An Interface between Grassmann manifolds and vector spaces

Lincon Souza[1], Naoya Sogi[1], Bernardo Gatto[2], Takumi Kobayashi[3], Kazuhiro Fukui[1,2]

[1] *Graduate School of Systems and Information Engineering, University of Tsukuba, Japan*

[2] *Center for Artificial Intelligence Research (C-AIR), University of Tsukuba, Japan*

[3] *National Institute of Advanced Industrial Science and Technology (AIST)*

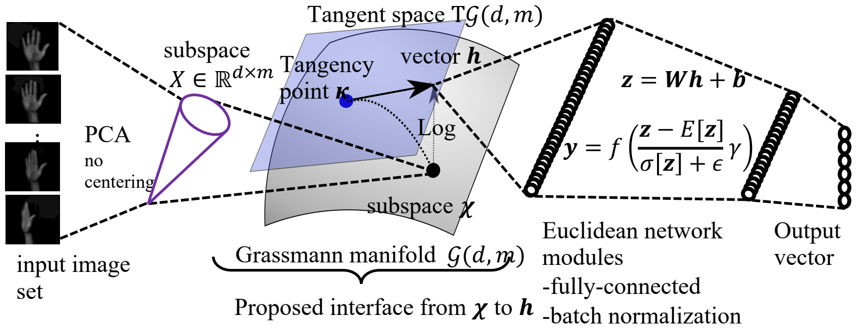
筑波大学 University of Tsukuba

AIST CENTER FOR ARTIFICIAL INTELLIGENCE RESEARCH

(1) Motivation and objective

- We propose the **Log module** to map data from a Grassmann manifold to a vector space.
- The **Grassmann manifold** is the set of subspaces of a vector space:
- It is a foundation for various types of machine learning tools using **subspace representation**.
- **Problems**:
  - Most standard machine learning methods cannot be promptly utilized on the Grassmann manifold, since they are constructed on Euclidean space.
  - It is hard to directly link the Grassmann manifold to deep neural network architectures.
- **Motivation:**
  - a subspace is well known as a practical and robust representation, being applied to numerous problems such as image set recognition, fine-grained classification and action recognition.
  - Abundance of well-established end-to-end network layers which work in Euclidean space, e.g. fully-connected, batch normalization, dropout.
- We attempt to fill these gaps by **connecting** those two representations.
- The **key idea** is to formulate the manifold logarithmic map (log) as an end-to-end **learnable model**.

(2) Proposed Log model



Definition of the Log module:

$$\boldsymbol{h} = \mathrm{vec}(\mathrm{Log}_{\boldsymbol{\kappa}}\boldsymbol{X})$$

Grassmann Log:

$$\mathrm{Log}_{\boldsymbol{\kappa}}\boldsymbol{X} = \boldsymbol{W}_{:m}\arctan(\boldsymbol{\Theta}_{:m})\boldsymbol{Z}_{:m}$$

$$(\boldsymbol{K}^\top\boldsymbol{X})^{-1}(\boldsymbol{K}^\top - \boldsymbol{K}^\top\boldsymbol{X}\boldsymbol{X}^\top) = \boldsymbol{W}\boldsymbol{\Theta}\boldsymbol{Z}^\top$$

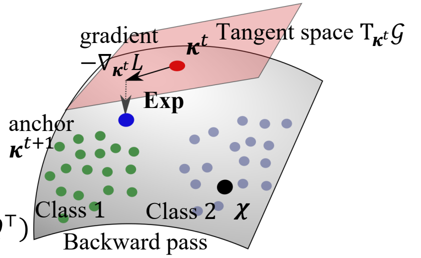
(3) Backward pass

Riemannian SGD update:

$$\boldsymbol{\kappa}^{t+1} = \mathrm{Exp}_{\boldsymbol{\kappa}^t} - \lambda\nabla_{\boldsymbol{\kappa}^t}L$$

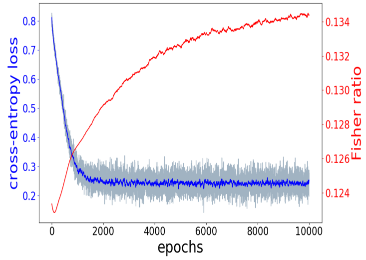
$$\nabla_{\boldsymbol{\kappa}^t}L = \boldsymbol{J}\boldsymbol{\Sigma}\boldsymbol{Q}^\top$$

$$\mathrm{Exp}_{\boldsymbol{K}}\lambda\nabla_{\boldsymbol{\kappa}^t}L = \mathrm{orth}(\boldsymbol{K}\boldsymbol{Q}(\cos\boldsymbol{\Sigma}\lambda)\boldsymbol{Q}^\top + \boldsymbol{J}\boldsymbol{Q}(\sin\boldsymbol{\Sigma}\lambda)\boldsymbol{Q}^\top)$$

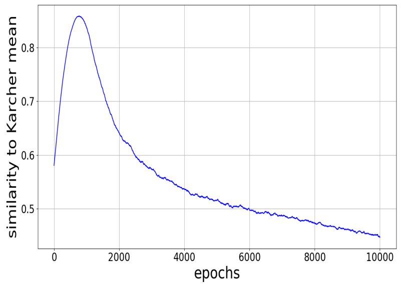


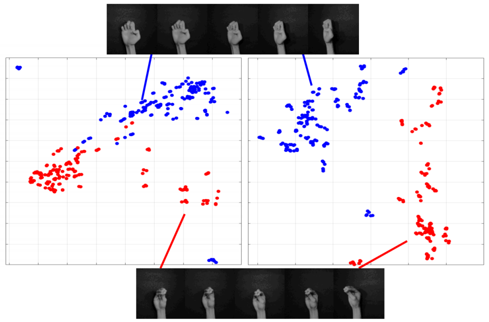
(4) Qualitative results

- Plot of Loss and Fisher ratio



- Plot Similarity to Karcher mean





- tSNE visualizations of 2 classes of hand shapes. The left plot denotes the tangent vectors at the Karcher mean, while the right plot shows the tangent vectors at the log model learned tangent space.

(5) Experimental results

- To evaluate the proposed method, we use the following databases and achieved the following recognition rates:

Hand Shape dataset

Method	Accuracy (%)
Karcher Log model	70.65
Log model	81.90
Conv+Log model	91.90
Resnet18+Log model	99.40

AFEW dataset

Method	Accuracy (%)
RSR-SPDML	30.12%
DCC	25.78%
GDA	29.11%
GGDA	29.45%
PML	28.98%
DeepO2P	28.54%
SPDNet	34.23%
GrNet-1	32.08%
GrNet-2	34.23%
Log Model	32.61%

(6) Conclusions

- We proposed in this paper the Grassmann log model to map subspace data to vector spaces while maximizing discrimination capability for classification.
- The Log model can be learned with RSGD along with other NN layers.
- Future works include the extension of this idea to other Riemannian manifolds; and to other applications, such as video retrieval, modeling of matrices in signal processing, and text modeling.

(7) Publications

[1] Lincon Souza, Naoya Sogi, Bernardo Gatto, Takumi Kobayashi, Kazuhiro Fukui, “An Interface between Grassmann manifolds and vector spaces”, CVPRW 2020.

[2] Lincon Souza, Bernardo Gatto, Jing-Hao Xue, Kazuhiro Fukui, “Enhanced Grassmann Discriminant Analysis with Randomized Time Warping for Motion Recognition”, Pattern Recognition, 2020.

[3] Lincon Souza, Bernardo Gatto, Kazuhiro Fukui, “Classification of Bioacoustic Signals with Tangent Singular Spectrum Analysis”, ICASSP 2019.