ME 3021 SYSTEM DYNAMICS AND CONTROL PROJECT

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11.5 HYDRAULIC SERVOMECHANISM CONTROL

Our fourth case study involves a feedback control system design for a hydraulic actuator. Hydraulic actuators have many applications ranging from robotics, earth-moving machinery, construction equipment, and aerospace [6, 7]. Figure 11.32 shows a schematic diagram of an electrohydraulic actuator (EHA) that consists of an electromechanical actuator (solenoid), spool valve, and hydraulic cylinder with piston. An input voltage signal is applied to the solenoid actuator (not shown in Fig. 11.32), which in turn moves the spool valve left or right in order to meter flow in and out of the hydraulic cylinder. If the spool-valve displacement y is positive (to the right) as shown in Fig. 11.32, fluid flows from the supply pressure PS through the valve orifice and into

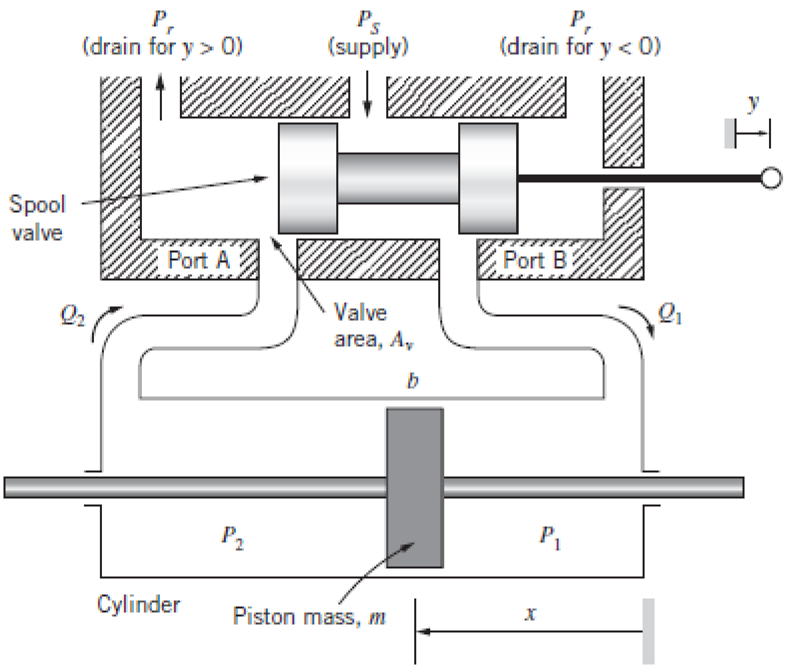


Figure 11.32 Schematic diagram of an electrohydraulic actuator.

the right-hand side of the cylinder. Consequently, if right-side cylinder pressure $P1$ is higher than pressure $P2$, the piston moves to the left resulting in a positive displacement x for the piston. When $y > 0$, fluid flows from the left-side cylinder (pressure $P2$) to the reservoir (drain) pressure Pr. Volumetric flow into and out of the right-side cylinder is $Q1$, while $Q2$ is the volumetric flow out of and into the left-side cylinder.

Our objective is to develop a feedback control system for the EHA that will automatically adjust the input voltage so that the piston stroke *x* reaches a desired target position. We want a fast piston response with good damping characteristics and very little overshoot. As an example, this case study might represent a hydraulic actuator for positioning an aerodynamic surface such as an airplane's elevator, aileron, or rudder. This type of actuator is called a servomechanism.

Mathematical Model

The complete mathematical model consists of the electromechanical (solenoid), hydraulic, and mechanical subsystems. Figure 11.33 shows a free-body diagram of the mechanical subsystem, which consists of the piston and load mass *m*. Piston displacement *x* is positive to the left as measured from the right end of the cylinder (see Fig. 11.32). Applying Newton's second law with a positive sign convention to the left yields

$$+\leftarrow \sum F = P_1 A - P_2 A - b\dot{x} = m\ddot{x} \qquad (11.33)$$

where *P*1 and *P*2 are the chamber pressures of the right and left sides of the cylinder, *A* is the area of the piston, and *b* is the viscous friction coefficient. Rearranging Eq. (11.33) so that all terms involving displacement *x* are on the left-hand side yields

$$m\ddot{x} + b\dot{x} = (P_1 - P_2)A \qquad (11.34)$$

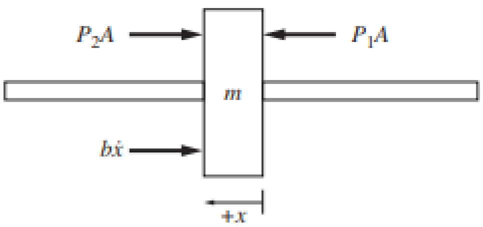


Figure 11.33 Free-body diagram of the mechanical subsystem.

Two pressure-rate equations are required for the two cylinder chambers

$$\dot{P}_1 = \frac{\beta}{V_1}(Q_1 - \dot{V}_1) \qquad (11.35)$$

$$\dot{P}_2 = \frac{\beta}{V_2}(Q_2 - \dot{V}_2) \qquad (11.36)$$

where β is the fluid bulk modulus, $Q1$ and $Q2$ are the volumetric-flow rates for chambers 1 and 2, and $V1$ and $V2$ are the volumes of cylinder chambers 1 and 2. Equations (11.35) and (11.36) are the basic pressure-rate equations derived in Chapter 4 for hydraulic systems with compressible fluids. The instantaneous volumes for cylinder chambers 1 and 2 depend on piston position x

$$V_1 = V_0 + Ax \tag{11.37}$$

$$V_2 = V_0 + A(L - x) \tag{11.38}$$

where $V0$ is the volume when $x = 0$ (piston is at the right end of the cylinder). The total stroke of the piston is L. The time rate of change of the two-cylinder volumes depends on the piston velocity: $\dot{V}1 = A\dot{x}$ and $\dot{V}2 = -A\dot{x}$.

Volumetric flow through the spool valve between the supply pressure PS and the cylinder (for eitherchamber 1 or 2) is modeled by the orifice-flow equation for hydraulic systems

$$Q_{1,2} = C_d A_v sgn(P_S - P_{1,2})\sqrt{\frac{2}{\rho}\left|P_S - P_{1,2}\right|} \tag{11.39}$$

where Av is the valve area, Cd is the discharge coefficient, and ρ is the fluid density. When spool-valve position $y > 0$, the supply pressure PS is connected to cylinder chamber $P1$ and Eq. (11.39) is used to compute $Q1$. When $y < 0$, the supply pressure PS is connected to $P2$ and Eq. (11.39) is used to compute $Q2$. In general, the supply pressure PS is always greater than the cylinder pressure $P1$ or $P2$, but the possibility for fluid flow *from* the cylinder back to the supply pressure is included by using the signum function. Volumetric flow through the spool valve between the cylinder (chamber 1 or 2) and the reservoir (drain) pressure Pr is also modeled by the orifice-flow equation

$$Q_{1,2} = -C_d A_v sgn(P_{1,2} - P_r)\sqrt{\frac{2}{\rho}\left|P_{1,2} - P_r\right|} \tag{11.40}$$

When $y < 0$, Eq. (11.40) models flow $Q1$ from chamber 1 to the drain and when $y > 0$ Eq. (11.40) models flow $Q2$ from chamber 2 to the drain. Equation (11.40) shows that $Q1$ (or $Q2$) is negative when $P1$ (or $P2$) is greater than the reservoir pressure and the fluid flows from the cylinder to the reservoir tank.

To complete the mathematical model, we must show the relationship for the electromagnetic solenoidused to position the spool valve. One common modeling method [6] is to use an underdamped, second-order transfer function to relate spool-valve position *y* to voltage input *e*in(*t*)

$$G(s) = \frac{Y(s)}{E_{in}(s)} = \frac{K_v \omega_n{}^2}{s^2 + 2\zeta\omega_n s + \omega_n{}^2} \tag{11.41}$$

Table 11.5 Parameters for the EHA

```
m = 12; % kg Piston and load mass
b = 215; % Viscous friction coefficient, b, N.s/m
A = 6.78*(10^(-4)); % Piston area, A,m2
V0 = 1.58*(10^(-4)); % Minimum chamber volume, V0,m3
L = 0.68; % Piston stroke, L, m
P_s = 18*10^6; % Supply pressure, PS, MPa
P_r = 0.12133*(10^(6)); %Reservoir (drain) pressure, Pr, MPa
beta = 729*(10^(6)); % Fluid bulk modulus, β  ,MPa
Cd = 0.72; % Valve discharge coefficient, Cd
rho = 899; % Fluid density, ρ  , kg/m3
h = 0.0094; % Valve orifice height, h, m
wn = 338; % Solenoid-valve undamped natural frequency, ωn , rad/s
zeta = 0.93; % Solenoid-valve damping ratio, ζ
Kv = 25*(10^(-6)); % Solenoid-valve DC gain, Kv  mm/V
K_HA = ((Cd*h)/A)*sqrt(P_s/rho); % hydraulic actuator gain
```

where ωn is the undamped natural frequency, ζ is the damping ratio, and *Kv* is the DC gain of the solenoid. In many cases, we can determine the second-order modeling parameters ωn, ζ, and *Kv* by open-loop experimental trials where we provide a step voltage signal *e*in(*t*) and measure the resulting valve position *y*(*t*).

Finally, the spool-valve orifice area *Av* is assumed to be a linear function of spool-valve position *y*

$$Av = h|y|$$

where *h* is the height of the valve opening. The absolute value of valve position *y* must be used because *y* can be positive (right) or negative (left). Valve orifice area is required in the nonlinear in/out-flow equations (11.39) and (11.40).

Equations (11.34)–(11.42) constitute the complete mathematical model of the EHA. Table 11.5 presents the numerical values for all parameters for the EHA system

Simulink Model

The complete EHA model is nonlinear and fairly complex, as indicated by Eqs. (11.34)–(11.42) and the multitude of system parameters summarized in Table 11.5. Just as we did with the pneumatic air-brake system, it is useful to identify and understand the state and input variables of the complete system before constructing the Simulink model. The mechanical modeling equation (11.34) is second-order and requires two state variables, piston position (x) and velocity ($\dot{x}$). Cylinder pressures $P1$ and $P2$ are the two inputs to the mechanical subsystem, as the differential pressure across the piston provides the actuation force. Equations (11.35) and (11.36) show that the complete hydraulic system is composed of two first-order nonlinear ODEs with two additional state variables $P1$ and $P2$. Piston position and velocity are required to compute the chamber volumes and time derivatives, which are both needed in the pressure-rate equations (11.35) and (11.36). Volumetric-flow rates $Q1$ and $Q2$ are also needed in the pressure-rate equations, and they are determined by the appropriate valve orifice- flow equations, Eq. (11.39) or (11.40). The orifice-flow equations depend on the supply pressure PS, reservoir pressure Pr, cylinder pressures, and the valve area $Av = h|y|$. Finally, the solenoid–valve Dynamics are modeled by the linear second-order transfer function (11.41) with source voltage $e\text{in}(t)$ as the input, and spool-valve displacement y as the output. Hence, the supply pressure PS, reservoir pressure Pr, and voltage input $e\text{in}(t)$ are the three input variables for the EHA system.

Figure 11.34 shows the Simulink model of the integrated EHA system. Note that the three system inputs are the voltage input $e\text{in}(t)$ and the supply and reservoir pressures. Spool-valve displacement y, supply and

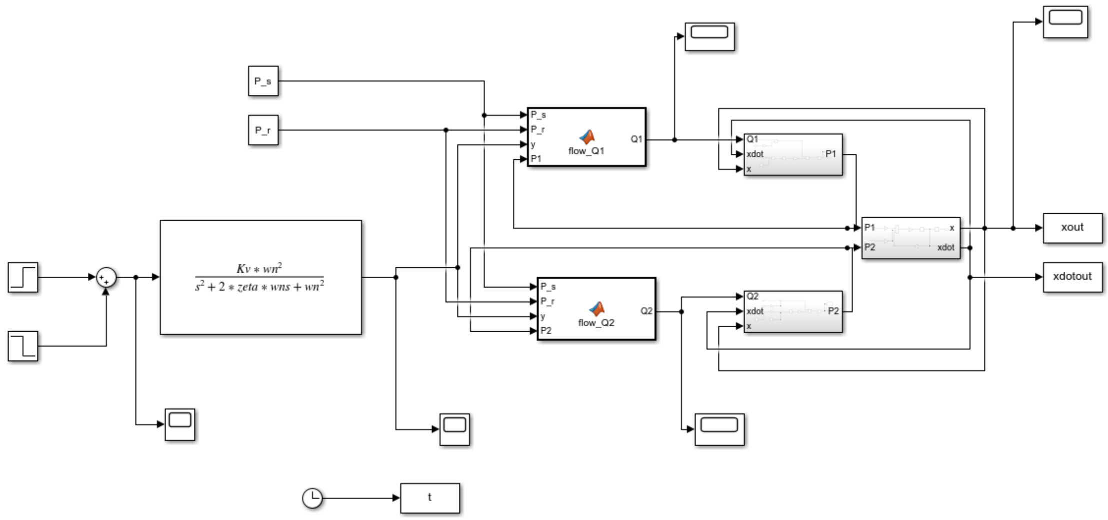


Figure 11.34 Simulink diagram for the EHA system.

reservoir pressures, and cylinder pressures are the four inputs to the two orifice-flow subsystems (recall that a valve displacement y will produce two orifice flows, one from PS to the cylinder and one flow from the cylinder to the reservoir). The two outputs of the orifice-flow subsystems are volumetric-flow rates $Q1$ and $Q2$, which are inputs to the two pressure-rate subsystems. Piston position and velocity, x and $\dot{x}$, are required for the volume and volume-rate terms in the two pressure-rate subsystems and consequently are also input variables. Cylinder pressures $P1$ and $P2$ are the two output variables from the pressure-rate subsystem that become the two inputs for the mechanical subsystem.

Because the valve-orifice-flow equations (11.39) and (11.40) are complicated nonlinear functions, they are contained in a user-defined M-file instead of as a simulation diagram. MATLAB M-file 11.4 shows the customized M-file flow_Q1.m, which computes the volumetric flow $Q1$ either from PS to $P1$ (for $y > 0$), or

MATLAB M-file 11.4

FLOW Q1

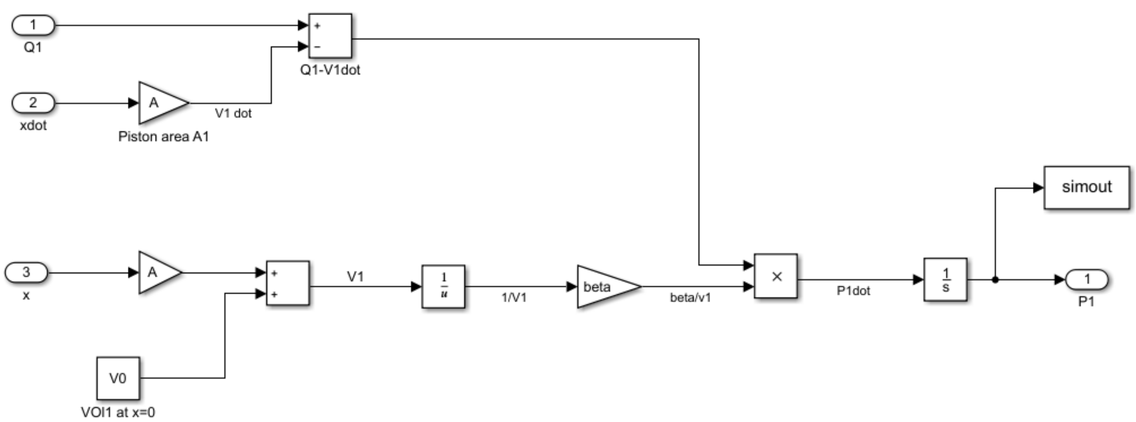
```
function Q1 = flow_Q1(P_s,P_r, y , P1)
u=[P_s, P_r, y , P1]'
                    % system parameter
h = 0.0094;                                     % height of valve opening, m
                   % hydraulic constants
Cd = 0.72;                                      % discharge coefficient
rho = 899;                                      % fluid density, kg/m^3
                    % System inputs
P_s = u(1);                                     % supply pressure, Pa
P_r = u(2);                                     % reservoir pressure, Pa
y = u(3);                                       % valve displacement, m
P1 = u(4);                                      % pressure in chamber 1, Pa
                    % Compute valve orifice area
Av = abs(y)*h;                                  % valve orifice area, m^2
                    % Determine if flow is from supply (y > 0), or if flow
                    % is out to reservoir pressure (y < 0)
if y >= 0
                    % Chamber 1 is connected to the supply, P_s
                    % (flow is positive if P_s > P1)
Q1 = Cd*Av*sign(P_s - P1)*sqrt( 2*abs(P_s - P1)/rho );
else
                     % Chamber 1 is connected to the reservoir, P_r
                     % (flow is negative if P1 > P_r)
Q1 = -Cd*Av*sign(P1 - P_r)*sqrt( 2*abs(P1 - P_r)/rho );
end
y=u;
```

FLOW Q2

```
function Q2 = flow_Q2(P_s,P_r, y , P2)
u=[P_s,P_r, y , P2]'
                                  % system parameter
h = 0.0094;                                      % height of valve opening, m
                                  % hydraulic constants
Cd = 0.72;                                       % discharge coefficient
rho = 899;                                       % fluid density, kg/m^3
                                  % System inputs
P_s = u(1);                                      % supply pressure, Pa
P_r = u(2);                                      % reservoir pressure, Pa
y = u(3);                                        % valve displacement, m
P2 = u(4);                                       % pressure in chamber 1, Pa
                                % Compute valve orifice area
Av = abs(y)*h;                                   % valve orifice area, m^2
                                % Determine if flow is from supply (y > 0), or if flow
                                % is out to reservoir pressure (y < 0)
if y <= 0
                                % Chamber 1 is connected to the supply, P_s
                                % (flow is positive if P_s > P1)
Q2 = Cd*Av*sign(P_s - P2)*sqrt( 2*abs(P_s - P2)/rho );
else
                                % Chamber 1 is connected to the reservoir, P_r
                                % (flow is negative if P1 > P_r)
Q2 = -Cd*Av*sign(P2 - P_r)*sqrt( 2*abs(P2 - P_r)/rho );
end
y = u;
```

from cylinder $P1$ to the reservoir Pr (for $y < 0$). The customized M-file flow_Q2.m is identical to flow_Q1, except that it computes the flow in/out of cylinder chamber 2 and hence uses pressure $P2$. The inner components of either orifice-flow subsystem shown in Fig. 11.34

consist of an Interpreted MATLAB Fcn. block (M-file flow_Q1 or flow_Q2) with four inputs (*PS*, *Pr*, *y*, and *P*1 or *P*2) and a single output signal (*Q*1 or *Q*2). Therefore, both hydraulic-flow subsystems are similar to the pneumatic valve-flow subsystem shown in Fig. 11.26. Figure 11.35 shows the inner details of the pressure-rate subsystem for the right-side chamber *P*1. The reader should be able to trace the signals and computations for volume *V*1, volume-rate $\dot{V}$ 1, and ultimately pressure-rate $\dot{P}$ 1. The second pressure-rate subsystem (for $\dot{P}$ 2) is nearly identical to Fig. 11.35, except that the volume computation is governed by Eq. (11.38). Finally, Fig. 11.36 shows the inner details of the mechanical subsystem. The reader should be able to identify the mechanical modeling equation (11.34) in the subsystem block diagram.



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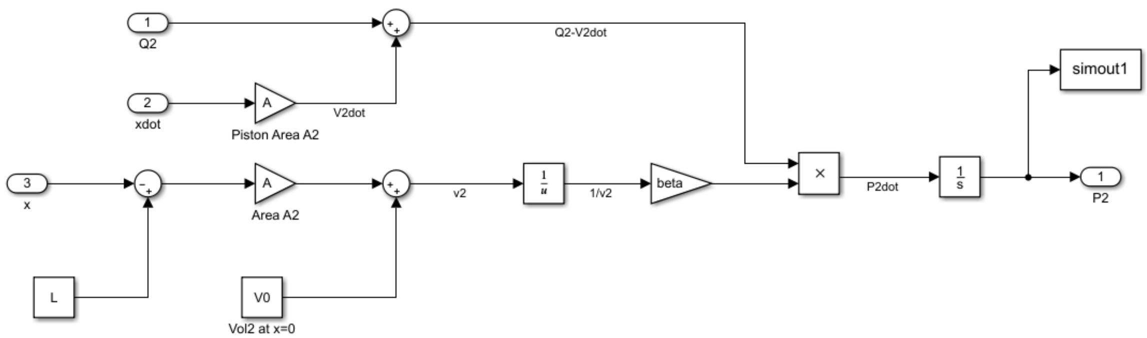

Figure 11.35 Cylinder pressure subsystem (chamber 1 and 2) for the EHA

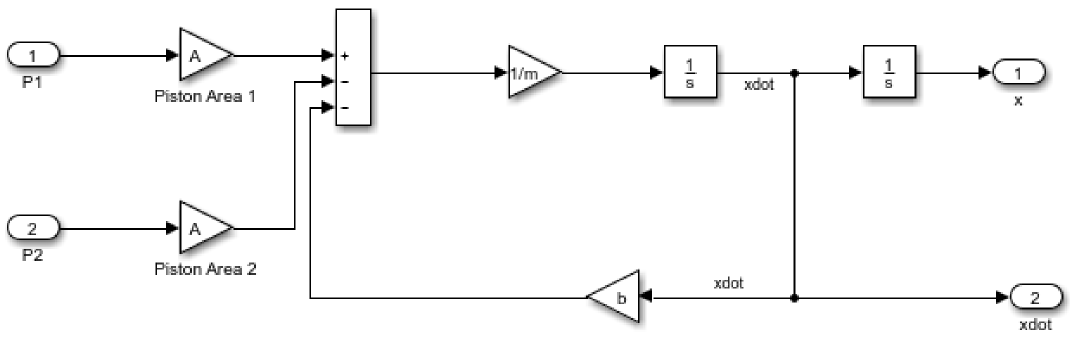

Figure 11.36 Mechanical subsystem for the EHA.

Pulse Response of the EHA

Next, we demonstrate the open-loop response of the EHA for a 10-V pulse input. We assume that the initial cylinder pressures are $P1 = P2 = 0.1PS$ (the value of the initial pressure has little effect on the response as long as the system is initially in equilibrium, or $P1 = P2$). The piston is initially at rest at the center of the cylinder, or $x(0) = 34$ cm. A constant (step) input voltage ($e$in = 10V) is applied at time $t = 0.5$ s and then stepped to zero at time $t = 1$ s in order to create a half-second pulse input.

Because the solenoid–valve subsystem is linear, it is relatively easy to compute its pulse response. Transfer function (11.41) shows that the DC gain of the solenoid is Kv and consequently the steady-state valve position is $e$in × Kv, or 10V× 2.5(10−5) m/V = 2.5(10−4) m (0.25mm). The settling time of the valve is $4/(\zeta\omega n) = 0.0127$s (12.7ms). Hence, the spool valve reaches its 0.2mm steady-state position very quickly and with little overshoot because its damping ratio is $\zeta = 0.93$. The valve closes in 12.7 ms after time $t = 1$ s when the pulse voltage is stepped to zero. Consequently, the valve response $y(t)$ is very nearly a pulse itself with a magnitude of 0.25mm for $0.5 < t < 1$ s.

Figure 11.37 shows the piston response $x(t)$ for the 10-V pulse input. The piston stroke exhibits a nearly linear increase from its initial position (34 cm) to its steady-state position (51.46 cm) during the 0.5 s pulse input. Figure 11.38 shows the volumetric-flow rates $Q1$

(into cylinder chamber 1) and *Q*2 (out of cylinder chamber 2). Note that during the "steady-state" phase of the valve opening, the magnitudes of the in- and out-flow rates are equal, which indicates that the differential pressure across the piston has reached a constant value. This phenomenon is confirmed by Fig. 11.39, which shows that the differential pressure exhibits a damped sinusoidal decay to a steady-state value of about 110,000 Pa during the pulse opening of the valve.

Linear EHA Model

Recall that our overall goal is to design an automatic feedback system for precise position control of the EHA. The full EHA model presented in the previous subsections is complex and highly nonlinear. Designing

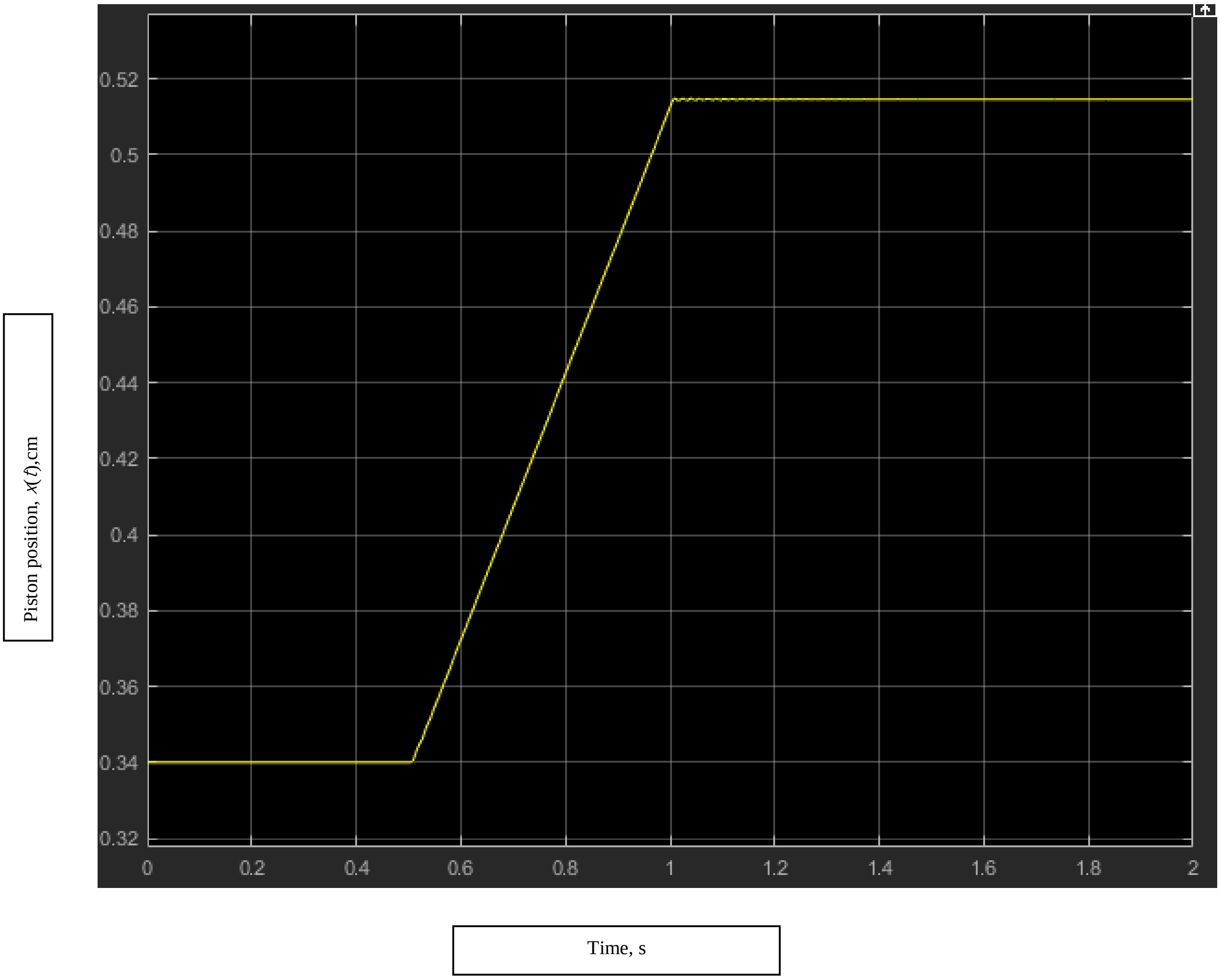

Figure 11.37 Piston stroke for 10-V pulse input.

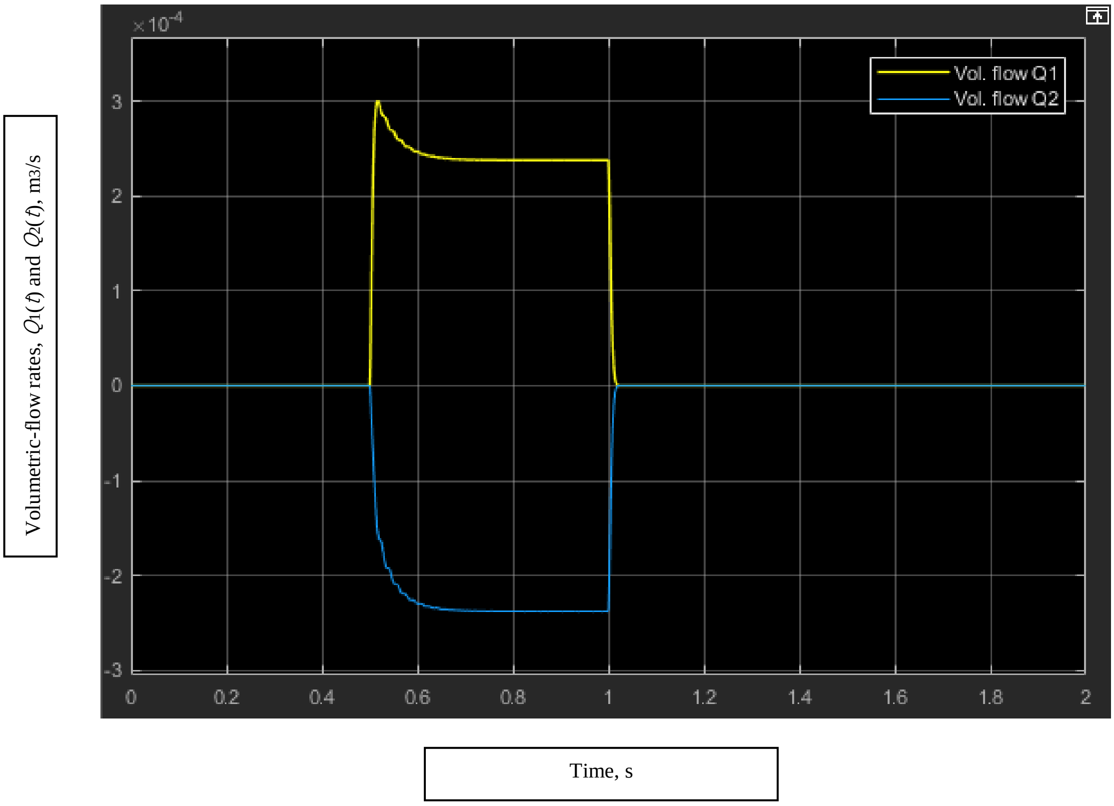

Figure 11.38 Volumetric-flow rates for 10-V pulse input.

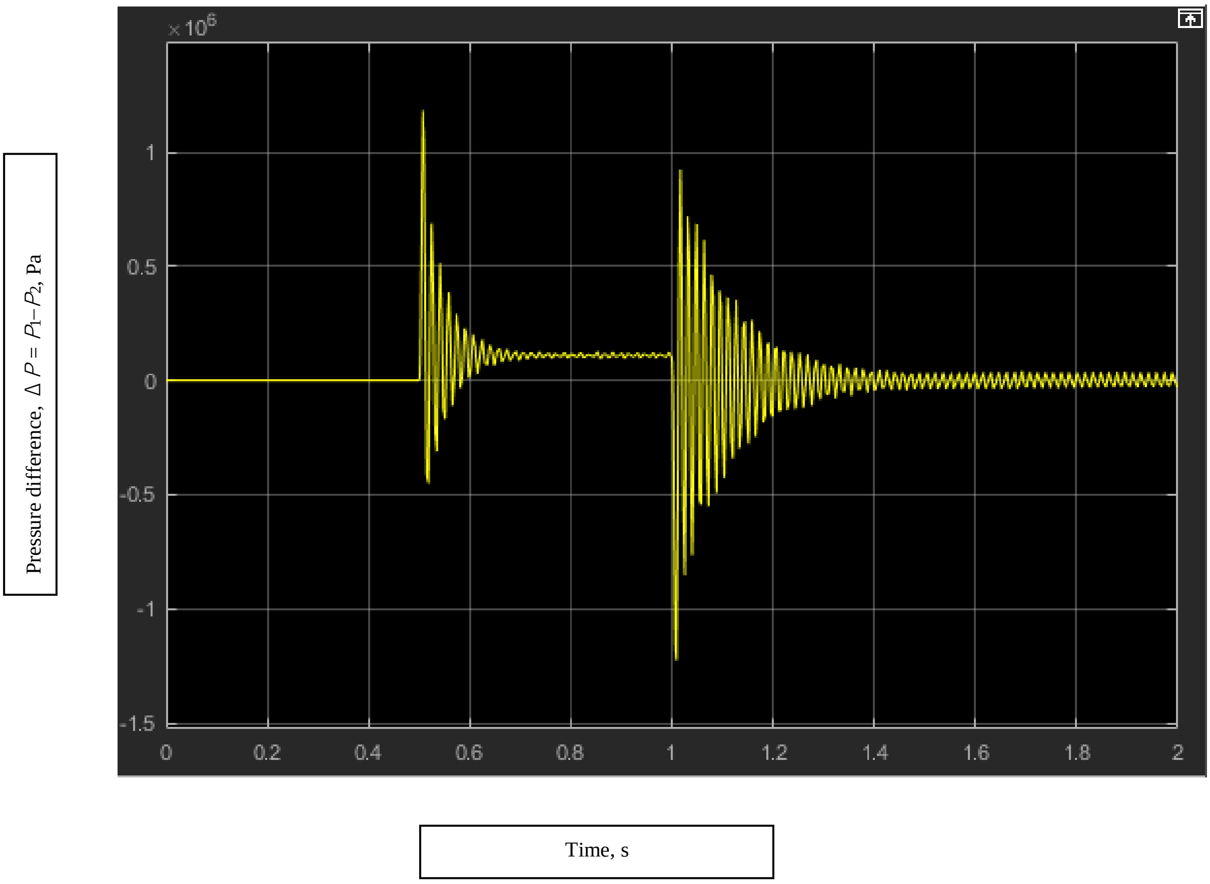

Figure 11.39 Differential pressure across the piston for 10-V pulse input.

a feedback control system becomes systematic when we have a *linear* plant model because we can make use of linear analysis tools such as the root-locus and frequency-response methods presented in Chapter 10. Therefore, it is to our advantage to develop a linear EHA model purely for the sake of control system design. It should be noted that we intend to test any potential control scheme designs with the full *nonlinear* EHA system dynamics. The following linearization steps are similar to the process presented by Ogata [8] for linearizing a hydraulic actuator. To begin the analysis, rewrite Eqs. (11.39) and (11.40) for the *magnitudes* of the volumetric flows with valve displacement $y > 0$

$$In - flow\ to\ chamber\ 1{:}\ Q_1 = C_d A_v \sqrt{\frac{2}{\rho}(P_S - P_1)} \quad (11.43)$$

$$Out - flow\ to\ chamber\ 2{:}\ Q_2 = C_d A_v \sqrt{\frac{2}{\rho}(P_2 - P_r)} \quad (11.44)$$

If we assume steady, incompressible flow where $Q1 = Q2$, then we can equate the two pressure differences contained in the radicands in Eqs. (11.43) and (11.44)

$$P_S - P_1 = P_2 - P_r \quad (11.45)$$

Let us define $\Delta P = P1 - P2$ as the differential pressure across the piston ($\Delta P > 0$ because fluid is flowing from chamber 2 to the reservoir). Substituting $P2 = P1 - \Delta P$ into Eq. (11.45) and solving for the supply pressure yields

$$P_S = P_1 + P_1 - \Delta P - P_r \quad (11.46)$$

Usually, the reservoir pressure Pr is much less than the other pressures and can be neglected. With this assumption, we obtain the following expression for cylinder pressure $P1$ from Eq. (11.46)

$$P_1 = \frac{P_S + \Delta P}{2} (11.47)$$

Substituting Eq. (11.47) for pressure $P1$ in Eq. (11.43) yields the following expression for flow rate $Q1$

$$Q_1 = C_d A_v \sqrt{\frac{2}{\rho}\left(P_S - \frac{P_S + \Delta P}{2}\right)}$$

Applying simple algebra and substituting Av = hy yields

$$Q_1 = C_d hy\sqrt{\frac{P_S - \Delta P}{\rho}} = f(y, \Delta P) \tag{11.48}$$

Equation (11.48) is a nonlinear function of valve position y and differential pressure ΔP. We can linearize flow rate about the reference states y∗ and ΔP∗

$$\delta Q_1 = \frac{\partial f}{\partial y}\bigg|\delta y + \frac{\partial f}{\partial(\Delta P)}\bigg|\delta(\Delta P) \tag{11.49}$$

where the partial derivatives of Eq. (11.48) are

$$\frac{\partial f}{\partial y}\bigg| = C_d h\sqrt{\frac{P_S - \Delta P}{\rho}}\bigg| \tag{11.50}$$

$$\frac{\partial f}{\partial(\Delta P)}\bigg| = \frac{-C_d hy}{2\rho}\left(\frac{P_S - \Delta P}{\rho}\right)^{-\frac{1}{2}}\bigg| \tag{11.51}$$

We select the reference (or nominal) states as $y* = 0$ and $\Delta P* = 0$ (no flow), and consequently the partial derivatives evaluated at the operating point are

$$\frac{\partial f}{\partial y}\bigg| = C_d h\sqrt{\frac{P_S}{\rho}}$$

$$\frac{\partial f}{\partial(\Delta P)}\bigg| = 0$$

Hence, the linear flow equation (11.49) becomes

$$\delta Q_1 = C_d h\sqrt{\frac{P_S}{\rho}}\,\delta y \tag{11.52}$$

The reader should note that the perturbation variables are relative to the reference values

$$\delta Q_1 = Q_1 - {Q_1}^* \tag{11.53}$$

$$\delta y = y - y^* \tag{11.54}$$

and because the reference condition is zero flow ($Q*1 = 0$ and $y* = 0$), we can use $\delta Q1 = Q1$ and $\delta y = y$ in the linearized flow equation (11.52)

$$Q_1 = C_d h \sqrt{\frac{P_S}{\rho}} y \tag{11.55}$$

If we assume steady, incompressible flow where $\dot{P}1 = 0$, the volumetric-flow rate $Q1$ is equal to the time derivative of chamber volume ($\dot{V}1 = A\dot{x}$) and Eq. (11.55) becomes

$$C_d h \sqrt{\frac{P_S}{\rho}} y = A\dot{x} \tag{11.56}$$

Solving Eq. (11.56) for the piston velocity, we obtain

$$\dot{x} = K_{HA} y \tag{11.57}$$

where the "hydraulic actuator" gain is

$$K_{HA} = \frac{C_d h}{A} \sqrt{\frac{P_S}{\rho}} \tag{11.58}$$

The linearized solution for the piston stroke x(t) is simply the integral of Eq. (11.57)

$$x(t) = x_0 + K_{HA} \int y dt \tag{11.59}$$

where $x0$ is the initial stroke at time $t = 0$. In other words, the complex, nonlinear EHA model shown in

Fig. 11.34 can be replaced by a single integrator block. Figure 11.40 shows a Simulink model of the linearized EHA system, which consists of the voltage input $e$in(t), the second-order linear solenoid–valve model, and the single integrator that represents the linear I/O equation relating spool-valve position y and piston stroke x. Of course, the linearized EHA model does not provide information about cylinder pressures.

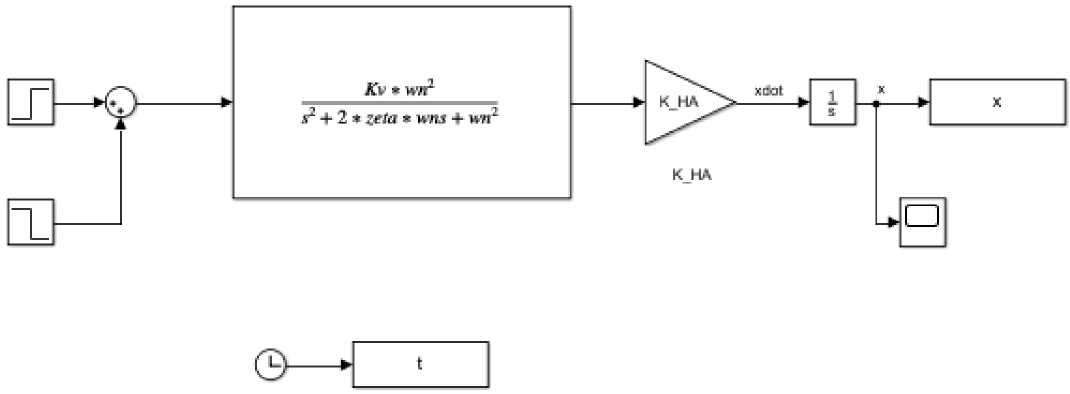

Figure 11.40 Simulink model of the linearized EHA system.

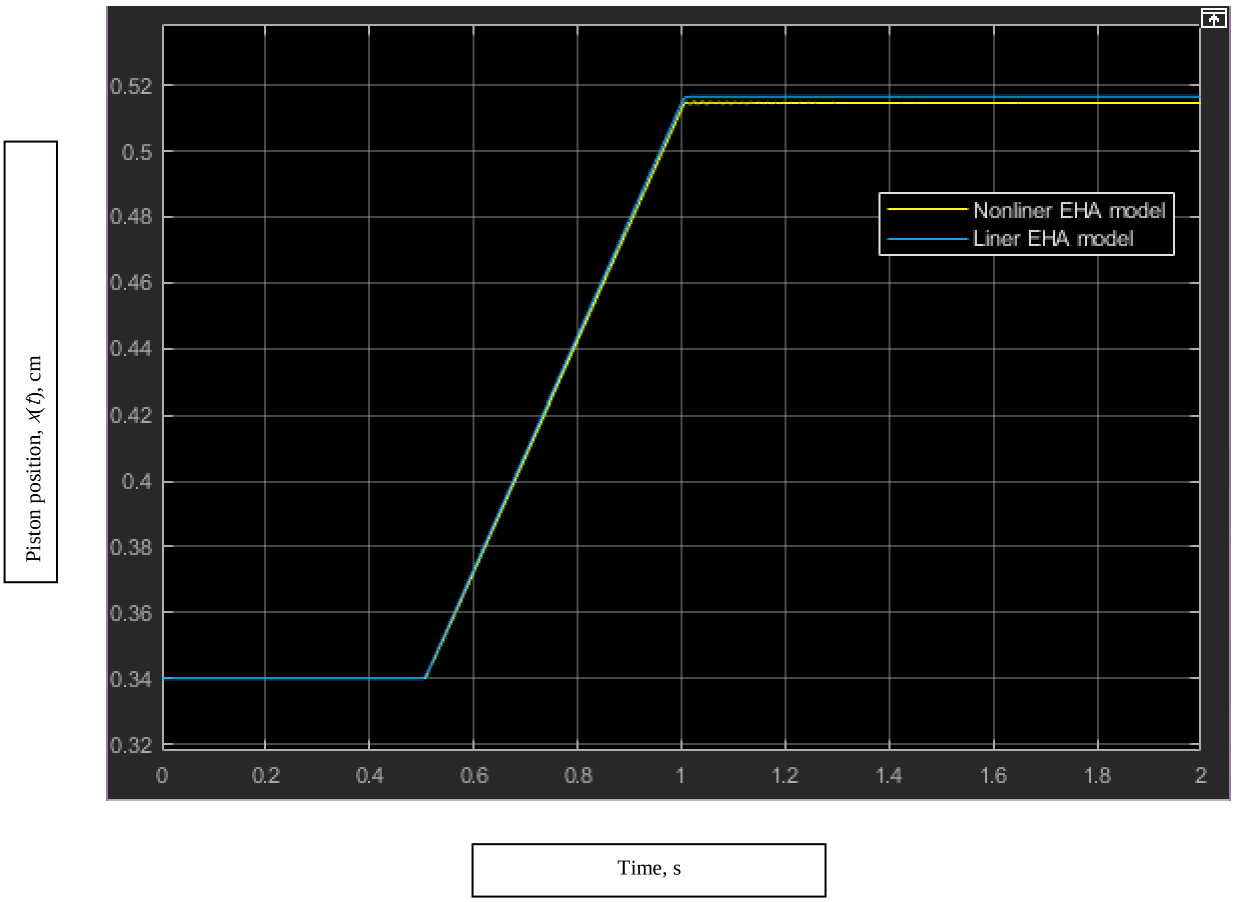

Figure 11.41 Piston position responses to a pulse input for nonlinear and linear EHA models.

We can compare the piston responses $x(t)$ from the full nonlinear EHA simulation (Fig. 11.34) and the extremely simple linear model (Fig. 11.40) for a 10-V pulse input. Using the nominal

EHA parameters in Table 11.5, the hydraulic actuator gain is determined to be KHA = 1412.5s−1. Hence, the linear model (11.57) predicts that the piston velocity is the product of KHA and valve displacement y, which yields $\dot{x}$ = 0.3492m/s for y = 2.5(10−4) m. Figure 11.41 shows the piston stroke $x(t)$ for a 10-V pulse input from the full nonlinear EHA model and the linear EHA model. The linear model response shows an excellent match with the nonlinear model response.

Feedback Control System Design

The goal of the feedback system is precise position control for a reference position (stroke) command. We begin with a proportional control scheme, where the solenoid-voltage signal $e$in(t) is proportional to the position error. Figure 11.42 shows a proportional feedback control system where $x$ref is the reference position command for the piston rod. Note that we are using the simple linear hydraulic actuator model, which is an integrator block with numerator KHA = 1412.5s−1. The proportional-control gain is KP, and it has units of

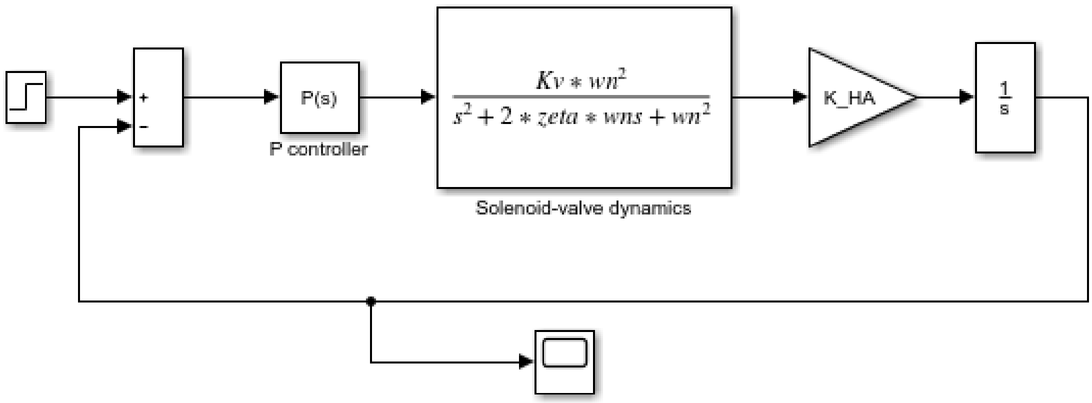

Figure 11.42 Proportional control for the linear EHA model.

V/m because it converts a position error (m) to a voltage signal. We can check the steady-state accuracy of the proposed control scheme by calculating the closed-loop transfer function

$$T(s) = \frac{K_p G(s)}{1+K_p G(s) H(s)} \tag{11.60}$$

where $G(s)$ is the forward transfer function

$$G(s) = \frac{4034}{s(s^2+628s+114244)} \tag{11.61}$$

and $H(s)$ is the feedback transfer function (unity in this case). Consequently, the closed-loop transfer function in Eq. (11.60) becomes

$$T(s) = \frac{4034Kp}{s^3 + 628s^2 + 114244s + 4034Kp} \quad (11.62)$$

Because the DC gain of the closed-loop transfer function, $T(s = 0)$, is unity for *any* positive value of the proportional gain *KP*, the proportional-control system will exhibit zero steady-state error for a constant reference position command. Recall that our steady-state error analysis in Chapter 10 showed that a type 1 system (i.e.one integrator in the forward transfer function) exhibits zero steady-state error for a constant input, and a finite steady-state error for a ramp input. Clearly, Fig. 11.42 shows that our linearized EHA model is a type 1 system because the hydraulic actuator model is an integrator.

The variation of the closed-loop response with gain *KP* can be determined by using the root-locus method. We can easily produce the root locus by using the following MATLAB commands:

```
sysG=tf(4034 , [1 628 114244 0 ]);          % forward transfer function
rlocus(sysG)                                 % plot root locus
```

The first command builds the forward transfer function $G(s)$, and the second command creates the root locus, which is shown in Fig. 11.43. The closed-loop poles begin at the open-loop roots (poles) when the proportional gain *KP* is zero. In this case, the open-loop poles are $s = 0$ (the integrator) and $s = -306 \pm j104.84$ (the solenoid–valve dynamics). Figure 11.43 shows that two closed-loop poles move from the two-complex open-loop poles to the negative real axis, and a single closed-loop pole moves to the left from the origin to a break-away point near −105 on the negative real axis. If the proportional gain *KP* is too high, two closed loop poles follow ±60∘ asymptotes and eventually cross the imaginary axis, rendering the closed-loop system unstable.

The root-locus diagram shown in Fig. 11.43 indicates that with the proper gain selection the three closed-loop poles can be large negative values, and consequently the closed-loop response will be extremely

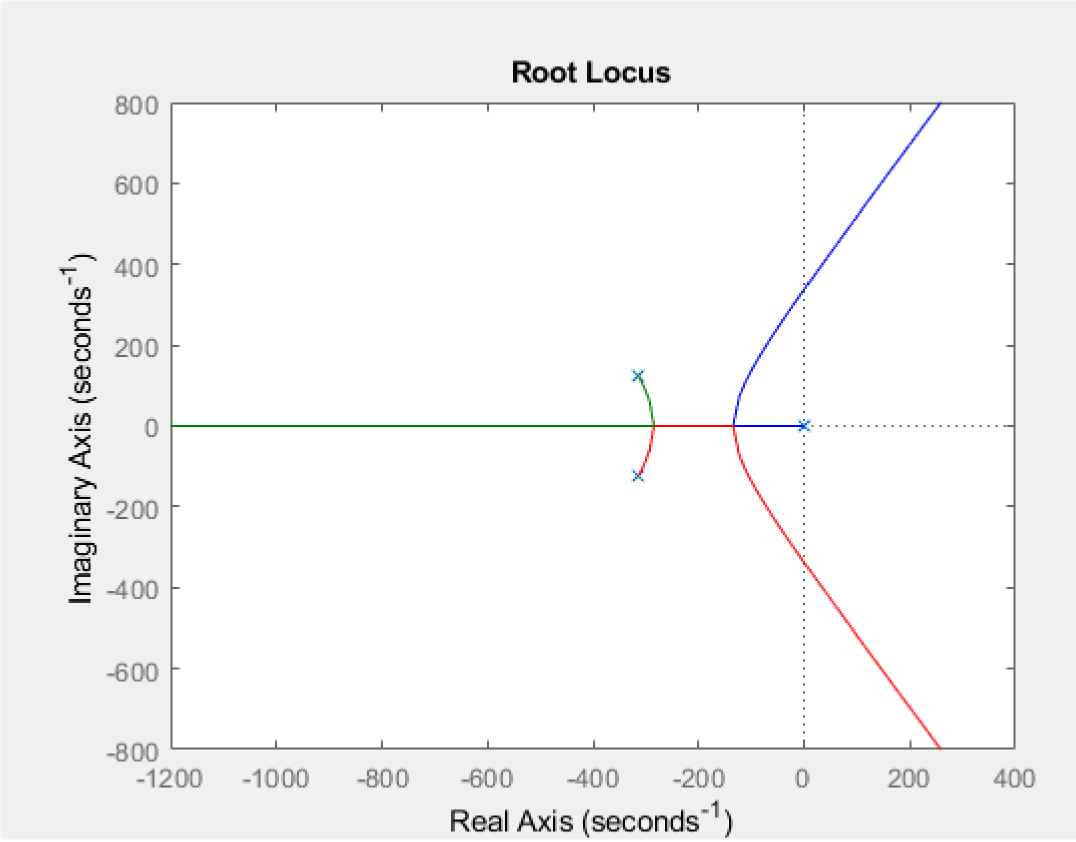

Figure 11.43 Root locus for the linear EHA model with proportional control.

fast and overdamped (no oscillations in the transient response). As an example, let the proportional gain be KP = 1550 V/m and compute the closed-loop poles using MATLAB's locus command

```
Kp=1550
CLpoles=rlocus(sysG,Kp)
```

The three closed-loop poles for this gain setting are $s = -357.13$, $s = -164.32$ and s=-106.54 Using the "slowest" closed-loop pole, $s = -106.54$, the "slowest" component of the closed-loop transient response will be *e^(−106.54*t)*, which decays to a steady-state value in about 0.0493 s. However, this idealized closed-loop response has its limitations owing to the large control gain. Figure 11.42 shows that the solenoid-voltage input $e_{in}(t)$ is the position error $x_{ref} - x$ multiplied by control gain KP. Consequently, if the position error is 11.3 cm (0.113 m) and KP = 1550 V/m, the voltage input will be e_{in} = 176.7V, which likely exceeds the capability of the solenoid. Hence, the proportional gain *KP* is limited by the voltage capacity of the solenoid–valve subsystem.

Assuming that 60V is the maximum acceptable voltage input to the solenoid, we can select a feasible control gain for the closed-loop system for a “nominal” position error. If the nominal position error is 0.17 m, the control gain is $KP = (60\text{V})/(0.17\text{m}) = 352.94$ V/m. Using this gain, the closed-loop poles are $s = -13.43$ and $s = -307.28 \pm j107.54$ Therefore, we expect the closed-loop response to reach its steady-state value in about 0.297 s. Figure 11.44 shows the closed-loop responses of the piston position for a reference position command *x*ref = 51 cm and control gain *KP* = 352.94 V/m. The initial piston position is 34 cm. The closed-loop responses of the nonlinear and linearized EHA models were simulated with Simulink using the proportional control scheme (the reader should note that the nonlinear hydraulic actuator shown in Fig. 11.34 is inserted as a subsystem in place of the simple integrator $1412.5/s$ in Fig. 11.42). Despite the complexity of the nonlinear EHA model, the nonlinear and linear closed-loop responses $x(t)$ shown in Fig. 11.44 are indistinguishable

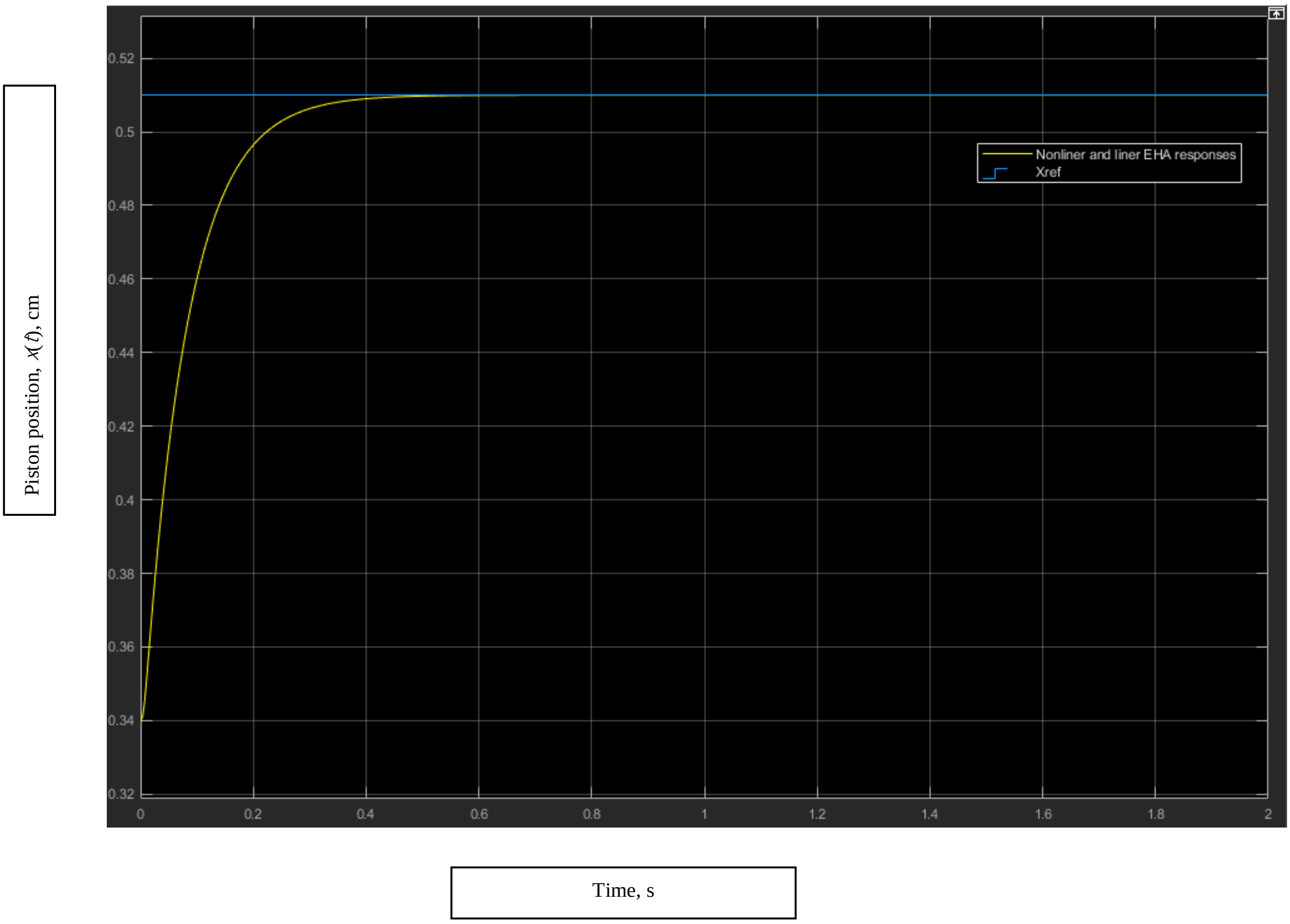

Figure 11.44 Closed-loop responses of the nonlinear and linear EHA models with $x$ref = 51cm and proportional gain KP = 352.94 V/m.

from each other. As the closed-loop system is well damped, there is little need for derivative feedback (i.e., a proportional-derivative or PD control scheme), and therefore the proportional control scheme provides adequate performance for a step reference input.

A second performance test for the control system is the ability to track a dynamic, periodic reference input. Figure 11.45 shows a Simulink diagram of the closed-loop control system with a sinusoidal reference input for $x$ref (t) (note that the *nonlinear* hydraulic actuator subsystem from Fig. 11.34 is included instead of the simple integrator actuator). The reference position $x$ref (t) is a sine function with a ±17-cm amplitude
about the piston's starting position of 34 cm. Note that we could construct a linear closed-loop Simulink model using the linearized EHA plant; that is, we would replace the nonlinear EHA plant in Fig. 11.45 with the integrator model $X(s)/Y(s) = K$HA$/s$.

Figure 11.46 shows the closed-loop responses of the nonlinear and linear hydraulic actuator models for the sinusoidal reference position $x$ref (t) with a frequency of 2 Hz. The proportional gain is KP = 300 V/m. Figure 11.46 clearly shows that the closed-loop frequency responses from the nonlinear and linear EHA models are indistinguishable from each other. Furthermore, the closed-loop response $x(t)$ using proportional

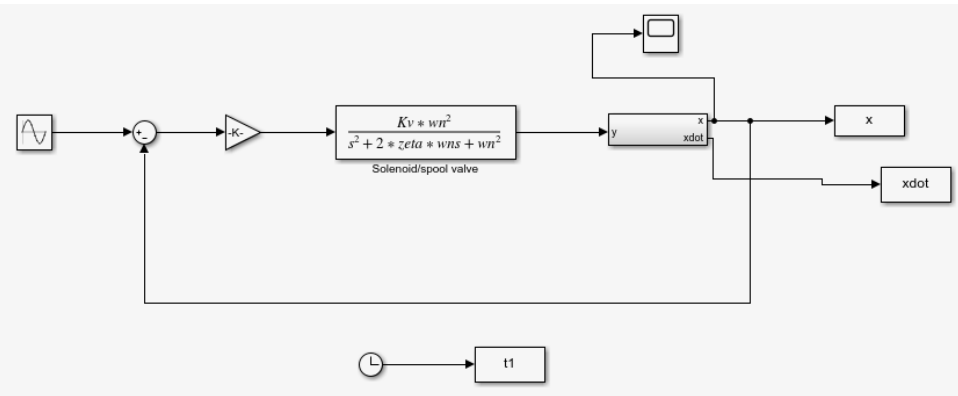

Figure 11.45 Closed-loop Simulink model with proportional control scheme and sinusoidal reference input.

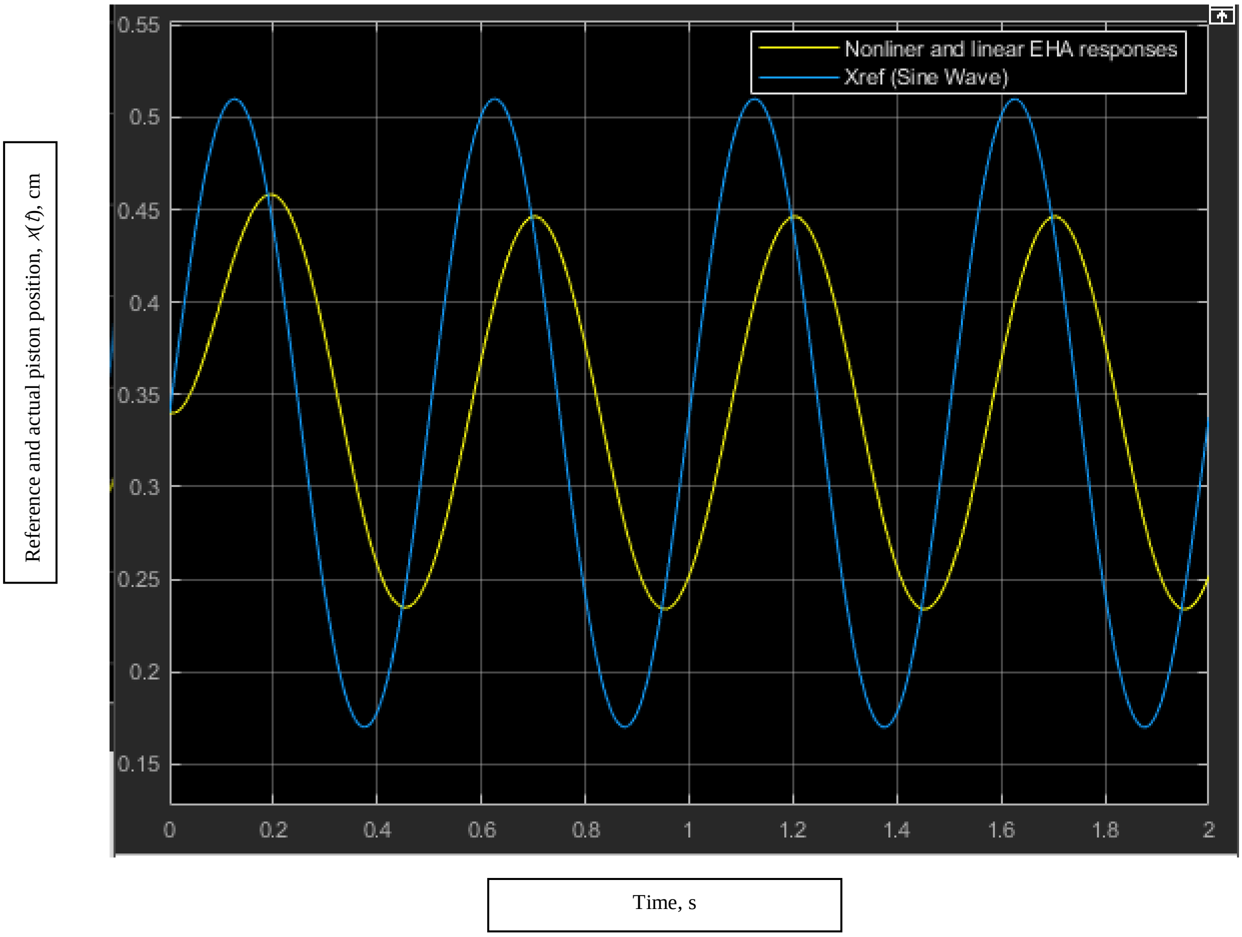

Figure 11.46 Closed-loop responses of the nonlinear and linear EHA models with sinusoidal $x$ref(t) and proportional gain KP = 300 V/m.

control exhibits significant lag with respect to the reference input. The voltage signal $e$in(t) oscillates between ±60V when the P-gain is KP = 300 V/m and therefore increasing the gain will cause the voltage input to exceed its limits.

We can use the following MATLAB commands to compute the magnitude and phase of the *closed-loop* frequency response of the linear EHA model:

```
Kp=300                                     % P-gain
sysT = tf(4034*Kp, [1 628 114244 4034*Kp]) % closed-loop transfer function T(s)
w=2*2*pi                                   % 2 Hz frequency, rad/s
[mag,phase] = bode(sysT,w)                 % closed-loop freq. response
hold on
grid on
```

The magnitude and phase are mag = 0.6672 and phase = -52.2255∘ (=−-0.912 rad). The magnitude value matches the simulation results shown in Fig. 11.46 because the output/input amplitude ratio is roughly (11.3 cm) ⁄ (17 cm) = 0.6647 (note that the amplitude is measured relative to the 34-cm mid-point of the hydraulic cylinder). We can use the phase lag to compute the time lag as 0.6419rad ⁄ (4π rad⁄s) = 0.053 s, which is the time delay between the peaks of the input and output sinusoids in Fig. 11.46.

One way to improve the tracking performance would be to replace the proportional controller with a *lead controller*. Recall that a lead controller approximates PD control and hence “anticipates” the reference signal due to the derivative term. We can replace the gain *KP* in Fig. 11.45 with the following lead controller

$$G_C(s) = \frac{K_{LF}(s + 10)}{s + 40} \tag{11.63}$$

where *K*LF is the “lead filter” gain. Figure 11.47 shows the closed-loop responses of the nonlinear and linear hydraulic actuator models using the lead controller with gain *K*LF = 1300 (the lead controller gain was selected so that its DC gain matches the P-gain *KP* = 300 V/m). Note that the output/input amplitude ratio

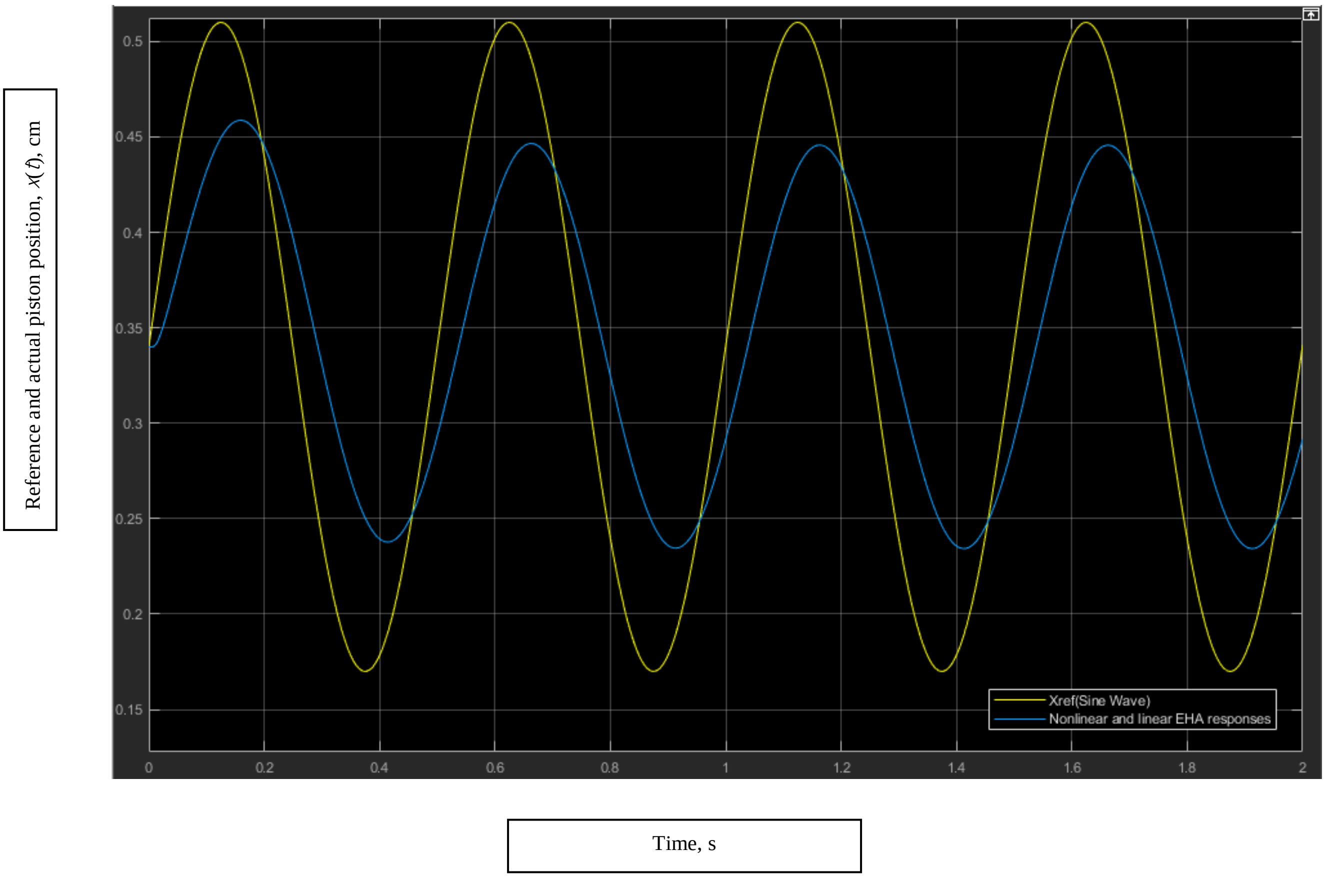

Figure 11.47 Closed-loop responses of the nonlinear and linear EHA models with sinusoidal *x*ref(*t*) and lead controller with gain *K*LF = 1300.

is roughly the same as the closed-loop response using P-control because the DC gain of the lead controller matches the P-gain *KP*. However, adding the lead controller has improved the closed-loop response because the time lag between input and output peaks has been reduced. We can use the MATLAB commands to compute the magnitude and phase angle:

```
%%
K_LF = 1300;                                   % lead filter gain
sysGc = tf([1 10],[1 40]);                     % lead filter GC(s)
sysG=tf(4034,[1 628 114244 0 ]);               % plant transfer function G(s)
sysTT = feedback(K_LF*sysGc*sysG,1)            % closed-loop transfer function T(s)
w = 2*2*pi;                                    % 2 Hz frequency, rad/s
[mag,phase] = bode(sysTT,w)
bode(sysTT)
legend("P-controller","Lead controller")
%%
```

The magnitude is 0.6698 (roughly 11.3/17, as expected) and the phase angle is –25.34∘ (–0.4274), which is less than half of the phase lag for the closed-loop system using proportional control. Hence, the time lag using the lead controller is 0.4274rad∕(4π rad/s) = 0.034 s, which is half the time lag for the P-control system.

Another way to illustrate the benefit of adding the lead controller is to observe the Bode diagrams of the respective *closed-loop* transfer functions. Figure 11.48 shows the Bode diagram of the EHA closed-loop transfer function $T(s)$ for the P-controller [i.e., Eq. (11.62)] with gain *KP* = 300 V/m and the lead controller (with gain *K*LF = 1300). Both controllers show the ability to closely track low-frequency input signals as the closed-loop magnitude is 0 dB (unity output/input ratio) and the phase angle is small. However, for input frequencies greater than 4 rad/s (about 0.6 Hz), the phase angle of the lead-controller system is greater than the phase of the P-control system. Therefore, the closed-loop EHA system with a lead controller can track a sinusoidal input with smaller phase (or time) lag when compared to the closed-loop P-control system. Note that we can read the magnitude and phase for ω = 12.57

rad/s (2 Hz) from the Bode diagram and estimate the closed-loop frequency responses shown in Figs. 11.46 and 11.47.

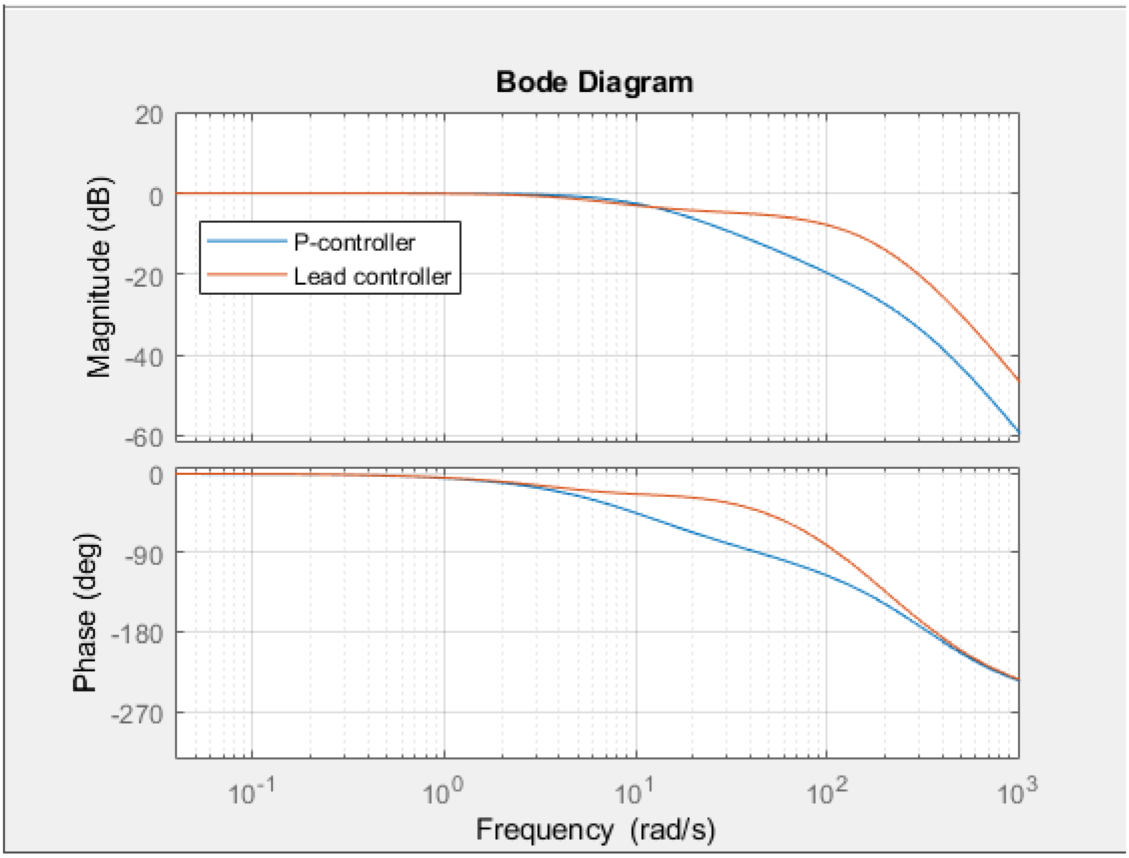

Figure 11.48 Bode diagram of the closed-loop EHA systems using proportional and lead controllers.

References

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