The Motivation, Specification, and Optimization of the Extended Schmidt Gnomonic (ESG) grid for Limited Area Applications

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Motivation

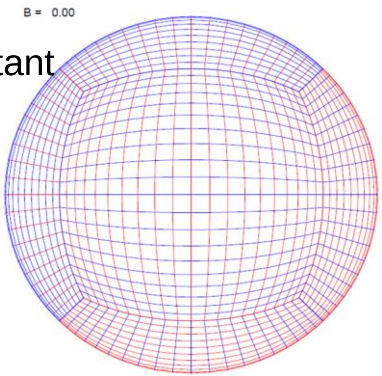
The FV3 Cubed Sphere model came with variants of the **gnomonic projection** grids spanning each of the six faces for the cube. In every case, the grid lines were great circles defining families of planes intersecting along axes through the center of the Earth (which we shall henceforth consider, for convenience, to be a unit sphere). Each variant spread the separation of the consecutive planes of a family of grid lines in different ways, so that their profiles of grid line separation between the cube-face median and the edge differed in characteristic ways. Additionally, the FV3 cube came with the globally smooth **Schmidt** focused refinement option – a conformal (angle-preserving) mapping of the sphere to itself.

But for large limited area domains, no paired combination of these two mapping options was found able to provide the sufficiently homogeneous, undistorted, grid we needed for numerical weather analysis and prediction. Therefore, we sought a method to expand the existing space of mapping parameters, that could then satisfy our limited-area needs.

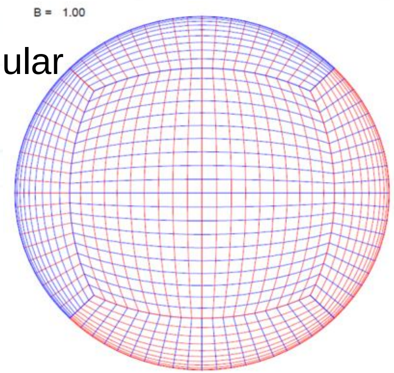
A continuous parameter for Gnomonic grid line spacing

On an arc-line, angular distance arccos(sqrt{**B**}) from the face center, the transverse grid lines which intersect it do so at a uniform spacing on the sphere (the **B**=0 case being obtained as a limit).

Equi-distant spacing B=0



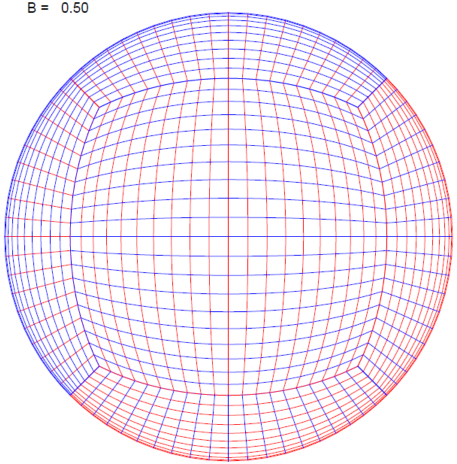
Equi-angular spacing B=1



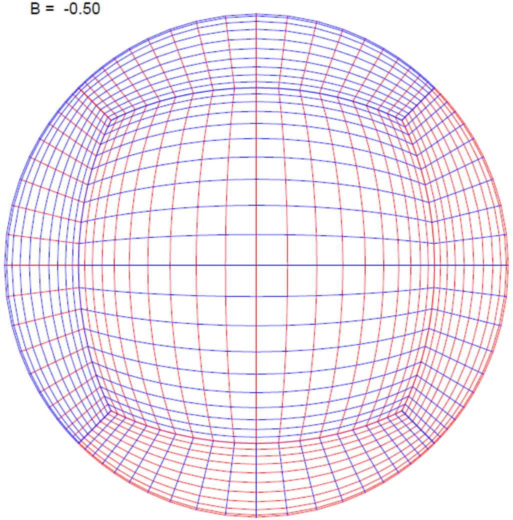
(Gnomonic grid arcs form great circles. When the successive planes of these great circles are uniformly spaced in angle at their common axis, we call this “equi-angular gnomonic”)

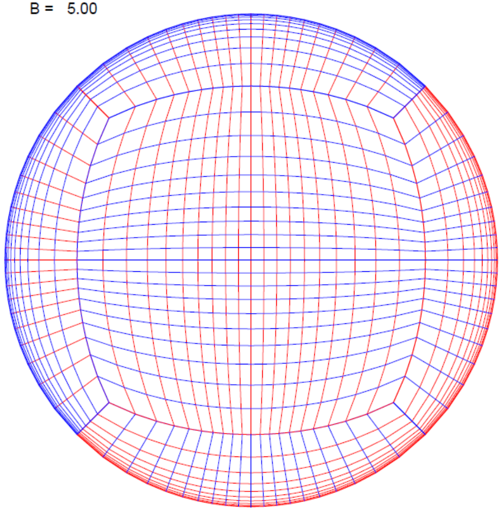
(lines equidistant on the gnomonic tangent plane (Sadourny, 1972))

With this purely geometrical definition of the spacing parameter, B, in the range [0,1] we can find the FV3-GFS grid used operationally, where the spacing is uniform along an edge of the cube, to be the case, B=0.5 =>



But ALGEBRAICALLY, there is no obstacle to <u>analytically-continuing</u> parameter B into the extended additional ranges, B<0 and B>1 as well!



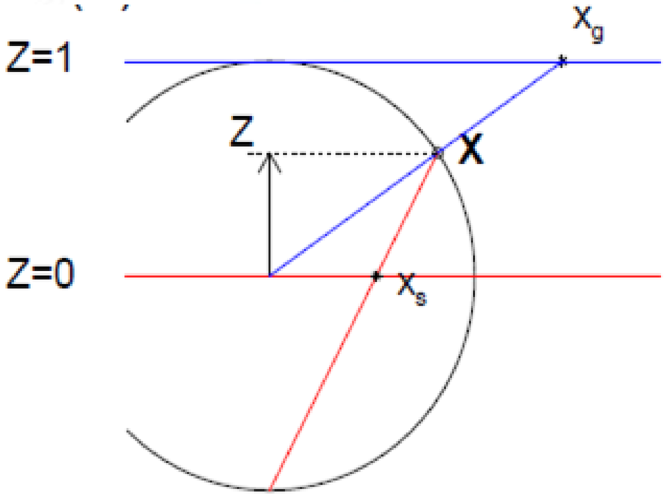


<= Although these extreme grids are not useful choices for a global grid, they do show that a greater parameter range is available when we construct a limited area grid.

Geometry of the Schmidt refinement

The FV3 cubed-sphere grid is equipped with a second parameter -- the Schmidt conformal refinement factor, S. It works as follows:

The gnomonic grid, formed by families of straight lines on the tangent plane (blue, in the figure), and centrally back-projected to a unit sphere (point **X**), are projected **stereographically**



If we **scale** this stereographic plane by the factor, 1/S, and back-project (stereographically) onto the **same** sphere, we shall have a conformally-distorted image of the original gnomonic grid in the vicinity of the 'north pole', but with a resolution enhanced by the factor S.

Schmidt generalized

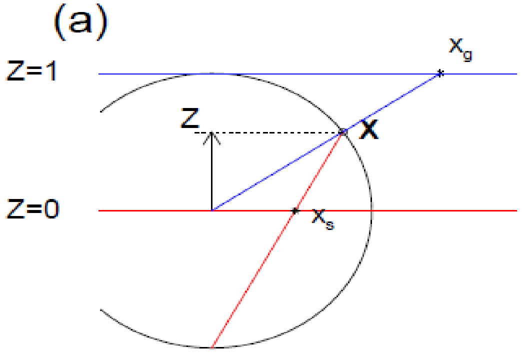
An equivalent picture is that the sphere on which the original gnomonic grid is constructed is of radius 1/S, and its stereographic projection onto the equatorial plane is **NOT scaled** before its back-projection onto the unit-radius ‘Earth’.

What if we replace parameter S by K= S^2?

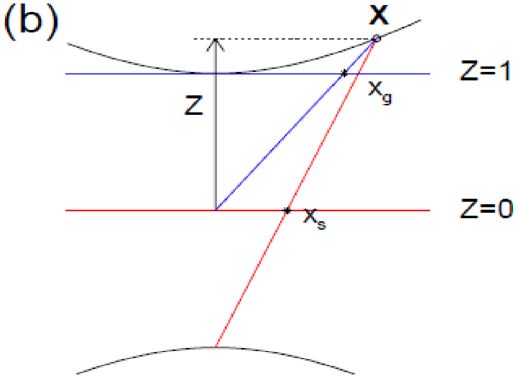
Geometry of Schmidt with negative K = S^2

When 1/S is the spherical radius, K=S^2 is the **GAUSSIAN CURVATURE**.
But there is no obstacle to allowing K to be a negative value!
For negative curvature, K, the original 'sphere' is actually a 'pseudo-sphere' or 'hyperbolic plane'.

Remarkably, gnomonic grids, with geodesic lines, and stereographic projections still occur for K<0.



Gnomonic and stereographic projections illustrated for a true sphere of curvature K=+1



Gnomonic and stereographic projections for a pseudo-sphere of curvature K = -1

Remarks

The conformal mappings of a sphere to itself to enhance the resolution of a global model locally, with enhancement factor, $S>0$, were proposed by F. Schmidt in 1977.

When the sphere is represented by some standard stereographic projection to the complex plane, then this continuous group of mappings are known as 'Mobius Transformations'.

By replacing the parameter by $K=S^2$,the traditional Schmidt mappings are retained for the $K>0$ range, but an extended range of parameter space, $K<0$, is now available. This, while not valid for the whole globe, is certainly valid for mapping gnomonic grids, with their line-spacing parameter, B, to limited areas.

However, to regularize the otherwise singular behavior of the mapping transformations near the special stereographic cases, $K=0$, we need to rescale the original line-spacing parameter, B, which we do by replacing it by:

$$A=KB.$$

Thus, our final parameter pair, defining the limited area Extended Schmidt Gnomonic (ESG) grid, uses the parameter pair, **(A,K)**.

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Optimizing the mapping parameters

In order to apply the extended Schmidt-transformed gnomonic mapping in an optimal way for a limited area domain we must first define an objective optimality criterion.
The Jacobian Matrix of the mapping at a point, relating the rate of change of the Earth-centered Cartesian 3-vector, **X**, with respect to the changes in the components of the map's coordinate 2-vector, **x**, is defined,

$$J_{i,j}(x) = \frac{\partial X_i}{\partial x_j}.$$

An associated symmetric 2x2 GRAM MATRIX is defined:

$$G(x) = J^T(x)J(x).$$

Our objective should be to minimize some kind of 'variance' of G, since this can serve as an objective measure of the map's overall inhomogeneity.

Quantifying the inhomogeneity of a map

The departure of the Gram matrix at each point from a constant multiple of the identity is a measure of the map's local deformation, or anisotropy. We could seek to minimize some integrated squared-measure of the departure of the Gram matrix from a constant, but there is another important diagnostic we should also consider: the variability of the areal resolution, which is related to the determinant of G. However, formulating the diagnostics of G directly is not as satisfactory as taking the diagnostics from the matrix logarithm, L, of G.

$$L(x) = \ln\left(G^{\frac{1}{2}}(x)\right) = \frac{1}{2}\ln(G(x))$$

Rescaling the map coordinates has an additive effect of L, so we can redefine L by subtracting the mean constant matrix part of it, and define a parameterized diagnostic Q of inhomogeneity:

$$Q_\gamma = \frac{\iint (1-\gamma)\mathrm{trace}(L(x)^2)+\gamma(\mathrm{trace}(L(x)))^2 dx_1 dx_2}{A}$$

Where the map-space area, *A*, of the rectangular domain is just:

$$A = \iint dx_1 dx_2 .$$

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$$Q_\gamma = \frac{\iint (1-\gamma)\mathrm{trace}(L(x)^2) + \gamma(\mathrm{trace}(L(x)))^2 dx_1 dx_2}{A}$$

When the parameter, γ, vanishes, the diagnostic, Q_γ, is measuring the variance of ALL components of inhomogeneity of the mapping equally; when $\gamma > 0$, it gives extra weight to the inhomogeneity of the areal resolution.

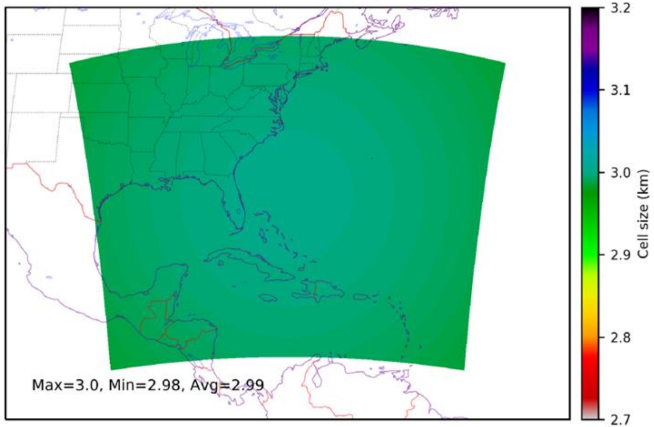
(Note, $\gamma < 1$ always.)

We typically choose a value, γ=0.8.

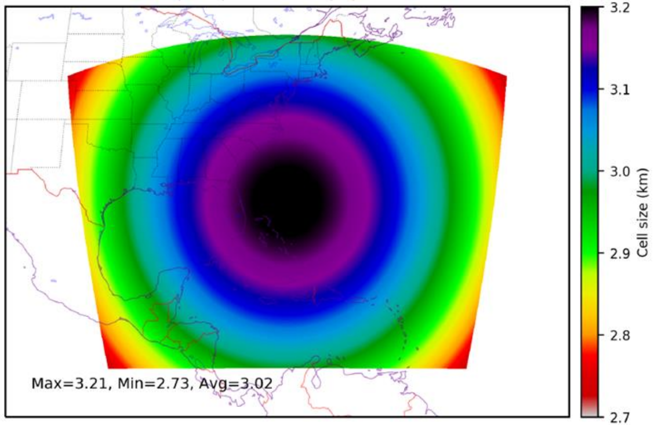
Grid parameter optimization minimizes Q_γ with respect to parameters (A,K).

Generally, this leads to a NEGATIVE optimal K.

Contours of grid cell size show the advantages of extending the parameter space



Extended Schmidt Gnomonic grid



Ordinary Gnomonic grid

(Figures kindly provided by Chan-Hoo Jeon)

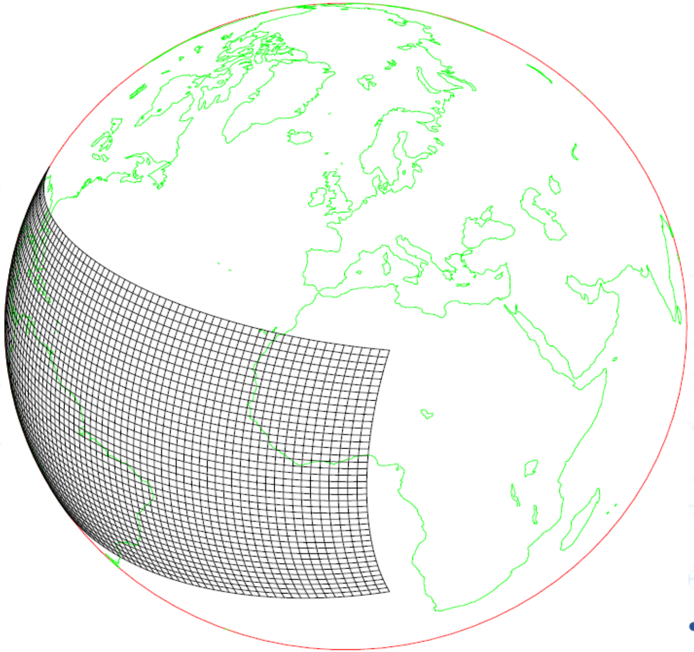
Remarks

The ESG grid allows rectangular domains of very large geographical widths without excessively large disparities of resolution.

The generally negative optimal **K** implies a slight relative ‘flaring’ of each grid, at its edges, compared to the ordinary family of gnomonic grids, whose corners are, contrastingly, blunt (obtuse angles).

The pinched (acute angle) corners of a fairly large ESG grid can be seen in the figure to the right.

The **A** parameter is of either sign, depending on the domain shape and size.



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Conclusion

We found a way to extend the existing space of two parameters of the FV3-cubed sphere Schmidt-Gnomonic grids by a form of analytic continuation, so that the old parameter range still remains a subset of the parameter range offered by the new ESG mapping.

The extension of the Schmidt parameter, to ‘imaginary S’ (i.e., negative K), becomes a valid selection, but ONLY in limited domains – not globally (where $K<0$ Schmidt has a singularity).

In limited domains, the freedom to apply a negative K parameter makes possible grids of surprisingly homogeneous resolution to be constructed over very wide rectangles.

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