CSE 4345 – Homework #5

Assigned: Thursday, March 7, 2019
Due: Thursday, March 21, 2019 at the end of class

Note the following about the homework:

1. You must show your work to receive credit.
2. If your submission has more than one page, staple the pages. If I have to staple it, the cost is 10 points.

Assignment:
Process

1. (do by hand) Approximate $\sqrt[3]{4}$ (i.e., the cube root of 4) using interval bisection and Newton's method. The function to use is
$$f(x) = x^3 - 4$$
We are only finding the real root here.

   (a) Use the interval bisection method with an initial interval of [1, 2]. You can stop after five iterations (consider the original interval as the first iteration, so find four more).

   (b) Use Newton's method with an initial guess of $x_0 = 1$. Show x_k to three decimal places on each iteration. Stop after producing x_4.

   To 3 decimal places, what is the correct answer according to Numpy?

2. (do by hand) Find a root of
$$f(x) = x^2 - 3x - 5$$
using

   (a) Newton's method with an initial guess of $x_0 = 0$. Show x_k to three decimal places on each iteration. Stop after producing x_4.

   (b) Newton's method with an initial guess of $x_0 = 3$. Show x_k to three decimal places on each iteration. Stop after producing x_4.

   The method you just used is a *numerical* method. An *analytical*, or closed-form, solution can be found using the quadratic equation. Use the quadratic equation to get the roots.

3. (Python solution) One of the applications of the SVD is to produce a least squares solution to a system of equations regardless of A's shape or rank. Another is to determine the effective rank of a matrix. We will use both of these for this problem.

   (a) Name your program `hw05.py`.

(b) On the course website is a file, `svd-data.csv`, that contains the data from a system of equations. The data file has m rows and $n+1$ columns. The last column of the data (i.e., column $n+1$) represents the $\vec{b}$ vector while the first n columns represent the $m \times n$ matrix A.

Your program will read the data file without hard-coding the dimensions of A or $\vec{b}$. If I were to provide a data file with a different number of rows or columns, your program should still function correctly.

(c) Do the following in the order given. Include a state indicating what the values represent.

i. Solve for $\vec{x}$ using $\vec{x} = (A^T A)^{-1} A^T \vec{b}$ and print the solution as a row vector.

ii. Print the 2-norm of the residual vector.

iii. Solve the linear system of equations $A^T A\vec{x} = A^T\vec{b}$ for $\vec{x}$ using `np.linalg.solve()` and print the solution as a row vector.

iv. Print the 2-norm of the residual vector.

v. Produce the SVD of A. Print the singular values as a row vector.

vi. Find the effective rank of A by counting the number of singular values greater than or equal to 0.1. Do not use Numpy's rank function. Print the effective rank.

vii. Solve for $\vec{x}$ using

$$\vec{x} = \sum_{i=1}^{r} \frac{\vec{u}_i^T \vec{b}}{\sigma_i} \vec{v}_i$$

where r is the effective rank of A that you found in the previous step. While this step could be vectorized, use loops to help us get same answer. Print the solution as a row vector.

viii. Print the 2-norm of the residual vector.

ix. Print the condition number of $A^T A$ based on the matrix 2-norm.

(d) The value of $\vec{x}$ that I used to produce $\vec{b}$ was $\vec{x} = [1, 1, 1, 1, 1, 1, 1]^T$. Why do you think none of the solutions match this very well? Include your answer as a comment in your code.

General requirements about the Python problems:

a) As a comment in your source code, include your name.

b) The Python program should do the work. Don't perform the calculations and then hard-code the values in the code or look at the data and hard-code to this data unless instructed to do so.

c) The program should not prompt the user for values, read from files unless instructed to do so, or print things not specified to be printed in the requirements.

To submit the Python portion, do the following:

a) Create a directory using your net ID in lowercase characters plus the specific homework. For example, if your net ID is `abc1234` and the homework is `hw04`, then the directory should be named `abc1234-hw04` (zero-pad the number if necessary to make it two digits).

b) Place your .py files in this directory.

c) Do not submit the data files unless instructed to do so.

d) Zip the directory, not just the files within the directory. You must use the zip format and the name of the file (using the example above) will be `abc1234-hw04.zip`.

e) Upload the zip'd file to Blackboard.