MARMARA UNIVERSITY

MECHANICAL SYSTEM DESIGN

DESIGN OF THE TWO STAGE GEARBOX – Project II

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INTRODUCTION AND PROJECT DESCRIPTION

I have been assigned to prepare a full report on designing a two stage gearbox that has XYZ rev/min speed as input, YZ kW power and (Z+2) reduction rate. (XYZ: Last three digit of your ID) My project report should convince the readers that this is a novel, functional, cost-effective, and feasible design.

My student number 150415055.Last three digits is 055**. 55 rev/min** input speed, **55 kW** input power and **7 reduction rates** are values that used this design project. The purpose of this project is to set out the basic design for an industrial gearbox. It should help us not familiar with gearboxes, lay out a reliable working design. And it is intended for the reader to use his own experience in selecting formulae, stress values etc., for gearbox components.

WORKING PRINCIPLE

A gearbox converts the rotational energy of the engine to a rotational speed appropriate for the wheels. Mechanical gearboxes do so with simple gears. Automatic gearboxes use more complicated planetary gearsets.

A car's engine cannot be attached directly to the wheels. Car engines can only operate in a relatively small RPM band, and the top speed they could achieve in this band is fairly low. Gearboxes change the ratio between the engine's rotational speed and the rotational speed of the wheels. At low speeds, cars rely on small gears. At higher speeds, larger gears are needed.

Manual gearboxes require the driver to select manually which gear to use. Automatic gearboxes, on the other hand, change to new gears automatically. When the engine's speed drops in an automatic car, its gearing is being changed. Traditionally, manual gearboxes were significantly more efficient than automatic gearboxes. Modern automatic gearboxes are far more efficient.

In general two stage gearbox means in which the torque is stepped down or up twice that is using two pairs of gears. But if we use this term in case of an automobile gearbox, then the meaning then a lot more comes into play. Gear boxes are used to get variable output torque as per requirement. Lower gear for higher torque and higher gear for lower torque (higher rpm). The torque is increased by transferring power from a smaller gear to a larger gear and torque is decreased (rpm in increased) and vice versa. In a normal car (5-speed) the engine power undergoes one transformation before reaching the axle.

TYPES OF GEARS

Spur Gear: Parallel and co-planer shafts connected by gears are called spur gears. The arrangement is called spur gearing. Spur gears have straight teeth and are parallel to the axis of the wheel. Spur gears are the most common type of gears. The advantages of spur gears are their simplicity in design, economy of manufacture and maintenance, and absence of end thrust. They impose only radial loads of the bearings. Spur gears are known as slow speed gears. If noise is not a serious design problem, spur gears can be used at almost any speed.

Figure1.1 Spur Gear

Helical Gear: Helical gears have their teeth inclined to the axis of the shafts in the form of a helix, hence the name helical gears. These gears are usually thought of as high speed gears. Helical gears can take higher loads than similarly sized spur gears. The motion of helical gears is smoother and quieter than the motion of spur gears. Single helical gears impose both radial loads and thrust loads on their bearings and so require the use of thrust bearings. The angle of the helix on both the gear and the must be same in magnitude but opposite in direction, i.e., a right hand pinion meshes with a left hand gear.

Figure 1.2 Helical Gear

Herringbone Gear: Herringbone gears resemble two helical gears that have been placed side by side. They are often referred to as "double helicals". In the double helical gears arrangement, the thrusts are counter-balanced. In such double helical gears there is no thrust loading on the bearings.

Figure 1.3 Herringbone Gear

Bevel Miter Gear: Intersecting but coplanar shafts connected by gears are called bevel gears. This arrangement is known as bevel gearing. Straight bevel gears can be used on shafts at any angle, but right angle is the most common. Bevel Gears have conical blanks. The teeth of straight bevel gears are tapered in both thickness and tooth height.

Spiral Bevel gears: In these Spiral Bevel gears, the teeth are oblique. Spiral Bevel gears are quieter and can take up more load as compared to straight bevel gears.

Figure 1.4 Spiral Bevel Gear

<u>Zero Bevel gear</u>: Zero Bevel gears are similar to straight bevel gears, but their teeth are curved lengthwise. These curved teeth of zero bevel gears are arranged in a manner that the effective spiral angle is zero.

<u>Worm Gear</u>: Worm gears are used to transmit power at 90° and where high reductions are required. The axes of worm gears shafts cross in space. The shafts of worm gears lie in parallel planes and may be skewed at any angle between zero and a right angle.In worm gears, one gear has screw threads. Due to this, worm gears are quiet, vibration free and give a smooth output.Worm gears and worm gear shafts are almost invariably at right angles.

Figure 1.5 Worm Gear

Rack and Pinion: A rack is a toothed bar or rod that can be thought of as a sector gear with an infinitely large radius of curvature. Torque can be converted to linear force by meshing a rack with a pinion: the pinion turns; the rack moves in a straight line. Such a mechanism is used in automobiles to convert the rotation of the steering wheel into the left-to-right motion of the tie rod(s). Racks also feature in the theory of gear geometry, where, for instance, the tooth shape of an interchangeable set of gears may be specified for the rack (infinite radius), and the tooth shapes for gears of particular actual radii then derived from that. The rack and pinion gear type is employed in a rack railway.

Figure 1.6 Rack and Pinion

Internal & External Gear:An *external gear* is one with the teeth formed on the outer surface of a cylinder or cone. Conversely, an *internal gear* is one with the teeth formed on the inner surface of a cylinder or cone. For bevel gears, an internal gear is one with the pitch angle exceeding 90 degrees. Internal gears do not cause direction reversal.

Face Gears:Face gears transmit power at (usually) right angles in a circular motion. Face gears are not very common in industrial application.

Figure 1.6 Face Gears

Sprockets:Sprockets are used to run chains or belts. They are typically used in conveyor systems.

CALCULATION

Input and Assumed Values

My student number 150415055. So last 3 digits are 055. According to that;

- n_{input}=55 rev/min (Input speed)
- P_{input}=55 kW (Input power)
- I_{total}=7 (Total Reduction rate)

This project is two stage gear box. So it has 4 gear. Twice of these gears are on same shaf(Gear 2 and Gear 3).

$$I_{total} = I_{12} \times I_{34} \Rightarrow I_{34} = \frac{I_{total}}{I_{12}} = \frac{7}{2} = \mathbf{3.5}$$

Teeth numbers and speed of gears should be known. Some values assumed in this calculation ;

- $n_1 = n_{input} = \mathbf{55}\ \mathrm{rev/min}$
- $I_{12} = \frac{n_1}{n_2} \Rightarrow n_2 = \frac{n_1}{I_{12}} = \frac{55}{2} = \mathbf{22.5}$ rev/min (Speed of gear 2)
- Speed of gear 2 and 3 are same because these gears are on same shaft .($n_3 = n_2 = 22.5$)
- $I_{34} = \frac{n_3}{n_4} \Rightarrow n_4 = \frac{n_3}{I_{34}} = \frac{22.5}{3.5} = \mathbf{6.43}\ \mathrm{rev/min}$ (Speed of gear 4)

Teet numbers of gear 1 and 3 are assumed , then another numbers are calculated according to reduction rates.

- N_1 = **10**, N_3 = **20**. (Assumed)
- $I_{12} = \frac{N_2}{N_1} \Rightarrow N_2 = N_1 \times I_{12} = 10 \times 2 = \mathbf{20}$ (Teeth number of gear 2)
- $I_{34} = \frac{N_4}{N_3} \Rightarrow N_4 = N_3 \times I_{34} = 20 \times 3.5 = \mathbf{70}$ (Teeth number of gear 4)

The efficiency of gears are assumed , because of power of the gears should be calculated.

- $\mu_1 = \mu_2$ = **0.90** (Stage 1 and 2 efficiencies are assumed)

Total gearbox efficiency caused power loses are ;

- $\mu_{total} = \mu_1 \times \mu_2 = 0.90 \times 0.90 = \mathbf{0.81}$

Powers for each gear ;

- P_1 = 55 kW
- $P_2 = P_3 = P_1 \times \mu_1$ = 55 × 0,90 = **49.5** kW
- $P_4 = P_{output} = P_1 \times \mu_{total}$ = 55 × 0,81 = **44.55** kW

Power loses for the system ;

- $P_{input} - P_{output} = 55 - 44.55 = \mathbf{10.45}$ kW

Torque of each gears are calculated . For calculate torque of gear 1 , angular velocity should be known;

- $w_1 = \frac{2\times\pi\times n_1}{60} = \frac{2\times\pi\times 55}{60} = 5.76 \ \frac{1}{sec}$
- $\tau_{G1} = \frac{P_1}{w_1} = \frac{55000}{5.76} = \mathbf{9548.6}$ N. m (Torque of gear 1)
- $\tau_{G2} = I_{12} \times \tau_{G1} \times \mu_1 = 2 \times 9548.6 \times 0.90 = \mathbf{17.187,48}$ N. m (Torque of gear 2)

Torques of gear 2 and 3 are same because these gears are on same shaft.

- $\tau_{G2} = \tau_{G3} = 17.187,48$ N. m
- $\tau_{G4} = I_{34} \times \tau_{G3} \times \mu_2 = 3.5 \times 17.187,48 \times 0.90 =$ $\mathbf{54.140,56}$ N. m (Torque for Gear 4)

Module Of Gearbox

Gear material should be selected because of stresses should known basically ;

- Material for gears is selected as **16 MnCr5**.

According to these material characteristic values are ;

- σ_t = 880 N/mm^2 (Tensile Strength)
- H_B = 1800 N/mm^2 (Brinell Hardness)
- E = 2.1×10^5 N/mm^2 (Modulus of Elasticity)
- σ_{sy} = 484 N/mm^2 (Yield Strength)
- q= 1.6
- $\sigma_{em} = \frac{\sigma_y}{q} = \frac{484}{1.6} = 302.5 \text{ N/mm}^2$
- ρ_{em} = 0.4 × H_B = 0.4 × 1800 = 720 N/mm^2

Also for calculations I need safety factor assumption => S=**1.5**

All calculations for stage 1 are made for founding hardness and stress control also another physıcal properties of gears .

Stage 1

φ=20° ,helix angle is selected 20 because my book has formed factor table according to 20° helix angle.

- Forming factor for φ=20° and N=10 from table 14-2 k_f = **0.201**

$$K_f = \frac{1}{k_f} = \frac{1}{0.201} = \mathbf{4.975}$$

$\varepsilon = 1.4$ (ε is selected between 1.1 to 1.4)

$$K_i = \frac{I_{34}+1}{I_{34}} = \frac{4.5+1}{4.5} = \mathbf{1.222}$$

Module of depth and surfaces are calculated ;

- $m = \sqrt[3]{\frac{2\times S\times\tau_{G1}\times K_f}{N_1\times\varphi\times\varepsilon\times\sigma_{em}}} = \sqrt[3]{\frac{2\times 1.5\times 9548.6.\times 4.975}{10\times 20\times 1.4\times 302.5\times 10^6}} = \mathbf{11.89\ mm}$ (Depth of teeth)

- $m = \sqrt[3]{\frac{2 \times S \times \tau_{G1} \times E \times K_i}{N_1^2 \times \varphi \times \varepsilon \times \rho_{em}^2}} = \sqrt[3]{\frac{2 \times 1.5 \times 9548.6 \times 2.1 \times 10^5 \times 10^6 \times 1.222}{10^2 \times 20 \times 1.4 \times (720 \times 10^6)^2}}$ =>
  $5.61 \times 10^{-4} m$ =**0.46mm**(Surface Teeth)

According to these two modules , I selected m =**12 mm** (Table 13-2).

- $d_1 = m \times N_1 = 12 \times 10 = \mathbf{120\ mm}$
- $d_2 = m \times N_2 = 12 \times 20 = \mathbf{240\ mm}$

These values are used for stress and hardness control ;

Stress and Hardness Control

- $F = \frac{2 \times S \times \tau_{G1}}{d_1} = \frac{2 \times 1.5 \times 9548.6}{120 \times 10^{-3}} = \mathbf{238715}\text{N}$
- $\sigma_{max} = K_f \times \frac{F}{m \times \varepsilon \times B}$
- $B = \varphi \times m = 20 \times 12 = \mathbf{240}\ \text{mm}$
- $\sigma_{max} = 4.975 \times \frac{\mathbf{238715}}{12 \times 1.4 \times 240} = \mathbf{294.55}\ \text{N/mm}^2$

σ_{em} =302.5 N/mm^2 and σ_{max} = 294.55 N/mm^2 , $\sigma_{max} \leq \sigma_{em}$. **There is no design problem.**

- $\rho_{max} = K_m \times K_\alpha \times K_\varepsilon \times \sqrt{\frac{F \times K_i}{B \times d_1}}$
- $K_m = \sqrt{0.35 \times E} = \sqrt{0.35 \times 2.1 \times 10^5} = 271.1088$
- $K_\alpha = \sqrt{\frac{1}{\sin\varphi \times \cos\varphi}} = \sqrt{\frac{1}{\sin 20 \times \cos 20}} = 1.76393$
- $K_\varepsilon = \frac{1}{\sqrt{\varepsilon}} = \frac{1}{\sqrt{1.4}} = 0.84515$

$$\rho_{max} = 271.1088 \times 1.76393 \times 0.84515 \times \sqrt{\frac{238715 \times 1.222}{240 \times 120}} = \mathbf{1286}\ \text{N/mm}^2$$

$\rho_{max} \leq \rho_{em}$, $\rho_{max} = \mathbf{1287}\ \text{N/mm}^2$, ρ_{em} =720 N/mm^2 so **there is** **design Problem.**

Module of the gearbox should selected m=20 Table (13-2)

- $d_1 = m \times N_1 = 20 \times 10 = \mathbf{200\ mm}$
- $d_2 = m \times N_2 = 20 \times 20 = \mathbf{400\ mm}$

These values are used for stress and hardness control ;

Stress and Hardness Control

- $F = \frac{2 \times S \times \tau_{G1}}{d_1} = \frac{2 \times 1.5 \times 9548.6}{200 \times 10^{-3}} = \mathbf{143229}\ \text{N}$
- $\sigma_{max} = K_f \times \frac{F}{m \times \varepsilon \times B}$
- $B = \varphi \times m = 20 \times 20 = \mathbf{400}\ \text{mm}$

- $\sigma_{max} = 4.975 \times \frac{143229}{20\times1.4\times400} = \mathbf{63.62}\ N/mm^2$

σ_{em} =302.5 N/mm^2 and σ_{max} = 63.62N/mm^2 , $\sigma_{max} \leq \sigma_{em}$. **There is no design problem.**

- $\rho_{max} = K_m \times K_\alpha \times K_\varepsilon \times \sqrt{\frac{F\times K_i}{B\times d_1}}$
- $K_m = \sqrt{0.35 \times E} = \sqrt{0.35 \times 2.1 \times 10^5} = 271.1088$
- $K_\alpha = \sqrt{\frac{1}{\sin\varphi\times\cos\varphi}} = \sqrt{\frac{1}{\sin20\times\cos20}} = 1.76393$
- $K_\varepsilon = \frac{1}{\sqrt{\varepsilon}} = \frac{1}{\sqrt{1.4}} = 0.84515$

$$\rho_{max} = 271.1088 \times 1.76393 \times 0.84515 \times \sqrt{\frac{143402 \times 1.222}{400 \times 200}} = \mathbf{598}\ N/mm^2$$

$\rho_{max} \leq \rho_{em}$, $\rho_{max} = \mathbf{598}\ N/mm^2$, ρ_{em} =720 N/mm^2 so **there is** **<u>no Design Problem</u>.**

<u>Stage 2</u>

All calculations are made for stage 2 again.

φ=20° ,helix angle is selected 20 because my book has formed factor table according to 20° helix angle.

- Forming factor for φ=20° and N=20 from table 14-2 k_f = **0.322.**

$$K_f = \frac{1}{k_f} = \frac{1}{0.201} = \mathbf{3.1055}$$

$\varepsilon = 1.4$ (ε is selected between 1.1 to 1.4)

$$K_i = \frac{I_{34}+1}{I_{34}} = \frac{6.43+1}{6.43} = \mathbf{1.16}$$

Module of depth and surfaces are calculated ;

- $m = \sqrt[3]{\frac{2\times S\times \tau_{G3}\times K_f}{N_1\times\varphi\times\varepsilon\times\sigma_{em}}} = \sqrt[3]{\frac{2\times1.5\times17208\times3.1055}{20\times20\times1.4\times302.5\times10^6}}$ = 9.81 mm(Depth of teeth)
- $m = \sqrt[3]{\frac{2\times S\times \tau_{G3}\times E\times K_i}{N_1^2\times\varphi\times\varepsilon\times\rho_{em}^2}} = \sqrt[3]{\frac{2\times1.5\times17208\times2.1\times10^5\times10^6\times1.16}{20^2\times20\times1.4\times(720\times10^6)^2}}$ = 12.93 mm(Surface Teeth)

According to these two modules , I selected m =**16 mm** (Table 13-2).

- $d_3 = m \times N_3 = 16 \times 20 = \mathbf{320\ mm}$
- $d_4 = m \times N_4 = 16 \times 70 = \mathbf{1120\ mm}$

These values are used for stress and hardness control ;

Stress and Hardness Control

- $F = \frac{2 \times S \times \tau_{G3}}{d_3} = \frac{2 \times 1.5 \times 17208}{320 \times 10^{-3}} = \mathbf{161325}\ N$
- $\sigma_{max} = K_f \times \frac{F}{m \times \varepsilon \times B}$
- $B = \varphi \times m = 20 \times 16 = \mathbf{320}\ mm$
- $\sigma_{max} = 4.975 \times \frac{161325}{16 \times 1.4 \times 320} = \mathbf{111.96}\ N/mm^2$

σ_{em} =302.5 N/mm^2 and σ_{max} = 111.96 N/mm^2 , $\sigma_{max} \leq \sigma_{em}$. **There is no design problem.**

- $\rho_{max} = K_m \times K_\alpha \times K_\varepsilon \times \sqrt{\frac{F \times K_i}{B \times d_3}}$
- $K_m = \sqrt{0.35 \times E} = \sqrt{0.35 \times 2.1 \times 10^5} = 271.1088$
- $K_\alpha = \sqrt{\frac{1}{\sin\varphi \times \cos\varphi}} = \sqrt{\frac{1}{\sin 20 \times \cos 20}} = 1.76393$
- $K_\varepsilon = \frac{1}{\sqrt{\varepsilon}} = \frac{1}{\sqrt{1.4}} = 0.84515$

$$\rho_{max} = 271.1088 \times 1.76393 \times 0.84515 \times \sqrt{\frac{161325 \times 1.16}{320 \times 320}} = \mathbf{546.819}\ N/mm^2$$

$\rho_{max} \leq \rho_{em}$, $\rho_{max} = \mathbf{560.783}\ N/mm^2$, ρ_{em} =720 N/mm^2 so **there is no design problem.**

Physical Parameters for Gears

Gear 1

- $N_1 = 10$
- m = 20 mm
- $t = \pi \times m = \pi \times 20 = 62.83$ mm
- $B = \varphi \times m = 20 \times 20 = 400$ mm
- $d_1 = m \times N_1 = 20 \times 10 = 200$ mm
- $d_{a1} = d_1 + (2 \times m) = 200 + (2 \times 20) = 240$ mm
- $d_{t1} = d_1 - (2.5 \times m) = 200 - (2.5 \times 20) = 150$ mm
- $\frac{d_1 + d_2}{2} = \frac{200+400}{2} = 300$ mm
- $a = \frac{d_{a1}}{2} - \frac{d_1}{2} = \frac{240}{2} - \frac{200}{2} = 10$mm
- $b = 1.2 \times m = 1.2 \times 20 = 24$ mm
- $h = a + b = 10 + 24 = 34$
- $c = b - a = 24 - 10 = 14$ mm
- **Gear 2**
- $N_2 = 20$
- m = 20 mm
- $t = \pi \times m = \pi \times 20 = 62.83$ mm
- $B = \varphi \times m = 20 \times 20 = 400$ mm

- $d_2 = m \times N_2 = 20 \times 20 = 400$ mm
- $d_{a2} = d_2 + (2 \times m) = 400 + (2 \times 20) = 440$
- $d_{t2} = d_2 - (2.5 \times m) = 400 - (2.5 \times 20) = 350$ mm
- $\frac{d_1+d_2}{2} = \frac{200+400}{2} = 300$ mm
- $a = \frac{d_{a2}}{2} - \frac{d_2}{2} = \frac{440}{2} - \frac{400}{2} = 10$mm
- $b = 1.2 \times m = 1.2 \times 20 = 24$mm
- $h = a + b = 10 + 24 = 34$mm
- $c = b - a = 24 - 10 = 14$ mm

Gear 3

- $m = 16$ mm
- $t = \pi \times m = \pi \times 16 = 50.26$ mm
- $B = \varphi \times m = 20 \times 16 = 320$ mm
- $d_3 = m \times N_3 = 16 \times 20 = 320$ mm
- $d_{a3} = d_3 + (2 \times m) = 320 + (2 \times 16) = 352$ mm
- $d_{t3} = d_3 - (2.5 \times m) = 320 - (2.5 \times 16) = 280$ mm
- $\frac{d_3+d_4}{2} = \frac{320+1120}{2} = 720$ mm
- $a = \frac{d_{a3}}{2} - \frac{d_3}{2} = \frac{352}{2} - \frac{320}{2} = 16$mm
- $b = 1.2 \times m = 1.2 \times 16 = 19.2$ mm
- $h = a + b = 16 + 19.2 = 35.2$ mm
- $c = b - a = 19.2 - 16 = 3.2$mm

Gear 4

- $m = 16$ mm
- $t = \pi \times m = \pi \times 16 = 50.26$ mm
- $B = \varphi \times m = 20 \times 16 = 320$ mm
- $d_4 = m \times N_4 = 16 \times 70 = 1120$ mm
- $d_{a4} = d_4 + (2 \times m) = 1120 + (2 \times 16) = 1152$ mm
- $d_{t4} = d_4 - (2.5 \times m) = 1120 - (2.5 \times 16) = 1080$ mm
- $\frac{d_3+d_4}{2} = \frac{320+1120}{2} = 720$ mm
- $a = \frac{d_{a4}}{2} - \frac{d_4}{2} = \frac{1152}{2} - \frac{1120}{2} = 16$ mm
- $b = 1.2 \times m = 1.2 \times 16 = 19.2$ mm
- $h = a + b = 16 + 19.2 = 35.2$ mm
- $c = b - a = 19.2 - 16 = 3.2$ mm

Forces applied shafts should be known for stress control of each 3 shafts.

FORCES ON SHAFTS

External forces: This forces due to torque directly **;**

- $F_{e21} = -F_{e12} = \frac{2 \times S \times \tau_{G1}}{d_1} = \frac{2 \times 1.5 \times 9548.6}{200 \times 10^{-3}} = \mathbf{143225}$ N

- $F_{e34} = -F_{e43} = \frac{2 \times S \times \tau_{G3}}{d_3} = \frac{2 \times 1.5 \times 17208}{320 \times 10^{-3}} = \mathbf{161325}\ \mathrm{N}$

Radial Forces:

- $F_{r21} = -F_{r12} = F_{e21} \times \tan\varphi = 143225 \times \tan 20 = \mathbf{52129}\ \mathrm{N}$
- $F_{r43} = -F_{r34} = F_{e34} \times \tan\varphi = 161325 \times \tan 20 = \mathbf{58717.5}\ \mathrm{N}$

Due to these radial and external forces , stress of each shafts should be calculated and if any parameter not suitable according to safety factor, design parameters should be changed.

BEARING ANALYSIS FOR SHAFT 1

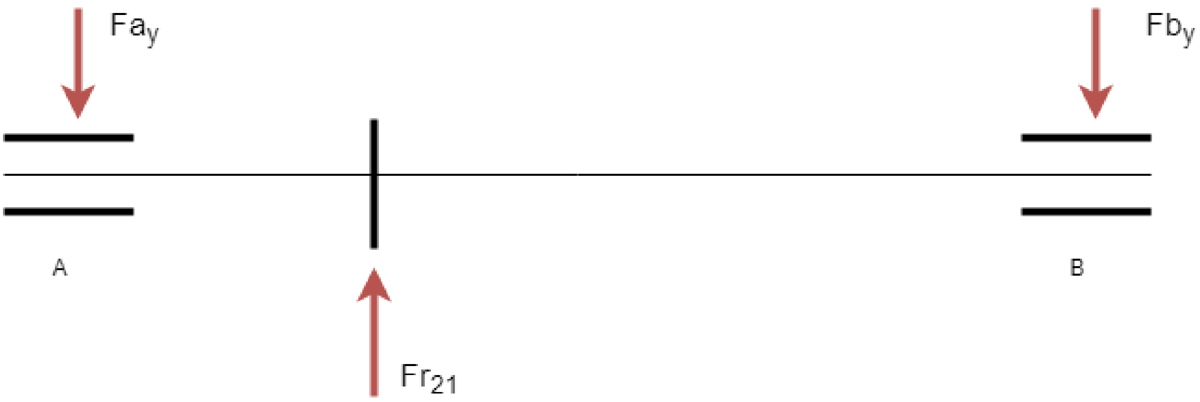
- Shaft length so distance between A and B is assumed ;

$$x_1 = \mathbf{150}\ \mathrm{mm}$$

- Distance between Gear 1 and bearing A assumed as;

$$x_2 = \mathbf{40}\ \mathrm{mm}$$

Force analyses on (x-y) plane



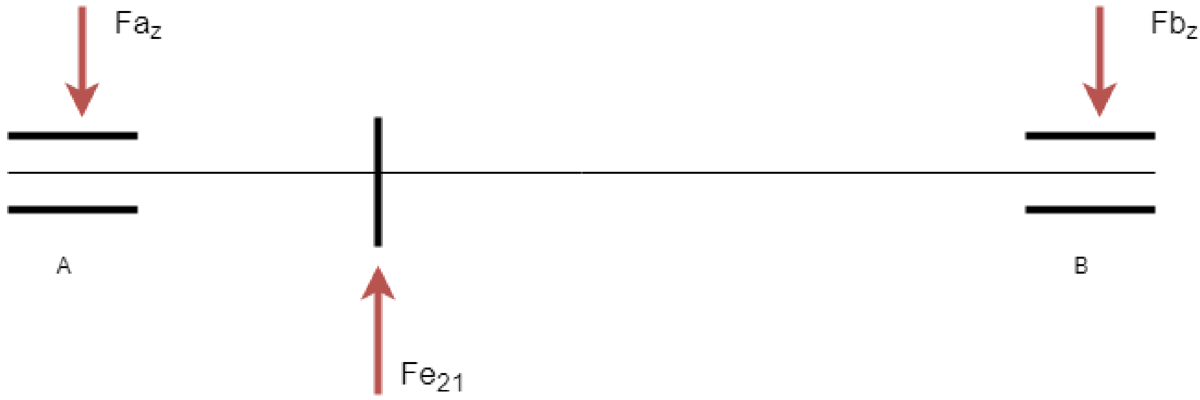
$\sum M_A = 0 \quad \rightarrow \quad F_{By} \times x_1 - F_{r21} \times x_2 = 0$

- $F_{By} = \frac{F_{r21} \times x_2}{x_1} = \frac{52193 \times 40}{150} = \mathbf{13818.13}\ \mathrm{N}$

$\sum F_y = 0 \quad \rightarrow \quad F_{Ay} + F_{By} - F_{r21} = 0$

- $F_{Ay} = F_{r21} - F_{By} = 52193 - 13818.13 = \mathbf{38375}\ \mathrm{N}$
- $M_{yG1} = F_{Ay} \times x_2 = 38375 \times 40 = 1535000\ \mathrm{N.mm} = \mathbf{1535}\ \mathrm{N.m}$

Force analyses on (x-z) plane



$\sum M_A = 0$ ➔ $F_{Bz} \times x_1 - F_{e21} \times x_2 = 0$

- $F_{Bz} = \frac{F_{e21} \times x_2}{x_1} = \frac{143400 \times 40}{150} = \mathbf{38240}\ \mathrm{N}$

$\sum F_z = 0$ ➔ $F_{Az} + F_{Bz} - F_{e21} = 0$

- $F_{Az} = F_{e21} - F_{Bz} = 143400 - 38240 = \mathbf{105160}\ \mathrm{N}$
- $M_{zG1} = F_{Az} \times x_2 = 105160 \times 40 = 4206400\ \mathrm{N.mm} = \mathbf{4206,4}\ \mathrm{N.m}$

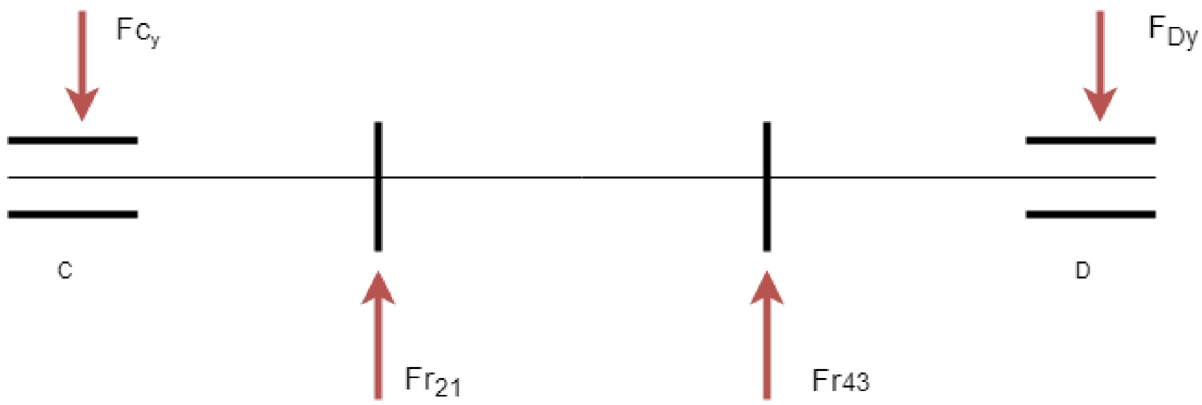
Total forces on bearings A and B

- $F_A = \sqrt{F_{Ay}^2 + F_{Az}^2} = \sqrt{38375^2 + 105160^2} = \mathbf{111973}\ \mathrm{N}$
- $F_B = \sqrt{F_{By}^2 + F_{Bz}^2} = \sqrt{13818.13^{\ 2} + 38240^2} = \mathbf{40659}\ \mathrm{N}$

BEARING ANALYSIS FOR SHAFT 2

- Distance between C and D assumed as $x_1 = \mathbf{180}$ mm.
- Distance between Gear 2 and bearing C assumed as $x_2 = \mathbf{40}$ mm.
- Distance between Gear 3 and bearing C assumed as $x_3 = \mathbf{140}$ mm.

Force analyses on (x-y) plane



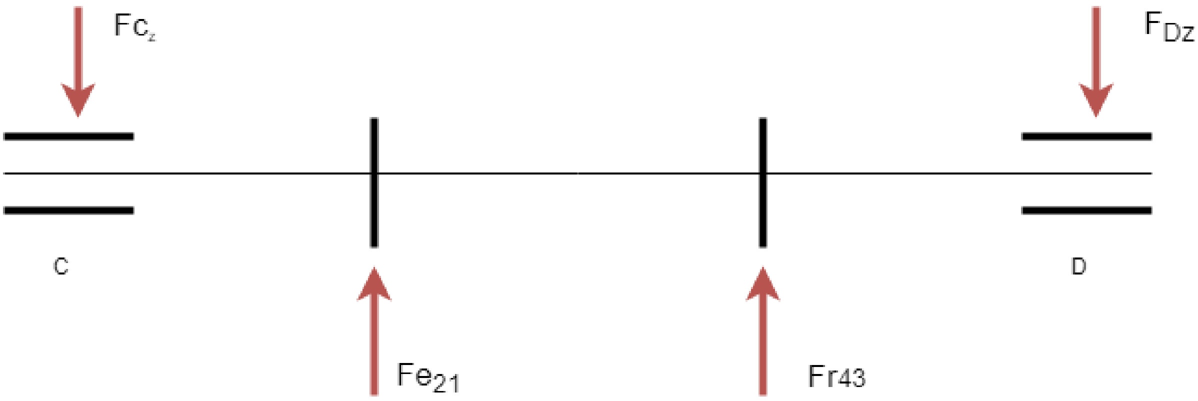
$\sum M_C = 0$ ➔ $F_{Dy} \times x_1 + F_{r12} \times x_2 - F_{r43} \times x_3 = 0$

- $F_{Dy} = \frac{F_{r43} \times x_3 - F_{r12} \times x_2}{x_1} = \frac{58717 \times 140 + 52193 \times 40}{180} = \mathbf{52267}\ \mathrm{N}$

$\sum F_y = 0$ ➔ $F_{Cy} + F_{r43} - F_{r12} - F_{Dy} = 0$

- $F_{Cy} = F_{r12} - F_{r43} + F_{Dy} = -52193 - 58717 + 52267 = \mathbf{-58643}\ \mathrm{N}$
- $M_{yG2} = F_{Cy} \times x_2 = -58643 \times 40 = -2345720\mathrm{N.mm} = \mathbf{-2345,72}\ \mathrm{N.m}$
- $M_{yG3} = F_{Cy} \times x_3 - F_{r12} \times (x_3 - x_2) = -58643 \times 140 + 52193 \times 100 = \mathbf{-2990720}\ \mathrm{N.mm}$

<u>Force analyses on (x-z) plane</u>



$\sum M_C = 0$ ➔ $F_{DZ} \times x_1 - F_{e12} \times x_2 - F_{e43} \times x_3 = 0$

- $F_{DZ} = \frac{F_{e43} \times x_3 + F_{e12} \times x_2}{x_1} = \frac{-161325 \times 140 - 143400 \times 40}{180} = \mathbf{-157341}\ \mathrm{N}$

$\sum F_z = 0$ ➔ $-F_{CZ} + F_{e43} + F_{e12} - F_{Dy} = 0$

- $F_{Cz} = F_{e12} + F_{e43} - F_{Dz} = -143400 - 161325 + 157341 = \mathbf{-147383}\ \mathrm{N}$
- $M_{zG2} = -F_{Cz} \times x_2 = 147383 \times 40 = 5895333\ \mathrm{N.mm} = \mathbf{5895.333}\mathrm{N.m}$
- $M_{zG3} = -F_{Cz} \times x_3 + F_{e12} \times (x_3 - x_2) = 147383 \times 140 - 143400 \times 100 = \mathbf{6293620}\ \mathrm{N.mm}$

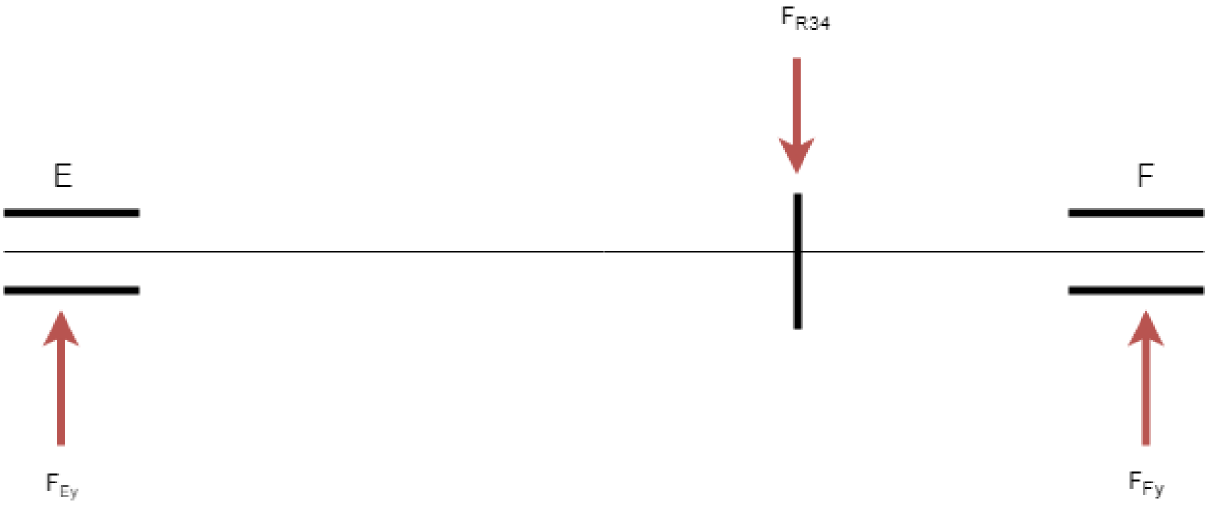
<u>Total forces on bearings C and D</u>

- $F_C = \sqrt{F_{Cy}^2 + F_{Cz}^2} = \sqrt{(-58643)^2 + (-147383)^2} = \mathbf{158621}\mathrm{N}$
- $F_D = \sqrt{F_{Dy}^2 + F_{Dz}^2} = \sqrt{52267^2 + (-157341)^2} = \mathbf{165795}\ \mathrm{N}$

BEARING ANALYSIS FOR SHAFT 3

- Distance between bearing E and bearing F assumed as $x_1 = \mathbf{150}$ mm.
- Distance between Gear 4 and bearing E assumed as $x_2 = \mathbf{110}$ mm.

Force analyses on (x-y) plane



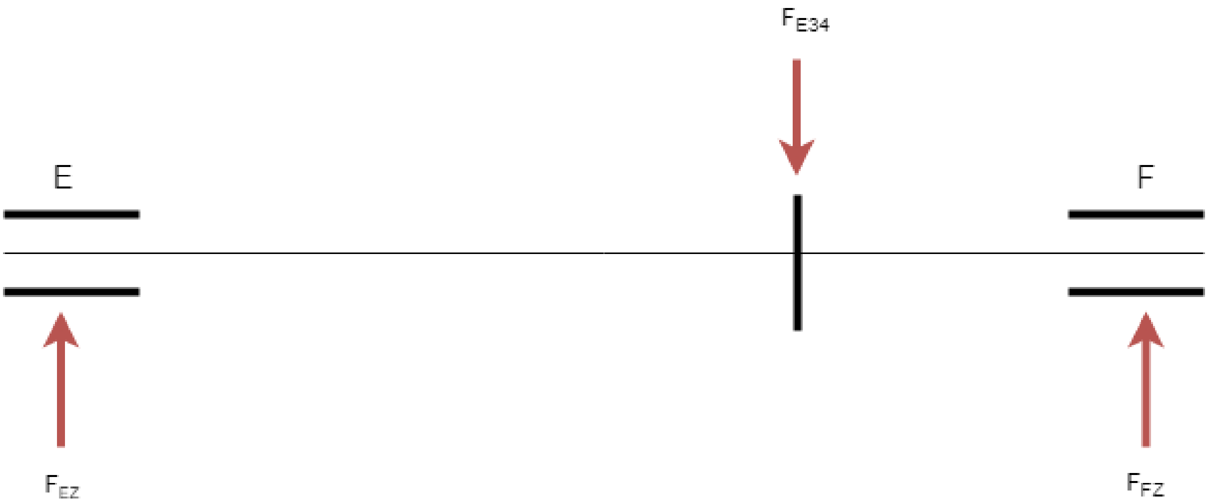
$\sum M_E = 0 \quad \Rightarrow \quad F_{Fy} \times x_1 - F_{r34} \times x_2 = 0$

- $F_{Fy} = \frac{F_{r34} \times x_2}{x_1} = \frac{-58717.5 \times 110}{150} = \mathbf{-43059.5}$ N

$\sum F_y = 0 \quad \Rightarrow \quad F_{Ey} + F_{Fy} - F_{r34} = 0$

- $F_{Ey} = F_{r34} - F_{Fy} = -58717.5 + 43059.5 = \mathbf{-15658}$ N
- $M_{yG4} = F_{Ey} \times x_2 = \mathbf{-15658} \times 110 = -1722380 \text{ N.mm} = \mathbf{-1722.38} \text{ N.m}$

Force analyses on (x-z) plane



$\sum M_E = 0 \quad \Rightarrow \quad F_{Fz} \times x_1 - F_{e34} \times x_2 = 0$

- $F_{Fz} = \frac{F_{e34} \times x_2}{x_1} = \frac{161325 \times 110}{150} = \mathbf{118305}\ \mathrm{N}$

$\sum F_z = 0 \quad \Rightarrow \quad F_{Ez} + F_{Fz} - F_{e34} = 0$

- $F_{Ez} = F_{e34} - F_{Fz} = 161325 - 118305 = \mathbf{43020}\ \mathrm{N}$
- $M_{zG4} = -F_{Ez} \times x_2 = -\mathbf{43020} \times 110 = -4732200\ \mathrm{N.mm} = -\mathbf{4732.2}\mathrm{N.m}$

Total forces on bearings E and F

- $F_E = \sqrt{F_{Ey}^2 + F_{Ez}^2} = \sqrt{(-15658\ )^2 + 43020^2} = \mathbf{45780.9}\ \mathrm{N}$
- $F_F = \sqrt{F_{Fy}^2 + F_{Fz}^2} = \sqrt{(-43059.5\ )^2 + 118305^2} = \mathbf{125897.3}\ \mathrm{N}$

My next step is selecting material of shafts and examine stresses each shaft according too factor of safety.

SHAFT 1 STRESS ANALYSIS

16 MnCr5

$\sigma_{tensile} = 882$ (MPa)

$\sigma_{yieled} = 440$ (MPa)

$\tau_{yield} = 360$ (MPa)

$S_{shaft} = 6$ (shaft safety factor)

$M_{G1} = M_{maxG1} = \sqrt{M_{zG1}^2 + M_{yG1}^2} = \sqrt{4206400^2 + 1535000^2} = \mathbf{4477725}\ \mathrm{N.mm}$

$\tau_{G1} = \mathbf{9560229}\ \ \mathrm{N.mm}$ (Torque of Gear 1)

$S = 1.5$, $\tau_{SG1} = S \times \tau_{G1} = 1.5 \times 9560229 = \mathbf{14340343}\ \mathrm{N.mm}$

$\tau_{sf} = \frac{\tau_{yield}}{S_{shaft}}$, $\tau_{sf} = \frac{360}{6} = 60\ \mathrm{Mpa}$

$d_1 \geq \sqrt[3]{\frac{16}{\pi} \times \frac{\tau_{SG1}}{\tau_{sf}}} = d_1 \geq \sqrt[3]{\frac{16}{\pi} \times \frac{14340343}{60}} = 107\ \mathrm{mm} = \ d_1 = 110\ \mathrm{mm}.$

$\mathbf{d_1 = 110mm}$ (**Diameter of shaft 1**)

$$A = \frac{\pi \times d_1^2}{4} = \frac{\pi \times 110^2}{4} = 9503\ \mathrm{mm}^2\ \text{(Shaft's cross sectional area)}$$

Bending Stress

$$W_b = \frac{\pi \times d_1^3}{32} \quad , \quad W_b = \frac{\pi \times 110^3}{32} = 130670\ mm^3$$

$$\sigma_b = \frac{M_{G1}}{W_b} \quad , \quad \sigma_b = \frac{4477725}{130670} = 34.26\ \mathrm{MPa}$$

Shear Stress

$$W_\tau = \frac{\pi \times d_1^3}{16} \quad , \quad W_\tau = \frac{\pi \times 110^3}{16} = \mathbf{261341}\ \mathrm{mm}^3$$

$$\tau_t = \frac{\tau_{SG1}}{W_\tau} \quad , \quad \tau_t = \frac{14340343}{261341} = \mathbf{54.87}\ \mathrm{MPa}$$

$$\tau_{st} = \tau_t \times S_{shaft} = 54.87 \times 6 = 329.23\ \mathrm{MPa}$$

$360 > 329.23$ (MPa) $\tau_{yield} > \tau_t$ **There is no design problem.**

$$\sigma_{max} = \sqrt{\sigma_b^2 + (2 \times \tau_t^2)} = \sqrt{34.26^2 + (2 \times 54.87^2)} = \mathbf{85.06}\ \mathrm{MPa}$$

$$\sigma_{Smax} = S \times \sigma_{max} = 1.5 \times 85.06 = \mathbf{127.6}\ \mathrm{MPa}$$

$440 > 127.6$ (MPa) $\sigma_{yield} > \sigma_{Smax}$ **There is no design problem.**

SHAFT 2 STRESS ANALYSIS

16 MnCr5

$\sigma_{tensile} = 882$ (MPa)

$\sigma_{yieled} = 440$ (MPa)

$\tau_{yield} = 360$ (MPa)

$S_{shaft} = 6$ (shaft safety factor)

$$M_{G2} = M_{maxG2} = \sqrt{M_{zG2}^2 + M_{yG2}^2} = \sqrt{589533^2 + 2345720^2} = \mathbf{6344868}\ \mathrm{N.mm}$$

$$M_{G3} = M_{maxG3} = \sqrt{M_{zG3}^2 + M_{yG3}^2} = \sqrt{6293620^2 + 2990720^2} = \mathbf{6968074}\ \mathrm{N.mm}$$

$$\tau_{G2} = \tau_{G3} = \mathbf{17208}\ \mathrm{N.m}$$

$$S = 1.5 \quad , \quad \tau_{SG2} = S \times \tau_{G2} = 1.5 \times 17208000 = \mathbf{25812000}\ \mathrm{N.mm}$$

$$\tau_{sf} = \frac{\tau_{yield}}{S_{shaft}} \quad , \quad \tau_{sf} = \frac{360}{6} = 60\ \mathrm{MPa}$$

$$d_2 \geq \sqrt[3]{\frac{16}{\pi} \times \frac{\tau_{SG2}}{\tau_{sf}}} = d_2 \geq \sqrt[3]{\frac{16}{\pi} \times \frac{\mathbf{25812000}}{60}} = 129.88 \text{ mm} = d_2 = 130 \text{ mm.}$$

$$\mathbf{d_2 = 130\ mm} \qquad (\textbf{Diameter of shaft 2})$$

$$A = \frac{\pi \times d_2^2}{4} = \frac{\pi \times 130^2}{4} = \mathbf{13273}\ \text{mm}^2 \quad \text{(Shaft's cross sectional area)}$$

<u>Bending Stress</u>

$$W_b = \frac{\pi \times d_2^3}{32} \qquad , \qquad W_b = \frac{\pi \times 130^3}{32} = \mathbf{215689}\ mm^3$$

$$\sigma_b = \frac{M_{G3}}{W_b} \qquad , \qquad \sigma_b = \frac{6968074}{215689} = \mathbf{32.3}\ \text{MPa}$$

<u>Shear Stress</u>

$$W_\tau = \frac{\pi \times d_2^3}{16} \qquad , \qquad W_\tau = \frac{\pi \times 130^3}{16} = \mathbf{431379}\text{mm}^3$$

$$\tau_t = \frac{\tau_{SG2}}{W_\tau} \qquad , \qquad \tau_t = \frac{25812000}{431379} = \mathbf{59.83}\ \text{MPa}$$

$$\tau_{st} = \tau_t \times S_{shaft} = 59.83 \times 6 = \mathbf{359}\ \text{MPa}$$

$360 > 359$ (MPa) , $\tau_{yield} > \tau_t$ **<u>There is no design problem.</u>**

$$\sigma_{max} = \sqrt{\sigma_b^2 + (2 \times \tau_t^2)} = \sqrt{32.3^{\ 2} + (2 \times 59.83^2)} = \mathbf{90.56}\ \text{MPa (Von Mises Stress)}$$

$$\sigma_{Smax} = S \times \sigma_{max} = 1.5 \times 90.56 = \mathbf{135.85}\ \text{MPa}$$

$440 > 135.85$ (MPa) $\sigma_{yield} > \sigma_{Smax}$ **<u>There is no design problem.</u>**

SHAFT 3 STRESS ANALYSIS

18 MnCr5

$\sigma_{tensile}$ = 882 (MPa)

σ_{yieled} = 440 (MPa)

τ_{yield} = 360 (MPa)

S_{shaft} = 6 (shaft safety factor)

$$\mathrm{M_{G4}} = \mathrm{M_{maxG4}} = \sqrt{\mathrm{M_{zG4}^2} + \mathrm{M_{yG4}^2}} = \sqrt{(-4732200\,)^2 + (-1722380\,)^2} = \mathbf{5035902}\ \mathrm{N.mm}$$

$\tau_{G4} = \mathbf{54206}\ \mathrm{N.m}$ (Torque of Gear4)

$S = 1.5$ $\quad\quad$ $\tau_{SG4} = S \times \tau_{G4} = 1.5 \times 54206000 = \mathbf{81309000}\ \mathrm{N.mm}$

$\tau_{sf} = \dfrac{\tau_{yield}}{S_{shaft}}$ $\quad\quad$ $\tau_{sf} = \dfrac{360}{6} = 60\ \mathrm{MPa}$

$$\mathrm{d_4} \geq \sqrt[3]{\frac{16}{\pi} \times \frac{\tau_{SG4}}{\tau_{sf}}} = \mathrm{d_4} \geq \sqrt[3]{\frac{16}{\pi} \times \frac{81309000}{60}} = 109\ \mathrm{mm} = \mathrm{d_3} = 110\ \mathrm{mm.}$$

$$\mathbf{d_3 = 110\ mm} \quad\quad (\textbf{Diameter of shaft 3})$$

Shaft's cross sectional area

$$\mathrm{A} = \frac{\pi \times d_3^2}{4} = \frac{\pi \times 110^2}{4} = \mathbf{9503}\ \mathrm{mm^2}\ \text{(Shaft's cross sectional area)}$$

Bending Stress

$W_b = \dfrac{\pi \times d_3^3}{32}$ $\quad\quad$ $W_b = \dfrac{\pi \times 110^3}{32} = \mathbf{130670}\ mm^3$

$\sigma_b = \dfrac{\mathrm{M_{G4}}}{\mathrm{W_b}}$ $\quad\quad$ $\sigma_b = \dfrac{5035902}{130670} = \mathbf{38.53}\ \mathrm{MPa}$

Shear Stress

$\mathrm{W_\tau} = \dfrac{\pi \times \mathrm{d_3^3}}{16}$ $\quad\quad$ $\mathrm{W_\tau} = \dfrac{\pi \times 110^3}{16} = \mathbf{261341}\ \mathrm{mm^3}$

$\tau_t = \dfrac{\tau_{SG4}}{\mathrm{W_\tau}}$ $\quad\quad$ $\tau_t = \dfrac{81309000}{261341} = \mathbf{311.12}\ \mathrm{MPa}$

$$\tau_{st} = \tau_t \times S_{shaft} = 311.12 \times 6 = 1866\ \mathrm{MPa}$$

$360 > 311.12 \quad (MPa) \qquad \tau_{yield} > \tau_t$ **There is no design problem.**

$$\sigma_{max} = \sqrt{\sigma_b^2 + (2 \times \tau_t^2)} = \sqrt{38.53^2 + (2 \times 311.12\ ^2)} = \mathbf{235}\ \text{MPa(Von Mises Stress)}$$

$$\sigma_{Smax} = S \times \sigma_{max} = 1.5 \times 235 = 352.5\ \text{MPa}$$

$440 > 325.5 \quad (MPa) \qquad \sigma_{yield} > \sigma_{Smax}$ **There is no design problem.**

Next step of this project is analysis the deflection of the shafts , this value should be in range safety.

SHAFT 1 DEFLECTION ANALYSIS

Strain Deflection

$$\delta_y = \frac{F_{r12} \times L^3}{48 \times E \times I} \rightarrow \delta_y = \frac{-52193 \times 150^3}{48 \times 2.1 \times 10^5 \times \frac{\pi \times 110^4}{64}} = \mathbf{-0.00243}\ \text{mm}$$

$$\delta_x = \frac{F_{e12} \times L^3}{48 \times E \times I} \rightarrow \delta_x = \frac{-143400 \times 150^3}{48 \times 2.1 \times 10^5 \times \frac{\pi \times 110^4}{64}} = \mathbf{-0.0068}\ \text{mm}$$

$$\delta_{max} = \sqrt{\delta_x^2 + \delta_y^2} \rightarrow \delta_{max} = \sqrt{\mathbf{0.00243}\ ^2 + \mathbf{0.0068}^2} = \mathbf{0.007}\ \text{mm}$$

Control of Deflection due to Strain

$\frac{\delta_{max}}{L} < \frac{1}{3000}\ ,\ \frac{0.007}{150} = 4.66 \times 10^{-5} < \frac{1}{3000}$ **This value in range so there is no design problem.**

Torsion Deflection

$$G = 8.1 \times 10^4 \text{MPa}$$

$$\varphi_{sf} = \frac{\tau_{yield}}{S_{shaft}} \times \frac{L}{r} \times \frac{1}{G} = \frac{360 \times 150}{6 \times 20 \times 8.1 \times 10^4} = 5.55 \times 10^{-3}$$

$$I_p = \frac{\pi \times d^4}{32} \rightarrow I_p = \frac{\pi \times 110^4}{32} = 14373768\ \text{mm}^4$$

Control of Deflection due to Torsion

For shaft safety $\varphi_{max} < \varphi_{sf}$ must be provided.

$$\varphi_{max} = \frac{M_{maxG1} \times L}{G \times I_p} \quad \rightarrow \quad \varphi_{max} = \frac{4477725 \times 150}{8.1 \times 10^4 \times 14373768} = 5.76 \times 10^{-4}\, rad$$

This value in range so there is no design problem.

SHAFT 2 DEFLECTION ANALYSIS

Strain Deflection

$$\delta_y = \frac{F_{r43} \times L^3}{48 \times E \times I} \quad \rightarrow \quad \delta_y = \frac{-52193 \times 150^3}{48 \times 2.1 \times 10^5 \times \frac{\pi \times 130^4}{64}} = \mathbf{-0.00126}\ \text{mm}$$

$$\delta_x = \frac{F_{e21} \times L^3}{48 \times E \times I} \quad \rightarrow \quad \delta_x = \frac{-143400 \times 150^3}{48 \times 2.1 \times 10^5 \times \frac{\pi \times 130^4}{64}} = \mathbf{-0.00348}\ \text{mm}$$

$$\delta_{max} = \sqrt{\delta_x^2 + \delta_y^2} \quad \rightarrow \quad \delta_{max} = \sqrt{0.00126^{\ 2} + 0.00348^2} = \mathbf{0.0037}\ \text{mm}$$

Control of Deflection due to Strain

$\frac{\delta_{max}}{L} < \frac{1}{3000}$, $\frac{0.0037}{150} = 2{,}47 \times 10^{-5} < \frac{1}{3000}$ **This value in range so there is no design problem.**

Torsion Deflection

$$G = 8.1 \times 10^4 \text{MPa}$$

$$\varphi_{sf} = \frac{\tau_{yield}}{S_{shaft}} \times \frac{L}{r} \times \frac{1}{G} = \frac{360 \times 150}{6 \times 20 \times 8.1 \times 10^4} = 5.55 \times 10^{-3}$$

$$I_p = \frac{\pi \times d^4}{32} \quad \rightarrow \quad I_p = \frac{\pi \times 130^4}{32} = 28039696\ \text{mm}^4$$

Control of Deflection due to Torsion

For shaft safety $\varphi_{max} < \varphi_{sf}$

$$\varphi_{max} = \frac{M_{maxG3} \times L}{G \times I_p} \quad \rightarrow \quad \varphi_{max} = \frac{6968074 \times 150}{8.1 \times 10^4 \times 14373768} = 4.59 \times 10^{-4}\, rad$$

This value in range so there is no design problem.

SHAFT 3 DEFLECTION ANALYSIS

Strain Deflection

$$\delta_y = \frac{F_{r34} \times L^3}{48 \times E \times I} \quad ➔ \quad \delta_y = \frac{-58717 \times 150^3}{48 \times 2.1 \times 10^5 \times \frac{\pi \times 110^4}{64}} = \mathbf{-0.00273}\ \text{mm}$$

$$\delta_x = \frac{F_{e34} \times L^3}{48 \times E \times I} \quad ➔ \quad \delta_x = \frac{-161325 \times 150^3}{48 \times 2.1 \times 10^5 \times \frac{\pi \times 110^4}{64}} = \mathbf{-0.0072}\ \text{mm}$$

$$\delta_{max} = \sqrt{\delta_x^2 + \delta_y^2} \quad ➔ \quad \delta_{max} = \sqrt{\mathbf{0.00273}\ ^2 + \mathbf{0.0072}^2} = \mathbf{0.0077}\ \text{mm}$$

Control of Deflection due to Strain

$\frac{\delta_{max}}{L} < \frac{1}{3000}$, $\frac{0.0077}{150} = 5.13 \times 10^{-5} < \frac{1}{3000}$ **This value in range so there is no design problem.**

Torsion Deflection

$$G = 8.1 \times 10^4 \text{MPa}$$

$$\varphi_{sf} = \frac{\tau_{yield}}{S_{shaft}} \times \frac{L}{r} \times \frac{1}{G} = \frac{360 \times 150}{6 \times 20 \times 8.1 \times 10^4} = 5.55 \times 10^{-3}$$

$$I_p = \frac{\pi \times d^4}{32} \quad ➔ \quad I_p = \frac{\pi \times 110^4}{32} = 14373768\ \text{mm}^4$$

Control of Deflection due to Torsion

For shaft safety $\varphi_{max} < \varphi_{sf}$

$$\varphi_{max} = \frac{M_{maxG4} \times L}{G \times I_p} \quad ➔ \quad \varphi_{max} = \frac{5035902 \times 150}{8.1 \times 10^4 \times 14373768} = 6.48 \times 10^{-4}\, rad$$

This value in range so there is no design problem.

- Last step of two stage gearbox design is analysis of keys. So for each 3 shafts ,keys should be use in ISO standarts.

Key & Keyway Dimensions - Millimeters										
Shaft Diameter "D"		Key Size Nominal		Keyway Width Hub "W"			Keyway Depth Hub "T2"		Keyway Radius "R"	
Over	Thru	Width "W"	Height "H"	Nominal	Min	Max	Min	Max	Min	Max
6	8	2	2	2	-.0125	+.0125	1.0	1.1	0.08	0.16
8	10	3	3	3	-.0125	+.0125	1.4	1.5	0.08	0.16
10	12	4	4	4	-.0150	+.0150	1.8	1.9	0.08	0.16
12	17	5	5	5	-.0150	+.0150	2.3	2.4	0.16	0.25
17	22	6	6	6	-.0150	+.0150	2.8	2.9	0.16	0.25
22	30	8	7	8	-.0180	+.0180	3.3	3.5	0.16	0.25
30	38	10	8	10	-.0180	+.0180	3.3	3.5	0.25	0.40
38	44	12	8	12	.-0215	+.0215	3.3	3.5	0.25	0.40
44	50	14	9	14	.-0215	+.0215	3.8	4.0	0.25	0.40
50	58	16	10	16	.-0215	+.0215	4.3	4.5	0.25	0.40
58	65	18	11	18	.-0215	+.0215	4.4	4.6	0.25	0.40
65	75	20	12	20	-.0260	+.0260	4.9	5.1	0.40	0.60
75	85	22	14	22	-.0260	+.0260	5.4	5.6	0.40	0.60
85	95	25	14	25	-.0260	+.0260	5.4	5.6	0.40	0.60
95	110	28	16	28	-.0260	+.0260	6.4	6.6	0.40	0.60
110	130	32	18	32	-.0310	+.0310	7.4	7.6	0.40	0.60
130	150	36	20	36	-.0310	+.0310	8.4	8.7	0.70	1.00
150	170	40	22	40	-.0310	+.0310	9.4	9.7	0.70	1.00
170	200	45	25	45	-.0310	+.0310	10.4	10.7	0.70	1.00
200	230	50	28	50	-.0310	+.0310	11.4	11.7	0.70	1.00
230	260	56	32	56	-.0370	+.0370	12.4	12.7	1.20	1.60
260	290	63	32	63	-.0370	+.0370	12.4	12.7	1.20	1.60
290	330	70	36	70	-.0370	+.0370	14.4	14.7	1.20	1.60
330	380	80	40	80	-.0370	+.0370	15.4	15.7	2.00	2.50

Figure 2.1 Key&Keyway Dimensions

- **Shaft 1 has 110 mm diameter**
- **Shaft 2 has 130 mm diameter.**
- **Shaft 3 has 110 mm diameter.**

So these values shown on this figure to determining key parameters.

The key material is selected to be **St70**,

$$\tau_e = 200\ [N/mm^2]$$

A safety factor of $S = 8$ so,

$$\tau_{em} = \frac{200}{8} = \mathbf{25}\ \left[\frac{N}{mm^2}\right]$$

Key Analysıs For Shaft 1

After examining table , key parameters are ;

$$d1 = 110mm\text{(Shaft 1)}$$

$$W = 32\ [mm]\ ;\ H = 18\ [mm]$$

$$T_1 = 7.4\ [mm]\ ;\ T_2 = 7.6\ [mm]$$

$$\tau_{em} = \frac{\frac{M_{d1-1}}{\frac{d_{01}}{2}}}{W \cdot l_{min}}$$

Thus, $l_{min} = \frac{\frac{M_{d1-1}}{d_{01}/2}}{W \cdot \tau_{em}} = \frac{\frac{\mathbf{4477725}}{100}}{32\times25} = \mathbf{55.97}[mm]$

- $l = 80[mm] > l_{min}$

$$\tau = \frac{\frac{M_{d1-1}}{d_{01}/2}}{W \cdot l} = \mathbf{90}\ [N/mm^2] < \tau_{allow}$$

Key Analysıs For Shaft 2

$$d2 = 130mm\text{(Shaft 2)}$$

$$W = 36\ [mm]\ ;\ H = 20\ [mm]$$

$$T_1 = 8.4\ [mm]\ ;\ T_2 = 8.7\ [mm]$$

Stage 1:

$$\tau_{em} = \frac{\frac{M_{d1-2}}{d_{02}/2}}{W \cdot l_{min}}$$

Thus, $l_{min} = \frac{\frac{M_{d1-1}}{d_{02}/2}}{W \cdot \tau_{em}} = \frac{\frac{\mathbf{6344868}}{400/2}}{36\times25} = \mathbf{35.24}[mm]$

$$l = 60\ [mm] > l_{min}$$

$$\tau = \frac{\frac{M_{d1-2}}{\frac{d_{02}}{2}}}{W \cdot l} = 57\ [N/mm^2] < \tau_{em}$$

Stage 2:

$$\tau_{em} = \frac{\frac{M_{d2-1}}{d_{03}/2}}{W \cdot l_{min}}$$

Thus, $l_{min} = \frac{\frac{M_{d2-1}}{d_{03}/2}}{W \cdot \tau_{em}} = \frac{\frac{\mathbf{6968074}}{\mathbf{320/2}}}{36 \times 25} = \mathbf{48.38}\ [mm]$

- $l = 60\ [mm] > l_{min}$

$$\tau = \frac{\frac{M_{d2-1}}{d_{03}/2}}{W \cdot l} = 23.88\ [N/mm^2] < \tau_{em}$$

Key Analysis For Shaft 3

$d_3 = 110mm$(Shaft 3)

$W = 32\ [mm]\ ;\ H = 18\ [mm]$

$T_1 = 7.4\ [mm]\ ;\ T_2 = 7.6\ [mm]$

$$\tau_{em} = \frac{\frac{M_{d4-1}}{d_{04}/2}}{W \cdot l_{min}}$$

Thus, $l_{min} = \frac{\frac{M_{d4-2}}{d_{04}/2}}{W \cdot \tau_{em}} = \frac{\frac{\mathbf{5035902}}{\mathbf{1120/2}}}{32 \times 25} = \mathbf{11.24}[mm]$

So, considering $l = 80\ [mm] > l_{min}$

$$\tau = \frac{\frac{M_{d4-2}}{d_{04}/2}}{W \cdot l} = 45[N/mm^2] < \tau_{em}$$

COST ANALYSIS

This calculation is assumed according to cost of steel kg and assumed mass of gears and shafts.

$$V = h.A = h.\pi r^2$$

$$M = \rho.V = \rho.h.\pi r^2$$

Cost of 1 Kg of steel is **24.10 TL (26.12.2019 Istanbul)**

Mass(Kg)	M_{gears}	M_{shafts}	M_{keys}	M_{total}
Mass(Kg)	5	20	3	28
Cost(TL)	120.5	482	72.3	674.8

$$\text{Total cost} = \text{Price of 1Kg of manufactured steel} * \text{Mtotal} + \text{Cost of } M_{total}\ .$$

$$Total\ Cost = 100\ TL(Assumed) * 28 + 674.8\ TL = \mathbf{3475}TL$$

CONCLUSION

In the project of the design of the two-stage gearbox, the input variables (n_{input}, $P_{input,}$ I_{total}) are determined according to the student numbers. My student number is 150415055. Hence, the input variables are;

- n_{input} = 55 rev/min (Input speed)
- P_{input} = 55 kW (Input power)
- I_{total} = 7 (Total Reduction rate)

There are some important variables that should be considered in the beginning of the design process of a gearbox such as the material from which the gears and shafts are made and the teeth number of gear 1 and gear 3.

At the beginning of the design process, any gear found at one stage should be known since the teeth numbers of these gears are very crucial for our further calculations. For this reason, the stage reduction rate is assumed and used to take advantage of the total reduction rate.

The material from which the gears are made of should be determined after these calculations. The material I have chosen for the gears is **16 MnCr5.** After that, I calculated the dimensions of gears and performed the stress control analysis to prevent any design problems.

The net forces and moments on x-z and x-y planes are determined and all of the forces on the shafts are calculated. After these calculations, the strength analysis is performed by controlling the strengths of shafts against forces due to axial and radial motions.

The material assumption should be made for three of the shafts and I selected them as;

Shaft 1: made of **16 MnCr5** material.

Shaft 2: made of **16 MnCr5** material.

Shaft 3: made of **18 MnCr5** material.

The stress analysis was done based on the characteristics of the strength values of these materials and no design problems were occurred at the end of the analysis.

Also, the shaft diameters were found according to the determined safety factors with these calculations.

After the determination of the shaft diameters. The key material is selected as **St70** as the result of the key analysis done for each shaft based on the Key & Keyway Dimensions table.

All of the dimensions required for the design process were determined with all these calculations and the design process of the gearbox is finished.

The cost analysis part is the last step of the project. The analysis is imperfect and unrealistic since I only knew the cost of the steel per kg which is not sufficient to perform the cost analysis. The price of the material should be known exactly since it will be used in the manufacturing process of the gearbox. However, this value is assumed as an approximate value and the total cost of the gearbox was obtained with this way.

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