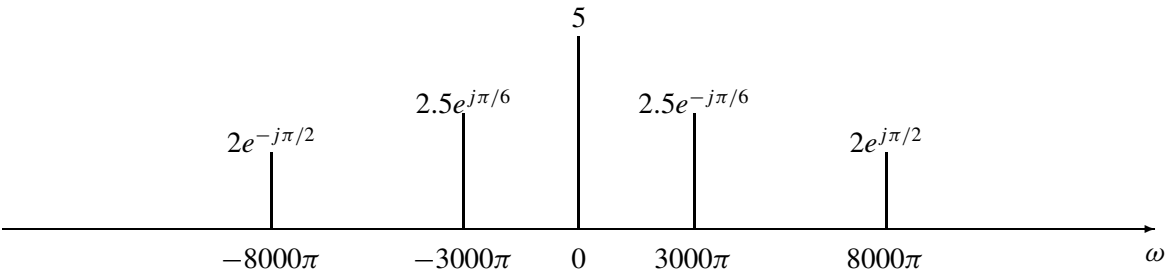


## PROBLEM:

A real signal  $x(t)$  has the following two-sided spectrum:



- (a) Write an equation for  $x(t)$  as a sum of cosines.
- (b) Plot the spectrum of the signal  $y(t) = 2x(t) - 3 \cos(5000\pi(t - 0.002))$ .

## PROBLEM:

In AM radio, the transmitted signal (voice or music) is modulated by a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta has a *carrier frequency* of 750 kHz. For example, if  $x(t)$  is the voice/music signal, then the transmitted signal would be:

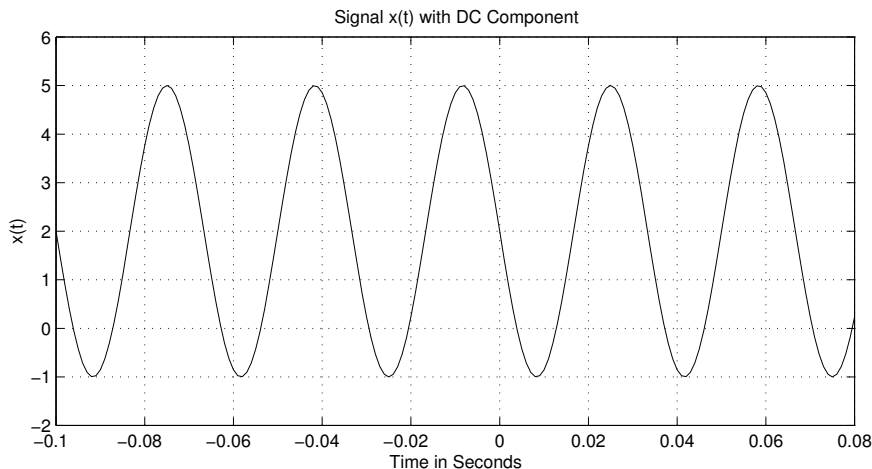
$$y(t) = [x(t) + A] \cos(2\pi(750 \times 10^3)t)$$

where  $A$  is a constant. ( $A$  is introduced to make the AM receiver design easier, in which case  $A$  must be chosen to be larger than the maximum value of  $v(t)$ .) Suppose that the signal that is to be transmitted is

$$x(t) = 3 \cos(2000\pi t + \pi/4) + \cos(4000\pi t + \pi/2)$$

Draw the spectrum for  $y(t)$  assuming a carrier at 750 kHz with  $A = 2$ . *Hint: Substitute for  $x(t)$  and expand  $y(t)$  into a sum of cosine terms of three different frequencies.*

## PROBLEM:



The above signal  $x(t)$  consists of a DC component plus a cosine signal. The terminology *DC component* means a component that is constant versus time.

- What is the frequency of the DC component? What is the frequency of the cosine component?
- Write an equation for the signal  $x(t)$ . You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the above graph.
- Expand the equation obtained in the previous part into a sum of positive and negative frequency complex exponential signals.
- Plot the two-sided spectrum of the signal  $x(t)$ . Show the complex amplitudes for each positive and negative frequency contained in  $x(t)$ .

### PROBLEM:

A linear-FM “chirp” signal is one that sweeps in frequency from  $\omega_1 = 2\pi f_1$  to  $\omega_2 = 2\pi f_2$  as time goes from  $t = 0$  to  $t = T_2$ . We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A \cos(\alpha t^2 + \beta t + \phi) \quad (1)$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument  $\psi(t)$  is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly*.

$$\omega_i(t) = \frac{d}{dt} \psi(t) \quad \text{radians/sec} \quad (2)$$

There are examples on the CD-ROM in the Chapter 3 demos.

- (a) For the linear-FM “chirp” in (1), determine formulas for the beginning instantaneous frequency ( $\omega_1$ ) and the ending instantaneous frequency ( $\omega_2$ ) in terms of  $\alpha$ ,  $\beta$  and  $T_2$ . For this problem, assume that the starting time of the “chirp” is  $t = 0$ .
- (b) For the “chirp” signal

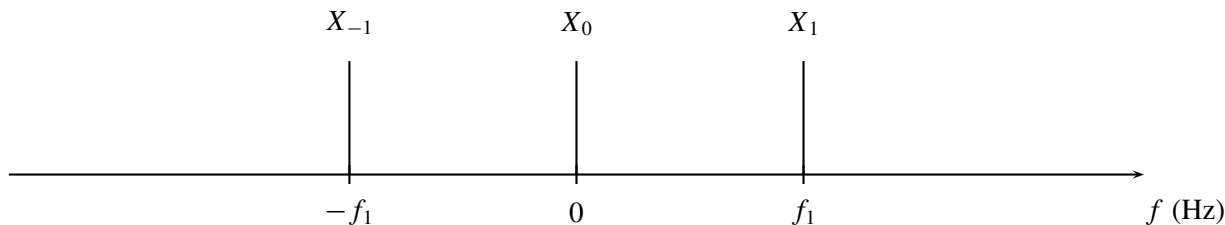
$$x(t) = \Re \left\{ e^{j2\pi(30t^2 - 30t)} \right\}$$

derive a formula for the *instantaneous* frequency versus time. Should your answer for the frequency be a positive number?

**PROBLEM:**

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal  $x(t)$  is given by  $x(t) = 3 \cos(250\pi t - \pi/6)$ , and its spectrum has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

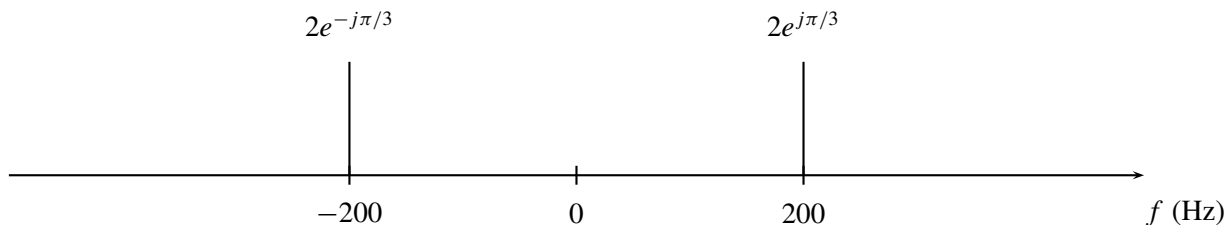
$f_1 =$

$X_0 =$

$X_1 =$

$X_{-1} =$

- (b) The spectrum of a signal  $x(t)$  has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for  $A$ ,  $f_0$ , and  $t_0$ ,

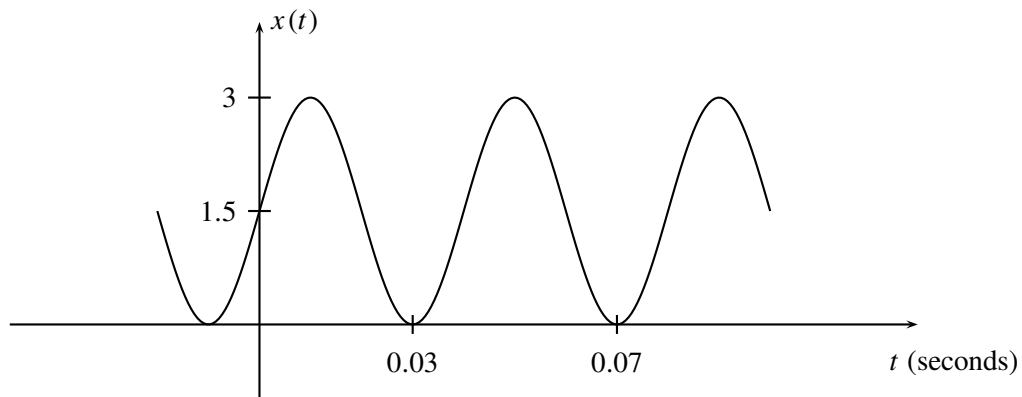
$A =$

$f_0 =$

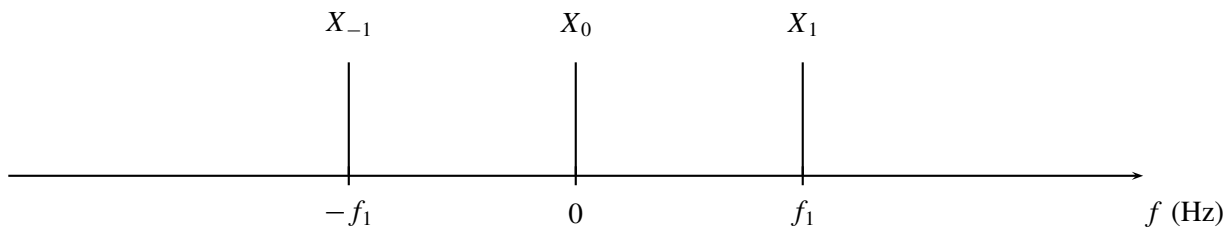
$t_0 =$

**PROBLEM:**

A signal  $x(t) = A \cos(2\pi f_1 t + \phi)$  is shown in the figure below,



The spectrum of  $x(t)$  has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

$f_1 =$

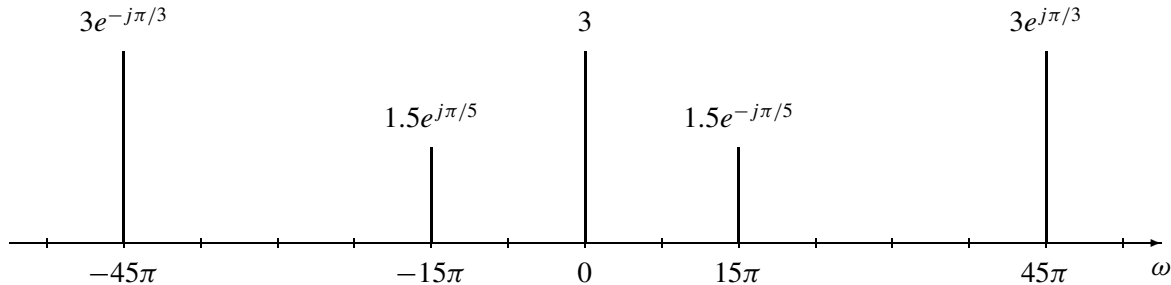
$X_0 =$

$X_1 =$

$X_{-1} =$

## PROBLEM:

The spectrum of a signal  $x(t)$  is shown in the following figure:



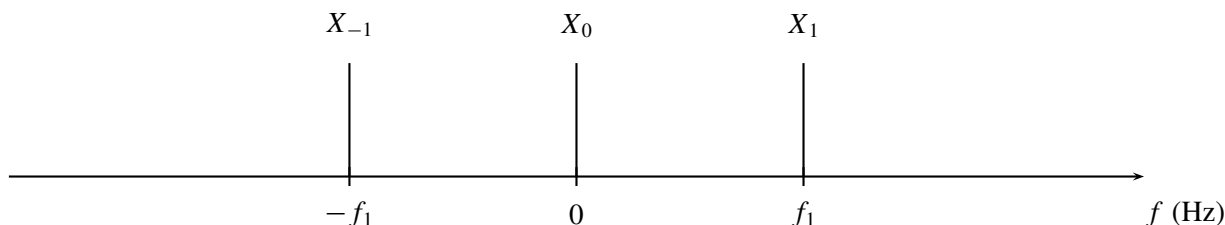
**Note that the frequency axis is radian frequency ( $\omega$ ) *not* cyclic frequency ( $f$ ).**

- Write an equation for  $x(t)$  in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of  $x(t)$ ?

**PROBLEM:**

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal  $x(t)$  is given by  $x(t) = 2 \cos(200\pi t + \pi/8)$ , and its spectrum has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

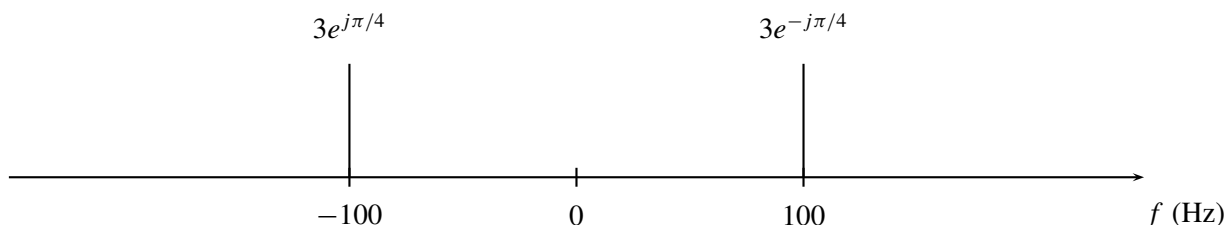
$f_1 =$

$X_0 =$

$X_1 =$

$X_{-1} =$

- (b) The spectrum of a signal  $x(t)$  has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for  $A$ ,  $f_0$ , and  $t_0$ ,

$A =$

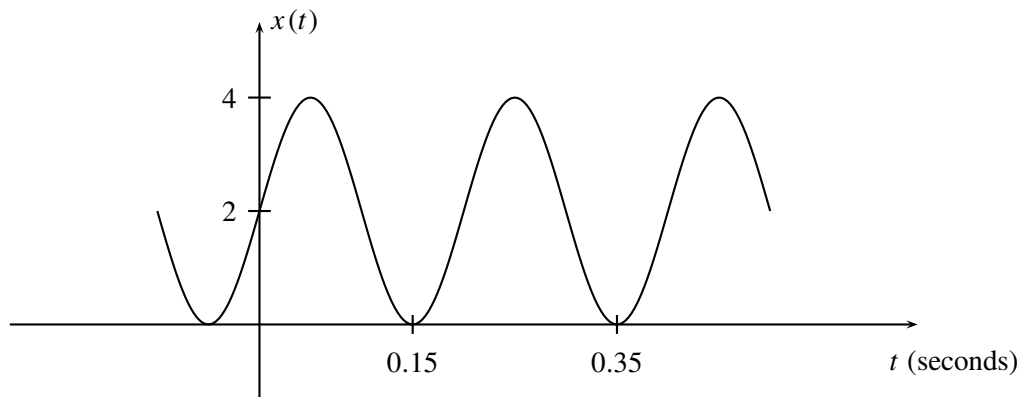
$f_0 =$

$t_0 =$

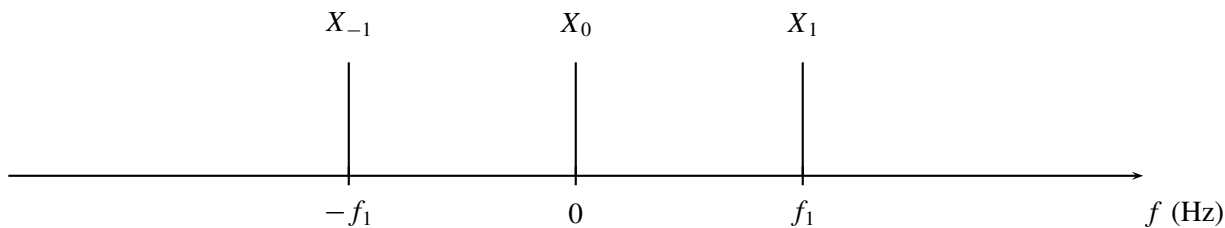


**PROBLEM:**

A signal  $x(t) = A \cos(2\pi f_1 t + \phi)$  is shown in the figure below,



The spectrum of  $x(t)$  has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

$$f_1 =$$

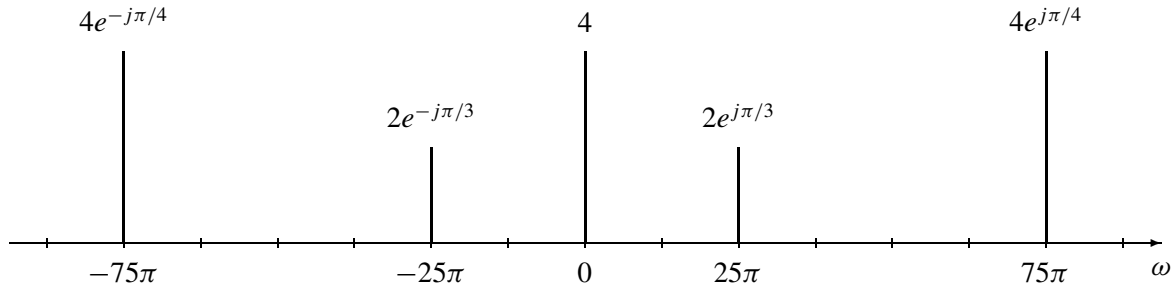
$$X_0 =$$

$$X_1 =$$

$$X_{-1} =$$

## PROBLEM:

The spectrum of a signal  $x(t)$  is shown in the following figure:



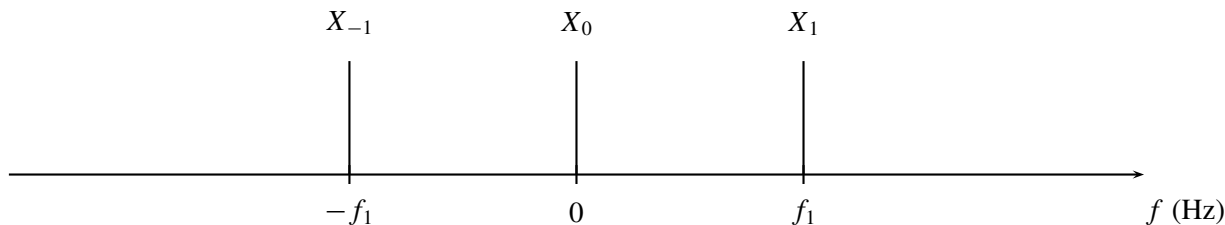
**Note that the frequency axis is radian frequency ( $\omega$ ) *not* cyclic frequency ( $f$ ).**

- Write an equation for  $x(t)$  in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of  $x(t)$ ?

**PROBLEM:**

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal  $x(t)$  is given by  $x(t) = 4 \cos(300\pi t - \pi/3)$ , and its spectrum has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

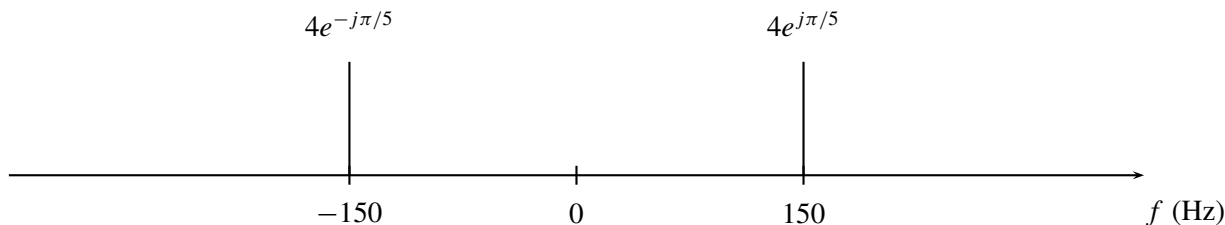
$f_1 =$

$X_0 =$

$X_1 =$

$X_{-1} =$

- (b) The spectrum of a signal  $x(t)$  has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for  $A$ ,  $f_0$ , and  $t_0$ ,

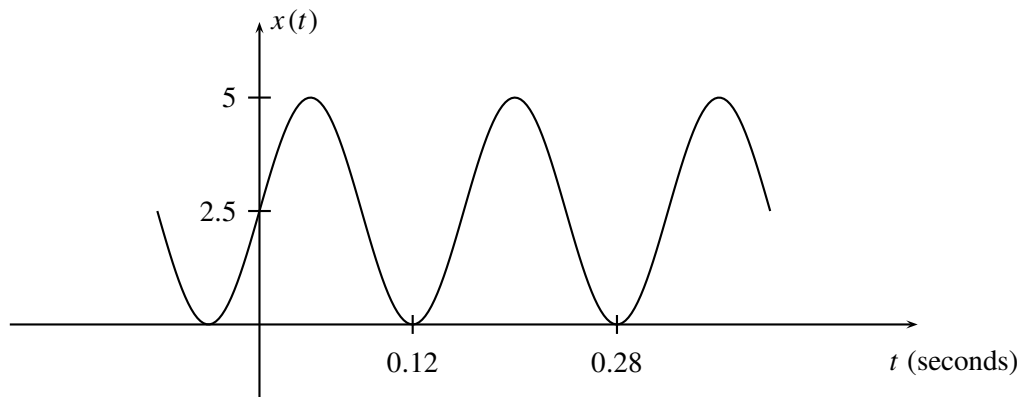
$A =$

$f_0 =$

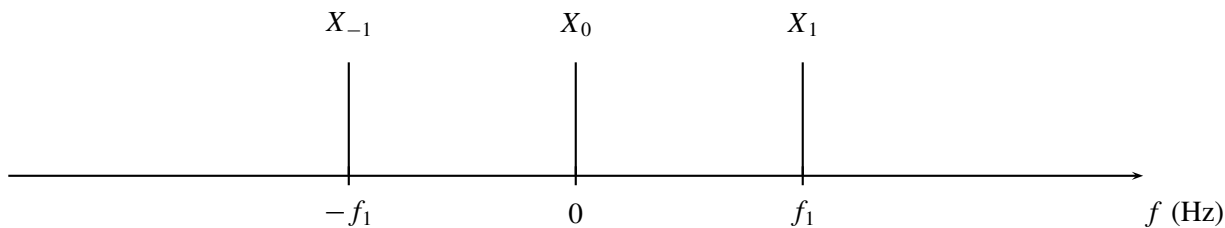
$t_0 =$

**PROBLEM:**

A signal  $x(t) = A \cos(2\pi f_1 t + \phi)$  is shown in the figure below,



The spectrum of  $x(t)$  has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

$f_1 =$

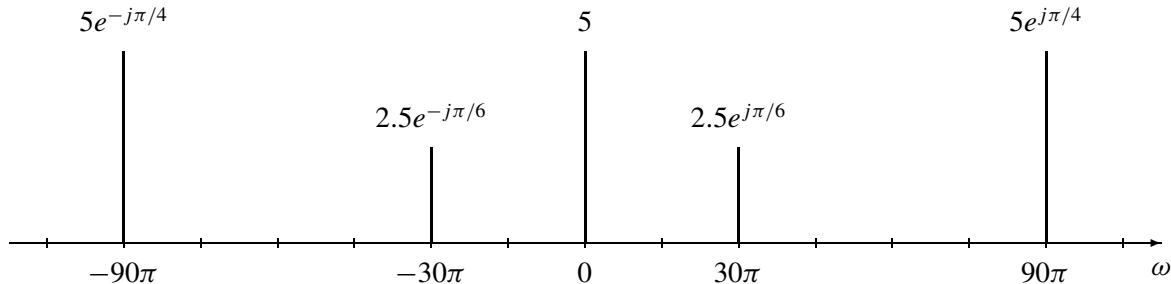
$X_0 =$

$X_1 =$

$X_{-1} =$

## PROBLEM:

The spectrum of a signal  $x(t)$  is shown in the following figure:



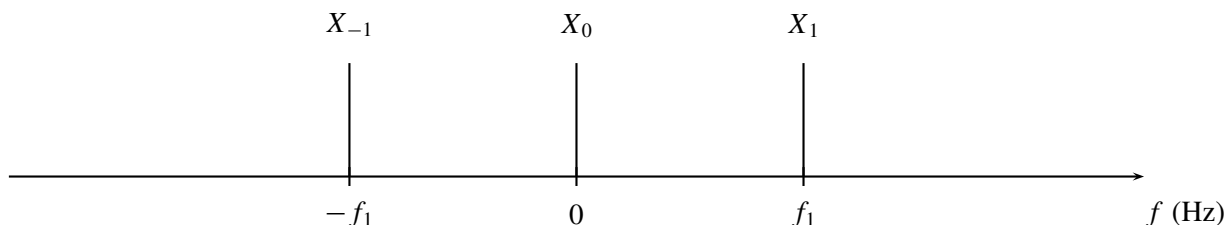
**Note that the frequency axis is radian frequency ( $\omega$ ) *not* cyclic frequency ( $f$ ).**

- Write an equation for  $x(t)$  in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of  $x(t)$ ?

**PROBLEM:**

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal  $x(t)$  is given by  $x(t) = 5 \cos(350\pi t - \pi/7)$ , and its spectrum has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

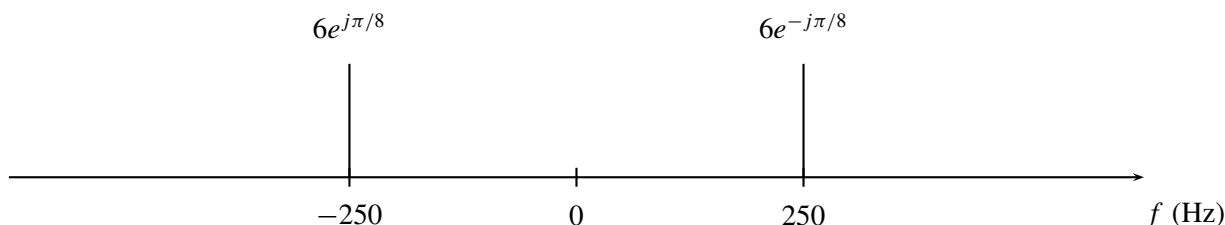
$f_1 =$

$X_0 =$

$X_1 =$

$X_{-1} =$

- (b) The spectrum of a signal  $x(t)$  has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for  $A$ ,  $f_0$ , and  $t_0$ ,

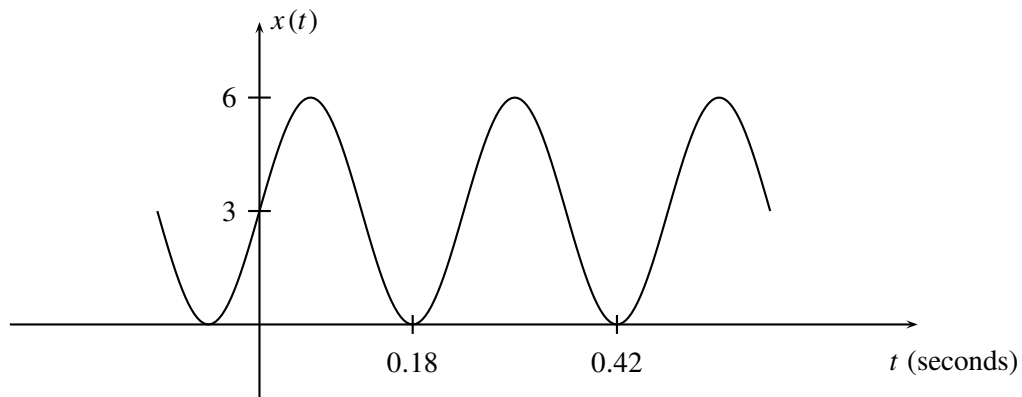
$A =$

$f_0 =$

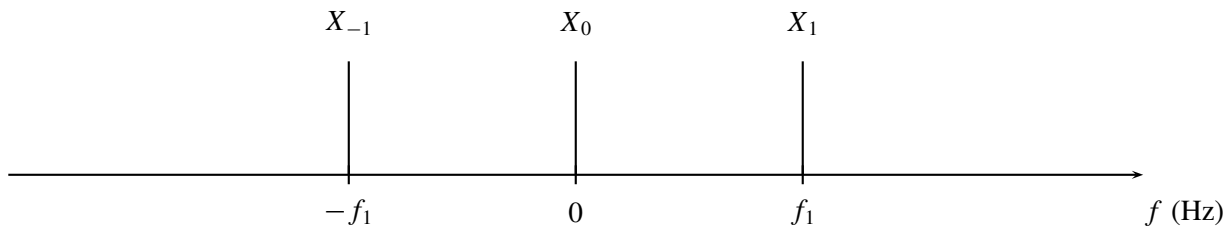
$t_0 =$

**PROBLEM:**

A signal  $x(t) = A \cos(2\pi f_1 t + \phi)$  is shown in the figure below,



The spectrum of  $x(t)$  has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

$$f_1 =$$

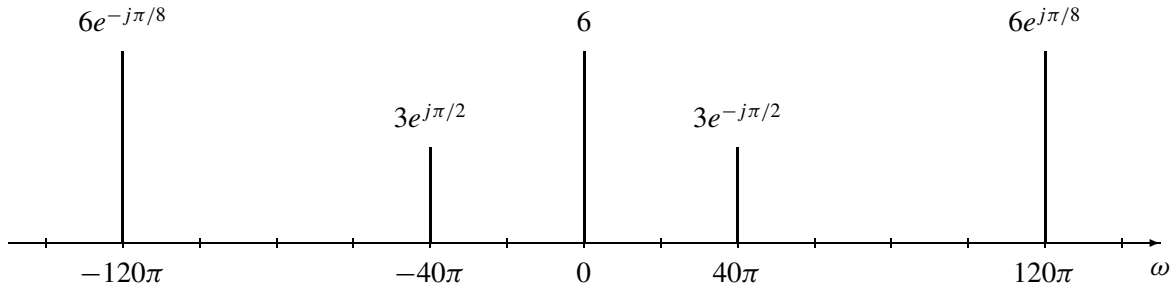
$$X_0 =$$

$$X_1 =$$

$$X_{-1} =$$

## PROBLEM:

The spectrum of a signal  $x(t)$  is shown in the following figure:



**Note that the frequency axis is radian frequency ( $\omega$ ) *not* cyclic frequency ( $f$ ).**

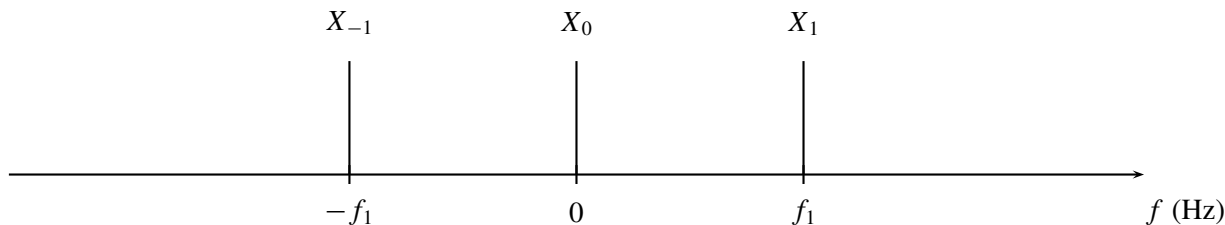
- Write an equation for  $x(t)$  in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of  $x(t)$ ?



**PROBLEM:**

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

- (a) A signal  $x(t)$  is given by  $x(t) = 3 \cos(400\pi t + 3\pi/16)$ , and its spectrum has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

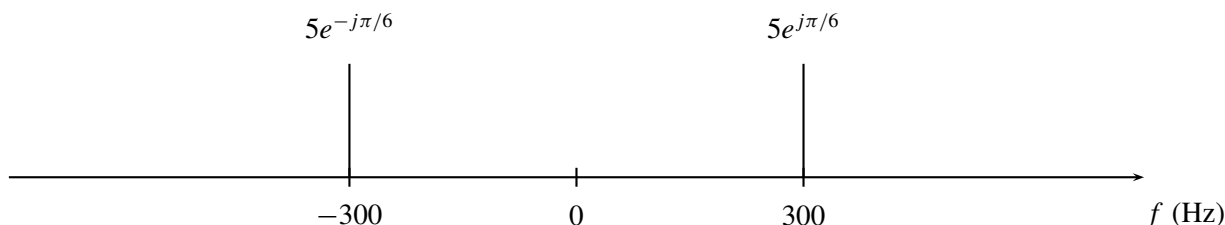
$f_1 =$

$X_0 =$

$X_1 =$

$X_{-1} =$

- (b) The spectrum of a signal  $x(t)$  has the form



Therefore, the signal has the form

$$x(t) = A \cos(2\pi f_0(t - t_0))$$

Determine the values for  $A$ ,  $f_0$ , and  $t_0$ ,

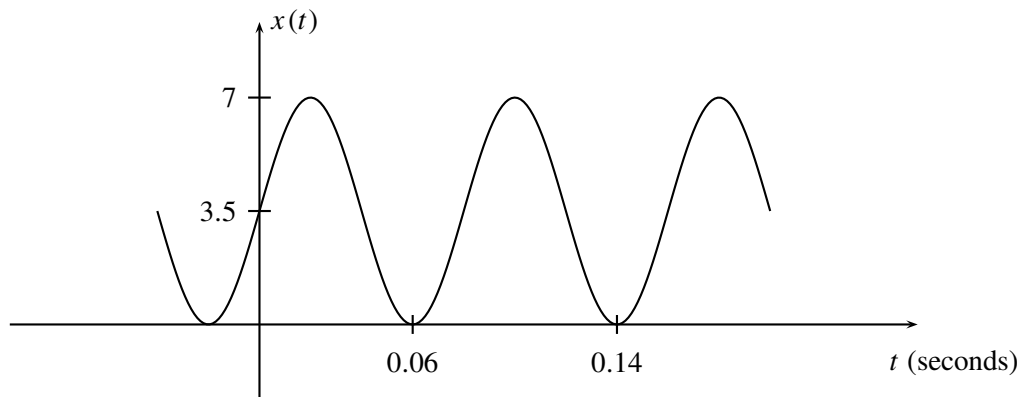
$A =$

$f_0 =$

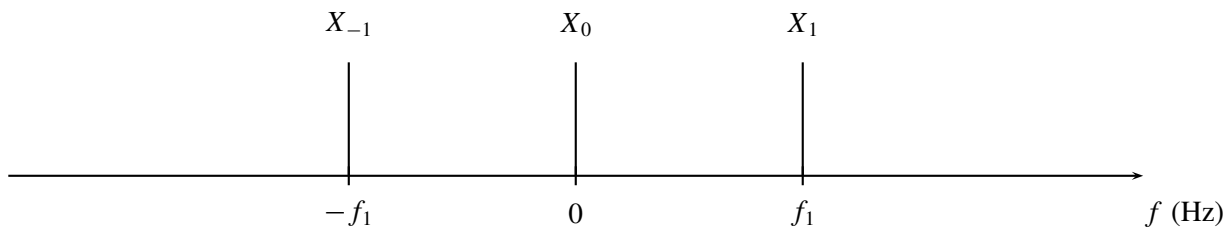
$t_0 =$

**PROBLEM:**

A signal  $x(t) = A \cos(2\pi f_1 t + \phi)$  is shown in the figure below,



The spectrum of  $x(t)$  has the form



Determine the values for  $f_1$ ,  $X_0$ ,  $X_1$ , and  $X_{-1}$ . Note that the frequencies  $f$  are given in Hertz.

$f_1 =$

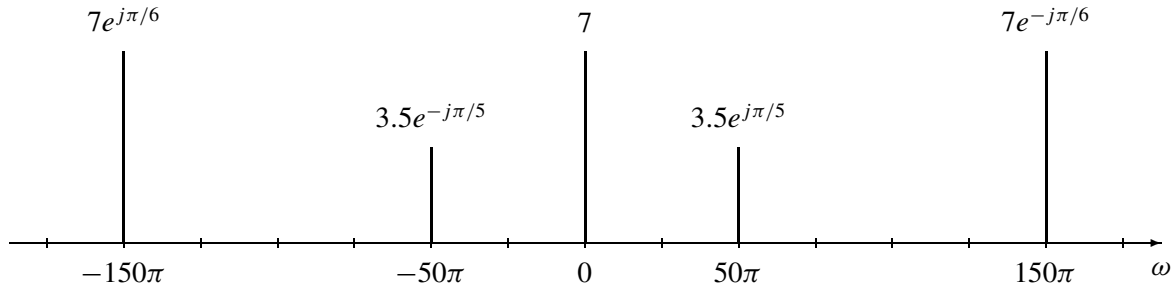
$X_0 =$

$X_1 =$

$X_{-1} =$

## PROBLEM:

The spectrum of a signal  $x(t)$  is shown in the following figure:



**Note that the frequency axis is radian frequency ( $\omega$ ) *not* cyclic frequency ( $f$ ).**

- Write an equation for  $x(t)$  in terms of cosine functions.
- This signal is periodic. What is the fundamental frequency and the corresponding period of  $x(t)$ ?

## PROBLEM:

A periodic signal,  $x(t)$ , is given by

$$x(t) = 1 + 3 \cos(300\pi t) + 2 \sin(500\pi t - \pi/4)$$

- (a) What is the period of  $x(t)$ ?
- (b) Find the Fourier series coefficients of  $x(t)$ .

## PROBLEM:

A periodic signal,  $x(t)$ , is given by

$$x(t) = 2 + \cos(150\pi t - \pi/6) + 2 \sin(450\pi t)$$

- (a) What is the period of  $x(t)$ ?
- (b) Find the Fourier series coefficients of  $x(t)$ .

## PROBLEM:

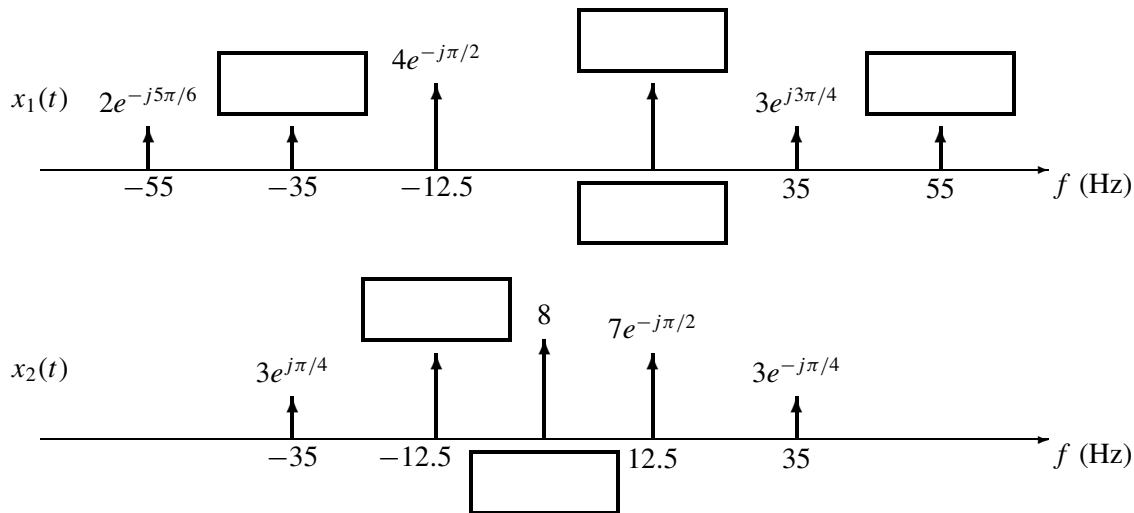
A periodic signal,  $x(t)$ , is given by

$$x(t) = 3 + \sin(200\pi t) + 3 \cos(600\pi t + \pi/3)$$

- (a) What is the period of  $x(t)$ ?
- (b) Find the Fourier series coefficients of  $x(t)$ .

## PROBLEM:

- (a) The incomplete spectra for two *real* signals  $x_1(t)$  and  $x_2(t)$  are shown in the following figures. Fill in the empty boxes for the missing components.



- (b) Write an equation for  $x_2(t)$  in terms of cosine functions.

## PROBLEM:

The signal  $x(t)$  is formed from the signal  $v(t)$  by AM modulation. Assume that

$$v(t) = 3 + 3 \cos(5t + \pi/3)$$

and that

$$x(t) = v(t) \cos(20t).$$

- (a) Draw the spectrum for  $v(t)$ .
- (b) Draw the spectrum for  $x(t)$ .



## PROBLEM:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form. Let

$$V = -\frac{1}{\sqrt{3}} - j.$$

- (a) Express  $jV$  in polar form. In addition plot  $jV$  as a vector.
- (b) Express the inverse of  $V$  in rectangular form. In addition plot  $\frac{1}{V}$  as a vector.
- (c) If  $Z = \frac{|V|}{V^*}$ , express  $Z$  in polar form. In addition plot  $Z$  as a vector.
- (d) Express  $\Re\{j^3 V e^{j15t}\}$  in the standard “cosine” form.