

# Trigonometry (1/5): Introduction and Overview

Introduction to Engineering Mathematics

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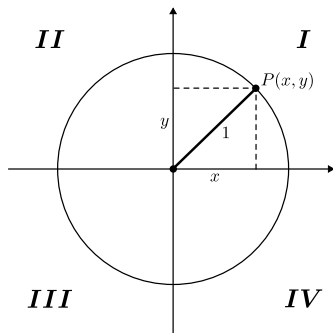
# Contents

- ① Angles and points on the unit circle
- ② Trigonometric functions as coordinates
- ③ Basic trigonometric identities
- ④ Graphs of trigonometric functions

## Angles and points on the unit circle

# The unit circle

- The circle of radius 1 in the  $xy$ -plane, centered on the origin.
- Equation:  $x^2 + y^2 = 1$
- Four quadrants:  $I$ ,  $II$ ,  $III$ ,  $IV$

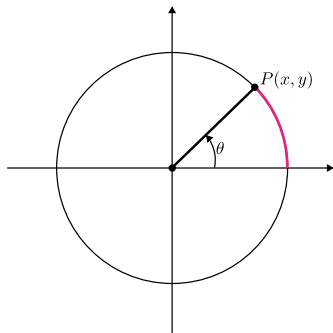


## Example

If  $P(\sqrt{3}/2, y)$  is a point on the unit circle, find the value of  $y$ .

## Angles and points on the unit circle

- Each point  $P(x, y)$  defines an angle  $\theta$  measured from the positive  $x$ -axis in counterclockwise direction.
- Angles measured in degrees or radians.
  - Value of  $\theta$  in radians: length of arc subtended by  $\theta$  (length of the red segment)



# Converting between angles and radians

General formula to convert between degrees and radians:

$$\text{degrees} \begin{array}{c} \xrightarrow{\times \frac{\pi}{180}} \\ \xleftarrow{\times \frac{180}{\pi}} \end{array} \text{radians}$$

	Degrees	Radians
Full circle	$360^\circ$	$2\pi$
Half circle	$180^\circ$	$\pi$
Quarter circle	$90^\circ$	$\pi/2$



## Negative angles

Measured from the positive  $x$ -axis, in clockwise direction.



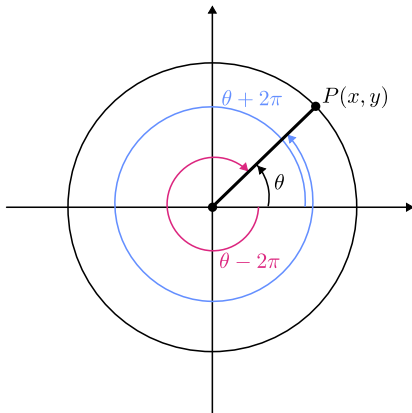


## Adding $2\pi$ to an angle

- Point  $P$  is determined by the angle  $\theta$ .
- $P$  stays same when adding  $\pm 2\pi$  to  $\theta$ .

$\Rightarrow$  All angles  $\theta + 2k\pi$  with  $k \in \mathbb{Z}$  give the same point  $P$ .

**Principal angle:**  $\theta$  such that  $-\pi < \theta \leq \pi$ .



# Trigonometric functions as coordinates

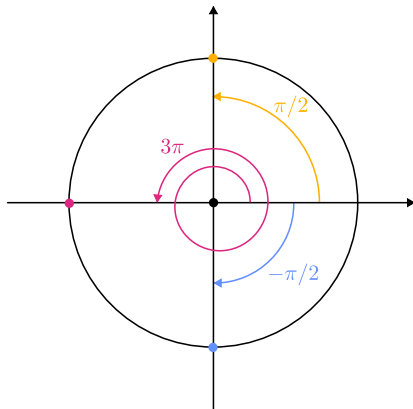
# Finding the coordinates of a point

Given an angle  $\theta$ , find the coordinates of  $P(x, y)$ .

①  $\theta = \pi/2$

②  $\theta = 3\pi$

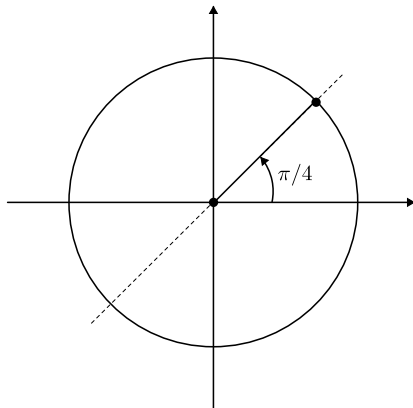
③  $\theta = -\pi/2$



# Finding the coordinates of a point

Slightly more involved case:

④  $\theta = \pi/4$



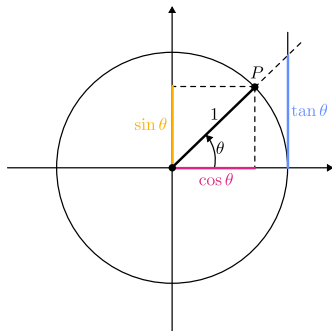
## Important angles

Angle	$x$ -coordinate	$y$ -coordinate
0	1	0
$\pi/6$	$\sqrt{3}/2$	$1/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$
$\pi/2$	0	1
$\pi$	-1	0
$2\pi$	1	0

# Trigonometric functions as coordinates

Let  $\theta$  be an angle with point  $P(x, y)$ .

Name	Notation	Definition
Cosine	$\cos \theta$	$x$
Sine	$\sin \theta$	$y$
Tangent	$\tan \theta$	$\frac{\sin \theta}{\cos \theta}$
Cotangent	$\cot \theta$	$\frac{\cos \theta}{\sin \theta}$
Cosecant	$\csc \theta$	$\frac{1}{\sin \theta}$
Secant	$\sec \theta$	$\frac{1}{\cos \theta}$



## Example

Given that  $\theta = \frac{\pi}{6}$ , find the values of all 6 trigonometric functions.

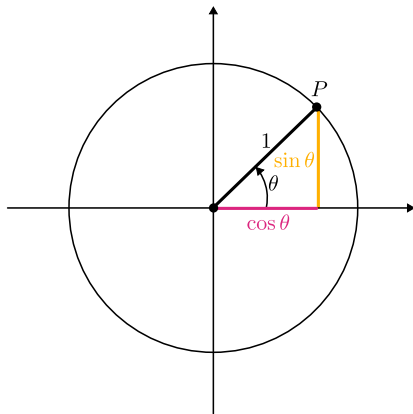
# Basic trigonometric identities



## Fundamental identity

- $P(x, y)$  is on the unit circle:  $x^2 + y^2 = 1$
- Put  $x = \cos \theta$  and  $y = \sin \theta$  to obtain the **fundamental identity**:

$$\cos^2 \theta + \sin^2 \theta = 1$$



## Aside: notation

Be very careful when you see  $\sin^k \theta$ .

- Positive exponent (**power**):

$$\sin^k \theta = (\sin \theta)^k.$$

- Negative exponent  $-1$  (**inverse function**):

$$\sin^{-1} y = \arcsin y.$$

## Fundamental identity: consequences

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

## Example

Suppose  $\cos \theta = -\frac{4}{5}$  and  $\theta$  is in quadrant III. Find  $\sin \theta$  and  $\tan \theta$ .

# Periodicity of sine and cosine

- Sine and cosine are  $2\pi$ -periodic:

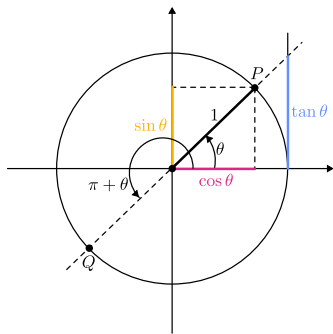
$$\sin(\theta \pm 2\pi) = \sin \theta$$

$$\cos(\theta \pm 2\pi) = \cos \theta$$

- The tangent is  $\pi$ -periodic:

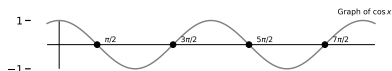
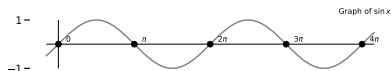
$$\tan(\theta \pm \pi) = \tan \theta$$

Example: Compute  $\tan\left(\frac{8093\pi}{4}\right)$



# Graphs of trigonometric functions

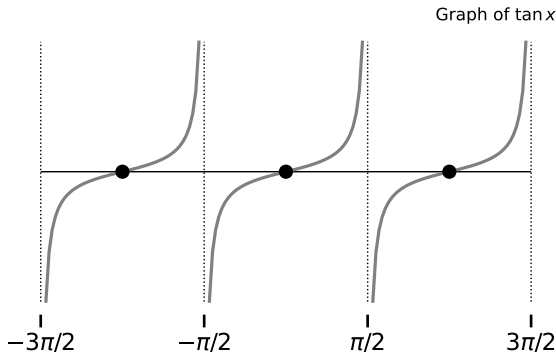
# Graphs of sine/cosine



The sine and cosine:

- Are defined for every real number.
- Oscillate between  $-1$  and  $+1$ .
- Repeat themselves every  $2\pi$  radians (**fundamental period**).

# Graph of the tangent function



The tangent function:

- Is defined for every real number, **except multiples of  $\pi/2$** .
- Can take on arbitrary values.
- Repeats itself every  $\pi$  radians (fundamental period).



## Example

Find the fundamental period of  $\sin(2x)$ .

