

Trigonometry (5/5): Inverse Trigonometric Functions

Introduction to Engineering Mathematics

Prof. Joris Vankerschaver

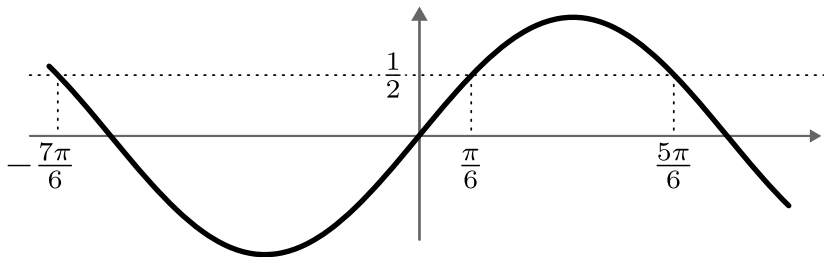
Overview

- 1 Definition of the inverse trigonometric functions
- 2 Examples

Inverting the sine function

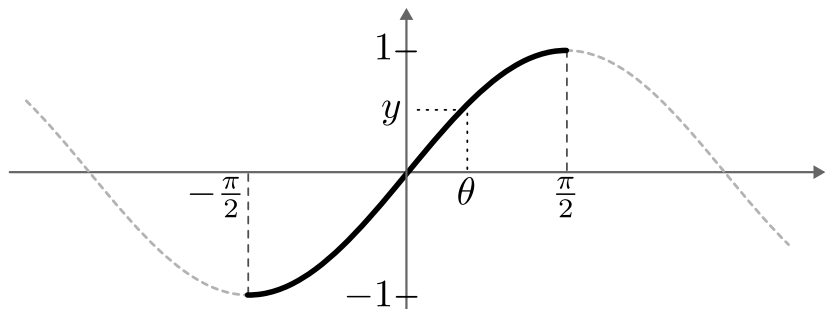
- The sine function turns angles into sine values.
- The **inverse sine** turns sine values back into angles.
- Notation: $\sin^{-1}(x)$, $\arcsin x$

Problem: Many values in the range correspond to the same angle!



Restricting the domain

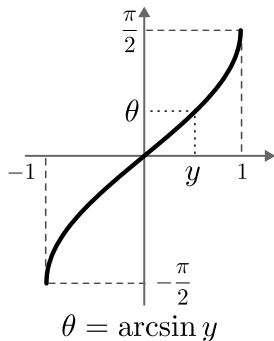
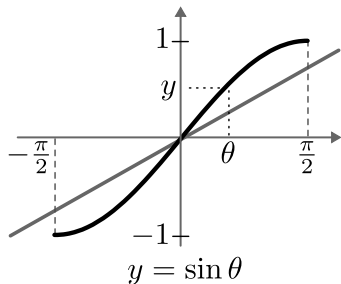
Solution: restrict the domain of the sine function so that there is exactly one angle corresponding to each value.



This gives us a meaningful way to define the inverse sine.

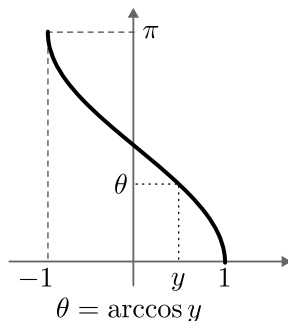
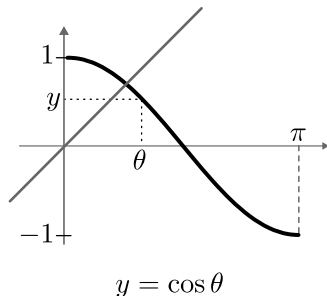
The inverse sine function

- Domain: $[-1, 1]$, range: $[-\pi/2, \pi/2]$
- Cancellation properties:
 - $\sin(\arcsin(y)) = y$ for all $y \in [-1, 1]$
 - $\arcsin(\sin(\theta)) = \theta$ for all $\theta \in [-\pi/2, \pi/2]$.



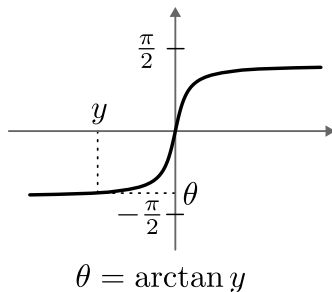
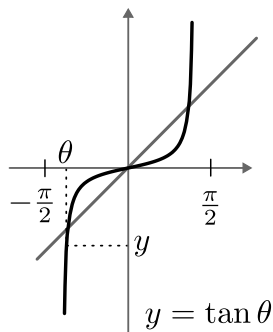
The inverse cosine function

- Domain: $[-1, 1]$, range: $[0, \pi]$
- Cancellation properties:
 - $\cos(\arccos(y)) = y$ for all $y \in [-1, 1]$
 - $\arccos(\cos(\theta)) = \theta$ for all $\theta \in [0, \pi]$



The inverse tangent function

- Domain: all of \mathbb{R} , range: $[-\pi/2, \pi/2]$
- Cancellation properties:
 - $\tan(\arctan(y)) = y$ for all $y \in \mathbb{R}$
 - $\arctan(\tan \theta) = \theta$ for all $\theta \in [-\pi/2, \pi/2]$



Notation (warning)

Don't confuse

$$\sin^k \theta = (\sin \theta)^k, \quad \text{for } k \geq 0$$

with

$$\sin^{-1} x = \arcsin x.$$

Example

Simplify the following expression: $\tan \left(\sin^{-1} \frac{2}{3} \right)$.

Example

Show that $\arccos x = \frac{\pi}{2} - \arcsin x$.