

Task

The goal of this assignment is to develop a mathematical model to predict the spread of COVID-19. The model is based on the rates of change of the number of susceptible, exposed, infected, and recovered individuals in a population. The model will be parameterized using data from the COVID-19 pandemic in Karnataka. Once the model is parameterized, it will be used to make predictions about the course of the pandemic. The assignment also investigates the effects of different control measures, such as social distancing and vaccination, on the spread of the disease.

1 Implementation

To analyze and predict the spread of COVID-19. First, I loaded the data from the dataset. Then, I initialized the model parameters and calculated the loss for the model. Then, I used gradient descent to minimize the loss function, and based on open-loop and closed-loop control, predictions were made accordingly. Finally, I have generated the plots to visualize data graphically.

1.1 Gradient Descent

To optimize parameters that minimizes the loss, I used the gradient descent method. It updates parameters β , R_0 , CIR_0 , I_0 , and E_0 based on their gradients. The updates are iteratively performed over a specified number of epochs = 100. The updates are done as follows:

$$\begin{aligned}\beta &= \beta - 0.00001 \frac{\partial Loss}{\partial \beta} \\ CIR_0 &= CIR_0 - 0.00001 \frac{\partial Loss}{\partial CIR_0} \\ R_0 &= R_0 - 0.00001 \frac{\partial Loss}{\partial R_0} \\ I_0 &= I_0 - 0.00001 \frac{\partial Loss}{\partial I_0} \\ E_0 &= E_0 - 0.00001 \frac{\partial Loss}{\partial E_0}\end{aligned}$$

1.2 Simulations

Here, to obtain the simulated values of S , E , I , and R . I have iterated for 43 days; in each iteration, I have calculated the values of S , E , I , and R using the following equations:

$$\begin{aligned}\Delta S(t) &= -\beta(t)S(t)\frac{I(t)}{N} - \varepsilon\Delta V(t) + \Delta W(t) \\ \Delta E(t) &= \beta(t)S(t)\frac{I(t)}{N} - \alpha E(t) \\ \Delta I(t) &= \alpha E(t) - \gamma I(t) \\ \Delta R(t) &= \gamma I(t) + \varepsilon\Delta V(t) - \Delta W(t).\end{aligned}$$

Used the mean incubation period $\alpha^{-1} = 5.8$ days, mean recovery period $\gamma^{-1} = 5$ days, vaccine efficacy $\varepsilon = 66\%$, and the total population $N = 70M$.

1.3 Loss Calculation

The loss is calculated as the root mean squared difference between the rolling averages of simulated data and observed data. This measures how closely the model's predictions align with the actual data. The loss

quantifies the model's accuracy in fitting the observed data. The Loss is calculated as follows:

$$L = \sqrt{\frac{1}{n} \sum_{i=1}^n (Avg_{sim}[i] - Avg_{obs}[i])^2}$$

2 Plotting

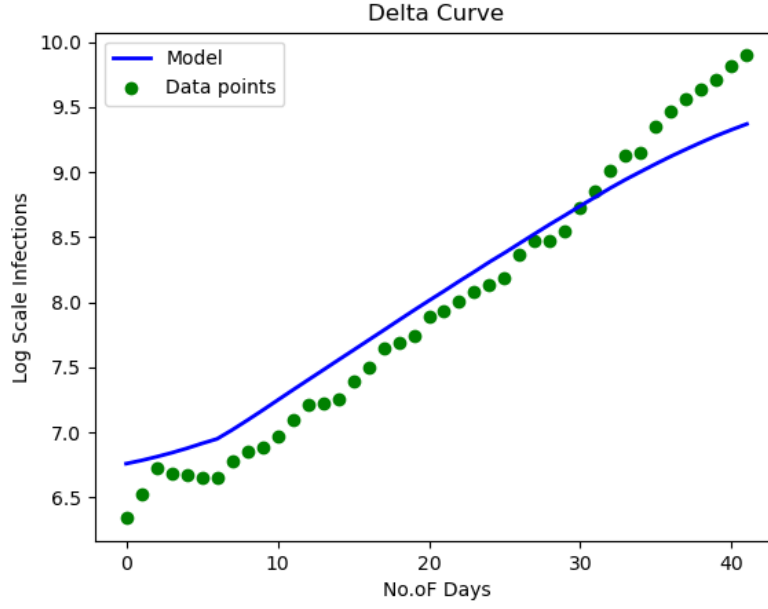


Fig. 1: Curve Fitting

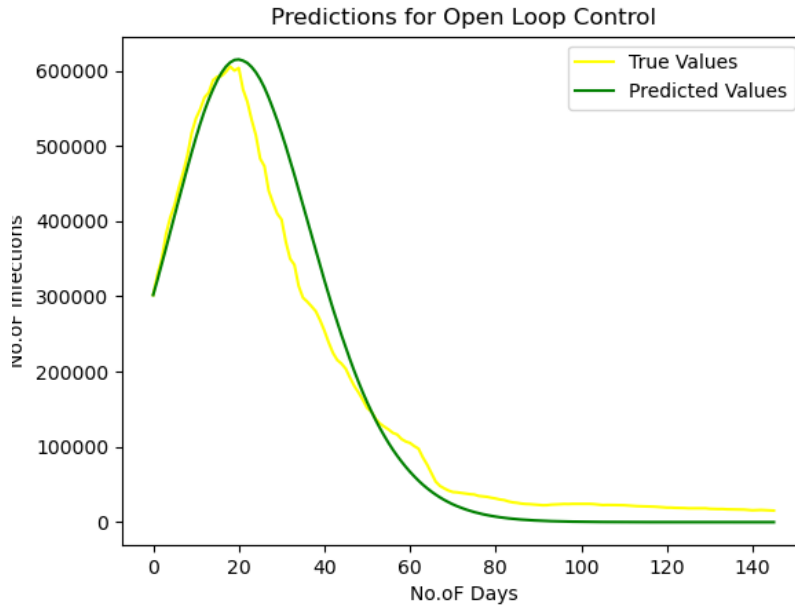


Fig. 2: predictions for Open loop control : β

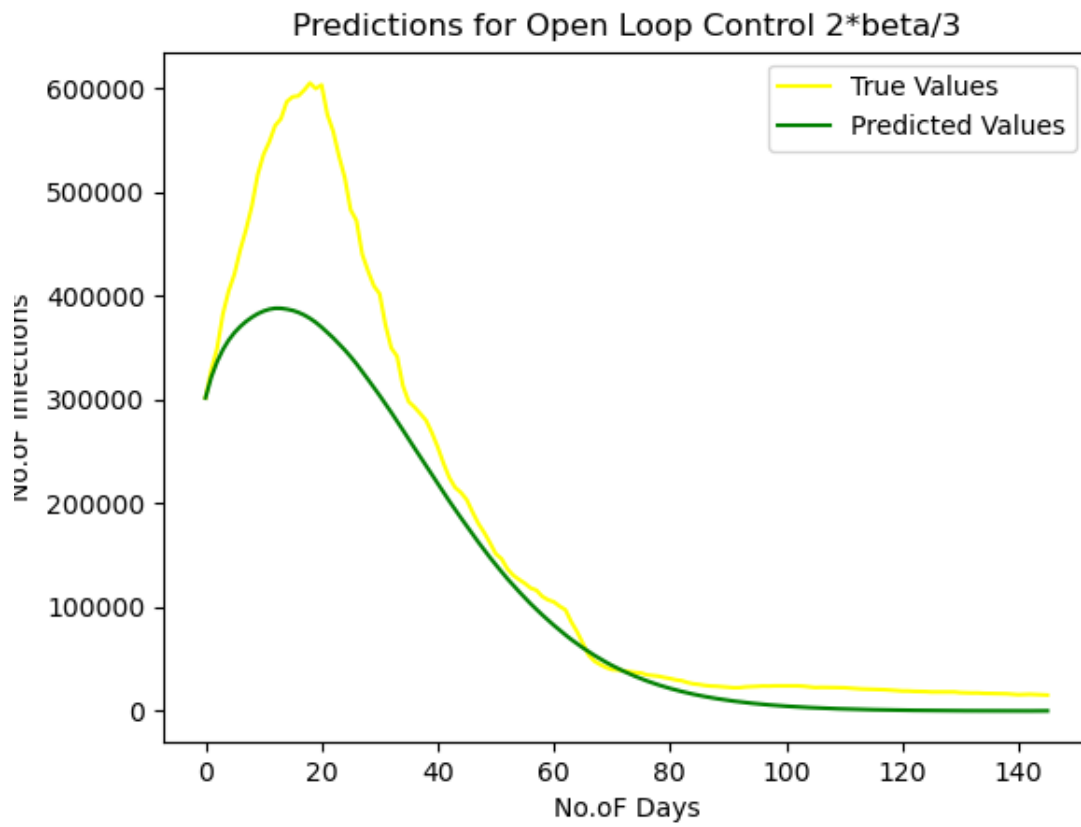


Fig. 3: predictions for Open loop control : $2 \cdot \beta/3$

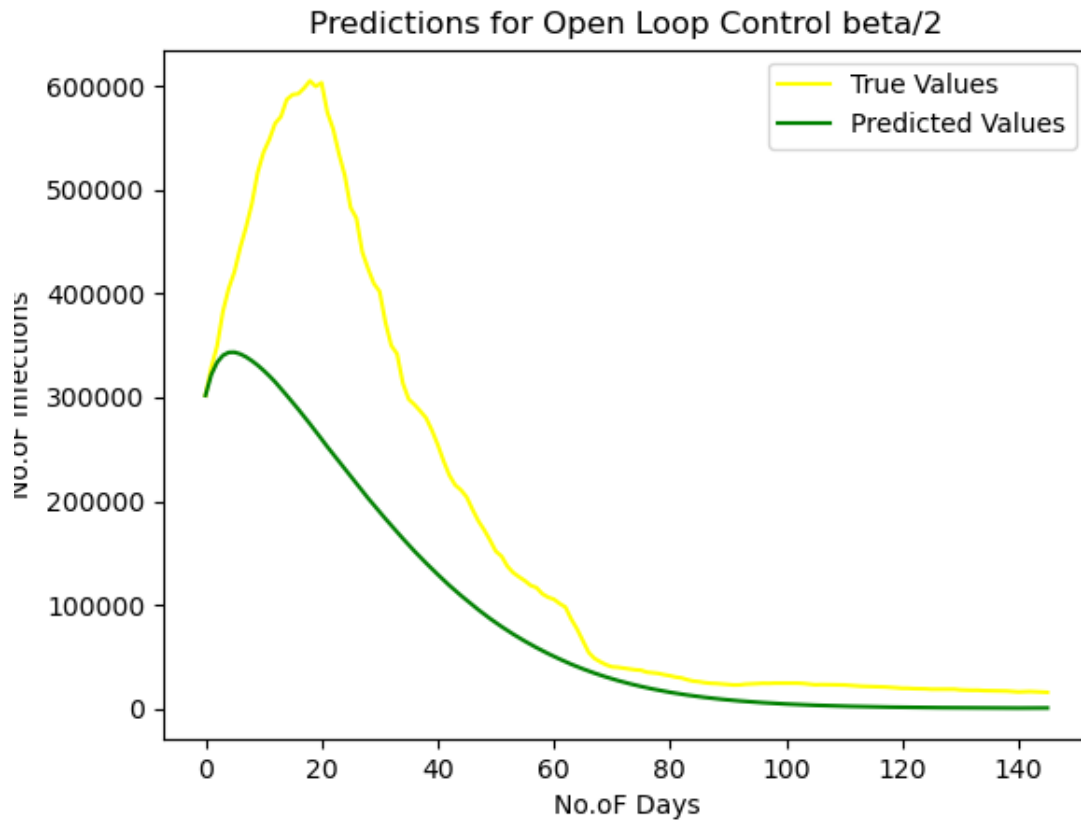


Fig. 4: predictions for Open loop control : $\beta/2$

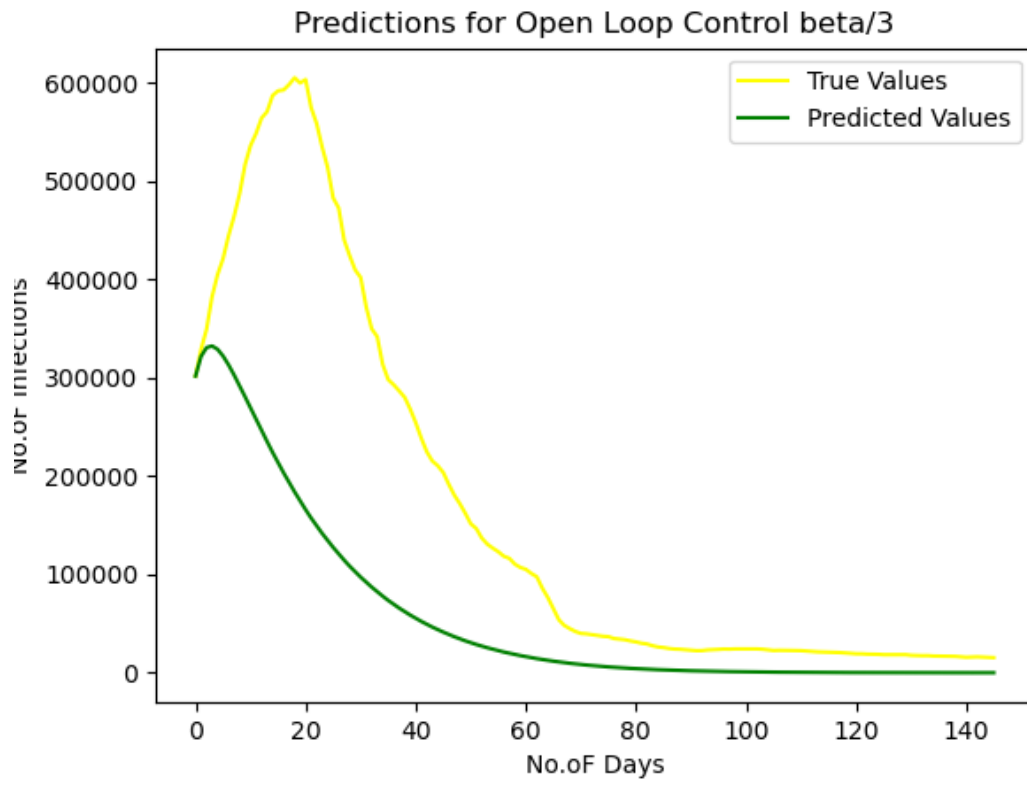


Fig. 5: predictions for Open loop control : $\beta/3$

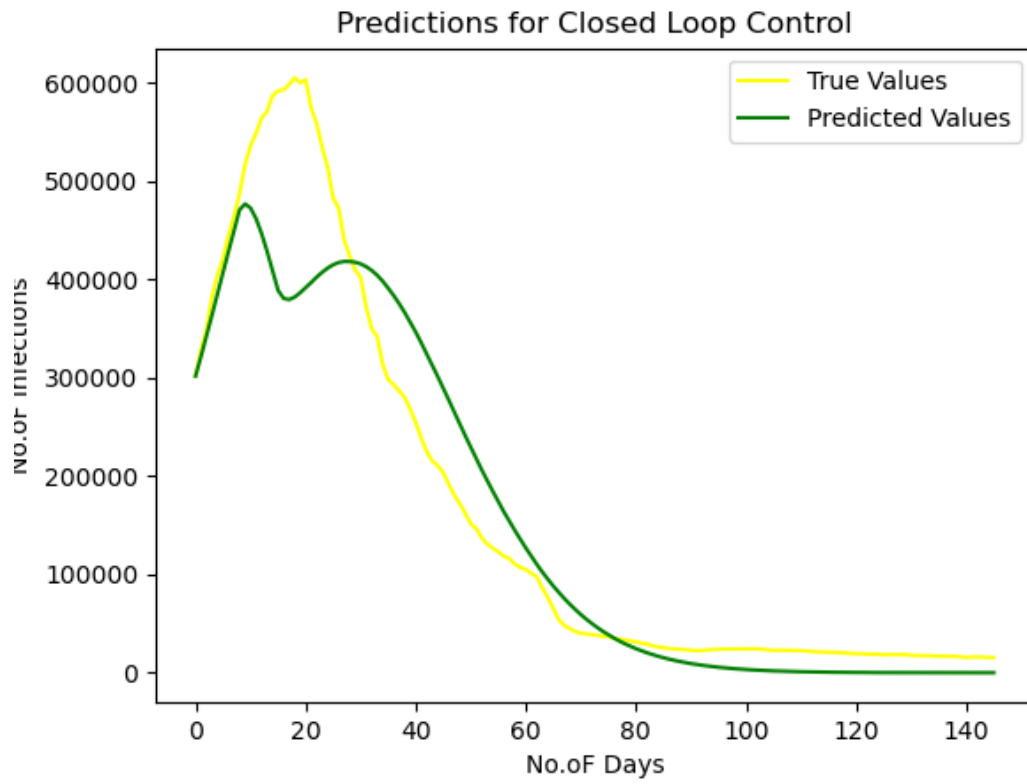


Fig. 6: predictions for Closed loop control

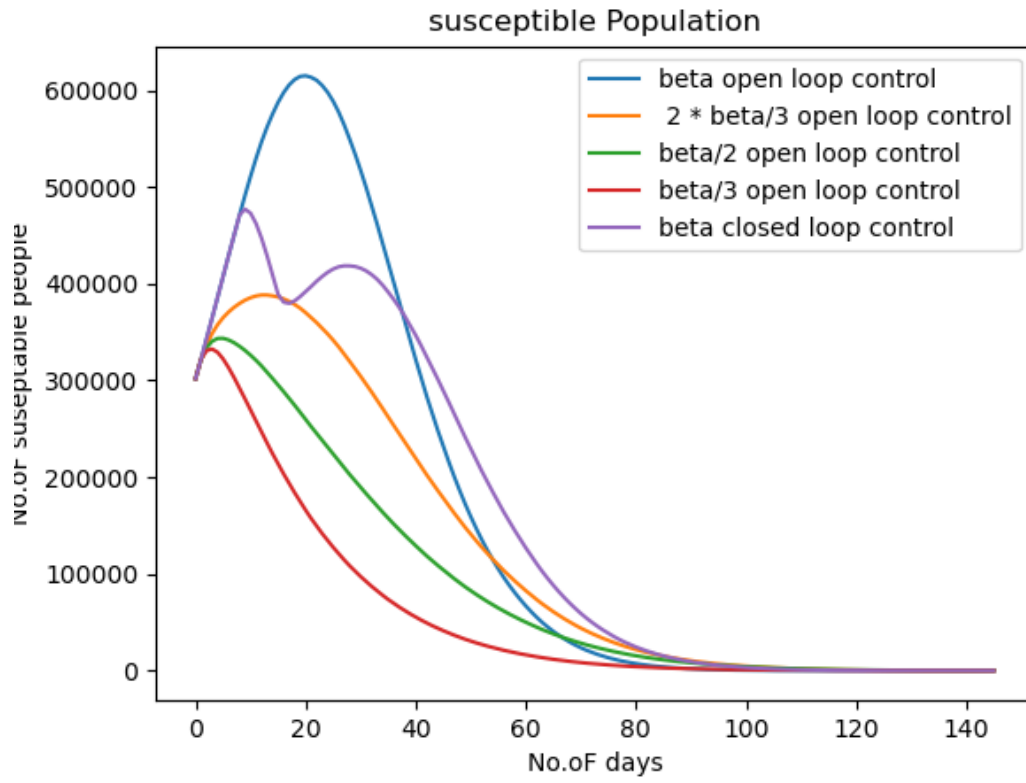


Fig. 7: susceptible population in all the five scenarios.

3 Results

The optimal unknown parameters are as follows:

The optimal β value is = 0.456

The optimal R_0 value is = 32%

The optimal CIR_0 value is = 14.17

The optimal I_0 value is = 0.34%

The optimal C_0 value is = 0.49%