

### PROBLEM:

A computer music student decided that he would create a new type of signal where the frequency moved as a function of time. In particular, he planned to generate a “chirp” signal—one that swept in frequency from  $\omega_1 = 2\pi f_1$  to  $\omega_2 = 2\pi$  as time goes from  $t = 0$  to  $t = T_2$ .

It turns out that the *instantaneous frequency* of the chirp can be derived from a derivative operation. If we define  $x(t)$  in the following manner:

$$x(t) = A \cos(\alpha t^2 + \beta t + \phi) \quad (1)$$

then we can think of this as the cosine function operating on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of  $\psi(t)$  is the *instantaneous frequency* which is also the audible frequency heard from the chirp.

$$\omega_i(t) = \frac{d}{dt}\psi(t) \quad \text{radians} \quad (2)$$

- (a) For the “chirp” in (1) determine formulas for the beginning frequency ( $\omega_1$ ) and ending frequency ( $\omega_2$ ) in terms of  $\alpha$ ,  $\beta$  and  $T_2$ .
- (b) For the “chirp” signal

$$x(t) = \Re\{e^{j(40t^2 + 27t + 13)}\}$$

derive a formula for the *instantaneous frequency*.

- (c) Make a plot of the *instantaneous frequency* (in Hertz) versus time over the range  $0 \leq t \leq 1$  sec.