Coordinate Geometry Introduction to Engineering Mathematics

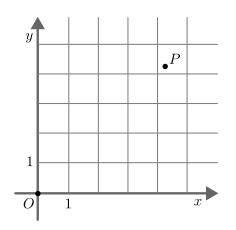
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What is coordinate geometry?

Studying geometry through coordinate calculations.

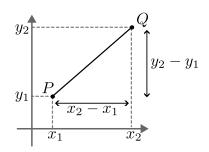




Example: distance between two points

Distance between P and Q:

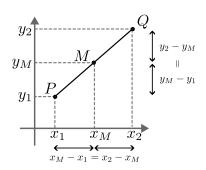
$$d(P,Q) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$



Example: midpoint between two points

Coordinates of midpoint between P and Q:

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right).$$

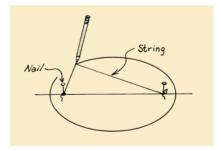


Locus of points

"Locus" = Set of points satisfying some condition.

- Circle: All points at given distance from a fixed center.
- Ellipse: All points for which the sum of distances to two fixed points (focal points) is constant
- Parabola: All points that are at equal distance from a fixed point and a given line (directrix)





Find the locus of points for which the distance to the x-axis is equal to the distance to the point (0,1).

Circles

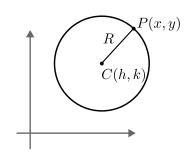
Locus of points P(x,y) at distance R from center C(h,k).

We have d(P,C)=R so that

$$\sqrt{(x-h)^2 + (y-k)^2} = R,$$

and by squaring

$$(x-h)^2 + (y-k)^2 = R^2$$



Find the equation of the circle that has the points (1,1) and (7,9) as end points of a diameter.

Find the center and radius of the circle given by $x^2 + y^2 - 6x + 2y + 8 = 0$.

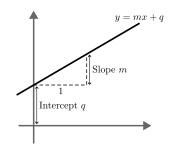
Lines

Line not parallel to the y-axis:

$$y = mx + q$$

with

- m: the **slope**
- ullet q: the intercept



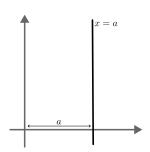
Lines

Line parallel to the y-axis:

$$x = a$$

with

 a: where the line intersects the x-axis



Finding the slope of a line

Take

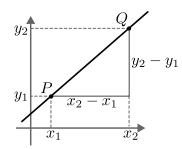
$$\bullet \ \Delta x = x_2 - x_1$$

$$\bullet \ \Delta y = y_2 - y_1$$

Then

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

" Δx steps to the right, Δy steps up/down."



Find the equation for the line through (1,5) and (2,7).

Properties

Equation for the line through (x_0, y_0) with slope m:

$$y - y_0 = m(x - x_0)$$

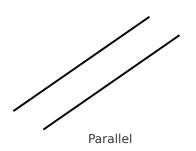
Equation for the line through the points (x_1,y_1) and (x_2,y_2) :

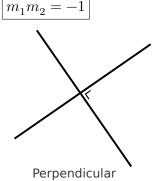
$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1).$$

Parallel/perpendicular lines

Two lines are ...

- parallel if their slopes are the same: $\boxed{m_1 = m_2}$
- **perpendicular** if their slopes satisfy:



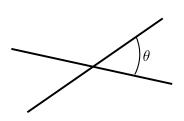


Angle between two lines

In general, the angle $\boldsymbol{\theta}$ between two lines is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|,$$

where m_1 and m_2 are the slopes of the lines.



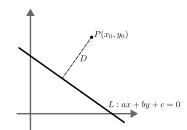
Given two lines $L_1:x+2y-3=0$ and $L_2:kx+y-5=0,$ for which value of k are L_1 and L_2 ...

- 1 Parallel?
- 2 Perpendicular?
- 3 At an angle of 45° ?

Distance of a point to a line

Distance between point $P(x_0,y_0)$ and line L:ax+by+c=0:

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}.$$



Different representations of lines

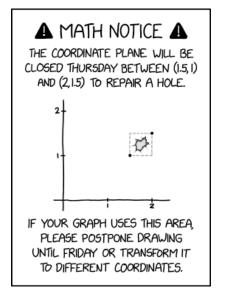
- Slope/intercept:
 - y = mx + q (not parallel to y-axis)
 - x = a (parallel)
- **2** Linear representation: ax + by + c = 0
- 3 Polar representation:
 - For line through the origin: $\tan \theta = m$
 - For line not through the origin:

$$r = \frac{q}{\sin \theta - m \cos \theta}$$

Exercise

Find the equation of the common tangent line between two touching circles given by

$$\begin{split} C_1 : x^2 + y^2 - 6x - 12y + 37 &= 0 \\ C_2 : x^2 + y^2 - 6y + 7 &= 0. \end{split}$$



Source: xkcd 2735