Digital implementation of guitar effects pedals

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1 Filters

1.1 1st order low-pass filter

The model used is a RC filter. The transfer function is given by

$$\frac{Y(s)}{X(s)} = \frac{\omega_c}{s + \omega_c} \tag{1}$$

where X is the input signal in the frequency domain, Y the output, ω_c the cutoff frequency in rad/s.

If the system is discretized with a sampling interval of T_s, and using the Tustin discretization method, the z-domain transfer function is given by

$$\frac{Y(z)}{X(z)} = K\frac{z + 1}{z - a} \tag{2}$$

where

$$K = \frac{\omega_c}{\omega_c + K_T} \tag{3}$$

$$a = \frac{K_T - \omega_c}{K_T + \omega_c} \tag{4}$$

given that $K_T = 2/T_s$

The difference equation is then

$$y[k] = ay[k - 1] + K(x[k] + x[k - 1]) \tag{5}$$

1.2 1st order high-pass filter

The transfer function of a 1st order high-pass filter is

$$\frac{Y(s)}{X(s)} = \frac{s}{s + \omega_c} \tag{6}$$

Using the Tustin discretization method gives

$$\frac{Y(z)}{X(z)} = K\frac{z - 1}{z - a} \tag{7}$$

where

$$K = \frac{K_T}{K_T + \omega_c} \tag{8}$$

$$a = \frac{K_T - \omega_c}{K_T + \omega_c} \tag{9}$$

The difference equation is

$$y[k] = ay[k - 1] + K(x[k] - x[k - 1]) \tag{10}$$

1.3 Band-pass filter

A band-pass filter can be implemented by cascading a low-pass and a high-filter. The transfer function is given by

$$\frac{Y(s)}{X(s)} = \frac{\omega_L}{s+\omega_L}\frac{s}{s+\omega_H} = \frac{\omega_L s}{s^2+(\omega_L+\omega_H)s+\omega_L\omega_H} \tag{11}$$

where ω_L and ω_H are the cut-off frequencies of the low-pass and high-pass filters, respectively.

The filter center frequency is at

$$w_c = \sqrt{\omega_L\omega_H} \tag{12}$$

One must pay attention that the cut-off frequencies respect:

$$\omega_H < \omega_L$$

Also, the frequency bandwidth is defined as

$$\omega_{BW} = \omega_L - \omega_H \tag{13}$$

The discretization is given by

$$\frac{Y(z)}{X(z)} = K\frac{z^2-1}{z^2+az+b} \tag{14}$$

where

$$K = \frac{K_T\omega_L}{K_T^2+K_T(\omega_H+\omega_L)+\omega_H\omega_L} \tag{15}$$

$$a = \frac{2(\omega_H\omega_L-K_T^2)}{K_T^2+K_T(\omega_H+\omega_L)+\omega_H\omega_L} \tag{16}$$

$$b = \frac{K_T^2-K_T(\omega_H+\omega_L)+\omega_H\omega_L}{K_T^2+K_T(\omega_H+\omega_L)+\omega_H\omega_L} \tag{17}$$

In the discrete time domain, the equation is

$$y[k] = -ay[k-1] - by[k-2] + K(x[k]-x[k-2]) \tag{18}$$

1.4 Band-stop filter

A band-stop fiter can be designed using the sum of a low-pass and a high-pass filter. The transfer function is given as

$$\frac{Y(s)}{X(s)} = \frac{\omega_L}{s+\omega_L} + \frac{s}{s+\omega_H} = \frac{s^2+2\omega_L s+\omega_H\omega_L}{s^2+(\omega_H+\omega_L)s+\omega_H\omega_L} \tag{19}$$

The center frequency is given by

$$\omega_c = \sqrt{\omega_H\omega_L} \tag{20}$$

and the frequency bandwidth is

$$\omega_{BW} = \omega_H - \omega_L \tag{21}$$

1.5 Second order low-pass filter

The transfer function is

$$\frac{Y(s)}{X(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{22}$$

where ω_n is the natural frequency and ζ is the damping factor.

In the z domain the transfer function is

$$\frac{Y(z)}{X(z)} = K\frac{z^2 + 2z + 1}{z^2 + az + b} \tag{23}$$

where

$$K = \frac{\omega_n^2}{K_T^2 + 2\zeta\omega_n K_t + \omega_n^2} \tag{24}$$

$$a = \frac{2(\omega_n^2 - K_T^2)}{K_T^2 + 2\zeta\omega_n K_t + \omega_n^2} \tag{25}$$

$$b = \frac{K_T^2 - 2\zeta\omega_n K_t + \omega_n^2}{K_T^2 + 2\zeta\omega_n K_t + \omega_n^2} \tag{26}$$

The difference equation is

$$y[k] = -ay[k-1] - by[k-2] + K\left(x[k] + 2x[k-1] + x[k-2]\right) \tag{27}$$

2 Drive

2.1 Distortion 1

It works by clipping the input signal. It differs from others because it uses a hard clipping.

Its mathematical definition is given by

$$y[k] = h(x[k])x[k] \tag{28}$$

where

$$h(x[k]) = \begin{cases} \frac{x_{\max}}{|x[k]|} & \text{if } |x[k]| > x_{\max} \\ 1 & \text{otherwise} \end{cases} \tag{29}$$

and $x_{\max} > 0$

2.2 Distortion 2

The input signal is multiplied by a variable gain. This gain is 1 up to a certain value of the input signal, then it changes its slope.

$$y[k] = h(x[k])x[k] \tag{30}$$

$$h(x[k]) = \begin{cases} 1 & \text{if } |x[k]| < x_{\text{threshold}} \\ m + (1-m)\frac{x_{\text{threshold}}}{|x|} & \text{otherwise} \end{cases} \tag{31}$$

The value $m \in \mathbb{R}$ represents the slope of the gain.

3 Modulation

3.1 Flanger

The flanger model is given by

$$y[k] = (1 - g)x[k] + gx[k - M[k]] \tag{32}$$

The term g is a gain and $M[k]$ represents a varying delay and is modulated by a low frequency oscillator (LFO).

In the case it is modulated by a sine wave is

$$M[k] = M_0(1 + A\sin(2\pi f k T_s)) \tag{33}$$

where M_0 is the mean delay, f is the "speed" ("rate") of the flanger, A is the "excursion" or maximum delay swing.

If the modulator is linear it is defined by

$$M[k] = M_0 A k T_s f \tag{34}$$

3.2 Tremolo

The tremolo effect can be performed by multiplying the input signal by a modulating one (with an offset). It is given by

$$y[k] = (1 - g)x[k] + gm[k]x[k] \tag{35}$$

where $m[k]$ is the modulating signal and g is a gain called depth, used to select the weiths between the original and the modulated signal. The modulating signal can use a low frequency sine wave

$$m[k] = \sin(2\pi f k T_s) \tag{36}$$

4 Source code

Source code publicly available at `https://github.com/adailtonjn68/Capitu-Pedal`