

# Theory of equations (1/2): Working with polynomials

Introduction to Engineering Mathematics

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# Overview

- Polynomial long division
- Synthetic division of polynomials

## Reminder: what is a polynomial?

Polynomial of degree  $n$ :

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

with

- $x$ : the unknown
- $a_0, \dots, a_n$ : the coefficients (in  $\mathbb{C}$  or  $\mathbb{R}$ )

## Reminder: Euclidian division of numbers

For every two integers  $p$  and  $d$ , we can write the fraction  $p/d$  as

$$\frac{p}{d} = q + \frac{r}{d},$$

with  $q$  the **quotient** and  $r$  the **remainder**. In other words

$$p = qd + r.$$

The numbers  $q$  and  $r$  can be found by *long division*.

Properties:

- The remainder  $r$  is always smaller than  $d$ .
- The quotient  $q$  and remainder  $r$  are unique.

## Example

If we divide 20 by 7, we get

$$\frac{20}{7} = 2 + \frac{6}{7},$$

or  $20 = 2 \times 7 + 6$ .

Therefore:

- Quotient:  $q = 2$
- Remainder:  $r = 6$ .

## Euclidian division of polynomials

For every two polynomials  $P(x)$  and  $D(x)$ , we can write the rational function  $P(x)/D(x)$  as

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)},$$

with  $Q(x)$  the **quotient** and  $R(x)$  the **remainder**. In other words

$$P(x) = Q(x)D(x) + R(x).$$

The polynomials  $Q(x)$  and  $R(x)$  can be found by *long division*.

Properties:

- The degree of  $R(x)$  is always smaller than the degree of  $D(x)$ .
- The quotient  $Q(x)$  and remainder  $R(x)$  are unique.

## Example

If we divide  $x^2 + 1$  by  $x - 1$ , we get:

$$\begin{aligned}\frac{x^2 + 1}{x - 1} &= \frac{x^2 - 1 + 2}{x - 1} \\ &= \frac{x^2 - 1}{x - 1} + \frac{2}{x - 1} \\ &= x + 1 + \frac{2}{x - 1}\end{aligned}$$

Therefore, the quotient is  $Q(x) = x + 1$  and the remainder is  $R(x) = 2$ .

## Polynomial long division

If  $P(x) = 3x^4 - x^3 + 2x^2 - 2x - 1$  and  $D(x) = x + 2$ , find the quotient and the remainder after dividing  $P(x)$  by  $D(x)$ .

Algorithm:

- 1 Divide leading term by leading term and write down result
- 2 Multiply result by divisor and transfer the result to the left
- 3 Subtract
- 4 Drop next term
- 5 Repeat steps 1-4.



## Example

Divide  $P(x) = 3x^4 - x^3 + 2x^2 - 2x - 1$  by  $D(x) = x + 2$ .

# Synthetic division

Algorithm:

- 1 Write down coefficients of  $P(x)$  on top
- 2 Write down coefficients of  $-D(x)$  (except the first) on left
- 3 Lower first coefficient
- 4 Multiply and put result back in the table
- 5 Add coefficients in next column
- 6 Repeat step 4-5 until done
- 7 Determine quotient and remainder

## Example

If  $P(x) = x^3 - 12x^2 - 42$  and  $D(x) = x - 3$ , find the quotient and the remainder after dividing  $P(x)$  by  $D(x)$ .

## Special case

$$\frac{x^n - 1}{x - 1} = x^{n-1} + x^{n-2} + \cdots + x + 1.$$

## Resources

Step-by-step walkthroughs of both algorithms (from the 2021-22 version of this course):

- Polynomial long division:  
<https://youtu.be/RyRqUg5oycE?t=499>
- Synthetic division: <https://youtu.be/NqQeMfGEzk4>

These videos can be found on Ufora as well.