# Trigonometry (1/5): Introduction and Overview Introduction to Engineering Mathematics

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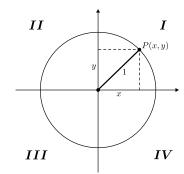
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# Angles and points on the unit circle

#### The unit circle

- The circle of radius 1 in the xy-plane, centered on the origin.
- Equation:  $x^2 + y^2 = 1$
- ullet Four quadrants: I, II, III, IV

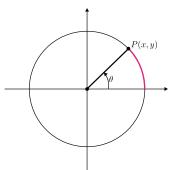


## Example

If  $P(\sqrt{3}/2,y)$  is a point on the unit circle, find the value of y.

# Angles and points on the unit circle

- Each point P(x,y) defines an angle  $\theta$  measured from the positive x-axis in counterclockwise direction.
- Angles measured in degrees or radians.
  - Value of  $\theta$  in radians: length of arc subtended by  $\theta$  (length of the red segment)

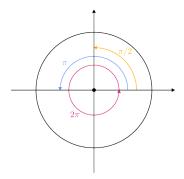


#### Converting between angles and radians

General formula to convert between degrees and radians:

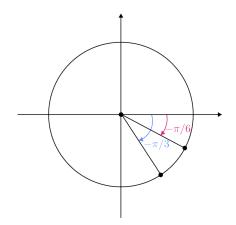
degrees 
$$\xrightarrow{\times \frac{\pi}{180}}$$
 radians

	Degrees	Radians
Full circle	$360^{\circ}$	$2\pi$
Half circle	$180^{\circ}$	$\pi$
Quarter circle	$90^{\circ}$	$\pi/2$



## Negative angles

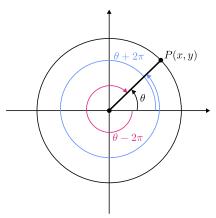
Measured from the positive x-axis, in clockwise direction.



#### Adding $2\pi$ to an angle

- Point P is determined by the angle  $\theta$ .
- P stays same when adding  $\pm 2\pi$  to  $\theta$ .
- $\Rightarrow$  All angles  $\theta + 2k\pi$  with  $k \in \mathbb{Z}$  give the same point P.

**Principal angle**:  $\theta$  such that  $-\pi < \theta \le \pi$ .

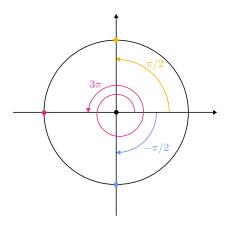


## Trigonometric functions as coordinates

# Finding the coordinates of a point

Given an angle  $\theta$ , find the coordinates of P(x,y).

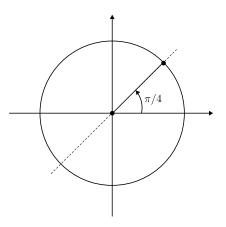
- $\bullet = \pi/2$
- $\theta = 3\pi$
- **3**  $\theta = -\pi/2$



# Finding the coordinates of a point

Slightly more involved case:

$$\bullet \theta = \pi/4$$



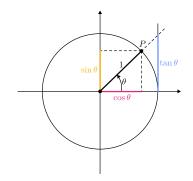
# Important angles

Angle	x-coordinate	y-coordinate
0	1	0
$\pi/6$	$\sqrt{3}/2$	1/2
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	1/2	$\sqrt{3}/2$
$\pi/2$	0	1
$\pi$	-1	0
$2\pi$	1	0

# Trigonometric functions as coordinates

Let  $\theta$  be an angle with point P(x,y).

Name	Notation	Definition
Cosine	$\cos \theta$	$\overline{x}$
Sine	$\sin \theta$	y
Tangent	$\tan \theta$	$\frac{\sin \theta}{\cos \theta}$
Cotangent	$\cot \theta$	$\frac{\cos \theta}{\sin \theta}$
Cosecant	$\csc \theta$	$\frac{1}{\sin \theta}$
Secant	$\sec \theta$	$\frac{1}{\cos \theta}$



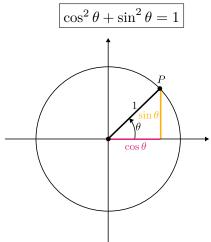
#### Example

Given that  $\theta=\frac{\pi}{6}$ , find the values of all 6 trigonometric functions.

# Basic trigonometric identities

## Fundamental identity

- P(x,y) is on the unit circle:  $x^2 + y^2 = 1$
- Put  $x=\cos\theta$  and  $y=\sin\theta$  to obtain the fundamental identity:



#### Aside: notation

Be very careful when you see  $\sin^k \theta$ .

• Positive exponent (power):

$$\sin^k \theta = (\sin \theta)^k.$$

• Negative exponent -1 (inverse function):

$$\sin^{-1} y = \arcsin y.$$

## Fundamental identity: consequences

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

#### Example

Suppose  $\cos\theta=-\frac{4}{5}$  and  $\theta$  is in quadrant III. Find  $\sin\theta$  and  $\tan\theta$ .

# Periodicity of sine and cosine

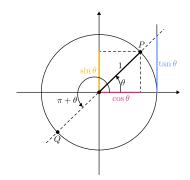
• Sine and cosine are  $2\pi$ -periodic:

$$\sin(\theta \pm 2\pi) = \sin \theta$$
$$\cos(\theta \pm 2\pi) = \cos \theta$$

• The tangent is  $\pi$ -periodic:

$$\tan(\theta \pm \pi) = \tan\theta$$

Example: Compute  $\tan\left(\frac{8093\pi}{4}\right)$ 



# Graphs of trigonometric functions

## Graphs of sine/cosine

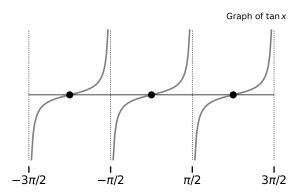




#### The sine and cosine:

- Are defined for every real number.
- Oscillate between -1 and +1.
- Repeat themselves every  $2\pi$  radians (fundamental period).

#### Graph of the tangent function



#### The tangent function:

- Is defined for every real number, **except multiples of**  $\pi/2$ .
- Can take on arbitrary values.
- Repeats itself every  $\pi$  radians (fundamental period).

## Example

Find the fundamental period of  $\sin(2x)$ .

