Computer Science 124: Programming Assignment 3

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Dynamic Programming Approach

For the purposes of this dynamic programming solution, we assume that we can add any numbers in the sequence in constant time. To solve the Number Partition problem, we set up an array S of size $n \times b$, using the following recurrence:

$$S(n', b') = 1 \text{ if } S(n' - 1, b') = 1 \text{ or } S(n' - 1, b' - x_{n'}) = 1$$

In this recurrence, n' and b' represent the iterators we use to traverse the list. As a base case, we set $S(1, b') = 1$ for $b' = x_1$ and 0 otherwise. The array represents whether or not the subset $x_1 \dots x_{n'}$ sums to each integer b'. For example, if $x_1 + x_2 = 42$, then $S(2, 42) = 1$. We also maintain another array of back-pointers, which indicate the last element of A that was added to achieve the current sum.

Now we find $\texttt{sum}_1 = \min_{b' \leq (b/2)} (b/2) - b' : S(n, b') = 1$. This represents the sum corresponding to the partition with minimum residue. Given $\texttt{sum}_1$, we can find $\texttt{sum}_2 = b - \texttt{sum}_1$. For the two values $\texttt{sum}_1$ and $\texttt{sum}_2$, we find subsets S_1 and S_2 by following the back-pointers to determine which elements of A were in each subset. The array is size $O(nb)$, and filling in each element takes time $O(1)$. Computing the minimum takes at most time $O(b)$, and determining the elements in each of the subsets takes time $O(n)$. Therefore, the algorithm overall takes time $O(nb)$.

Karmarkar-Karp Runtime

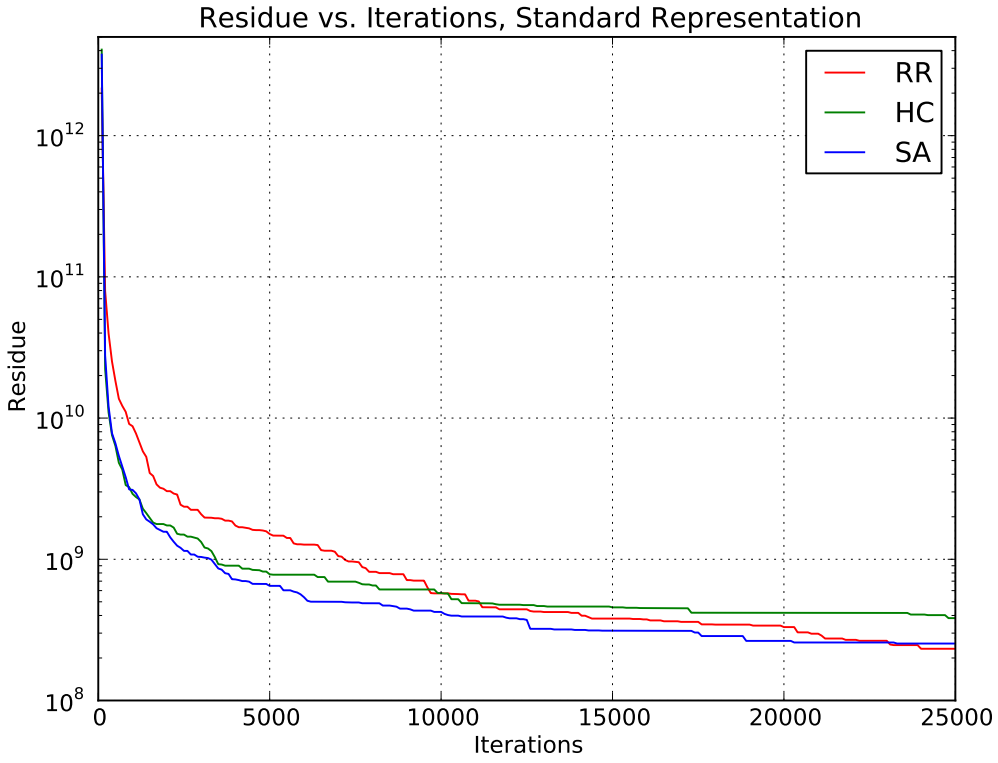
Assuming the values in A are small enough that arithmetic operations take one step, the Karmarkar-Karp algorithm can be implemented in $O(n \log n)$ steps by using a max-heap. We take an array containing the sequence A and turn it into a max-heap using the standard algorithm, which takes time $O(n \log n)$. At each step of the iteration, we remove the two largest elements in the heap, which takes time $O(2 \log n)$. Finding the absolute difference between the elements and adding them back into the heap takes time $O(\log n)$. At most we conduct these last two steps $n/2$ times, which is on the order of $O(n \log n)$. Therefore, the algorithm takes $O(n \log n)$ time overall.

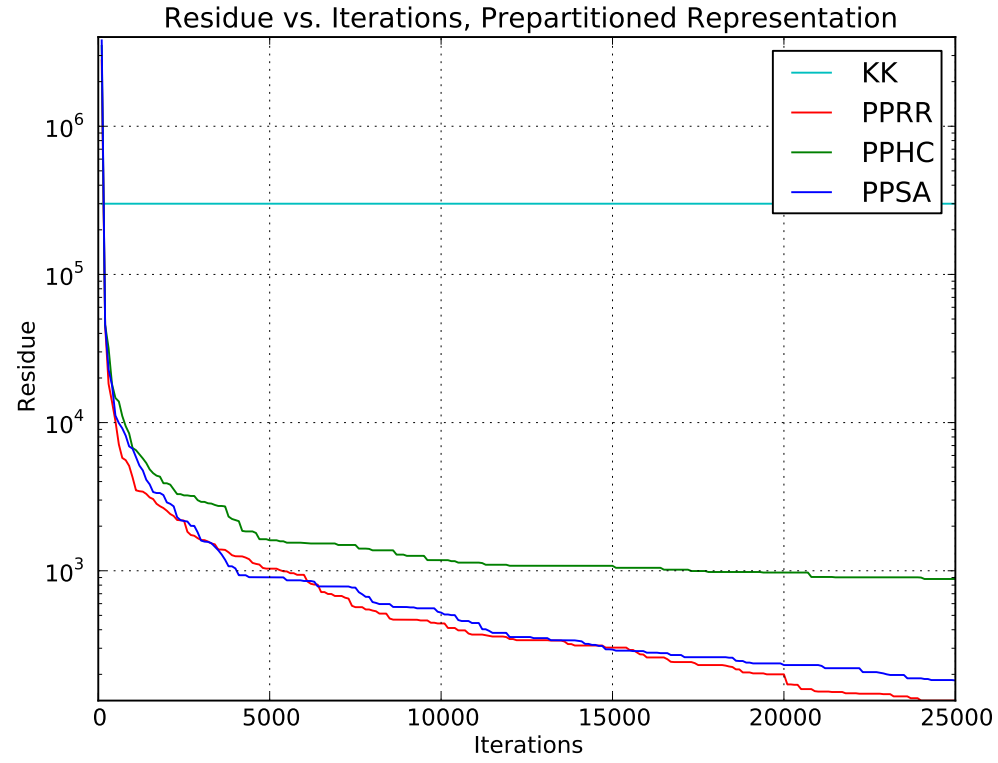
Experimental Results

The `kk.py` file implements the Karmarkar-Karp algorithm and prints the residual given an input file containing integers listed one per line. We chose to implement the three local search algorithms in a separate file, `localsearch.py`. This file generates 50 random instances of the number partition problem, and runs each algorithm and representation combination on each problem. After each iteration, the program outputs a row of of preformatted LaTeXcontaining the residuals returned by each method. `localsearch.py` also writes the residue after each 100 iterations for each method to a file named `graph_output.txt`. This data was used to generate the graphs of residue over iterations shown on the page after the following table.

This table lists the residuals given by each requested approach, for 50 random instances. In both the table and the following graphs, KK denotes Karmarkar-Karp, RR denotes Repeated Random, HC denotes Hill Climbing, SA denotes Simulated Annealing, and the prefix PP denotes the fact that the prepartitioning representation was used, as opposed to the standard representation.

	KK	RR	HC	SA	$PPRR$	$PPHC$	$PPSA$
1	91427	406443511	33120495	106819349	7	25	183
2	63283	1322701821	362972551	35642873	133	1368	16
3	163041	93821615	742852931	157347421	505	904	10
4	466207	12169019	425819983	80189599	173	203	91
5	79396	92608552	1288477952	165452038	14	669	413
6	354912	50143172	28740116	13372450	334	476	38
7	504732	117424542	1939594240	2658604	12	852	146
8	2427505	38227327	19063717	116059393	45	3752	28
9	295062	124982314	15396276	39236998	144	1297	3
10	54487	105614619	80998711	3028187	159	28	120
11	472160	610554254	148668126	502410648	16	274	366
12	193792	223582218	145309958	634207712	40	320	170
13	88942	2629020	281037788	106171114	38	210	88
14	744435	174778463	556639737	98844361	309	966	780
15	103848	213825332	423850736	77765850	196	6552	62
16	236759	187639653	612563149	47176221	231	127	109
17	40674	163072572	269202562	678378576	146	173	47
18	11607	97594747	63574105	1115642287	29	423	31
19	206345	336398345	241835911	261384795	377	228	404
20	144220	171380008	147280714	211426364	74	316	466
21	179183	56841275	215627505	34772327	711	1175	833
22	547724	109350376	746490910	895155752	34	1594	676
23	18603	47037011	249169143	22661999	57	536	54
24	55538	98347184	952841924	180238514	364	3596	456
25	250158	628183176	37787026	8068206	274	1354	68
26	49573	74364503	109226183	216974149	249	87	113
27	22252	431902502	152102174	124785836	126	166	72
28	317662	192482156	158367406	97609500	40	4472	8
29	398754	46841870	638127884	377046968	196	371	325
30	256496	40624802	734552746	52301636	58	238	412
31	292734	135278880	623007776	923965820	20	1361	375
32	35834	277278876	121073534	408387908	140	66	124
33	214098	66802796	72067434	316818602	80	238	324
34	86784	89496906	609371060	887727188	86	313	89
35	12991	133914399	96724547	159623497	747	191	69
36	475647	763046555	263310249	63586997	131	237	121
37	166296	316026416	214499626	6419464	184	311	135
38	86289	171707805	439576957	156811839	25	510	28
39	301285	160285245	204811639	250547555	25	88	282
40	1181594	203836722	234938892	15023280	200	5	11
41	250436	212086428	900111716	104629314	86	613	29
42	827960	180641356	51438100	772114620	266	98	332
43	775095	353128717	199636161	723722829	199	894	146
45	299169	380575959	1006943279	662677875	1	6293	69
44	216304	434131204	593919092	40518992	0	270	206
46	34292	493256096	203807418	39013290	118	14	320
47	121705	5488317	137323677	193375229	177	202	28
48	572099	104463749	624225553	212992145	221	429	861
49	130764	393339398	699410876	23561622	100	1	57
50	83224	476586246	33612740	230095668	46	208	6
Mean	300067.54	232458760.58	383022059.7	253088869.22	158.86	901.88	204.0


Residue vs. Iterations, Standard Representation
RR
HC
SA
Residue
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
0
5000
10000
15000
20000
25000
Iterations


Residue vs. Iterations, Prepartitioned Representation
KK
PPRR
PPHC
PPSA
Residue
10^{6}
10^{5}
10^{4}
10^{3}
0
5000
10000
15000
20000
25000
Iterations

Note that the Karmarkar-Karp mean residual is not shown on the first of the above graphs, since it was so small relative to the residuals output by the other methods that it was not easily visible on the graph. The Karmarkar-Karp mean residual is shown on the second graph.

Discussion

Evaluation of Representations and Algorithms

We begin by comparing the results of the two representations of the Number Partition problem, then comparing results for each local search algorithm within both representations.

The result residues for the standard representation are significantly worse than those for prepartitioning. They are also significantly worse that the residues returned by the Karmarkar-Karp algorithm. The performance of all the local search algorithms likely suffers because the standard representation insists on partitioning the sequence A into two parts from the very beginning, whereas the prepartitioning result allows elements to be grouped into up to n sets, where n is the number of elements in A. This allows for more generality in representing partitions of the set, and results in far more progress when we use the local search algorithms.

For both representations, the repeated random and simulated annealing algorithms returned solutions with approximately equal residues. The repeated random algorithm slightly outperformed the simulated annealing algorithm, likely because we used so many iterations (25,000) that the algorithm was able to eventually generate a very good solution. In the prepartitioning representation, this approach succeeded because of the nature of the representation: on each iteration, the x_i were grouped into up to n different sets, allowing for very different sets to be generated on each iteration.

The hill climbing algorithm underperformed the other two local search algorithms likely due to the main flaw with hill climbing algorithms: the possibility of getting stuck at local minima. Although the hill climbing approach was very successful on some iterations, on others it stalled and was unable to escape from a local minimum it foundd. This phenomenon is clear on the prepartitioning graph. On average across 50 trials, the hill climbing algorithm tended to level off after about 5000 iterations.

As expected, the simulated annealing algorithm was able to avoid the problem of the hill climbing algorithms. The SA algorithm implements the same general approach as hill climbing, but sometimes randomly moves to a worse location, with probability degrading over time. This avoids the pitfall of hill climbing. Even if the simulated annealing algorithm gets stuck at a local minimum for some time, it will escape from that minimum with some probability. This leads to far better results than hill climbing alone.

Using Karmarkar-Karp as a Starting Point

We could use a solutiona from the Karmarkar-Karp as a starting point for either the hill climbing or simulated annealing algorithms. The randomized algorithm simply generates a new solution every time, so having a different starting point would not make a difference in its results, unless the randomly generated results were always worse than the KK result (as is the case with the standard representation). Hill climbing and simulated annealing, on the other hand, rely on iteratively decreasing the residue; as a result, picking a better starting point will give the algorithms a leg up in the process of iterating to a better solution. In particular, the standard representation algorithms could have achieved much better results if they began with the KK result. The results for the prepartitioning representation would likely have been similar in the end, but could have been achieved in significantly fewer iterations.