Derivatives (2/2): Applications Introduction to Engineering Mathematics

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Implicit differentiation

- We now know how to compute the derivative of a function y=f(x).
- But what if we can't write y as a function of x?

Example: Find y'(x) if x and y are related by $x^2 + y^2 = 1$.

Two ways of solving:

- **1** The direct way (hard): find y(x) and differentiate
- 2 Via implicit differentiation (easy): differentiate both sides

Example

Find an equation for the tangent line to the curve defined by $x^2 + xy + 2y^3 = 4$ at the point P(-2,1).

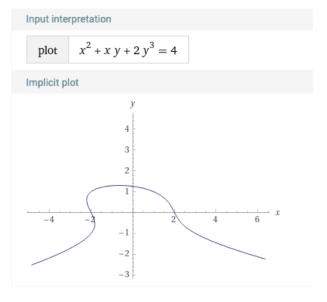


Figure made with Wolfram Alpha.

Velocity and acceleration

We will consider only objects moving along a straight line (one-dimensional motion)

- Position x(t)
- Velocity v(t)

Recall from physics 1: v(t) is the **first derivative** of x(t). Why?

Average and instantaneous velocity, speed

• The average velocity $v_{\rm avg}(t)$ over an interval [t,t+h] is the change in distance divided by the change in time:

$$v_{\rm avg}(t) = \frac{\Delta x}{\Delta t} = \frac{x(t+h) - x(t)}{h}.$$

• The instantaneous velocity v(t) is the limit as $h \to 0$:

$$v(t) = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h} = \frac{dx}{dt} = \dot{x}(t).$$

• The speed s(t) is the magnitude of the velocity: $s(t) = \vert v(t) \vert.$

Characteristics of velocity

- v(t) > 0: moving to the **right** (x(t) increases)
- v(t) < 0: moving to the **left** (x(t) decreases)
- ullet v(t)=0: instantaneously at rest

Acceleration

The **acceleration** a(t) is the rate of change of the velocity:

$$a(t) = \dot{v}(t) = \ddot{x}(t).$$

Characteristics:

- a(t) > 0: v(t) increases
- a(t) < 0: v(t) decreases

Speeding up and slowing down:

- $a(t) \cdot v(t) > 0$: speeding up
- $a(t) \cdot v(t) < 0$: slowing down

Example

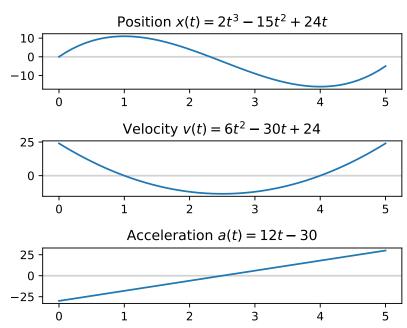
A particle P moves along the $x\mbox{-axis}$ with position given by $x(t)=2t^3-15t^2+24t.$

- **1** Find v(t) and a(t).
- 2 In which direction is P moving at t=2? Is P speeding up or slowing down?
- ${f 3}$ When is P instantaneously at rest? When does its velocity instantaneously not change?

Example (continued)

- 4 When is P moving to the left/right?
- f 5 When is P speeding up/slowing down?

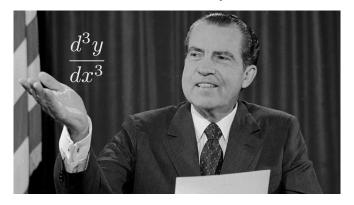
Example (continued)



Historical aside

In the fall of 1972, President Nixon announced that "the rate of increase of inflation was decreasing".

- Probably the first time a sitting president used the 3rd derivative.
- The 3rd derivative is also know as "jerk".



Source: Fermat's library