Chapter 14

Thermomechanics

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In this chapter, we consider coupled thermo-mechanical (TM) processes in a porous medium. For a heat transport problem in any medium, the governing equation is given by

$$\rho C_p T' = -\nabla \mathbf{q}_{\mathrm{T}} + Q_{\mathrm{T}}(\boldsymbol{x}, t),\, \boldsymbol{x} \in \mathbb{R}^3 \tag{14.1}$$

where ρ is medium density, $C_p(T)$ is the specific heat capacity, Q_{T} is the heat source and $\mathbf{q}_{\mathrm{T}}$ is the heat flux, which takes the form

$$\mathbf{q}_{\mathrm{T}} = -K_e \nabla, T \tag{14.2}$$

for solid and

$$\mathbf{q}_{\mathrm{T}} = -K_e \nabla T + n \sum_{\gamma}^{phase} (\rho^\gamma C_p^\gamma) T \mathbf{v},\, \gamma = \text{liquid, gaseous} \tag{14.3}$$

for porous media, considering advective and diffusive fluxes with K_e as the heat conductivity. For porous media, the specific heat capacity consists of portions of solid, liquid and gaseous phases as

$$\rho C_p = \sum_{\gamma}^{phase} (\rho^\gamma C_p^\gamma) \tag{14.4}$$

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where γ specifies a solid, liquid or gaseous phase. The boundary conditions are given by

$$\mathbf{q}_{\mathrm{T}} \cdot \mathbf{n} = q^{\mathrm{T}}_{\Gamma}, \text{ or } \quad T = T_{\Gamma}, \forall \boldsymbol{x} \in \partial\Omega \tag{14.5}$$

and the initial condition reads

$$T(\boldsymbol{x}, t) = T_0(\boldsymbol{x}), \forall \boldsymbol{x} \in \Omega \tag{14.6}$$

with $\mathbf{n}$, the normal direction at $\boldsymbol{x} \in \partial\Omega$

For the mechanical process, the total strain rate $\Delta\boldsymbol{\epsilon}$ can be decomposed into its elastic (reversible) and thermal components,

$$\Delta\boldsymbol{\epsilon} = \mathbb{C}(\Delta\boldsymbol{\epsilon}^e - \alpha\,\mathbf{I}\Delta T) \tag{14.7}$$

where $\mathbb{C}$ is the constitutive tensor, α is the linear thermal expansion coefficient, $\mathbf{I}$ is the identity tensor and ΔT is temperature change. With the generalized Hook's law, the total stress with the thermal effect can be expressed as

$$\Delta\boldsymbol{\sigma} = \mathbb{C}(\Delta\boldsymbol{\epsilon} - \alpha\,\mathbf{I}\Delta T) \tag{14.8}$$

where $\boldsymbol{\sigma}$ is the stress tensor. The volume of a solid increases or decreases with temperature changes and homogeneous bodies expand evenly in each direction by increasing temperatures. In this case no variation of the stresses occurs. If the deformation of the solid is prevented, the stresses increase or decrease with temperature changes. This phenomenon can be easily calculated by analytical solutions of the Hooke's linear elastic model. The equations of the mechanical behaviour base on the Hooke's law for linear elastic materials are:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) + \alpha\Delta T \tag{14.9}$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right) + \alpha\Delta T \tag{14.10}$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) + \alpha\Delta T \tag{14.11}$$

where $\varepsilon_{i,\, i=x,y,z}$ are strains, σ_i are stresses, E is Young's modulus and ν is Poisson's ratio.

14.1 Thermoelastic Stress Analysis in Homogeneous Material (3 D)

14.1.1 Definition

The top and bottom of a solid body that consists of one homogeneous material are heated. The aim of this calculation is to find out the isotropic state of stress

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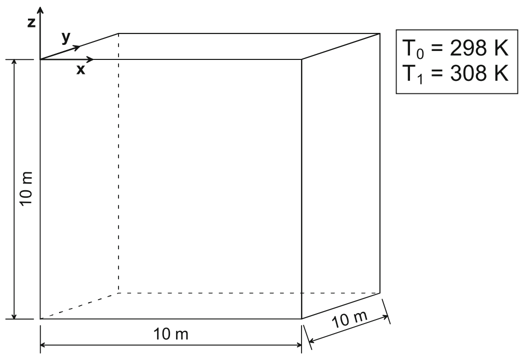

Figure 14.1: Calculation area with one material

Table 14.1: Model parameters

Symbol	Parameter	Value	Unit
T_0	Initial temperature (before heating)	298	K
T_1	Temperature after heating	308	K
ρ	Density of the solid	2,200	kg m^{-3}
E	Young's modulus of the solid	25	GPa
ν	Poisson ratio	0.27	–
α	Linear thermal expansion	6.0×10^{-6}	K^{-1}
c	Specific heat capacity	1.0	J kg^{-1} K^{-1}
λ	Thermal conductivity	1.0	W m^{-1} K^{-1}

that is reached after the whole solid is heated. Figure 14.1 shows a sketch of the calculation area assuming a homogeneous solid, a constant temperature in the whole body at the beginning and a heating of the top and the bottom of the body at about 10 K. Linear elastic material behaviour, isotropic thermal expansion and no gravity effect are assumed. The xy-plane is the horizontal plane. The height of the body is in the z-direction. The dimensions of this 3 D-model are 10 m in all directions. As deformations in the x- and y-directions are suppressed, the increasing temperature evokes stresses within the solid. The parameters used for the solid represent the material behaviour of concrete (Table 14.1).

14.1.2 Solution

Analytical Solution

The analytical solution can be derived from the time independent equation (14.9)–(14.11) with the assumptions of no deformation and an isotropic thermal expansion:

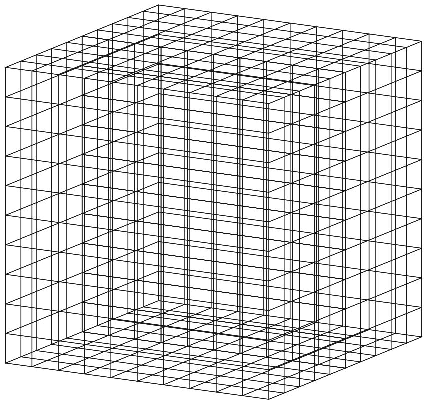

Figure 14.2: Mesh for TM coupling homogeneous material 3D model

$$
\begin{aligned}
\varepsilon_i &\equiv 0 \\
\sigma_x &= \sigma_y = \sigma_z = -\frac{\alpha \Delta T E}{1 - 2\nu}
\end{aligned}
\tag{14.12}
$$

Equation (14.12) provides the stresses after heating the solid and shows an isotropic state of stress.

Numerical Solution

The dimensions of this 3 D-model are 10 m in all directions. Deformations perpendicular to the outer surfaces are suppressed. The initial temperature in the whole area is 298 K. At the top and at the bottom of the model, the thermal boundary conditions are set with a temperature of 308 K. Thereby the heating of the solid to about 10 K is simulated. A mesh with 1,000 hexahedral elements and 1,331 nodes is used for the simulation. The time duration is divided in 384 time steps with a constant time step size of 900 seconds. This means that heating of the solid within 4 days is simulated. The calculation model is sketched in Fig. 14.2.

14.1.3 Results

The calculation of temporal development of the stresses in the centre of the model (at node 665) is presented in Fig. 14.3. The results of the 3 D simulation show an exact agreement with the analytical solutions.

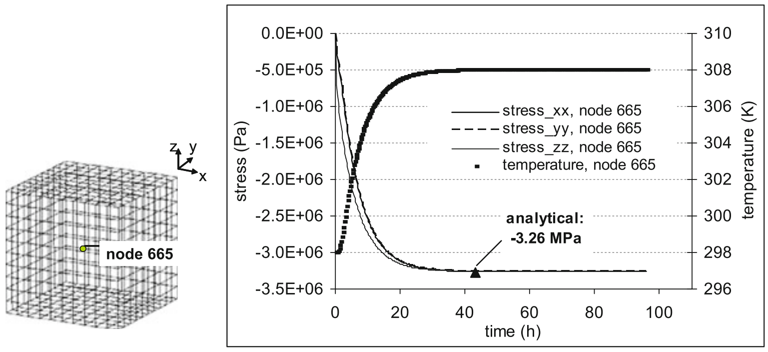

Figure 14.3: Temporal stress development in the centre of the calculation model (node 665)

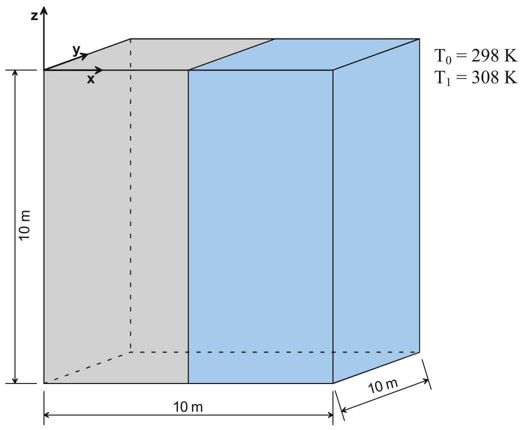

Figure 14.4: Calculation area with two different materials

14.2 Thermoelastic Stress Analysis in Composite Materials (3 D)

14.2.1 Definition

If there are two materials with different thermal expansions, the volume changes of the materials will be uncommon. The material with the higher thermal expansion expands more than the material with the lower thermal expansion. If deformations at the outer boundaries are prevented, different states of stress will occur in these two materials. But the stresses perpendicular to the parting plane must be equal. The values of the stresses as a result of temperature changes can also easily be calculated by the Hooke's linear elastic model. The aim of this simulation is to specify the stresses at several areas in the solid. Figure 14.4 shows a sketch of the calculation area. The model parameters are given in Table 14.2.