

# Coordinate Geometry

## Introduction to Engineering Mathematics

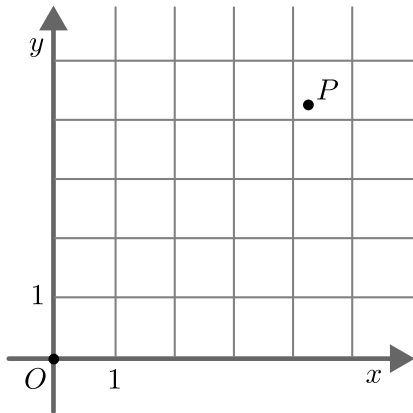
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# What is coordinate geometry?

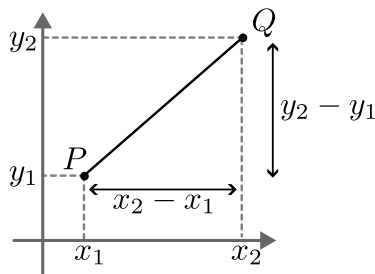
Studying geometry through coordinate calculations.



## Example: distance between two points

Distance between  $P$  and  $Q$ :

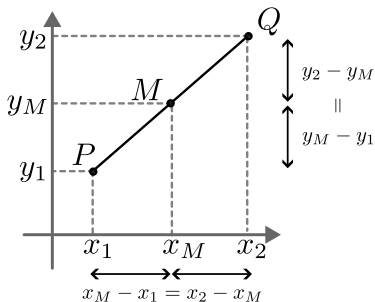
$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



## Example: midpoint between two points

Coordinates of midpoint between  $P$  and  $Q$ :

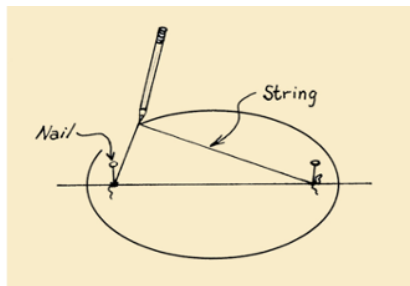
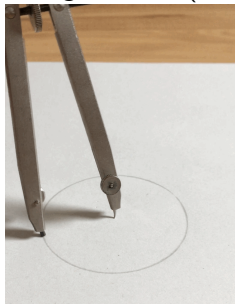
$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



# Locus of points

“Locus” = Set of points satisfying some condition.

- **Circle:** All points at given distance from a fixed center.
- **Ellipse:** All points for which the sum of distances to two fixed points (focal points) is constant
- **Parabola:** All points that are at equal distance from a fixed point and a given line (directrix)



## Example

Find the locus of points for which the distance to the  $x$ -axis is equal to the distance to the point  $(0, 1)$ .

# Circles

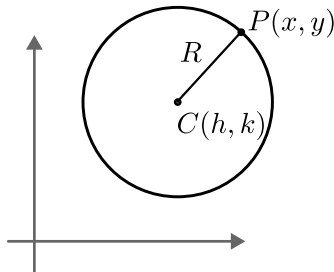
Locus of points  $P(x, y)$  at distance  $R$  from center  $C(h, k)$ .

We have  $d(P, C) = R$  so that

$$\sqrt{(x - h)^2 + (y - k)^2} = R,$$

and by squaring

$$\boxed{(x - h)^2 + (y - k)^2 = R^2}$$





## Example

Find the equation of the circle that has the points  $(1, 1)$  and  $(7, 9)$  as end points of a diameter.

## Example

Find the center and radius of the circle given by

$$x^2 + y^2 - 6x + 2y + 8 = 0.$$

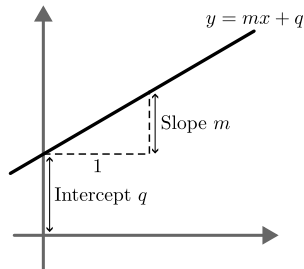
# Lines

Line not parallel to the  $y$ -axis:

$$y = mx + q$$

with

- $m$ : the **slope**
- $q$ : the **intercept**



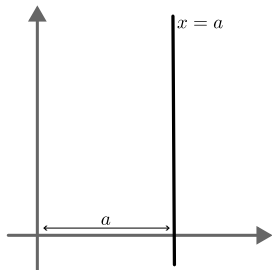
# Lines

Line parallel to the  $y$ -axis:

$$x = a$$

with

- $a$ : where the line intersects the  $x$ -axis



## Finding the slope of a line

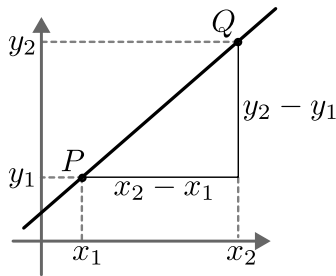
Take

- $\Delta x = x_2 - x_1$
- $\Delta y = y_2 - y_1$

Then

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

“ $\Delta x$  steps to the right,  $\Delta y$  steps up/down.”



## Example

Find the equation for the line through  $(1, 5)$  and  $(2, 7)$ .

# Properties

Equation for the line through  $(x_0, y_0)$  with slope  $m$ :

$$y - y_0 = m(x - x_0)$$

Equation for the line through the points  $(x_1, y_1)$  and  $(x_2, y_2)$ :

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

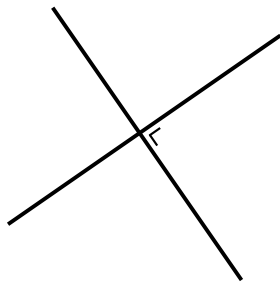
## Parallel/perpendicular lines

Two lines are ...

- **parallel** if their slopes are the same:  $m_1 = m_2$
- **perpendicular** if their slopes satisfy:  $m_1 m_2 = -1$



Parallel



Perpendicular

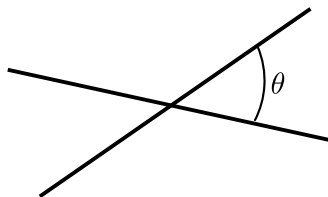


## Angle between two lines

In general, the angle  $\theta$  between two lines is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|,$$

where  $m_1$  and  $m_2$  are the slopes of the lines.



## Example

Given two lines  $L_1 : x + 2y - 3 = 0$  and  $L_2 : kx + y - 5 = 0$ , for which value of  $k$  are  $L_1$  and  $L_2$  ...

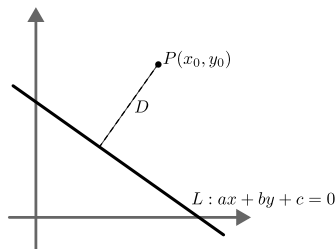
- ① Parallel?
- ② Perpendicular?
- ③ At an angle of  $45^\circ$ ?



## Distance of a point to a line

Distance between point  $P(x_0, y_0)$  and line  $L : ax + by + c = 0$ :

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}.$$



# Different representations of lines

## ① Slope/intercept:

- $y = mx + q$  (not parallel to  $y$ -axis)
- $x = a$  (parallel)

## ② Linear representation: $ax + by + c = 0$

## ③ Polar representation:

- For line through the origin:  $\tan \theta = m$
- For line not through the origin:

$$r = \frac{q}{\sin \theta - m \cos \theta}$$

## Exercise

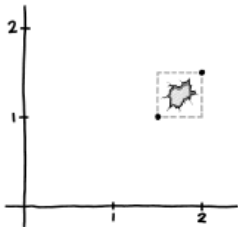
Find the equation of the common tangent line between two touching circles given by

$$C_1 : x^2 + y^2 - 6x - 12y + 37 = 0$$

$$C_2 : x^2 + y^2 - 6y + 7 = 0.$$

## ⚠ MATH NOTICE ⚠

THE COORDINATE PLANE WILL BE  
CLOSED THURSDAY BETWEEN  $(1.5, 1)$   
AND  $(2, 1.5)$  TO REPAIR A HOLE.



IF YOUR GRAPH USES THIS AREA,  
PLEASE POSTPONE DRAWING  
UNTIL FRIDAY OR TRANSFORM IT  
TO DIFFERENT COORDINATES.

Source: xkcd 2735