BLM3620 Digital Signal Processing*

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*Based on lecture notes from Ali Can Karaca & Ahmet Elbir

Lecture #13 – z - Transform

- Introduce the z-Trasnform
- Examples
- Transform Example
- MATLAB Applications

Review & Recall

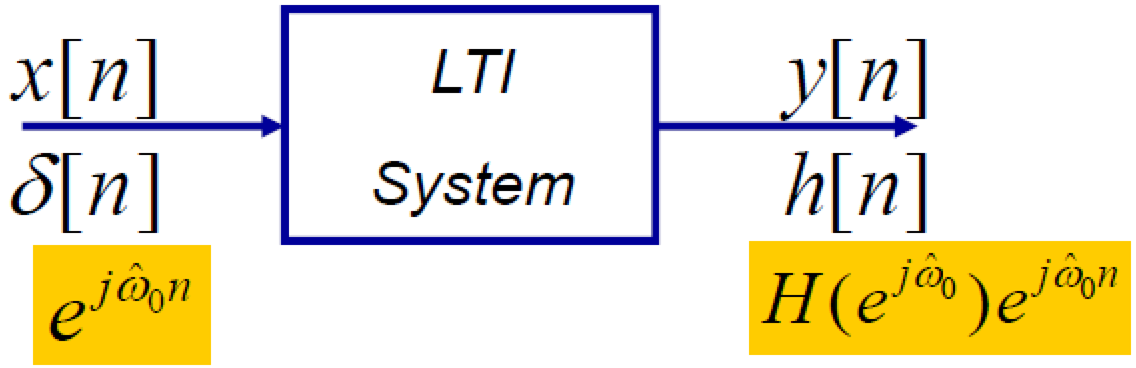
- The FFT is simply an algorithm for efficiently calculating the DFT
- Computational efficiency of an N-Point FFT:
  - DFT: N^2 Complex Multiplications
  - FFT: $(N/2)\log_2(N)$ Complex Multiplications

N	DFT Multiplications	FFT Multiplications	FFT Efficiency
256	65,536	1,024	64 : 1
512	262,144	2,304	114 : 1
1,024	1,048,576	5,120	205 : 1
2,048	4,194,304	11,264	372 : 1
4,096	16,777,216	24,576	683 : 1

Discrete Fourier Transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn}$$

Recap: Frequency Response $H(e^{j\hat{\omega}})$



$$\boxed{H(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\hat{\omega}n}}$$

$$\boxed{\text{Periodic}: H(e^{j(\hat{\omega}+2\pi)}) = H(e^{j\hat{\omega}})}$$

$$\boxed{y[n] = A \cdot \left|H(e^{j\hat{\omega}_0})\right| \cos(\hat{\omega}_0 n + \varphi + \angle H(e^{j\hat{\omega}_0}))}$$

Recap: Transfer Function/System Function $H(z)$

Given impulse response of an LTI system h[n], the transfer function can be found as:

$$\boxed{H[z] = \sum_{k=-\infty}^{\infty} h[k]\, z^{-k}}$$

$$z^n \longrightarrow \boxed{\text{System}} \longrightarrow H(z)z^n$$

$= e^{j\hat{w}})$

Z transform can be computed if FT exists or not.

Easy to compute the system output.

Mostly used with block diagrams to illustrate the system.

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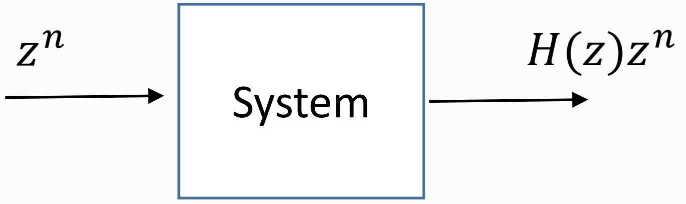

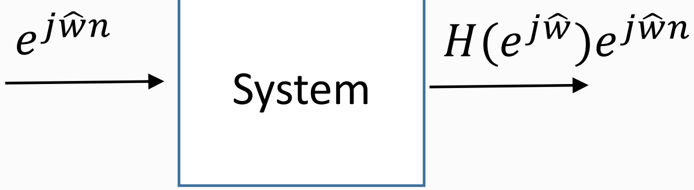

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Three Domains

$$\{b_k\}$$

TIME-DOMAIN

FREQ-DOMAIN

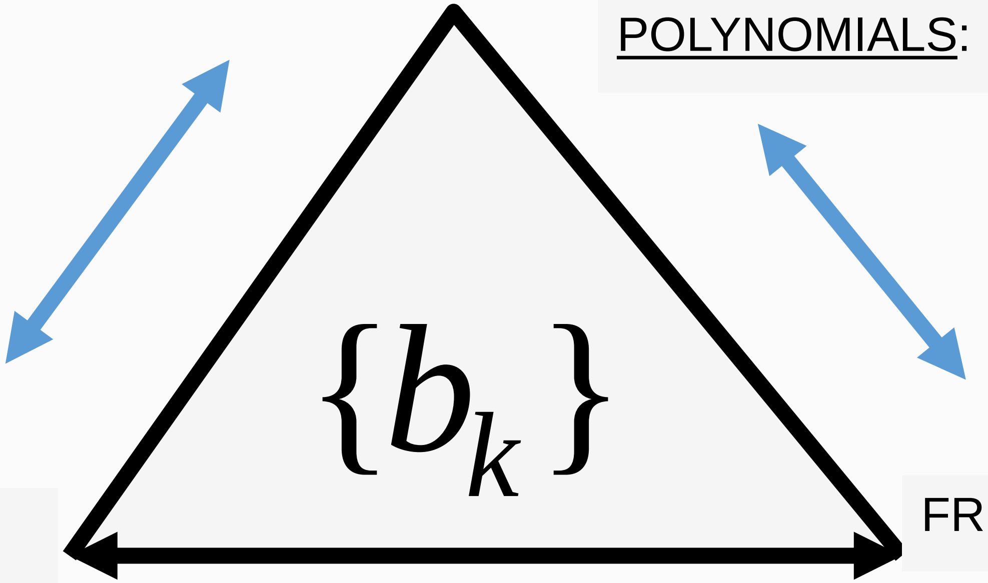
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

Three Domains

Z-TRANSFORM-DOMAIN

<u>POLYNOMIALS</u>: H(z)



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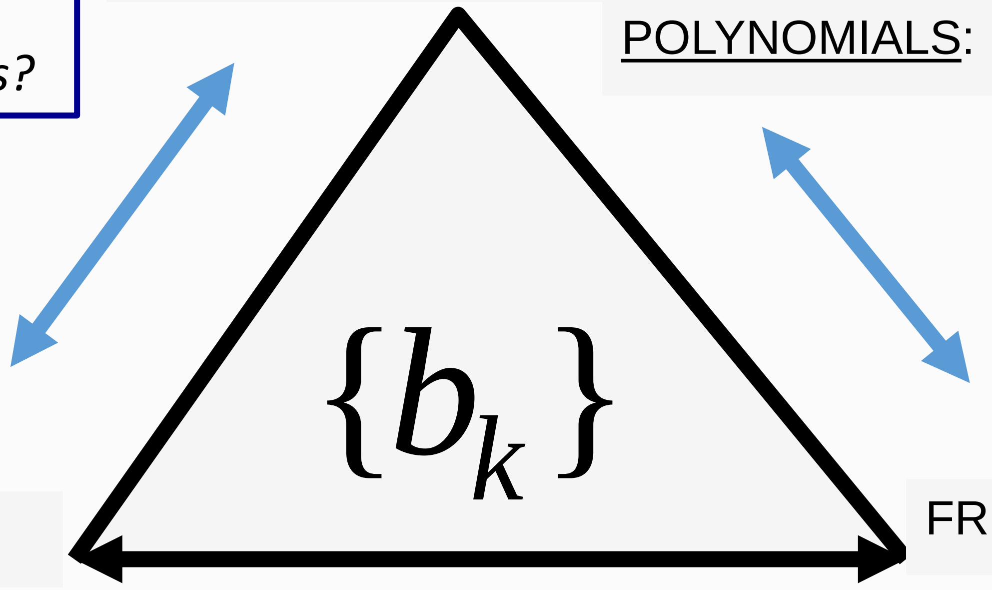
Algebra Domain:
Best of both worlds?

Z-TRANSFORM-DOMAIN

<u>POLYNOMIALS</u>: H(z)



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Z-Transform DEFINITION

- POLYNOMIAL Representation of LTI SYSTEM:

- EXAMPLE:

$$\boxed{H(z) = \sum_{n} h[n] z^{-n}}$$

APPLIES to any SIGNAL

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$$H(z) = 2z^{-0} + 0z^{-1} - 3z^{-2} + 0z^{-3} + 2z^{-4}$$

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$$= 2 - 3z^{-2} + 2z^{-4}$$

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$$\{h[n]\} = \{\,2, 0, -3, 0, 2\,\}$$

$$\begin{aligned} H(z) &= 2z^{-0} + 0z^{-1} - 3z^{-2} + 0z^{-3} + 2z^{-4} \\ &= 2 - 3z^{-2} + 2z^{-4} \\ &= 2 - 3(z^{-1})^2 + 2(z^{-1})^4 \end{aligned}$$

POLYNOMIAL in z^{-1}

Transfer Fonksiyonu

$y[n]-y[n-1]=x[n]+2x[n-1]$ sisteminin transfer fonksiyonunu bulunuz.

Fark denkleminde her iki tarafın z-dönüşümü alındığında

$$Y(z)-z^{-1}Y(z)=X(z)+2z^{-1}X(z)$$

elde edilmektedir. Sistemin transfer fonksiyonu

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+2z^{-1}}{1-z^{-1}}$$

olarak bulunmaktadır.

Credit by Sarp Ertürk

Blok Diyagram Gösterimi

$y[n]-4.5y[n-1]+2y[n-2]=x[n]$ şeklinde tanımlanmış sistemin blok diyagramını çizelim.

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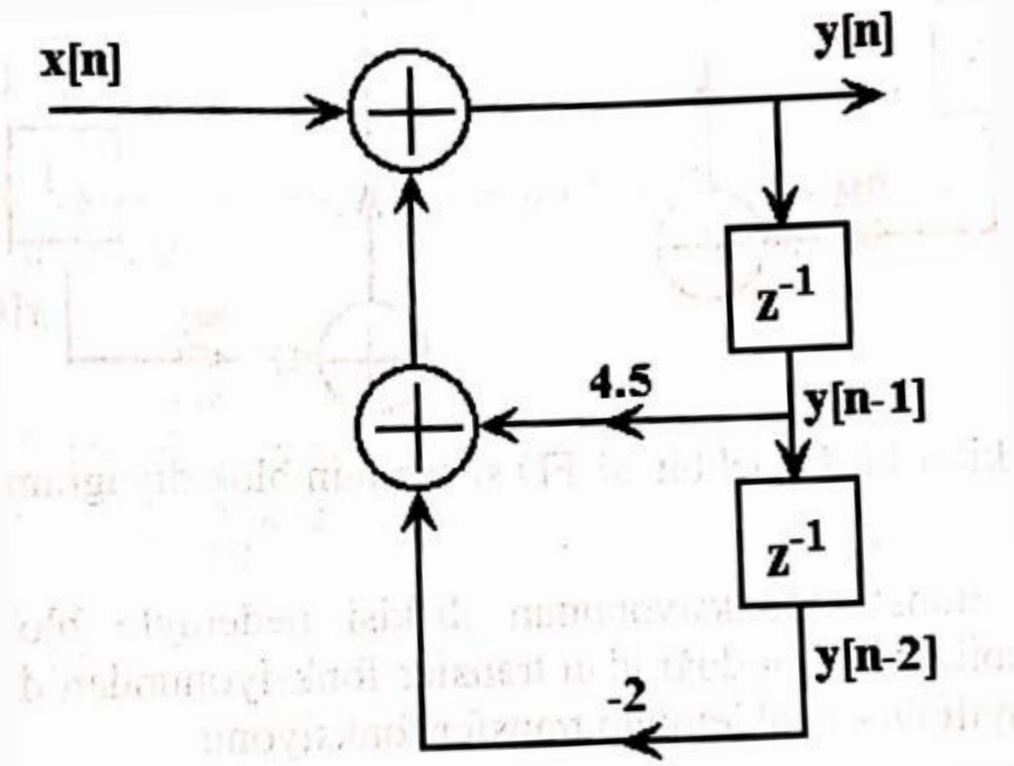

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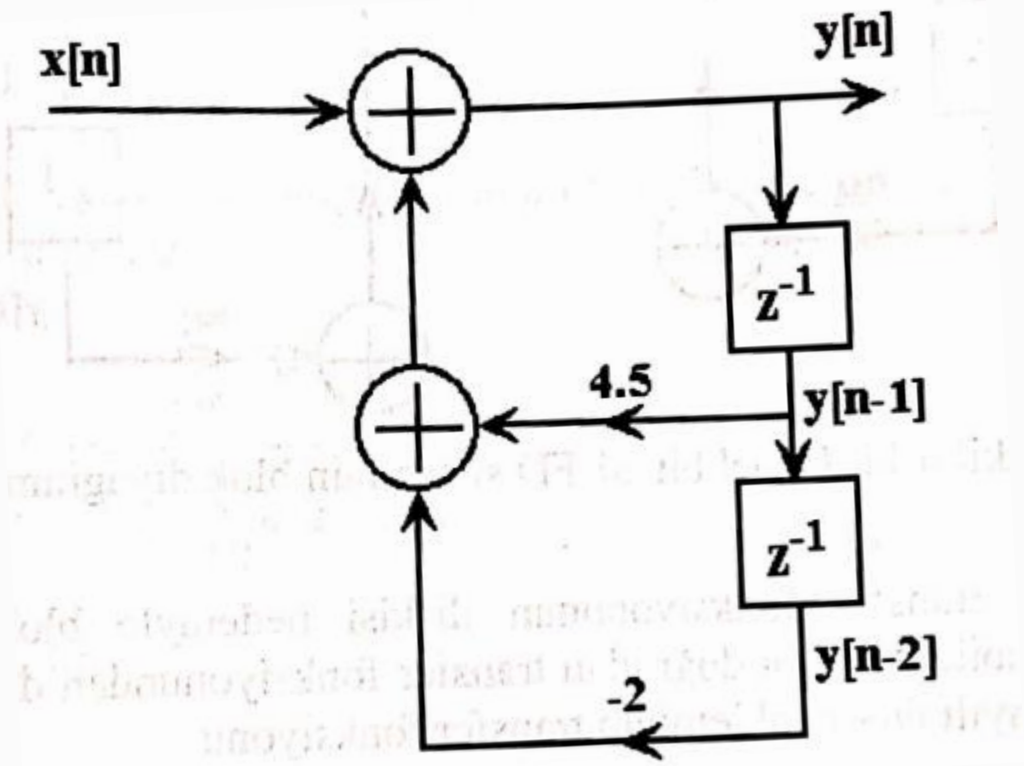
$$H(z)=\frac{1}{1-4.5z^{-1}+2z^{-2}}$$



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Z-Transform EXAMPLE

$$H(z) = \sum_{n} h[n] z^{-n}$$

- ANY SIGNAL has a z-Transform:

$$X(z) = \sum_{n} x[n] z^{-n}$$

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Example 7.1

n	$n < -1$	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	2	4	6	4	2	0	0

$$H(z) = \sum_n h[n] z^{-n}$$

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$$X(z) = ?$$

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Z-Transform EXAMPLE

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Example 9.2

$$X(z) = 1 - 2z^{-1} + 3z^{-3} - z^{-5}$$

EXPONENT GIVES TIME LOCATION

$$x[n] = ?$$

Example 9.2

$$X(z) = 1 - 2z^{-1} + 3z^{-3} - z^{-5}$$

EXPONENT GIVES TIME LOCATION

$$x[n] = ?$$

$$x[n] = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ -2 & n = 1 \\ 0 & n = 2 \\ 3 & n = 3 \\ 0 & n = 4 \\ -1 & n = 5 \\ 0 & n > 5 \end{cases}$$

$$x[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-3] - \delta[n-5]$$

Example 9.2

$$X(z) = 1 - 2z^{-1} + 3z^{-3} - z^{-5}$$

EXPONENT GIVES TIME LOCATION

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$$x[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-3] - \delta[n-5]$$

Z-Transform Property: Delay Property

A delay of one sample multiplies the z-transform by z^{-1}.

$$x[n-1] \quad \Longleftrightarrow \quad z^{-1}X(z)$$

Time delay of n_0 *samples multiplies the z-transform by* z^{-n_0}

$$x[n-n_0] \quad \Longleftrightarrow \quad z^{-n_0}X(z)$$

Example:

Find z-transform of shifted impulse $x[n] = 3\delta[n-5]$.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

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Fourier transform?

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Fourier transform?

$$X\left(e^{j\Omega}\right) = X[z]\Big|_{z=e^{jw}} = 3e^{-j5w}$$

- CALLED the **SYSTEM FUNCTION**
  - because h[n] is same as $\{b_k\}$

SYSTEM FUNCTION

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} = \sum_{k=0}^{M} h[k] z^{-k}$$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

CONVOLUTION

FIR DIFFERENCE EQUATION

Z-Transform of FIR Filter

- Get H(z) DIRECTLY from the $\{b_k\}$
- Example 7.3 in the book:

$$y[n] = 6x[n] - 5x[n-1] + x[n-2]$$

$$\{b_k\} = \{6, -5, 1\}$$

$$\boxed{H(z) = \sum b_k z^{-1} = 6 - 5z^{-1} + z^{-2}}$$

GENERAL I/O PROBLEM

- Input is x[n], find y[n]   (for FIR, h[n])
- How to combine X(z) and H(z) ?

Example 7.5

$$x[n] = \delta[n-1] - \delta[n-2] + \delta[n-3] - \delta[n-4]$$

$$\text{and} \quad h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$$

$$X(z) = 0 + 1z^{-1} - 1z^{-2} + 1z^{-3} - 1z^{-4}$$

$$\text{and} \quad H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

FIR Filter = CONVOLUTION

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

CONVOLUTION

FIR Filter = CONVOLUTION

x[n], X(z)	0	+1	-1	+1	-1			
h[n], H(z)	1	2	3	4				
	0	+1	-1	+1	-1			
		0	+2	-2	+2	-2		
			0	+3	-3	+3	-3	
				0	+4	-4	+4	-4
y[n], Y(z)	0	+1	+1	+2	+2	-3	+1	-4

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

CONVOLUTION

CONVOLUTION PROPERTY

- PROOF:

$$y[n] = x[n] * h[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

$$Y(z) = \sum_{k=0}^{M} h[k]\left(z^{-k}X(z)\right)$$

$$= \left(\sum_{k=0}^{M} h[k]z^{-k}\right)X(z) = H(z)X(z).$$

MULTIPLY
z-TRANSFORMS

CONVOLUTION EXAMPLE

- **MULTIPLY** the z-TRANSFORMS:

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$$\text{and} \quad H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

$$\boxed{y[n] = h[n] * x[n] \quad \leftrightarrow \quad Y(z) = H(z)X(z)}$$

CONVOLUTION EXAMPLE

- Finite-Length input x[n]
- FIR Filter (L=4)

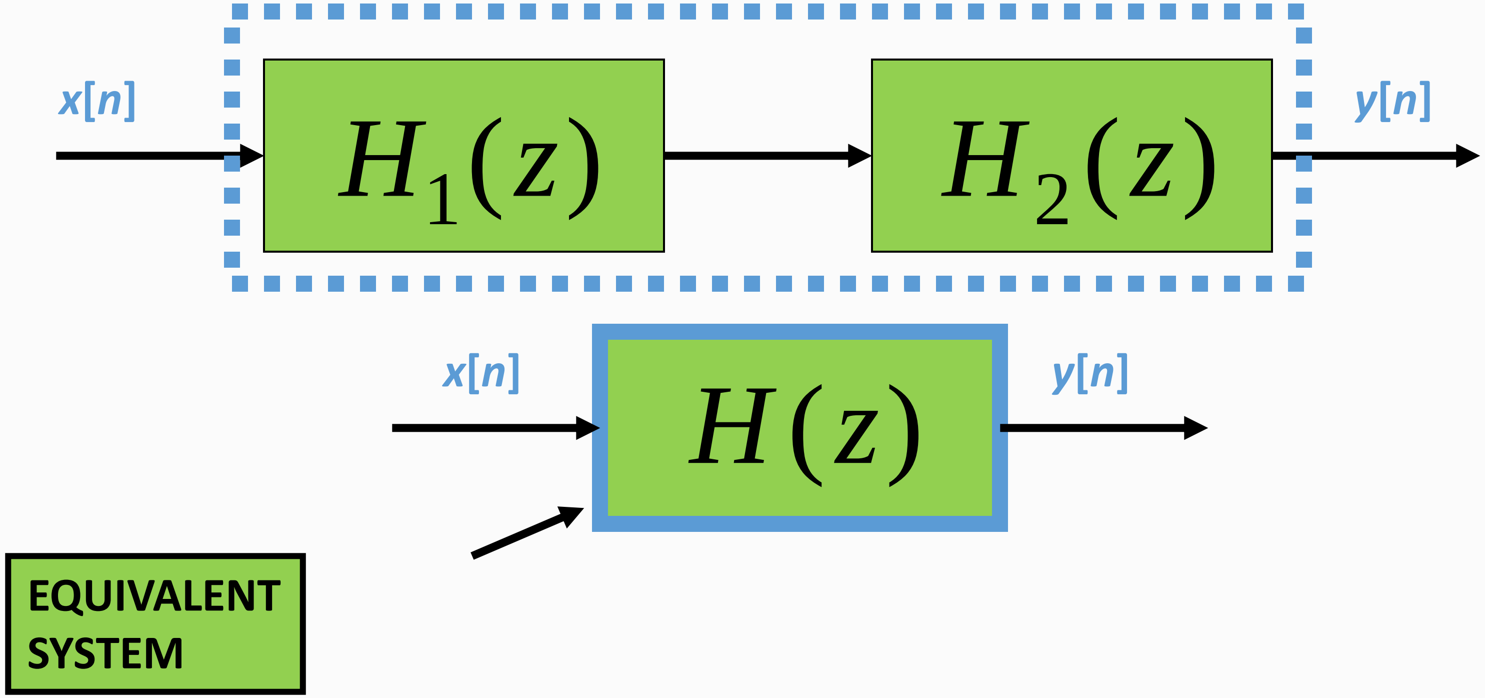
MULTIPLY Z-TRANSFORMS

$$
\begin{aligned}
Y(z) &= H(z)X(z) \\
&= (1 + 2z^{-1} + 3z^{-2} + 4z^{-3})(z^{-1} - z^{-2} + z^{-3} - z^{-4}) \\
&= z^{-1} + (-1 + 2)z^{-2} + (1 - 2 + 3)z^{-3} + (-1 + 2 - 3 + 4)z^{-4} \\
&\quad + (-2 + 3 - 4)z^{-5} + (-3 + 4)z^{-6} + (-4)z^{-7} \\
&= z^{-1} + z^{-2} + 2z^{-3} + 2z^{-4} - 3z^{-5} + z^{-6} - 4z^{-7}
\end{aligned}
$$

***y*[*n*] = ?**

Z-Transform Property: CASCADE EQUIVALENT

- Multiply the System Functions



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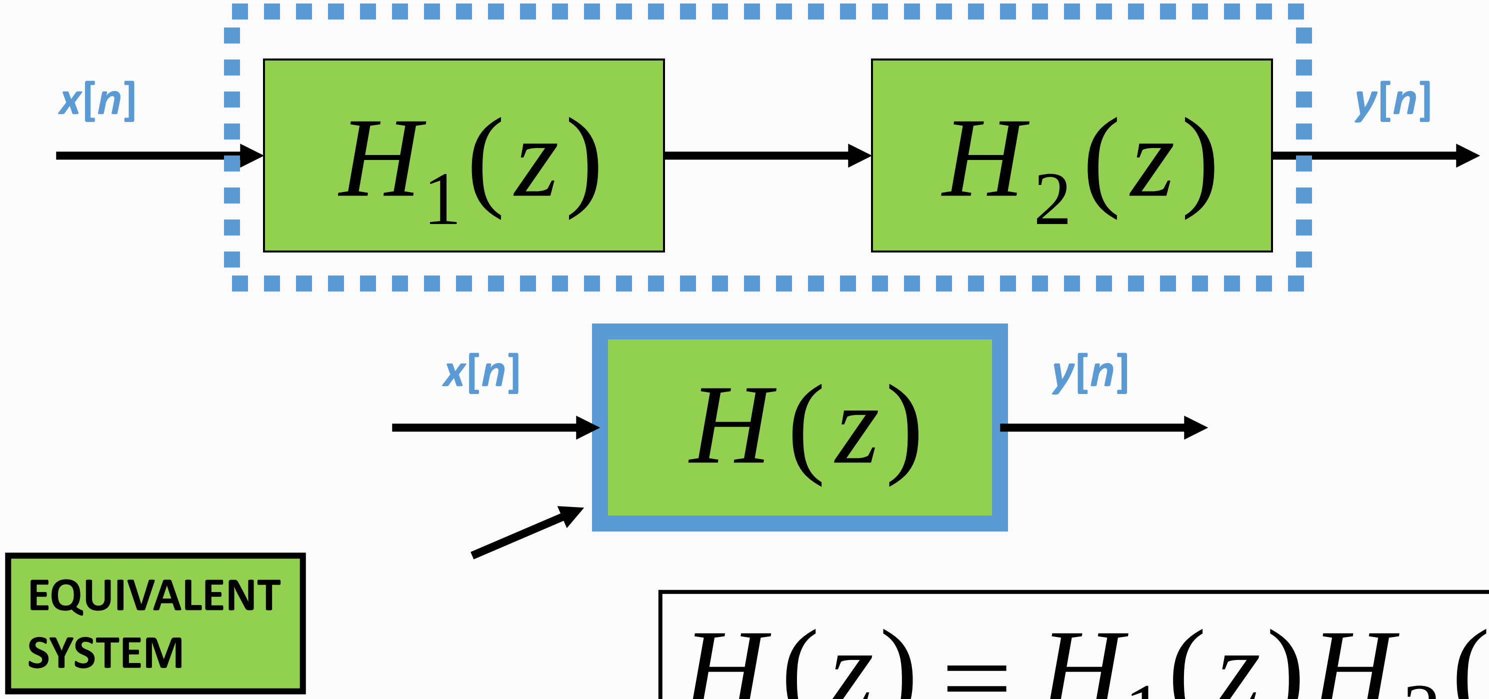
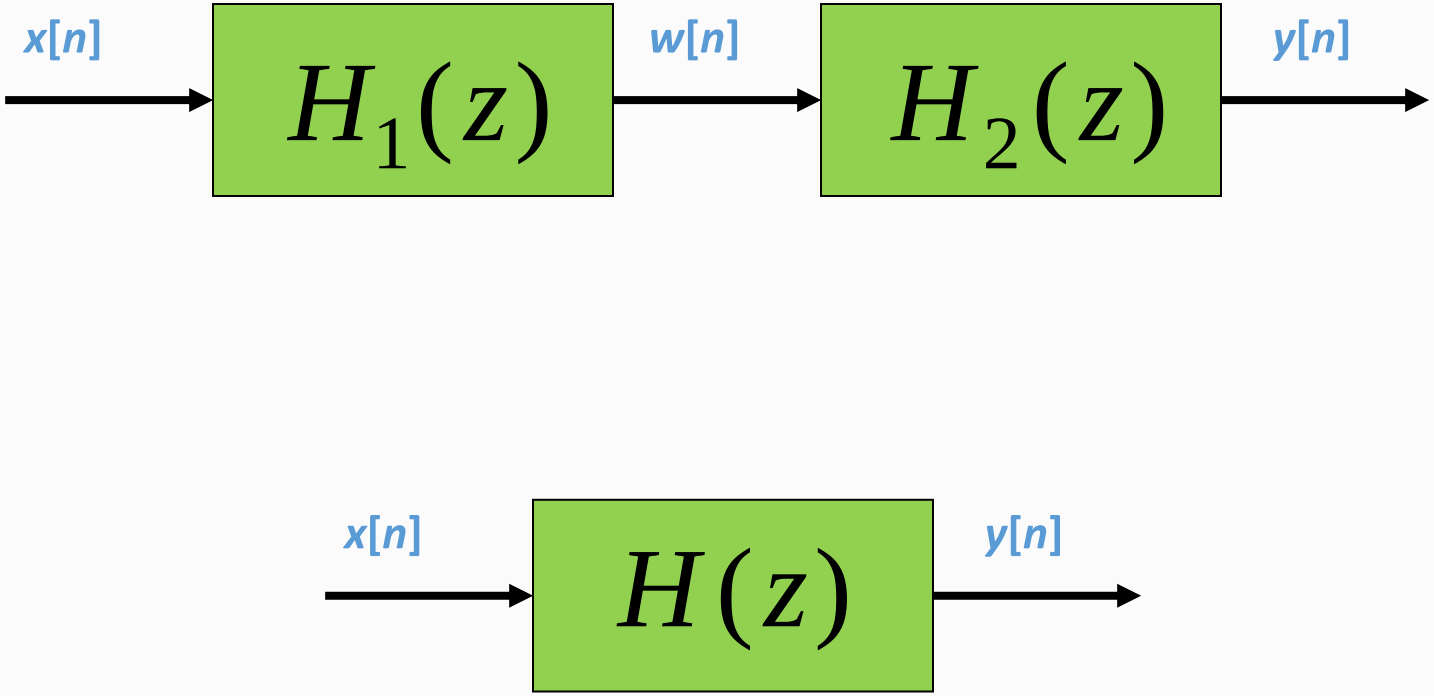
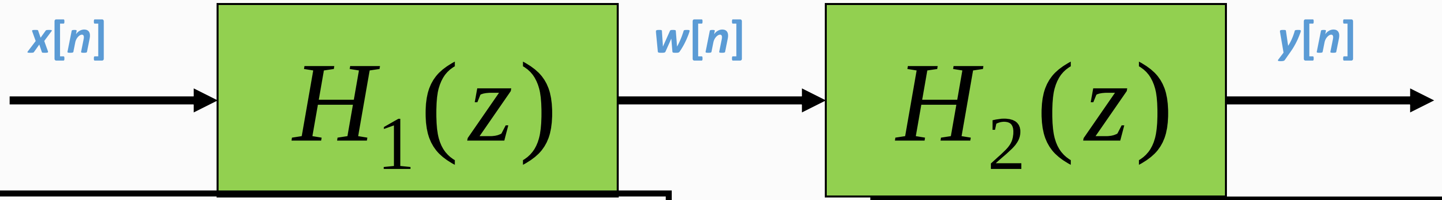


$$H(z) = H_1(z)H_2(z)$$

CASCADE EXAMPLE

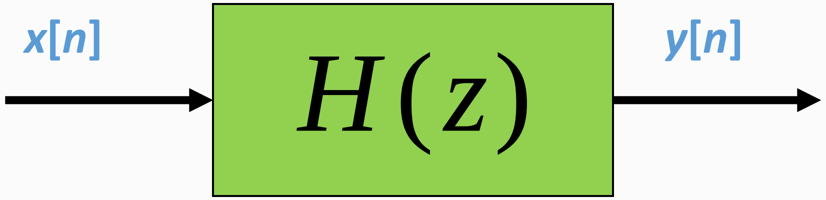


CASCADE EXAMPLE

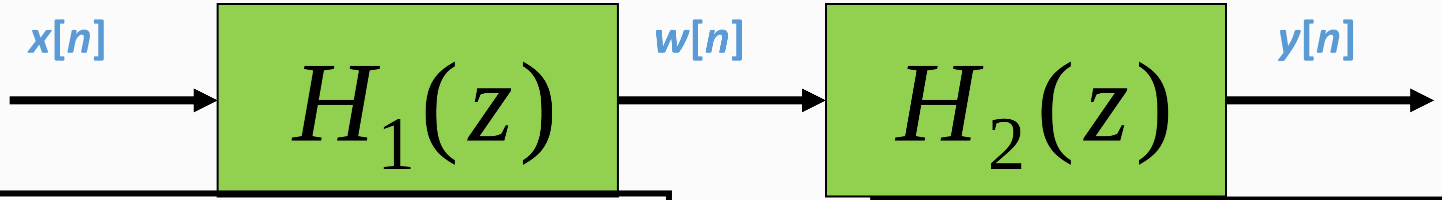


$$\boxed{w[n] = x[n] - x[n-1]}$$

$$\boxed{y[n] = w[n] + w[n-1]}$$



CASCADE EXAMPLE

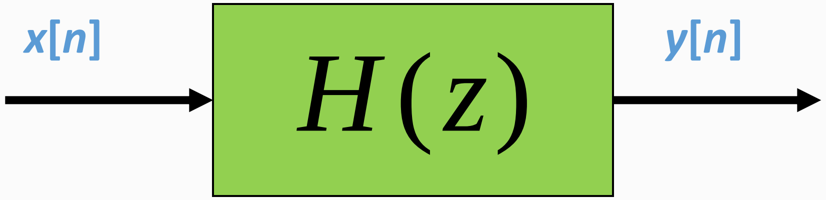


$$w[n] = x[n] - x[n-1]$$

$$H_1(z) = 1 - z^{-1}$$

$$y[n] = w[n] + w[n-1]$$

$$H_2(z) = 1 + z^{-1}$$



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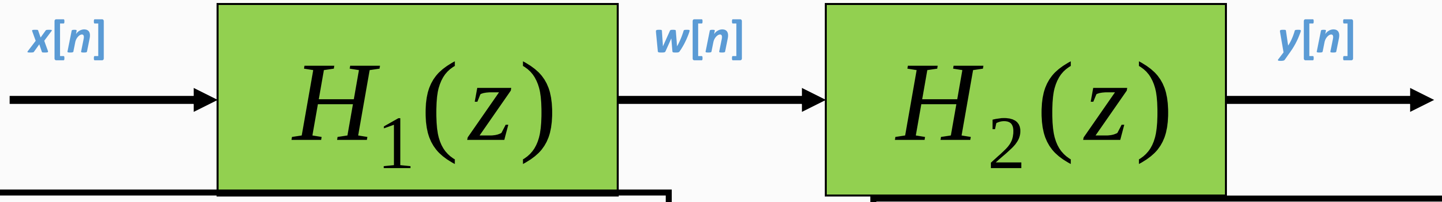

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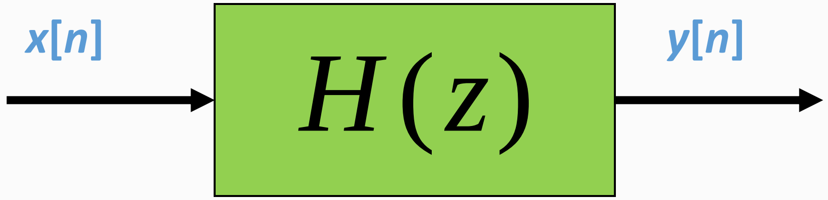
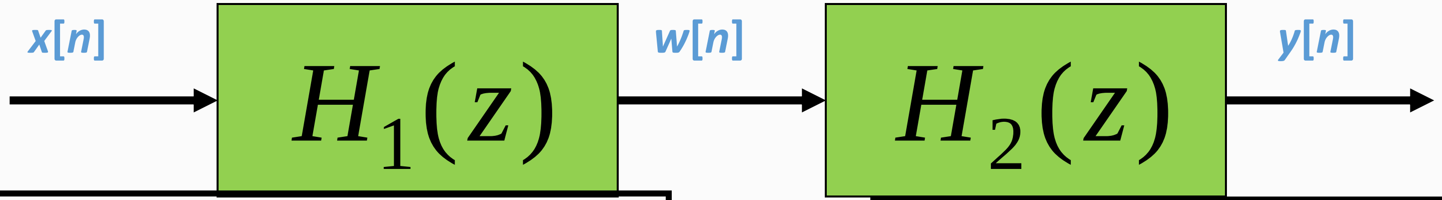


$$H(z) = (1 - z^{-1})(1 + z^{-1}) = 1 - z^{-2}$$

CASCADE EXAMPLE



$$\boxed{w[n] = x[n] - x[n-1]}$$

$$\boxed{y[n] = w[n] + w[n-1]}$$

$$\boxed{H_1(z) = 1 - z^{-1}}$$

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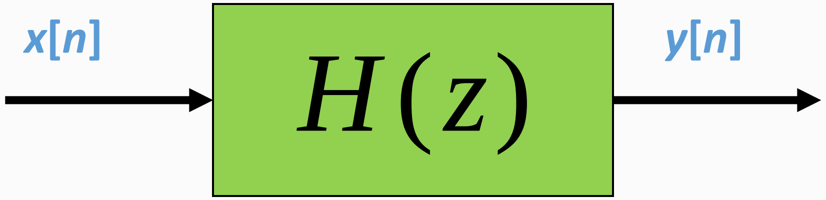

$$\boxed{H(z) = (1 - z^{-1})(1 + z^{-1}) = 1 - z^{-2}}$$

$$\boxed{y[n] = x[n] - x[n-2]}$$

Region of Convergence (ROC)

- Give a sequence, the set of values of z for which the z-transform converges, i.e., $|X(z)|<\infty$, is called the region of convergence.

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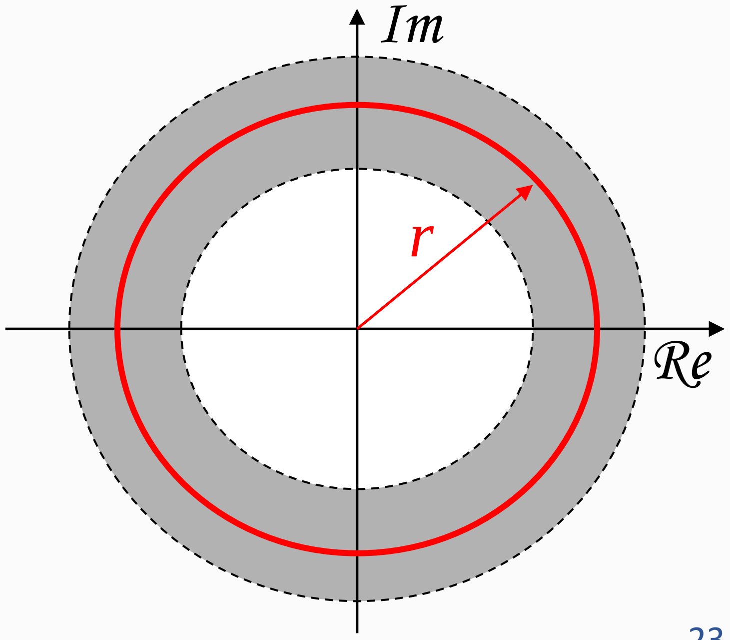
$$|X(z)| = \left| \sum_{n=-\infty}^{\infty} x(n) z^{-n} \right| = \sum_{n=-\infty}^{\infty} |x(n)|\,|z|^{-n} < \infty$$

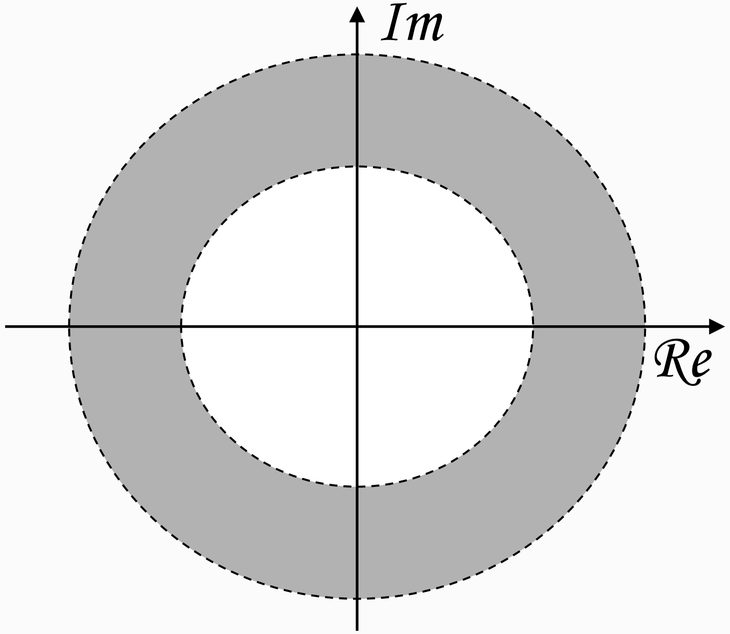
ROC is centered on origin and consists of a set of rings.

$$ROC = \{ z = re^{j\omega} \mid R_{x-} < r < R_{x+} \}$$



Stable Systems

- A stable system requires that its Fourier transform is uniformly convergent.



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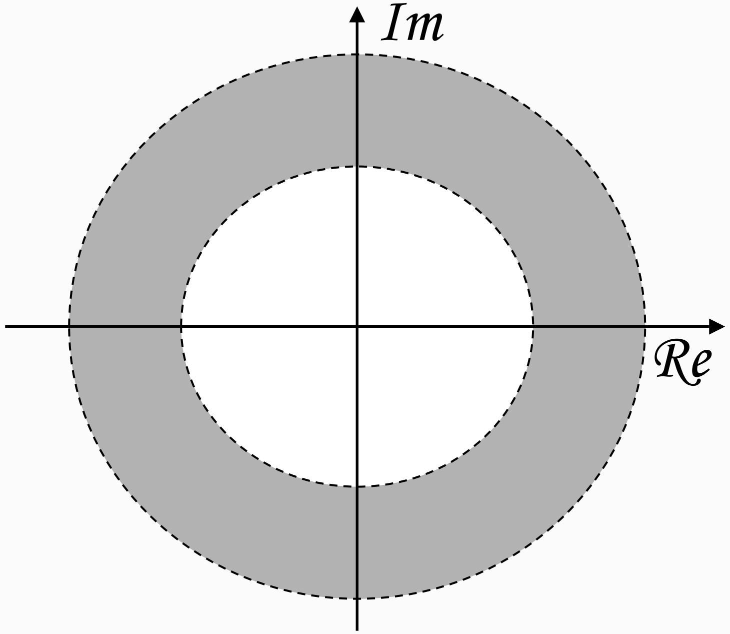

- Fact: Fourier transform is to evaluate *z*-transform on a unit circle.

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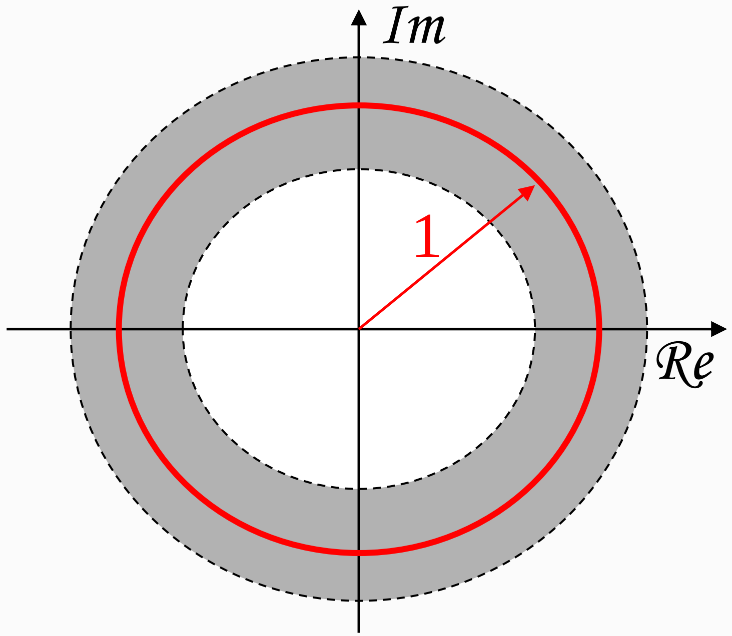

- Fact: Fourier transform is to evaluate z-transform on a unit circle.
- A stable system requires the ROC of z-transform to include the unit circle.

Example: A right sided Sequence

$$\boxed{x(n) = a^n u(n)}$$

$$X(z) = \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n}$$

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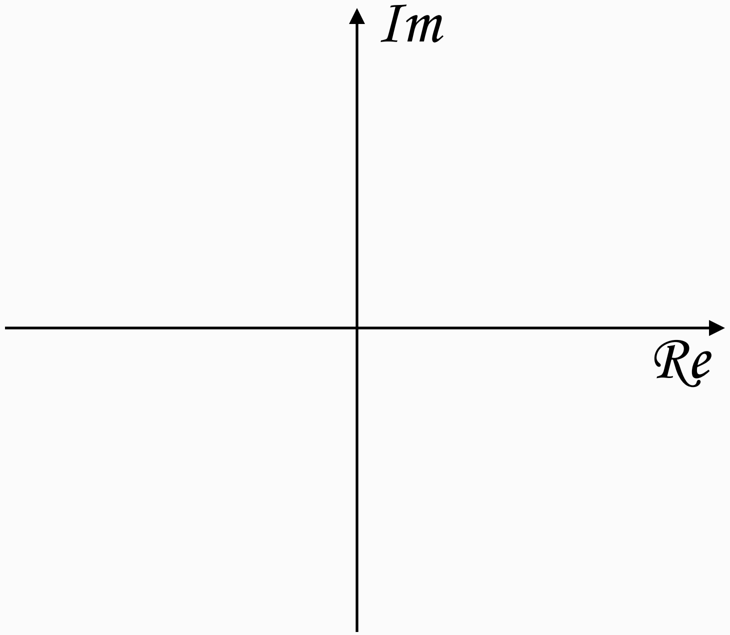

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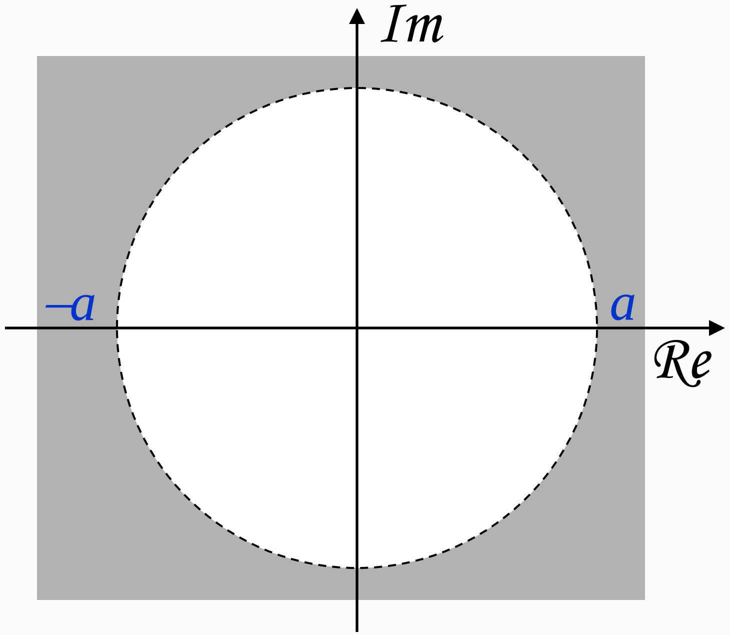

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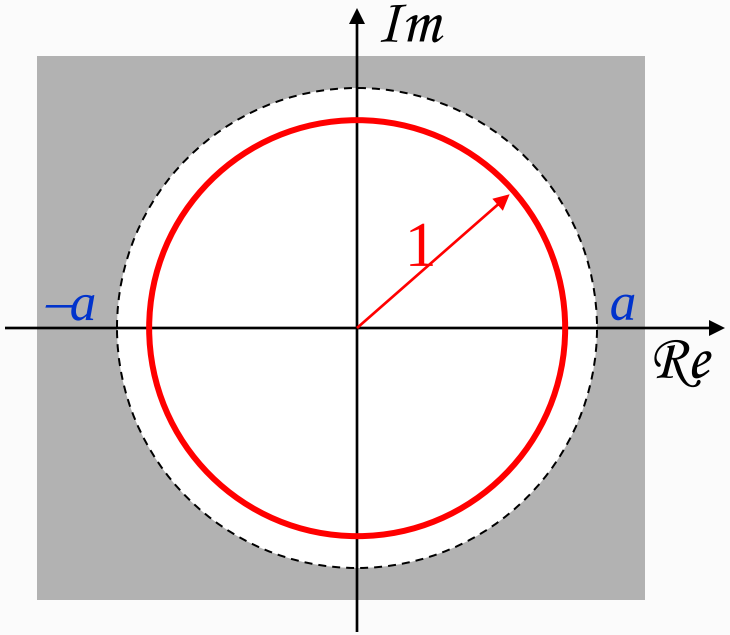

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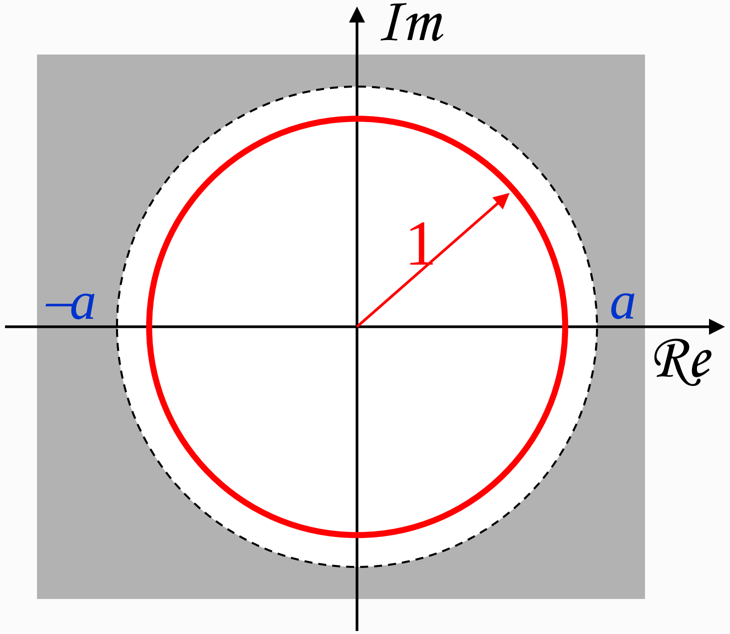

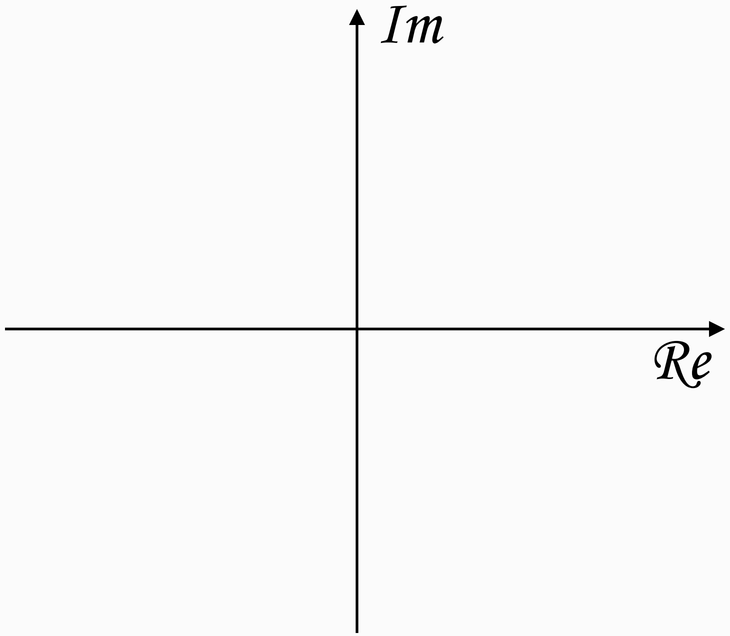

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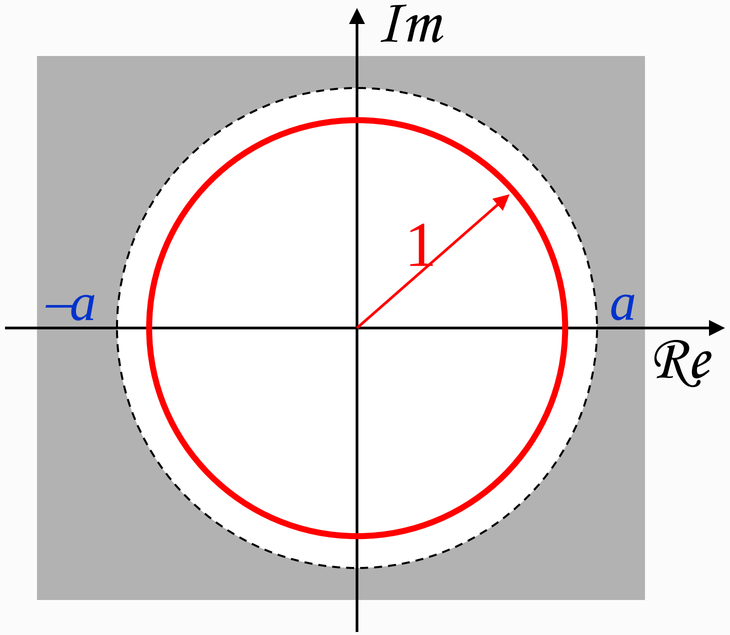

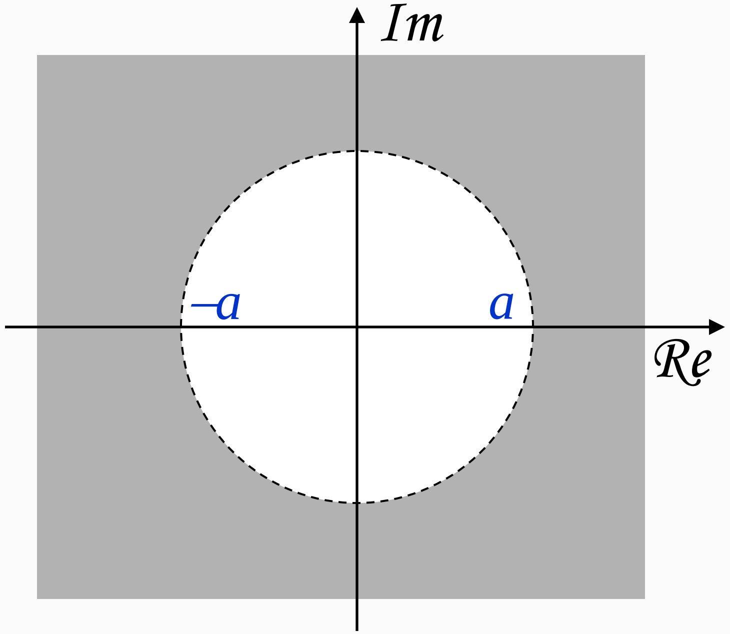

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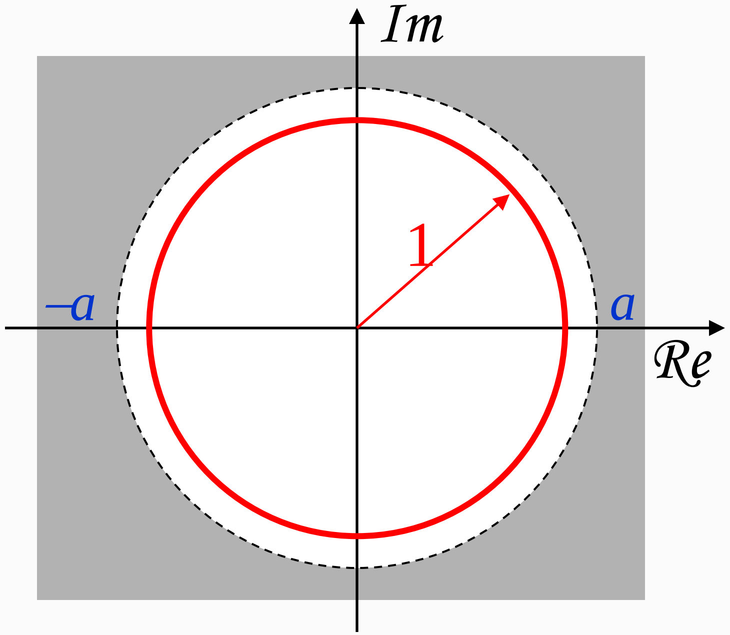

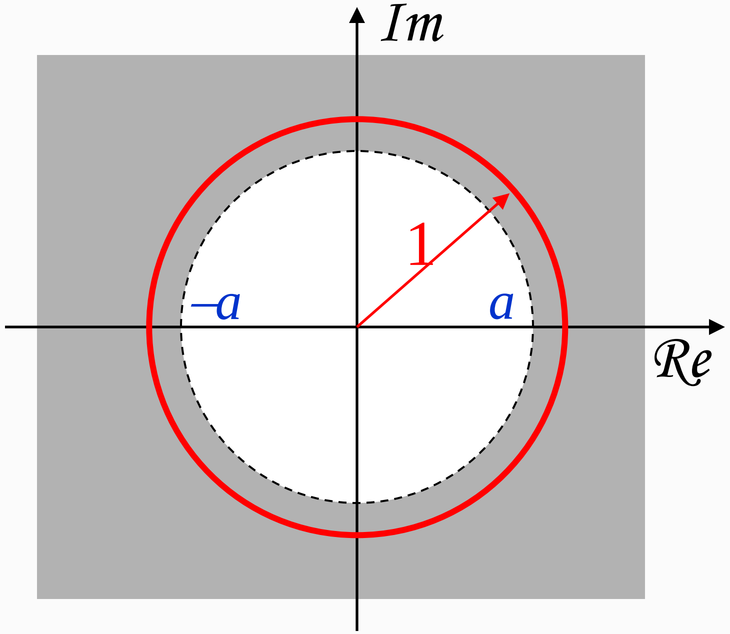

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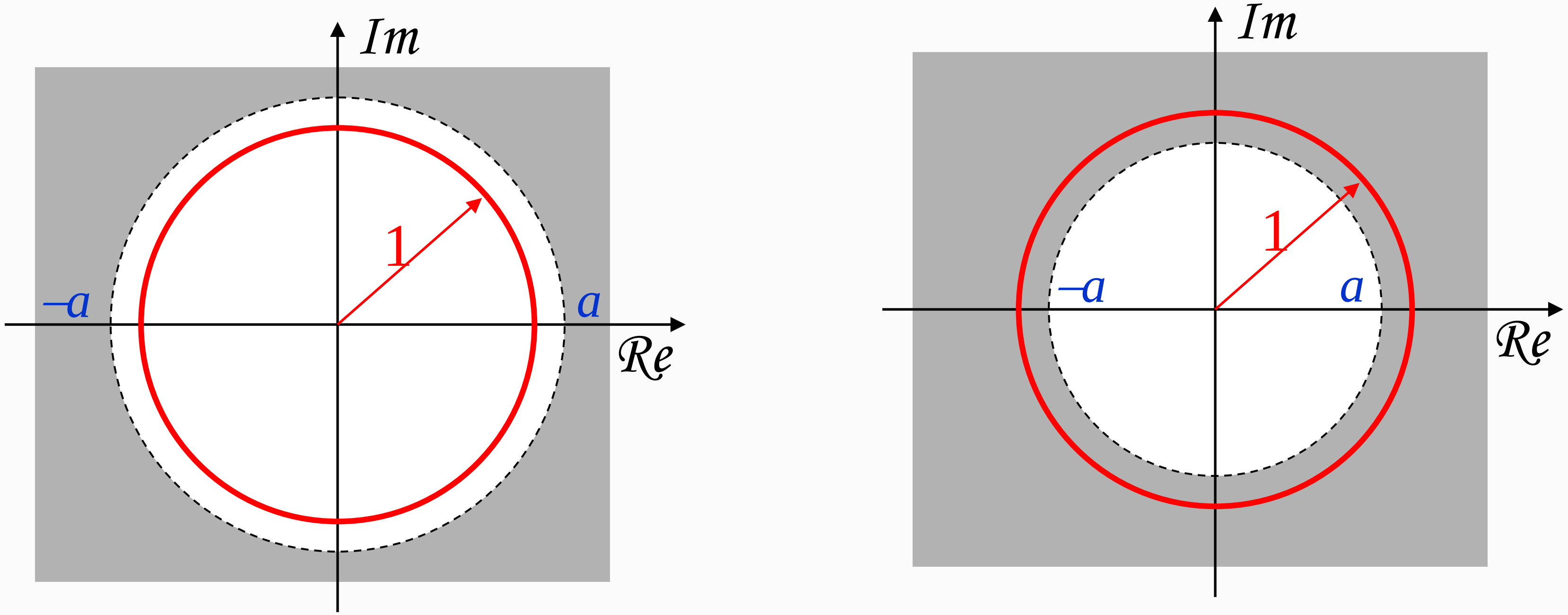
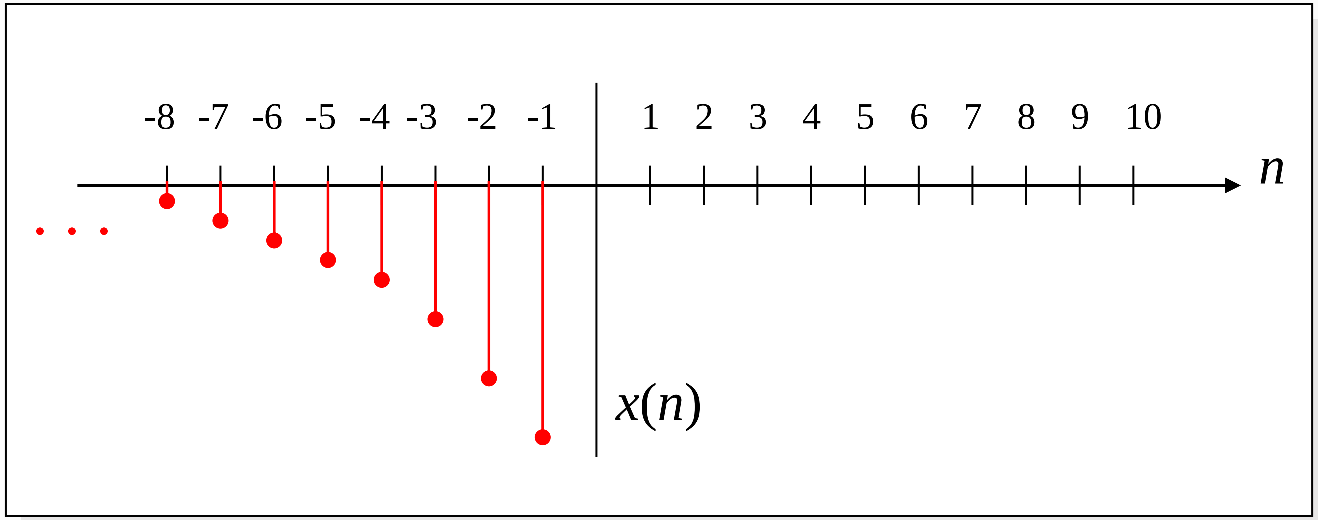


Another Example

$$x(n) = -a^n u(-n-1)$$



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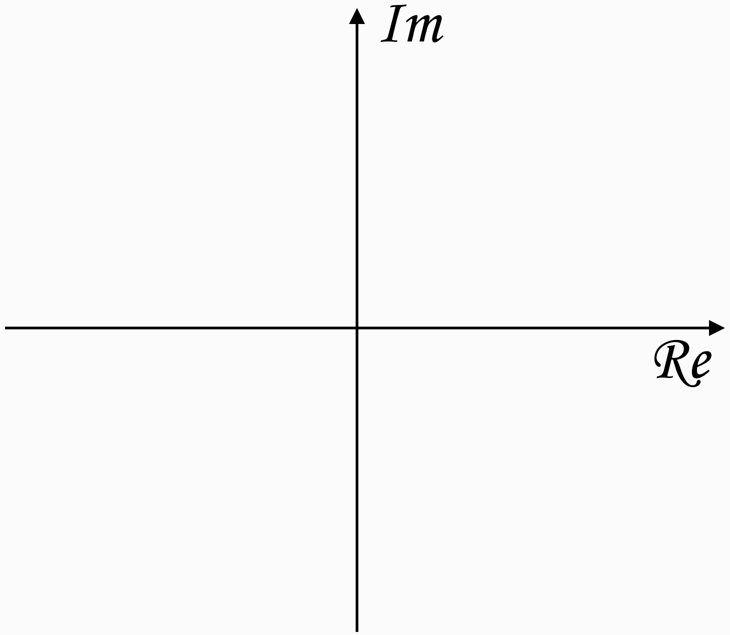

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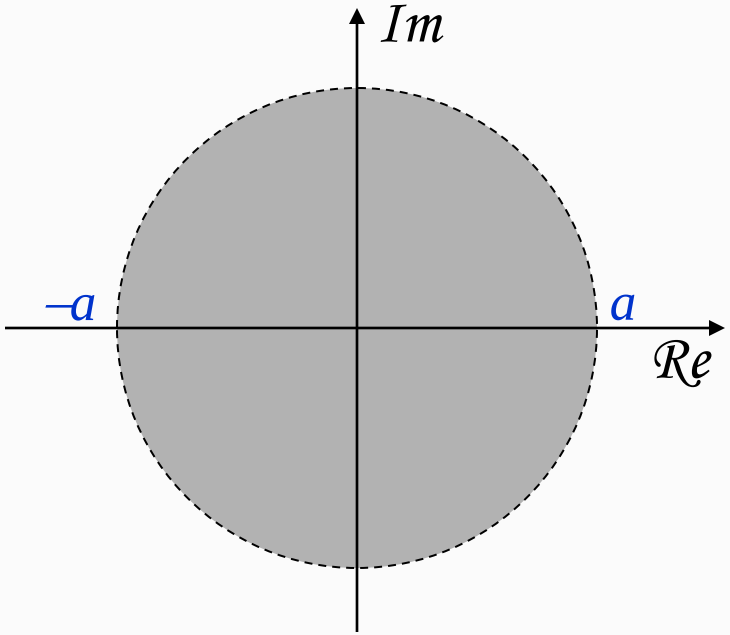

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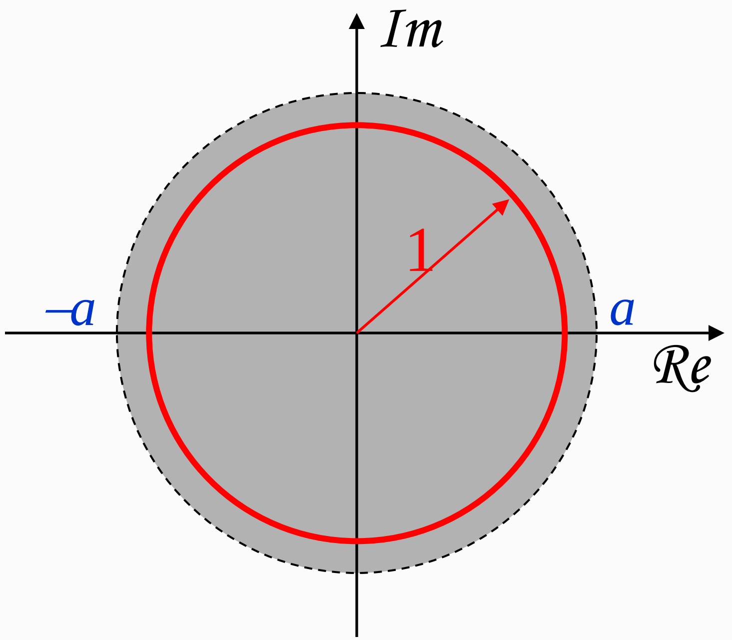

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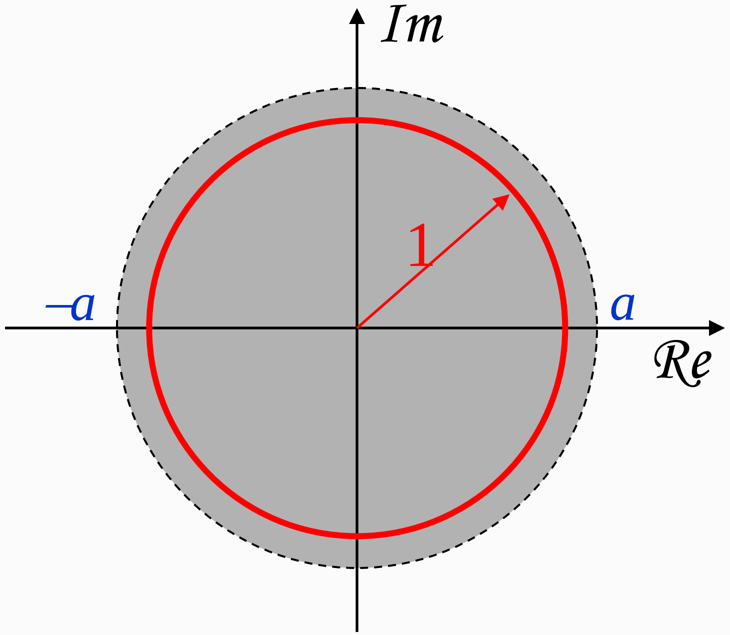

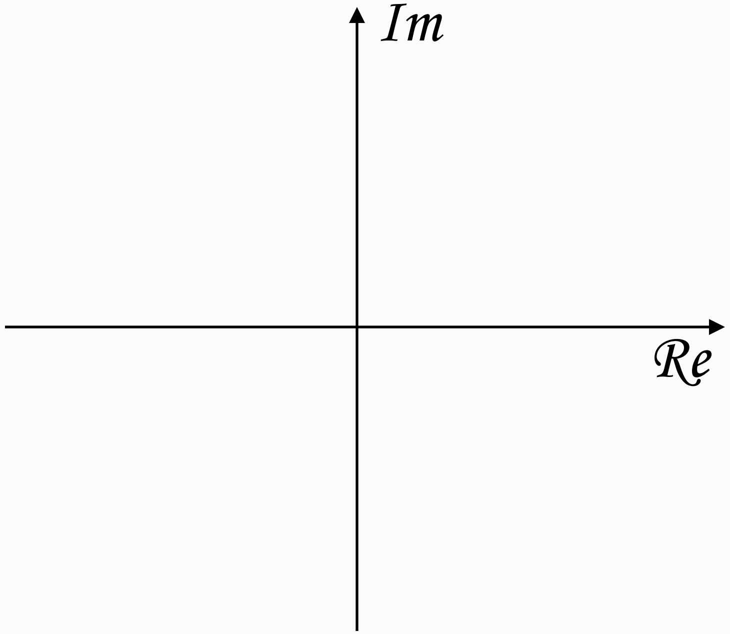

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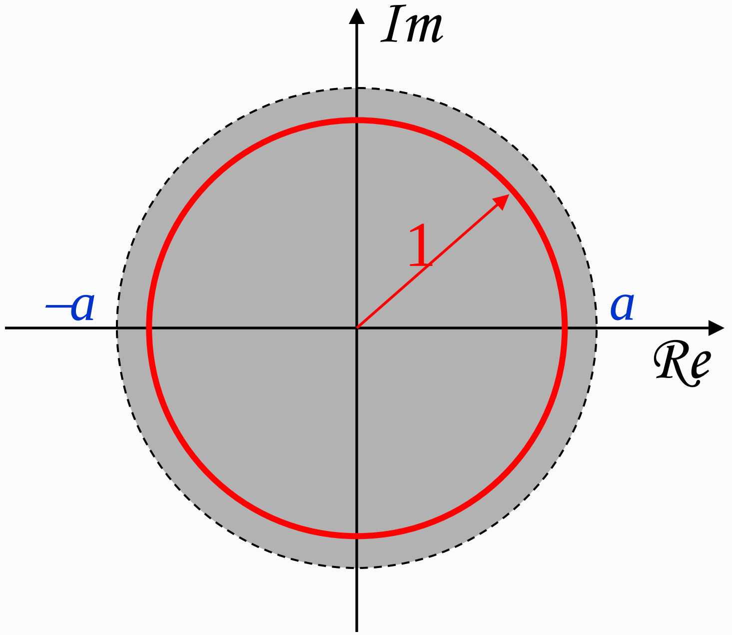

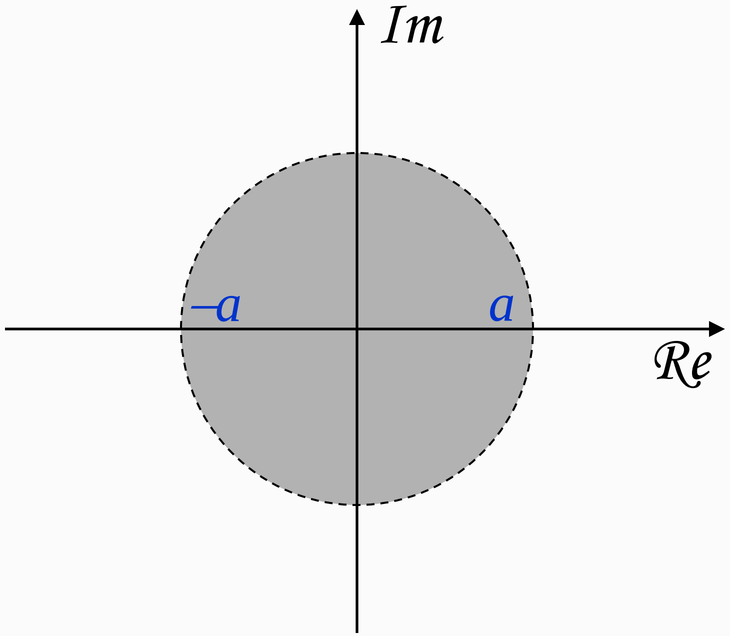

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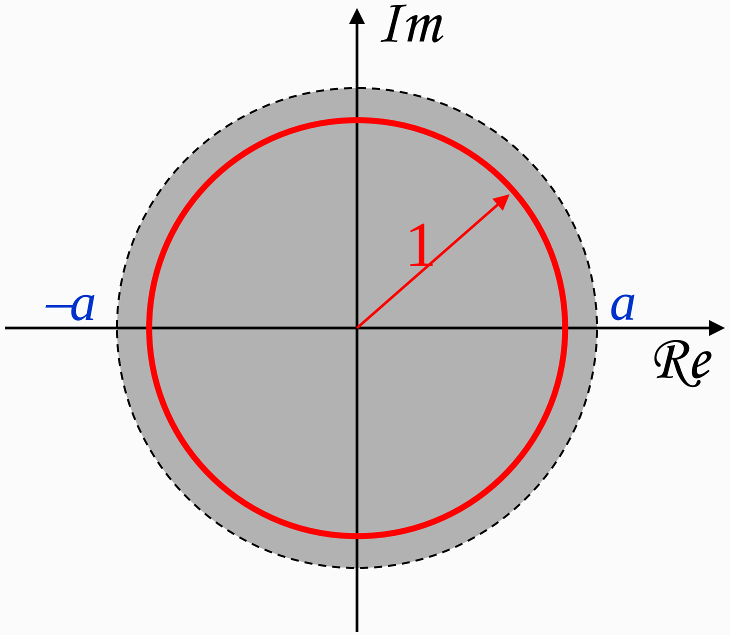

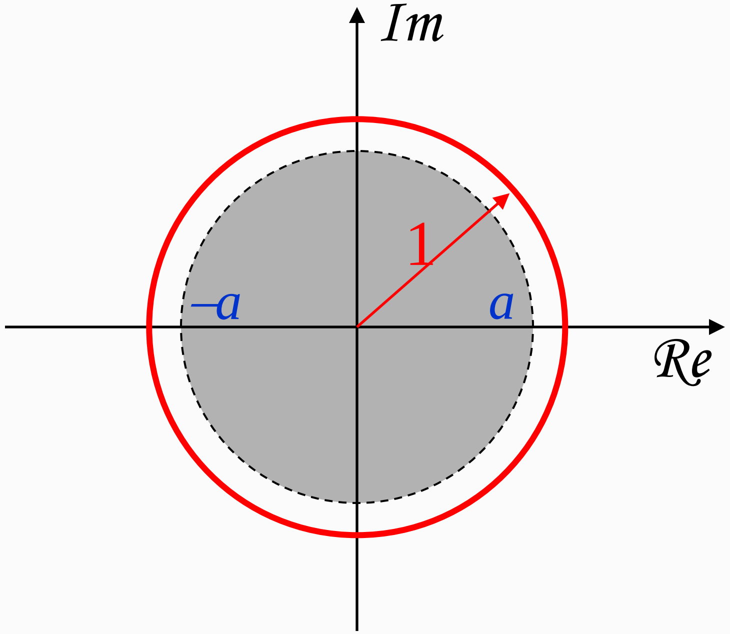

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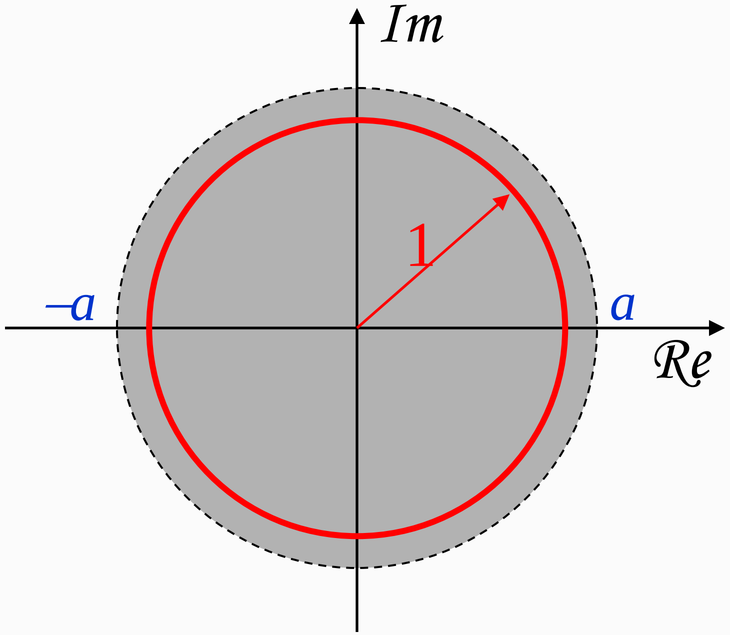

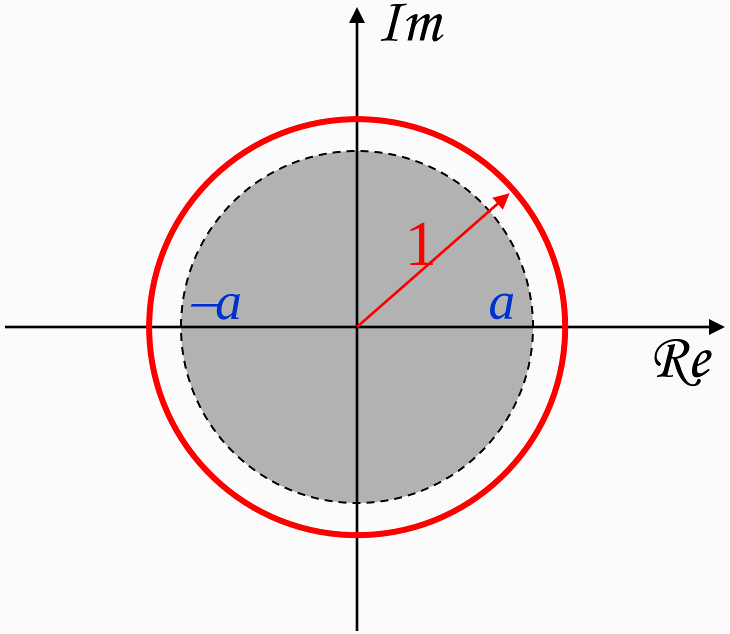

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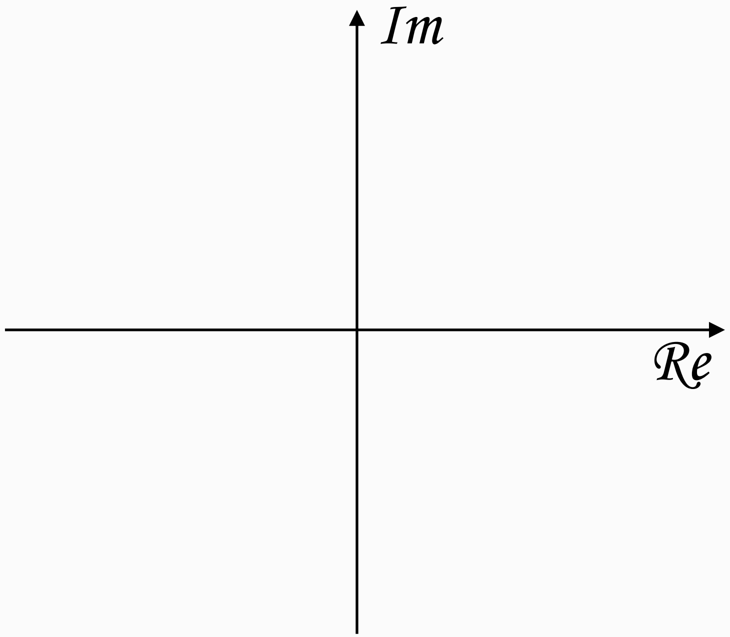

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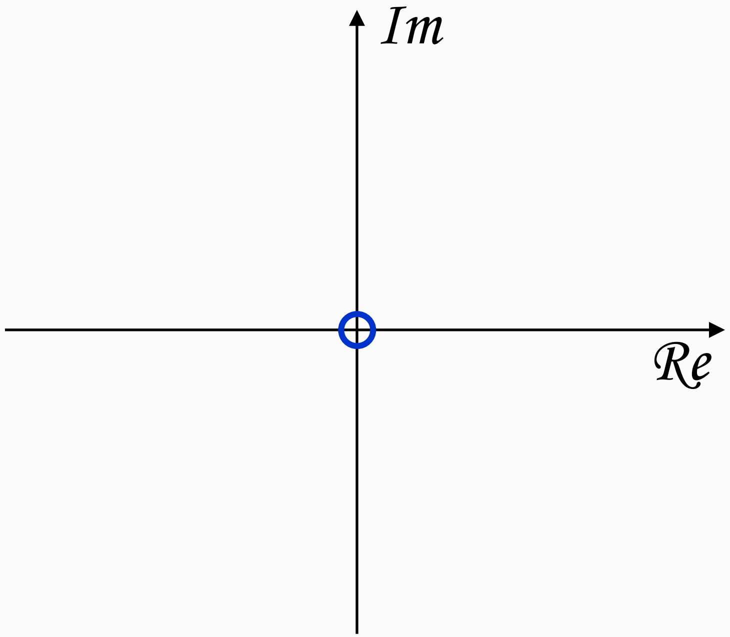

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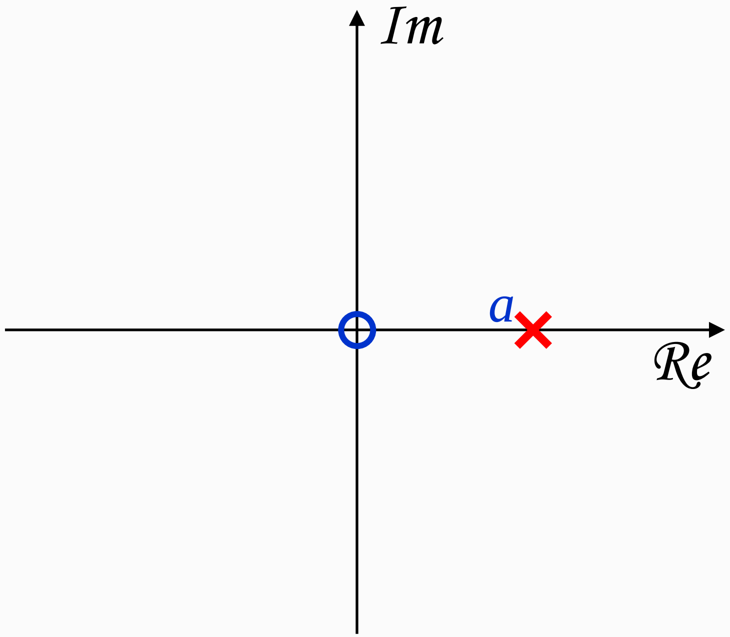

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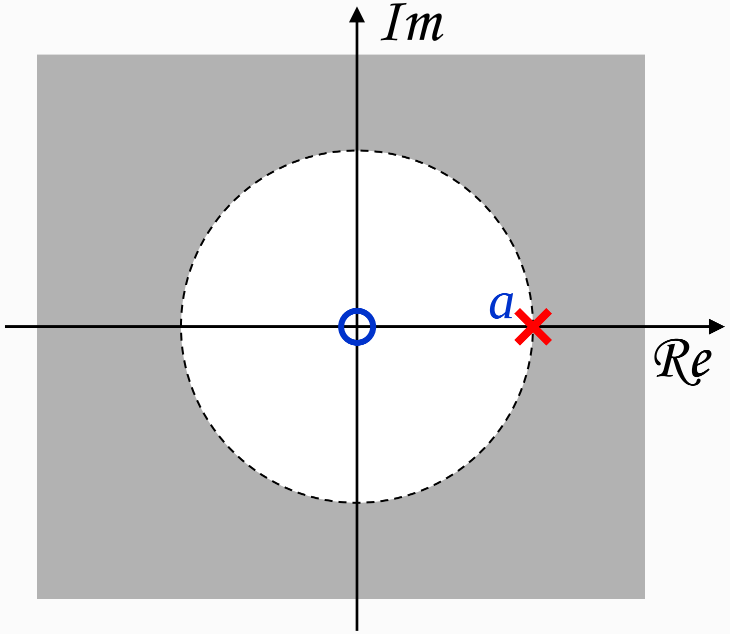

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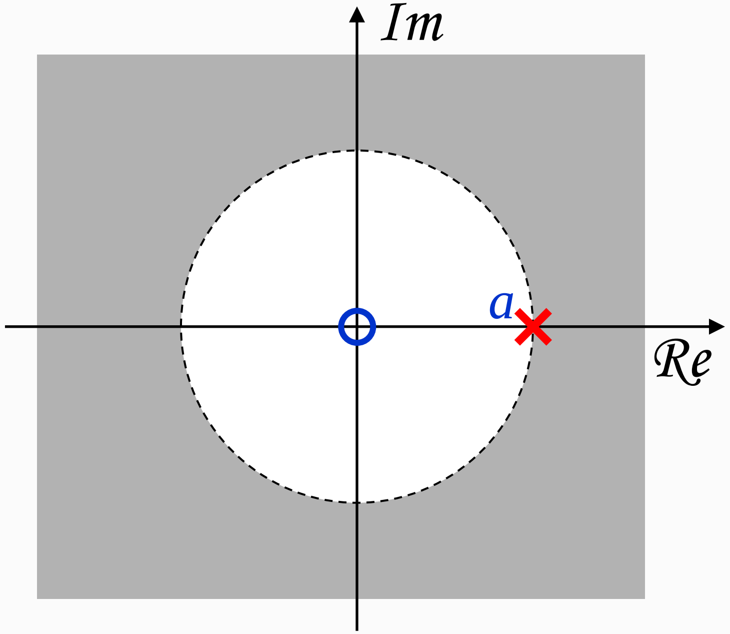

ROC is bounded by the pole and is the exterior of a circle.

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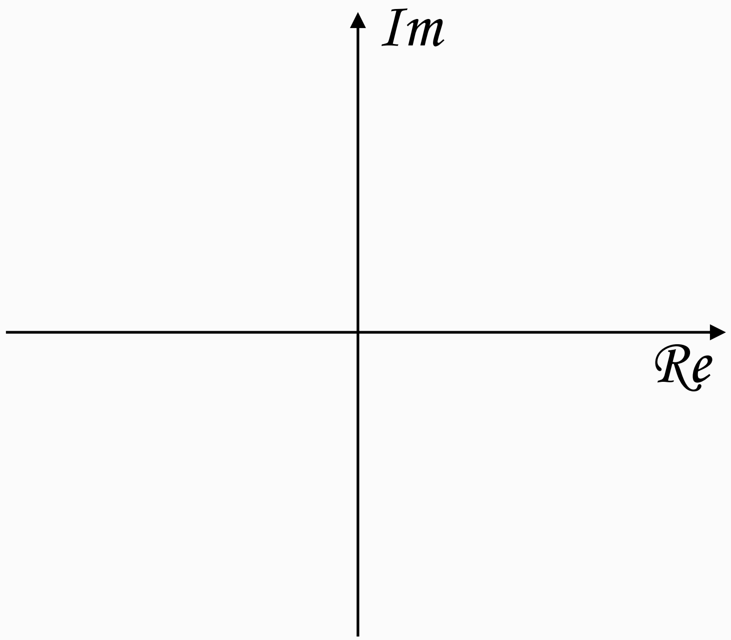

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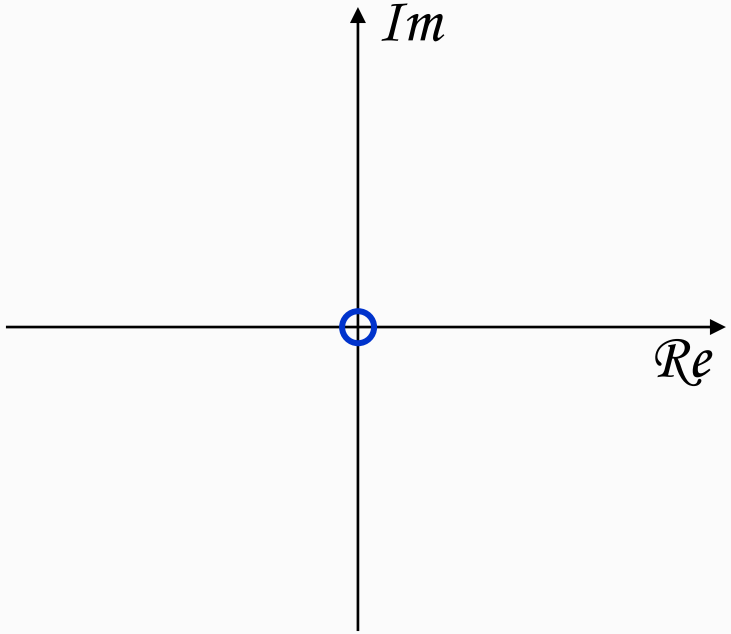

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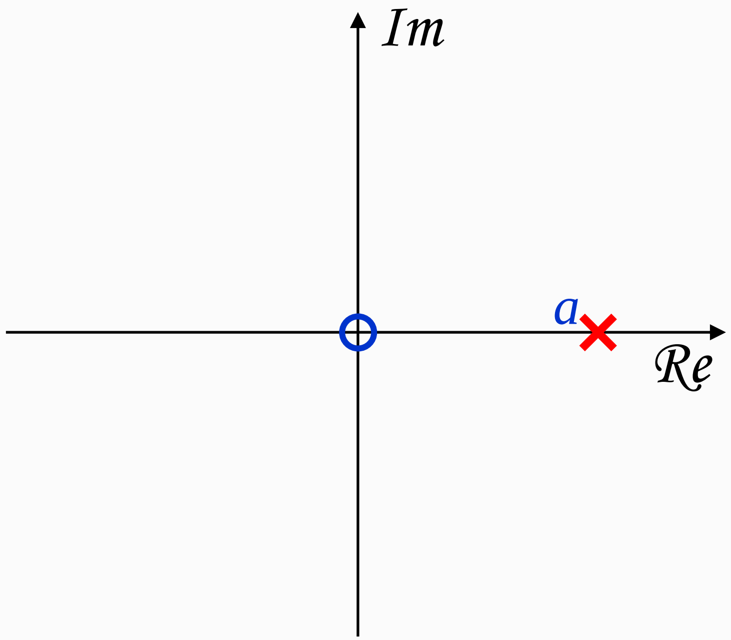

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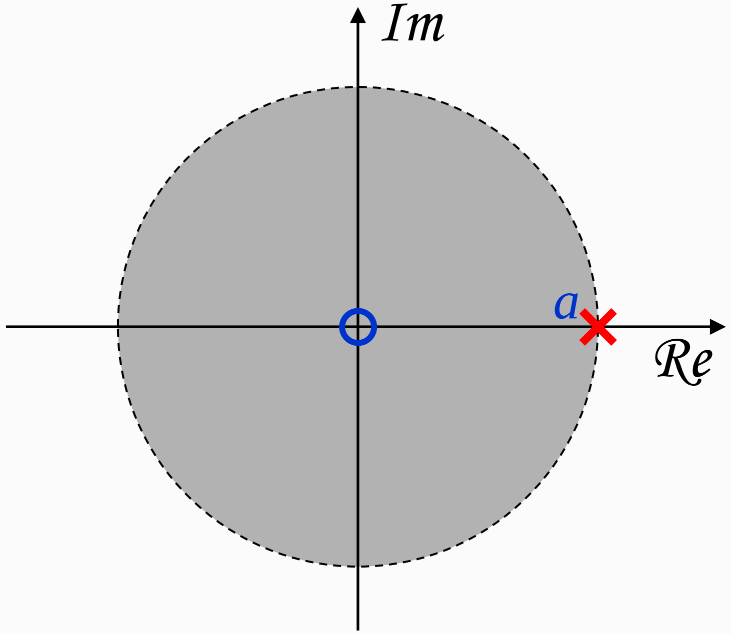

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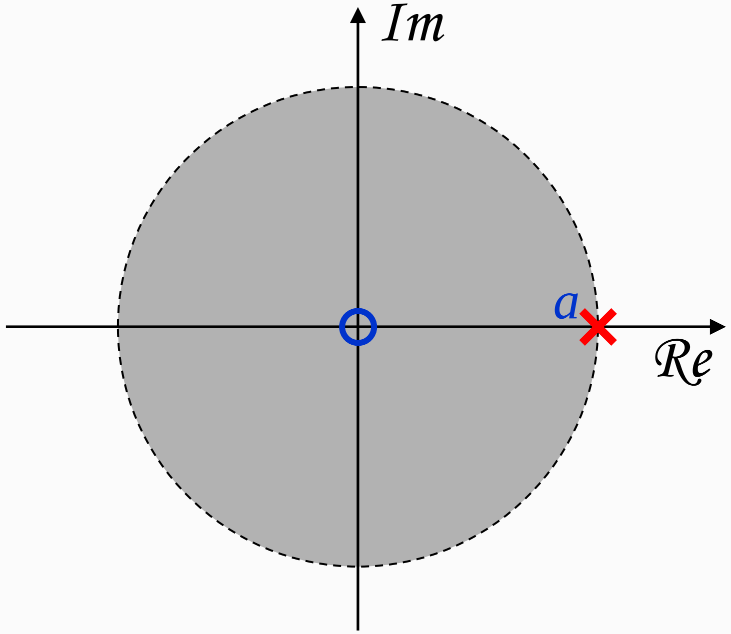

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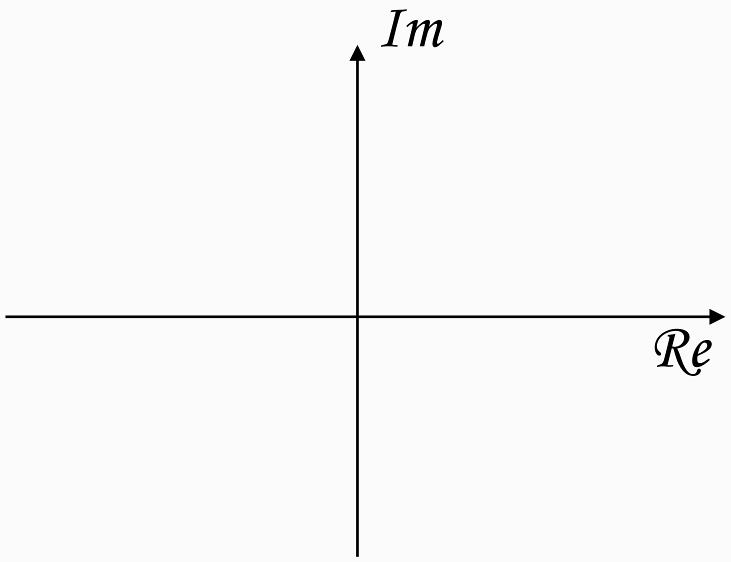

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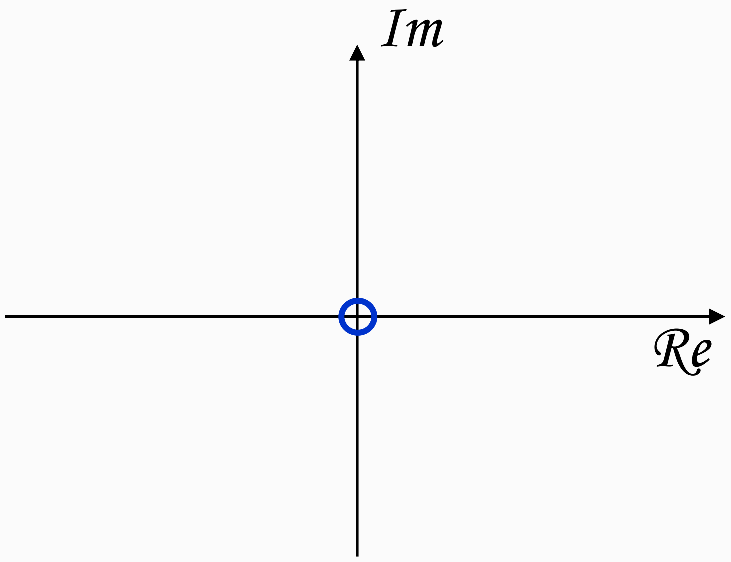

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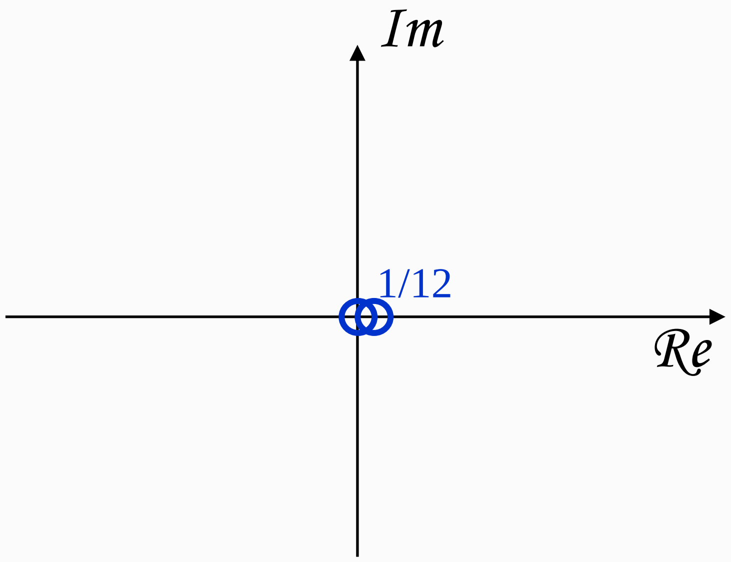

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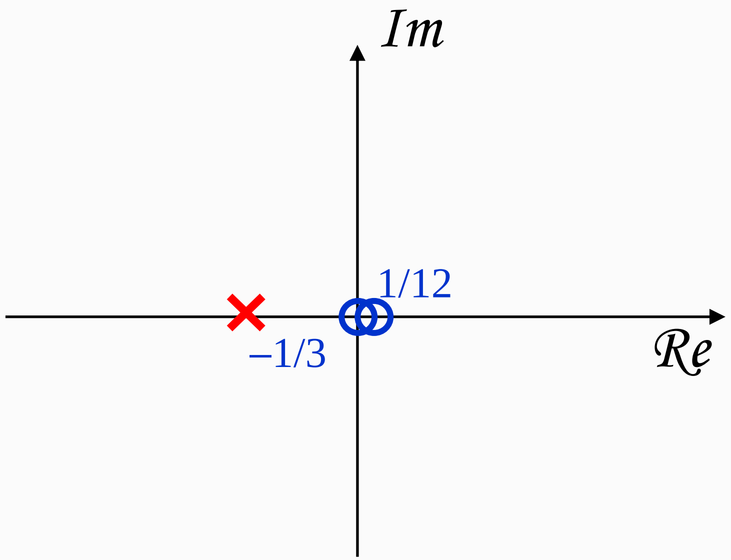

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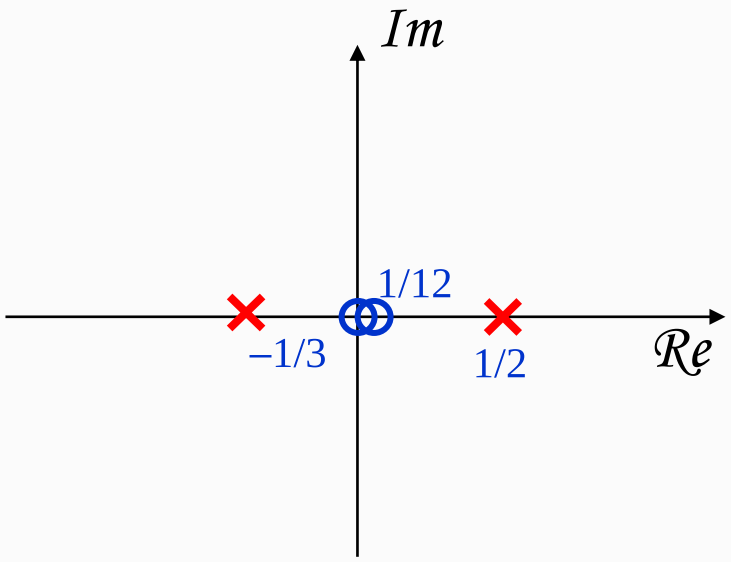

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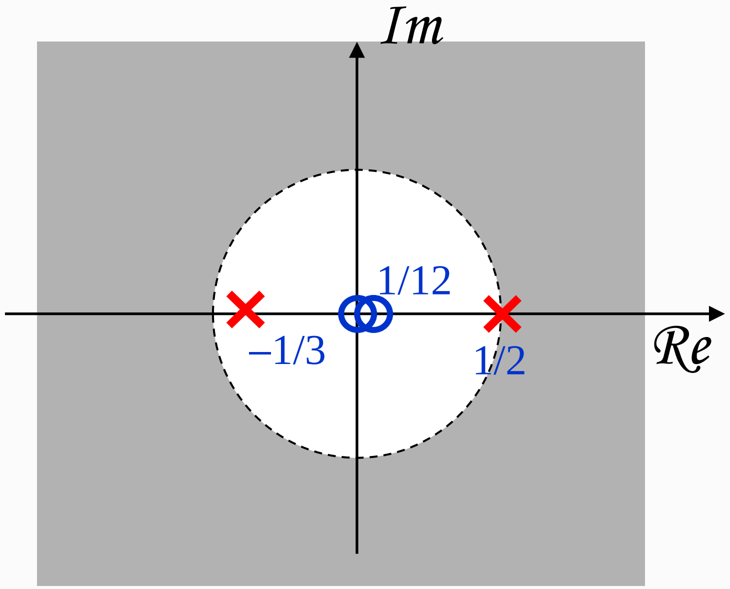

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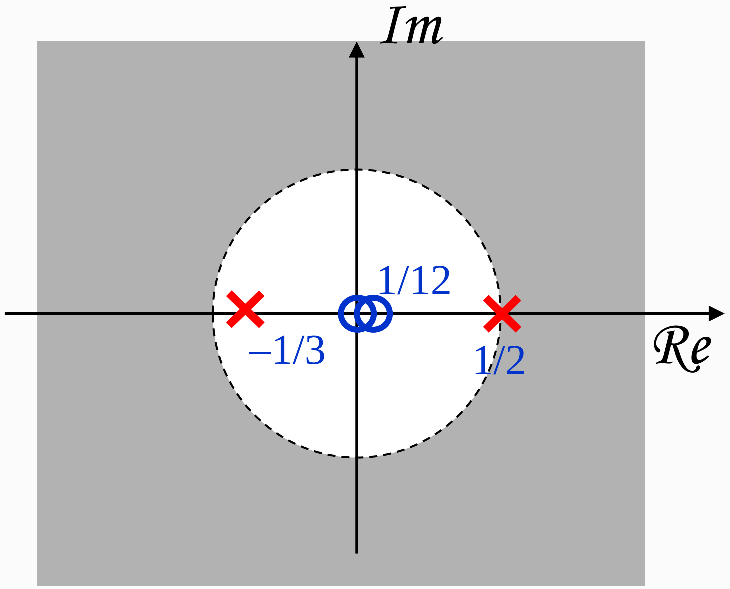

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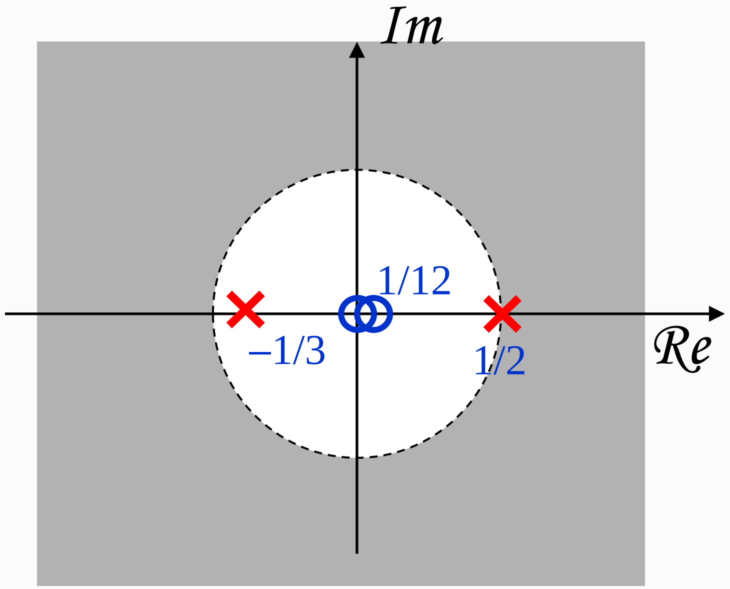

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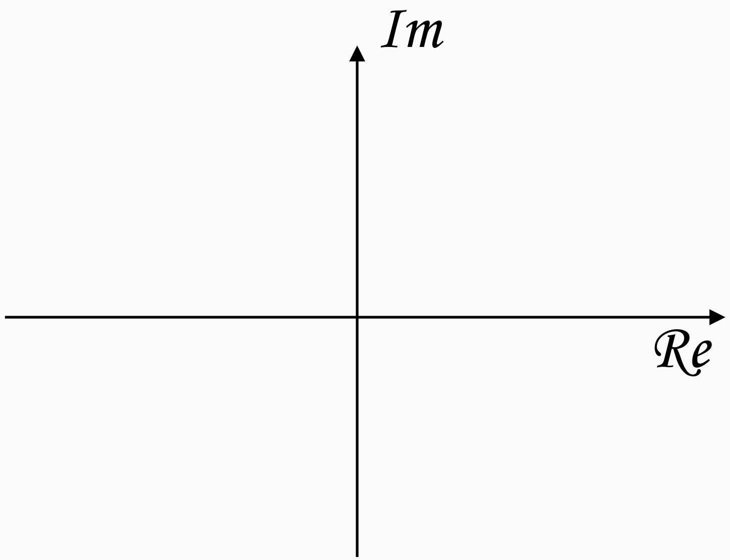

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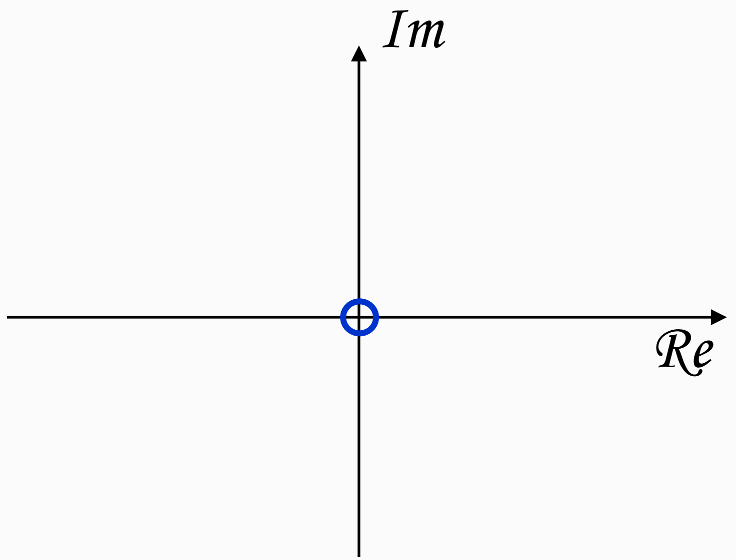

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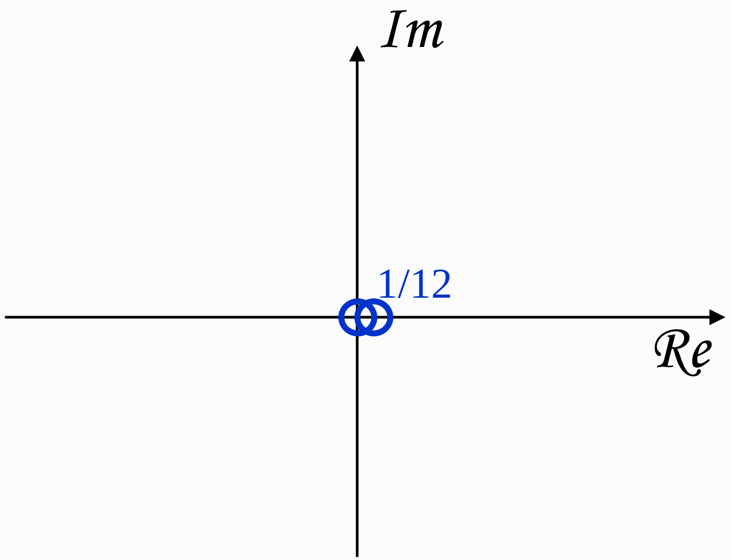

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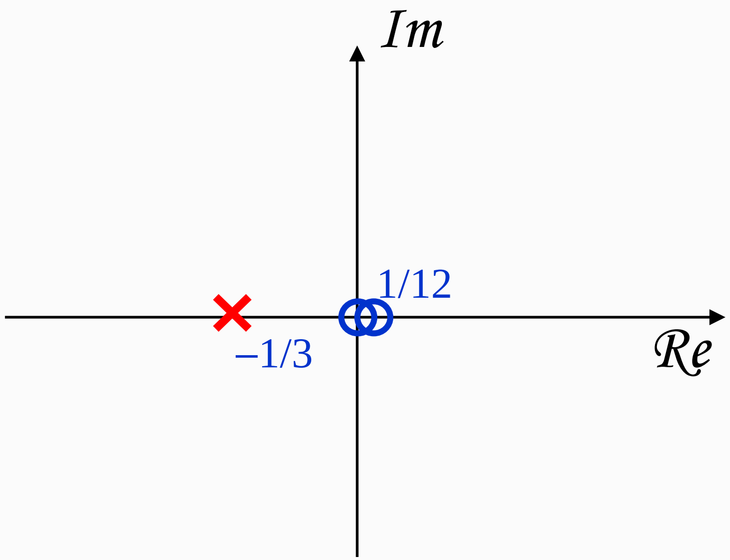

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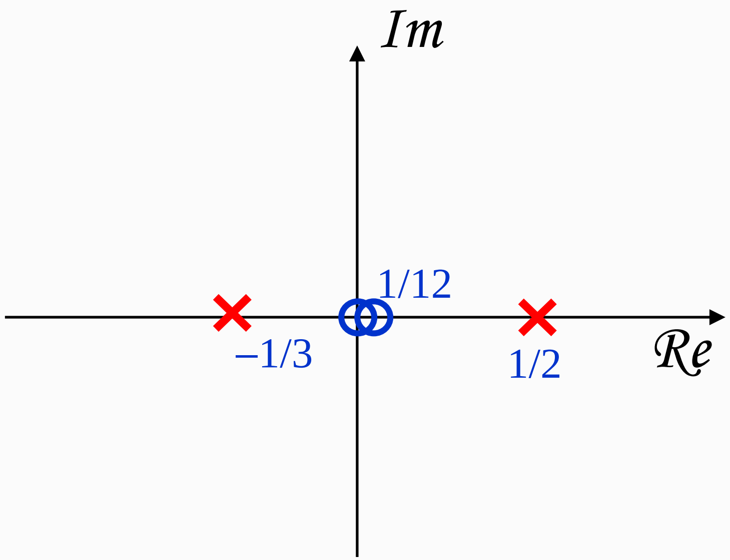

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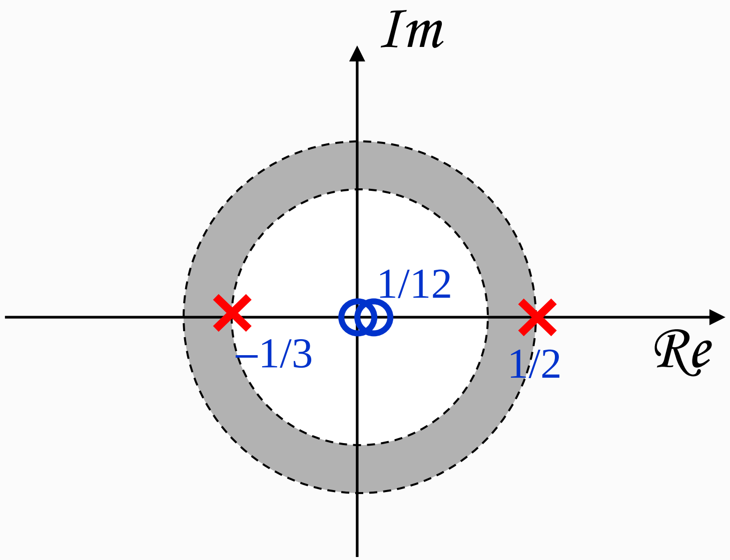

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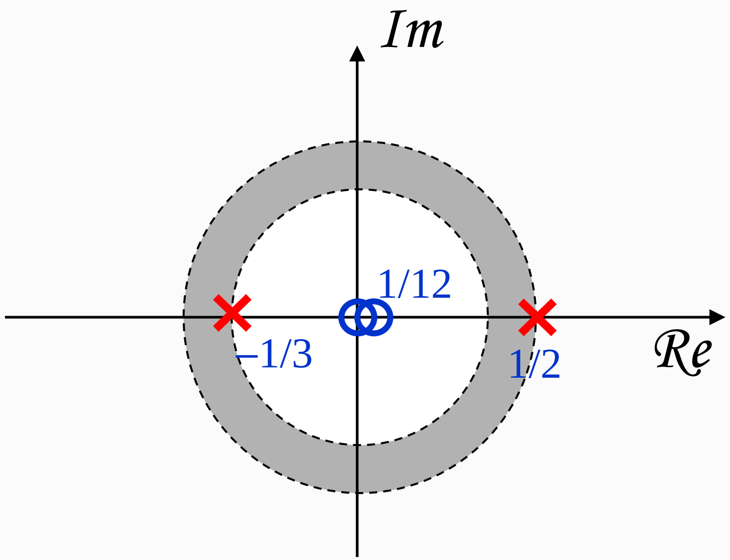

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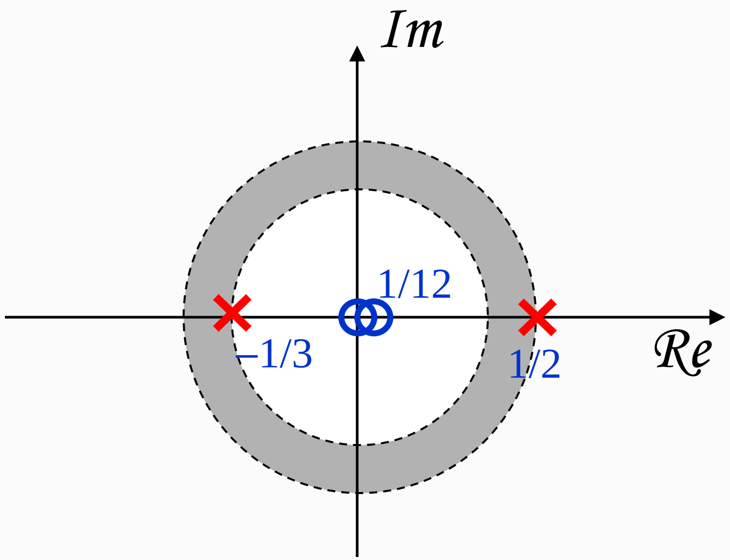

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Represent z-transform as a Rational Function

$$X(z) = \frac{P(z)}{Q(z)}$$

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Sıfır / Kutup Gösterimi

$X(z) = \dfrac{1-0.64z^{-2}}{1-0.2z^{-1}-0.08z^{-2}}$ için kutup-sıfır grafiğini çizelim.

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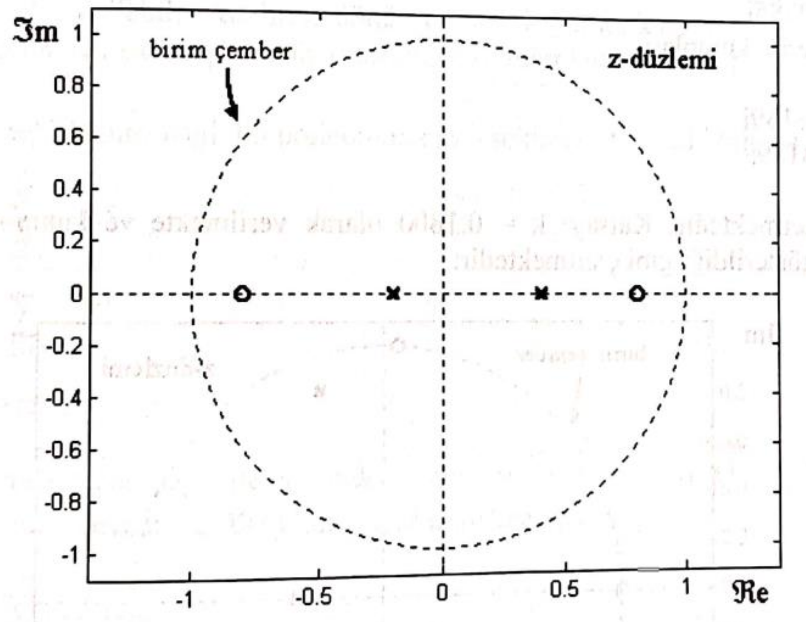

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POLES of H(z)

- Find z, where
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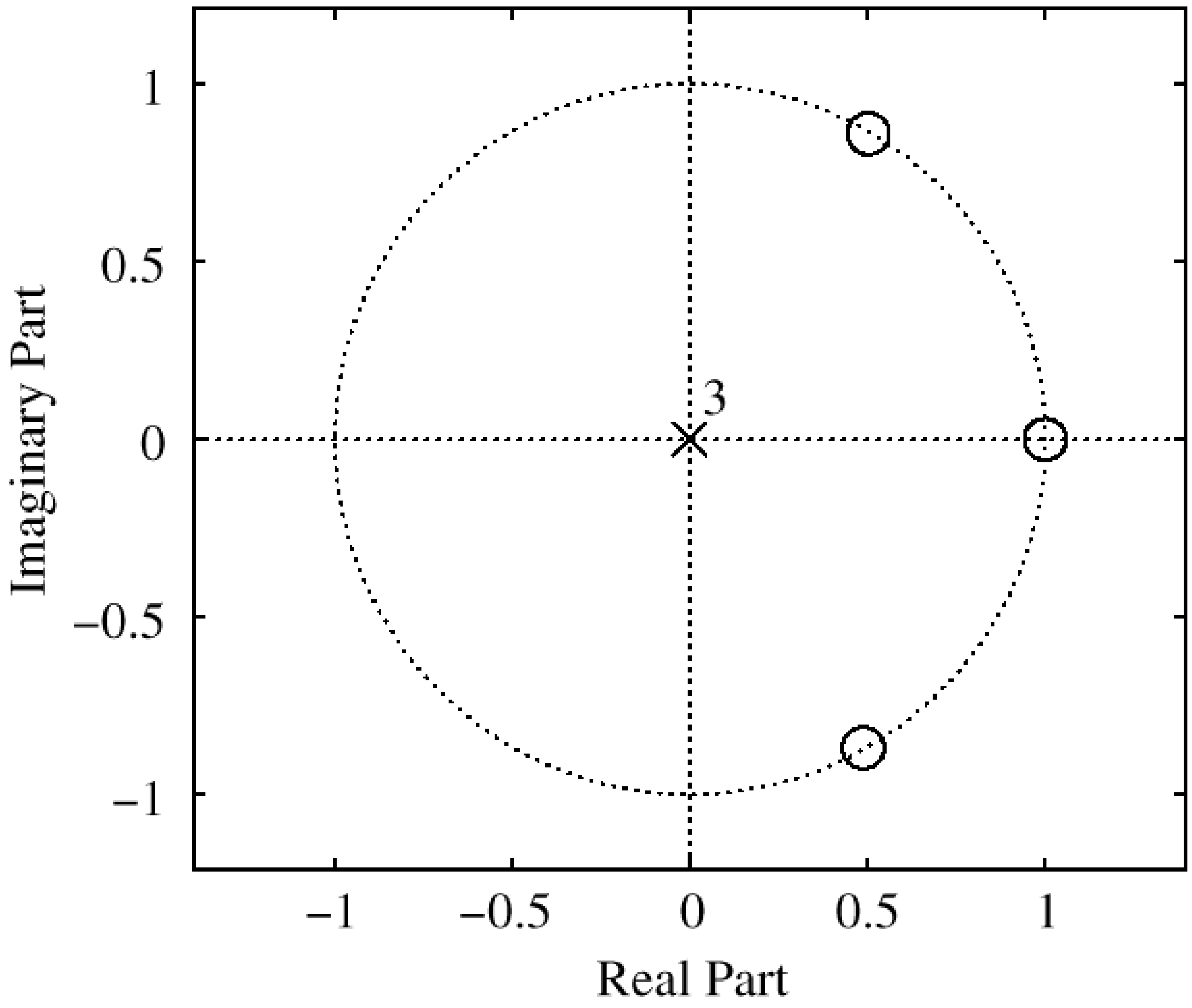
$$\boxed{e^{\pm j\pi/3}}$$

PLOT ZEROS in z-DOMAIN



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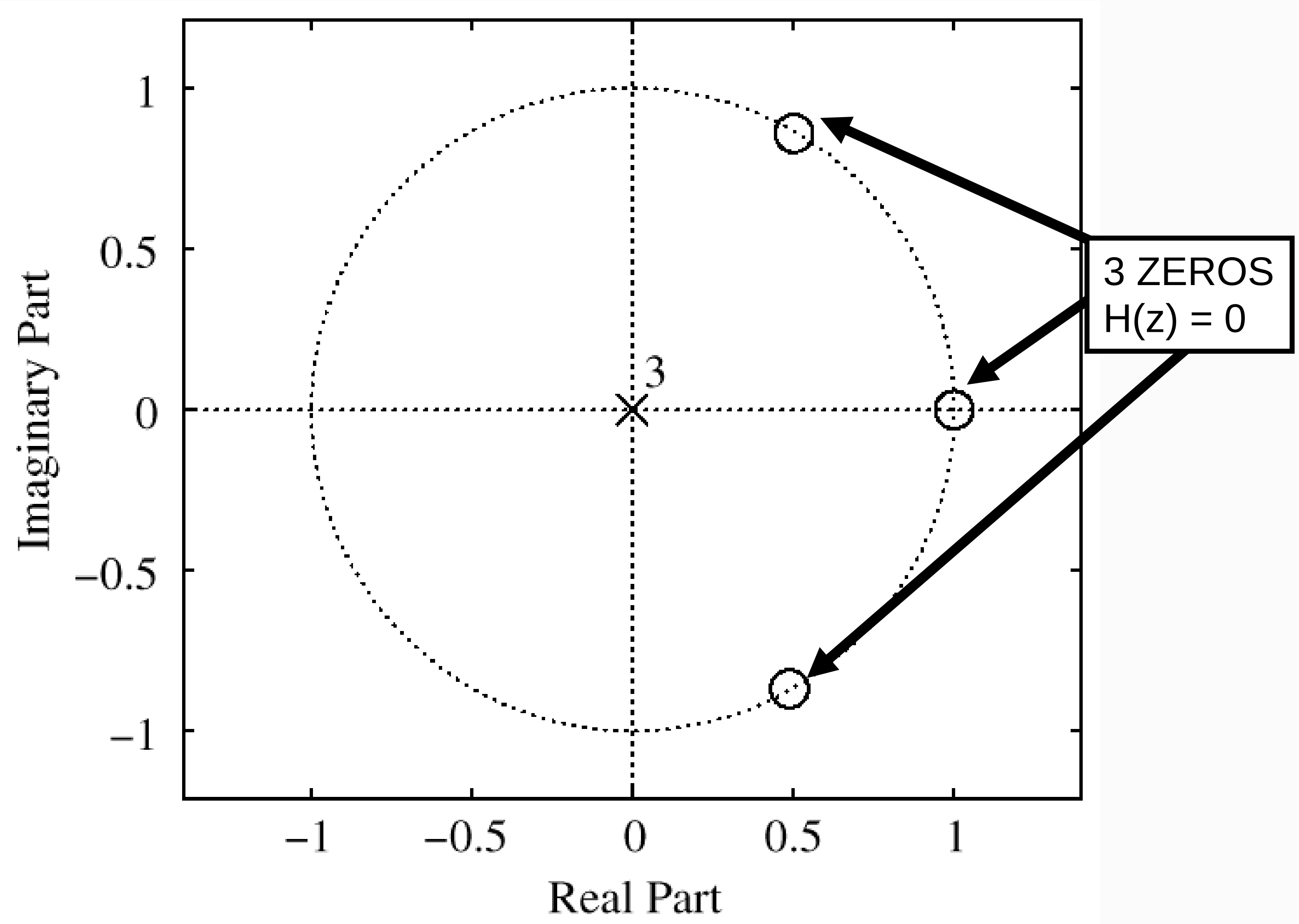

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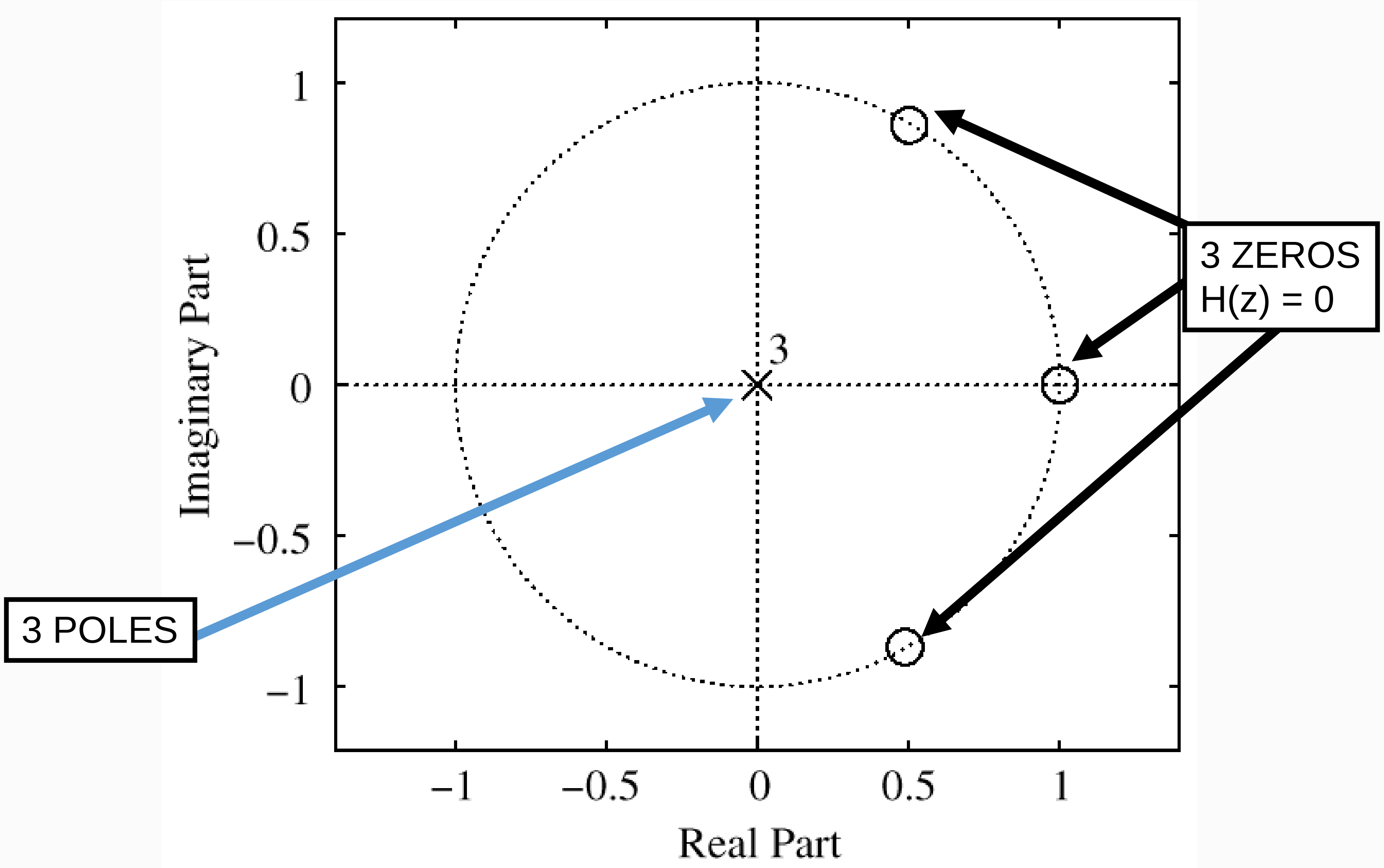

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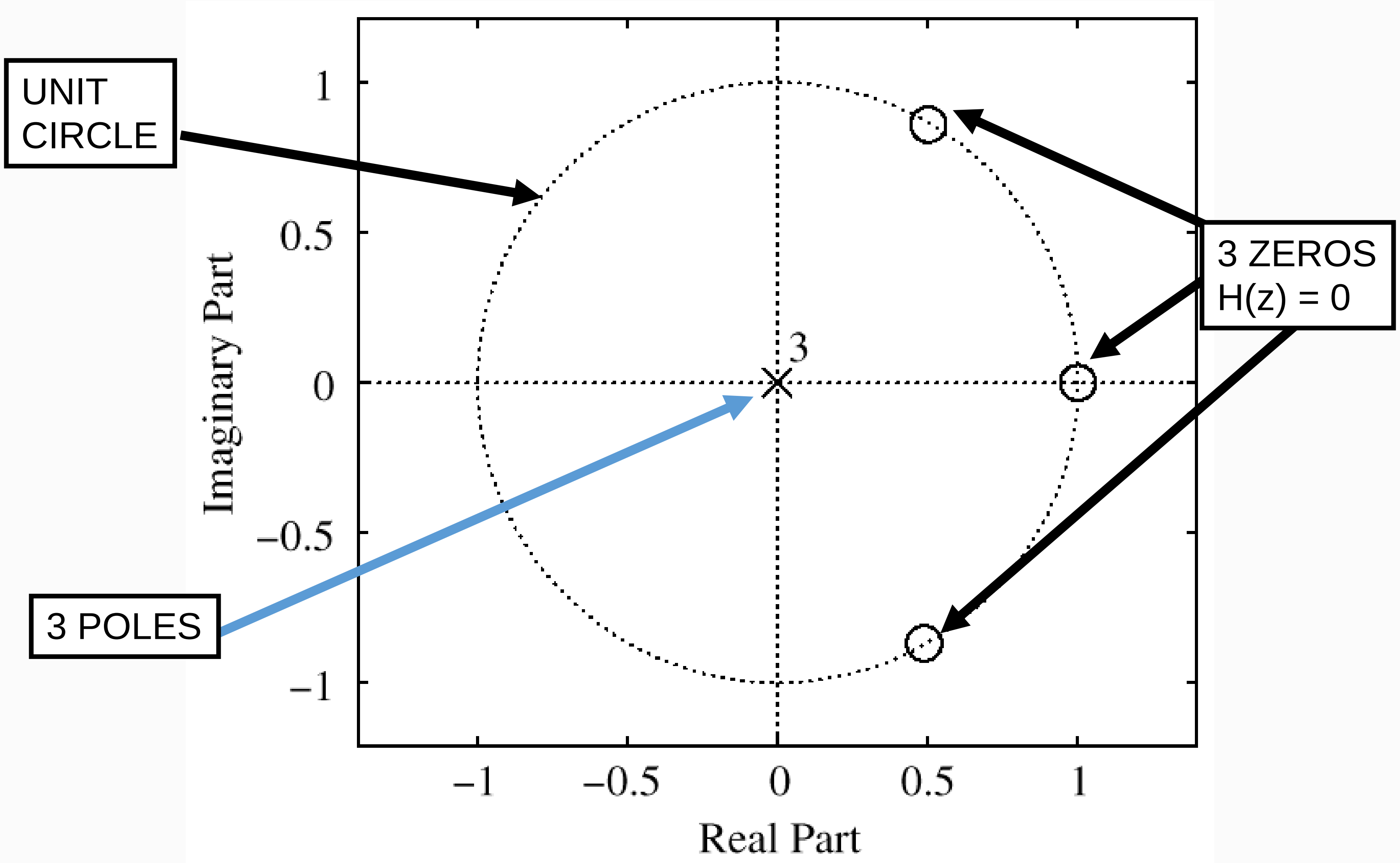
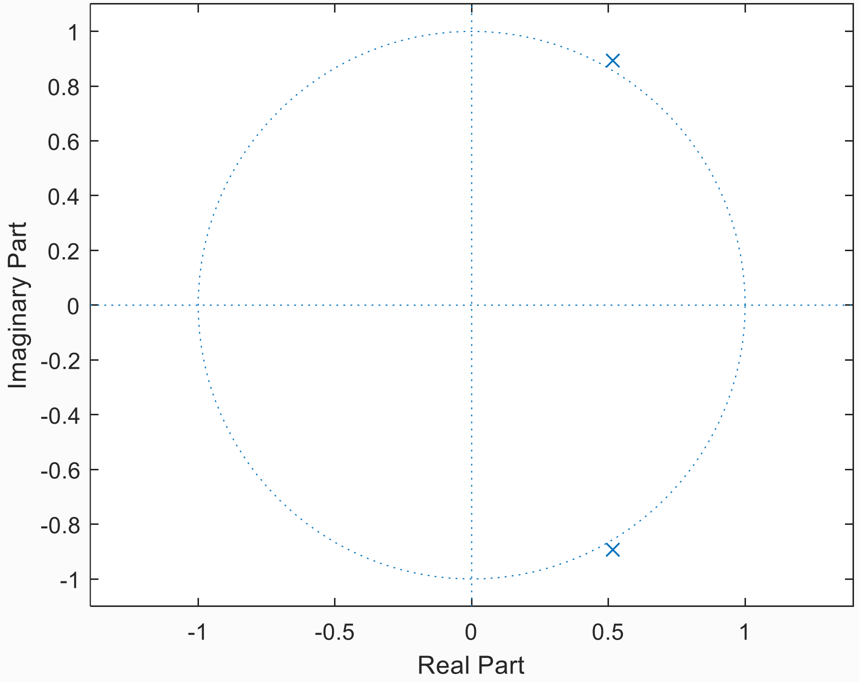


MATLAB Code

$$H(z) = \frac{z^2}{z^2 - 0.97z + 0.94}$$




```
%% Z transform
sym z;
Hz = 1/( 1 - 0.97*z^(-1) + 0.94*z^(-2));

[N,D] = numden(Hz);
pay = roots([0 0 100]);
payda = roots([94 -97 100]);

%% Z-plane
zplane(pay, payda);
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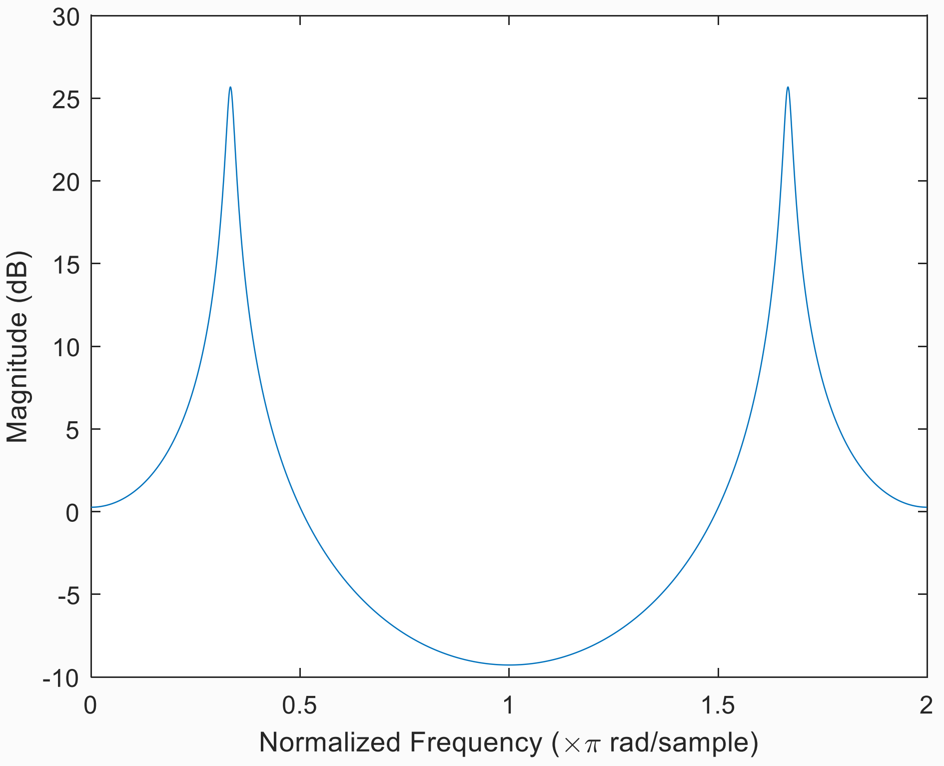


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%% Z-plane
zplane(pay, payda);

%% Freq. Response
[h,w] = freqz([0 0 100],[94 -97
100],'whole',2001);
plot(w/pi,20*log10(abs(h)))
ax = gca;
ax.XTick = 0:.5:2;
xlabel('Normalized Frequency’)
ylabel('Magnitude (dB)')
```

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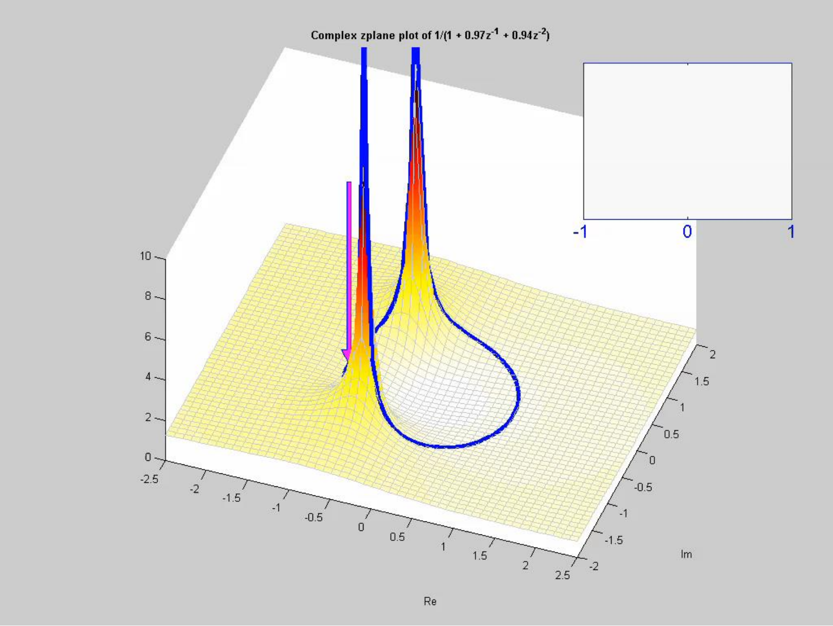


```
%% Z transform
syms a b;
Hz = 1/( 1 - 0.97*(a+1j*b)^(-1) + 0.94*(a+1j*b)^(-
2));

fsurf(abs(Hz));
xlim([-1 1]);
ylim([-1 1]);

hold on;
omega = 0:pi/100:2*pi;
s = exp(-1j*omega);
plot3(real(s),imag(s),2*ones(201,1),'LineWidth',3);
```

Exercise-1

EXERCISE 9.2: Determine the system function $H(z)$ of an FIR filter whose impulse response is

$$h[n] = \delta[n] - 7\delta[n-2] - 3\delta[n-3]$$

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.

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SOLUTION to EXERCISE 9.2: **DSP First 2e**

$$H(z) = \sum_{n=0}^{3} h[n]\, z^{-n}$$

EQUALS ONE AT n=0

$$= \sum_{n=0}^{3} \left(\delta[n] - 7\delta[n-2] - 3\delta[n-3] \right) z^{-n}$$

$$= z^{-0} - 7z^{-2} - 3z^{-3}$$

$$H(z) = 1 - 7z^{-2} - 3z^{-3}$$

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Exercise-2

EXERCISE 9.5: Use z-transforms to combine the following cascaded systems

$$w[n] = x[n] + x[n-1]$$
$$y[n] = w[n] - w[n-1] + w[n-2]$$

into a single difference equation for $y[n]$ in terms of $x[n]$.

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SOLUTION to EXERCISE 9.5: DSP First 2e

$$w[n] = x[n] + x[n-1]$$
$$W(z) = X(z) + z^{-1}X(z)$$

DELAY PROPERTY

$$y[n] = w[n] - w[n-1] + w[n-2]$$
$$Y(z) = W(z) - z^{-1}W(z) + z^{-2}W(z)$$
$$Y(z) = (1 - z^{-1} + z^{-2})W(z)$$
$$= (1 - z^{-1} + z^{-2})(1 + z^{-1})X(z)$$
$$= (1 + z^{-3})X(z)$$

MULTIPLY POLYNOMIALS

$$Y(z) = X(z) + z^{-3}X(z)$$
$$y[n] = x[n] + x[n-3]$$

Exercise-3

Example 9-9: Consider a DTFT function expressed as

$$H(e^{j\hat{\omega}}) = (1 + \cos 2\hat{\omega})e^{-j3\hat{\omega}}$$

Using the inverse Euler formula for the cosine term gives

$$H(e^{j\hat{\omega}}) = \left(1 + \frac{e^{j2\hat{\omega}} + e^{-j2\hat{\omega}}}{2}\right)e^{-j3\hat{\omega}} = (e^{j\hat{\omega}})^{-3} + \tfrac{1}{2}(e^{j\hat{\omega}})^{-1} + \tfrac{1}{2}(e^{j\hat{\omega}})^{-5}$$

Making the substitution $e^{j\hat{\omega}} = z$ gives

$$H(z) = z^{-3} + \tfrac{1}{2}z^{-1} + \tfrac{1}{2}z^{-5} = \tfrac{1}{2}z^{-1} + z^{-3} + \tfrac{1}{2}z^{-5}$$

If the impulse response of the system is needed, the inverse z-transform gives

$$h[n] = \tfrac{1}{2}\delta[n-1] + \delta[n-3] + \tfrac{1}{2}\delta[n-5]$$

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