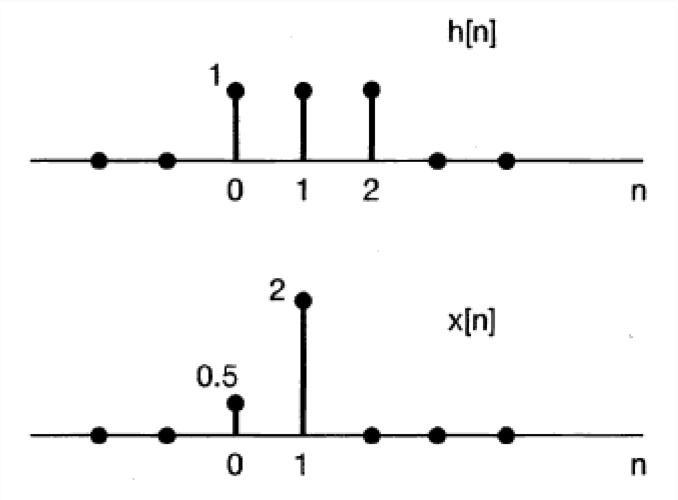
n = 1 :

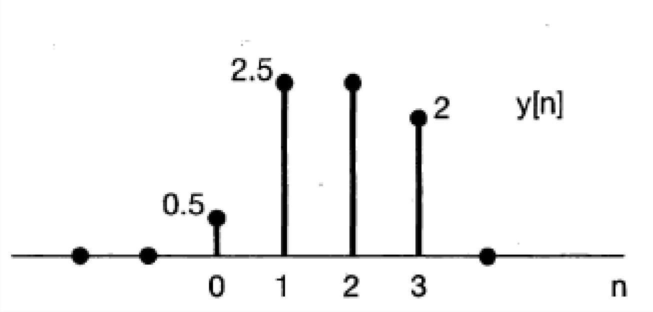
$$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k] = \sum_{k=0}^{1} x[k]h[1-k] = x[0]h[1] + x[1]h[0] = 0.5 * 1 + 2 * 1 = 2.5$$

n = 2 :

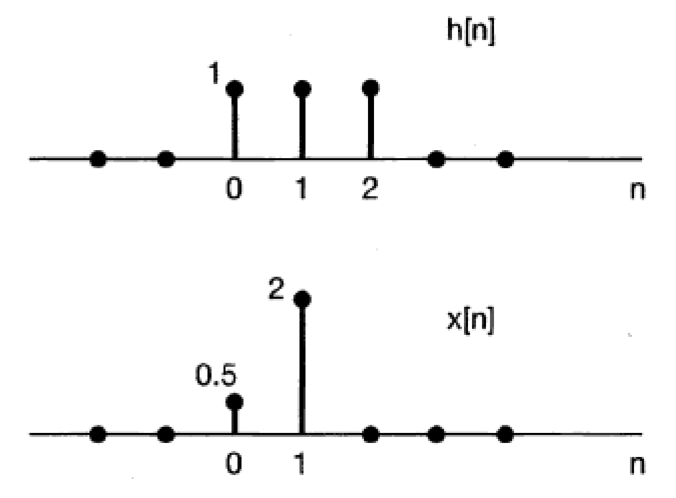
$$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k] = \sum_{k=0}^{1} x[k]h[2-k] = x[0]h[2] + x[1]h[1] = 0.5 * 1 + 2 * 1 = 2.5$$

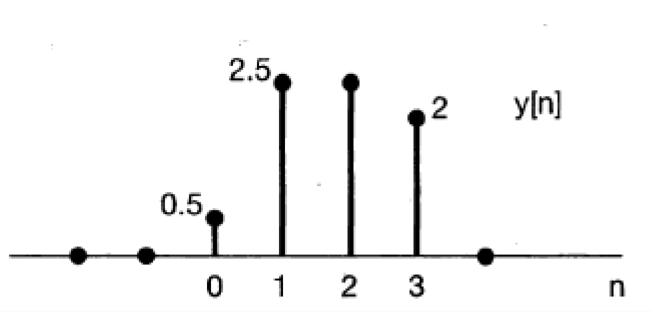
Example 1





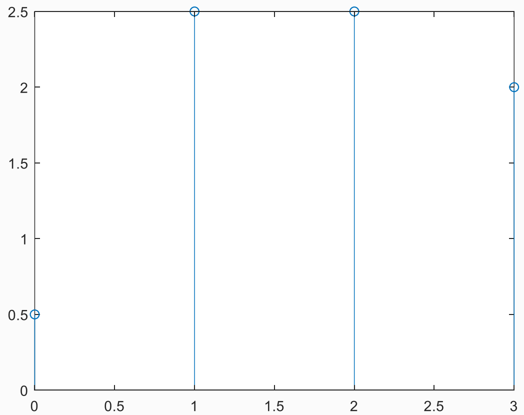
Example 1



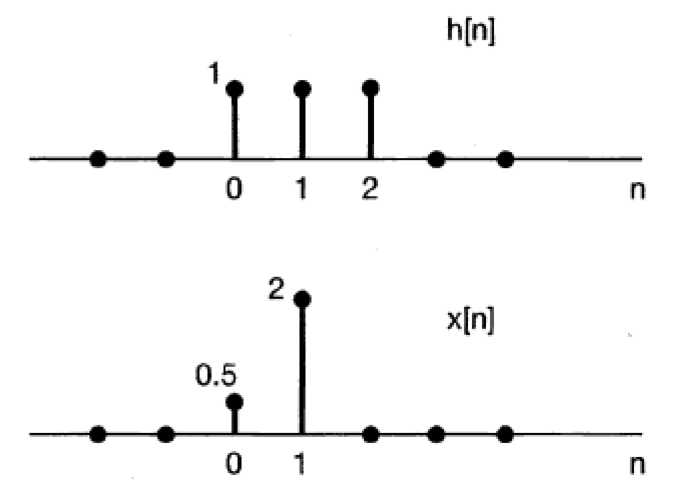


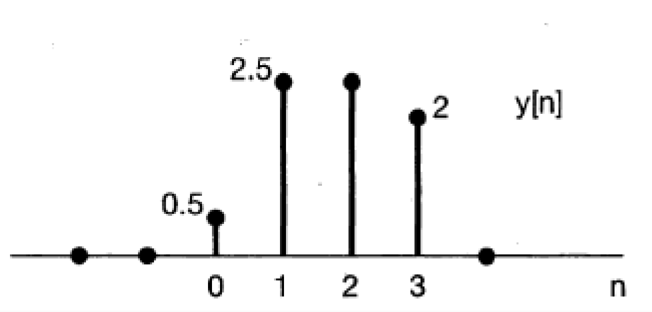

```
h = [1 1 1];
x = [0.5 2];

y = conv (x,h);
stem(0:length(h)+length(x)-2,y);
```



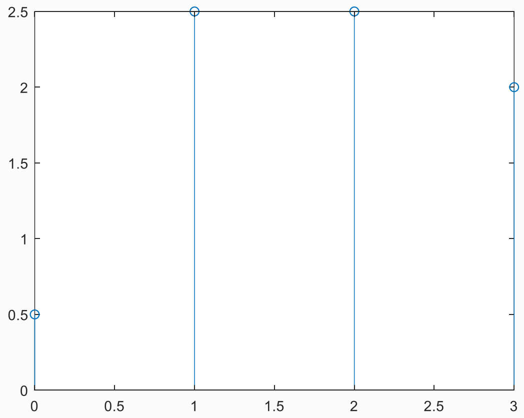
Example 1





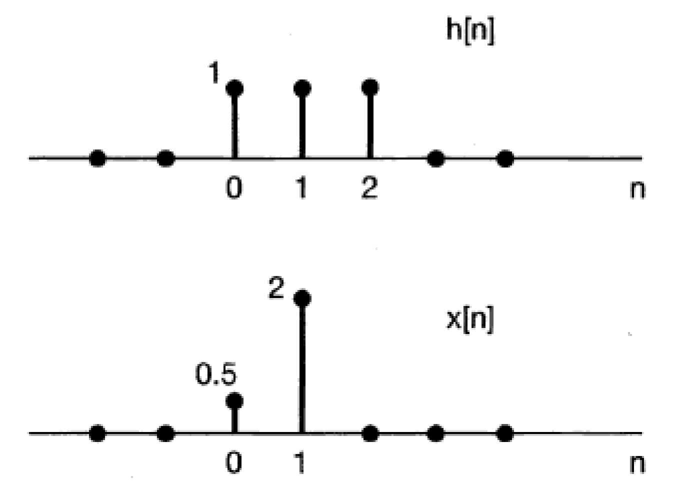

```
h = [1 1 1];
x = [0.5 2];

y = conv (x,h);
stem(0:length(h)+length(x)-2,y);
```



Check the **Associativity property?**

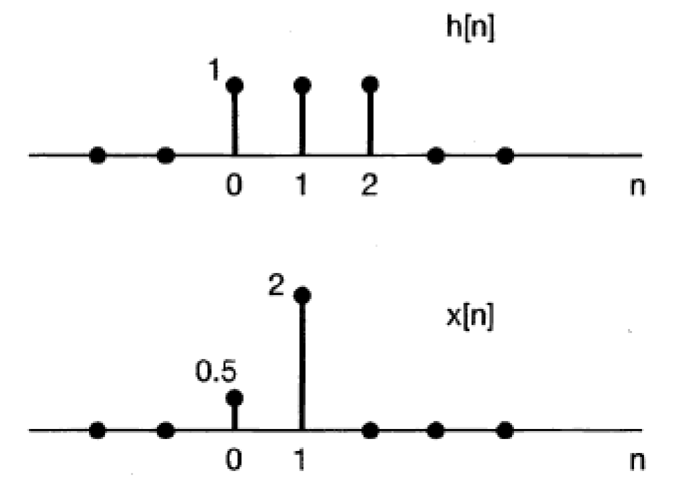
Example 2 (Table Approach)



Graphical approach will be explained next week!!

Example 2 (Table Approach)

$$y[n] = \sum_{k=0}^{2} h[k]x[n-k] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2]$$

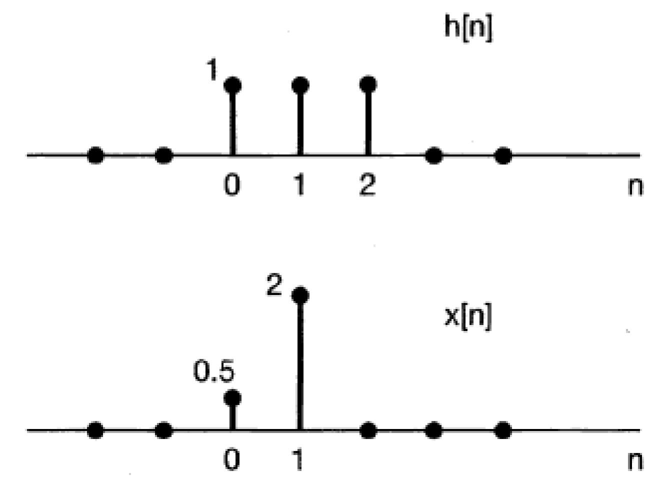


Graphical approach will be explained next week!!

Example 2 (Table Approach)

$$y[n] = \sum_{k=0}^{2} h[k]x[n-k] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2]$$

n	n<0	0	1	2	3	4	5
x[n]	0	1	1	1	0	0	0
h[n]	0	0.5	2	0	0	0	0
h[0]x[n]	0	0.5	0.5	0.5	0	0	0
h[1]x[n-1]	0	0	2	2	2	0	0
h[2]x[n-2]	0	0	0	0	0	0	0
y[n]	0	0.5	2.5	2.5	2	0	0



Graphical approach will be explained next week!!

Another Example of Convolution without fewer Math

Find the impulse response of a system which is given below.

$$y[n] = x[n] + 0.4x[n-1600] \qquad h[n] = \delta[n] + 0.4\delta[n-1600]$$

Another Example of Convolution without fewer Math

Find the impulse response of a system which is given below.

$$y[n] = x[n] + 0.4x[n-1600] \qquad\qquad h[n] = \delta[n] + 0.4\delta[n-1600]$$

* Find the output when the input signal $x[n] = \cos(5\pi n)$ is applied to the system.

Another Example of Convolution without fewer Math

Find the impulse response of a system which is given below.

$$y[n] = x[n] + 0.4x[n-1600] \qquad h[n] = \delta[n] + 0.4\delta[n-1600]$$

* Find the output when the input signal $x[n] = \cos(5\pi n)$ is applied to the system.

$$y[n] = h[n] * x[n]$$

Another Example of Convolution without fewer Math

Find the impulse response of a system which is given below.

$$y[n] = x[n] + 0.4x[n - 1600] \qquad h[n] = \delta[n] + 0.4\delta[n - 1600]$$

* Find the output when the input signal $x[n] = \cos(5\pi n)$ is applied to the system.

$$y[n] = h[n] * x[n]$$

$$y[n] = (\delta[n] + 0.4\delta[n - 1600]) * \cos(5\pi n)$$

Another Example of Convolution without fewer Math

Find the impulse response of a system which is given below.

$$y[n] = x[n] + 0.4x[n - 1600] \qquad h[n] = \delta[n] + 0.4\delta[n - 1600]$$

* Find the output when the input signal $x[n] = \cos(5\pi n)$ is applied to the system.

$$y[n] = h[n] * x[n]$$

$$y[n] = (\delta[n] + 0.4\delta[n - 1600]) * \cos(5\pi n)$$

***Using Distributive Property**

$$y[n] = \delta[n] * \cos(5\pi n) + 0.4\delta[n - 1600] * \cos(5\pi n)$$

Another Example of Convolution without fewer Math

Find the impulse response of a system which is given below.

$$y[n] = x[n] + 0.4x[n-1600] \qquad h[n] = \delta[n] + 0.4\delta[n-1600]$$

* Find the output when the input signal $x[n] = \cos(5\pi n)$ is applied to the system.

$$y[n] = h[n] * x[n]$$

$$y[n] = (\delta[n] + 0.4\delta[n-1600]) * \cos(5\pi n)$$

***Using Distributive Property**

$$y[n] = \delta[n] * \cos(5\pi n) + 0.4\delta[n-1600] * \cos(5\pi n)$$

***Using Identity Element of Convolution Property**

Convolution with an Impulse

$$x[n] * \delta[n-n_0] = x[n-n_0]$$

Another Example of Convolution without fewer Math

Find the impulse response of a system which is given below.

$$y[n] = x[n] + 0.4x[n - 1600] \qquad h[n] = \delta[n] + 0.4\delta[n - 1600]$$

* Find the output when the input signal $x[n] = \cos(5\pi n)$ is applied to the system.

$$y[n] = h[n] * x[n]$$

$$y[n] = (\delta[n] + 0.4\delta[n - 1600]) * \cos(5\pi n)$$

***Using Distributive Property**

$$y[n] = \delta[n] * \cos(5\pi n) + 0.4\delta[n - 1600] * \cos(5\pi n)$$

***Using Identity Element of Convolution Property**

$$y[n] = \cos(5\pi n) + 0.4\cos(5\pi(n - 1600))$$

Convolution with an Impulse

$$x[n] * \delta[n - n_0] = x[n - n_0]$$

Let's implement this filter to the audio

What does this system do?

$$y[n] = x[n] + 0.4x[n-1600]$$

```
clc; clear all;
%%
[x,Fs] = audioread('myRecording.wav');
x = x(:,1);
sound(x,Fs);
%%
h = zeros(1,1601);
h(1)=1;
h(1601)=0.8;
%%
lenOutput = length(x)+length(h)-1;
y = zeros(1,lenOutput);

for n = 1:length(x)
    for k = 1:length(h)
        if (n-k)>1
          y(n) =  y(n)+ h(k)*x(n-k);
        end
    end
end
figure (2); stem(1:lenOutput, y));
sound(y,Fs);
```

Let’s implement this filter to the audio

What does this system do?

$$y[n] = x[n] + 0.4x[n - 1600]$$

$$h[n] = \delta[n] + 0.4\delta[n - 1600]$$

```
clc; clear all;
%%
[x,Fs] = audioread('myRecording.wav');
x = x(:,1);
sound(x,Fs);
%%
h = zeros(1,1601);
h(1)=1;
h(1601)=0.8;
%%
lenOutput = length(x)+length(h)-1;
y = zeros(1,lenOutput);

for n = 1:length(x)
    for k = 1:length(h)
        if (n-k)>1
          y(n) =  y(n)+ h(k)*x(n-k);
        end
    end
end
figure (2); stem(1:lenOutput, y));
sound(y,Fs);
```

Let’s implement this filter to the audio

What does this system do?

$$y[n] = x[n] + 0.4x[n-1600]$$

$$h[n] = \delta[n] + 0.4\delta[n-1600]$$

Write a convolution code that applies this filter to a piano sound.

```
clc; clear all;
%%
[x,Fs] = audioread('myRecording.wav');
x = x(:,1);
sound(x,Fs);
%%
h = zeros(1,1601);
h(1)=1;
h(1601)=0.8;
%%
lenOutput = length(x)+length(h)-1;
y = zeros(1,lenOutput);

for n = 1:length(x)
    for k = 1:length(h)
        if (n-k)>1
          y(n) =  y(n)+ h(k)*x(n-k);
        end
    end
end
figure (2); stem(1:lenOutput, y));
sound(y,Fs);
```

Let's implement this filter to the audio

What does this system do?

$$y[n] = x[n] + 0.4x[n - 1600]$$

$$h[n] = \delta[n] + 0.4\delta[n - 1600]$$

Write a convolution code that applies this filter to a piano sound.

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

```
clc; clear all;
%%
[x,Fs] = audioread('myRecording.wav');
x = x(:,1);
sound(x,Fs);
%%
h = zeros(1,1601);
h(1)=1;
h(1601)=0.8;
%%
lenOutput = length(x)+length(h)-1;
y = zeros(1,lenOutput);

for n = 1:length(x)
    for k = 1:length(h)
        if (n-k)>1
          y(n) =  y(n)+ h(k)*x(n-k);
        end
    end
end
figure (2); stem(1:lenOutput, y));
sound(y,Fs);
```

Let’s implement this filter to the audio

What does this system do?

$$y[n] = x[n] + 0.4x[n-1600]$$

$$h[n] = \delta[n] + 0.4\delta[n-1600]$$

Write a convolution code that applies this filter to a piano sound.

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

h[n] = [1 0 0 0 0 … 0 0 0 … 1] -> length of h is 1601

$$y[n] = \sum_{k=0}^{1600} h[k]x[n-k]$$

```
clc; clear all;
%%
[x,Fs] = audioread('myRecording.wav');
x = x(:,1);
sound(x,Fs);
%%
h = zeros(1,1601);
h(1)=1;
h(1601)=0.8;
%%
lenOutput = length(x)+length(h)-1;
y = zeros(1,lenOutput);

for n = 1:length(x)
    for k = 1:length(h)
        if (n-k)>1
          y(n) =  y(n)+ h(k)*x(n-k);
        end
    end
end
figure (2); stem(1:lenOutput, y));
sound(y,Fs);
```

Let’s implement this filter to the audio

What does this system do?

$$y[n] = x[n] + 0.4x[n-1600]$$

$$h[n] = \delta[n] + 0.4\delta[n-1600]$$

Write a convolution code that applies this filter to a piano sound.

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

h[n] = [1 0 0 0 0 … 0 0 0 … 1] -> length of h is 1601

$$y[n] = \sum_{k=0}^{1600} h[k]x[n-k]$$

```
clc; clear all;
%%
[x,Fs] = audioread('myRecording.wav');
x = x(:,1);
sound(x,Fs);
%%
h = zeros(1,1601);
h(1)=1;
h(1601)=0.8;
%%
lenOutput = length(x)+length(h)-1;
y = zeros(1,lenOutput);

for n = 1:length(x)
    for k = 1:length(h)
        if (n-k)>1
          y(n) =  y(n)+ h(k)*x(n-k);
        end
    end
end
figure (2); stem(1:lenOutput, y));
sound(y,Fs);
```

Just code, no R, L, C circuit elements!!

Convolution in 2D

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Delta fonksiyonu

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Kaydır ve çıkart

$$\begin{bmatrix} -k/8 & -k/8 & -k/8 \\ -k/8 & k+1 & -k/8 \\ -k/8 & -k/8 & -k/8 \end{bmatrix}$$

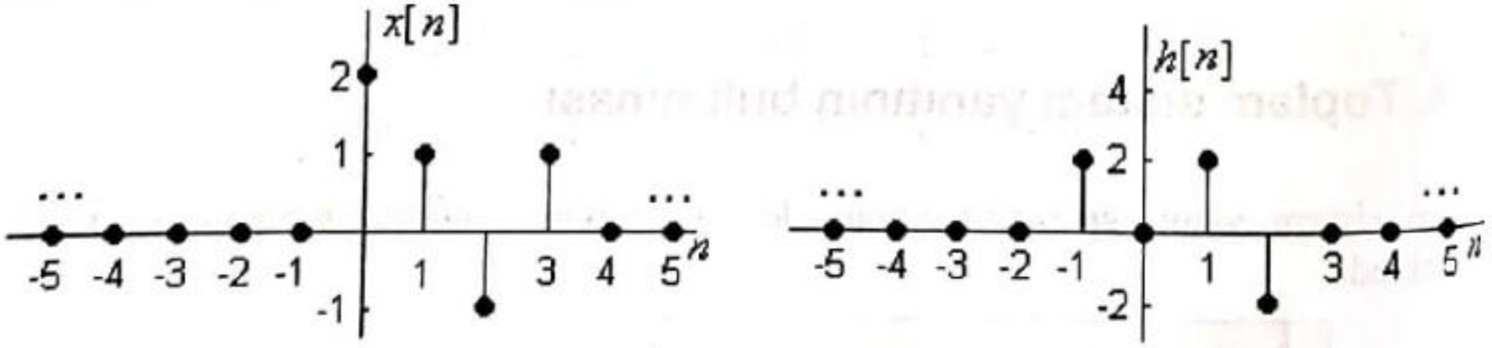
Kenar pekiştirme

Mart 2012

MATLAB -> conv2 function

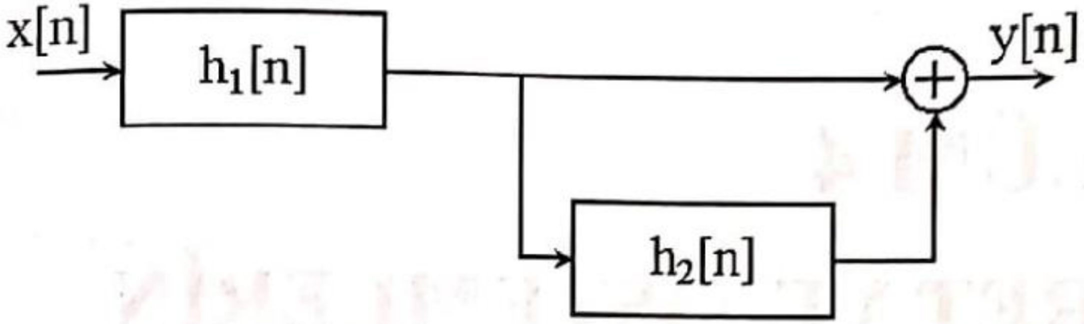
Homework

3.4. Şekil 3.17'de gösterilen dürtü yanıtı ve giriş işareti için sistemin çıkışını bulunuz. Bu sistem kararlı mıdır? Bu sistem nedensel midir?



Şekil 3.17. Problem 3.4. için giriş işareti ve dürtü yanıtı.

3.5. Şekil 3.18'de gösterilen sistem düzeneğinde $h_1[n]=(0.2)^n u[n]$ ve $h_2[n]=\delta[n-1]$ olarak verildiğine göre eşdeğer sistemin dürtü yanıtını bulunuz.



Şekil 3.18. Problem 3.5. için sistem düzeneği