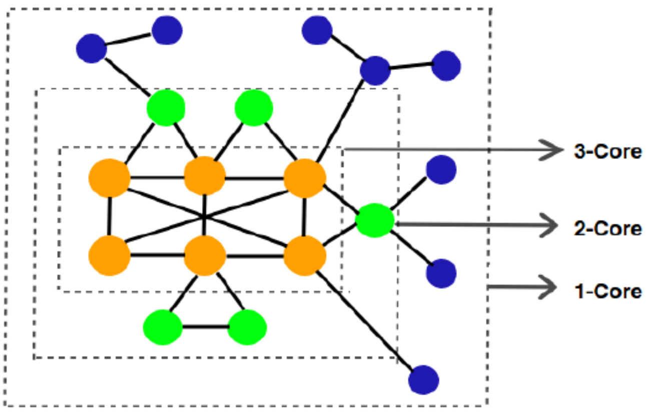
Exercises for model transformations and additional properties extensions

1. Write a function to determine the k-core decomposition of a given graph G. The k-core of a graph is given by the largest induced subgraph of G in which every vertex has degree at least k. The coreness of a node v is the largest value of k such that there is a k-core of G containing v. The k-core decomposition is equivalent to identify the coreness of each node in G.

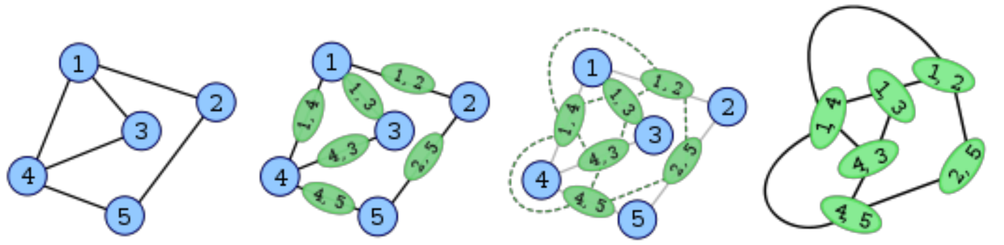


We can obtain the different k-cores of a graph using a modified version of the code to obtain induced subgraphs on a number of nodes from Exercise 4 Task1.Try to fill in the missing code in the provided structure:

```
my_coreness(g) {
  #save the original vertex number
  V(g)$idx=1:length(V(g))
  #initialize vector to store coreness
  cores=rep(NA, length(V(g))
  #get degrees
  ind_degree=degree(g)
  k=1
  while(length(ind_degree)>0){

    #if there are nodes with a degree smaller or equal to k
      # set the coreness of this vertices to k and remove them
      # from the graph update the degree of the vertices
    #else
      #increase k by one
  }
  return(cores)
}
```

2. A line graph of a graph $G = (V, E)$ is the graph H for which $V(H) = E(G)$ and two nodes in H are adjacent if the corresponding edges are adjacent in G. Write a function to create a line graph of a given graph G. Are centralities of edges in H related to centralities of the end-nodes in G? Justify your answer.



Graph G — Nodes in L(G) constructed from edges in G — Added edges in L(G) — The line graph L(G)

The line graph has as many nodes as there are edges in the original graph. One way to solve this is to generate a zero adjacency matrix with $n = m = |E|$. Finish the code structure according to the comments:

```
line_graph <- function(g){
     ec <- length(E(g))
     adj <- matrix(0, ec, ec)
     #loop over upper triangle of adjacency matrix and
     #check for each entry if the two corresponding edges
     #are adjacent
     l <- graph_from_adjacency_matrix(adj, mode =
"undirected")
     return(l)
}
```

Homework

1. A graph complement of G is the graph $\bar{G}$ for which $V(\bar{G}) = V(G)$ and $E(\bar{G}) = E(K_n)\backslash E(G)$. Are the respective centrality measures in G and $\bar{G}$ negatively correlated? Justify your answer either by testing on graphs or demonstrate it in general. What is the degree distribution of $\bar{G}$ if the one of G is known?

2. Write a function that for a given graph G identifies all P_4 (paths on 4 nodes) on nodes u, v, w, x such that (u, v), (v, w), (w, x) are the only edges in the subgraph induced by the 4 nodes. Are there more induced P_4 (paths on 4 nodes) on Barabasi-Albert than Erdos-Renyi random graphs with the same number of nodes and edges?