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Oblivious Routing using Learning Methods

Ufuk Usubütün, Murali Kodialam, T.V. Lakshman and Shivendra Panwar

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Making Oblivious Routing Practical

Context

Routing in Networks

High volume,

Highly variable traffic

Motivation

Traffic Oblivious Routing

Stable and Robust Routing Solution

Far less complex handling of highly varying traffic

Compatible with current day networks

Contribution

Adversarial Learning Method

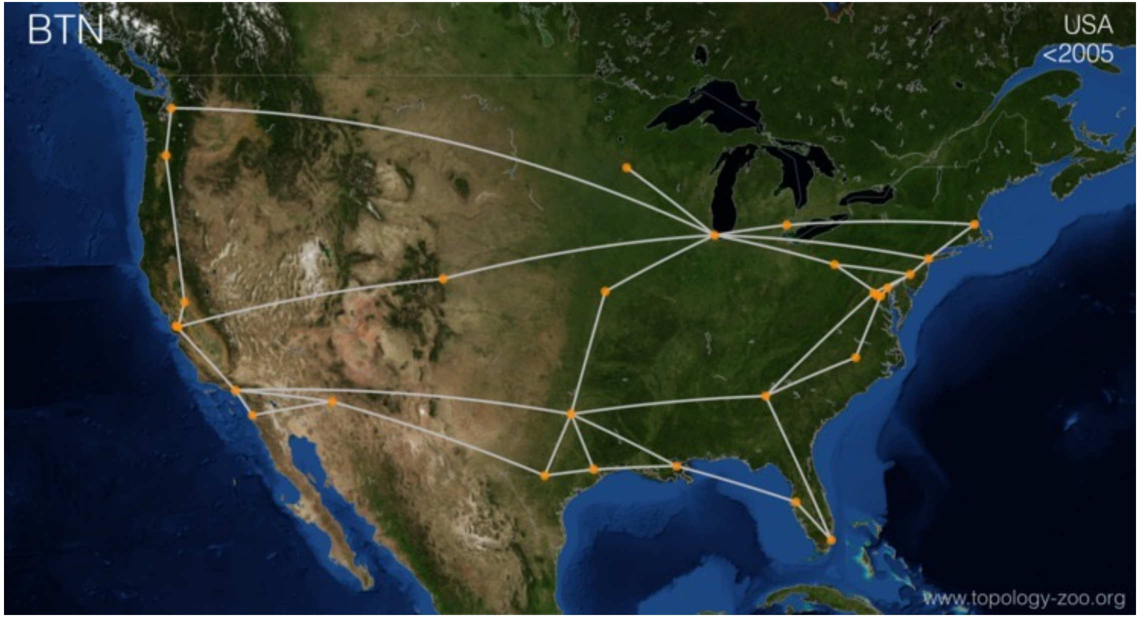
New, fast and parallelized ways to solve optimization problems

Provide performance guarantees

Routing in Networks

How to optimally route flows?

- How to satisfy all demands and constraints?



How to optimally route flows?

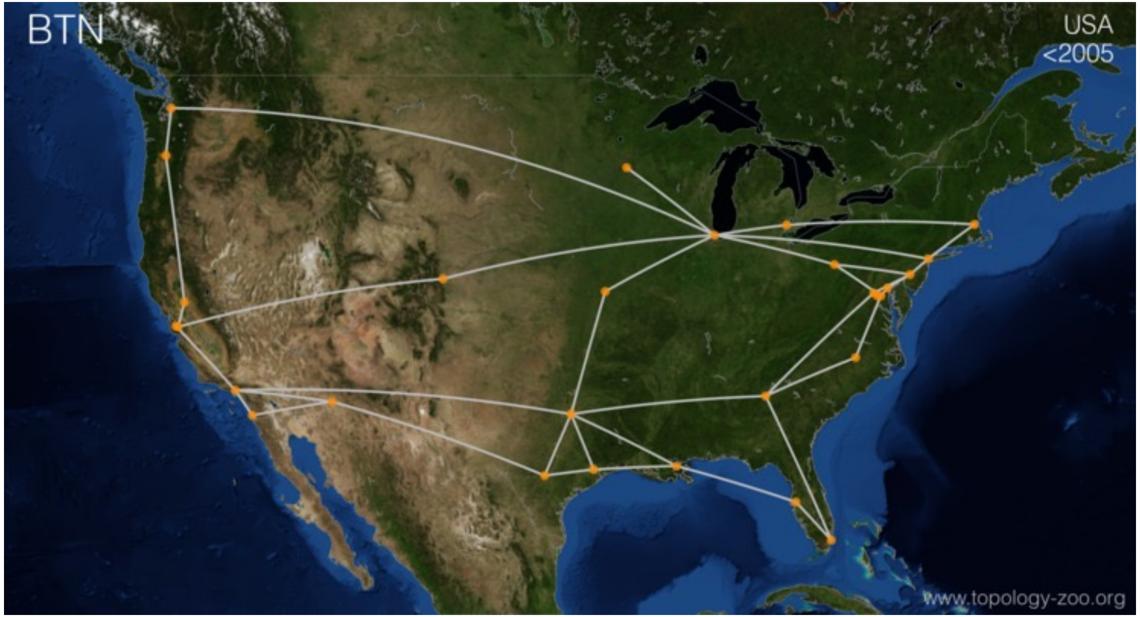
- ❑ How to satisfy all demands and constraints?
  - ❑ Solve a nice optimization problem if we can forecast what traffic to expect

min (max link utilization)

for traffic matrix:

$$[t_{ij}] = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1N} \\ t_{21} & t_{22} & & t_{2N} \\ \vdots & & \ddots & \vdots \\ t_{N1} & t_{N2} & \cdots & t_{NN} \end{bmatrix}$$

By assigning routes to flows



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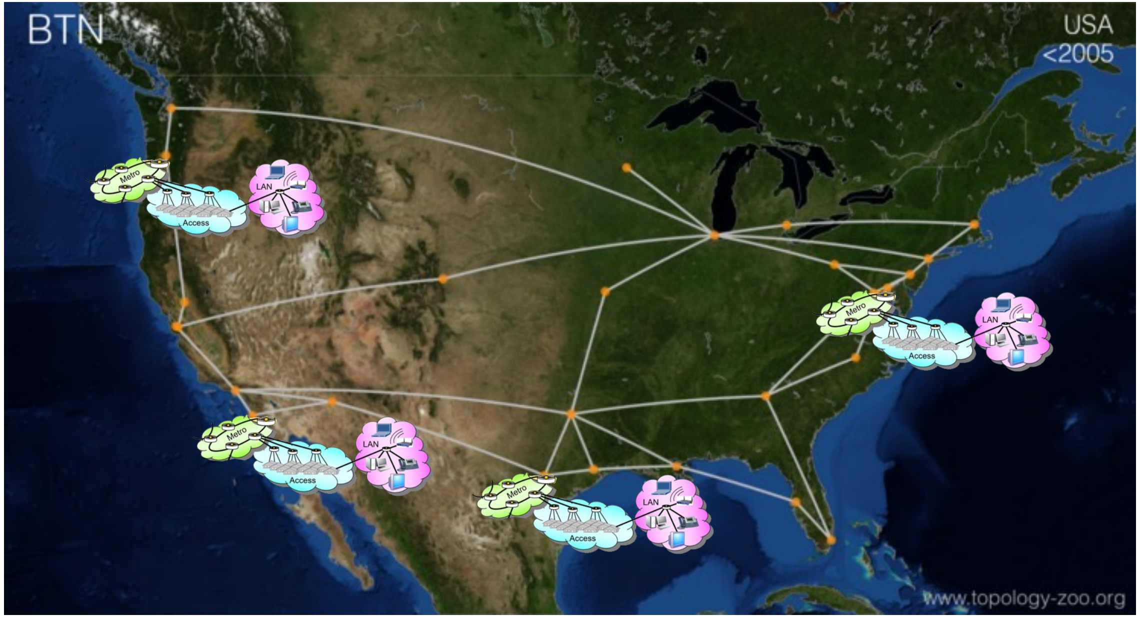
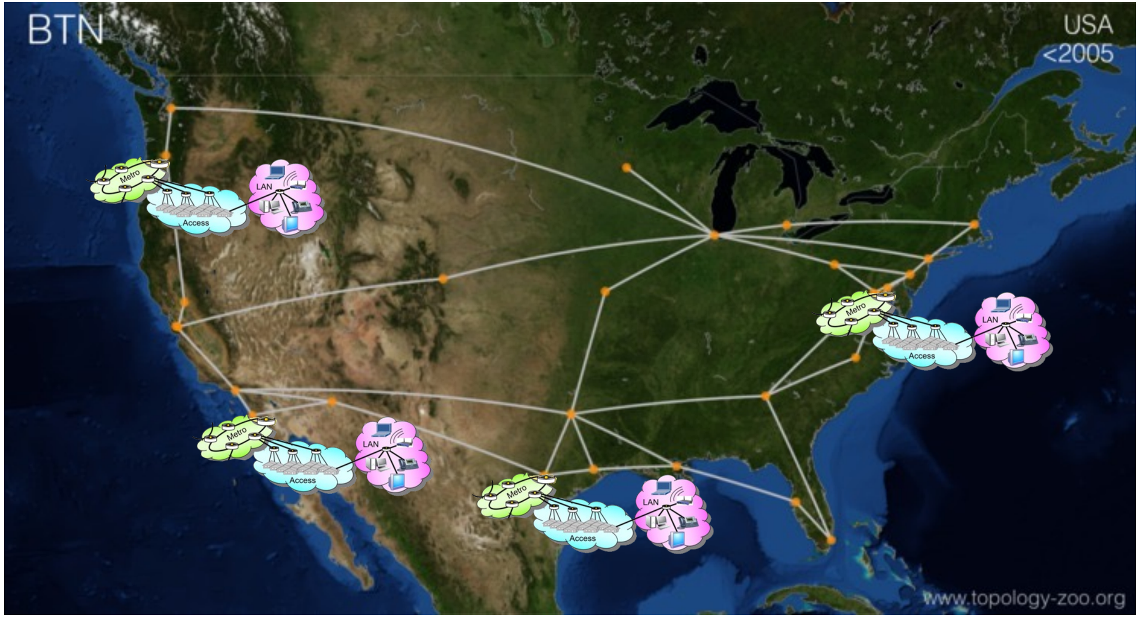
Traffic Patterns Today are Highly Variable and Harder to Predict

- ❑ High number of users
- ❑ Diverse applications
- ❑ Failures in adjacent networks
- ❑ Data Centers
- ❑ Edge Clouds
- ❑ Virtualization and Migration



How to optimize if we can’t forecast traffic?

- ❑ Over-provisioning without performance guarantees?



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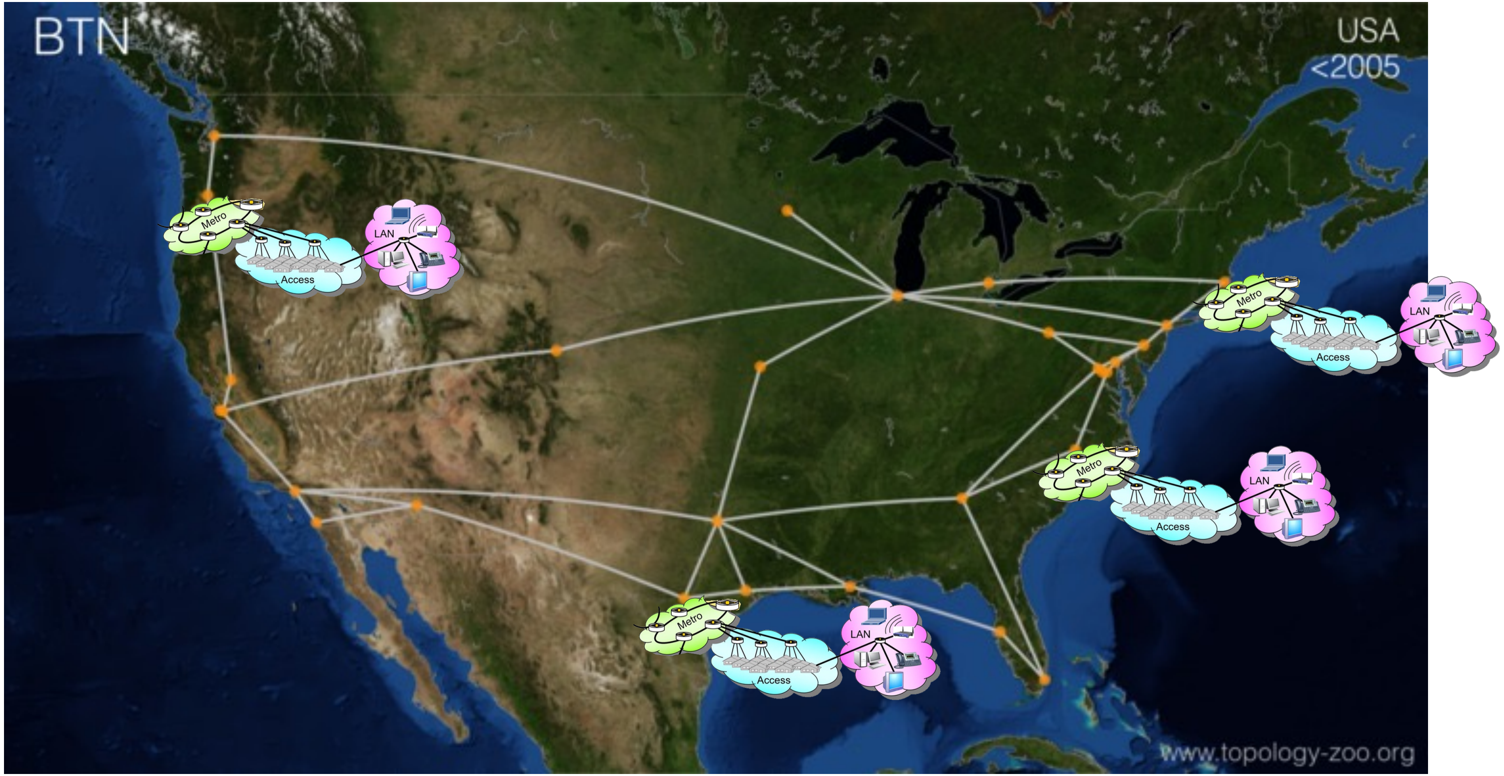
Oblivious Routing

with Hose Constraints

We know link capacities



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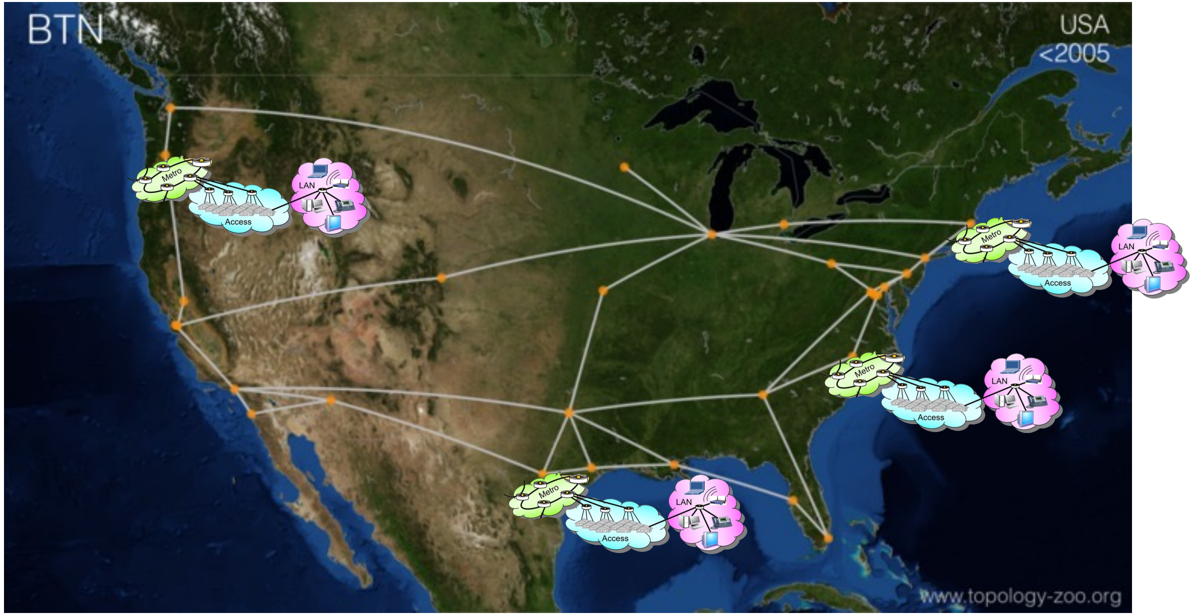
We know one more thing about the network:



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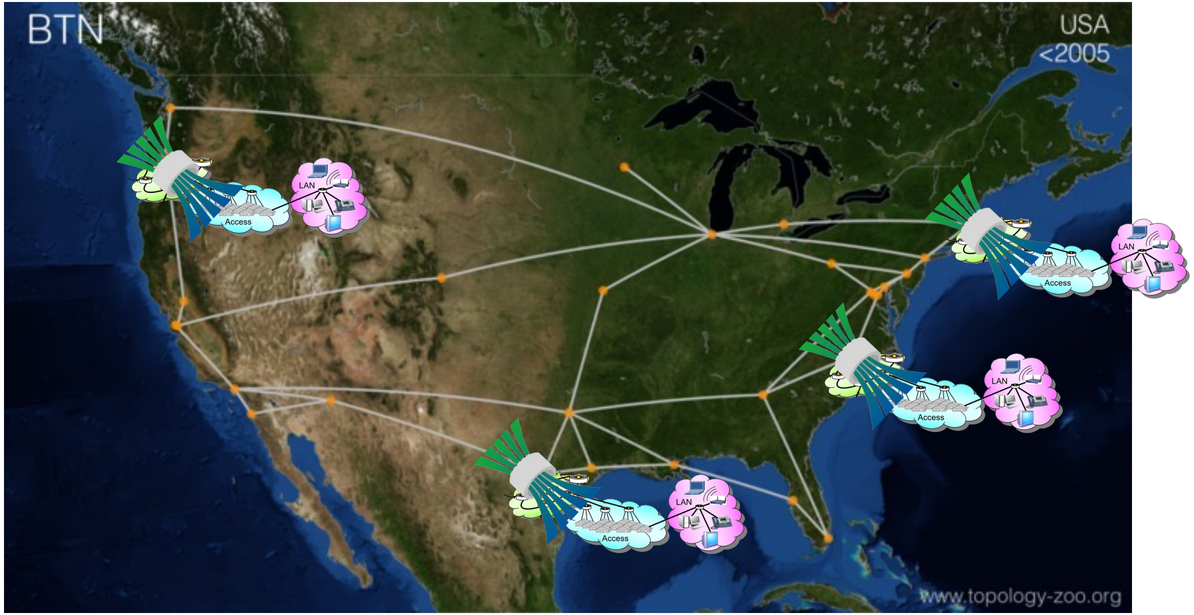
We know one more thing about the network:

- The capacity of the physical connection of the network with the outside: The Hose



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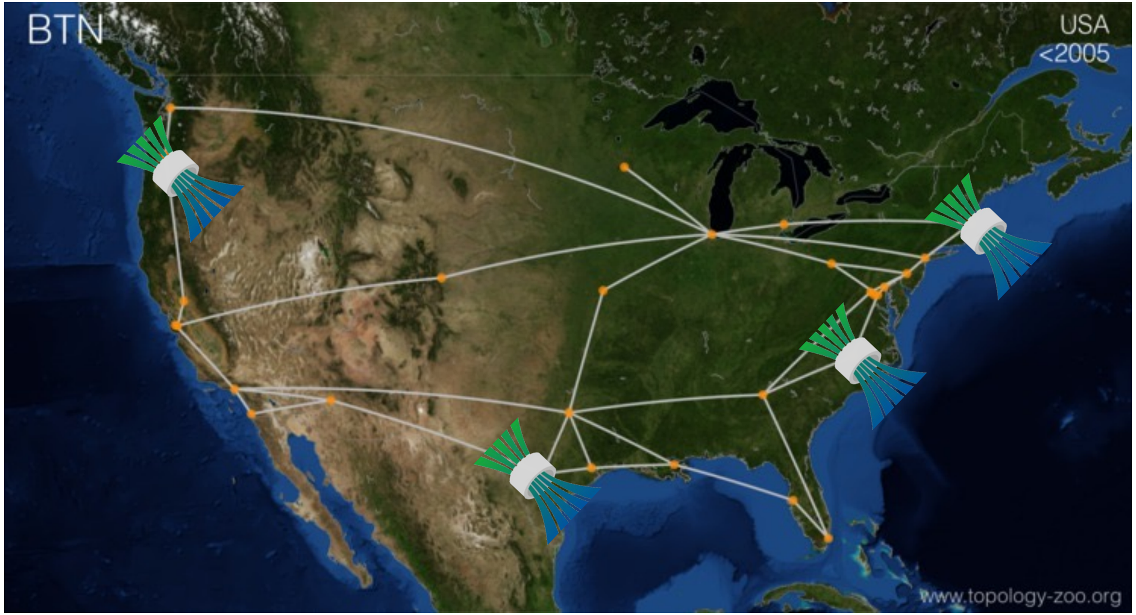

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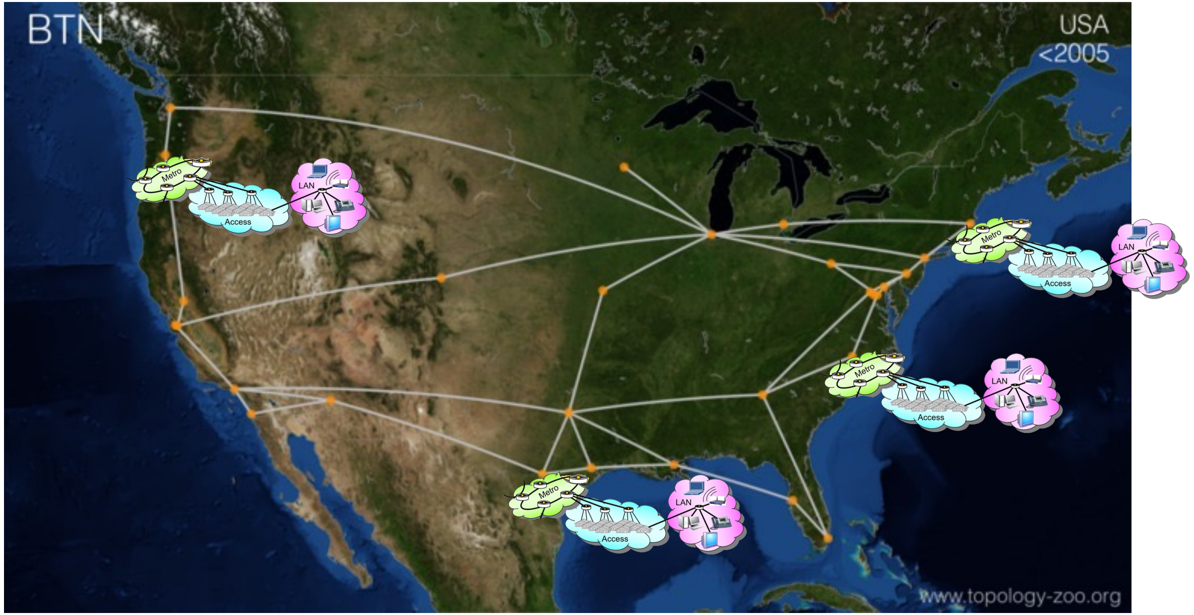

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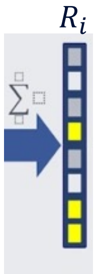

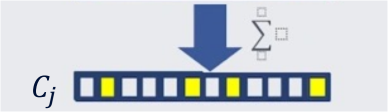

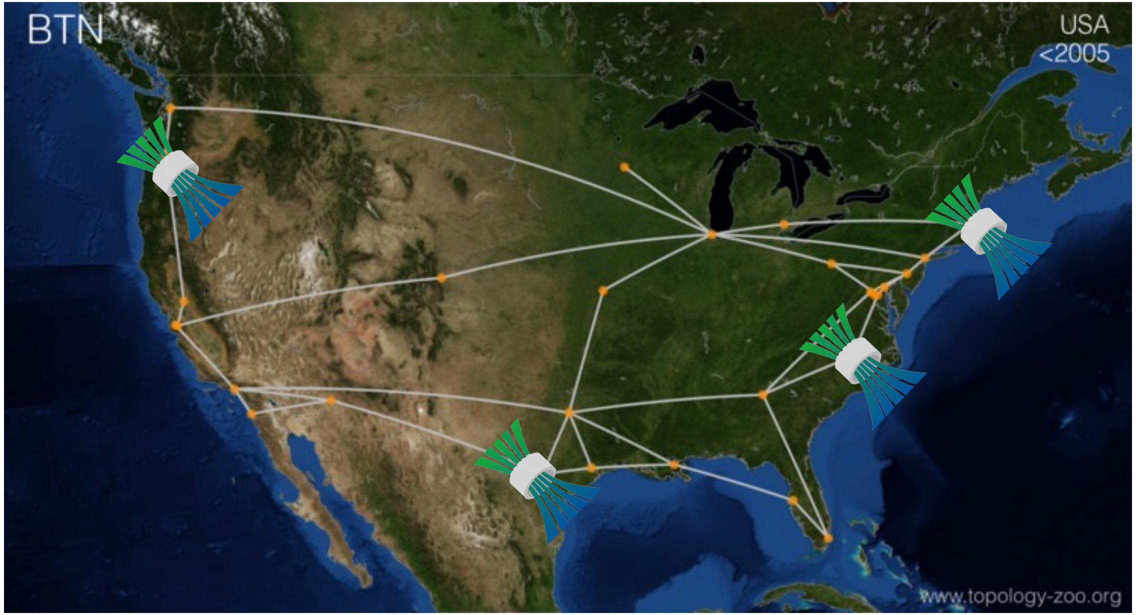

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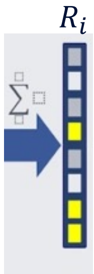
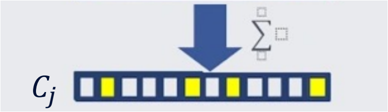
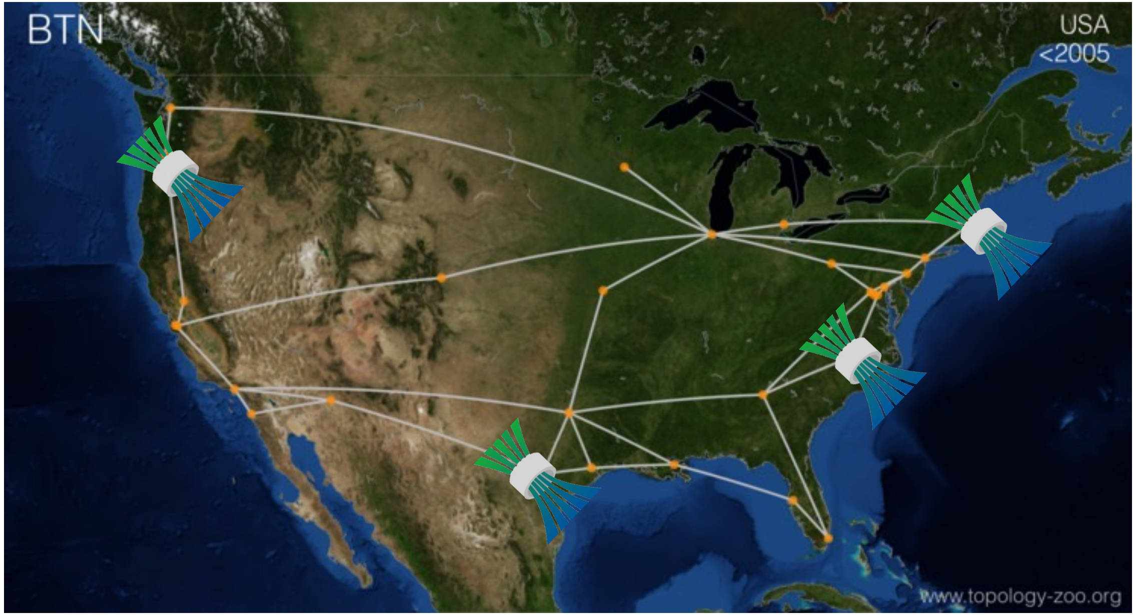




$$\mathcal{T}(\vec{\mathcal{R}},\vec{\mathcal{C}}) = \left\{ [t_{ij}] \,\middle|\, \sum_{j \neq i} t_{ij} \leq R_i \text{ and } \sum_{j \neq i} t_{ji} \leq C_i \;\forall\, i \right\}$$



Find a fixed routing rule by solving for all possible traffic matrices

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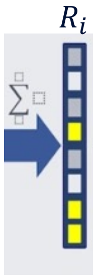

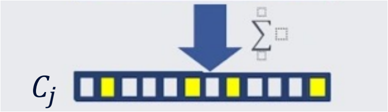

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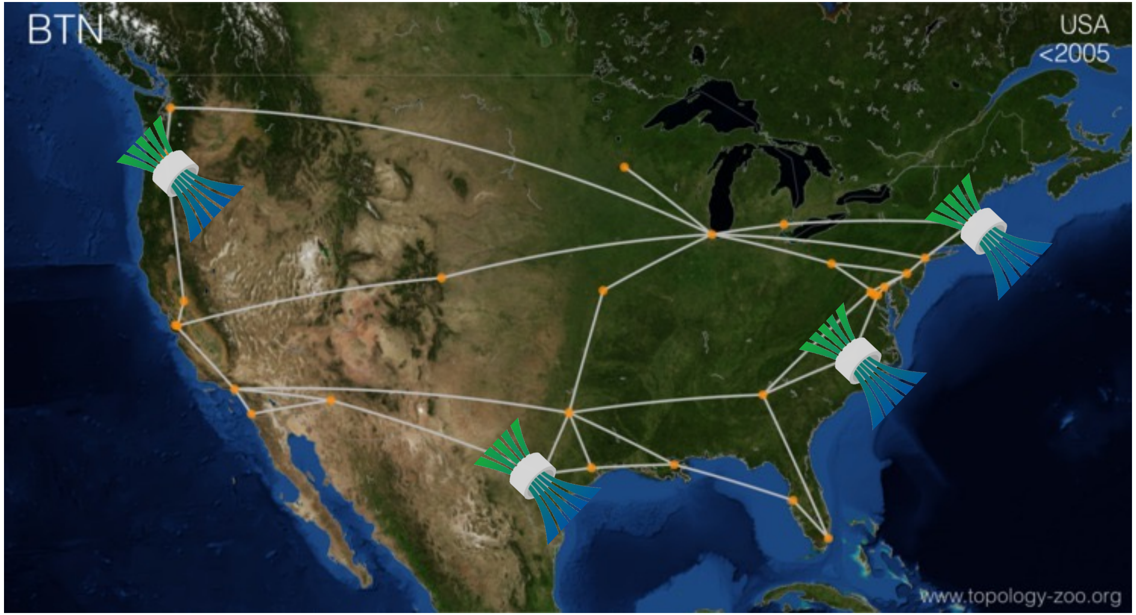

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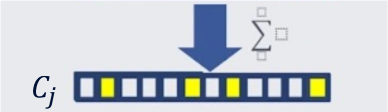

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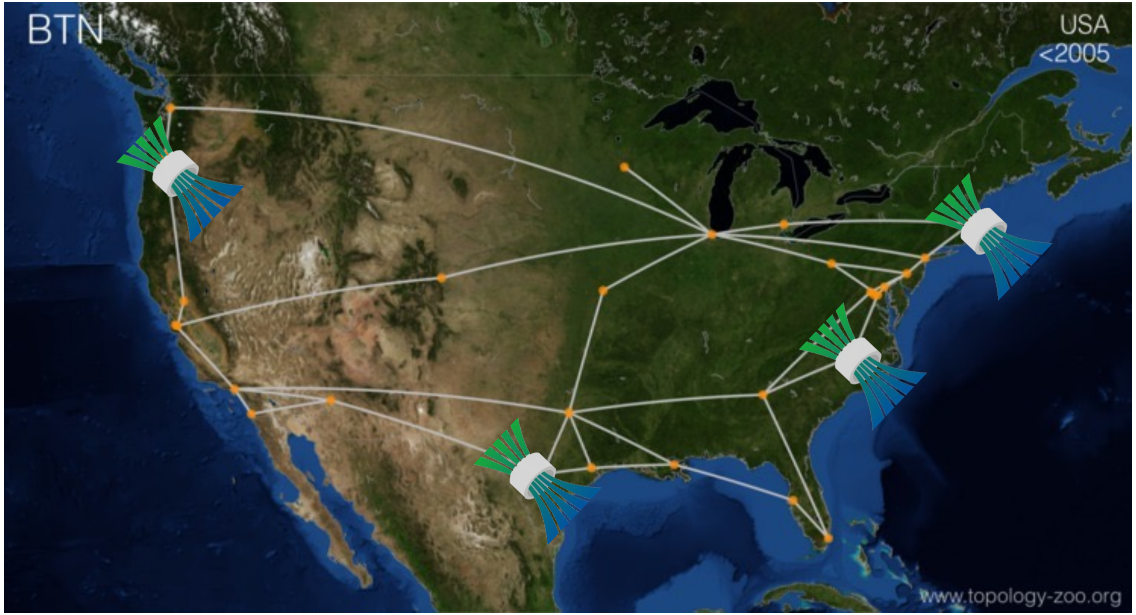

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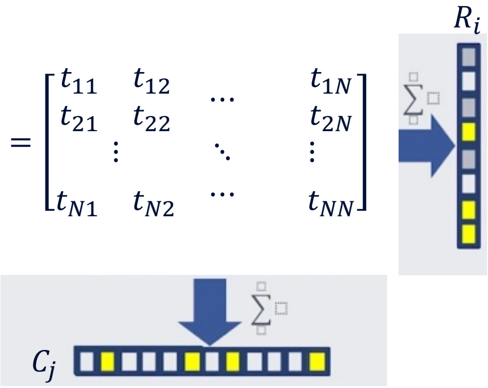

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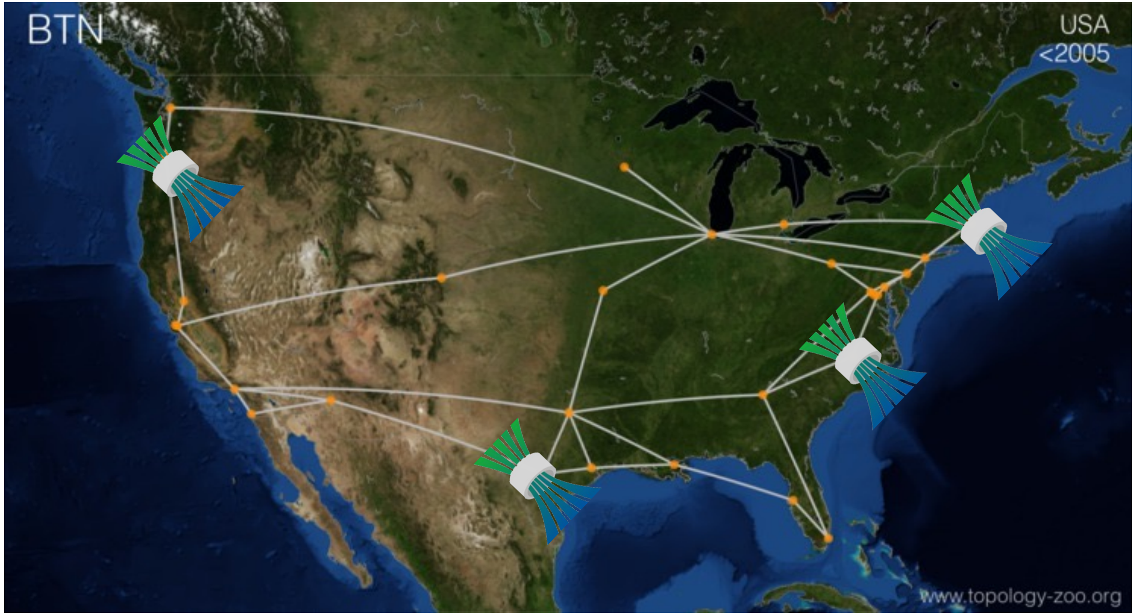

- ❑ Stable and robust scheme reducing network complexity
- ❑ However, we need to optimize for infinitely many traffic matrices. A complex optimization problem!

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Learning Methods

Machine Learning is a Natural Way of Solving Large Problems

- High performance, highly parallelized and optimized ways of solving **unconstrained** optimization problems
  - Using variants of Gradient Descent

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  - Yes!

How?

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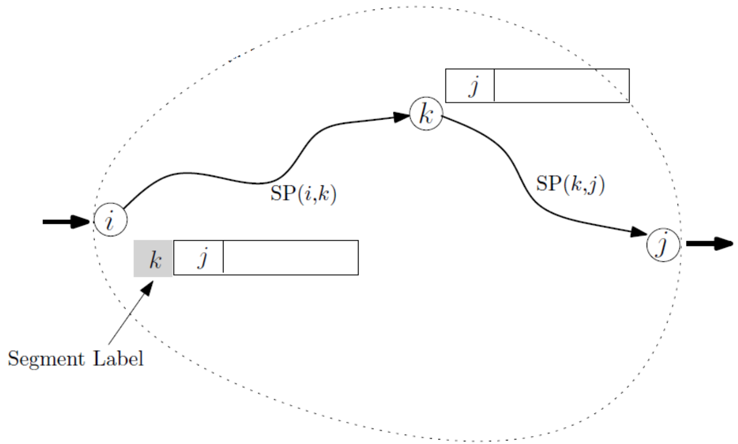
  - **Segment Routing** is a recent and widely deployed non-shortest path technique



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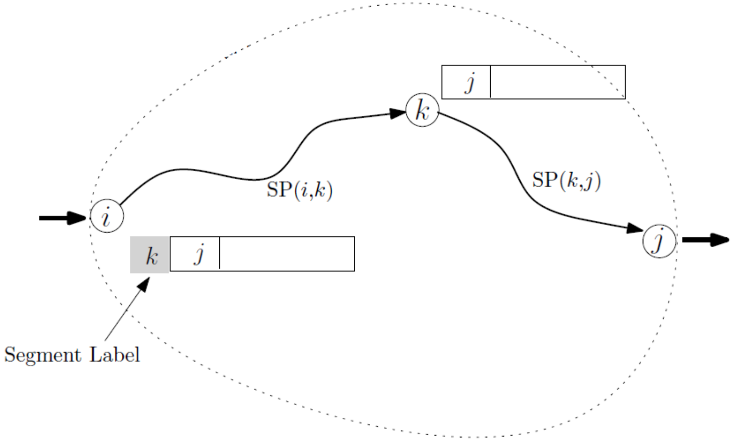

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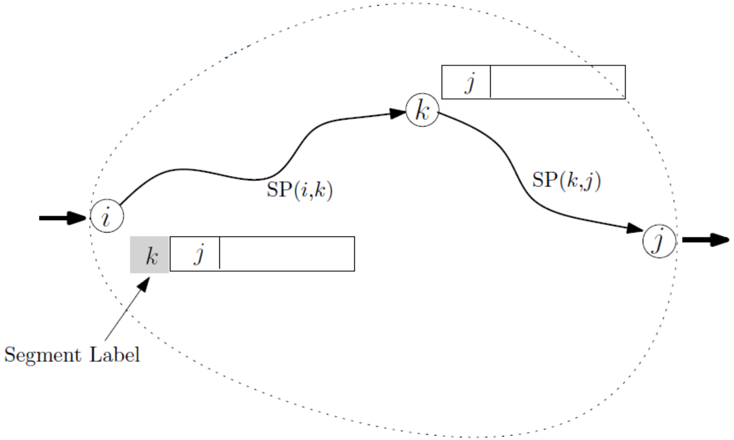
- We embed constraints into the objective function!
  - Segment Routing renders paths enumerable!
  - *Softmax* function trick!
    (see paper for details!)



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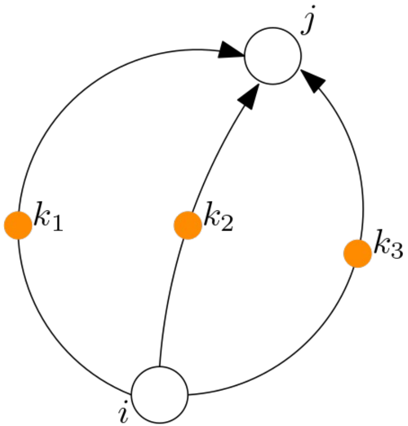
How to Solve an Oblivious Routing Problem Using ML?

- Follow an adversarial approach



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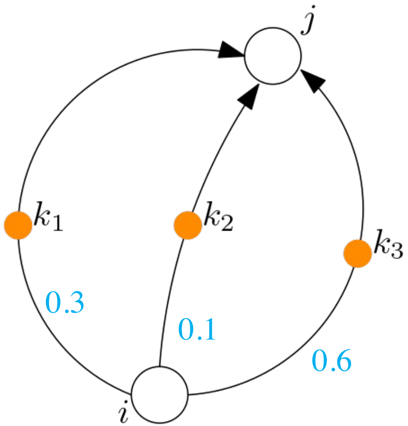

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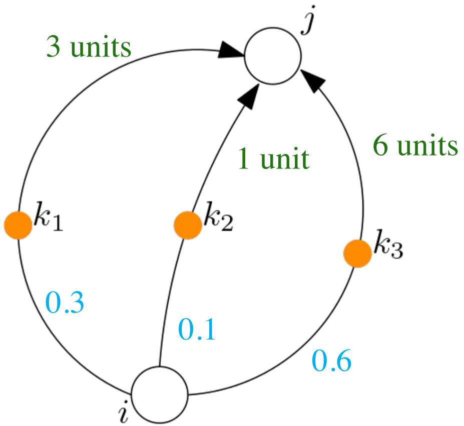

Traffic $i \rightarrow j : 10\ units$

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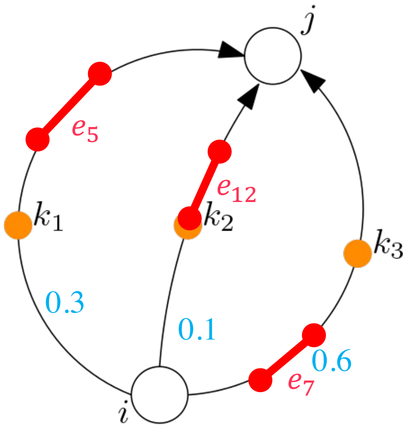

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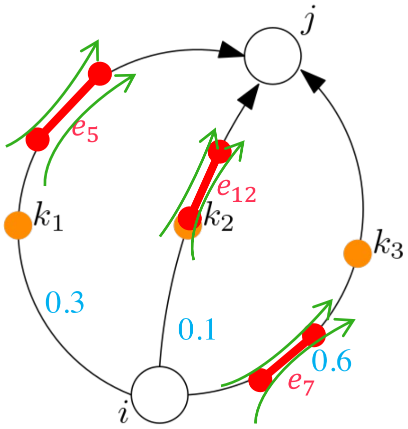

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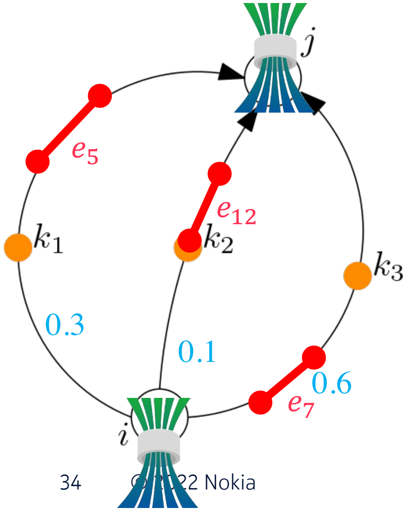
Adversarial Step

Generate the worst possible traffic matrices $[t_{ij}]^{(l)}$ for the current routing fractions α_{ij}^{k} and add to the set of bad matrices $\mathcal{L}$



Generate a traffic matrix corresponding to each edge:

- from the hose constraints
- that maximizes the load

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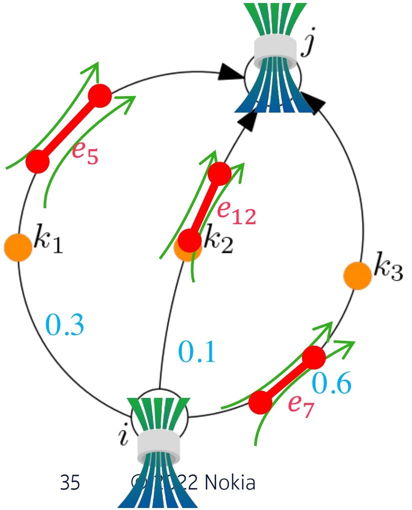

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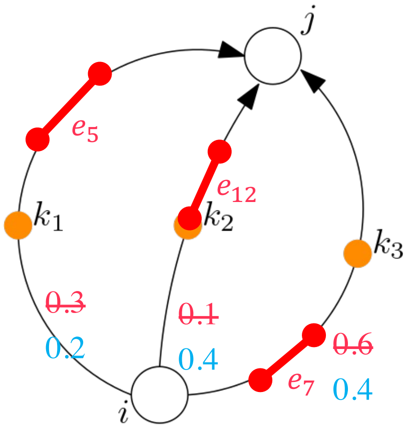

Update routing fractions of each $i \rightarrow j$ pair

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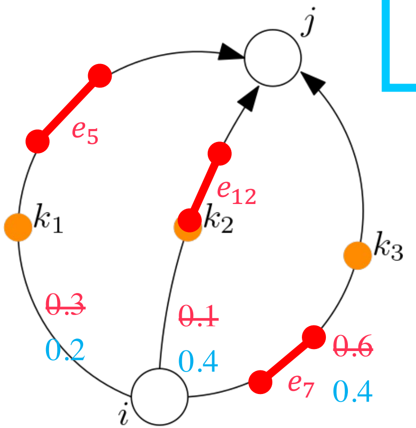

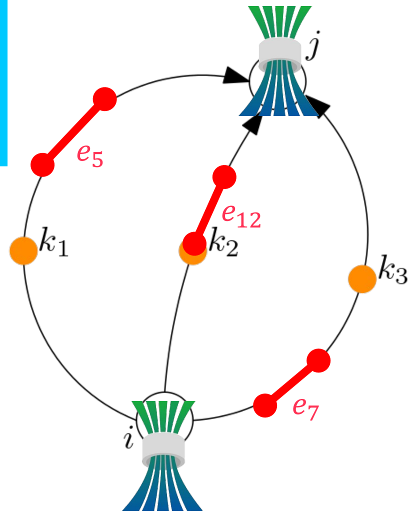

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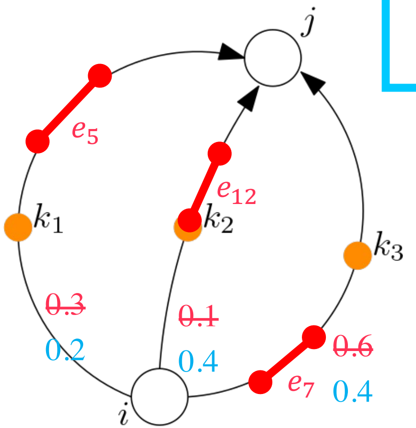
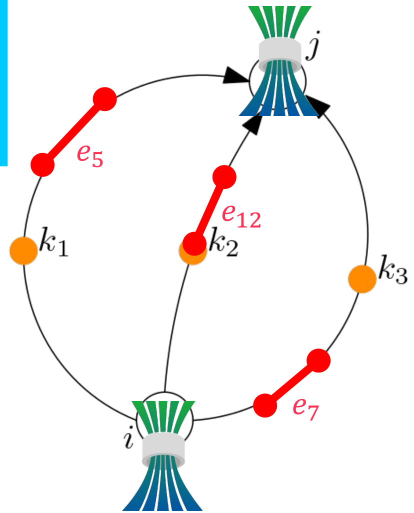
Stop when performance upper and lower bounds converge





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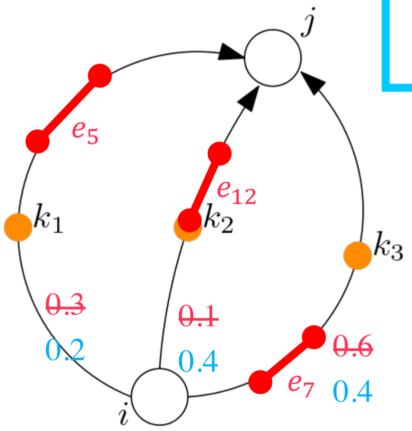
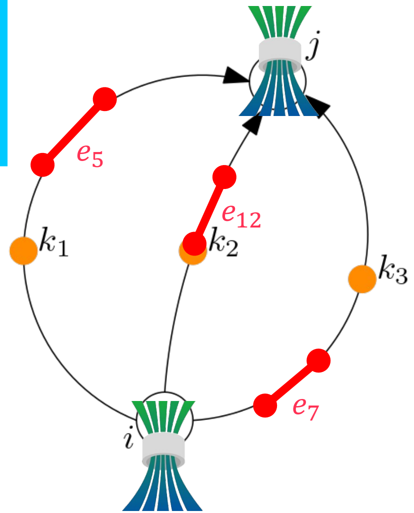


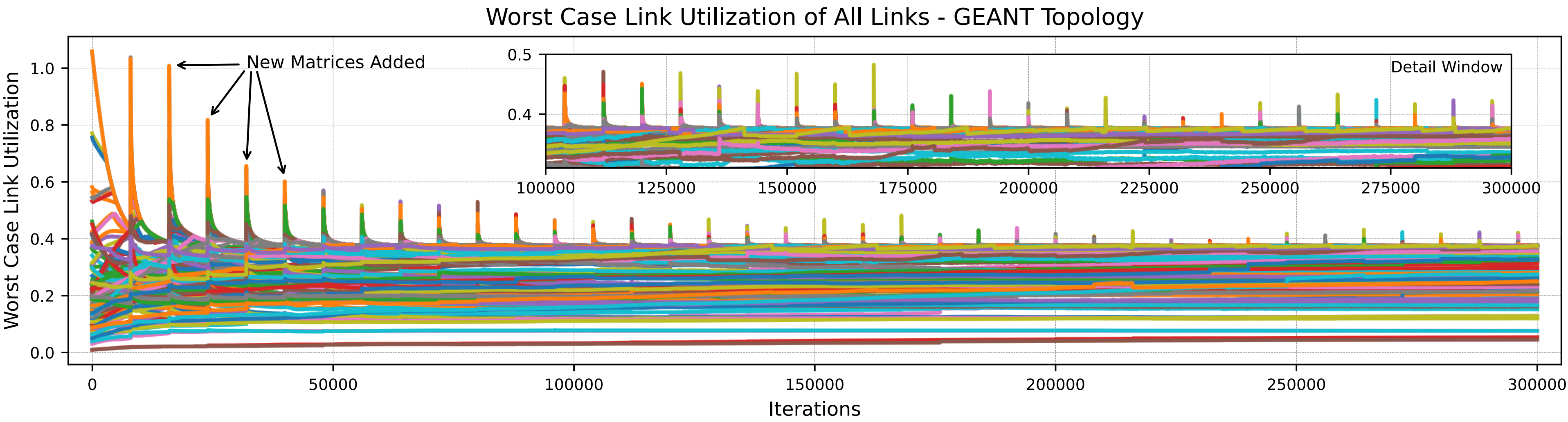
- Optimization step is solved with gradient descent.



Results

The Adversarial Approach converges to the optimal solution

- Worst case utilization of each link on the network is depicted as split fractions are tuned.
- Adversarial traffic matrices are periodically added.



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We Can Solve Large Scale Problems Using the ML Approach

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- ❑ Our method is compatible with:
  - ❑ Legacy shortest path routing extensions (e.g., MPLS)
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- Also available in our paper:
  - Two computationally simple relaxations of this problem –providing bounding solutions

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Thank You

Oblivious Routing using Learning Methods

Ufuk Usubutun

usubutun@nyu.edu

Backup Slides

How do we incorporate 2-Segment Routing into the problem formulation?

- 2-Segment Routing: $i \rightarrow \mathrm{k} \rightarrow j$
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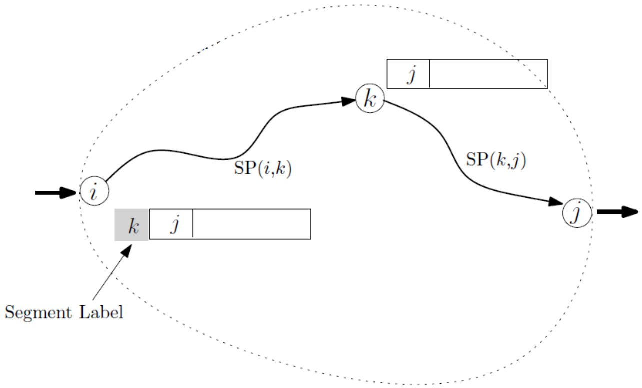

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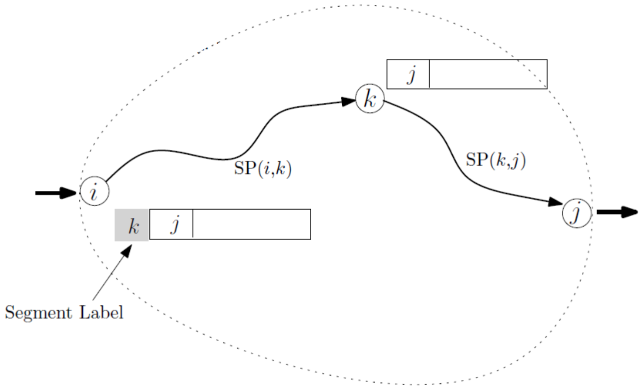

- Define an indicator function $f_{ij}(e) \in \{0,1\}$ where e is an edge
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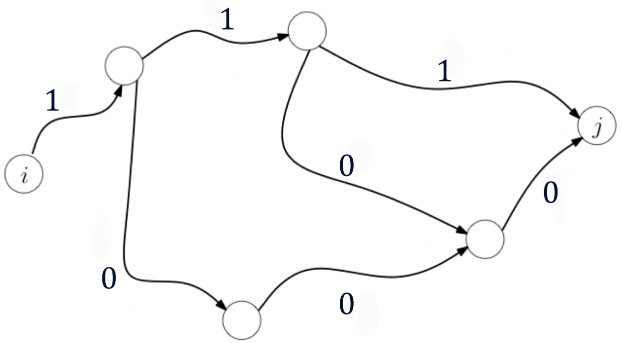
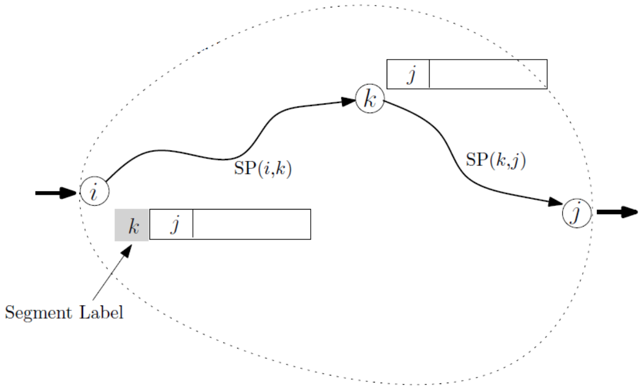


How do we incorporate 2-Segment Routing into the problem formulation?

- 2-Segment Routing: $i \rightarrow \mathrm{k} \rightarrow j$
  - Choose an intermediate node k
  - Set what fraction of $i \rightarrow j$ should go through k
  - $\sum_k \alpha_{ij}^k = 1\ ,\ \forall i, j$



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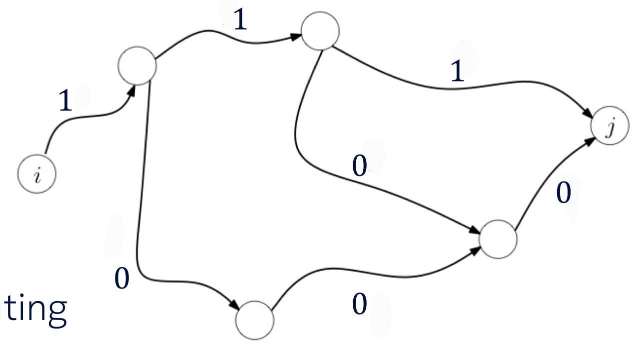
- Define $g_{ij}^k(e) = f_{ik}(e) + f_{kj}(e)$ to accommodate 2-Segment Routing



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ECMP Extension

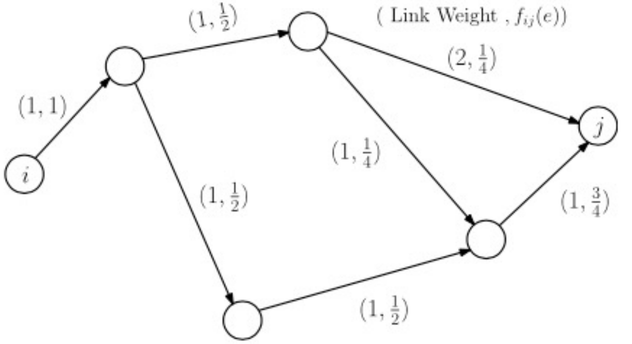
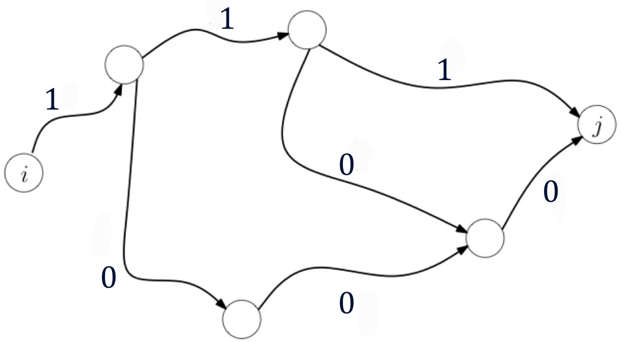


Fig. 1. Definition of $f_{ij}(e)$ with ECMP. The first number next to the link represents the link weight and the second number is $f_{ij}(e)$. The shortest path length is 4 and there are three shortest paths.



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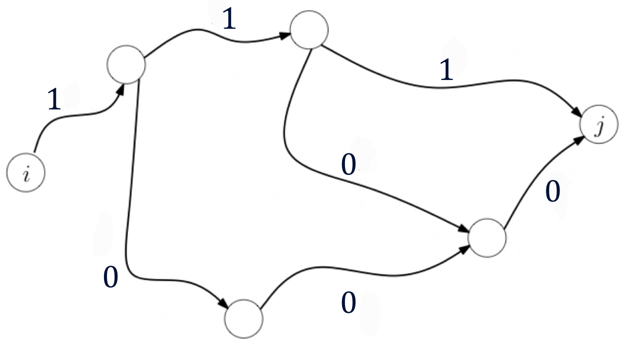

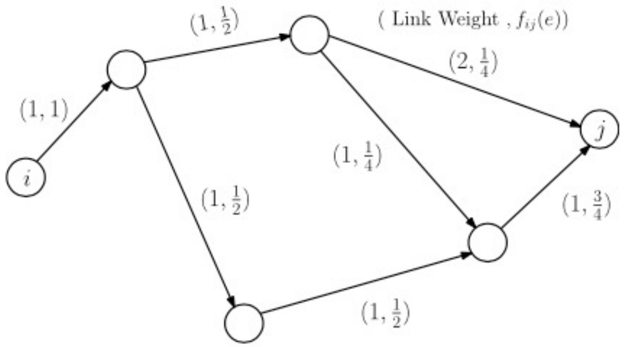

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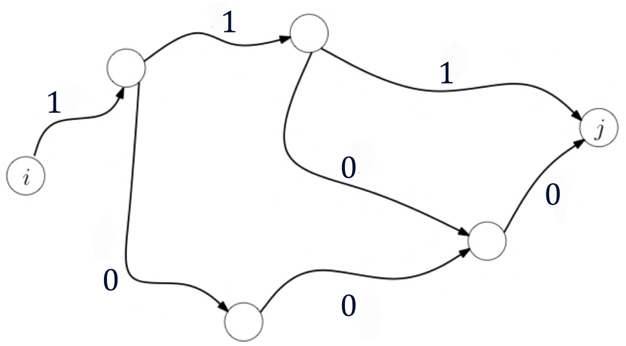
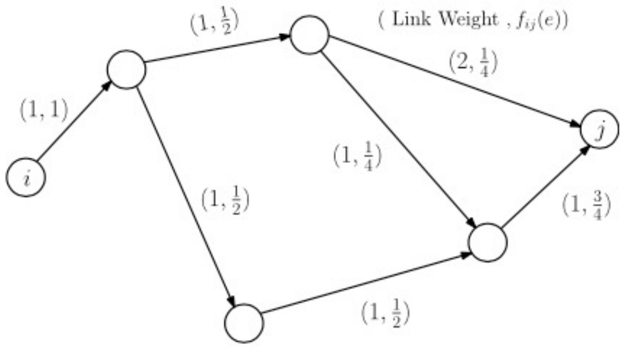


- ECMP Extension



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Formulating a 2-Segment Routing Problem

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- Total traffic from i to j traveling over node k:

$$t_{ij}\alpha_{ij}^{k}$$

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How to Solve an Oblivious Routing Problem Using ML?

- Follow an adversarial approach

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Adversarial Step

Generate the worst possible traffic matrices $[t_{ij}]^{(l)}$ for the current routing fractions α_{ij}^k and add to the set of bad matrices $\mathcal{L}$

$$\mathcal{T}(\vec{\mathcal{R}},\vec{\mathcal{C}}) = \left\{ [t_{ij}] \,\middle|\, \sum_{j\neq i} t_{ij} \leq R_i \text{ and } \sum_{j\neq i} t_{ji} \leq C_i \; \forall\, i \right\}$$

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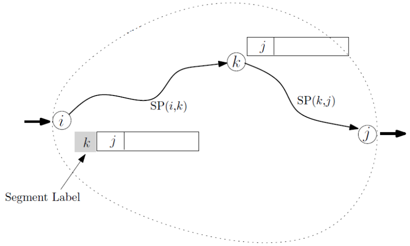

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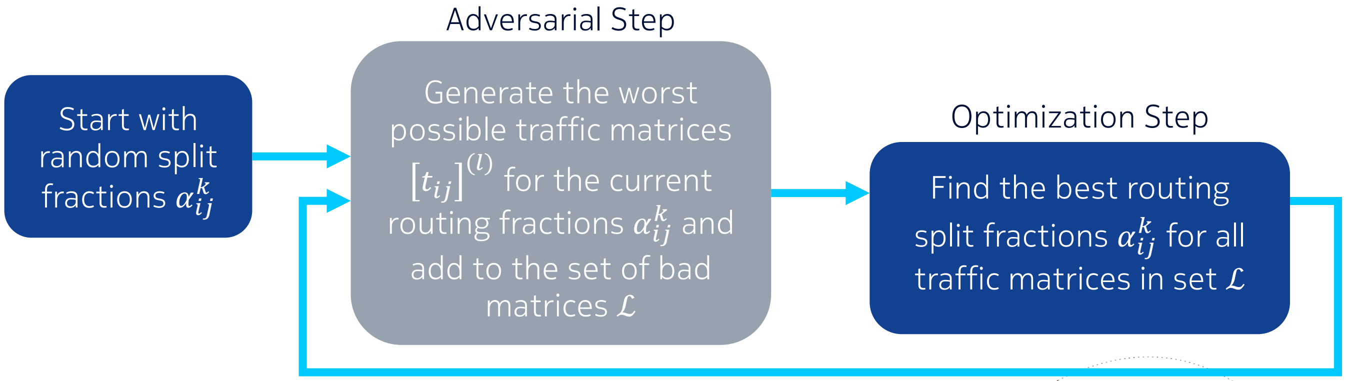

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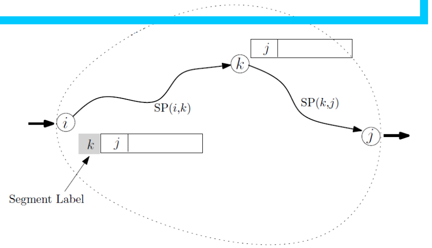

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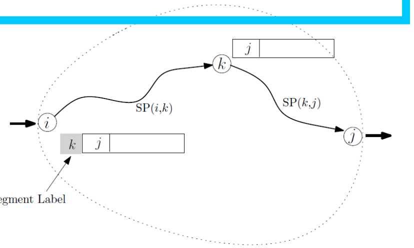

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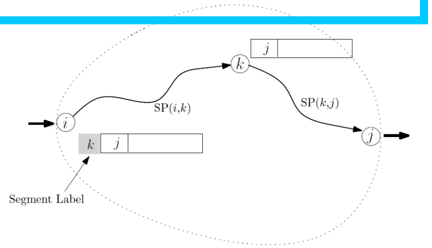

- Optimization step is solved with gradient descent.

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Formulating a 2-Segment Oblivious Routing Problem

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- Let the capacity of a given link e is u_e. The optimization problem becomes:

$$\min_{\alpha_{ij}^k} \mu \;\; \text{s.t.}$$

$$\sum_{ijk} t_{ij} \alpha_{ij}^k g_{ij}^k(e) \le \mu u_e \,, \qquad \forall i,j, \forall e, \forall [t_{ij}] \in T(R,C)$$

$$\sum_k \alpha_{ij}^k = 1 \,, \qquad \forall i,j$$

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- How do we work with infinitely many traffic matrices?

How to Solve an Oblivious Routing Problem Using ML?

The Optimization Step

- Reshape the 2-segment routing problem to embed some constraints into the objective function

$$\min_{\alpha_{ij}^k} \mu \ \text{s.t.}$$
$$\sum_{ijk} t_{ij}^{(l)} \alpha_{ij}^k g_{ij}^k(e) \le \mu u_e \,, \qquad \forall i,j, \forall l, \forall e$$
$$\sum_k \alpha_{ij}^k = 1 \,, \qquad \forall i,j$$
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$\Rightarrow$

$$\min_{\alpha_{ij}^k}\{\max_{e,l} \frac{\sum_{ijk} t_{ij}^{(l)} \alpha_{ij}^k g_{ij}^k(e)}{u_e}\} \ \text{s.t.}$$
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$$\sum_k \alpha_{ij}^k = 1\,, \qquad \forall i,j$$
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- Use the *softmax* function to keep solutions in the feasible set.

$$\alpha_{ij}^k = SM(y_{ij}^k) = \frac{e^{y_{ij}^k}}{\sum_k e^{y_{ij}^k}}, \qquad \forall i,j$$

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Generate worst case matrices:
The Adversarial Step

Given a choice of split fractions α_{ij}^{k}

Generate one worst case traffic matrix for each link e

Append all new matrices to set $\mathcal{L}$

$$[\mathrm{t}_{ij}]^{(l)} = arg\,max_{[t_{ij}]} \sum_{ijk} t_{ij} \alpha_{ik}^{k} g_{ij}^{k}(e) \quad \text{s.t.}$$

$$\sum_{i} t_{ij} \le C_j \,, \qquad \forall j$$

$$\sum_{j} t_{ij} \le R_i \,, \qquad \forall i$$

$$t_{ij} \ge 0 \,, \qquad \forall i, j$$

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