



**Department of Mathematics
School of Natural Sciences**

**Course Catalog and Syllabi
Undergraduate Courses**

Academic Year 2018-19 onwards

CCC 101 – Mathematics in India	MAT 330 – Partial Differential Equations
CCC 801 – Art of Numbers	MAT 332 – Geometry of Curves and Surfaces
CCC 805 – Data Analysis and Business Modelling Using Excel	MAT 360 – Linear Algebra II
MAT 020 – Elementary Calculus I	MAT 386 – Dynamical Systems
MAT 084 – Basic Probability & Statistics	MAT 388 – Optimization I
MAT 100 – Foundations	MAT 390 – Introduction to Mathematical Finance
MAT 101 – Calculus I	MAT 420 – Probability and Measure
MAT 102 – Calculus II	MAT 424 – Complex Analysis
MAT 103 – Mathematical Methods I	MAT 440 – Elementary Number Theory
MAT 104 – Mathematical Methods II	MAT 442 – Graph Theory
MAT 110 – Computing	MAT 444 – Basic Category Theory
MAT 140 – Discrete Structures	MAT 484 – Advanced Statistics
MAT 160 – Linear Algebra I	MAT 490 – Discrete Time Finance
MAT 184 – Probability	
MAT 205 – Mathematical Methods III	MAT 499 – Project
MAT 220 – Real Analysis I	MAT 522 – Fourier Analysis
MAT 221 – Real Analysis II	MAT 542 - Cryptography
MAT 230 – Ordinary Differential Equations	MAT 543 – Error Correcting Codes
MAT 240 – Algebra I	MAT 544 – Combinatorial Design Theory
MAT 241 – Algebra II	MAT 584 – Stochastic Processes
MAT 280 – Numerical Analysis I	

Notes

- CCC courses are part of the "Common Core Curriculum" of the SNU undergraduate program. These are half-semester courses with minimal prerequisites, open to any undergraduate student of SNU.
- MAT course codes starting 0 through 5 designate undergraduate courses.
- MAT course codes starting with 5 designate advanced undergraduate courses that can also serve as graduate courses.
- MAT course codes starting 6 and above designate graduate courses (Masters and PhD).
- See the Undergraduate Prospectus for credit requirements and core/elective designations.
- Elective courses are offered based on student and faculty interest.

CCC 101 – Mathematics in India

Open to all undergraduate students.

Credits (Lec:Tut:Lab) = 1.5:0:0 (3 lectures weekly over a half-semester)

Prerequisites: None

Overview: Mathematics had a rich history in ancient and medieval India. Indian mathematicians made original contributions to algebra, number theory and geometry; while the Kerala school made fundamental discoveries related to differential calculus and infinite series two centuries before their full development by Newton and Leibniz. This course will provide an overview of the story of mathematics in India, and also incorporate the social context and the connections with other civilizations.

Detailed Syllabus: Issues of dating, translation and interpretation; prehistory; the ancient civilizations of Egypt, Iraq, China and America; Indus Valley Civilization; Mathematics in the Vedas and Puranas; Pythagoras theorem; Applications to grammar, logic, astronomy and technology; Medieval mathematicians and schools of mathematics; Universities; Invention of Zero; Trigonometry; Rates of change; π ; Connections with Greece, China and the Arabs; The Kerala school.

References:

1. *Mathematics in India* by Kim Plofker, Princeton University Press.
2. *Studies in the History of Indian Mathematics* by C S Seshadri (ed.), Hindustan Book Agency.
3. *Contributions to the History of Indian Mathematics* by Gerard G Emch et al (ed.), Hindustan Book Agency.
4. *History of Mathematics* by Carl B Boyer and Uta C Merzbach, Wiley.
5. *Mathematics and its History* by John Stillwell, Springer.

CCC 801 – Art of Numbers

Open to all undergraduate students.

Credits (Lec:Tut:Lab) = 1.5:0:0 (3 lectures weekly over a half-semester)

Prerequisites: None

Overview: This course deals with two aspects of numbers. In the first part of the course we will take up some unexplored patterns that exist in nature, study them and understand some of their applications. The second part looks at numbers as carriers of information about our lives. Here we learn how to analyze and present data in ways that help us make sense of our lives. We'll use the spreadsheet program in Open Office to analyze the data in depth.

Detailed Syllabus:

Part A: Fun with Numbers

1. Moessner's Magic
2. Permutation, Combinations
3. Pascal Triangle, Binomial Theorem
4. Fibonacci Sequence
5. Some applications

Part B: Handling Data

1. Interacting with real time data
2. Descriptive Statistics like mean, median, mode, range, standard deviation, percentiles, quartiles
3. Introduction to a Spreadsheet program (Open Office or Excel)
4. Charts – Bar Charts, Histograms, Line Charts, Pie Charts
5. Simulations
6. Case Studies

References:

1. *The Book of Numbers* by John Horton Conway, Richard K. Guy. 2nd edition, Copernicus.
2. *The Heart of Mathematics: An Invitation to Effective Thinking* by Edward B. Burger, Michael Starbird. 3rd edition, Wiley.
3. *The Visual Display Of Quantitative Information* by Edward Tufte. 2nd edition, Graphics Press.
4. *Excel 2007 for Starters: The Missing Manual* by Matthew MacDonald. Shroff/O'Reilly.

5. *Analyzing Business Data with Excel* by Gerald Knight. Shroff/O'Reilly.

CCC 805 – Data Analysis and Business Modelling Using Excel

Open to all undergraduate students.

Credits (Lec:Tut:Lab) = 1.5:0:0 (3 lectures weekly over a half-semester)

Prerequisites: None

Overview: If you work for a multinational, a small company, a government organization or an NGO, it is very likely that you will be using a spreadsheet program like Excel to summarize, report and analyze data. You may also be involved in building analytic models to help your employer increase profit, reduce cost, or manage operations more efficiently. Efficient use of tools and methods available in Excel can save you hours of time and improve approaches for analyzing important business problems.

Detailed Syllabus:

1. Excel Basics
2. Developing charts in Excel
3. Some useful functions like sum, average, Index, Match, Vlookup, offset, Indirect, etc.
4. Interest and Amortization using functions like PV, FV, NPV, etc.
5. Data Handling Wizards
6. Data Handling Functions like sumif, countif, Dsum, Dcount, etc.
7. Cash Flow Analysis
8. Sensitivity Analysis
9. Optimization using Solver
10. Linear Regression
11. Pivot Tables

References:

1. *Microsoft Excel 2007: Data analysis and Business Modeling* by Wayne L. Winston, Prentice Hall of India, 2007.
2. *Excel 2007 Formulas* by John Walkenbach, Wiley, 2007.
3. *Favourite Excel 2007 Tips & Tricks* by John Walkenbach, Wiley, 2007.

MAT 020 Elementary Calculus I

Core course for B.Sc. (Research) Biotechnology. Only available as UWE with prior permission of Department of Mathematics.

Credits (Lec:Tut:Lab): (3:1:0)

Overview: This course is mainly targeted at undergraduates who did not take Mathematics at +2 level in school, and now need to quickly acquire basic calculus skills in order to satisfy their major requirements. For example, the purpose may be to enable them to take a Probability course which requires basic concepts from calculus. The course will emphasize geometric meaning rather than formal proof.

Students who need greater rigour as well as more computational skills should take MAT 101 Calculus I.

Detailed Syllabus:

1. Functions: Real line and its subsets, real functions, graphs, polynomials, rational functions, real powers, trigonometric functions, roots, boundedness, monotonicity, composition of functions, inverse functions.
2. Limits and Continuity: Algebra of limits, left and right limits, limits involving infinity, continuity, left and right continuity, types of discontinuity.
3. Differentiation: Rates of change, tangents to graphs, first and higher derivatives, algebra of differentiation, chain rule, exponentials and logarithms.
4. Applications of differentiation: Exponential growth and decay, intervals of increase and decrease, first and second derivative tests, curve sketching.
5. Integration: Definite and indefinite integrals, Fundamental Theorem of Calculus, substitution, integration by parts, trigonometric integrals, improper integrals.
6. First-Order Differential Equations: Separable differential equations, logistic growth.

References:

1. *Short Calculus*, by Serge Lang, Springer.
2. *Essential Calculus – Early Transcendentals*, by James Stewart. Cengage, India Edition.

MAT 084 Basic Probability and Statistics

Core course for B.Sc. (Research) Biotechnology. Only available as UWE with prior permission of Department of Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures +1 tutorial weekly)

Prerequisites: MAT 020 (Elementary Calculus) or MAT 101 (Calculus I)

Overview: Probability is the means by which we model the inherent randomness of natural phenomena. This course provides an introduction to a range of techniques for understanding randomness and variability, and for understanding relationships between quantities. The concluding portions on Statistics take up the problem of testing our theoretical models against actual data, as well as applying the models to data in order to make decisions. This course will act as an introduction to probability and statistics for students from natural sciences, social sciences and humanities.

Detailed Syllabus:

1. **Describing data:** scales of measurement, frequency tables and graphs, grouped data, stem and leaf plots, histograms, frequency polygons and ogives, percentiles and box plots, graphs for two characteristics
2. **Summarizing data:** Measures of the middle: mean, median, mode; Measures of spread: variance, standard deviation, coefficient of variation, percentiles, interquartile range; Chebyshev's inequality, normal data sets, Measures for relationship between two characteristics; Relative risk and Odds ratio
3. **Elements of Probability:** Sample space and events, basic definitions and rules of probability, conditional probability, Bayes' theorem, independent events
4. **Sampling:** Population and samples, reasons for sampling, methods of sampling, standard error, Population parameter and sample statistic
5. **Special random variables and their distributions:** Bernoulli, Binomial, Poisson, Uniform, Normal, Exponential, Gamma, distributions arising from the Normal: Chi-square, t, F
6. **Distributions of Sampling statistics:** Sampling distribution of the mean, The central limit theorem, Determination of sample size, standard deviation versus standard error, the sample variance, sampling distributions from a normal population, sampling from a finite population
7. **Estimation:** Maximum likelihood estimator; Interval estimates; Estimating the confidence interval for population mean, variance and proportions; Confidence intervals for the difference between independent means
8. **Hypothesis testing:** Null and alternate hypothesis; Significance levels; Type I and Type II errors; Tests based on Normal, t, F and Chi-Square distributions for

testing of mean, variance and proportions, Tests for independence of attributes, Goodness of fit; Non-parametric tests: the sign test, the Signed Rank test, Wilcoxon Rank-Sum Test.

9. **Analysis of variance:** Comparing three or more means: One-way analysis of variance, Two-factor analysis of variance, Two-way analysis of variance with interaction
10. **Correlation and Regression:** Correlation, calculating correlation coefficient, coefficient of determination, Spearman's rank correlation; Linear regression, Least square estimation of regression parameters, distribution of the estimators, assumptions and inferences in regression; analysis of residuals: assessing the model; transforming to linearity; weighted least squares; polynomial regression

Main References:

- *Introduction to Probability and Statistics for Engineers and Scientists* by Sheldon Ross, 2nd edition, Harcourt Academic Press.

Other References:

- *Basic and Clinical Biostatistics* by Beth Dawson-Saunders and Robert G. Trapp, 2nd edition, Appleton and Lange.
- *John E. Freund's Mathematical Statistics with Applications* by I. Miller & M. Miller, 7th edition, Pearson, 2011.

MAT 100 – Foundations

Core course for B.Sc. (Research) Mathematics. Not available as UWE.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: None

Overview: Introduction to modern mathematical language and reasoning: Sets and Logic, Proof strategies, Functions, Induction.

Detailed Syllabus:

1. **Sentential Logic:** Deductive reasoning, negation of a sentence, conjunction and disjunction of sentences, equivalence of sentences, truth tables, logical connectives.
2. **Sets:** Operations on sets, Venn diagrams, cartesian product, quantifiers.
3. **Proof Strategies:** Direct proofs, proofs involving negations, conditionals, conjunctions, and disjunctions, existence and uniqueness proofs, proofs involving equivalence.
4. **Relations and Functions:** Ordered pairs, equivalence relations, equivalence classes, partitioning of a set, functions as many-one relations, graphs of functions, one-one functions, onto functions, inverse of a function, images and inverse images of sets.
5. **Mathematical Induction:** Division algorithm, principle of mathematical induction, well ordering principle, strong induction, principle of recursive definition.
6. **More on Sets:** Finite and infinite sets, countable and uncountable sets.

References:

1. *Book of Proof* by Richard Hammack. www.people.vcu.edu/~rhammack/BookOfProof/
2. *Mathematical Thinking* by Keith Devlin, Lightning Source.
3. *How to Prove It* by Daniel J. Velleman, Cambridge University Press.
4. *Mathematical Writing* by Franco Vivaldi, Springer.
5. *Proofs and Fundamentals* by Ethan D. Bloch, Springer.

MAT 101 – Calculus I

Core course for B.Sc. (Research) programs in Mathematics, Physics and Economics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Class XII mathematics or MAT020 (Elementary Calculus I)

Overview: This course covers one variable calculus and applications. It forms the base for subsequent courses in advanced vector calculus and real analysis as well as for applications in probability, differential equations, optimization, etc. One of the themes of the course is to bring more rigour to the formulas and techniques students may have learned in school.

Detailed Syllabus:

1. **Real Number System:** The axioms for **N** and **R**, mathematical induction.
2. **Integration:** Area as a set function, integration of step functions, upper and lower integrals, integrability of bounded monotone functions, basic properties of integration, polynomials, trigonometric functions.
3. **Continuous Functions:** Functions, limits, continuity, Intermediate Value Theorem, Extreme Value Theorem, integrability of continuous functions, Mean Value Theorem for integrals.
4. **Differentiation:** Tangent line, rates of change, derivative as function, algebra of derivatives, implicit differentiation, related rates, linear approximation, differentiation of inverse functions, derivatives of standard functions (polynomials, rational functions, trigonometric and inverse trigonometric functions), absolute and local extrema, First Derivative Test, Rolle's Theorem, Mean Value Theorem, concavity, Second Derivative Test, curve sketching.
5. **Fundamental Theorem of Calculus:** Antiderivatives, Indefinite Integrals, Fundamental Theorem of Calculus, Logarithm and Exponential functions, techniques of integration.
6. **Polynomial Approximations:** Taylor polynomials, remainder formula, indeterminate forms and L'Hopital's rule, limits involving infinity, improper integrals.
7. **Ordinary Differential Equations:** 1st order and separable, logistic growth, 1st order and linear.

References:

1. *Calculus, Volume I*, by Tom M Apostol, Wiley.
2. *Introduction to Calculus and Analysis I* by Richard Courant and Fritz John, Springer
3. *Essential Calculus – Early Transcendentals*, by James Stewart. Cengage, India Edition.
4. *Calculus with Analytic Geometry* by G F Simmons, McGraw-Hill

MAT 102 – Calculus II

Core course for B.Sc. (Research) programs in Mathematics, Physics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 101 (Calculus I)

Overview: The first part is an introduction to multivariable differential calculus. The second part deals with sequences and series and their applications in approximating functions. The concepts and techniques covered here are used extensively in the social and natural sciences as well as in engineering to study systems with many dimensions.

Detailed Syllabus:

1. **Differential calculus in several variables:** Functions of several variables, level curves and surfaces, limits and continuity, partial derivatives, directional derivatives, tangent planes, differentiability, chain rule, gradient, Lagrange multipliers, extreme values and saddle points, 2nd derivative test. (6 weeks)
2. **Sequences and Series of Numbers:** Limits of sequences, algebra of limits, series, divergence test, comparison and limit comparison tests, integral test, alternating series test, absolute convergence, root & ratio tests, Dirichlet and Abel tests. (4 weeks)
3. **Sequences and Series of Functions:** Sequences and series of functions, pointwise convergence, power series, interval of convergence, Abel's theorem, Taylor series, differentiation and integration of power series, applications to Taylor series calculations, trigonometric series, Fourier series, convergence and sum of a Fourier series, half-range expansions, error estimates. (5 weeks)

Main References:

- *Calculus, Volume II*, by Tom M Apostol, Wiley.
- *Essential Calculus – Early Transcendentals* by James Stewart, Cengage, India Edition.
- *Calculus and Analytic Geometry* by G B Thomas and R L Finney, 9th edition, Pearson.
- *Basic Multivariable Calculus* by J E Marsden, A J Tromba and A Weinstein, 1st edition, Springer (India), 2011.
- *Calculus* by Ken Binmore and Joan Davies, 1st edition, Cambridge, 2010.

MAT 132 – Vector Calculus and Geometry

Core course for B.Sc. (Research) programs in Mathematics, Physics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 101 (Calculus I)

Overview:

Detailed Syllabus:

1. **Double and triple integrals:** Double integrals over rectangles, double integrals over general regions, double integrals in polar coordinates, center of mass, triple integrals, triple integrals in cylindrical coordinates, triple integrals in spherical coordinates, change of variables
2. **Vector Integration:** Vector fields, line integrals, fundamental theorem, independence of path, Green's theorem.

MAT 103 – Mathematical Methods I

Core course for all B.Tech. Optional for B.Sc. (Research) Chemistry. Not open as UWE.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Class XII Mathematics.

Overview: In this course we study multi-variable calculus. Concepts of derivatives and integration will be developed for higher dimensional spaces. This course has direct applications in most engineering applications.

Detailed Syllabus:

1. Review of high school calculus.
2. Parametric curves (Vector functions): plotting, tangent, arc-length, polar coordinates, derivatives and integrals.
3. Functions of several variables: level curves and surfaces, differentiation of functions of several variables, gradient, unconstrained and constrained optimization.
4. Double and triple integrals: integrated integrals, polar coordinates, cylindrical and spherical coordinates, change of variables.
5. Vector fields, divergence and curl, Line and surface integrals, Fundamental Theorems of Green, Stokes and Gauss.

References:

1. A Banner, *The Calculus Lifesaver*, Princeton University Press.
2. James Stewart, *Essential Calculus – Early Transcendentals*, Cengage.
3. G B Thomas and R L Finney, *Calculus and Analytic Geometry*, Addison-Wesley.
4. Erwin Kreyszig, *Advanced Engineering Mathematics*, Wiley.

MAT 104 – Mathematical Methods II

Core course for all B.Tech. Programs. Optional for B.Sc. (Research) Chemistry. Not available as UWE.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Class XII Mathematics

Overview: We will study Ordinary Differential Equations which are a powerful tool for solving many science and engineering problems. This course also covers some basic linear algebra which is needed for systems of ODEs.

Detailed Syllabus:

1. First order ODEs: separable, exact, linear
2. Second order ODEs: homogeneous and nonhomogeneous linear, linear with constant coefficients, Wronskian, undetermined coefficients, variation of parameters
3. Laplace transform: definition and inverse, linearity, shift, derivatives, integrals, initial value problems, time shift, Dirac's delta function and partial fractions, convolution, differentiation and integration of transform
4. Matrices: operations, inverse, determinant, eigenvalues and eigenvectors, diagonalization
5. Systems of ODEs: superposition principle, Wronskian, constant coefficient systems, phase plane, critical points, stability

References:

1. James Stewart, *Essential Calculus – Early Transcendentals*, Cengage.
2. Erwin Kreyszig, *Advanced Engineering Mathematics*, Wiley.

MAT 110 – Computing

Core course for B.Sc. (Research) Mathematics. Optional for B.Sc. (Research) Economics. Not available for B.Tech. students. Others may credit it as UWE with permission from Department of Mathematics. Does not count towards Minor in Mathematics.

Credits (Lec:Tut:Lab)= 3:0:1 (Three lecture hours and two lab hours weekly)

Prerequisites: Class XII Mathematics

Overview: This course aims to empower the students in data abstraction, algorithm design and performance estimation. In the process they shall learn the art of programming – a pretty useful skill to have! Programming in C and Matlab will be taught.

Detailed Syllabus:

1. Basic programming constructs: conditional statements, functions, loops, arrays, structures, pointers.
2. Linear data structures: Linked list, queue, stack
3. Trees and Graphs: basic operations
4. Searching and Hashing: Linear search, Binary search, tree search, hash tables
5. Sorting: Insertion sort, bubble sort, merge sort, heap sort
6. Introduction to MATLAB programming.

References:

1. B. Kolman, R. Busby, and S. Ross, *Discrete Mathematical Structures*, PHI, 2012
2. Jeri R. Hanly, Ekliot B. Koffmain, *Problem Solving and Program Design in C*, Pearson, 2009
3. A. Aho, J. Hopcroft, D. Ullman, *Data structures and Algorithms*, Addison-Wesley, 1983
4. A. Aho, and D. Ullman, *Foundations of Computer Science*, Comp. Sci. Press, 1992
5. T. Cormen and C. Leiserson, *Introduction to Algorithms*, MIT Press, 2009
6. N. Kalicharan, *Data Structures in C*, CreateSpace Independent Publishing, 2008
7. A. Tenenbaum, *Data Structures using C*, PHI, 2003

MAT 140 – Discrete Structures

Elective course for B.Sc. (Research) Mathematics. Major elective course for B.Sc. (Research) Mathematics. Not open to B.Tech. Computer Science majors or any other student who has taken CSD205.

Credits: 3:0:1 (3 lectures and 1 tutorial weekly)

Prerequisites: Class XII Mathematics

Overview: This course covers finite processes and the formal structures used to describe them. The first part consists of a detailed exploration of relations and functions. The second part takes up graph theory and abstract algebra.

Detailed Syllabus:

1. **Relations and Digraphs:** Paths in relations and digraphs, Properties of relations, Equivalence relations and equivalence classes, Operations on relations, Connection between relations and some data structures, Transitive Closure and Warshall's algorithm.
2. **Recursion:** Division algorithm, gcd and lcm of two integers, Congruencies, Pigeonhole principle, Recurrence relations.
3. **Functions:** Frequently encountered functions in computer science, Permutation functions.
4. **Partial Orders:** Posets, Extremal elements in a poset, Lattice, Finite Boolean algebras, Functions on Boolean algebras, Karnaugh maps, Logic gates, Digital circuits.
5. **Trees:** Labeled trees, Tree searching, undirected trees, isomorphic trees, Minimal spanning trees, Prim's algorithm.
6. **Graphs:** Euler paths and circuits, Hamiltonian paths and circuits, isomorphic graphs, Transport networks, Matching problems, Colouring graphs.
7. **Introduction to Abstract Algebra:** Binary operations, Semi groups, Groups, Subgroups, Normal subgroups, Cyclic groups, Permutation Groups, Rings and Fields, Finite Fields.

References:

1. Bernard Kolman, Robert Busby, Sharon C. Ross, *Discrete Mathematical Structures*, Pearson Education, New Delhi.
2. Joseph A. Gallian, *Contemporary Abstract Algebra*, Narosa Publishing House, 4th Edition.
3. Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Tata McGraw-Hill, New Delhi.
4. C. L. Liu, D. P. Mohapatra, *Elements of Discrete Mathematics*, Tata McGraw-Hill, New Delhi.

5. J.P. Tremblay and R. Manohar, *Discrete Mathematical Structures with Applications to Computer Science*, 1st edition, Tata McGraw-Hill, New Delhi, 2001.

MAT 160 – Linear Algebra I

Core course for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Class XII Mathematics

Overview: Linear Algebra provides the means for studying several quantities simultaneously. A good understanding of Linear Algebra is essential in almost every area of higher mathematics, and especially in applied mathematics. A CAS such as Maxima/Matlab will be used throughout the course for computational purposes.

Detailed Syllabus:

1. Matrices and Linear Systems
2. Vector Spaces and Linear Transformations
3. Inner Product Spaces
4. Determinant
5. Eigenvalues and Eigenvectors, Diagonalization
6. Quadratic Forms and Positive Definite Matrices
7. Applications chosen from: Numerical aspects, Difference equations, Markov matrices, Least squares.

References:

1. Linear Algebra by Jim Hefferon <http://joshua.smcvt.edu/linearalgebra>
2. *Linear Algebra and its Applications* by Gilbert Strang, 4th edition, Cengage.
3. *Linear Algebra and its Applications* by David C. Lay, 3rd edition, Pearson.
4. *Linear Algebra: A Geometric Approach* by S. Kumaresan, PHI, 2011.
5. *Elementary Linear Algebra* by Howard Anton and Chris Rorres, 9th edition, Wiley.
6. *Linear Algebra: An Introductory Approach* by Charles Curtis, Springer.
7. *Matrix Analysis and Applied Linear Algebra* by Carl D Meyer, SIAM.
8. Videos of lectures by Prof Gilbert Strang: *18.06 Linear Algebra, Spring 2010*. (Massachusetts Institute of Technology: MIT OpenCourseWare), <http://ocw.mit.edu>

MAT 184 – Probability

Core course for B.Sc. (Research) Mathematics, Economics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Any one of Calculus I (MAT 101) or Elementary Calculus (MAT 020) or Mathematical Methods I (MAT 103) or Basic Probability & Statistics (MAT 084)

Overview: Probability is the means by which we model the inherent randomness of natural phenomena. This course introduces you to a range of techniques for understanding randomness and variability, and for understanding relationships between quantities. The concluding portions on Statistics take up the problem of testing our theoretical models against actual data, as well as applying the models to data in order to make decisions. This course is a prerequisite for later courses in Advanced Statistics, Stochastic Processes and Mathematical Finance, as well as for the Minor in Data Analytics.

Detailed Syllabus:

1. **Probability:** Classical probability, axiomatic approach, conditional probability, independent events, addition and multiplication theorems with applications, Bayes' theorem.
2. **Random Variables:** Probability mass function, probability density function, cumulative density function, expectation, variance, standard deviation, mode, median, moment generating function.
3. **Some Distributions and their Applications:** Uniform (discrete and continuous), Bernoulli, Binomial, Poisson, Exponential, Normal.
4. **Joint Distributions:** Joint and marginal distributions, independent random variables, IIDs, conditional distributions, covariance, correlation, moment generating function.
5. **Sequences of Random Variables:** Markov's Inequality, Chebyshev's Inequality, Law of Large Numbers, Central Limit Theorem.

References:

1. *A First Course in Probability* by Sheldon Ross, 6th edition, Pearson.
2. *Introduction to Probability and Statistics for Engineers and Scientists* by Sheldon Ross, 2nd edition, Harcourt Academic Press.
3. *Theory and Problems of Probability and Statistics* by Murray R Spiegel and Ray Meddis, Schaum's Outlines.
4. *John E. Freund's Mathematical Statistics with Applications* by I. Miller & M. Miller, 7th edition, Pearson, 2011.
5. *Elementary Probability Theory: With Stochastic Processes and an Introduction to Mathematical Finance* by Kai Lai Chung and Farid Aitsahlia, 4th edition, Springer

International Edition, 2004.

MAT 205 – Mathematical Methods III

Core course for B.Tech. except Computer Science. Not available as UWE.

Credits (Lec:Tut:Lab)= 3:0:0 (3 lectures weekly)

Prerequisites: MAT 103 (Mathematical Methods I)

Overview: Probability is the means by which we model the inherent randomness of natural phenomena. This course introduces you to a range of techniques for understanding randomness and variability, and for understanding relationships between quantities. The concluding portions on Statistics take up the problem of testing our theoretical models against actual data, as well as applying the models to data in order to make decisions.

Detailed Syllabus:

1. **Probability:** sample space and events, classical and axiomatic probability, permutations and combinations, conditional probability, independence, Bayes' formula
2. **Random Variables:** discrete and continuous probability distributions, mean and variance, binomial and Poisson, normal, joint distributions, covariance, correlation and regression (linear)
3. **Mathematical Statistics:** exploring data, random samples, point estimation, Central limit theorem, Maximum likelihood, chi-square, t and F-distributions, confidence intervals, hypothesis testing

References:

1. *Advanced Engineering Mathematics* by Erwin Kreyszig, Wiley.
2. *Introduction to Probability and Statistics for Engineers and Scientists* by Sheldon Ross, 2nd edition, Harcourt Academic Press.
3. *Theory and Problems of Beginning Statistics* by L. J. Stephens, Schaum's Outline Series, McGraw-Hill
4. *John E. Freund's Mathematical Statistics with Applications* by I. Miller & M. Miller, 7th edition, Pearson, 2011.

MAT 220 – Real Analysis I

Core course for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Calculus I (MAT 101)

Overview: This course provides a rigorous base for the geometric facts and relations that we take for granted in one-variable Calculus. The main ingredients include sequences; series; continuous and differentiable functions on \mathbf{R} ; their various properties and some highly applicable theorems. This is the foundational course for further study of topics in pure or applied Analysis, such as Metric Spaces, Complex Analysis, Numerical Analysis, and Differential Equations.

Detailed Syllabus:

1. **Fundamentals:** Review of \mathbf{N} , \mathbf{Z} and \mathbf{Q} , order, sup and inf, \mathbf{R} as a complete ordered field, Archimedean property and consequences, intervals and decimals. Functions: Images and pre-images, Cartesian product, Cardinality.
2. **Sequences:** Convergence, bounded, monotone and Cauchy sequences, subsequences, \limsup and \liminf .
3. **Series:** Infinite Series: Cauchy convergence criterion, Infinite Series of non-negative terms, comparison and limit comparison, integral test, p-series, root and ratio test, power series, alternating series, absolute and conditional convergence, rearrangement.
4. **Continuity:** Limits of functions, continuous functions, Extreme Value Theorem, Intermediate Value Theorem, monotonic functions, uniform continuity.
5. **Differentiation:** Differentiable functions on \mathbf{R} , local maxima, local minima, Mean Value Theorems, L'Hopital's Rule, Taylor's Theorem.

References:

1. *A Basic Course in Real Analysis* by Ajit Kumar and S Kumaresan. CRC Press, 2014.
2. *Introduction to Real Analysis* by R G Bartle & D R Sherbert, John Wiley & Sons, Singapore, 2/e (or later editions), 1994.
3. *Elementary Analysis: The Theory of Calculus* by Kenneth A Ross. Springer India, 2004.
4. *Analysis I* by Terence Tao. Hindustan Book Agency. 2nd Edition, 2009.
5. *Principles of Mathematical Analysis* by Walter Rudin. McGraw-Hill. 3rd Edition, 2006.

6. *Mathematical Analysis* by Tom Apostol, Narosa, New Delhi, 2/e, 2002.
7. *Calculus, Volume 1*, by Tom Apostol, Wiley India. 2nd Edition, 2011.

Department of Mathematics
School of Natural Sciences

MAT 221 – Real Analysis II

Core course for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab): 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Real Analysis I (MAT 220)

Overview: Continuing the work done in MAT 220 of understanding the rigor behind one-variable Differential Calculus, this course dwells on various aspects of Integration as well as functions on higher dimensional spaces. We discuss sequences and series of functions; uniform convergence and consequences; some important approximation theorems for continuous functions; rigorous discussions of some special functions; and finally the world of functions of several variables.

Detailed Syllabus:

1. **Integration:** Upper and lower Riemann integrals, basic properties of Riemann integral, Riemann integrability of continuous and monotone functions, non-Riemann integrable functions, Fundamental Theorem of Calculus and consequences.
2. **Sequences and Series of Functions:** Pointwise and uniform convergence, uniform convergence and continuity, series of functions, Weierstrass M-test, uniform convergence and integration, uniform convergence and differentiation, equicontinuous families of functions, Stone-Weierstrass Theorem.
3. **Some Special Functions:** Power Series, the exponential, logarithmic and trigonometric functions.
4. **Topology of \mathbb{R}^n :** Open and closed sets, continuous functions, completeness, compactness, connectedness.
5. **Functions of Several Variables:** Derivatives, partial and directional derivatives, Chain Rule, Inverse Function Theorem.

References:

1. *Analysis II* by Terence Tao. Hindustan Book Agency. 2nd Edition, 2009.
2. *Principles of Mathematical Analysis* by Walter Rudin. McGraw-Hill. 3rd Edition, 2006.
3. *Real Mathematical Analysis* by Charles C Pugh. Springer India. 2004.
4. *Mathematical Analysis* by Tom Apostol, Narosa, New Delhi, 2/e, 2002.

5. *Calculus, Volume 2*, by Tom Apostol, Wiley India. 2nd Edition, 2011.

MAT 230 – Ordinary Differential Equations

Core course for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 101 Calculus I or MAT 103 Mathematical Methods II

Overview: Ordinary Differential Equations are fundamental to many areas of science. In this course we learn how to solve large classes of them, how to establish that solutions exist in others, and to find numerical approximations when exact solutions can't be achieved. Further, many phenomena which undergo changes with respect to time or space can be studied using differential equations. In this course we will also see many examples of mathematical modeling using differential equations.

Detailed Syllabus:

1. First Order ODEs: Modelling, Geometrical Meaning, Solution techniques
2. Second and Higher Order Linear ODEs: Modelling, Geometrical Meaning, Solution techniques
3. Numerical Techniques
4. Existence of Solutions of Differential Equations
5. Systems of ODEs: Phase Plane and Qualitative Methods
6. Laplace Transforms
7. Series Solutions

References:

1. Erwin Kreyszig, *Advanced Engineering Mathematics*, 9th edition, Wiley India, 2012.
2. G.F. Simmons and S. Krantz, *Differential Equations: Theory, Technique, and Practice*, McGraw Hill Publishing Company, 2006.
3. J. Polking, D. Arnold, A. Boggess, *Differential Equations*, Pearson, 2005.
4. C. Henry Edwards and David E. Penney, *Differential Equations and Boundary Value Problems: Computing And Modeling*, 3rd edition, Pearson, 2010.
5. Hirsch, Morris W., Stephen Smale, and Robert L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*. Academic Press, 2012.

MAT 240 – Algebra I

Core course for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Class XII Mathematics or MAT100 (Foundations)

Overview: Algebraic structures like groups, rings, integral domains, fields, modules and vector spaces are present in almost all mathematical applications as well as in development of more complicated structures in mathematics. The basic building block of these structures is the group. This, a first course in Abstract Algebra, concentrates mainly on groups and their basic properties. If there is time, we shall also take a brief look at Rings.

It is desirable that the student already have a basic understanding of sets, relations, functions, binary operations, equivalence relations, and sets of numbers.

Detailed Syllabus:

1. Groups: Definition, Examples and Elementary Properties.
Subgroups: Subgroup Tests, Subgroups Generated by Sets, Cyclic Groups, Classification of Subgroups of Cyclic Groups, Cosets and Lagrange's Theorem.
2. Normal Subgroups and Quotient Groups, Homomorphisms, Isomorphisms and Automorphisms of a Group.
Conjugates, centre, centralizer, normalizer.
Cayley's Theorem.
Direct Products, Finite Abelian Groups.
3. Permutation Groups: Definition, Examples and Properties, Symmetric Group of n Letters (S_n), Alternating Group on n Letters (A_n).
4. (If time permits) Rings, Homomorphisms, Ideals and Quotient Rings, Integral Domains.

References:

1. *Contemporary Abstract Algebra* by Joseph A. Gallian, 4th edition. Narosa, 1999.
2. *Algebra* by Michael Artin, 2nd Edition. Prentice Hall India, 2011.
3. *Topics in Algebra* by I.N. Herstein, 2nd Edition. Wiley India, 2006.
4. *A First Course in Abstract Algebra* by John B. Fraleigh, 7th Edition. Pearson, 2003.
5. *Undergraduate Algebra* by Serge Lang, 2nd Edition. Springer India, 2009.
6. *Abstract Algebra* by David S. Dummit and Richard M. Foote, 3rd Edition. John Wiley and Sons, 2011.

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MAT 241 – Algebra II

Core course for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab): 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Algebra I (MAT 240)

Overview: The course continues the work done in MAT 240 on the one hand by extending the study of groups to include group actions and applications, and on the other by studying the algebraic structures of rings and fields.

Detailed Syllabus:

1. **Groups** – Definition, subgroups, cyclic groups, homomorphisms, normal subgroups, semi-direct products, group actions, Sylow theorems.
2. **Rings** – Definition and examples of rings, ideals, quotient rings, maximal ideals, prime ideals, ring homomorphisms, integral domains, Euclidean domains, PID, UFD.
3. **Polynomial Rings and Fields** – Polynomial rings, irreducible polynomials, definition and examples of fields, characteristic, field extensions, finite fields.

References:

1. I. N. Herstein, *Topics in Algebra*, 2/e, Wiley Eastern, 1994.
2. Bhattacharya, Jain and Nagpaul, *Basic Abstract Algebra*, 2nd edition, CUP, 1995.
3. Joseph A. Gallian, *Contemporary Abstract Algebra*, 4th edition, Narosa, 1999.
4. M. Artin, *Algebra*, 2nd edition. Prentice Hall India, 2011.
5. Dummit and Foote, *Abstract Algebra*, 3rd edition, Wiley.
6. Serge Lang, *Undergraduate Algebra*, 2nd edition. Springer India, 2009.
7. Thomas W. Hungerford, *Algebra*, GTM 73, Springer India, 2004.

MAT 280 – Numerical Analysis I

Core course for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab) = 3:0:1 (3 lectures and 1 two-hour lab weekly)

Prerequisites: Class XII Mathematics

Overview: Numerical Analysis takes up the problems of practical computation that arise in various areas of mathematics, physics and engineering. The focus is on analyzing the numerical methods and algorithms for obtaining approximate solutions, error estimates and rate of convergence, and implementation by computer programs.

Detailed Syllabus:

1. **Solving Equations:** Iterative methods, Bisection method, Secant method, Newton-Raphson method, Rates of convergence, Roots of polynomials.
2. **Interpolation:** Lagrange and Hermite interpolation, Interpolating polynomials using difference operators.
3. **Numerical Differentiation:** Methods based on interpolation, methods based on finite difference operators.
4. **Numerical Integration:** Newton-Cotes formula, Gauss quadrature, Chebyshev's formula.
5. **Systems of Linear Equations:** Direct methods (Gauss elimination, Gauss-Jordan method, LU decomposition, Cholesky decomposition), Iterative methods (Jacobi, Seidel, and Relaxation methods)
6. **Labs:** Computational work using C, Python or Matlab.

References:

1. E. Suli and D. Mayers, *Introduction to Numerical Analysis*, Cambridge University Press, 2003.
2. R.L. Burden and J.D. Faires, *Numerical Analysis*, Cengage Learning, 9th Edition, 2010.
3. M.K. Jain, S.R.K. Iyengar, and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New Age International Ltd., 1999.
4. J.H. Mathews and K. Fink, *Numerical Methods using Matlab*, PHI Learning, 4th Edition, 2003.

MAT 330 – Partial Differential Equations

Core course for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 102 Calculus II or MAT 103 Mathematical Methods I. MAT 230 Ordinary Differential Equations or MAT 104 Mathematical Methods II.

Overview: Many physics principles like conservation of mass, momentum, energy, when applied to real life scenarios, take the form of PDEs. In this course we will learn how basic physics concepts together with simple calculus translate into mathematical models of many engineering problems in the form of PDEs. We will learn some well-known techniques to solve these problems in simple settings. We will also learn approximation techniques which will be needed in cases where it is impossible to get analytical solutions.

Detailed Syllabus:

We will aim to cover Chapters 1, 2, 4, 5, 6, and 8 of the book by Strauss. This material will be supplemented with exercises from other prescribed texts and Matlab exercises. The list of topics covered is:

1. Definition of PDEs, well posedness, initial value and boundary value problems
2. Examples of PDEs, classification of PDEs
3. Wave equation, diffusion equation
4. Source terms
5. Boundary conditions and their impact on solution
6. Fourier Series and their use in solving PDEs
7. Harmonic equations and their solution
8. Numerical methods

References:

- *Partial Differential Equations, an Introduction* , Second Edition, by Walter A. Strauss
- *Applied Partial Differential Equations* by Paul DuChateau, David Zachmann
- *Partial Differential Equations for Scientists and Engineers*, by Stanley J. Farlow

MAT 332 – Geometry of Curves and Surfaces

A Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 102 Calculus II or MAT 103 Mathematical Methods I. And MAT 160.

Overview: This course combines the traditional approach to learn the basic concepts of curves and surfaces with the symbolic manipulative abilities of Mathematica. Students will learn and study the classical curves/surfaces as well as more interesting curves/surfaces using computer methods. For example, to see the effect of change of parameter, the student will explore and observe with the help of Mathematica and then the mathematical proof of the observation will be developed in the class.

Detailed Syllabus:

- 1- Curves in the plane: Length of a curve, Vector fields along curves. Famous plane curves: cycloids, lemniscates of Bernoulli, cardioids, catenary, cissoid of Diocles, tractrix, clothoids, pursuit curves.
- 2- Regular curve, curvature of a curve in a plane, curvature and torsion of a curve in \mathbb{R}^3 . Determining a plane curve from given curvature.
- 3- Global properties of plane curves: Four vertex theorem, Isoperimetric inequality.
- 4- Curves on Sphere. Loxodromes on spheres, animation of curves on a sphere.
- 5- Review of calculus in Euclidean space.
- 6- Surfaces in Euclidean spaces: Patches in \mathbb{R}^3 , local Gauss map, Regular surface, Tangent vectors.
- 7- Example of surfaces: Graphs of a function of two variables, ellipsoid, stereographic ellipsoid, tori, paraboloid, seashells.
- 8- Orientable and Non-orientable surfaces. Mobius strip, Klein Bottle.
- 9- The shape operator, normal curvature, Gaussian and mean curvature, fundamental forms.
- 10- Surfaces of revolution

References:

1. *Modern Differential Geometry of Curves and Surfaces with Mathematica, Third Edition* by Elsa Abbena, Simon Salamon, Alfred Gray.
2. *Elementary Differential Geometry* by A.N. Pressley, Springer Undergraduate Mathematics Series.
3. *Differential Geometry of Curves and Surfaces* by Manfredo DoCarmo.

MAT 360 – Linear Algebra II

Core course for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab): 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Linear Algebra I (MAT 160)

Overview: In MAT 260 we studied real and complex linear transformations up to the diagonalizability of symmetric operators. In this course we take up vector spaces over arbitrary fields and more advanced results on expressing linear transformations by simple matrices.

Detailed Syllabus:

1. **Linear Equations** – Systems of linear equations, matrices, elementary row operations and row reduction.
2. **Vector spaces** – Abstract vector spaces, subspaces, dimension, coordinates.
3. **Linear transformations** – Matrix representations, change of basis, linear functionals and the double dual, transpose.
4. **Determinants** – Commutative rings, determinant function, permutations, properties.
5. **Canonical Forms** – Characteristic values, invariant subspaces, simultaneous diagonalization and triangulation, invariant direct sums, Primary Decomposition Theorem, cyclic subspaces, Rational Form, Jordan Form.
6. **Inner Product Spaces** – Linear functionals and adjoints, unitary and normal operators, spectral theory.

References:

1. Kenneth Hoffman and Ray Kunze, *Linear Algebra*, 2nd edition, PHI Learning.
2. Friedberg, Insel and Spence, *Linear Algebra*, 4th edition, PHI Learning
3. Sheldon Axler, *Linear Algebra Done Right*, 2nd edition, Springer International Edition
4. Paul Halmos, *Finite Dimensional Vector Spaces*, 2nd edition, Springer International Edition
5. Paul Halmos, *Linear Algebra Problem Book*, Mathematical Association of America, 1995.

MAT 386 – Dynamical Systems

Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab): 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 102 Calculus II or MAT 103 Mathematical Methods I. And MAT 104 Mathematical Methods II or MAT 160 Linear Algebra I.

Overview:

Detailed Syllabus:

Basic Concepts: Discrete and continuous dynamical systems. Linear and nonlinear systems and principle of superposition. Linear and nonlinear forces. Concepts of evolution, iterations, orbits, fixed points, periodic and aperiodic (chaotic) orbits. Basics of Linear Algebra: Symmetric & Skew-symmetric matrices, matrix norm and singular value decomposition. Eigenvalues, left and right eigenvectors, and dynamical interpretation. Canonical forms; simple and non-simple canonical systems. System of Equations.

Stability Analysis:

Stability of a fixed point and classification equilibrium states (for both discrete and continuous systems). Concept of bifurcation and classification of bifurcations. Concepts of Lyapunov stability & Asymptotic stability of orbits. Phase Portraits of various Linear and Nonlinear systems. Hopf bifurcation. Concept of attractors and repellers, limit cycles and torus.

Phenomena of Bifurcation:

Definition of bifurcation. Bifurcations in one, two and higher dimensional systems. Hopf, Period doubling, Saddle node, Transcritical bifurcations. Feigenbaum's number. Local and Global bifurcations. Homoclinic & Hetero-clinic points and orbits. Poincaré-Bendixson Theorem. Conservative and Dissipative Systems.

Investigation Tools & Chaos Theory:

Time Series and Phase Plane Analysis, Poincaré Map & Section. Lyapunov Characteristic Exponents. Hamiltonian Systems and concept of integrability and non-integrability. Concept of **Chaos** and **Chaotic evolution** of a Dynamical System. Measure of Chaos. Routs to Chaos.

Applications:

Applications of Dynamical Systems, (to Physics, Biology, Economics with Examples).

Mathematical Models. Population Dynamics. Investigation of Evolutionary Phenomena in Logistic Map, Lotka-Volterra System, Duffing Oscillator, Oscillation of Nonlinear Pendulum, Predator-Prey Systems etc.

Tutorial:

Drawing orbits of a system for given initial values. Clear Demonstration of Linear and Nonlinear Systems. Calculation of fixed points for given system and examine their stabilities (discrete and continuous). Drawing time series graphs, phase portraits for regular and chaotic systems. Cobweb Plots. Calculations of Eigenvalues and Eigenvectors corresponding to any fixed point. Plotting Bifurcation diagrams of 1 and 2 dimensional systems. Calculations of Lyapunov exponent.

Software such as MATHEMATICA / MATLAB will be used as needed.

References:

1. *Nonlinear Systems*, by P. G. Drazin, Cambridge University Press India.
2. *An Introduction to Chaotic Dynamical Systems* by R. L. Devaney, Addison Wesley, 1989.
3. *Chaos in Dynamical Systems*, by Edward Ott, Cambridge University Press, 2002
4. *Chaotic Dynamics – An Introduction*, by G. L. Baker and J. P. Gollub, Cambridge University Press, 1996.
5. *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields*, by J. Guckenheimer and P. Holmes, Springer, 1983.

MAT 388 – Optimization I

A Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab): 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 160 (Linear Algebra) or MAT 104 (Mathematical Methods II)

Overview: Optimization deals with the problem of establishing the best & worst cases for a given situation. This course deals mostly with the special case of linear programming, which is commonly applied to problems of business and economics as well as industrial problems in transportation, energy and telecommunication.

Detailed Syllabus:

1. Mathematical modeling and optimization problem formulation
2. Application of optimization (linear case)
3. Geometry of linear optimization
4. Simplex method
5. Duality theory
6. Sensitivity analysis
7. Robust optimization
8. Graphs and Network flow problems
9. Discrete optimization or Integer programming formulations
10. Non-linear optimization – introduction and applications

Main References:

1. *Linear Programming* by G. Hadley, Narosa, 2000
2. *Understanding and Using Linear Programming* by J. Matousek and B. Gärtner, Springer, 2006
3. *Introduction to Linear Optimization* by D. Bertsimas and J. Tsitsiklis, Athena Scientific, 1997
4. *Theory of Linear and Integer Programming* by A. Schrijver, Wiley, 1998
5. *Operations Research: An Introduction* by H. Taha, Pearson, 2012

MAT 390 – Introduction to Mathematical Finance

A Major Elective for B.Sc. (Research) Mathematics. Cross-listed as **FAC201**.

Credits (Lec:Tut:Lab): 3:0:1 (3 lectures and 1 two-hour lab weekly)

Prerequisites: MAT 184 Probability or MAT 205 Mathematical Methods III or CSD209.

Overview: Mathematical Finance is a modern study area where mathematical methods are used to create and add immense value in a practical environment. The aim of this course is twofold. First, to discuss the mathematical models that have driven the explosion of financial services and products over the last 30 years or so. Second, to use spreadsheet programs to work with actual data. This course is also the gateway to our Specialization in Mathematical Finance.

Detailed Syllabus:

1. **Basic concepts:** Bonds and shares, risk versus profit, return and interest, time value of money, arbitrage.
2. **Fixed Income Securities:** Net Present Value and Internal Rate of Return, price and yield of a bond, term structures, duration, immunization.
3. **Mean-Variance Analysis:** Random returns, efficient portfolios, feasible set, Markowitz model, Two Fund and One Fund Theorems, Capital Asset Pricing Model and applications.
4. **Forwards, Futures and Swaps:** Replicating portfolios, futures on assets without income, futures on assets with fixed income or dividend yield, hedging with futures, currency futures, stock index futures, forward rate agreements, interest rate swaps, currency swaps, equity swaps.
5. **Stock Price Models:** Geometric Brownian Motion, Binomial Tree.
6. **Options:** Call and put options, put-call parity, Binomial Options Pricing Model, dynamic hedging, risk-neutral valuation, Black-Scholes formula, trading strategies.
7. **Labs:** Microsoft Excel and VBA.

References:

1. *Principles of Finance with Excel 2nd edition* by Simon Benninga, Oxford University Press, 2010.
2. *Mathematics for Finance* by M Capinski and T Zastawniak, Springer (International Edition), 2003.
3. *The Calculus of Finance* by Amber Habib, Universities Press, 2011.
4. *Options, Futures and Other Derivatives 7th edition* by John C Hull and Sankarshan Basu, Pearson 2009.

5. *Investment Science* by David Luenberger, Oxford University Press (Indian Edition), 1997.
6. *An Elementary Introduction to Mathematical Finance 2nd edition* by Sheldon Ross, Cambridge University Press (Indian Edition), 2005.

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MAT 420 –Measure and Probability

A Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 220 Real Analysis I. And one of MAT 184 Probability or MAT 205 Mathematical Methods III or CSD209.

Overview: This sequel to an introductory course on probability provides a rigorous look at the subject which opens up many more applications, especially to stochastic processes. This course is compulsory for students opting for the Specialization in Mathematical Finance.

Detailed Syllabus:

1. Introduction to measures and probability triples
2. Random variables: independence, limit events, expectation
3. Inequalities and convergence, Laws of large numbers
4. Distributions, change of variables
5. Limit Theorems, Differentiation of expectation, Moment generating functions, Fubini's theorem
6. Weak convergence
7. Characteristic functions, Central Limit Theorem and generalizations, Method of Moments
8. Lebesgue and Hahn decompositions
9. Conditional probability and expectation

References:

1. *A First Look at Rigorous Probability Theory* by J S Rosenthal, 2nd edition, World Scientific Publishing, 2006.
2. *Measure, Integral and Probability* by M Capinski and E Kopp, 2nd edition, Springer.
3. *Probability and Random Processes* by G R Grimmett and D R Stirzaker, 2nd edition, Oxford University Press.

MAT 424 – Complex Analysis

A Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 221 Real Analysis II

Overview: This course covers the basic principles of differentiation and integration with complex numbers. Topics will be taught in a computational and geometric way. Knowledge of topology of euclidean space and calculus of several real variables will be assumed.

Detailed Syllabus:

1. Algebraic properties of complex numbers, modulus, complex conjugate, roots of complex numbers, regions.
2. Functions of a complex variable, limits, continuity.
3. Differentiation, Cauchy-Riemann equations, harmonic functions, polar coordinates.
4. Exponential function, logarithm, branch and derivative of logarithm, complex exponents, trigonometric functions, hyperbolic functions, inverse hyperbolic functions.
5. Derivatives of curve $w(t)$ in complex plane, Definite integral of functions $w(t)$, Contours, Contour Integrals, Antiderivatives, Modulus of Contour integrals, Cauchy Goursat theorem.
6. Simply and multiply connected domain, Cauchy Integral Formula and applications, Liouville's theorem, maximum modulus principle.
7. Convergence of series, Power Series, Laurent series, Residues, Cauchy's Residue theorem, Singularities, Zeroes of analytic functions, Behaviour of function near singularities.

References:

1. James W Brown and Ruel V Churchill, *Complex Variables and Applications*, 8th edition, Tata McGraw-Hill, 2009.
2. H A Priestley, *Introduction to Complex Analysis*, 2nd edition, Oxford University Press. 2003.
3. J Bak and D J Newman, *Complex Analysis*, 2nd edition, Springer, 2008.
4. M J Ablowitz and A S Fokas, *Complex Variables: Introduction and Applications*, 2nd edition, Cambridge University Press India, 2006.

MAT 440 – Elementary Number Theory

A Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: Class XII Mathematics

Overview: This introductory course to Number Theory is also the entry point to the specialization in applications of algebra.

Detailed Syllabus:

1. **Divisibility:** Definition and properties of Divisibility, Division Algorithm, Greatest Common Divisor, Least Common Multiple, Euclidean Algorithm, Linear Diophantine Equations.
2. **Primes and their Distribution:** Sieve of Eratosthenes, Euclid's Theorem, Prime Number Theorem (statement only), Goldbach Conjecture, Twin Primes, Fermat Primes, Mersenne Primes, The Fundamental Theorem of Arithmetic, Euclid's Lemma, Divisibility, gcd and lcm in terms of prime factorizations, Dirichlet's Theorem on primes in arithmetic progressions (statement only).
3. **Theory of Congruences:** Residue Classes, Linear congruences in one variable, Euclid's algorithm Chinese Remainder Theorem, Wilson's Theorem, Fermat's Theorem, Pseudoprimes and Carmichael Numbers, Euler's Theorem, Primality Testing, The Pollard Rho Factoring Method, Complete residue system.
4. **Applications of Congruences:** Divisibility tests, Modular Designs, Check Digits, The p-Queens Puzzle, Round-Robin Tournaments, The Perpetual Calendar.
5. **Arithmetic Functions:** Multiplicative Functions, Moebius function, Moebius inversion formula, The number-of-divisors and sum-of-divisors functions, Euler phi function, Greatest Integer Function, Carmichael conjecture, Perfect numbers, characterization of even perfect numbers, Dirichlet product, Riemann Zeta function.
6. **Group of Units and Quadratic Residues:** Primitive roots, Group of units, Quadratic Residues and Non-Residues, Legendre symbol, Euler's Criterion, Gauss' Lemma, Law of Quadratic Reciprocity.
7. **Sums of Squares:** Sums of Two squares, Sums of Three squares and Sums of Four squares.

References:

- David M. Burton *Elementary Number Theory*, Tata McGraw-Hill.
- Gareth A. Jones and J. Mary Jones *Elementary Number Theory*, Springer Undergraduate Mathematics Series.
- Thomas Koshy *Elementary Number Theory with Applications*, 2nd Edition, Academic Press.
- Kenneth Rosen *Elementary Number Theory and its Applications*, 5th Edition, McGraw Hill.

- G. H. Hardy and E. M. Wright *An Introduction to the Theory of Numbers*, 5th edition, Oxford University Press.

MAT 442 – Graph Theory

A Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 160 Linear Algebra I

Overview: Graphs are fundamental objects in combinatorics. The results in graph theory, in addition to their theoretical value, are increasingly being applied to understand and analyze systems across a broad domain of enquiry, including natural sciences, social sciences and engineering. The course does not require any background of the learner in graph theory. The emphasis will be on the axiomatic foundations and formal definitions, together with the proofs of some of the central theorems. Few applications of these results to other disciplines would be discussed.

Detailed Syllabus:

Unit 1

- Definitions of Graph, Digraph, Finite and Infinite Graph, Degree of a Vertex, Degree Sequence, Walk, Path, Cycle, Clique.
- Operations on graphs, Complement of a graph, Subgraph, Connectedness, Components, Isomorphism.
- Regular graph, Complete graph, Bipartite graph, Cyclic graph, Euler graph, Hamiltonian path and circuit, Tree, Cut set, Spanning tree.

Unit 2

- Planar graph, Colouring, Covering, Matching, Factorization, Independent sets.

Unit 3

- Graphs and relations, Adjacency matrix, Incidence matrix, Laplacian matrix, Spectral properties of graphs, Matrix tree theorem, Automorphism group of a graph.

Unit 4

- DFS, BFS for minimal spanning tree, Kruskal, Prim and Dijkstra algorithms.

References:

- D. West, *Introduction to Graph Theory*, 2nd ed., PHI Learning, New Delhi, 2009.
- N. Deo, *Graph Theory: With Application to Engineering and Computer Science*, PHI Learning, New Delhi, 2012.
- C. D. Godsil and G. Royle, *Algebraic Graph Theory*, Springer, New Delhi, 2013.
- B. Kolman, R.C. Busby, S.C. Ross, *Discrete Mathematical Structures*, 6th ed., PHI Learning, New Delhi, 2012.
- F. Harary, *Graph Theory*, Narosa, New Delhi, 2012.
- J.A. Bondy and U.S.R. Murty, *Graph Theory*, Springer, New Delhi, 2013.

- R.J. Wilson, *Introduction to Graph Theory*, 4th ed., Pearson Education, New Delhi, 2003.

MAT 444 – Basic Category Theory

A Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 160 Linear Algebra I and MAT 240 Algebra I

Overview: Category theory is a branch of mathematics which studies and isolates the fundamental structures, underlying constructions and techniques appearing across different areas of mathematics, physics and computer science. The goal of this course is to exhibit the power of category theory as a language for understanding and formalizing common concepts occurring in various branches of mathematics and computers.

Detailed Syllabus:

Unit 1: The category of sets

Sets and functions, Commutative diagrams, Products and coproducts, Finite limits in **Set**, Finite colimits in **Set**, other notions in **Set**.

Unit 2: Categories and functors, without admitting it

Monoids, Groups, Graphs, Orders, Databases: schemas and instances.

Unit 3: Basic Category theory

Categories and functors, natural transformations, Categories and schemas are equivalent (**Cat** \sim **Sch**), Yoneda's lemma, Limits and colimits.

Unit 4: Categories at work

Adjoint functors, Categories of functors, Monads.

References:

1. Steven Awodey, *Category Theory*. Oxford University Press, 2006.
2. Lawvere & Schanuel, *Conceptual Mathematics: A First Introduction to Categories*. Cambridge University Press.
3. Michael Barr and Charles Wells, *Category Theory for Computing Science*, Centre de recherches mathématiques CRM, 1999.
4. Benjamin Pierce, *Basic Category Theory for Computer Scientists*. MIT Press Cambridge
5. Saunders Mac Lane, *Categories for the Working Mathematician*. (the standard reference)
6. David I Spivak, *Category Theory for Scientists*. MIT Press; 1st edition

<http://ocw.mit.edu/courses/mathematics/18-s996-category-theory-for-scientists-spring-2013/textbook/>

MAT 484 – Advanced Statistics

A Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 184 Probability

Overview: Regression, the most widely used statistical technique, estimates relationships between independent (explanatory) variables and a dependent (outcome) variable. In this course you will learn different ways of estimating the parameter of the statistical models, criteria for these estimations, and then use them for deriving the coefficients of the regression models, use software (**R**) to implement them, learn what assumptions underlie the models, learn how to test whether your data meet those assumptions and what can be done when those assumptions are not met, and develop strategies for building and understanding useful models.

Detailed Syllabus:

1. **Review:** Introduction, Descriptive Statistics; Sampling Distributions. Graphical representation of data, Basic distributions, properties, fitting, and their uses;
2. **Estimation:** Point and interval estimation, Histogram and Kernel density estimation, Sufficiency, Exponential family, Bayesian methods, Moment methods, Least squares, Maximum likelihood estimation;
3. **Criteria for estimation:** UMVUE, Large sample theory Consistency; asymptotic normality, Confidence intervals, Elements of hypothesis testing; Neyman-Pearson Theory, UMP tests, Likelihood ratio and related tests, Large sample tests;
4. **Linear Models:** Simple and Multiple linear regression, Analysis and Inference.

References:

1. *Mathematical Statistics: Basic Ideas and Selected Topics* by Peter J. Bickel and Kjell A. Doksum
2. *Testing Statistical Hypotheses* by Erich L. Lehmann
3. *Statistical Decision Theory: Foundations, Concepts and Methods* by James O. Berger

MAT 490 – Discrete Time Finance

A Major Elective for B.Sc. (Research) Mathematics.

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial/lab weekly)

Prerequisites: MAT 184 Probability (MAT 390 Introduction to Mathematical Finance is recommended but is not a compulsory requirement.)

Overview: This course serves two purposes. On the one hand, it introduces various theoretical notions in the simpler setting of discrete time and sets the stage for continuous time finance. On the other, it has a strong computational aspect and the student learns to implement models using Excel or Matlab.

Detailed Syllabus:

1. Binomial pricing models
2. Conditional expectation, Martingales, Markov Processes
3. Risk-neutral probability measure
4. American derivatives
5. Random Walks
6. Interest rate models and derivatives
7. Implementation of models in Excel/Matlab

References:

1. Steven E Shreve, *Stochastic Calculus for Finance I: The Binomial Asset Pricing Model*, Springer 2004.
2. Les Clewlow and Chris Strickland, *Implementing Derivatives Models*, Wiley 1998.
3. John C Hull, *Options, Futures and Other Derivatives*, 8th edition, Pearson, 2013.
4. Rudiger Seydel, *Tools for Computational Finance*, 5th edition, Springer, 2012.



Department of Mathematics
School of Natural Sciences

MAT 499 – Undergraduate Thesis II

Core course for B.Sc. (Research) Mathematics

Credits (Lec:Tut:Lab)= 0:8:0 (12 credits over one or two semesters)

Prerequisites: None

Overview: This is a compulsory final semester/year course for students majoring in Mathematics. The grade will be awarded at the end of the academic year. A student pursuing a Specialization within mathematics must choose a Project in the area of Specialization.

MAT 542 – Cryptography

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures +1 tutorial weekly)

Prerequisites: Programming knowledge in C/Python/Matlab (e.g. as in MAT 110 or MAT 210 or CSD 101), MAT 240 or MAT 260.

Overview: Cryptography is a key technology in electronic security systems. Modern cryptographic techniques have diverse applications, like digital signatures of documents or secure internet transactions. This course is a basic introduction to the mathematical foundations of modern cryptographic algorithms and their implementation and analysis.

Detailed Syllabus:

1. **Mathematical Foundations:** Number theory, group theory, modular arithmetic, matrices, linear transformations.
2. **Introduction to Cryptography:** Objectives, attacks, cryptographic protocols, provable security.
3. **Encryption:** Symmetric and asymmetric cryptosystems, classical ciphers – Caesar shift, Affine, Vigenere, Hill - stream and block ciphers, cryptanalysis.
4. **Probability:** Probability, conditional probability, birthday paradox, Vernam one-time pad, random and pseudorandom numbers.
5. **AES:** Feistel ciphers, DES, AES, security analysis.
6. **Public Key Cryptosystems:** Objectives, RSA cryptosystem, Rabin encryption, Diffie-Hellman key exchange, ElGamal cryptosystem, attacks, factoring.
7. **Applications:** Hash functions, their relation to compression functions, birthday attacks, MACs, Digital signatures – RSA, ElGamal and DSA.
8. **Discrete Logarithms:** DL problem, Shanks baby-step giant-step, Pollard rho, Pohlig-Hellman, index calculus.

References:

1. *Introduction to Cryptography (Undergraduate Texts in Mathematics)* by Johannes A. Buchmann, Springer, 2nd edition, 2004.
2. *Applied Cryptography* by Bruce Schneier, 2nd edition, Wiley, 1996.
3. *Introduction to Cryptography* by H. Delfs and H. Knebl, Springer, 2nd edition, 2010.
4. *A Course in Number Theory and Cryptography (Graduate Texts in Mathematics)* by Niel Koblitz, Springer, 2nd edition, 1994.
5. *An Introduction to Mathematical Cryptography (Undergraduate Texts in Mathematics)* by J. Hoffstein, J. Pipher and J.H. Silverman, Springer, 2008.

MAT 543 – Error Correcting Codes

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 260 Linear Algebra I

Overview: This course is an introduction to the *mathematical* theory of error correcting codes. Interested students can use the introduction to develop and implement error correcting codes. It can be taken as a stepping stone to research in this area. The course is of interest to those mathematicians who would like to work in algebra and number theory; as well to those who are looking at applications of algebra. Computer Scientists and Electronics or Communications Engineers should find this course of value as it gives the mathematical foundations of coding theory.

Detailed Syllabus:

1. Error Correcting Codes, definition and examples; the main coding theory problem.
2. Basic Number Theory; finite fields; vector spaces; subspaces; linear transformations; quotient spaces; linear independence and dimension.
3. Linear codes, introduction and examples.
4. Encoding and decoding of linear codes.
5. Dual codes, parity-check matrix and syndromes.
6. Hamming codes.
7. Perfect codes.
8. Cyclic codes.

References:

1. *A First Course in Coding Theory* by Raymond Hill, Clarendon Press, Oxford, 1986.
2. *Error Correcting Codes* by W. Wesley Peterson & E. J. Weldon, Jr., MIT Press, 2nd Edition, USA, 1972.
3. *Numbers, Groups and Codes* by J. F. Humphreys & M. Y. Prest, CUP, UK, 1991.
4. *Coding Theory* by Jacobus H van Lint, Springer Verlag, LNM 201, USA, 1973.
5. *The Mathematics of Coding Theory* by Paul Garrett, Pearson Education, USA, 2004.

MAT 544 – Combinatorial Design Theory

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 260 (Linear Algebra) or MAT 202 (Mathematical Methods)

Overview:

Detailed Syllabus:

1. Introduction to Design Theory: Basic definitions and properties, Incidence matrices, Fisher's Inequality.
2. Symmetric BIBD's
 - a. Intersection Property, Residual and Derived BIBD's, Projective planes and Geometries
 - b. The Bruck-Ryser-Chowla Theorem
3. Difference Sets and Automorphisms: Quadratic residue difference sets, Singer difference sets.
4. Hadamard Matrices and Designs: An equivalence between Hadamard matrices and BIBD's, Conference matrices and Hadamard matrices, Bent Functions
5. Latin Squares: Steiner Triple systems, Orthogonal Latin Squares, MOL's, Orthogonal arrays
6. PBIBD's: Connection of PBIBD's to Association Schemes and Distance regular graphs.
7. Applications of Combinatorial Design Theory: Medicine, Agriculture, Visual cryptography, Information Security, Statistical designs.

References:

Online Material: <http://designtheory.org/library/>

1. Douglas R. Stinson, *Combinatorial Designs: Construction and Analysis*, Springer.
2. Anne Penfold Street and Deborah J. Street, *Combinatorics of Experimental Design*, Oxford Science Publications.
3. D. Raghavarao, Lakshmi V. Padgett, *Block Designs: Analysis, Combinatorics and Applications*, World Scientific.
4. P.J. Cameron and J.H. Van Lint, *Designs, Graphs, Codes and their links*, Cambridge University Press.
5. J.H. Van Lint and R.M. Wilson, *A Course in Combinatorics*, Second edition, Cambridge University Press.

MAT 584 – Stochastic Processes

Credits (Lec:Tut:Lab)= 3:1:0 (3 lectures and 1 tutorial weekly)

Prerequisites: MAT 420 Probability and Measure.

Overview: The course emphasizes applications. It is intended for Mathematics students interested in Probability and Analysis, as well as CSE and Electrical Engineering Majors. MAT 584 is compulsory for students completing the Mathematical Finance specialization in the B.Sc. (Research) Mathematics program.

Detailed Syllabus:

1. Review of Probability
2. Conditional Expectation
3. Random Walks
4. Markov Chains
5. Basic Limit Theorems of Markov Chains and Applications
6. Martingales in Discrete Time
7. Martingale Inequalities and Convergence
8. Stochastic Processes in Continuous time
9. Applications

References:

- *Introduction to Probability Models* by Sheldon M. Ross, 10th edition, Elsevier India, 2010
- *Basic Stochastic Processes - A Course Through Exercises* by Z. Brzezniak and T. Zastawniak, Springer India, 2009