

# Trigonometry (5/5): Inverse Trigonometric Functions

Introduction to Engineering Mathematics

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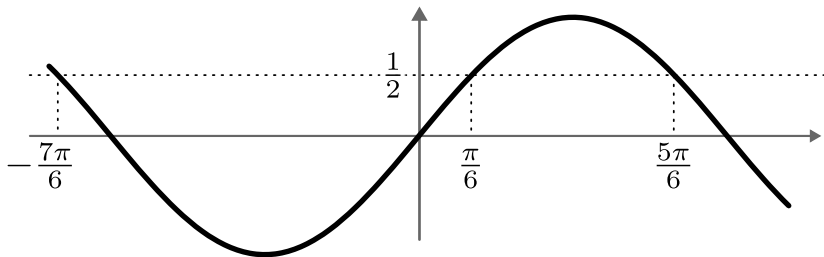
# Overview

- 1 Definition of the inverse trigonometric functions
- 2 Examples

## Inverting the sine function

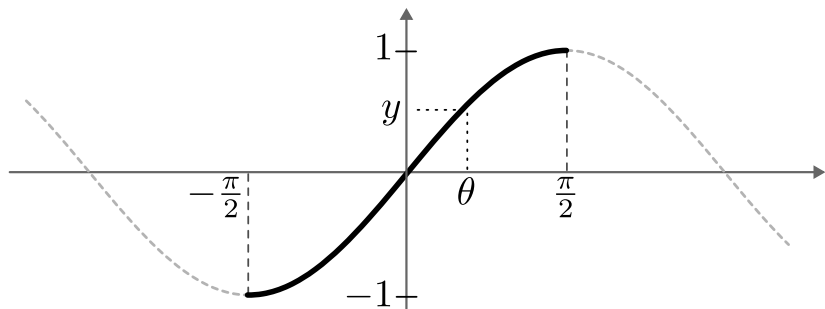
- The sine function turns angles into sine values.
- The **inverse sine** turns sine values back into angles.
- Notation:  $\sin^{-1}(x)$ ,  $\arcsin x$

**Problem:** Many values in the range correspond to the same angle!



## Restricting the domain

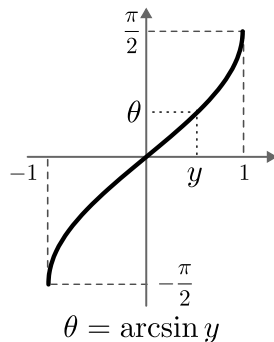
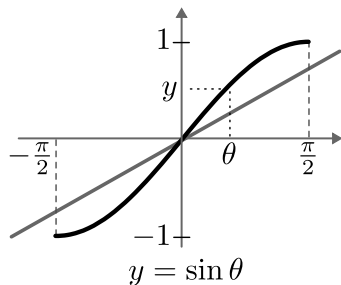
Solution: restrict the domain of the sine function so that there is exactly one angle corresponding to each value.



This gives us a meaningful way to define the inverse sine.

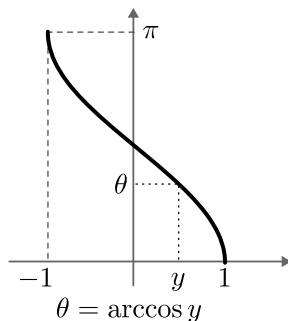
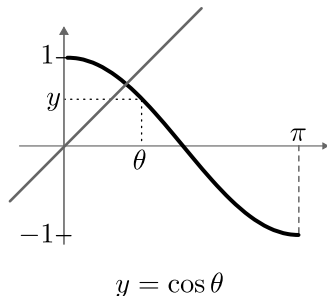
# The inverse sine function

- Domain:  $[-1, 1]$ , range:  $[-\pi/2, \pi/2]$
- Cancellation properties:
  - $\sin(\arcsin(y)) = y$  for all  $y \in [-1, 1]$
  - $\arcsin(\sin(\theta)) = \theta$  for all  $\theta \in [-\pi/2, \pi/2]$ .



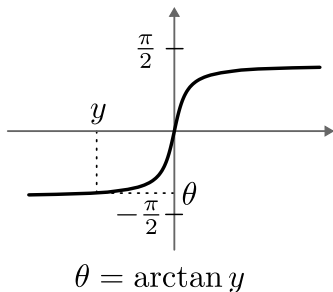
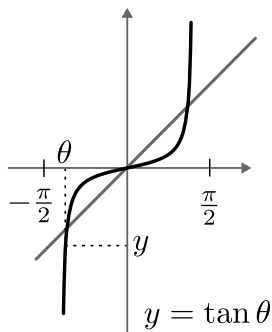
# The inverse cosine function

- Domain:  $[-1, 1]$ , range:  $[0, \pi]$
- Cancellation properties:
  - $\cos(\arccos(y)) = y$  for all  $y \in [-1, 1]$
  - $\arccos(\cos(\theta)) = \theta$  for all  $\theta \in [0, \pi]$



# The inverse tangent function

- Domain: all of  $\mathbb{R}$ , range:  $[-\pi/2, \pi/2]$
- Cancellation properties:
  - $\tan(\arctan(y)) = y$  for all  $y \in \mathbb{R}$
  - $\arctan(\tan \theta) = \theta$  for all  $\theta \in [-\pi/2, \pi/2]$



## Notation (warning)

Don't confuse

$$\sin^k \theta = (\sin \theta)^k, \quad \text{for } k \geq 0$$

with

$$\sin^{-1} x = \arcsin x.$$



## Example

Simplify the following expression:  $\tan \left( \sin^{-1} \frac{2}{3} \right)$ .

## Example

Show that  $\arccos x = \frac{\pi}{2} - \arcsin x$ .