

# Derivatives (2/2): Applications

## Introduction to Engineering Mathematics

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# Implicit differentiation

- We now know how to compute the derivative of a function  $y = f(x)$ .
- But what if we can't write  $y$  as a function of  $x$ ?

Example: Find  $y'(x)$  if  $x$  and  $y$  are related by  $x^2 + y^2 = 1$ .

Two ways of solving:

- 1 The direct way (hard): find  $y(x)$  and differentiate
- 2 Via **implicit differentiation** (easy): differentiate both sides

## Example

Find an equation for the tangent line to the curve defined by  $x^2 + xy + 2y^3 = 4$  at the point  $P(-2, 1)$ .

### Input interpretation

plot  $x^2 + x y + 2 y^3 = 4$

### Implicit plot



Figure made with Wolfram Alpha.

# Velocity and acceleration

We will consider only objects moving along a straight line (one-dimensional motion)

- Position  $x(t)$
- Velocity  $v(t)$

Recall from physics 1:  $v(t)$  is the **first derivative** of  $x(t)$ . Why?

## Average and instantaneous velocity, speed

- The *average velocity*  $v_{\text{avg}}(t)$  over an interval  $[t, t + h]$  is the change in distance divided by the change in time:

$$v_{\text{avg}}(t) = \frac{\Delta x}{\Delta t} = \frac{x(t + h) - x(t)}{h}.$$

- The *instantaneous velocity*  $v(t)$  is the limit as  $h \rightarrow 0$ :

$$v(t) = \lim_{h \rightarrow 0} \frac{x(t + h) - x(t)}{h} = \frac{dx}{dt} = \dot{x}(t).$$

- The *speed*  $s(t)$  is the magnitude of the velocity:  $s(t) = |v(t)|$ .

## Characteristics of velocity

- $v(t) > 0$ : moving to the **right** ( $x(t)$  increases)
- $v(t) < 0$ : moving to the **left** ( $x(t)$  decreases)
- $v(t) = 0$ : instantaneously at rest



# Acceleration

The **acceleration**  $a(t)$  is the rate of change of the velocity:

$$a(t) = \dot{v}(t) = \ddot{x}(t).$$

Characteristics:

- $a(t) > 0$ :  $v(t)$  increases
- $a(t) < 0$ :  $v(t)$  decreases

Speeding up and slowing down:

- $a(t) \cdot v(t) > 0$ : speeding up
- $a(t) \cdot v(t) < 0$ : slowing down

## Example

A particle  $P$  moves along the  $x$ -axis with position given by  $x(t) = 2t^3 - 15t^2 + 24t$ .

- 1 Find  $v(t)$  and  $a(t)$ .
- 2 In which direction is  $P$  moving at  $t = 2$ ? Is  $P$  speeding up or slowing down?
- 3 When is  $P$  instantaneously at rest? When does its velocity instantaneously not change?

## Example (continued)

- ④ When is  $P$  moving to the left/right?
- ⑤ When is  $P$  speeding up/slowing down?

## Example (continued)

Position  $x(t) = 2t^3 - 15t^2 + 24t$



Velocity  $v(t) = 6t^2 - 30t + 24$



Acceleration  $a(t) = 12t - 30$



## Historical aside

In the fall of 1972, President Nixon announced that “the rate of increase of inflation was decreasing”.

- Probably the first time a sitting president used the 3rd derivative.
- The 3rd derivative is also known as “jerk”.

