Qiskit Notes - Circuits

In [1]:
```
from IPython.display import HTML, display

def set_background(color):
    script = (
        "var cell = this.closest('.jp-CodeCell');"
        "var editor = cell.querySelector('.jp-Editor');"
        "editor.style.background='{}';"
        "this.parentNode.removeChild(this)"
    ).format(color)

    display(HTML('<img src onerror="{}">'.format(script)))
```

Circuit Basics

A basic workflow using Qiskit consists of two stages: **Build** and **Run**.

Build allows you to make different quantum circuits that represent the problem you are solving

Run that allows you to run them on different backends

In [2]:
```
import numpy as np
from qiskit import QuantumCircuit
```

Building the Circuit

1st: Create the circuit

In [3]:
```
circ = QuantumCircuit(3)
```

After creating the circuit with its registers, one can add gate (**operations**) to manipulate the register.

Ex: three-qubit GHZ state

$$|\psi\rangle = (|000\rangle + |111\rangle)/\sqrt(2)$$

By default, qubits are initialized to $|0\rangle$.

$$|000\rangle \rightarrow (H \otimes I \otimes I)|000\rangle = (|000\rangle + |100\rangle)/\sqrt(2) \rightarrow \mathrm{CNOT}_{01}(|000\rangle + |100\rangle)/\sqrt(2) = (|000\rangle + |110\rangle)/\sqrt(2) \rightarrow \mathrm{CNOT}_{02}(|000\rangle + |100\rangle)/\sqrt(2) = (|000\rangle + |111\rangle)/\sqrt(2)$$

In [4]:
```
#add H-gate to put qubit in superposition
circ.h(0)

circ.cx(0,1)

circ.cx(0,2)
```

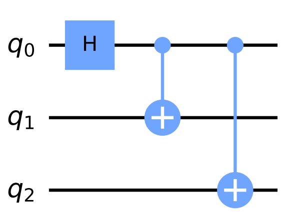
Out[4]:
```
<qiskit.circuit.instructionset.InstructionSet at 0x7fb1b80738e0>
```

Visualize Circuit

`QuantumCircuit.draw()`

In [5]:
```
circ.draw('mpl')
```

Out[5]:



When representing the state of a multi-qubit system, the tensor order used in Qiskit is different than that used in most physics textbooks. Suppose there are n qubits, and qubit j is labeled Q_j; In Qiskit, n^{th} qubit is on the left side of the tensor product, with the basis vectors labeled $Q_{n-1} \otimes \cdots \otimes Q_1 \otimes Q_0$. For instance, if $q_0, q_1,$ and $q2$ are on the states $0, 0, 1$, textbooks would represent this as $|001\rangle$, while in Qiskit this is represented as $|100\rangle$. This affects matrix representation. C_X, for instance, becomes

$$C_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Simulating circuits

To simulate a circuit we use the `quant_info` module in Qiskit.

The simulator returns the quantum state: 2^n vector in C, n qubits.

Two stages to the simulator:

1. set the input state
2. evolve the state by the quantum circuit

In [6]:

```
from qiskit.quantum_info import Statevector

## Set the initial state of the sim using from_int
n=3
state = Statevector.from_int(0,2**n)

## Evolve the state by the QC
state = state.evolve(circ)

## draw using latex
state.draw('latex')
```

Out[6]:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In [7]:

```
# or, alternatively
from qiskit.visualization import array_to_latex

array_to_latex(state)
```

Out[7]:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

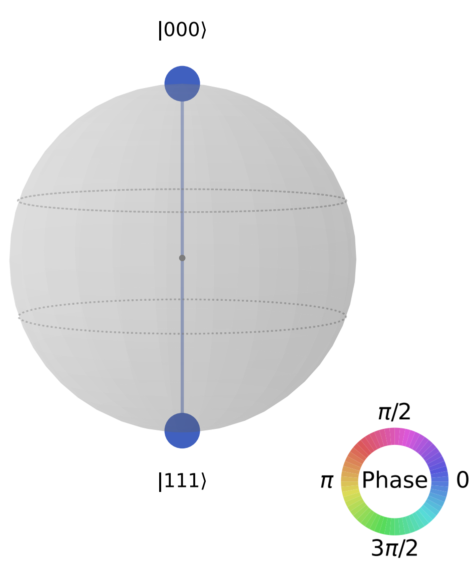
Visualization toolbox

- plot qsphere
- hinton representing the R and C components of the state density matrix ρ.

In [8]:

```
state.draw('qsphere')
```

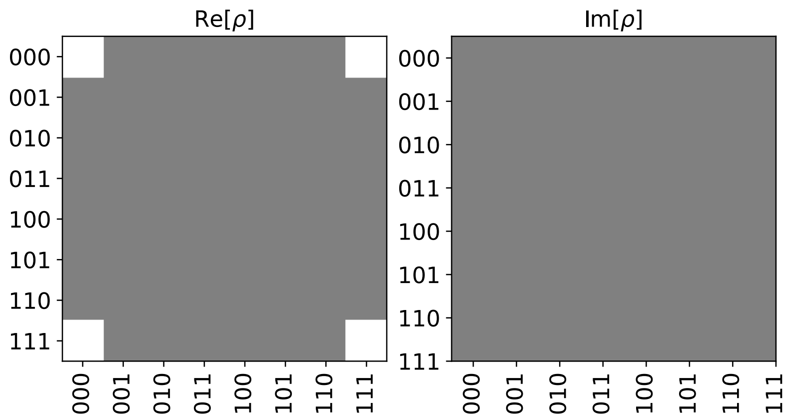
Out[8]:



In [9]:

```
state.draw('hinton')
```

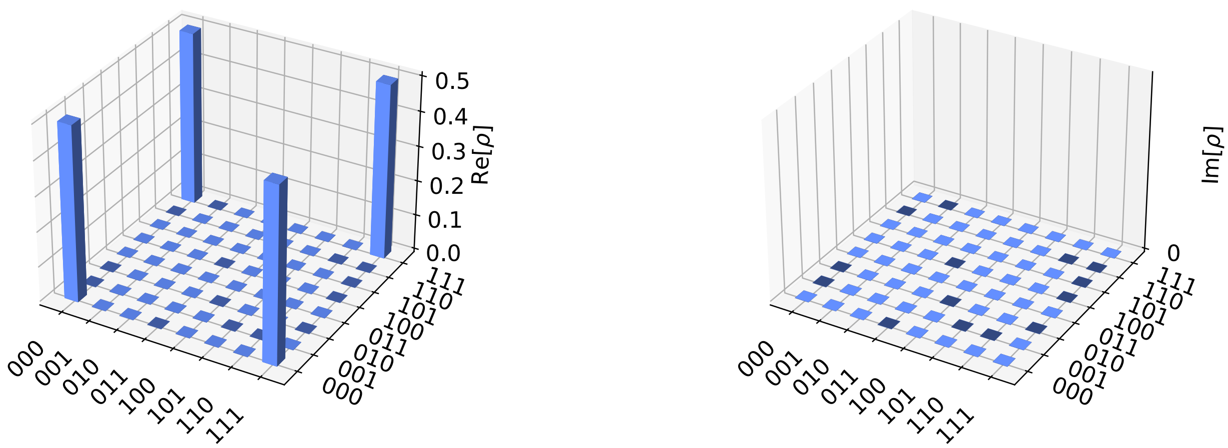
Out[9]:



In [10]:

```
from qiskit.visualization import plot_state_city
plot_state_city(state)
```

Out[10]:



Unitary representation of a circuit

Operator method to make an unitary operator for the circuit: $2^n \times 2^n$ matrix.

In [11]:

```
from qiskit.quantum_info import Operator

U = Operator(circ)

U.data
```

Out[11]:

```
array([[ 0.70710678+0.j,  0.70710678+0.j,  0.        +0.j,
         0.        +0.j,  0.        +0.j,  0.        +0.j,
         0.        +0.j,  0.        +0.j],
       [ 0.        +0.j,  0.        +0.j,  0.        +0.j,
         0.        +0.j,  0.        +0.j,  0.        +0.j,
         0.70710678+0.j, -0.70710678+0.j],
       [ 0.        +0.j,  0.        +0.j,  0.70710678+0.j,
         0.70710678+0.j,  0.        +0.j,  0.        +0.j,
         0.        +0.j,  0.        +0.j],
       [ 0.        +0.j,  0.        +0.j,  0.        +0.j,
         0.        +0.j,  0.70710678+0.j, -0.70710678+0.j,
         0.        +0.j,  0.        +0.j],
       [ 0.        +0.j,  0.        +0.j,  0.        +0.j,
         0.        +0.j,  0.70710678+0.j,  0.70710678+0.j,
         0.        +0.j,  0.        +0.j],
       [ 0.        +0.j,  0.        +0.j,  0.70710678+0.j,
        -0.70710678+0.j,  0.        +0.j,  0.        +0.j,
         0.        +0.j,  0.        +0.j],
       [ 0.        +0.j,  0.        +0.j,  0.        +0.j,
         0.        +0.j,  0.        +0.j,  0.        +0.j,
         0.70710678+0.j,  0.70710678+0.j],
       [ 0.70710678+0.j, -0.70710678+0.j,  0.        +0.j,
         0.        +0.j,  0.        +0.j,  0.        +0.j,
         0.        +0.j,  0.        +0.j]])
```

In [12]:

```
array_to_latex(U)
```

Out[12]:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

OpenQASM backend

The simulators are useful because they provide info about the **state output** of the ideal circuit and the **matrix representation**. However, in real experiments, we need to **measure** each qubit (usually in the computational basis $|0\rangle$ and $|1\rangle$) to gain information about the state, causing the system to collapse to classical bits.

In the example above, suppose we make measurements of each qubit in the GHZ state:

$$|\psi\rangle = (|000\rangle + |111\rangle)/\sqrt{(2)}$$

and let xyz denote the bitsttring that results (recall that in Qiskit, $x =$ result of q_2, etc.)

Most significant bit (MSB) on the left and Least significant bit (LSB) on the right. This is why QIskit uses a *non-standard tensor product order*.

Probability is given by

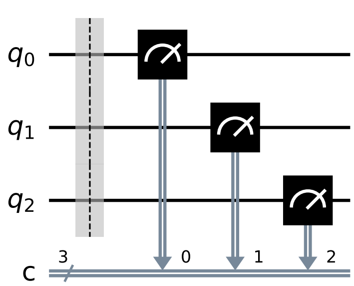
$$P(xyz) = |\langle xyz|\psi\rangle|^2$$

To simulate this, we use a different Aer backend.

In [13]:

```
meas = QuantumCircuit(3,3)
meas.barrier(range(3))

#map the measurement into classical bits
meas.measure(range(3), range(3)) ## q-> c

meas.draw('mpl')

# # The Qiskit circuit object supports composition.
# # Here the meas has to be first and front=True (putting it before)
# # as compose must put a smaller circuit into a larger one.
```

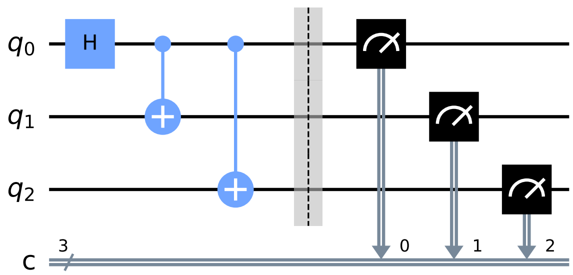
Out[13]:



In [14]:

```
# The Qiskit circuit object supports composition.
# Here the meas has to be first and front=True (putting it before)
# as compose must put a smaller circuit into a larger one.
qc = meas.compose(circ, range(3), front=True)

qc.draw()
```

Out[14]:



This circuit adds a classical register, and three measurements that are used to map the outcome of qubits to the classical bits.

To simulate this circuit, we use the `qasm_simulator` in Qiskit Aer. Each run of this circuit will yield either the bitstring 000 or 111. To build up **statistics about the distribution of the bitstrings** (to, e.g., estimate $Pr(|000\rangle)$), we need to repeat the circuit many times.

The number of times the circuit is repeated can be specified in the **execute function, via the shots keyword**.

In [15]:

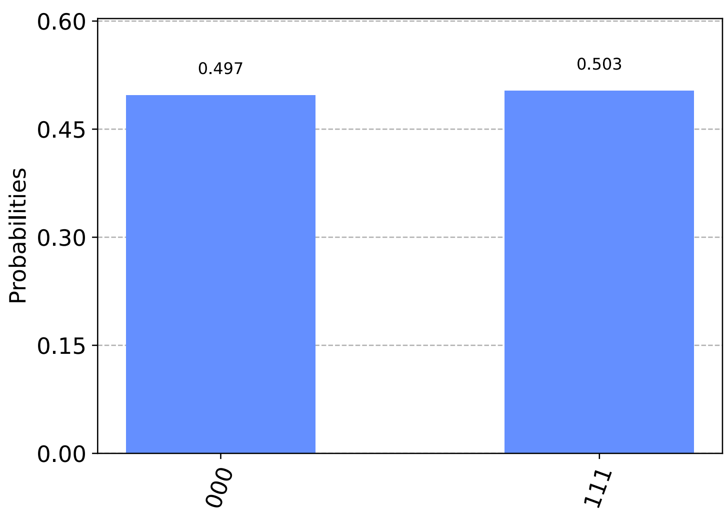
```
# Adding the transpiler to reduce the circuit to QASM instructions
# supported by the backend
from qiskit import transpile

# use Aer's qasm_simulator
from qiskit.providers.aer import QasmSimulator
backend = QasmSimulator()

# first, we need to transpile the instructions of the QC
# to the low-level QASM instructions used by the backend
qc_compiled = transpile(qc, backend)

# Execute the circuit on the qasm simulator
# We've set the n of repeates to 1024, which is the default
job_sim = backend.run(qc_compiled, shots=1024)

result_sim = job_sim.result()

counts = result_sim.get_counts()
print(counts)
```

```
{'000': 509, '111': 515}
```

In [16]:

```
from qiskit.visualization import plot_histogram
plot_histogram(counts)
```

Out[16]:



Qiskit Visualizations

In [17]:

```
from qiskit import *
from qiskit.visualization import plot_histogram
from qiskit.tools.monitor import job_monitor
```

Plot histogram

To visualize the data from a quantum circuit run on a **real device** or `qasm_simulator` : `plot_histogram(data)`

Ex: 2-qubit Bell state

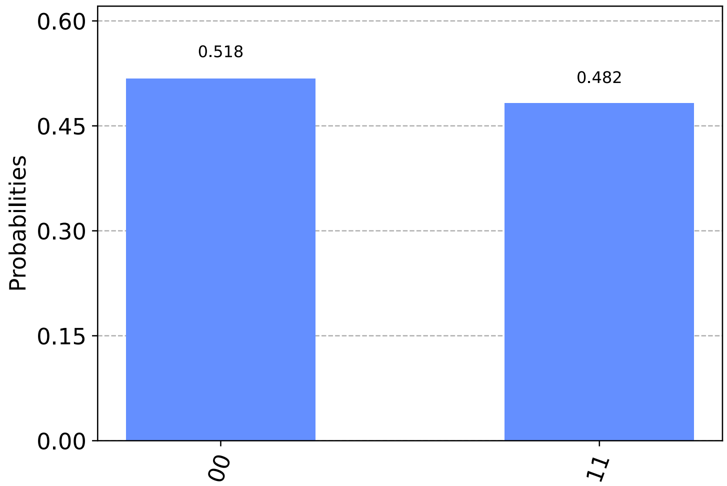
In [18]:

```
bell = QuantumCircuit(2,2)
```

```
bell.h(0)
bell.cx(0,1)

meas = QuantumCircuit(2,2)
meas.measure(range(2),range(2))

# execute the QC
backend = BasicAer.get_backend('qasm_simulator')
circ = bell.compose(meas)
result = backend.run(transpile(circ, backend), shots=1024).result()
counts = result.get_counts()
print(counts)

plot_histogram(counts)
```

{'00': 530, '11': 494}

Out[18]:



Options when plotting a histogram

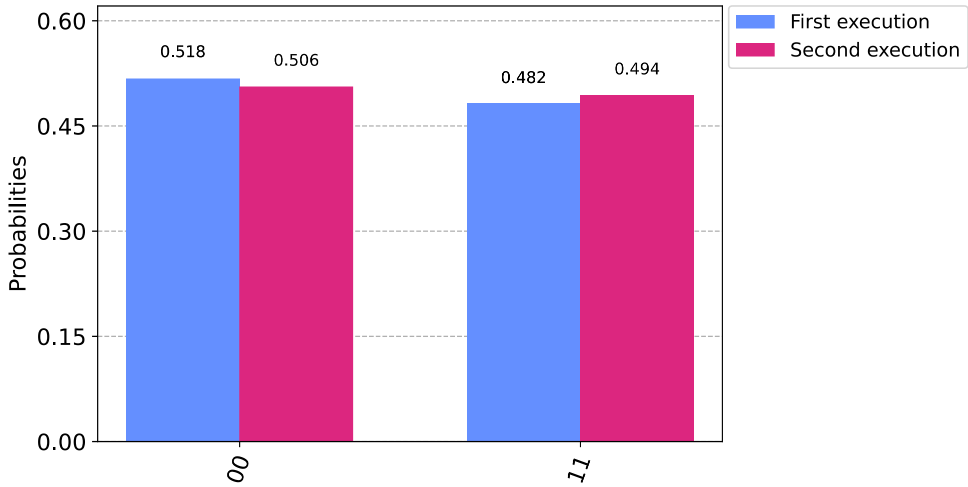
The `plot_histogram()` has a few options to adjust the output graph.

- `legend` kwarg: used to provide a label for the executions. It takes a list of strings use to label each execution's results. This is mostly useful when plotting multiple execution results in the same histogram.
- `sort` kwarg: used to adjust the order the bars in the histogram are rendered. It can be set to either ascending order with `asc` or descending order with `desc` .
- `number_to_keep` kwarg: takes an integer for the number of terms to show, the rest are grouped together in a single bar called rest.
- `color` kwarg: adjusts the color of the bars, which either takes a string or a list of strings for the colors to use for the bars for each execution.
- `bar_labels` kwarg: adjusts whether labels are printed above the bars or not.
- `figsize` kwarg: takes a tuple of the size in inches to make the output figure.

In [19]:

```
# Execute 2-qubit Bell state again

second_result = backend.run(transpile(circ, backend), shots=1024).result()
second_counts = second_result.get_counts()

legend = ['First execution', 'Second execution']
plot_histogram([counts, second_counts], legend=legend)
```

Out[19]:

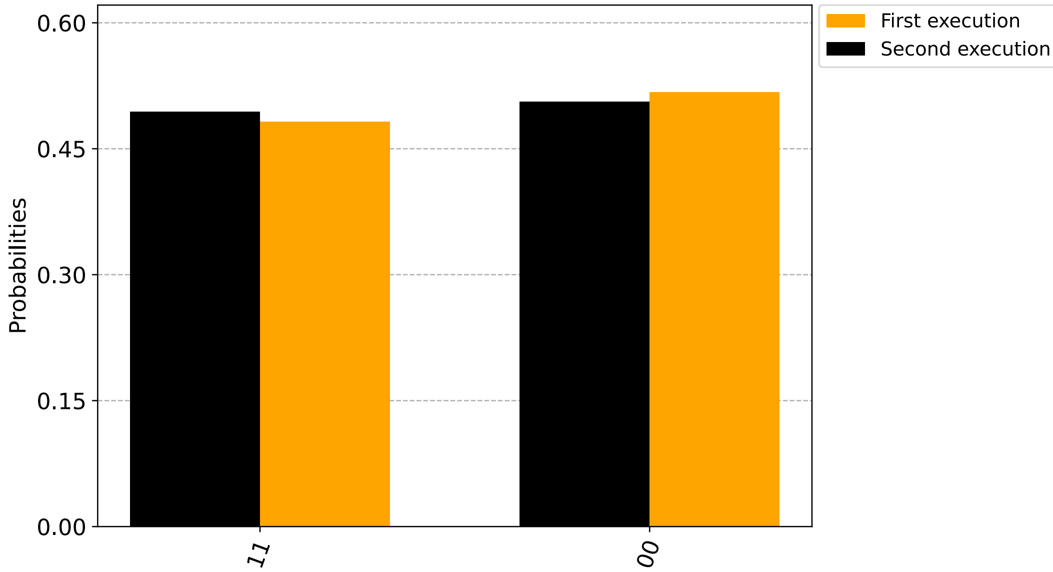


In [20]:

```
plot_histogram([counts, second_counts], legend=legend, sort='desc',
```

```
            figsize=(8,6), color=['orange', 'black'], bar_labels=False)
## and can obviously savefig, etc
```

Out[20]:



Plot State

In many situations you want to see the state of a quantum computer, for debugging. Here we assume you have this state (either from *simulation* or *state tomography*) and the goal is to visualize the quantum state. **This requires exponential resources, so we advise to only view the state of small quantum systems**.

There are several functions:

`plot_state_city(quantum_state)`

`plot_state_qsphere(quantum_state)`

`plot_state_paulivec(quantum_state)`

`plot_state_hinton(quantum_state)`

`plot_bloch_multivector(quantum_state)`

A quantum state is either a state Hermitian matrix ρ or a statevector ψ, such that the state matrix is given by

$$\rho = |\psi\rangle\langle\psi|$$

The state matrix is, in fact, more general, as it can represent *mixed states*, i.e., positive sum of statevectors,

$$\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k|$$

The visualization functions are:

- `plot_state_city` : The standard view for quantum states where the real and imaginary (imag) parts of the state matrix are plotted like a city.
- `plot_state_qsphere` : The Qiskit unique view of a quantum state where the amplitude and phase of the state vector are plotted in a spherical ball. The amplitude is the thickness of the arrow and the phase is the color. For mixed states it will show different 'qsphere' for each component.
- `plot_state_paulivec` : The representation of the state matrix using Pauli operators as the basis $\rho = \sum_{q=0}^{d^2-1} p_j P_j / d$.
- `plot_state_hinton` : Same as 'city' but where the size of the element represents the value of the matrix element.
- `plot_bloch_multivector` : The projection of the quantum state onto the single qubit space and plotting on a bloch sphere.

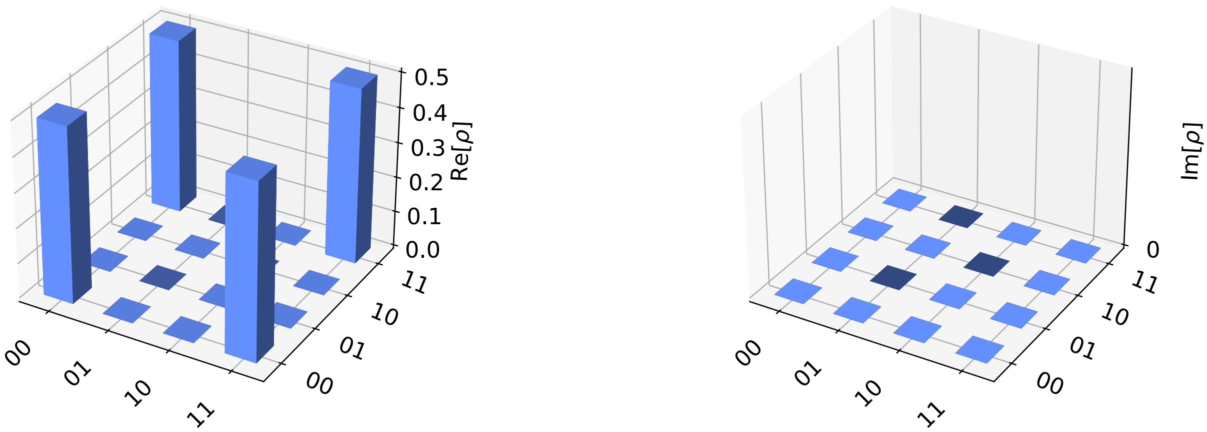
In [21]:

```
from qiskit.visualization import plot_state_city, plot_state_qsphere
from qiskit.visualization import plot_state_paulivec, plot_state_hinton
from qiskit.visualization import plot_bloch_multivector
```

In [22]:

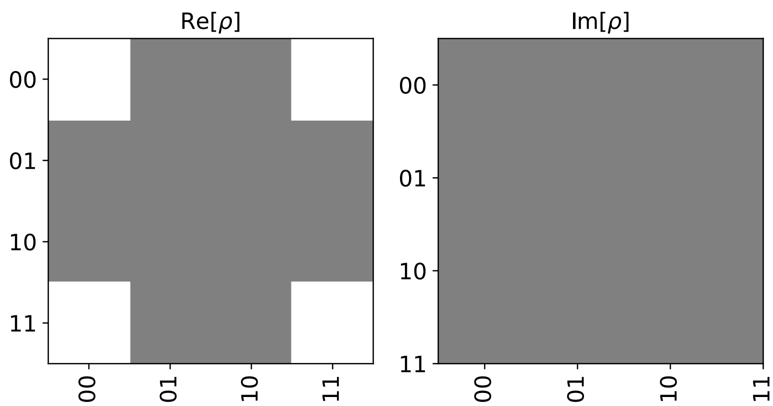
```
# execute the quantum circuit
backend = BasicAer.get_backend('statevector_simulator')
result = backend.run(transpile(bell, backend)).result()
psi = result.get_statevector(bell)

plot_state_city(psi)
```

Out[22]:



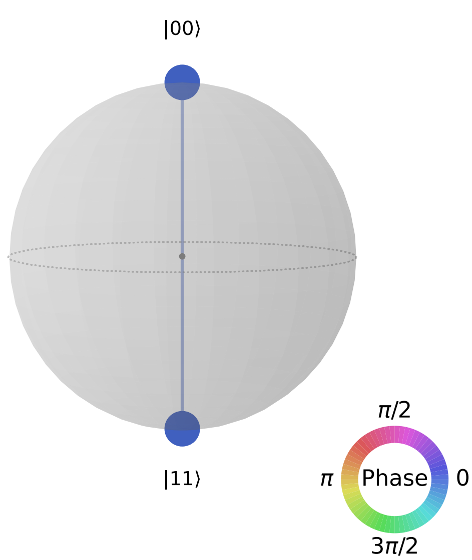
In [23]:
```
plot_state_hinton(psi)
```

Out[23]:



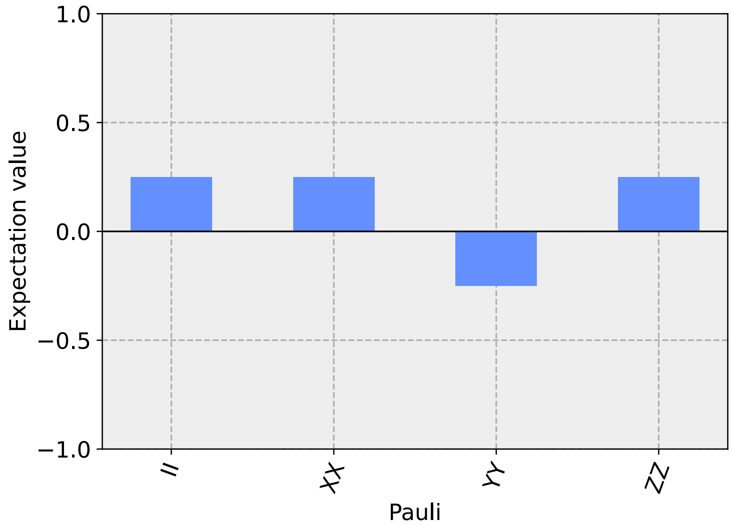
In [24]:
```
plot_state_qsphere(psi)
```

Out[24]:



In [25]:
```
plot_state_paulivec(psi)
```

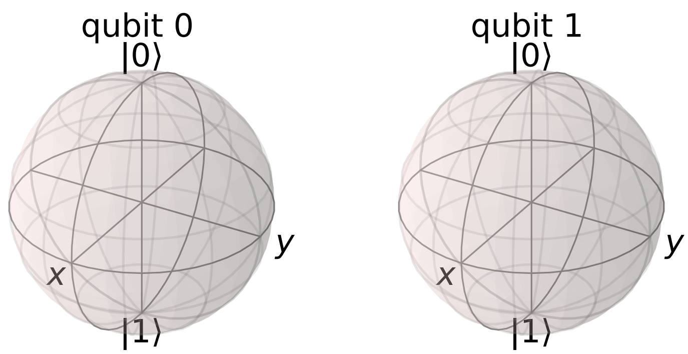
Out[25]:



In [26]:

```
plot_bloch_multivector(psi)
```

Out[26]:



Here we see that there is *no information* about the quantum state in the single qubit space as *all vectors are zero*.

Options when using state plotting functions

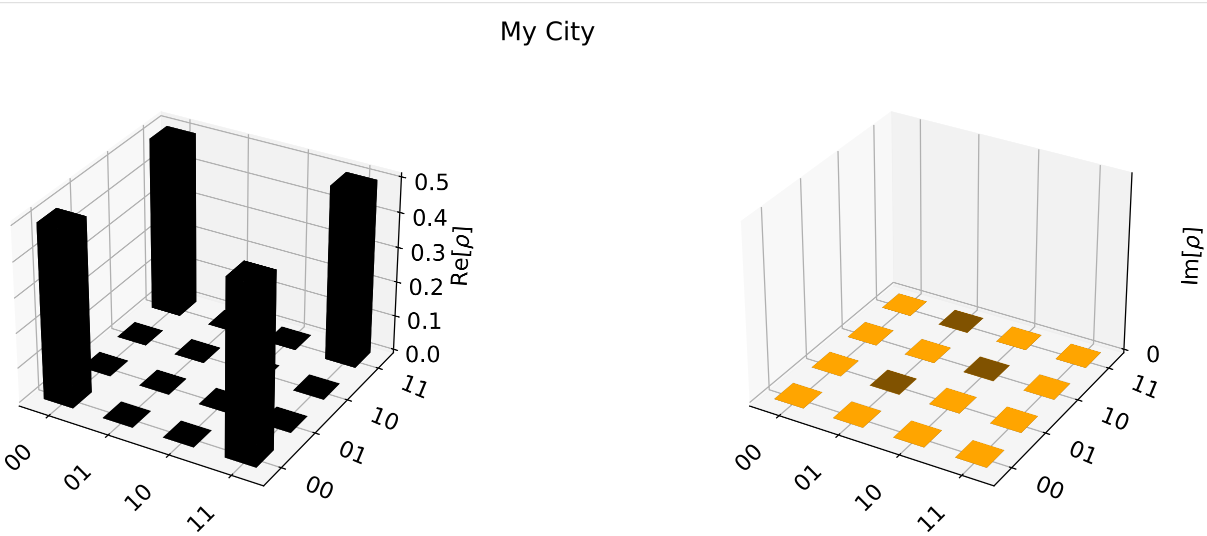
plot_state_city()

- `title (str)` : a string that represents the plot title
- `figsize (tuple)` : figure size in inches (width, height).
- `color (list)` : a list of len=2 giving colors for real and imaginary components of matrix elements.

In [27]:

```
plot_state_city(psi, title="My City", color=['black', 'orange'])
```

Out[27]:

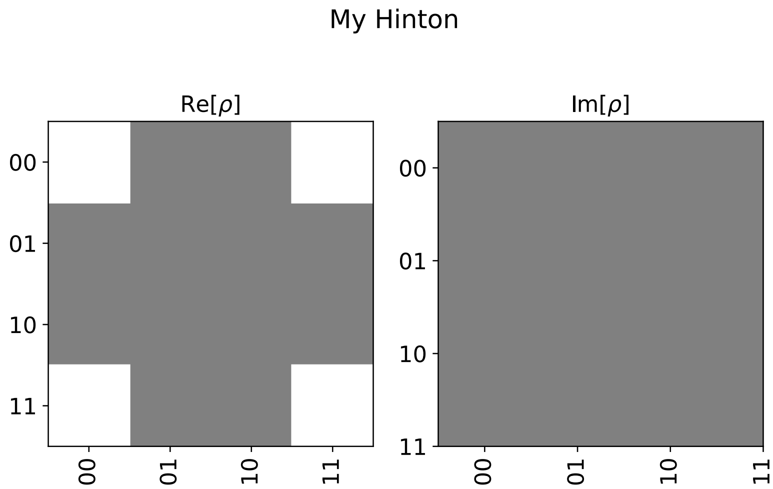


plot_state_hinton()

- `title (str)` : a string that represents the plot title
- `figsize (tuple)` : figure size in inches (width, height).

In [28]:
```
plot_state_hinton(psi, title='My Hinton')
```

Out[28]:

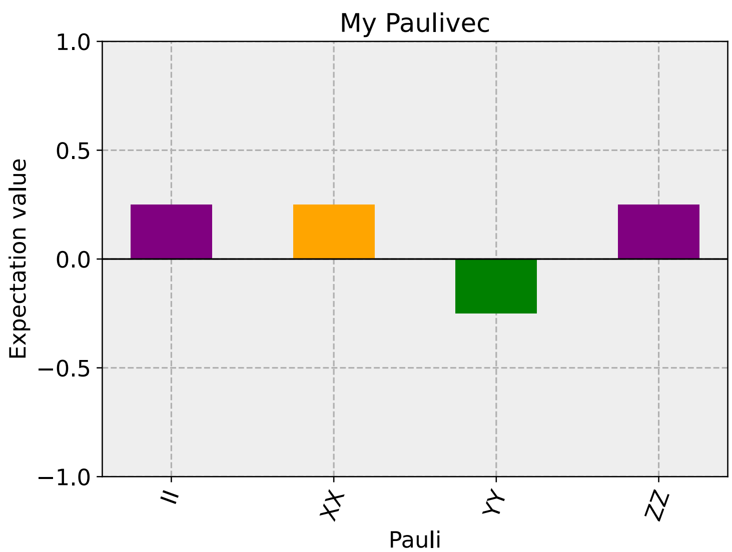


plot_state_paulivec()

- `title (str)` : a string that represents the plot title
- `figsize (tuple)` : figure size in inches (width, height).
- `color (list)` : color of the expectation value bars.

In [29]:
```
plot_state_paulivec(psi, title='My Paulivec', color=['Purple', 'Orange', 'Green'])
```

Out[29]:



plot_state_qsphere()

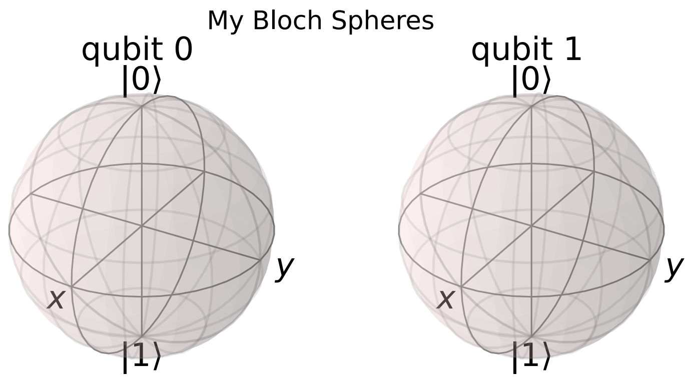
- `figsize (tuple)` : figure size in inches (width, height).

plot_bloch_multivector()

- `title (str)` : a string that represents the plot title
- `figsize (tuple)` : figure size in inches (width, height).

In [30]:
```
plot_bloch_multivector(psi, title='My Bloch Spheres')
```

Out[30]:



Plot Bloch Vector

Standard way of plotting a quantum system of a **single qubit** with the input as a Bloch vector.

Bloch Vectors:

$$x = \mathrm{Tr}[X\rho], \quad y = \mathrm{Tr}[Y\rho], \quad z = \mathrm{Tr}[Z\rho],$$

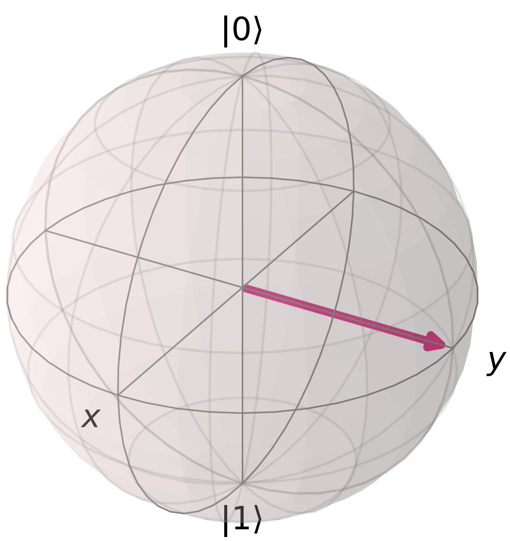
In [31]:

```
from qiskit.visualization import plot_bloch_vector
```

In [32]:

```
plot_bloch_vector([0,1,0])
```

Out[32]:



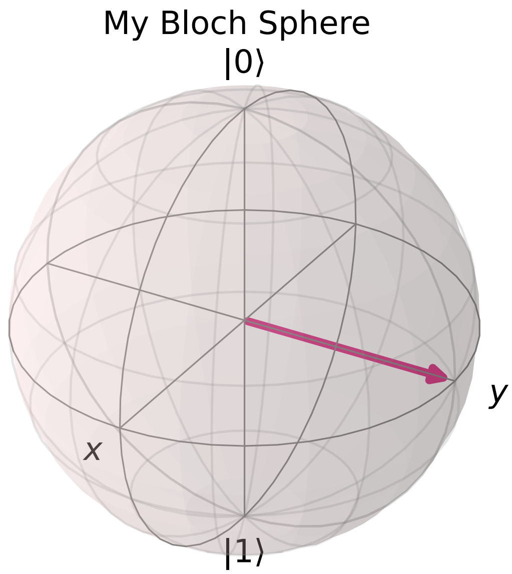
Options for plot_bloch_vector()

- `title (str)` : a string that represents the plot title
- `figsize (tuple)` : figure size in inches (width, height).

In [33]:

```
plot_bloch_vector([0,1,0], title='My Bloch Sphere')
```

Out[33]:



In [ ]:

```

```

In [ ]:

```

```

Summary of Quantum Operations

Review of the different operations that are available in Qiskit Terra:

- Single-qubit quantum gates
- Multi-qubit quantum gates
- Measurements
- Reset
- Conditionals
- State initialization

We will also see how to use the three different simulators:

- `unitary_simulator`
- `qasm_simulator`
- `statevector_simulator`

In [34]:

```
import matplotlib.pyplot as plt
import numpy as no
from math import pi
```

In [35]:

```
from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister, transpile
from qiskit.tools.visualization import circuit_drawer
from qiskit.quantum_info import state_fidelity
from qiskit import BasicAer

backend = BasicAer.get_backend('unitary_simulator')
```

Single-Qubit Quantum States

Single qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}, \quad \alpha, \beta \in \mathrm{C}, \quad \text{with } |\alpha|^2 + |\beta|^2 = 1 \text{ and } |\psi\rangle := e^{i\delta}|\psi\rangle$$

requiring only two real numbers to describe a single qubit quantum state.

A convenient representation is

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin\theta/2|1\rangle, \quad \text{where } 0 \le \theta \le \pi,\ 0 \le \phi < 2\pi$$

showing a one-to-one correspondence between qubit states (C^2) and points in the surface of a unit sphere (R^3): **Bloch sphere**.

Quantum gates/operations are usually repesented as matrices.

- acting on a single qubit: 2×2 unitary matrix $U \longrightarrow |\psi'\rangle = U|\psi\rangle$

A general unitary must be able to take $|0\rangle$ to the above state,

$$U = \begin{pmatrix} \cos(\theta/2) & a \\ e^{i\phi}\sin\theta/2 & b \end{pmatrix}$$

where a and $bin\mathbb{C}$ are constrained by unitarity, $U^\dagger U = I$, for all $0 \leq \theta \leq \pi,\ 0 \leq \phi < 2\pi$. This implies $a \to -e^{i\lambda}\sin\theta/2$ and $b \to e^{i(\lambda+\phi)}\cos\theta/2, 0 \leq \lambda < 2\pi$,

$$U = \begin{pmatrix} \cos(\theta/2) & -e^{i\lambda}\sin\theta/2 \\ e^{i\phi}\sin\theta/2 & e^{i(\lambda+\phi)}\cos\theta/2 \end{pmatrix}$$

which is **the most general form of a single qubit unitary**.

In [ ]:

Single-Qubit Gates

The single-qubit gates available are:

- u gates
- Identity gate
- Pauli gates
- Clifford gates
- $C3$ gates
- Standard rotation gates

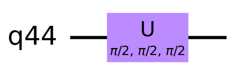
In [36]:
```
q = QuantumRegister(1)
```

u gates

$$u3(\theta, \phi, \lambda) = U(\theta, \phi, \lambda)$$

In [37]:
```
qc = QuantumCircuit(q)
qc.u(pi/2, pi/2, pi/2, q)
qc.draw()
```

Out[37]:



In [38]:
```
job = backend.run(transpile(qc, backend))
job.result().get_unitary(qc, decimals=3)
```

Out[38]:
```
array([[ 0.707+0.j   , -0.   -0.707j],
       [ 0.   +0.707j, -0.707+0.j   ]])
```

u2-gate

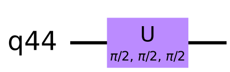
The $u2(\phi, \lambda) = u3(\pi/2, \phi, \lambda)$ gate has the form

$$u2(\phi, \lambda) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -e^{i\lambda} \\ e^{i\phi} & e^{i(\lambda+\phi)} \end{pmatrix}$$

which is useful to create superpositions.

In [39]:
```
qc = QuantumCircuit(q)
#qc.u2(pi/2, pi/2, q) ## deprecated
qc.u(pi/2, pi/2, pi/2, q)
qc.draw()
```

Out[39]:



In [40]:
```
job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```

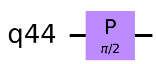
Out[40]:
```
array([[ 0.707+0.j   , -0.   -0.707j],
       [ 0.   +0.707j, -0.707+0.j   ]])
```

p- (or u1-) gate

$$p(\lambda) = u(0, 0, \lambda) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{pmatrix}$$

which is useful to apply a quantum phase.

In [41]:
```
qc = QuantumCircuit(q)
qc.p(pi/2,q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[41]:
```
array([[1.+0.j, 0.+0.j],
       [0.+0.j, 0.+1.j]])
```

In [ ]:

Identity gate

$I = u0(1)$

In [42]:
```
qc = QuantumCircuit(q)
qc.id(q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```

Out[42]:
```
array([[1.+0.j, 0.+0.j],
       [0.+0.j, 1.+0.j]])
```

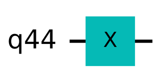
In [ ]:

Pauli gates

X: bit-flip gate

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = u3(\pi, 0, \pi)$$

In [43]:
```
qc = QuantumCircuit(q)
qc.x(q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[43]:
```
array([[0.+0.j, 1.+0.j],
       [1.+0.j, 0.+0.j]])
```

Y: bit- and phase flip gate

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = u3(\pi, \pi/2, \pi/2)$$

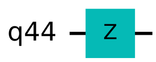
In [44]:
```
qc = QuantumCircuit(q)
qc.y(q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```

Out[44]:
```
array([[ 0.+0.j, -0.-1.j],
       [ 0.+1.j,  0.+0.j]])
```

Z: phase flip gate

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = u1(\pi) = u3(0, 0, \pi)$$

In [45]:
```python
qc = QuantumCircuit(q)
qc.z(q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[45]:
```
array([[ 1.+0.j,  0.+0.j],
       [ 0.+0.j, -1.+0.j]])
```

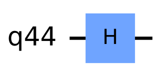
In [ ]:
```python

```

Clifford gates

Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = u2(0, \pi) = u3(\pi/2, 0, \pi)$$

In [46]:
```python
qc = QuantumCircuit(q)
qc.h(q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```

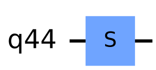


Out[46]:
```
array([[ 0.707+0.j,  0.707-0.j],
       [ 0.707+0.j, -0.707+0.j]])
```

S, or $\sqrt{Z}$ phase, gate

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = u1(\pi/2) = u3(0, 0, \pi/2)$$

In [47]:
```python
qc = QuantumCircuit(q)
qc.s(q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



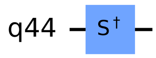
Out[47]:
```
array([[1.+0.j, 0.+0.j],
       [0.+0.j, 0.+1.j]])
```

$S^\dagger$, or conjugate of $\sqrt{Z}$ phase, gate

$$S^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} = u1(-\pi/2) = u3(0, 0, -\pi/2)$$

In [48]:
```python
qc = QuantumCircuit(q)
qc.sdg(q)
display(qc.draw())
```

```
job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



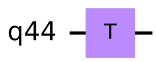
Out[48]:
```
array([[1.+0.j, 0.+0.j],
       [0.+0.j, 0.-1.j]])
```

$C3$ gates

T, or $\sqrt{S}$ phase, gate

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{2}}(1+i) \end{pmatrix} = u1(\pi/4) = u3(0,0,\pi/4)$$

In [49]:
```
qc = QuantumCircuit(q)
qc.t(q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```

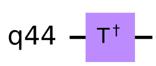


Out[49]:
```
array([[1.    +0.j    , 0.    +0.j    ],
       [0.    +0.j    , 0.707+0.707j]])
```

$T^\dagger$, or conjugate of $\sqrt{S}$ phase, gate

$$T^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{2}}(1-i) \end{pmatrix} = u1(-\pi/4) = u3(0,0,-\pi/4)$$

In [50]:
```
qc = QuantumCircuit(q)
qc.tdg(q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[50]:
```
array([[1.    +0.j    , 0.    +0.j    ],
       [0.    +0.j    , 0.707-0.707j]])
```

Standard rotation gates

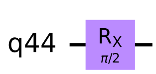
Defined around the Paulis $P = X, Y, Z$,

$$R_P(\theta) = \exp(-i\theta P/2) = \cos(\theta/2)I - i\sin(\theta/2)P$$

Rotation around X-axis

$$R_X(\theta) = \cos(\theta/2)I - i\sin(\theta/2)X = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix} = u3(\theta, -\pi/2, -\pi/2)$$

In [51]:
```
qc = QuantumCircuit(q)
qc.rx(pi/2,q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



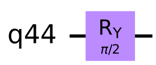
Out[51]:
```
array([[ 0.707+0.j    , -0.    -0.707j],
       [ 0.    -0.707j,  0.707+0.j    ]])
```

Rotation around Y-axis

$$R_Y(\theta) = \cos(\theta/2)I - i\sin(\theta/2)Y = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix} = u3(\theta, 0, 0)$$

In [52]:

```
qc = QuantumCircuit(q)
qc.ry(pi/2,q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[52]:

```
array([[ 0.707+0.j, -0.707+0.j],
       [ 0.707+0.j,  0.707+0.j]])
```

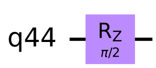
Rotation around Z-axis

Global phase only

$$R_Y(\theta) = \cos(\theta/2)I - i\sin(\theta/2)Z = \begin{pmatrix} \cos(\theta/2) - i\sin(\theta/2) & 0 \\ 0 & \cos(\theta/2) + i\sin(\theta/2) \end{pmatrix} = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix} = u1(\theta)$$

In [53]:

```
qc = QuantumCircuit(q)
qc.rz(pi/2,q)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[53]:

```
array([[0.707-0.707j, 0.   +0.j   ],
       [0.   +0.j   , 0.707+0.707j]])
```

In [ ]:

Multi_Qubit Gates

The two-qubit gates are:

- controlled Pauli gates
- controlled Hadamard gate
- controlled rotation gates
- controlled phase gate
- controlled u3 gate
- swap gate

The three-qubit gates are:

- Toffoli gate
- Fredkin gate

Mathematical Preliminaries

Space of a QC grows exponentially with the number of qubits n: complex vector space has dimension $d = 2^n$. **Tensor product** glues together operators and basis vectors.

Consider a 2-qubit system. Given that A and B are operators acting on a single qubit, the joint operator $A \otimes B$ acting on two qubits is:

$$A \otimes B = \begin{pmatrix} A_{00}\begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} & A_{01}\begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} \\ A_{10}\begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} & A_{11}\begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} \end{pmatrix}$$

where A_{jk} and B_{lm} are the matrix elements of A and B, respectively.

Analogously, the basis vectors for the 2-qubit system are formed using the tensor product of basis vectors for a single qubit:

$$|00\rangle = \begin{pmatrix} 1\begin{pmatrix}1\\0\end{pmatrix} \\ 0\begin{pmatrix}1\\0\end{pmatrix} \end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad |01\rangle = \begin{pmatrix} 1\begin{pmatrix}0\\1\end{pmatrix} \\ 0\begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} |10\rangle = \begin{pmatrix} 0\begin{pmatrix}1\\0\end{pmatrix} \\ 1\begin{pmatrix}1\\0\end{pmatrix} \end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad |11\rangle = \begin{pmatrix} 0\begin{pmatrix}0\\1\end{pmatrix} \\ 1\begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

The state of a n-qubit system can be described using the n-fold tensor product of single-qubit basis vectors, $|0\rangle \otimes \cdots \otimes |0\rangle = |0\cdots 0\rangle$

Basis ordering: remember that in Qiskit q_0 is the rightmost one, contrary to the standard books notation. **This affects the matrix representation** in Qiskit.

Controlled operation on qubits

Controlled gates: A common multi-qubit gate involves the application of a gate to one qubit, conditioned on the state of another qubit.

In [ ]:

```

```

Two-qubit Gates

The two-qubit gates are:

- controlled Pauli gates
- controlled Hadamard gate
- controlled rotation gates
- controlled phase gate
- controlled u3 gate
- swap gate

Most of the que-qubit gates are of the controlled type (the `swap` -gate being the exception).

In general: C_U acts to apply the single-qubit unitary U to 2nd qubit when the state of the first one is $|1\rangle$.

Suppose U has a matrix representation:

$$U = \begin{pmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{pmatrix}$$

Suppose the **control qubit** is **qubit 0** (the one on the RHS of the tensor product in Qiskit notation). Then:

$$C_U: \underset{\text{qubit 1}}{|0\rangle} \otimes \underset{\text{qubit 0}}{|0\rangle} \longrightarrow |0\rangle \otimes |0\rangle C_U: \underset{\text{qubit 1}}{|0\rangle} \otimes \underset{\text{qubit 0}}{|1\rangle} \longrightarrow U|0\rangle \otimes |0\rangle C_U: \underset{\text{qubit 1}}{|1\rangle} \otimes \underset{\text{qubit 0}}{|0\rangle} \longrightarrow |0\rangle \otimes |0\rangle C_U: \underset{\text{qubit 1}}{|1\rangle} \otimes \underset{\text{qubit 0}}{|1\rangle} \longrightarrow U|0\rangle \otimes |0\rangle$$

,

which, in matrix form, corresponds to

$$C_U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & u_{00} & 0 & u_{01} \\ 0 & 0 & 1 & 0 \\ 0 & u_{10} & 0 & u_{11} \end{pmatrix}$$

To work out these matrix elements, let

$$C_{(jk),(lm)} = \left(\underset{\text{qubit 1}}{\langle j|} \otimes \underset{\text{qubit 0}}{\langle k|} \right) C_U \left(\underset{\text{qubit 1}}{|l\rangle} \otimes \underset{\text{qubit 0}}{|m\rangle|} \right)$$

compute the action of C_U and the inner products.

If the **control qubit** is **qubit 1** :

$$C_U: \underset{\text{qubit 1}}{|0\rangle} \otimes \underset{\text{qubit 0}}{|0\rangle} \longrightarrow |0\rangle \otimes |0\rangle C_U: \underset{\text{qubit 1}}{|0\rangle} \otimes \underset{\text{qubit 0}}{|1\rangle} \longrightarrow |0\rangle \otimes |0\rangle C_U: \underset{\text{qubit 1}}{|1\rangle} \otimes \underset{\text{qubit 0}}{|0\rangle} \longrightarrow |0\rangle \otimes U|0\rangle C_U: \underset{\text{qubit 1}}{|1\rangle} \otimes \underset{\text{qubit 0}}{|1\rangle} \longrightarrow |0\rangle \otimes U|0\rangle$$

,

which, in matrix form, corresponds to

$$C_U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & u_{00} & u_{01} \\ 0 & 0 & u_{10} & u_{11} \end{pmatrix}$$

In [54]:

```
q = QuantumRegister(2)
```

Controlled Pauli gates

Controlled-X (Controlled-NOT) gate

Flips the target if the control is in $|1\rangle$.

If we take the **MSB** as the control (`cx(q[1],q[0])`), the matrix is

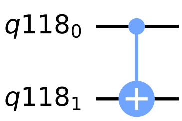
$$C_X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If we take the **LSB** as the control (`cx(q[0],q[1])`), the matrix is

$$C_X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

In [55]:

```
qc = QuantumCircuit(q)
qc.cx(q[0],q[1])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[55]:

```
array([[1.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 1.+0.j],
       [0.+0.j, 0.+0.j, 1.+0.j, 0.+0.j],
       [0.+0.j, 1.+0.j, 0.+0.j, 0.+0.j]])
```

Controlled-Y gate

Applies the Y-gate to the target if the control is in $|1\rangle$.

If we take the **MSB** as the control (`cy(q[1],q[0])`), the matrix is

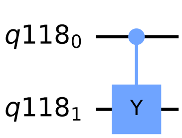
$$C_Y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

If we take the **LSB** as the control (`cy(q[0],q[1])`), the matrix is

$$C_Y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 1 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

In [56]:

```
qc = QuantumCircuit(q)
qc.cy(q[0],q[1])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[56]:

```
array([[1.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.-1.j],
       [0.+0.j, 0.+0.j, 1.+0.j, 0.+0.j],
       [0.+0.j, 0.+1.j, 0.+0.j, 0.+0.j]])
```

Controlled-Z (Controlled-phase) gate

Applies the Z-gate to the target if the control is in $|1\rangle$.

If we take the **MSB** as the control (`cz(q[1],q[0])`), the matrix is

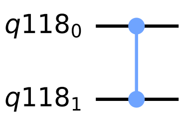
$$C_Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

If we take the **LSB** as the control (`cz(q[0],q[1])`), the matrix is

$$C_Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In [57]:

```
qc = QuantumCircuit(q)
qc.cz(q[0],q[1])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[57]:

```
array([[ 1.-0.j,  0.+0.j,  0.+0.j,  0.+0.j],
       [ 0.+0.j,  1.-0.j,  0.+0.j,  0.+0.j],
       [ 0.+0.j,  0.+0.j,  1.-0.j,  0.+0.j],
       [ 0.+0.j,  0.+0.j,  0.+0.j, -1.+0.j]])
```

In [ ]:

Controlled Hadamard gate

Applies the Hadamard-gate to the target if the control is in $|1\rangle$.

If we take the **MSB** as the control (`ch(q[1],q[0])`), the matrix is

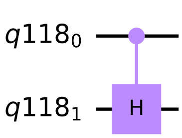
$$C_H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

If we take the **LSB** as the control (`ch(q[0],q[1])`), the matrix is

$$C_H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

In [58]:

```
qc = QuantumCircuit(q)
qc.ch(q[0],q[1])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[58]:

```
array([[ 1.   -0.j,  0.   +0.j,  0.   +0.j,  0.   +0.j],
       [ 0.   +0.j,  0.707+0.j,  0.   +0.j,  0.707-0.j],
       [ 0.   +0.j,  0.   +0.j,  1.   -0.j,  0.   +0.j],
       [ 0.   +0.j,  0.707+0.j,  0.   +0.j, -0.707+0.j]])
```

Controlled rotation gates

Controlled rotation around Z-axis

Applies the Z-gate to the target if the control is in $|1\rangle$.

If we take the **MSB** as the control (`crz(q[1],q[0])`), the matrix is

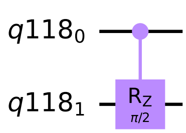
$$C_{R_Z}(\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{-i\lambda/2} & 0 \\ 0 & 0 & 0 & e^{-i\lambda/2} \end{pmatrix}$$

If we take the **LSB** as the control (`crz(q[0],q[1])`), the matrix is

$$C_{R_Z}(\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i\lambda/2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\lambda/2} \end{pmatrix}$$

In [59]:

```
qc = QuantumCircuit(q)
qc.crz(pi/2, q[0],q[1])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```

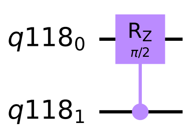


Out[59]:

```
array([[1.    +0.j    , 0.    +0.j    , 0.    +0.j    , 0.    +0.j    ],
       [0.    +0.j    , 0.707-0.707j, 0.    +0.j    , 0.    +0.j    ],
       [0.    +0.j    , 0.    +0.j    , 1.    +0.j    , 0.    +0.j    ],
       [0.    +0.j    , 0.    +0.j    , 0.    +0.j    , 0.707+0.707j]])
```

In [60]:

```
qc = QuantumCircuit(q)
qc.crz(pi/2, q[1],q[0])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[60]:

```
array([[1.    +0.j    , 0.    +0.j    , 0.    +0.j    , 0.    +0.j    ],
       [0.    +0.j    , 1.    +0.j    , 0.    +0.j    , 0.    +0.j    ],
       [0.    +0.j    , 0.    +0.j    , 0.707-0.707j, 0.    +0.j    ],
       [0.    +0.j    , 0.    +0.j    , 0.    +0.j    , 0.707+0.707j]])
```

In [ ]:

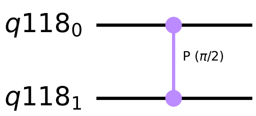
Controlled phase rotation

Performs a phase rotation if both qubits are in the $|11\rangle$ state. The matrox look the same for **MSB** and **LSB**:

$$C_p(\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\lambda/2} \end{pmatrix}$$

In [61]:

```
qc = QuantumCircuit(q)
qc.cp(pi/2, q[0],q[1])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[61]:
```
array([[1.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 1.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 1.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.+1.j]])
```

In [ ]:

Controlled u3 gate

Perform controlled-$u3$ rotation on the target qubit if the control qubit is $|1\rangle$.

If we take the **MSB** as the control (`cu3(q[1],q[0])`), the matrix is

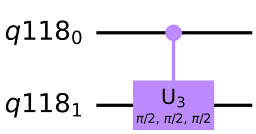
$$C_u(\theta,\phi,\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{-i(\phi+\lambda)/2}\cos(\theta/2) & -e^{-i(\phi-\lambda)/2}\sin(\theta/2) \\ 0 & 0 & e^{i(\phi-\lambda)/2}\sin(\theta/2) & e^{i(\phi+\lambda)/2}\cos(\theta/2) \end{pmatrix}$$

If we take the **LSB** as the control (`cu3(q[0],q[1])`), the matrix is

$$C_u(\theta,\phi,\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i(\phi+\lambda)/2}\cos(\theta/2) & 0 & -e^{-i(\phi-\lambda)/2}\sin(\theta/2) \\ 0 & 0 & 1 & 0 \\ 0 & e^{i(\phi-\lambda)/2}\sin(\theta/2) & 0 & e^{i(\phi+\lambda)/2}\cos(\theta/2) \end{pmatrix}$$

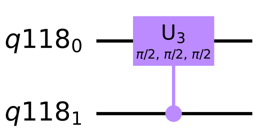
In [62]:
```
qc = QuantumCircuit(q)
qc.cu3(pi/2, pi/2, pi/2, q[0],q[1])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```

```
<ipython-input-62-5cc2df86b7ca>:2: DeprecationWarning: The QuantumCircuit.cu3 method is deprecated as of 0.16.0. It will be removed no earlier than 3 months after the release date. You should use the QuantumCircuit.cu method instead, where cu3(θ,φ,λ) = cu(θ,φ,λ,0).
  qc.cu3(pi/2, pi/2, pi/2, q[0],q[1])
```



Out[62]:
```
array([[ 1.   +0.j   ,  0.   +0.j   ,  0.   +0.j   ,  0.   +0.j   ],
       [ 0.   +0.j   ,  0.707+0.j   ,  0.   +0.j   ,  0.   -0.707j],
       [ 0.   +0.j   ,  0.   +0.j   ,  1.   +0.j   ,  0.   +0.j   ],
       [ 0.   +0.j   , -0.   +0.707j,  0.   +0.j   , -0.707-0.j   ]])
```

In [63]:
```
qc = QuantumCircuit(q)
qc.cu3(pi/2, pi/2, pi/2, q[1],q[0])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



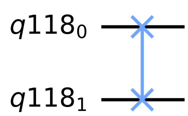
Out[63]:
```
array([[ 1.   +0.j   ,  0.   +0.j   ,  0.   +0.j   ,  0.   +0.j   ],
       [ 0.   +0.j   ,  1.   +0.j   ,  0.   +0.j   ,  0.   +0.j   ],
       [ 0.   +0.j   ,  0.   +0.j   ,  0.707+0.j   ,  0.   -0.707j],
       [ 0.   +0.j   ,  0.   +0.j   , -0.   +0.707j, -0.707-0.j   ]])
```

Swap gate

The SWAP gate exchanges the two qubits.

$$\mathrm{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In [64]:
```
qc = QuantumCircuit(q)
qc.swap(q[0],q[1])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```



Out[64]:
```
array([[1.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 1.+0.j, 0.+0.j],
       [0.+0.j, 1.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 1.+0.j]])
```

In [ ]:

Three-qubit Gates

The three-qubit gates are:

- Toffoli gate
- Fredkin gate

For three qubits, the basis vectors are ordered as:

$$|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle$$

which, in bitstrings, represent the integers $0, \cdots, 7$.

In Qiskit:

$$|abc\rangle = \underset{\text{qubit 2}}{|a\rangle} \otimes \underset{\text{qubit 1}}{|b\rangle} \otimes \underset{\text{qubit 0}}{|c\rangle}$$

Toffoli gate

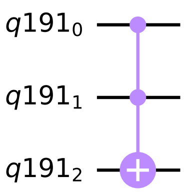
Flips the 3rd qubit if the first two (**LSB**) are both $|1\rangle$.

$$|abc\rangle = |bc \oplus a\rangle \otimes |b\rangle \otimes |c\rangle$$

In matrix form:

$$C_{CX} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In [65]:
```
q = QuantumRegister(3)

qc = QuantumCircuit(q)
qc.ccx(q[0],q[1],q[2])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```




```
Out[65]: array([[1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j]])
```

In [ ]:

Fredkin gate - Controlled swap gate

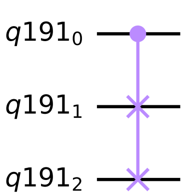
Exchanges the second and third qubits if the first qubit (**LSB**) is $|1\rangle$.

$$|abc\rangle \rightarrow \begin{cases} |bac\rangle & \text{if } c = 1 \\ |abc\rangle & \text{if } c = 0 \end{cases}$$

In matrix form:

$$C_{\text{SWAP}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In [66]:
```
qc = QuantumCircuit(q)
qc.cswap(q[0],q[1],q[2])
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_unitary(qc, decimals=3)
```




```
Out[66]: array([[1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j, 0.+0.j],
       [0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 0.+0.j, 1.-0.j]])
```

In [ ]:

Non-unitary Operations

Now that we have gone through all the unitary operations in quantum circuits, we also have access to non-unitary operations. These include

- measurements,
- reset of qubits, and
- classical conditional operations.

In [67]:
```
q = QuantumRegister(1)
c = ClassicalRegister(1)
```

Measurement

We don't have access to all the information when we make a measurement in a quantum computer. The quantum state is projected onto the standard basis.

Below are two examples showing a circuit that is prepared in a basis state and the quantum computer prepared in a superposition state.

In [68]:
```
qc = QuantumCircuit(q,c)
qc.measure(q,c)
display(qc.draw)

backend = BasicAer.get_backend('qasm_simulator')
job = backend.run(transpile(qc,backend))
job.result().get_counts(qc)

## The simulator predicts that 100 percent of the time the classical register returns 0.
```

```
<bound method QuantumCircuit.draw of <qiskit.circuit.quantumcircuit.QuantumCircuit object at 0x7fb1cc55a610>>
```

Out[68]:
```
{'0': 1024}
```

In [69]:
```
qc = QuantumCircuit(q,c)
qc.h(q)
qc.measure(q,c)
display(qc.draw)

backend = BasicAer.get_backend('qasm_simulator')
job = backend.run(transpile(qc,backend))
job.result().get_counts(qc)

# The simulator predicts that 50 percent of the time the classical register returns 0 or 1.
```

```
<bound method QuantumCircuit.draw of <qiskit.circuit.quantumcircuit.QuantumCircuit object at 0x7fb1cc56a340>>
```

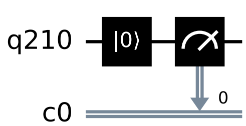
Out[69]:
```
{'1': 509, '0': 515}
```

In [ ]:

Reset of qubits

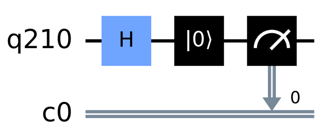
It is also possible to reset qubits to the $|0\rangle$ state in the middle of computation. Note that **reset is not a Gate operation, since it is irreversible**.

In [70]:
```
qc = QuantumCircuit(q,c)
qc.reset(q[0])
qc.measure(q,c)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_counts()
```



Out[70]:
```
{'0': 1024}
```

In [71]:
```
qc = QuantumCircuit(q,c)
qc.h(q[0])
qc.reset(q[0])
qc.measure(q,c)
display(qc.draw())

job = backend.run(transpile(qc,backend))
job.result().get_counts()
```



Out[71]:
```
{'0': 1024}
```

Here we see that for both of these circuits the simulator always predicts that the output is 100 percent in the 0 state.

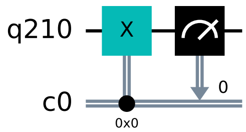
In [ ]:

Classical conditional operations

It is also possible to do **operations conditioned on the state of the classical register**.

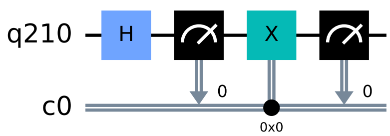
In [72]:

```
qc = QuantumCircuit(q,c)
qc.x(q[0]).c_if(c,0) ##
qc.measure(q,c)
display(qc.draw())

job=backend.run(transpile(qc,backend))
job.result().get_counts(qc)
```



Out[72]: {'1': 1024}

In [73]:
```
qc = QuantumCircuit(q,c)
qc.h(q)
qc.measure(q,c)
qc.x(q[0]).c_if(c,0) ##
qc.measure(q,c)
display(qc.draw())

job=backend.run(transpile(qc,backend))
job.result().get_counts(qc)
```



Out[73]: {'1': 1024}

The **classical bit by the first measurement is random** but the conditional operation results in the **qubit being deterministically put into** $|1\rangle$.

In [ ]:

Arbitrary Initialization

What if we want to initialize a qubit register to an arbitrary state?

n qubits, 2^n amplitudes, normalized to 1. For instance:

$$|\psi\rangle = \frac{i}{4}|000\rangle + \frac{1}{\sqrt{8}}|001\rangle + \frac{1+i}{4}|010\rangle + \frac{1+2i}{\sqrt{8}}|101\rangle + \frac{1}{4}|110\rangle$$

In [1]:
```
## Initializing a 3-qubit quantum state
import math

#list with the desired vector in terms of the basis states
desired_vector=[
    1/math.sqrt(16)*complex(0,1),
    1/math.sqrt(8)*complex(1,0),
    1/math.sqrt(16)*complex(1,1),
    0,
    0,
    1/math.sqrt(8)*complex(1,2),
    1/math.sqrt(16)*complex(1,0),
    0
    ]

q = QuantumRegister(3)

qc = QuantumCircuit(q)

qc.initialize(desired_vector, [q[0],q[1],q[2]]) # desired vector,list of qubits

qc.draw()
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-1-14f2eb91ad22> in <module>
     14     ]
     15
---> 16 q = QuantumRegister(3)
     17
     18 qc = QuantumCircuit(q)

NameError: name 'QuantumRegister' is not defined
```

In [75]:
```
backend = BasicAer.get_backend('statevector_simulator')
job = backend.run(transpile(qc, backend))
```

```
qc_state = job.result().get_statevector(qc)
array_to_latex(qc_state)
```

Out[75]:

$$\begin{bmatrix} \frac{1}{4}i & \frac{1}{\sqrt{8}} & \frac{1}{4}(1+i) & 0 & 0 & \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{2}}i & \frac{1}{4} & 0 \end{bmatrix}$$

Fidelity

Fidelity is useful to check whether two states are the same or not. For quantum (pure) states $|\psi_1\rangle$ and $|\psi_2\rangle$, the fidelity is

$$F(|\psi_1\rangle, |\psi_2\rangle) = |\psi_1\langle|\psi_2\rangle|^2 = \begin{cases} 1 & \text{if states are the same} \\ < 1 & \text{if states are different} \end{cases}$$

In [76]:

```
state_fidelity(desired_vector, qc_state)
```

Out[76]: 1.0

In [ ]:

Further details

How does the desired state get generated behind the scenes? There are multiple methods for doing this. Qiskit uses a method proposed by Shende et al, [Synthesis of Quantum Logic Circuits, 2006].

The idea is to assume the quantum register to have started from our desired state, and construct a circuit that takes it to the $|00\cdots0\rangle$ state. The initialization circuit is then the reverse of such circuit.

To take an arbitrary quantum state to the zero state in the computational basis, we perform an **iterative procedure that disentangles qubits from the register one-by-one**. We know that any arbitrary single-qubit state $|\rho\rangle$ can be taken to $|0\rangle$ using

1. a ϕ-degree rotation around the Z-axis
2. followed by a θ-degree rotation around the Y-axis

$$R_y(-\theta)R_z(-\phi)|\rho\rangle = re^{it}|0\rangle.$$

Since now we are dealing with n qubits, we must factorize the state vector to separate the **LSB**

$$\begin{aligned} |\psi\rangle &= \alpha_{0_0}|0\cdots00\rangle + \alpha_{0_1}|0\cdots01\rangle + \cdots + \alpha_{2^{n-1}-1_0}|1\cdots10\rangle + \alpha_{2^{n-1}-1_1}|1\cdots11\rangle \\ &= |0\cdots00\rangle(\alpha_{0_0}|0\rangle + \alpha_{0_1}|1\rangle) + \cdots + |1\cdots1\rangle(\alpha_{2^{n-1}-1_0}|0\rangle + \alpha_{2^{n-1}-1_1}|1\rangle) \\ &= |0\cdots00\rangle|\rho_0\rangle + \cdots + |1\cdots1\rangle|\rho_{2^{n-1}-1}\rangle \end{aligned}$$

And now each fo the $|\rho_{2^{n-1}-1}\rangle$ cand be taken to $|0\rangle$ by finding the appropriate ϕ and θ angles. For all states: unitary that disentangles the LSB:

$$U = \begin{pmatrix} R_y(-\theta_0)R_z(-\phi_0) & & & \\ & R_y(-\theta_0)R_z(-\phi_0) & & \\ & & \ddots & \\ & & & R_y(-\theta_{2^{n-1}-1})R_z(-\phi_{2^{n-1}-1}) \end{pmatrix}$$

and

$$U|\psi\rangle = \begin{pmatrix} r_0 e^{it_0} \\ r_1 e^{it_1} \\ \vdots \\ r_{2^{n-1}-1} e^{it_{2^{n-1}-1}} \end{pmatrix} \otimes |0\rangle$$

U can be implemented as a **quantum multiplexor** gate (defined in the reference by Shende above), since it is a block diagonal matrix. In the quantum multiplexor formalism, a block diagonal matrix of size $2^n \times 2^n$ consisting of 2^s blocks is equivalent to a multiplexor with s selected qubits and $n - s$ data qubits. It can be implemented after recursive decomposition to primitive gates of `cx`, `rz`, and `ry`.

In [ ]:

In [ ]: