Trigonometry (1/5): Introduction and Overview

Introduction to Engineering Mathematics

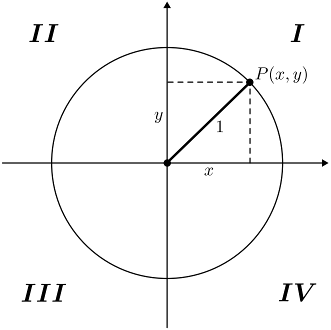
Prof. Joris Vankerschaver

Table of contents

Angles and points on the unit circle

The unit circle

- The circle of radius 1 in the xy-plane, centered on the origin.
- Equation: $x^2 + y^2 = 1$
- Four quadrants: I, II, III, IV

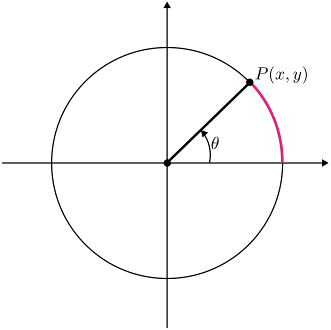


Example

If $P(\sqrt{3}/2, y)$ is a point on the unit circle, find the value of y.

Angles and points on the unit circle

- Each point $P(x, y)$ defines an angle θ measured from the positive x-axis in counterclockwise direction.
- Angles measured in degrees or radians.
  - Value of θ in radians: length of arc subtended by θ (length of the red segment)

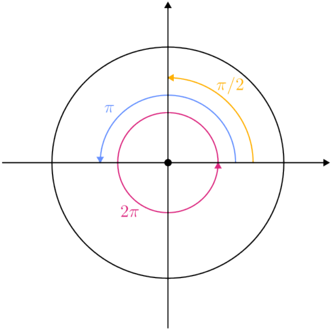


Converting between angles and radians

General formula to convert between degrees and radians:

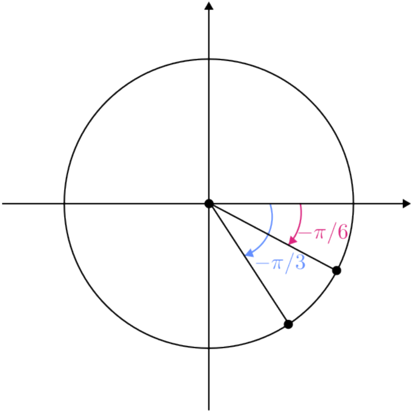
$$\text{degrees} \underset{\times \frac{180}{\pi}}{\overset{\times \frac{\pi}{180}}{\rightleftarrows}} \text{radians}$$

	Degrees	Radians
Full circle	360°	2π
Half circle	180°	π
Quarter circle	90°	$\pi/2$



Negative angles

Measured from the positive x-axis, in clockwise direction.

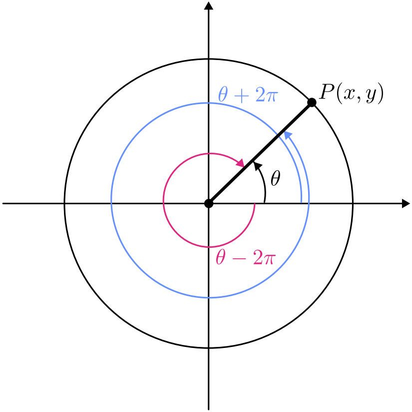


Adding 2π to an angle

- Point P is determined by the angle θ.
- P stays same when adding $\pm 2\pi$ to θ.

$\Rightarrow$ All angles $\theta + 2k\pi$ with $k \in \mathbb{Z}$ give the same point P.

Principal angle: θ such that $-\pi < \theta \leq \pi$.

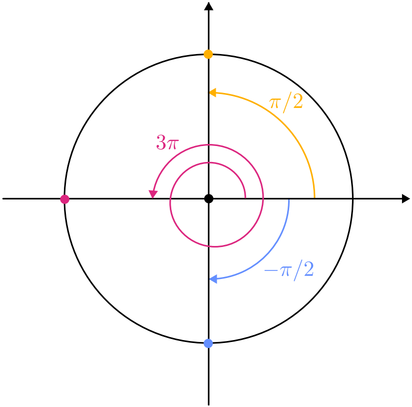


Trigonometric functions as coordinates

Finding the coordinates of a point

Given an angle θ, find the coordinates of $P(x, y)$.

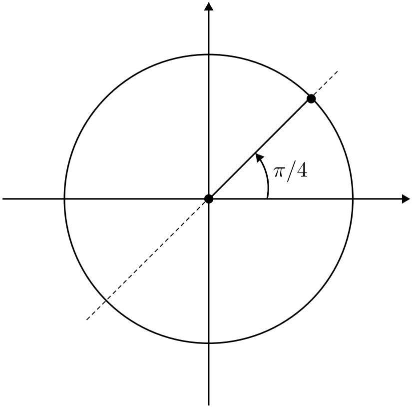
1. $\theta = \pi/2$
2. $\theta = 3\pi$
3. $\theta = -\pi/2$



Finding the coordinates of a point

Slightly more involved case:

4. $\theta = \pi/4$



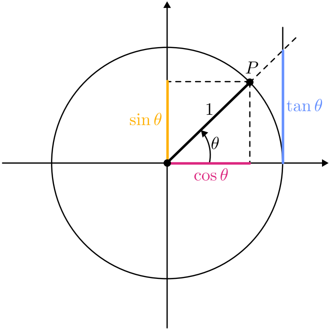
Important angles

Angle	x-coordinate	y-coordinate
0	1	0
$\pi/6$	$\sqrt{3}/2$	$1/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$
$\pi/2$	0	1
π	-1	0
2π	1	0

Trigonometric functions as coordinates

Let θ be an angle with point $P(x, y)$.

Name	Notation	Definition
Cosine	$\cos\theta$	x
Sine	$\sin\theta$	y
Tangent	$\tan\theta$	$\frac{\sin\theta}{\cos\theta}$
Cotangent	$\cot\theta$	$\frac{\cos\theta}{\sin\theta}$
Cosecant	$\csc\theta$	$\frac{1}{\sin\theta}$
Secant	$\sec\theta$	$\frac{1}{\cos\theta}$



Example

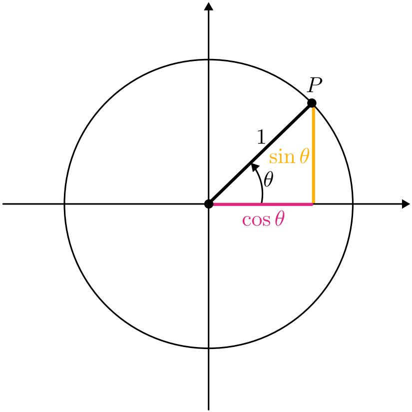
Given that $\theta = \dfrac{\pi}{6}$, find the values of all 6 trigonometric functions.

Basic trigonometric identities

Fundamental identity

- $P(x, y)$ is on the unit circle: $x^2 + y^2 = 1$
- Put $x = \cos\theta$ and $y = \sin\theta$ to obtain the **fundamental identity**:

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$



Aside: notation

Be very careful when you see $\sin^k \theta$.

- Positive exponent (**power**):

$$\sin^k \theta = (\sin \theta)^k.$$

- Negative exponent -1 (**inverse function**):

$$\sin^{-1} y = \arcsin y.$$

Fundamental identity: consequences

$$1 + \tan^2\theta = \sec^2\theta \qquad\qquad 1 + \cot^2\theta = \csc^2\theta$$

Example

Suppose $\cos\theta = -\frac{4}{5}$ and θ is in quadrant III. Find $\sin\theta$ and $\tan\theta$.

Periodicity of sine and cosine

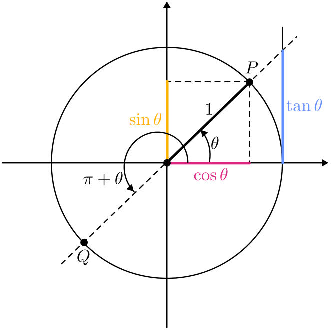
- Sine and cosine are 2π-periodic:

$$\sin(\theta \pm 2\pi) = \sin\theta$$
$$\cos(\theta \pm 2\pi) = \cos\theta$$

- The tangent is π-periodic:

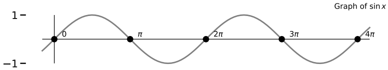
$$\tan(\theta \pm \pi) = \tan\theta$$

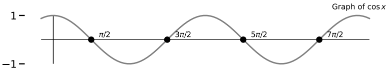
Example: Compute $\tan\left(\dfrac{8093\pi}{4}\right)$



Graphs of trigonometric functions

Graphs of sine/cosine

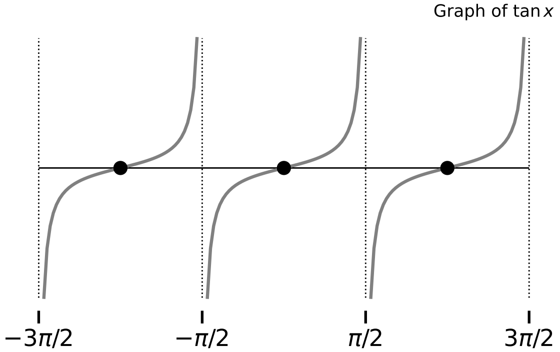




The sine and cosine:

- Are defined for every real number.
- Oscillate between -1 and $+1$.
- Repeat themselves every 2π radians (**fundamental period**).

Graph of the tangent function



The tangent function:

- Is defined for every real number, **except multiples of** $\pi/2$.
- Can take on arbitrary values.
- Repeats itself every π radians (fundamental period).

Example

Find the fundamental period of $\sin(2x)$.