

Coordinate Geometry

Introduction to Engineering Mathematics

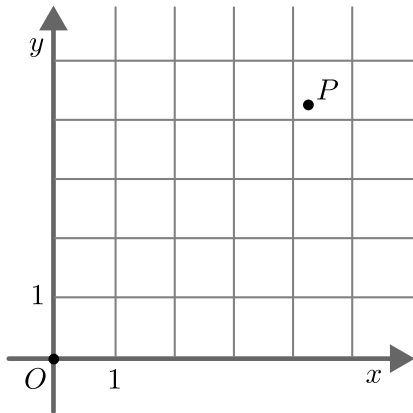
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What is coordinate geometry?

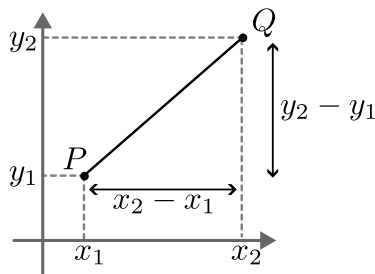
Studying geometry through coordinate calculations.



Example: distance between two points

Distance between P and Q :

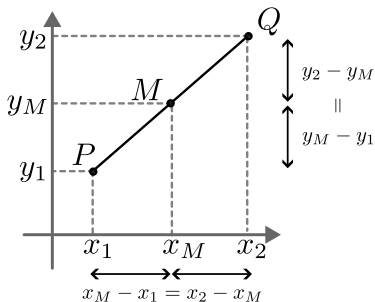
$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example: midpoint between two points

Coordinates of midpoint between P and Q :

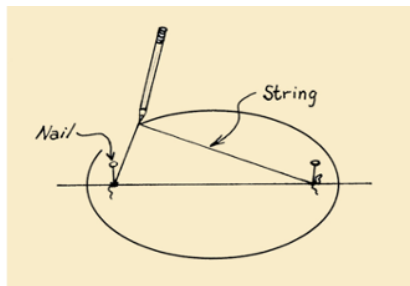
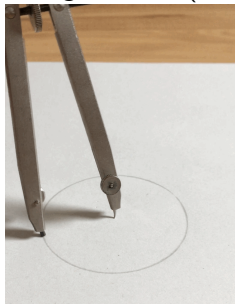
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



Locus of points

“Locus” = Set of points satisfying some condition.

- **Circle:** All points at given distance from a fixed center.
- **Ellipse:** All points for which the sum of distances to two fixed points (focal points) is constant
- **Parabola:** All points that are at equal distance from a fixed point and a given line (directrix)



Example

Find the locus of points for which the distance to the x -axis is equal to the distance to the point $(0, 1)$.

Circles

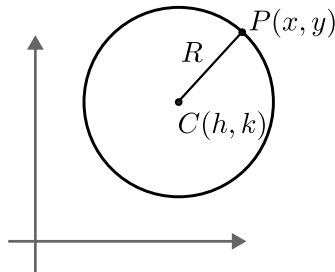
Locus of points $P(x, y)$ at distance R from center $C(h, k)$.

We have $d(P, C) = R$ so that

$$\sqrt{(x - h)^2 + (y - k)^2} = R,$$

and by squaring

$$\boxed{(x - h)^2 + (y - k)^2 = R^2}$$



Example

Find the equation of the circle that has the points $(1, 1)$ and $(7, 9)$ as end points of a diameter.

Example

Find the center and radius of the circle given by

$$x^2 + y^2 - 6x + 2y + 8 = 0.$$

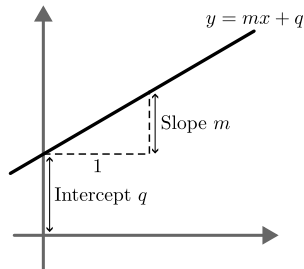
Lines

Line not parallel to the y -axis:

$$y = mx + q$$

with

- m : the **slope**
- q : the **intercept**



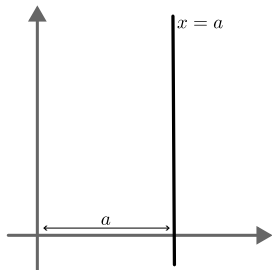
Lines

Line parallel to the y -axis:

$$x = a$$

with

- a : where the line intersects the x -axis



Finding the slope of a line

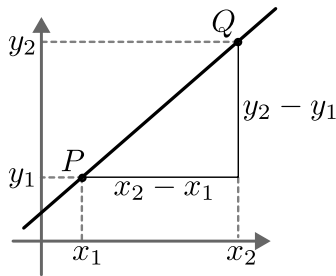
Take

- $\Delta x = x_2 - x_1$
- $\Delta y = y_2 - y_1$

Then

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

“ Δx steps to the right, Δy steps up/down.”



Example

Find the equation for the line through $(1, 5)$ and $(2, 7)$.

Properties

Equation for the line through (x_0, y_0) with slope m :

$$y - y_0 = m(x - x_0)$$

Equation for the line through the points (x_1, y_1) and (x_2, y_2) :

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

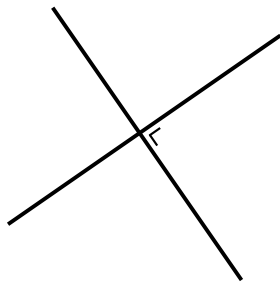
Parallel/perpendicular lines

Two lines are ...

- **parallel** if their slopes are the same: $m_1 = m_2$
- **perpendicular** if their slopes satisfy: $m_1 m_2 = -1$



Parallel



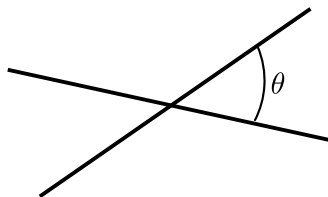
Perpendicular

Angle between two lines

In general, the angle θ between two lines is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|,$$

where m_1 and m_2 are the slopes of the lines.



Example

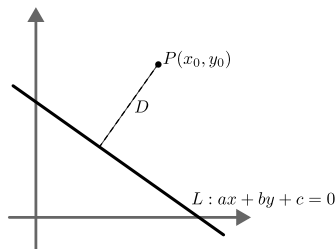
Given two lines $L_1 : x + 2y - 3 = 0$ and $L_2 : kx + y - 5 = 0$, for which value of k are L_1 and L_2 ...

- ① Parallel?
- ② Perpendicular?
- ③ At an angle of 45° ?

Distance of a point to a line

Distance between point $P(x_0, y_0)$ and line $L : ax + by + c = 0$:

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}.$$



Different representations of lines

① Slope/intercept:

- $y = mx + q$ (not parallel to y -axis)
- $x = a$ (parallel)

② Linear representation: $ax + by + c = 0$

③ Polar representation:

- For line through the origin: $\tan \theta = m$
- For line not through the origin:

$$r = \frac{q}{\sin \theta - m \cos \theta}$$

Exercise

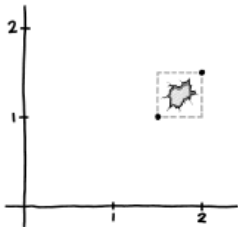
Find the equation of the common tangent line between two touching circles given by

$$C_1 : x^2 + y^2 - 6x - 12y + 37 = 0$$

$$C_2 : x^2 + y^2 - 6y + 7 = 0.$$

⚠ MATH NOTICE ⚠

THE COORDINATE PLANE WILL BE
CLOSED THURSDAY BETWEEN $(1.5, 1)$
AND $(2, 1.5)$ TO REPAIR A HOLE.



IF YOUR GRAPH USES THIS AREA,
PLEASE POSTPONE DRAWING
UNTIL FRIDAY OR TRANSFORM IT
TO DIFFERENT COORDINATES.

Source: xkcd 2735