

Association Rule Mining

- **Frequent Itemsets, Association Rules**
- **Apriori Algorithm**
- **Compact Representation of Frequent Itemsets**
- **FP-Growth Algorithm: An Alternative Frequent Itemset Generation Algorithm**
- **Evaluation of Association Patterns**

Frequent Pattern

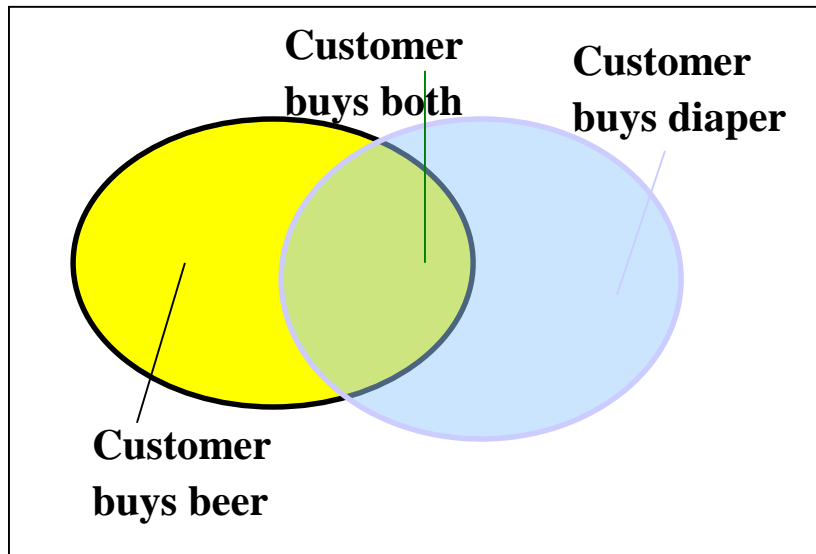
- **Frequent Pattern:** a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set.
- For example, a set of items, such as milk and bread, that appear frequently together in a transaction data set is a *frequent itemset*.
- A subsequence, such as buying first a PC, then a digital camera, and then a memory card, if it occurs frequently in a shopping history database, is a *(frequent) sequential pattern*.
- A substructure can refer to different structural forms, such as subgraphs, subtrees, or sublattices, which may be combined with itemsets or subsequences. If a substructure occurs frequently, it is called a *(frequent) structured pattern*.

Frequent Pattern Market Basket Analysis

- **Frequent Pattern:** a pattern that occurs frequently in a data set.
 - A set of items that appear frequently together in a transaction data set is called as a *frequent itemset*.
- An example of *frequent itemset mining* is **market basket analysis**.
 - This process analyzes customer buying habits by finding associations between the different items that customers place in their “shopping baskets”.
 - If we think of the universe as the set of items available at the store, then each item has a Boolean variable representing the presence or absence of that item.
 - Each basket can then be represented by a Boolean vector of values assigned to these variables.
 - The Boolean vectors can be analyzed for buying patterns that reflect items that are frequently associated or purchased together.
 - These *patterns* can be represented in the form of *association rules*.

Basic Concepts: Frequent Patterns

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- **itemset**: A set of one or more items
 - **k-itemset** $X = \{x_1, \dots, x_k\}$
- **(absolute) support of X**: Frequency of an itemset X.
 - Absolute Support of {Beer} is 3
- **(relative) support of X** is the fraction of transactions that contains X (i.e., the probability that a transaction contains X).
 - Relative Support of {Beer} is 3/5
- An itemset X is **frequent** if X's support is no less than a **minsup** threshold.

Basic Concepts: Association Rules

Association Rule

- An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets

Association Rule Mining:

- Find all the rules $X \rightarrow Y$ with **minimum support** and **minimum confidence**
 - **support**, probability that a transaction contains $X \cup Y$: $P(X \cup Y)$
 - Fraction of transactions that contain both X and Y
 - **confidence**, conditional probability that a transaction having X also contains Y :
 $P(Y/X) = \text{support}(X \cup Y) / \text{support}(X)$
 - Measures how often items in Y appear in transactions that contain X

Basic Concepts: Association Rules

Association Rule

- An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets

Association Rule Mining:

- Find all the rules $X \rightarrow Y$ with **minimum support** and **minimum confidence**

Let $\text{minsup} = 50\%$, $\text{minconf} = 50\%$

Frequent Patterns:

$\{\text{Beer}\}:3$, $\{\text{Nuts}\}:3$, $\{\text{Diaper}\}:4$, $\{\text{Eggs}\}:3$,
 $\{\text{Beer, Diaper}\}:3$

Association Rules:

- $\{\text{Beer}\} \rightarrow \{\text{Diaper}\}$ (60%, 100%)
- $\{\text{Diaper}\} \rightarrow \{\text{Beer}\}$ (60%, 75%)

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

Why Use Support and Confidence?

- **Support** is an important measure because a rule that has very low support may occur simply by chance.
 - A low support rule may be uninteresting from a business perspective because it may not be profitable to promote items that customers seldom buy together
 - For these reasons, support is often used to eliminate uninteresting rules
- **Confidence** measures the reliability of the inference made by a rule.
 - For a given rule $X \rightarrow Y$, the higher the confidence, the more likely it is for Y to be present in transactions that contain X.
- Association analysis results should be interpreted with caution.
 - The inference made by an association rule does not necessarily imply causality.
 - Instead, it suggests a strong co-occurrence relationship between items in the antecedent and consequent of the rule.

Association Rule Mining Task

- Given a set of transactions T , the goal of association rule mining is to find all rules having
 - **support** $\geq \textit{minsup}$ threshold
 - **confidence** $\geq \textit{minconf}$ threshold
- Brute-force approach:
 - List all possible association rules
 - Compute the support and confidence for each rule
 - Prune rules that fail the *minsup* and *minconf* thresholds

\Rightarrow **Computationally not feasible!**

Mining Association Rules

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

$\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$ (s=0.4, c=0.67)
 $\{\text{Milk, Beer}\} \rightarrow \{\text{Diaper}\}$ (s=0.4, c=1.0)
 $\{\text{Diaper, Beer}\} \rightarrow \{\text{Milk}\}$ (s=0.4, c=0.67)
 $\{\text{Beer}\} \rightarrow \{\text{Milk, Diaper}\}$ (s=0.4, c=0.67)
 $\{\text{Diaper}\} \rightarrow \{\text{Milk, Beer}\}$ (s=0.4, c=0.5)
 $\{\text{Milk}\} \rightarrow \{\text{Diaper, Beer}\}$ (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset: {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Association Rule Mining

- The problem of mining association rules can be reduced to that of mining frequent itemsets.
- In general, association rule mining can be viewed as a *two-step process*:
 1. **Find all frequent itemsets**: By definition, each of these itemsets will occur at least as frequently as a predetermined minimum support count, **minsup**.
 - Generate all itemsets whose **support** \geq **minsup**
 2. **Generate strong association rules from the frequent itemsets**: By definition, these rules must satisfy minimum support and minimum confidence.
 - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

Association Rules - Example

Transactions

A,B,D

A,B,C,D

A

A,B,C

B,C

B

$\text{minsup} = 0.5$

$\text{minconf} = 0.7$

- Find frequent itemsets and association rules satisfying minsup and minconf .

Association Rules - Example

Transactions

A,B,D

A,B,C,D

A

A,B,C

B,C

B

$\text{minsup} = 0.5$

$\text{minconf} = 0.7$

- Find frequent itemsets and association rules satisfying minsup and minconf .

Frequent Itemsets:

1-itemsets: {A} $\text{support}(\{A\}) = 4/6$

 {B} $\text{support}(\{B\}) = 5/6$

 {C} $\text{support}(\{C\}) = 3/6$

2-itemsets: {A,B} $\text{support}(\{A,B\}) = 3/6$

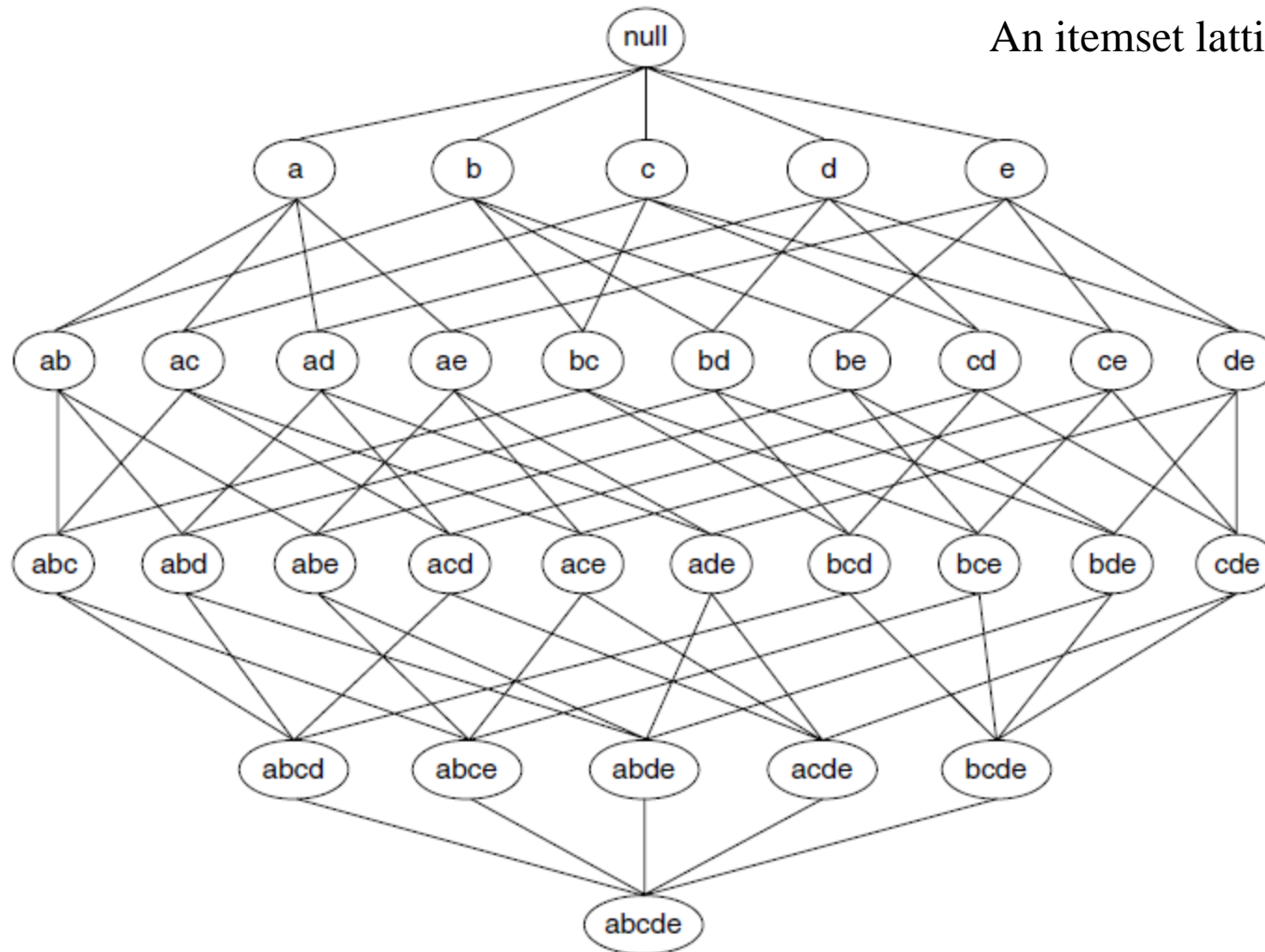
 {B,C} $\text{support}(\{B,C\}) = 3/6$

Association Rules:

$A \rightarrow B$ $\text{conf}(A \rightarrow B) = 3/4$

$C \rightarrow B$ $\text{conf}(C \rightarrow B) = 3/3$

Frequent Itemset Generation

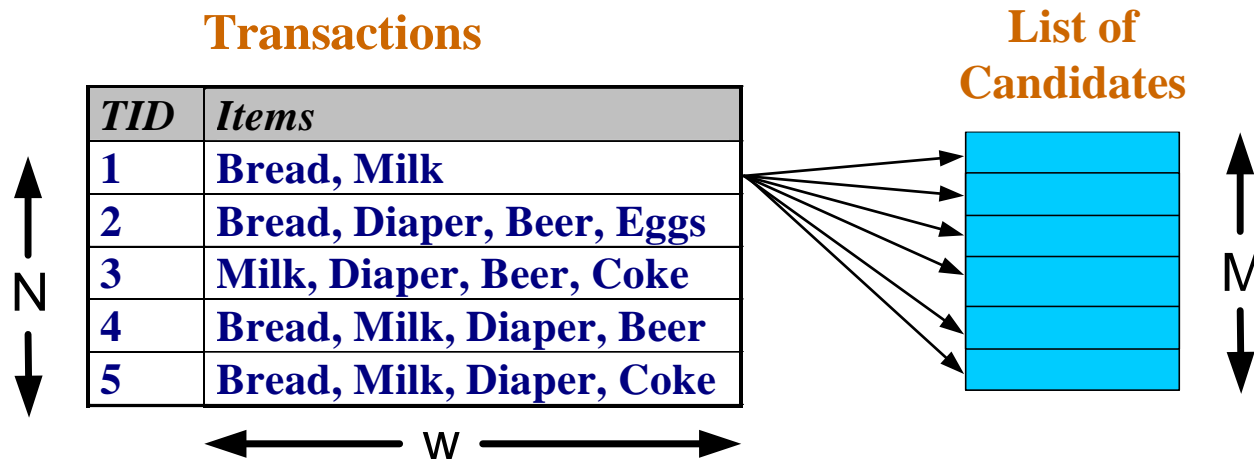


Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- **Brute-force approach:**

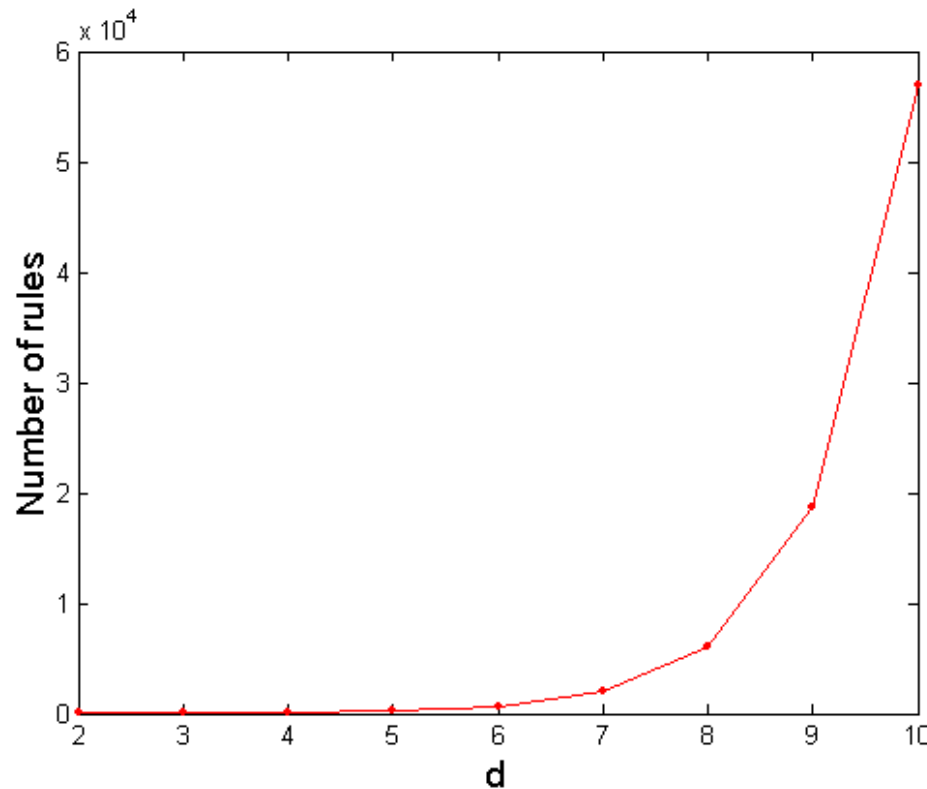
- Each itemset in the lattice is a **candidate** frequent itemset
- Count the support of each candidate by scanning the database



- Match each transaction against every candidate
- Complexity $\sim O(NMW) \Rightarrow$ **Expensive since $M = 2^d$!!!**

Computational Complexity

- Given d unique items:
 - Total number of itemsets = 2^d
 - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1} \left[\binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \right]$$
$$= 3^d - 2^{d+1} + 1$$

If $d=6$, $R = 602$ rules

Frequent Itemset Generation Strategies

- Reduce the **number of candidates** (M)
 - Complete search: $M=2^d$
 - Use pruning techniques to reduce M
 - The **Apriori principle** is an effective way to eliminate some of the candidate itemsets without counting their support values.
- Reduce the **number of comparisons** (NM)
 - Use efficient data structures to store the candidates or transactions
 - No need to match every candidate against every transaction
- Reduce the **number of transactions** (N)
 - Reduce size of N as the size of itemset increases

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Reducing Number of Candidates

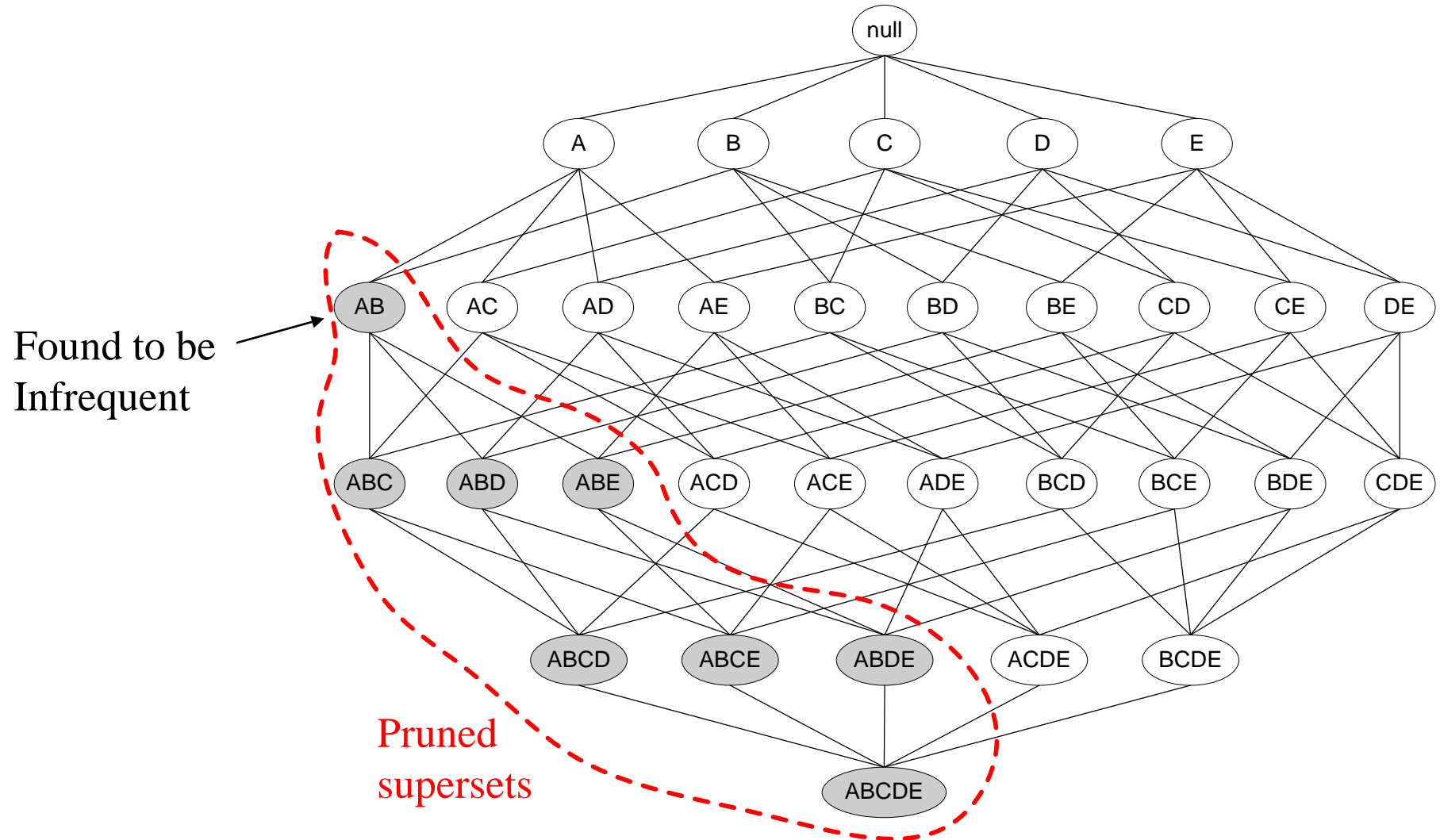
Apriori Principle

- **Apriori Principle:** If an itemset is frequent, then all of its subsets must also be frequent.
- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

- Support of an itemset never exceeds the support of its subsets
- This is known as the **anti-monotone** property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk



Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

Generate 1-itemset candidates

Illustrating Apriori Principle

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk



Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3 \\ 6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

Eliminate infrequent 1-itemset candidates

Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)



Itemset
{Bread,Milk}
{Bread, Beer }
{Bread,Diaper}
{Beer, Milk}
{Diaper, Milk}
{Beer,Diaper}

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3 \\ 6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

Generate 2-itemset candidates

Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)



Itemset	Count
{Bread,Milk}	3
{Beer, Bread}	2
{Bread,Diaper}	3
{Beer,Milk}	2
{Diaper,Milk}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3 \\ 6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

Eliminate infrequent 2-itemset candidates

Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)



Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,
 ${}^6C_1 + {}^6C_2 + {}^6C_3$
 $6 + 15 + 20 = 41$
 With support-based pruning,
 $6 + 6 + 4 = 16$



Triplets (3-itemsets)

Itemset
{ Beer, Diaper, Milk}
{ Beer,Bread,Diaper}
{Bread, Diaper, Milk}
{ Beer, Bread, Milk}

Generate 3-itemset candidates

Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)



Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

$$6 + 6 + 1 = 13$$



Triplets (3-itemsets)

Itemset	Count
{ Beer, Diaper, Milk}	2
{ Beer,Bread, Diaper}	2
{Bread, Diaper, Milk}	2
{Beer, Bread, Milk}	1

Prune 3-itemset candidates with infrequent 2-itemsets
Eliminate infrequent 3-itemset candidates

Apriori Algorithm:

Finding Frequent Itemsets Using Candidate Generation

- **Apriori pruning principle**: If there is any itemset which is infrequent, its superset should not be generated/tested!

Apriori Algorithm: F_k : frequent k-itemsets L_k : candidate k-itemsets

- Let $k=1$
- Generate $F_1 = \{\text{frequent 1-itemsets}\}$
- Repeat until F_k is empty
 - **Candidate Generation**: Generate L_{k+1} from F_k
 - **Candidate Pruning**: Prune candidate itemsets in L_{k+1} containing subsets of length k that are infrequent
 - **Support Counting**: Count the support of each candidate in L_{k+1} by scanning the DB
 - **Candidate Elimination**: Eliminate candidates in L_{k+1} that are infrequent, leaving only those that are frequent $\Rightarrow F_{k+1}$

Apriori Algorithm:

Candidate Generation: $F_{k-1} \times F_{k-1}$ Method

- Merge two frequent $(k-1)$ -itemsets if their first $(k-2)$ items are identical
- $F_3 = \{ABC, ABD, ABE, ACD, BCD, BDE, CDE\}$
 - Merge(ABC, ABD) = ABCD
 - Merge(ABC, ABE) = ABCE
 - Merge(ABD, ABE) = ABDE
 - Do not merge(ABD, ACD) because they share only prefix of length 1 instead of length 2
- $L_4 = \{ABCD, ABCE, ABDE\}$ is the set of candidate 4-itemsets **generated**

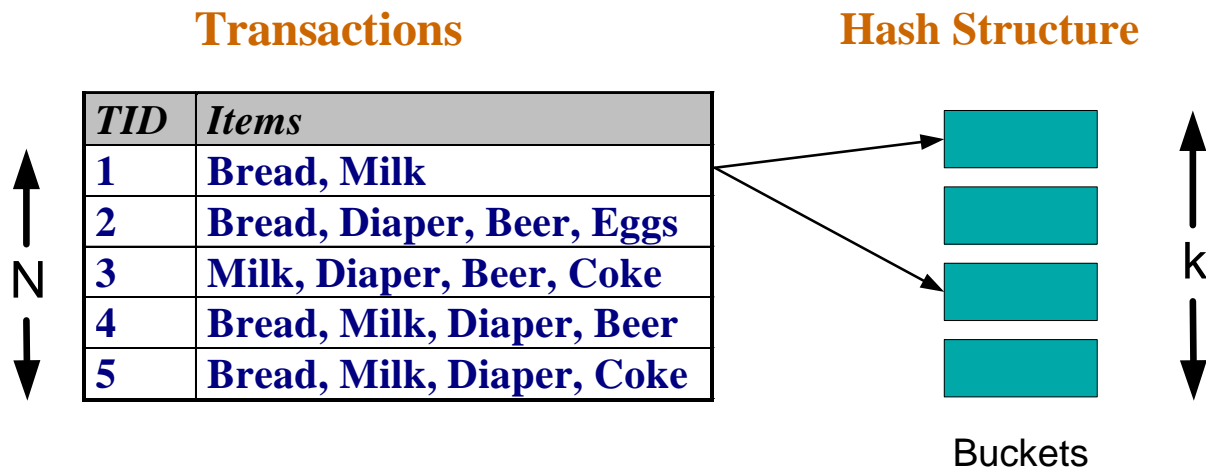
Apriori Algorithm: Candidate Pruning

- Let $F_3 = \{ABC, ABD, ABE, ACD, BCD, BDE, CDE\}$ be the set of frequent 3-itemsets
- $L_4 = \{ABCD, ABCE, ABDE\}$ is the set of **candidate 4-itemsets generated**
- Candidate pruning
 - Prune ABCE because ACE and BCE are infrequent
 - Prune ABDE because ADE is infrequent
- After **candidate pruning**: $L_4 = \{ABCD\}$

Apriori Algorithm:

Support Counting of Candidate Itemsets

- Scan the database of transactions to determine the support of each candidate itemset
 - Must match every candidate itemset against every transaction, which is an expensive operation
- To reduce the number of comparisons, store the candidates in a hash structure
 - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets



Apriori Algorithm

Algorithm: Apriori. Find frequent itemsets using an iterative level-wise approach based on candidate generation.

Input:

- D , a database of transactions;
- min_sup , the minimum support count threshold.

Output: L , frequent itemsets in D .

Method:

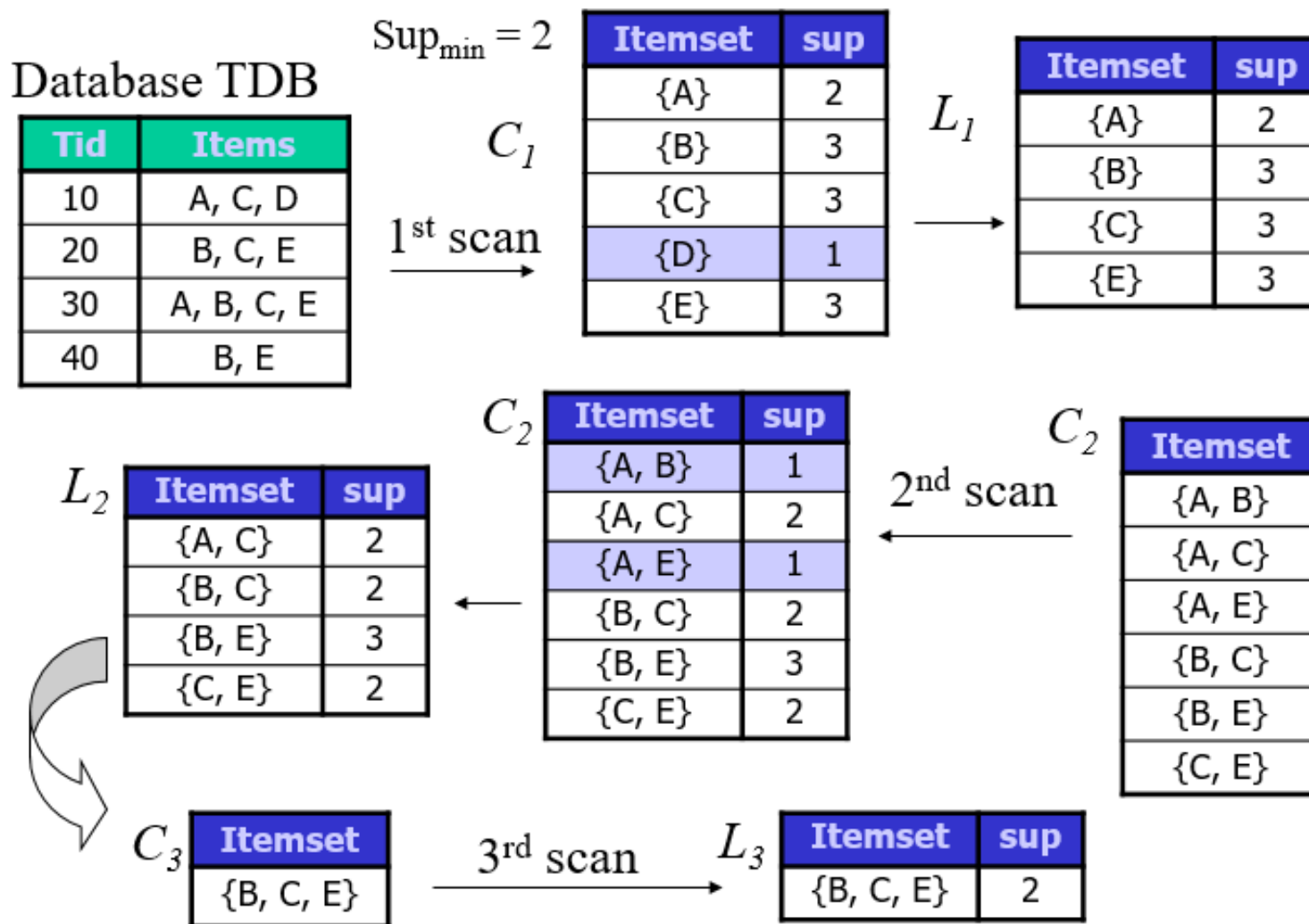
```
 $L_1 = \text{find\_frequent\_1-itemsets}(D);$ 
for ( $k = 2; L_{k-1} \neq \phi; k++$ ) {
     $C_k = \text{apriori\_gen}(L_{k-1});$ 
    for each transaction  $t \in D$  { // scan  $D$  for counts
         $C_t = \text{subset}(C_k, t);$  // get the subsets of  $t$  that are candidates
        for each candidate  $c \in C_t$ 
             $c.\text{count}++;$ 
    }
     $L_k = \{c \in C_k | c.\text{count} \geq min\_sup\}$ 
}
return  $L = \cup_k L_k;$ 
```

Apriori Algorithm

```
procedure apriori_gen( $L_{k-1}$ :frequent  $(k-1)$ -itemsets)
  for each itemset  $l_1 \in L_{k-1}$ 
    for each itemset  $l_2 \in L_{k-1}$ 
      if  $(l_1[1] = l_2[1]) \wedge (l_1[2] = l_2[2])$ 
         $\wedge \dots \wedge (l_1[k-2] = l_2[k-2]) \wedge (l_1[k-1] < l_2[k-1])$  then {
           $c = l_1 \bowtie l_2$ ; // join step: generate candidates
          if has_infrequent_subset( $c, L_{k-1}$ ) then
            delete  $c$ ; // prune step: remove unfruitful candidate
          else add  $c$  to  $C_k$ ;
        }
  return  $C_k$ ;
```

```
procedure has_infrequent_subset( $c$ : candidate  $k$ -itemset;
   $L_{k-1}$ : frequent  $(k-1)$ -itemsets); // use prior knowledge
  for each  $(k-1)$ -subset  $s$  of  $c$ 
    if  $s \notin L_{k-1}$  then
      return TRUE;
  return FALSE;
```


Apriori Algorithm - An Example



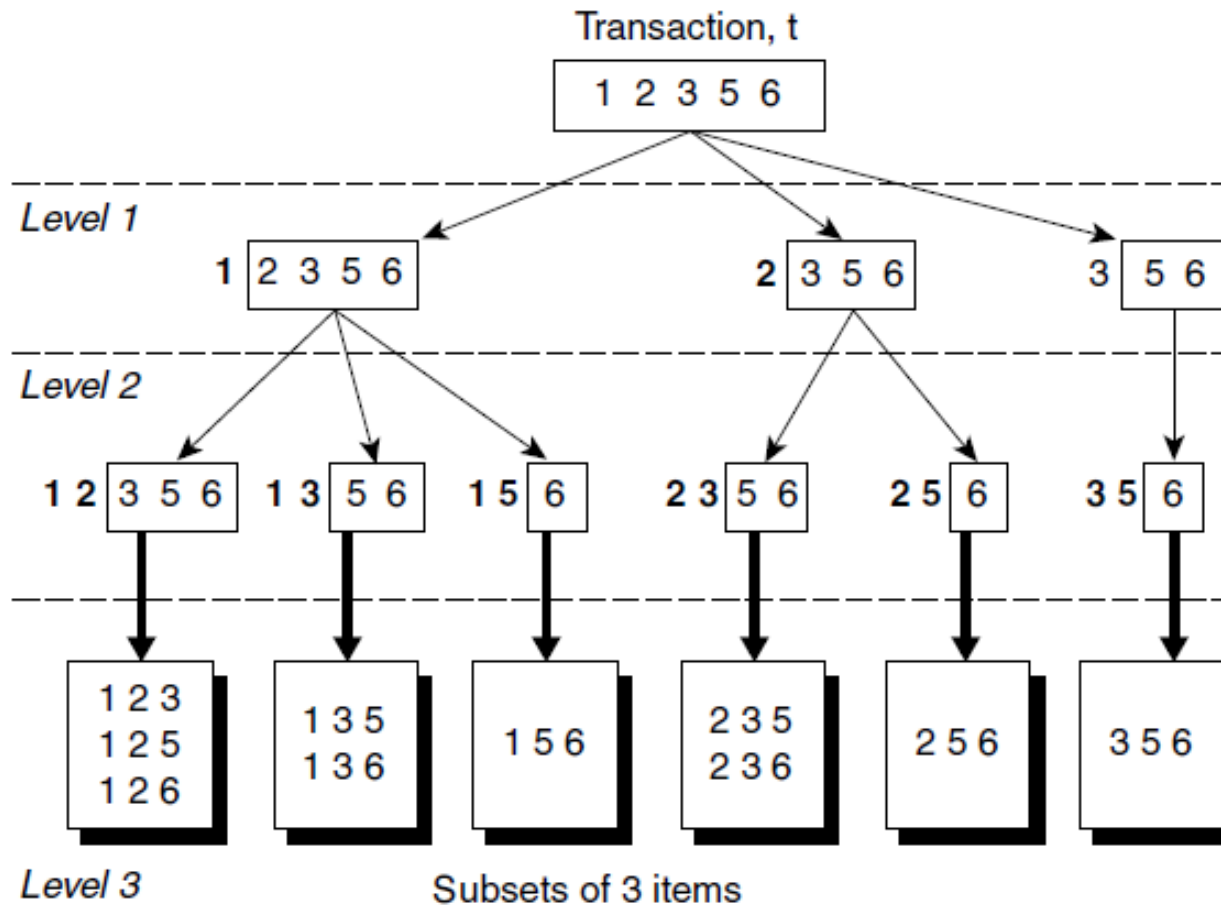
Support Counting Using Hash Tree

- Why counting supports of candidates a problem?
 - The total number of candidates can be very huge
 - One transaction may contain many candidates
 - Must match every candidate itemset against every transaction, which is an expensive operation
- Method:
 - Candidate itemsets are stored in a *hash-tree*
 - *Leaf node* of hash-tree contains a list of itemsets and counts
 - *Interior node* contains a hash table
 - *Subset function*: finds all the candidates contained in a transaction

Support Counting Using Hash Tree

Subset Operation

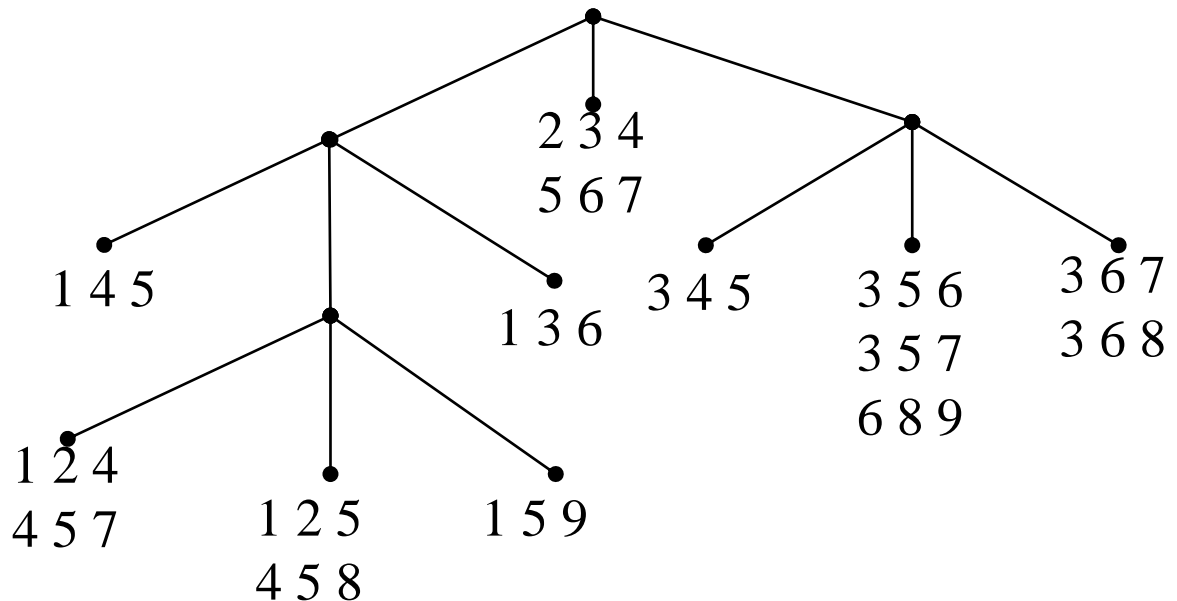
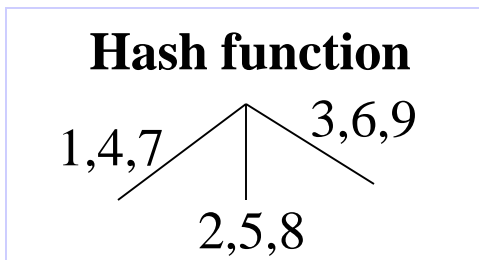
- Enumerating subsets of three items from a transaction t



Support Counting Using Hash Tree

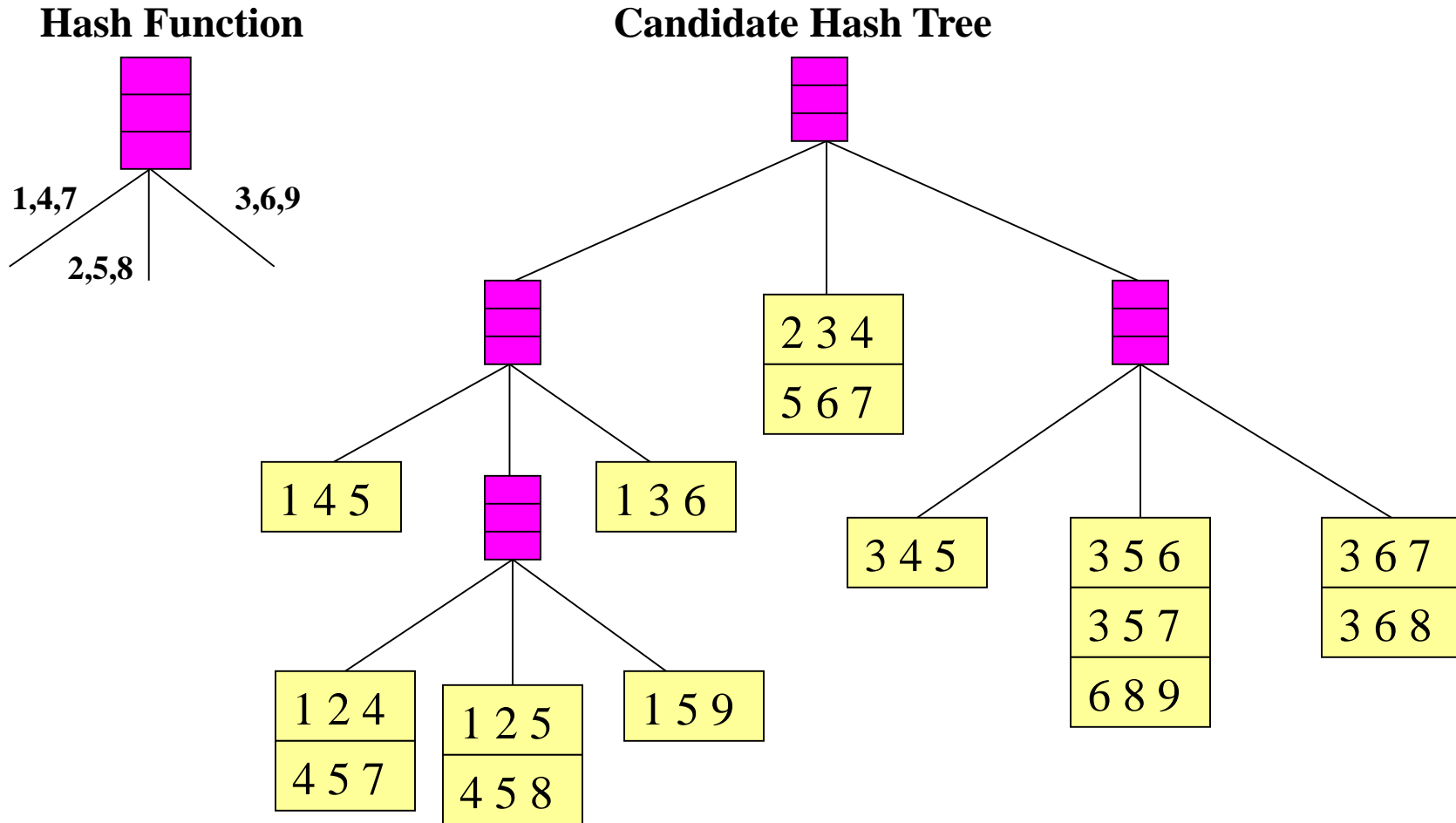
Generate Candidate Hash Tree

- Suppose you have 15 candidate itemsets of length 3:
 $\{1\ 4\ 5\}$, $\{1\ 2\ 4\}$, $\{4\ 5\ 7\}$, $\{1\ 2\ 5\}$, $\{4\ 5\ 8\}$, $\{1\ 5\ 9\}$, $\{1\ 3\ 6\}$, $\{2\ 3\ 4\}$, $\{5\ 6\ 7\}$,
 $\{3\ 4\ 5\}$, $\{3\ 5\ 6\}$, $\{3\ 5\ 7\}$, $\{6\ 8\ 9\}$, $\{3\ 6\ 7\}$, $\{3\ 6\ 8\}$
- We need: Hash function
 - HashFunc: **mod 3**



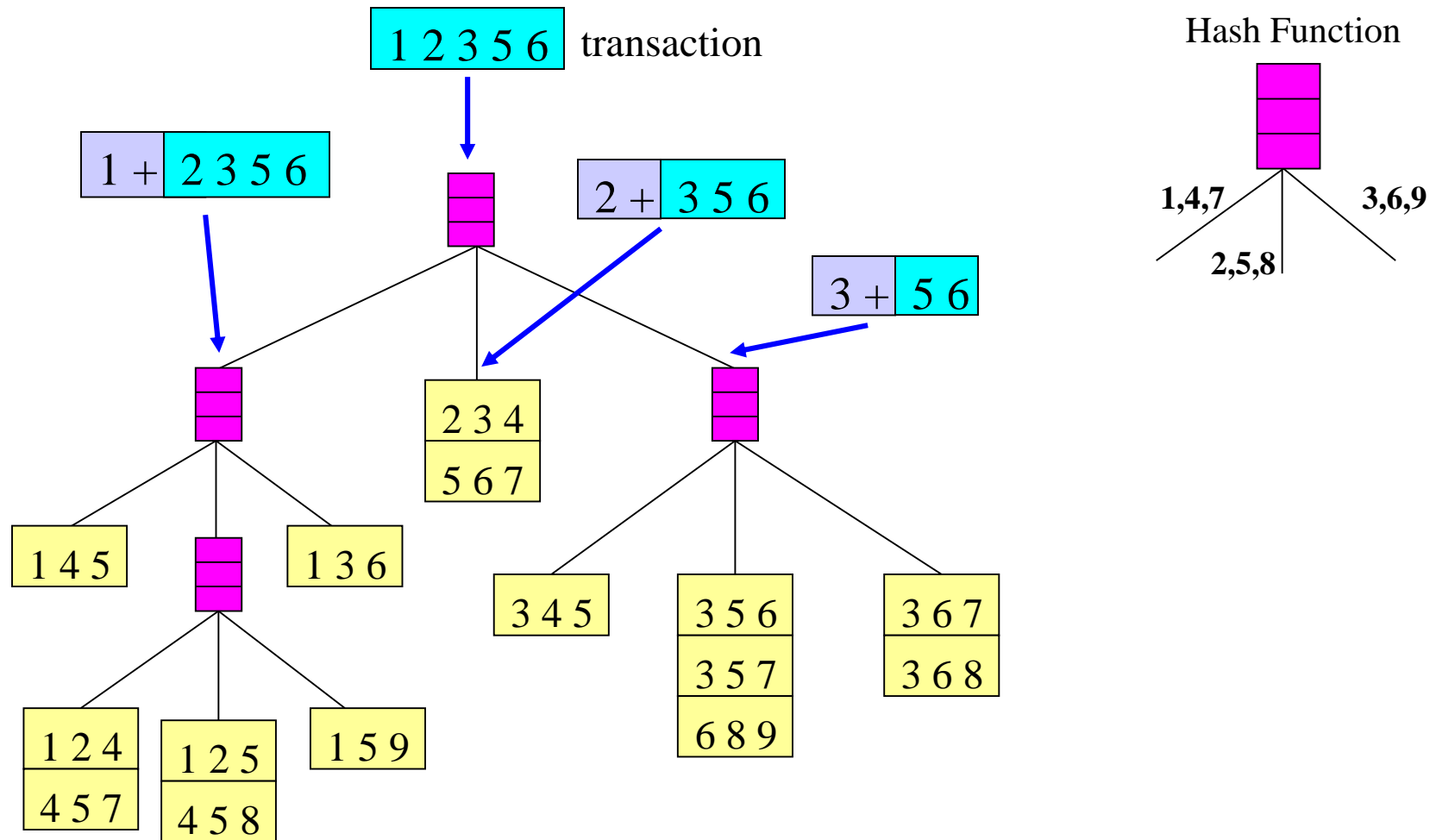
Support Counting Using Hash Tree

Generate Candidate Hash Tree



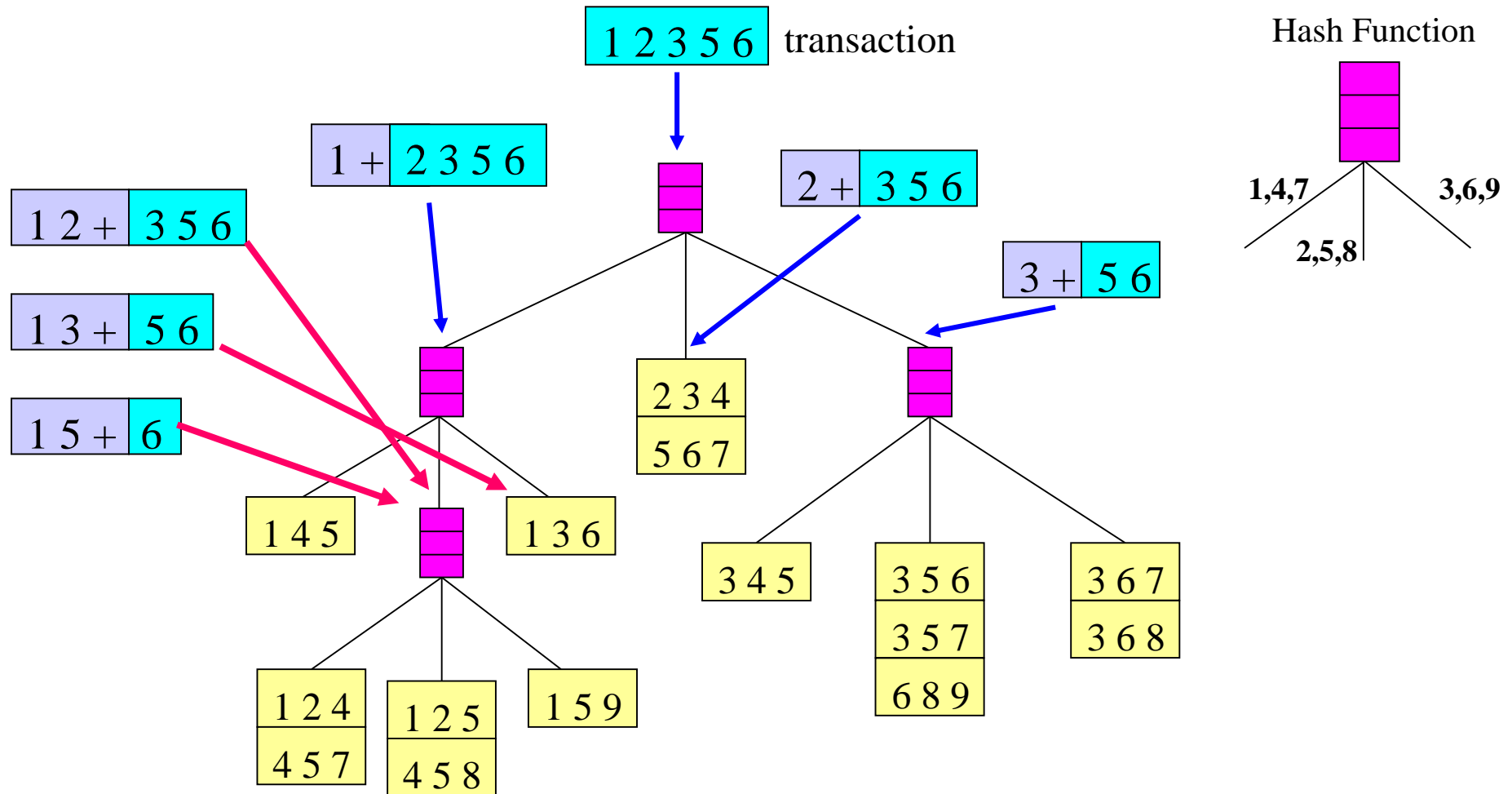
Support Counting Using Hash Tree

Traverse Candidate Hash Tree to Update Support Counts



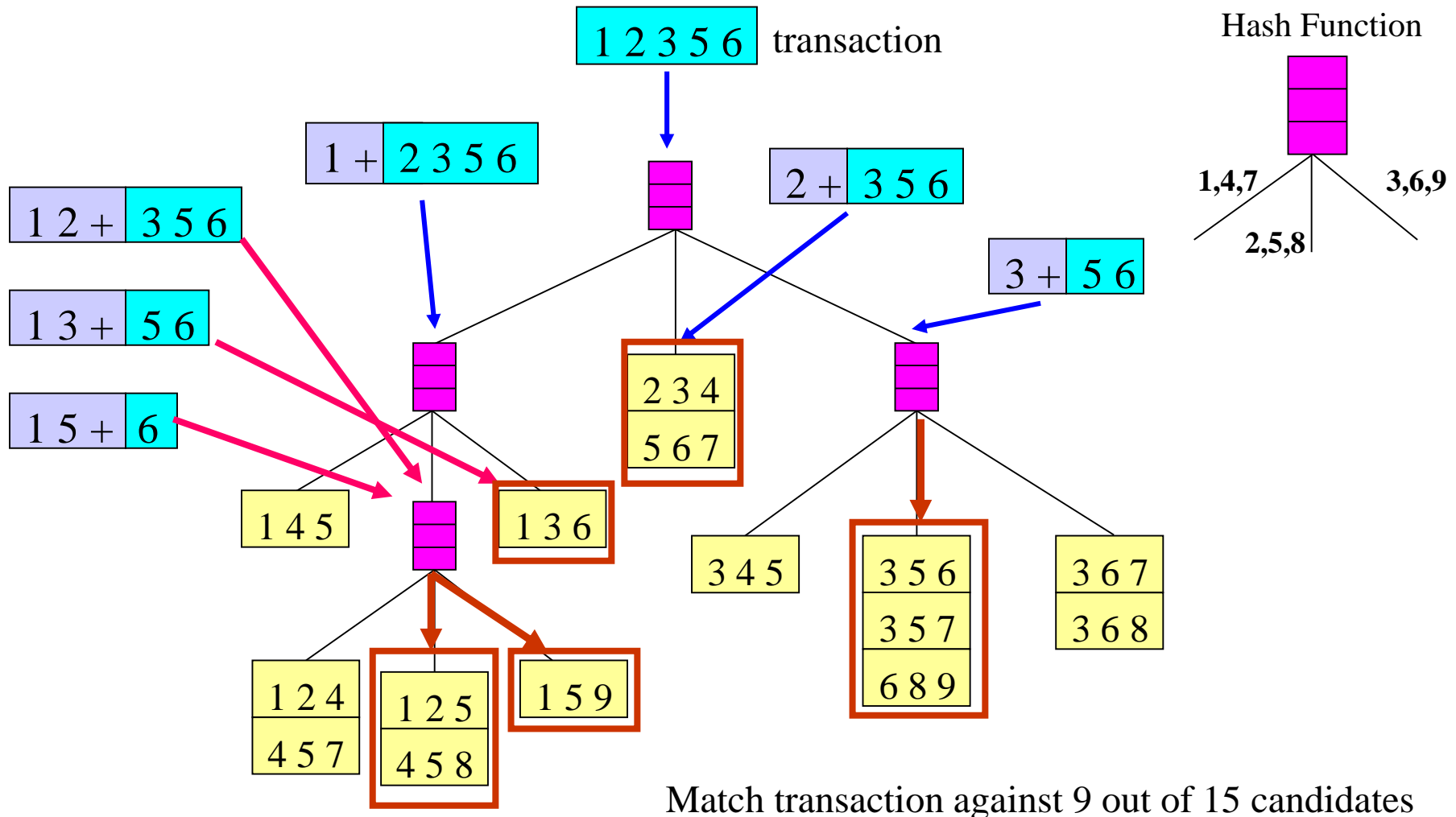
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Traverse Candidate Hash Tree to Update Support Counts



Support Counting Using Hash Tree

Traverse Candidate Hash Tree to Update Support Counts

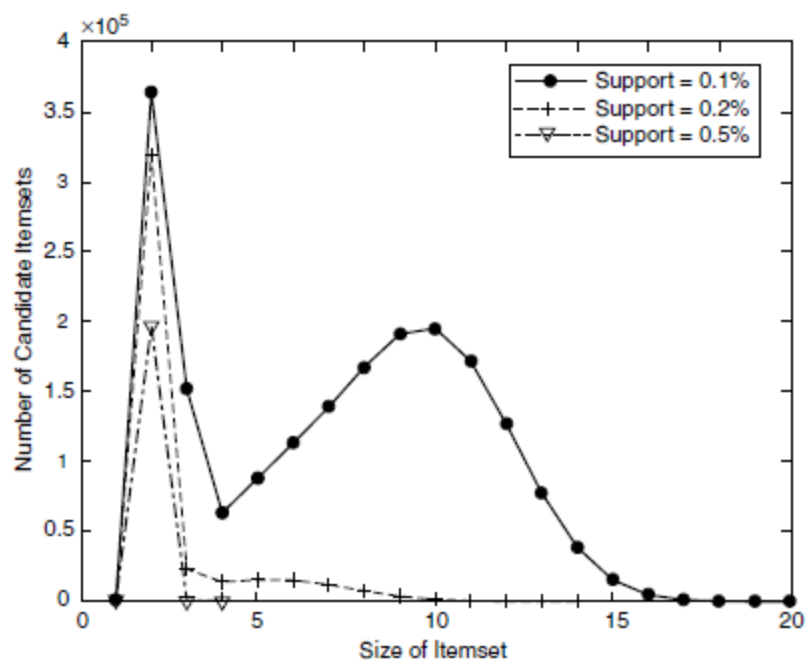


Factors Affecting Complexity of Apriori Algorithm

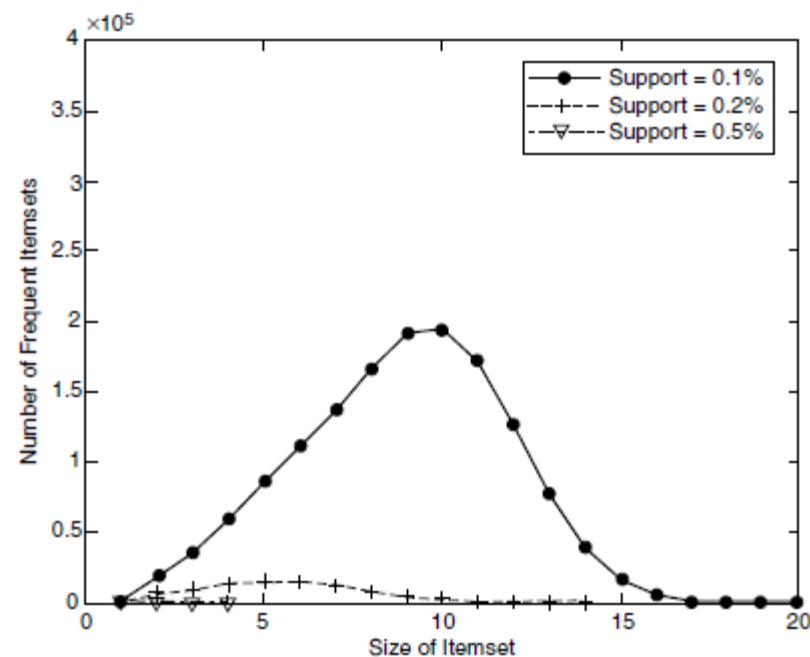
- **Choice of minimum support threshold**
 - lowering support threshold results in more frequent itemsets
 - this may increase number of candidates and max length of frequent itemsets
- **Dimensionality (number of items) of the data set**
 - more space is needed to store support count of each item
 - if number of frequent items also increases, both computation and I/O costs may also increase
- **Size of database**
 - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- **Average transaction width**
 - transaction width increases with denser data sets
 - This may increase max length of frequent itemsets and number of subsets in a transaction increases with its width

Effect of Support Threshold

- Effect of support threshold on the number of candidate and frequent itemsets



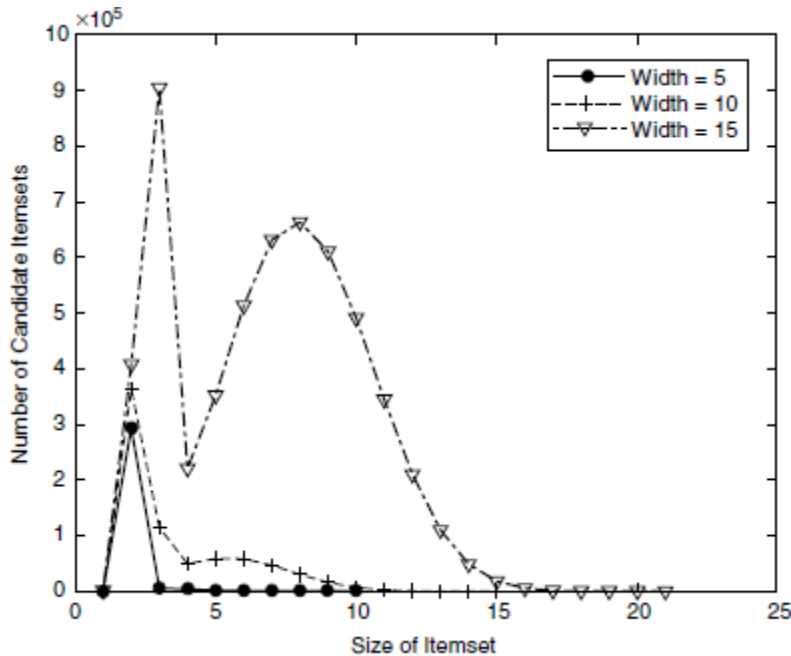
Number of candidate itemsets



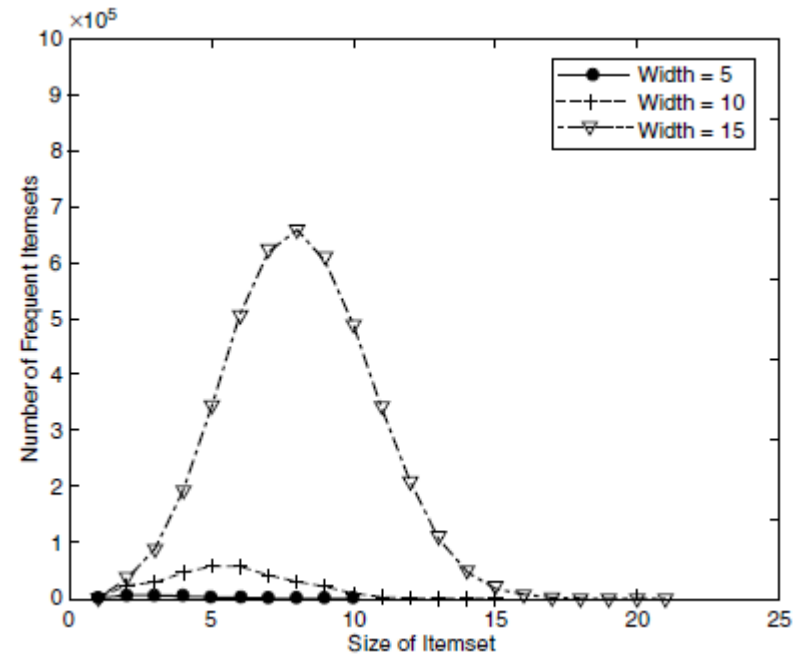
Number of frequent itemsets

Effect of Average Transaction Width

- Effect of average transaction width on the number of candidate and frequent itemsets



Number of candidate itemsets



Number of frequent itemsets

Effect of Support Distribution

- How to set the appropriate *minsup* threshold?
 - If *minsup* is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products)
 - If *minsup* is set too low, it is computationally expensive and the number of itemsets is very large
- Using a single minimum support threshold may not be effective

Multiple Minimum Support

- How to apply multiple minimum supports?
 - $MS(i)$: minimum support for item i
 - e.g.: $MS(\text{Milk})=5\%$, $MS(\text{Coke}) = 3\%$,
 $MS(\text{Broccoli})=0.1\%$, $MS(\text{Salmon})=0.5\%$
 - $MS(\{\text{Milk}, \text{Broccoli}\}) = \min (MS(\text{Milk}), MS(\text{Broccoli}))$
 $= 0.1\%$
 - Challenge: Support is no longer anti-monotone
 - Suppose: $\text{Support}(\text{Milk}, \text{Coke}) = 1.5\%$ and
 $\text{Support}(\text{Milk}, \text{Coke}, \text{Broccoli}) = 0.5\%$
 - $\{\text{Milk}, \text{Coke}\}$ is infrequent but $\{\text{Milk}, \text{Coke}, \text{Broccoli}\}$ is frequent

Multiple Minimum Support

- Order the items according to their minimum support (in ascending order)
 - e.g.: $MS(\text{Milk})=5\%$, $MS(\text{Coke}) = 3\%$,
 $MS(\text{Broccoli})=0.1\%$, $MS(\text{Salmon})=0.5\%$
 - Ordering: Broccoli, Salmon, Coke, Milk
- Need to modify Apriori such that:
 - L_1 : set of frequent items
 - F_1 : set of items whose support is $\geq MS(1)$
where $MS(1)$ is $\min_i(MS(i))$
 - C_2 : candidate itemsets of size 2 is generated from F_1
instead of L_1

Multiple Minimum Support

- Modifications to Apriori:
 - In traditional Apriori,
 - A candidate $(k+1)$ -itemset is generated by merging two frequent itemsets of size k
 - The candidate is pruned if it contains any infrequent subsets of size k
 - Pruning step has to be modified:
 - Prune only if subset contains the first item
 - e.g.: Candidate={Broccoli, Coke, Milk} (ordered according to minimum support)
 - {Broccoli, Coke} and {Broccoli, Milk} are frequent but {Coke, Milk} is infrequent
 - Candidate is not pruned because {Coke, Milk} does not contain the first item, i.e., Broccoli.

Rule Generation in Apriori Algorithm

- Given a frequent itemset L , find all non-empty subsets $f \subset L$ such that **candidate rule** $f \rightarrow L - f$ satisfies the minimum confidence requirement

- If $\{A,B,C,D\}$ is a frequent itemset, candidate rules:

$ABC \rightarrow D$	$ABD \rightarrow C$	$ACD \rightarrow B$	$BCD \rightarrow A$
$D \rightarrow ABC$	$C \rightarrow ABD$	$B \rightarrow ACD$	$A \rightarrow BCD$

,

$AB \rightarrow CD$	$AC \rightarrow BD$	$AD \rightarrow BC$
$CD \rightarrow AB$	$BD \rightarrow AC$	$BC \rightarrow AD$

- If $|L| = k$, then there are $2^k - 2$ candidate association rules
 - (ignoring $L \rightarrow \emptyset$ and $\emptyset \rightarrow L$)

Rule Generation in Apriori Algorithm

- How to efficiently generate rules from frequent itemsets?
- In general, confidence does not have an anti-monotone property
 $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$
- But confidence of rules generated from the same itemset has an anti-monotone property

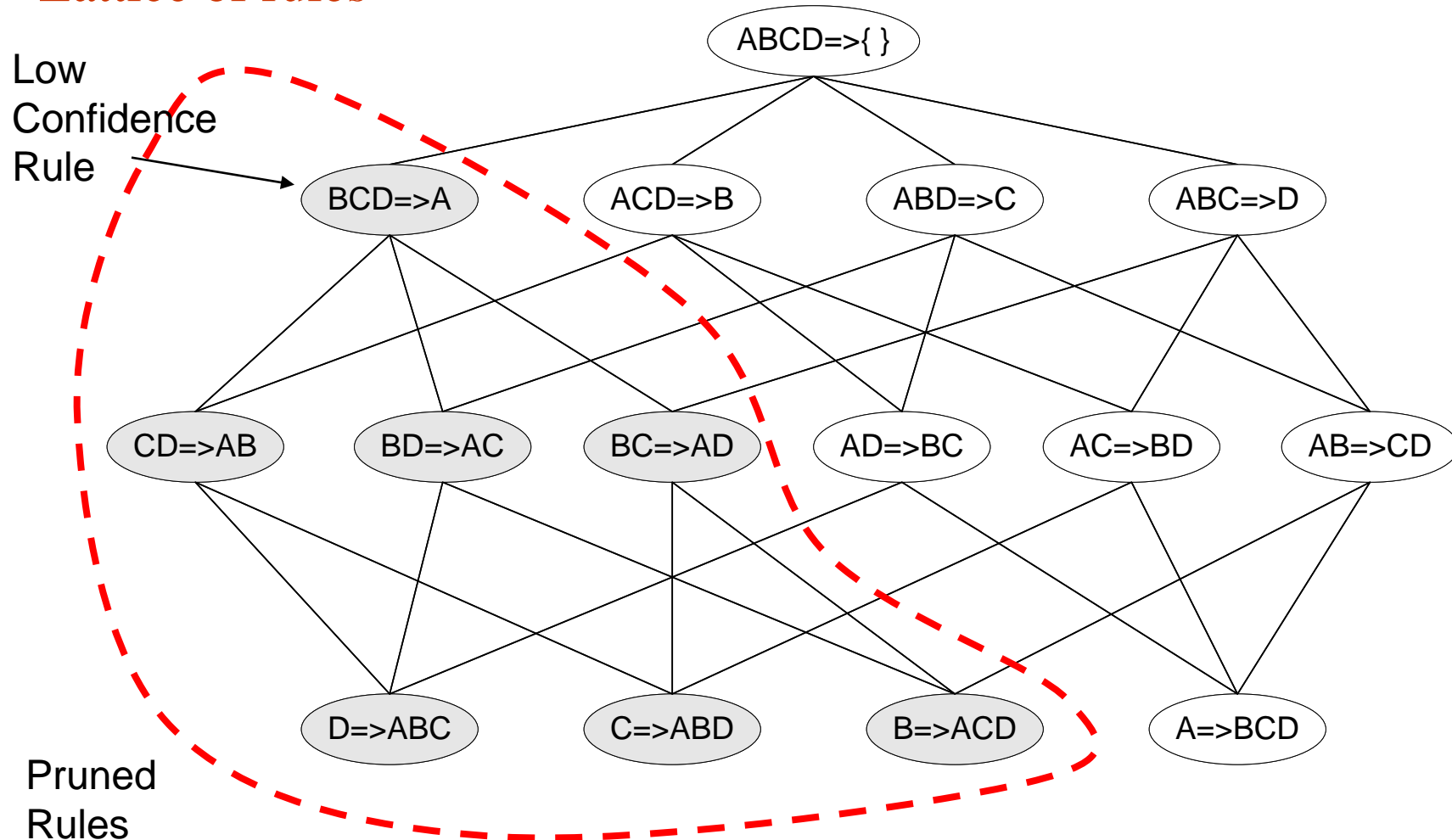
- E.g., Suppose $\{A, B, C, D\}$ is a frequent 4-itemset:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

- Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation in Apriori Algorithm

Lattice of rules



- **Frequent Itemsets, Association Rules**
- **Apriori Algorithm**
- **Compact Representation of Frequent Itemsets**
- **FP-Growth Algorithm: An Alternative Frequent Itemset Generation Algorithm**
- **Evaluation of Association Patterns**

Compact Representation of Frequent Itemsets

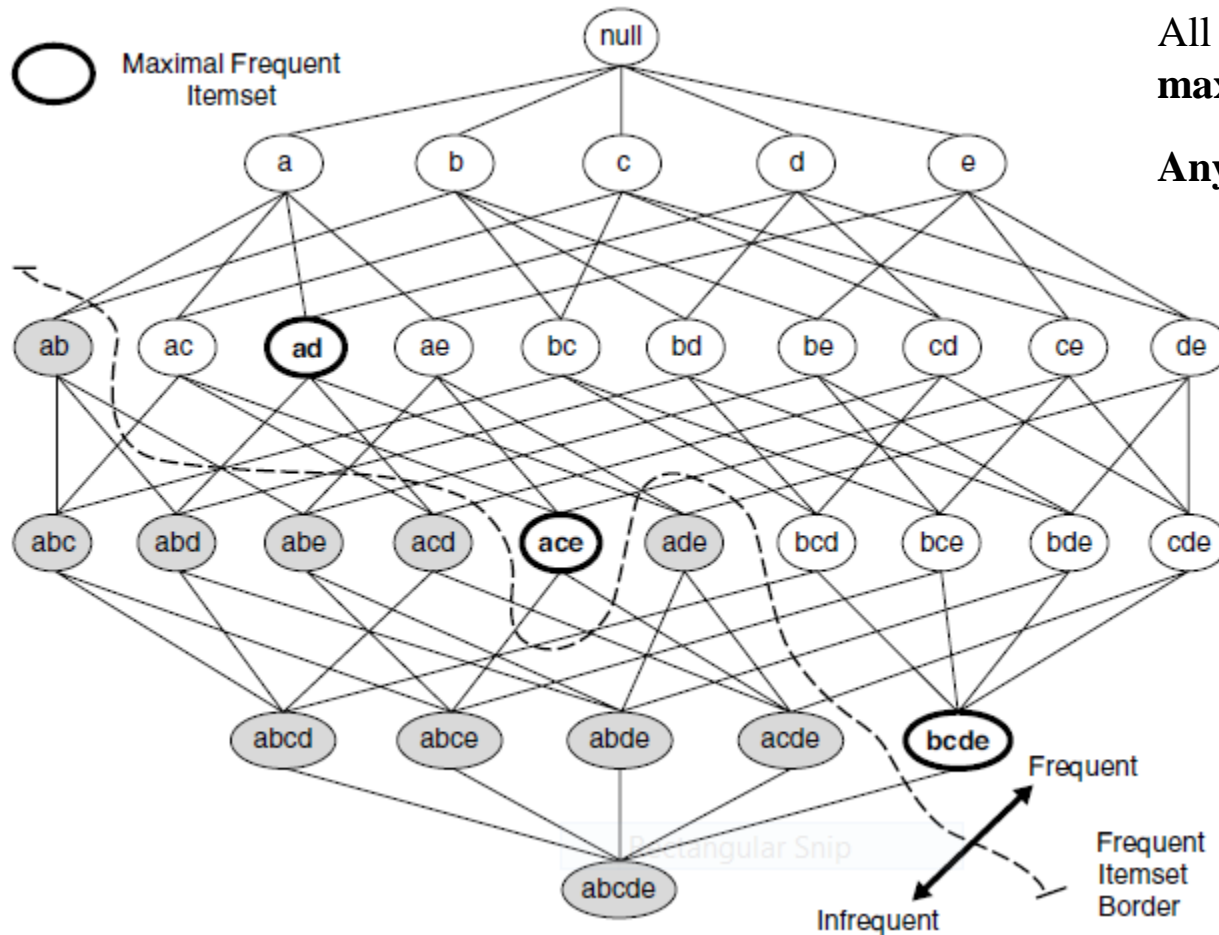
- The number of frequent itemsets produced from a transaction data set can be very large.
- Some produced itemsets can be redundant because they have identical support as their supersets
- It is useful to identify a small representative set of itemsets from which all other frequent itemsets can be derived. ➔ Need a compact representation
 - **Maximal Frequent Itemsets** and
 - **Closed Frequent Itemsets**

Maximal Frequent Itemsets

Maximal Frequent Itemset: A maximal frequent itemset is defined as a frequent itemset for which none of its immediate supersets are frequent.

- Maximal frequent itemsets effectively provide a compact representation of frequent itemsets.
- Maximal frequent itemsets form the smallest set of itemsets from which all frequent itemsets can be derived.

Maximal Frequent Itemsets



All frequent itemsets can be derived from maximal frequent itemsets **ad**, **ace**, **bcde**.

Any frequent itemset \subseteq **a maximal frequent itemset**

Maximal Frequent Itemsets

- Despite providing a compact representation, maximal frequent itemsets do not contain the support information of their subsets.
- For example, the support of the maximal frequent itemsets $\{a, c, e\}$, $\{a, d\}$, and $\{b, c, d, e\}$ do not provide any hint about the support of their subsets.
- An additional pass over the data set is therefore needed to determine the support counts of the non-maximal frequent itemsets.
- It might be desirable to have a minimal representation of frequent itemsets that preserves the support information. ➔ **Closed Frequent Itemsets**

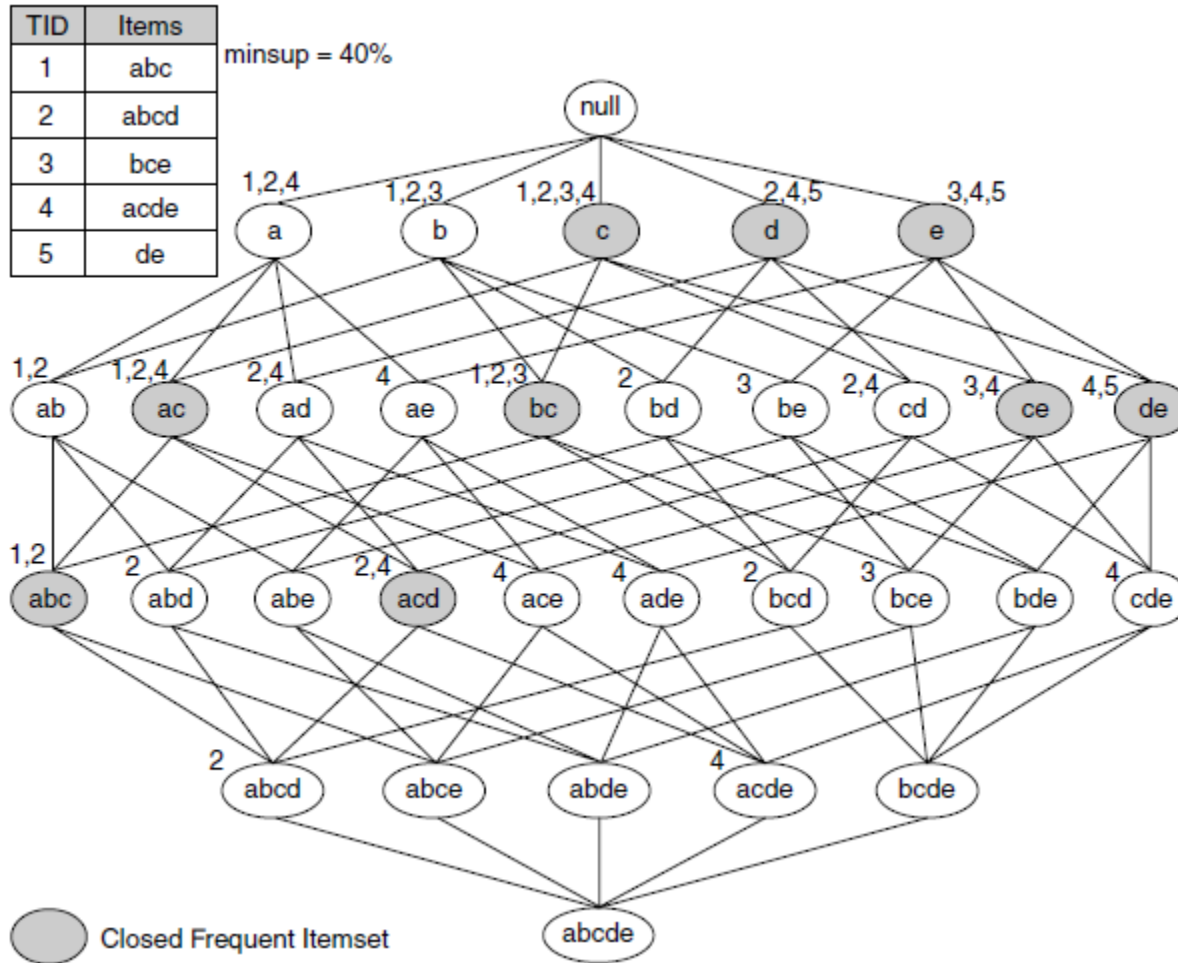
Closed Frequent Itemsets

Closed Itemset: An itemset X is **closed** if none of its immediate supersets has exactly the same support count as X .

- **Closed itemsets** provide a minimal representation of itemsets without losing their support information.
- Put another way, X is not closed if at least one of its immediate supersets has the same support count as X .

Closed Frequent Itemset: An itemset is a closed frequent itemset if it is **closed** and its support is greater than or equal to *minsup*.

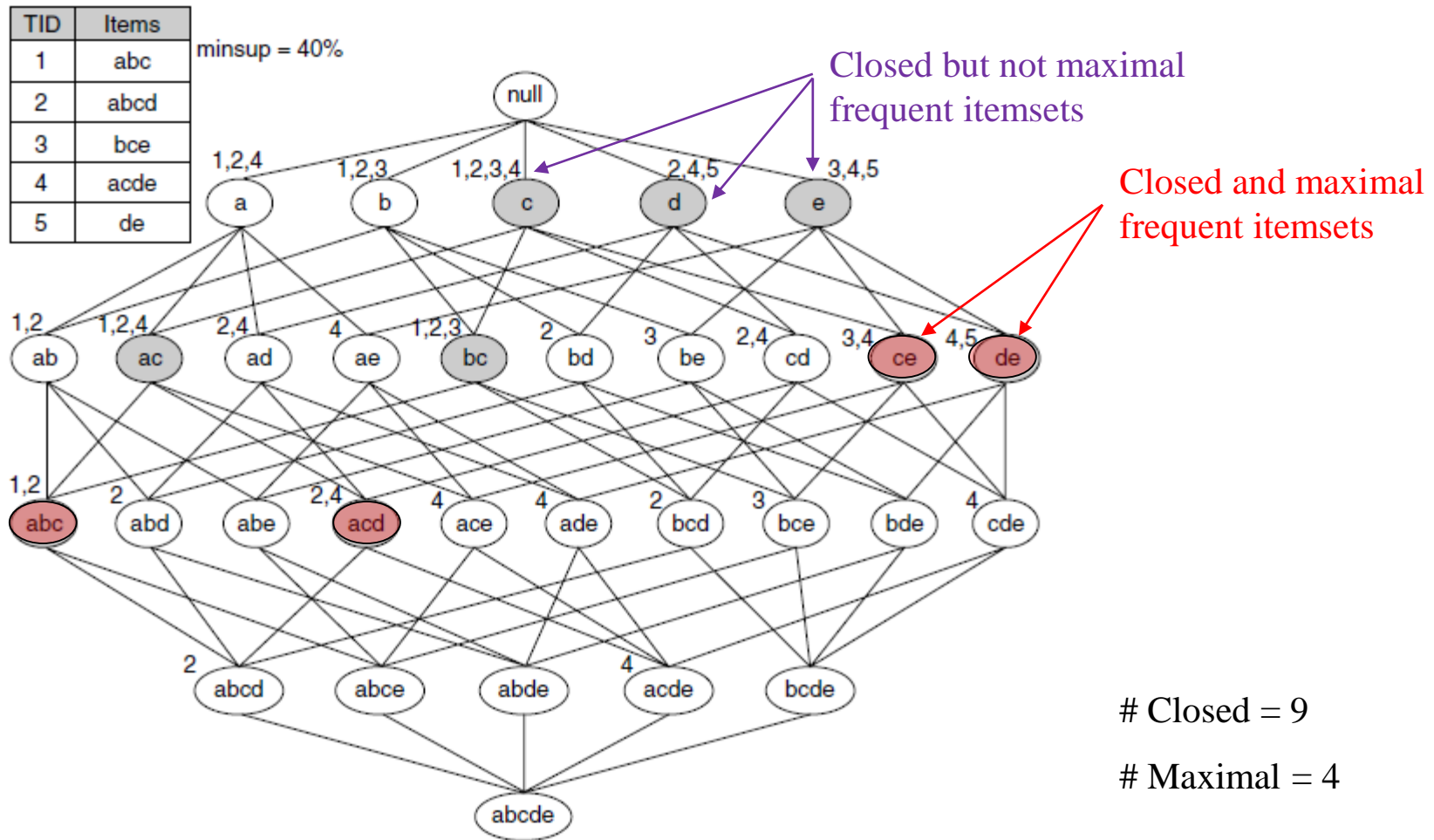
Closed Frequent Itemsets



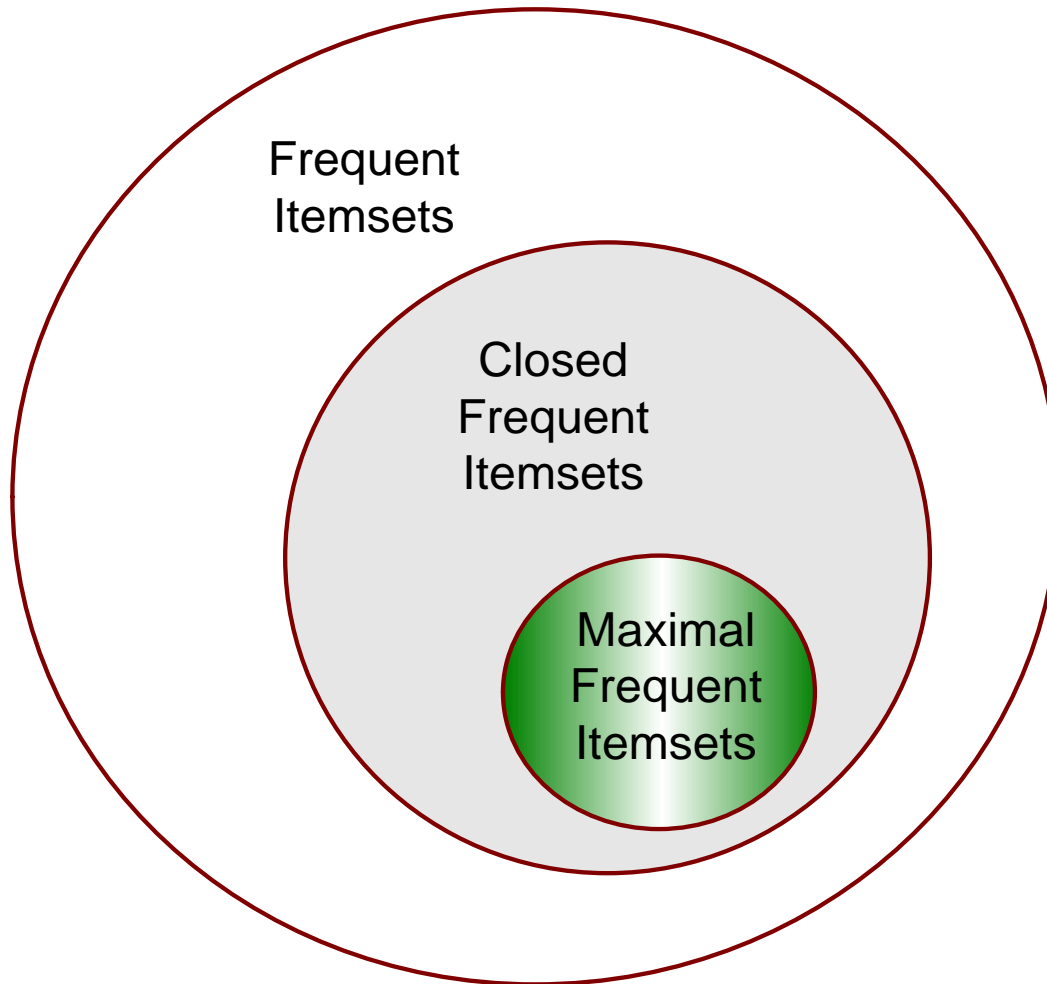
All subsets of a closed frequent itemset are frequent and their supports is greater than or equal to the support of that closed frequent itemset.

For example, all subsets of a closed frequent itemset **abc** are frequent and their supports \geq support of **abc**.

Maximal vs Closed Itemsets



Maximal vs Closed Itemsets



- **Frequent Itemsets, Association Rules**
- **Apriori Algorithm**
- **Compact Representation of Frequent Itemsets**
- **FP-Growth Algorithm: An Alternative Frequent Itemset Generation Algorithm**
- **Evaluation of Association Patterns**

FP-Growth (Frequent Pattern Growth) Algorithm

- **FP-growth algorithm** that takes a radically different approach to discovering frequent itemsets.
 - The algorithm does not subscribe to the generate-and-test paradigm of Apriori
- **FP-growth algorithm** encodes the data set using a compact data structure called an **FP-tree** and extracts frequent itemsets directly from this structure.
 - Use a compressed representation of the database using an FP-tree
 - Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets

FP-Tree Construction

- An FP-tree is a compressed representation of the input data.
- It is constructed by reading the data set one transaction at a time and mapping each transaction onto a path in the FP-tree.
 - Different transactions can have several items in common, their paths may overlap.
 - The more the paths overlap with one another, the more compression we can achieve using the FP-tree structure.

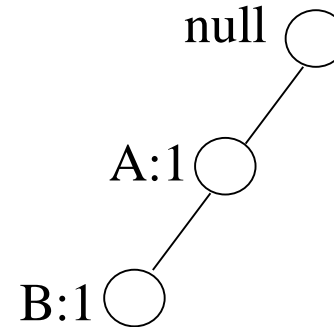
FP-Tree Construction

- Each node in the tree contains the label of an item along with a counter that shows the number of transactions mapped onto the given path.
 - Initially, the FP-tree contains only the root node represented by the null symbol.
 - Every transaction maps onto one of the paths in the FP-tree.
- The size of an FP-tree is typically smaller than the size of the uncompressed data because many transactions in market basket data often share a few items in common.
 - best-case scenario, all transactions have same set of items
 - ➔ FP-tree contains only a single branch.
 - worst-case scenario happens when every transaction has a unique set of items
 - ➔ FP-tree is effectively the same as the size of the original data.
 - physical storage requirement for FP-tree is higher because it requires additional space to store pointers between nodes and counters for each item.

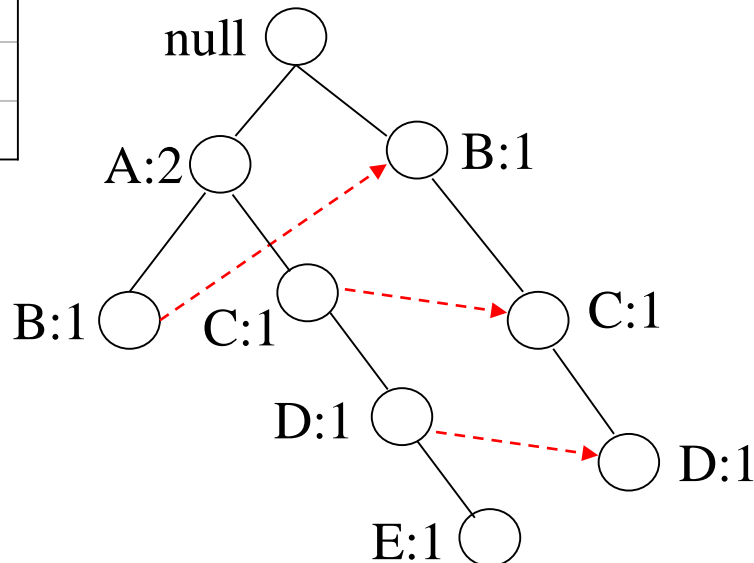
FP-Tree Construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

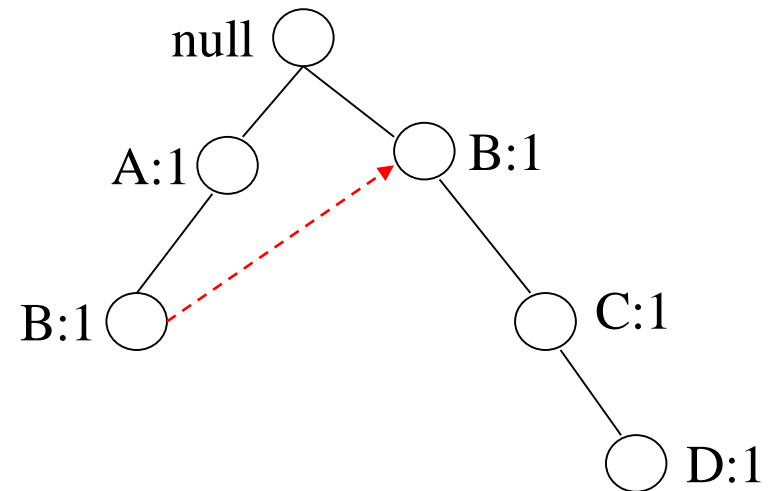
After reading TID=1:



After reading TID=3:



After reading TID=2:



FP-Tree Construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

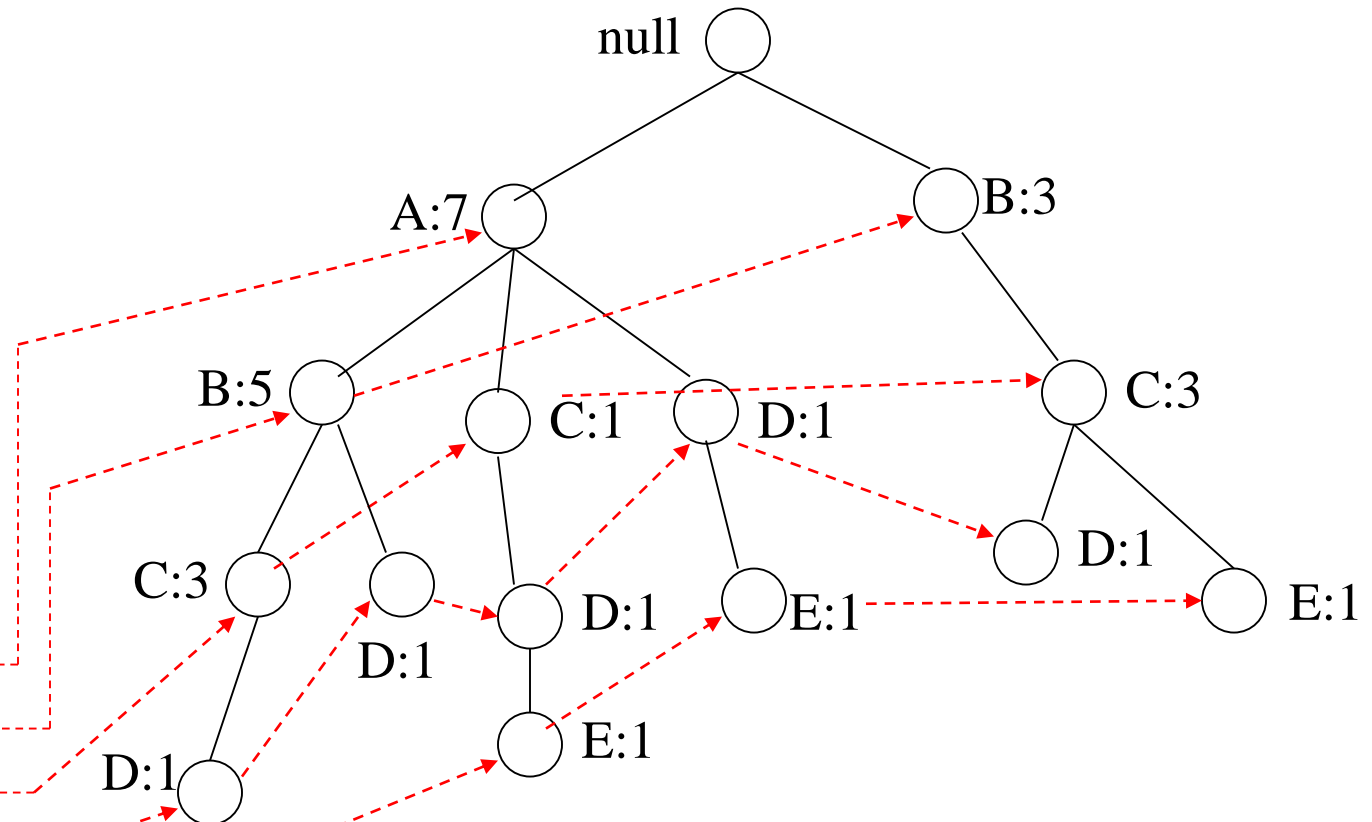
Transaction
Database

After reading all transactions:

Header table

Item	Pointer
A	
B	
C	
D	
E	

sorted



Pointers are used to assist frequent
itemset generation

Frequent Itemset Generation in FP-Growth Algorithm

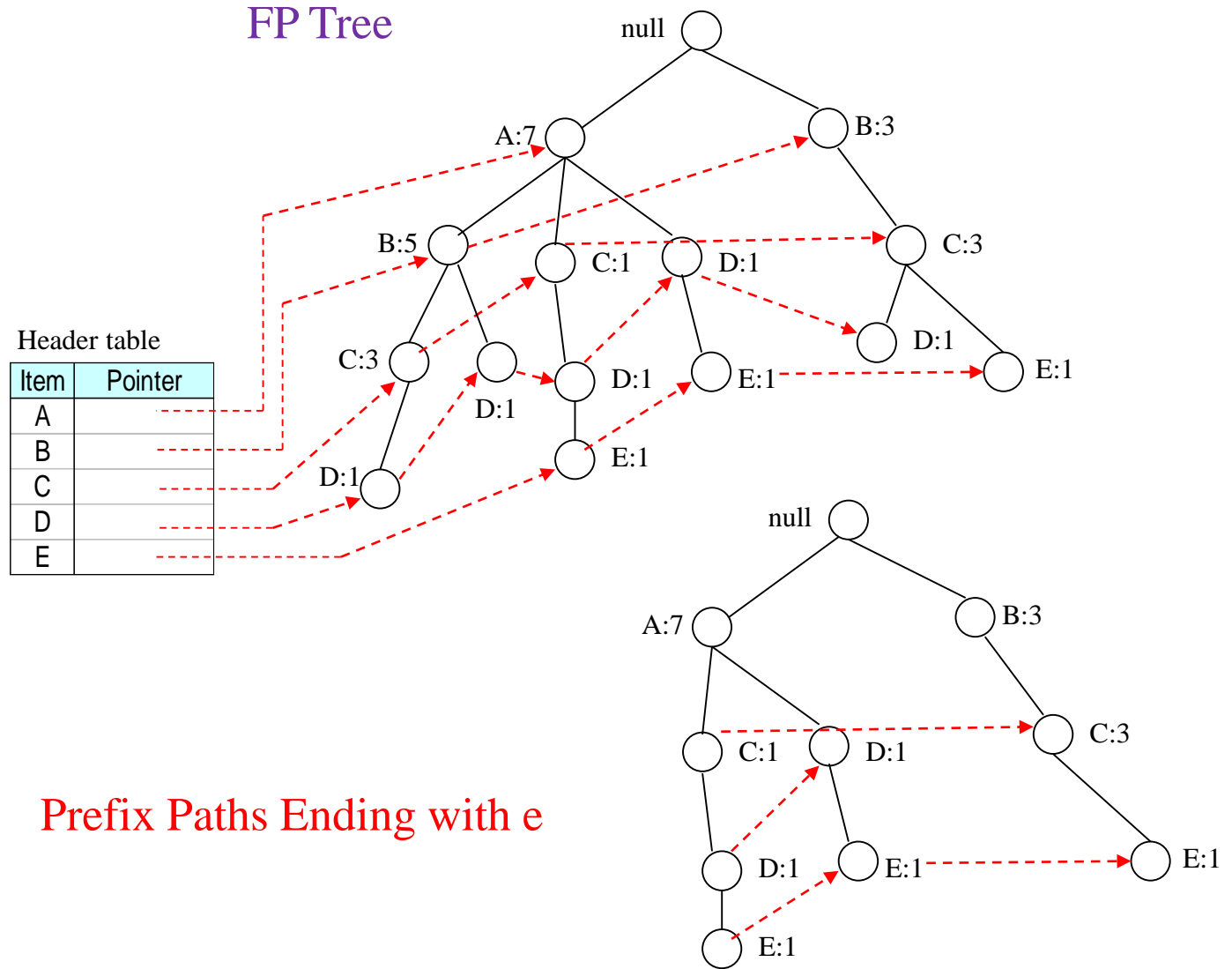
- FP-growth is an algorithm that generates frequent itemsets from an FP-tree by exploring the tree in a bottom-up fashion.
 - This bottom-up strategy for finding frequent itemsets ending with a particular item is equivalent to the suffix-based approach
 - Since every transaction is mapped onto a path in the FP-tree, we can derive the frequent itemsets ending with a particular item, say **e**, by examining only the paths containing node **e**.
 - The algorithm looks for frequent itemsets ending in **e** first, followed by **d**, **c**, **b**, and finally, **a**.
- FP-growth finds all the frequent itemsets ending with a particular suffix by employing a divide-and-conquer strategy to split the problem into smaller subproblems.
 - To find all frequent itemsets ending in **e**, we must first check whether the itemset **{e}** itself is frequent.
 - If it is frequent, we consider the subproblem of finding frequent itemsets ending in **de**, followed by **ce**, **be**, and **ae**.
 - In turn, each of these subproblems are further decomposed into smaller subproblems.
 - By merging the solutions obtained from the subproblems, all the frequent itemsets ending in **e** can be found.

Finding Frequent Itemsets Ending with **e**

1. The first step is to gather all the paths containing node **e**. These initial paths are called *prefix paths*
2. From the prefix paths, the support count for **e** is obtained by adding the support counts associated with node **e**. Assuming that the minimum support count is 2, **{e}** is declared a frequent itemset because its support count is 3.
3. Because **{e}** is frequent, the algorithm has to solve the subproblems of finding frequent itemsets ending in **de**, **ce**, **be**, and **ae**. Before solving these subproblems, it must first convert the prefix paths into a *conditional FP-tree*, which is structurally similar to an FP-tree, except it is used to find frequent itemsets ending with a particular suffix.
 - First, the support counts along the prefix paths must be updated because some of the counts include transactions that do not contain item **e**.
 - The prefix paths are truncated by removing the nodes for **e**.
 - After updating the support counts along the prefix paths, some of the items may no longer be frequent
 - the node b appears only once and has a support count equal to 1, which means that there is only one transaction that contains both b and e. Item b can be safely ignored from subsequent analysis because all itemsets ending in be must be infrequent.
4. FP-growth uses the conditional FP-tree for **e** to solve the subproblems of finding frequent itemsets ending in **de**, **ce**, and **ae**.

Prefix Paths Ending with e

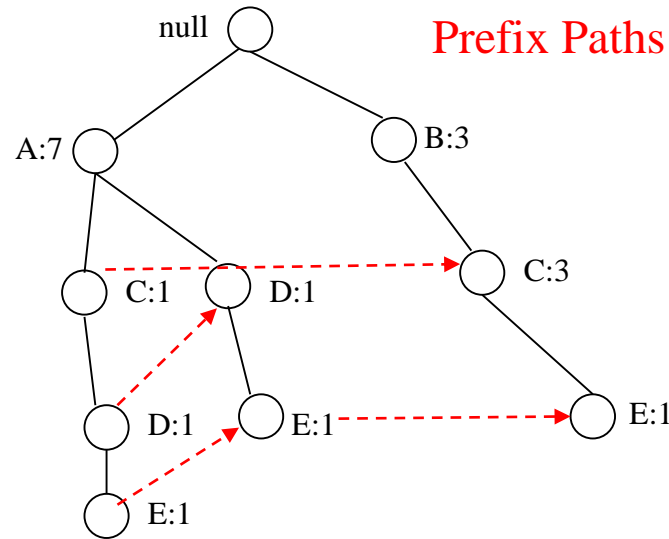
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}



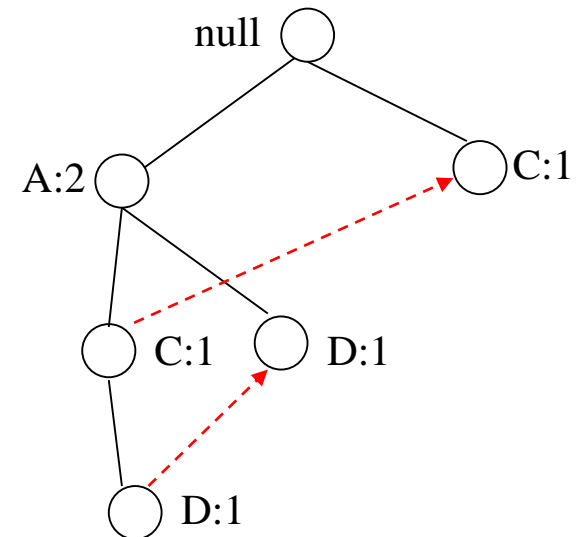
Conditional FP-Tree for e

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2



Conditional FP-Tree for e



To create Conditional FP-Tree for e

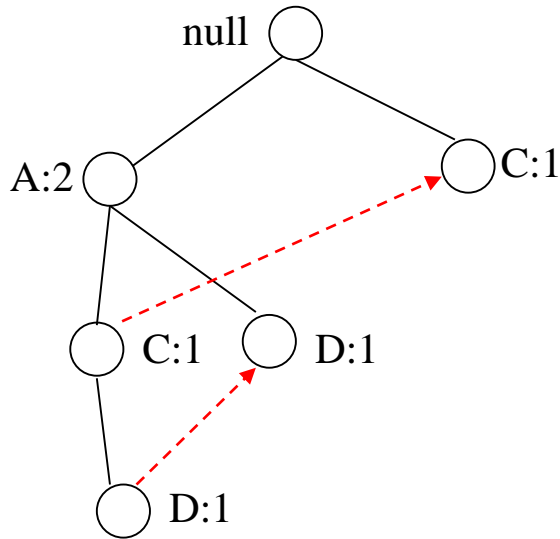
- Update support counts because paths without e are removed
- e is frequent (support=3), Remove e nodes from prefix paths
- Remove infrequent nodes

Conditional FP-Tree for de

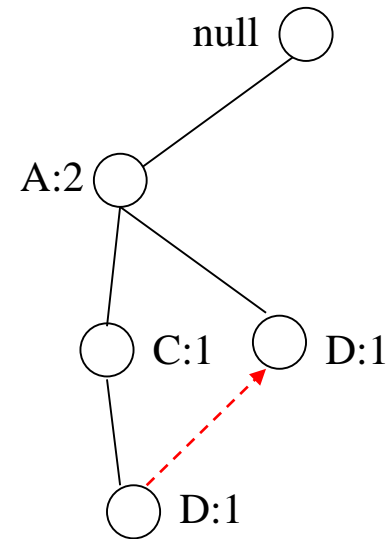
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2

Conditional FP-Tree for e

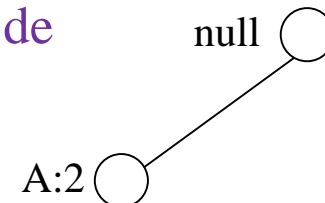


Prefix Paths Ending with de



de is frequent (support=2)

Conditional FP-Tree for de

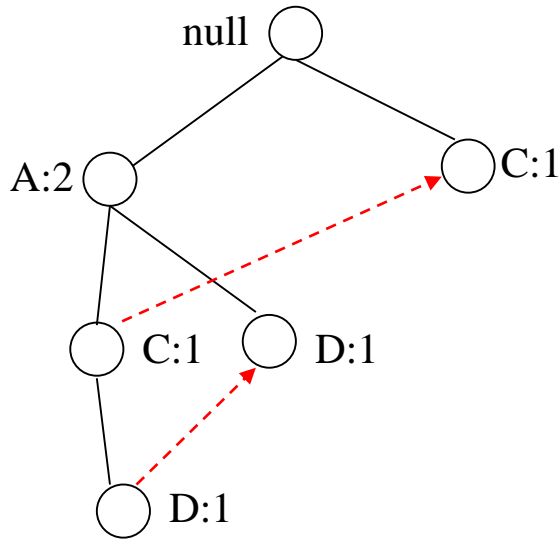


Conditional FP-Tree for ce

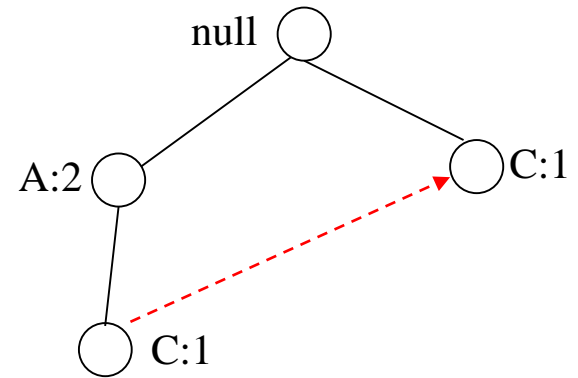
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2

Conditional FP-Tree for e

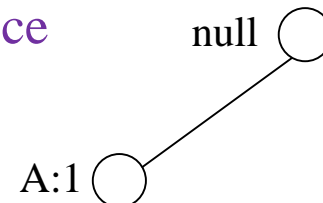


Prefix Paths Ending with ce



ce is frequent (support=2)

Conditional FP-Tree for ce

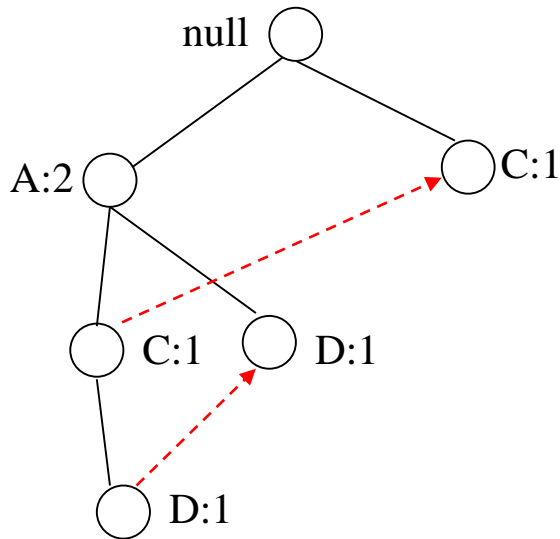


Conditional FP-Tree for ae

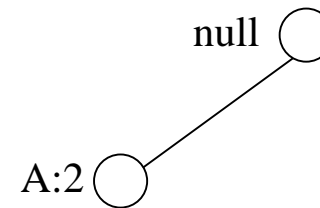
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2

Conditional FP-Tree for e



Prefix Paths Ending with ae



ae is frequent (support=2)

Conditional FP-Tree for ae

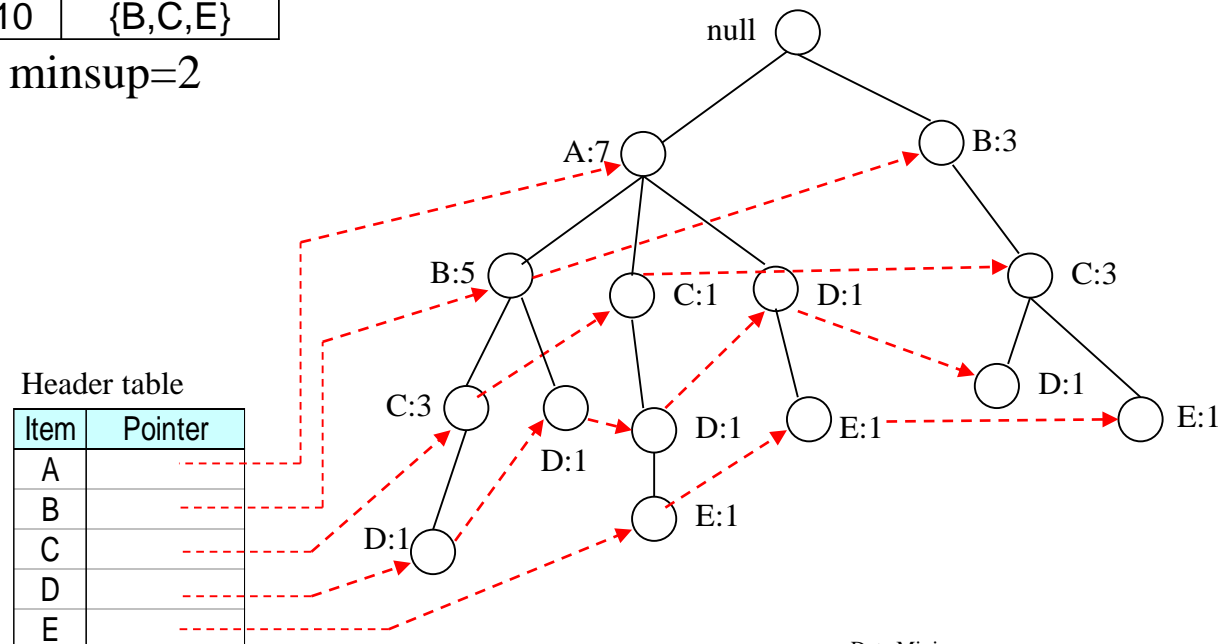


Frequent Itemsets Ordered by Suffixes

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2

Suffix	Frequent Itemsets
E	{E}, {D,E}, {A,D,E}, {C,E}, {A, E},
D	{D}, {C,D}, {B,C,D}, {A,C,D}, {B,D}, {A,B,D}, {A,D}
C	{C}, {B,C}, {A,B,C}, {A,C}
B	{B}, {A,B}
A	{A}



- **Frequent Itemsets, Association Rules**
- **Apriori Algorithm**
- **Compact Representation of Frequent Itemsets**
- **FP-Growth Algorithm: An Alternative Frequent Itemset Generation Algorithm**
- **Evaluation of Association Patterns**

Evaluation of Association Patterns

- Association rule algorithms tend to produce too many rules
 - many of them are *uninteresting* or *redundant*
 - $\{A,B\} \rightarrow \{D\}$ is **Redundant** if $\{A,B,C\} \rightarrow \{D\}$ and $\{A,B\} \rightarrow \{D\}$ have same support & confidence
 - An association rule $X \rightarrow Y$ is **redundant** if there exists another rule $X' \rightarrow Y'$, where X is a subset of X' and Y is a subset of Y' , such that the support and confidence for both rules are identical.
- *Interestingness measure* can be used to prune/rank the derived patterns
- In the original formulation of association rules, support & confidence are the only measures used

Computing Interestingness Measure

- Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	\overline{Y}	
X	f_{11}	f_{10}	f_{1+}
\overline{X}	f_{01}	f_{00}	f_{0+}
	f_{+1}	f_{+0}	$ T $

f_{11} : support of X and Y

f_{10} : support of X and \overline{Y}

f_{01} : support of \overline{X} and Y

f_{00} : support of \overline{X} and \overline{Y}

Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea \rightarrow Coffee

Confidence = $P(\text{Coffee}|\text{Tea}) = 0.75 = \text{support}(\{\text{Tea}, \text{Coffee}\}) / \text{support}(\{\text{Tea}\})$

but $P(\text{Coffee}) = 0.9$

\Rightarrow Although confidence is high, rule is misleading

$\Rightarrow P(\text{Coffee}|\overline{\text{Tea}}) = 0.9375$

Measure for Association Rules

- So, what kind of rules do we really want?
 - Confidence($X \rightarrow Y$) should be sufficiently high
 - To ensure that people who buy X will more likely buy Y than not buy Y
 - Confidence($X \rightarrow Y$) > support(Y)
 - Otherwise, rule will be misleading because having item X actually reduces the chance of having item Y in the same transaction
 - Is there any measure that capture this constraint?
 - Answer: Yes. There are many of them.

Statistical Independence

- Population of 1000 students
 - 600 students know how to swim (S)
 - 700 students know how to bike (B)
 - 420 students know how to swim and bike (S,B)
 - $P(S \wedge B) = 420/1000 = 0.42$
 - $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$
 - $P(S \wedge B) = P(S) \times P(B) \Rightarrow$ Statistical independence
 - $P(S \wedge B) > P(S) \times P(B) \Rightarrow$ Positively correlated
 - $P(S \wedge B) < P(S) \times P(B) \Rightarrow$ Negatively correlated

Statistical-Based Measures for Interestingness

- Statistical-Based Measures use statistical dependence information.
- Two of them are **Lift** and **Interest** (they are equal).

$$\text{Lift} = P(Y|X) / P(Y)$$

$$\text{Interest} = P(X,Y) / P(X) P(Y)$$

$$\begin{aligned}\text{Lift}(A,B) &= \text{conf}(A \rightarrow B) / \text{support}(B) \\ &= \text{support}(A \cup B) / \text{support}(A) \text{support}(B)\end{aligned}$$

$$\text{Interest}(A,B) = \text{support}(A \cup B) / \text{support}(A) \text{support}(B)$$

$$\text{Interest}(A,B) \begin{cases} = 1 & \text{if } A \text{ and } B \text{ are independent} \\ > 1 & \text{if } A \text{ and } B \text{ are positively correlated} \\ < 1 & \text{if } A \text{ and } B \text{ are negatively correlated} \end{cases}$$

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea \rightarrow Coffee

Confidence = $P(\text{Coffee}|\text{Tea}) = 0.75 = \text{support}(\{\text{Tea}, \text{Coffee}\}) / \text{support}(\{\text{Tea}\})$

but $P(\text{Coffee}) = 0.9$

$\rightarrow \text{Lift} = 0.75/0.9 = 0.8333$ (< 1 , therefore is negatively correlated)

Example: Lift/Interest

- *play basketball* \Rightarrow *eat cereal* [40%, 66.7%] is misleading
 - The overall % of students eating cereal is 75% > 66.7%.
- *play basketball* \Rightarrow *not eat cereal* [20%, 33.3%] is more accurate, although with lower support and confidence

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

$$\text{lift}(B, C) = \frac{2000 / 5000}{3000 / 5000 * 3750 / 5000} = 0.89$$

$$\text{lift}(B, \neg C) = \frac{1000 / 5000}{3000 / 5000 * 1250 / 5000} = 1.33$$

Limitations of Interest Factor

- We expect the words *data* and *mining* to appear together more frequently than the words *compiler* and *mining* in a collection of computer science articles.

	p	\bar{p}	
q	880	50	930
\bar{q}	50	20	70
	930	70	1000

	r	\bar{r}	
s	20	50	70
\bar{s}	50	880	930
	70	930	1000

Contingency tables for word pairs $\{p, q\}$ and $\{r, s\}$.

- The interest factor for $\{p, q\}$ is 1.02 and for $\{r, s\}$ is 4.08.
 - Although p and q appear together in 88% of the documents, their interest factor is close to 1, which is the value when p and q are statistically independent.
 - On the other hand, the interest factor for $\{r, s\}$ is higher than $\{p, q\}$ even though r and s seldom appear together in the same document.
 - Confidence is perhaps the better choice in this situation because it considers the association between p and q (94.6%) to be much stronger than that between r and s (28.6%).

Different Measures

- There are lots of measures proposed in the literature
- Some measures are good for certain applications, but not for others
- What criteria should we use to determine whether a measure is good or bad?

#	Measure	Formula
1	ϕ -coefficient	$\frac{P(A,B) - P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j, B_k) - \max_j P(A_j) - \max_k P(B_k)}{2 - \max_j P(A_j) - \max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\bar{A},\bar{B})}{P(A,\bar{B})P(\bar{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\bar{A}\bar{B}) - P(A,\bar{B})P(\bar{A},B)}{P(A,B)P(\bar{A}\bar{B}) + P(A,\bar{B})P(\bar{A},B)} = \frac{\alpha - 1}{\alpha + 1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\bar{A}\bar{B})} - \sqrt{P(A,\bar{B})P(\bar{A},B)}}{\sqrt{P(A,B)P(\bar{A}\bar{B})} + \sqrt{P(A,\bar{B})P(\bar{A},B)}} = \frac{\sqrt{\alpha} - 1}{\sqrt{\alpha} + 1}$
6	Kappa (κ)	$\frac{P(A,B) + P(\bar{A},\bar{B}) - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i, B_j) \log \frac{P(A_i, B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i) \log P(A_i), -\sum_j P(B_j) \log P(B_j))}$
8	J-Measure (J)	$\max \left(P(A, B) \log \left(\frac{P(B A)}{P(B)} \right) + P(\bar{A}\bar{B}) \log \left(\frac{P(\bar{B} \bar{A})}{P(\bar{B})} \right), \right.$ $\left. P(A, B) \log \left(\frac{P(A B)}{P(A)} \right) + P(\bar{A}B) \log \left(\frac{P(\bar{A} \bar{B})}{P(\bar{A})} \right) \right)$
9	Gini index (G)	$\max \left(P(A)[P(B A)^2 + P(\bar{B} A)^2] + P(\bar{A})[P(B \bar{A})^2 + P(\bar{B} \bar{A})^2] \right.$ $\left. - P(B)^2 - P(\bar{B})^2, \right.$ $\left. P(B)[P(A B)^2 + P(\bar{A} B)^2] + P(\bar{B})[P(A \bar{B})^2 + P(\bar{A} \bar{B})^2] \right.$ $\left. - P(A)^2 - P(\bar{A})^2 \right)$
10	Support (s)	$P(A, B)$
11	Confidence (c)	$\max(P(B A), P(A B))$
12	Laplace (L)	$\max \left(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2} \right)$
13	Conviction (V)	$\max \left(\frac{P(A)P(\bar{B})}{P(\bar{A}B)}, \frac{P(B)P(\bar{A})}{P(\bar{B}A)} \right)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A, B) - P(A)P(B)$
17	Certainty factor (F)	$\max \left(\frac{P(B A) - P(B)}{1 - P(B)}, \frac{P(A B) - P(A)}{1 - P(A)} \right)$
18	Added Value (AV)	$\max(P(B A) - P(B), P(A B) - P(A))$
19	Collective strength (S)	$\frac{P(A,B) + P(\bar{A}\bar{B})}{P(A)P(B) + P(\bar{A})P(\bar{B})} \times \frac{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A,B) - P(\bar{A}\bar{B})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A) + P(B) - P(A,B)}$
21	Klogsen (K)	$\sqrt{P(A,B) \max(P(B A) - P(B), P(A B) - P(A))}$

Properties of A Good Measure

3 properties a good measure M must satisfy:

- $M(A,B) = 0$ if A and B are statistically independent
- $M(A,B)$ increase monotonically with $P(A,B)$ when $P(A)$ and $P(B)$ remain unchanged
- $M(A,B)$ decreases monotonically with $P(A)$ [or $P(B)$] when $P(A,B)$ and $P(B)$ [or $P(A)$] remain unchanged