

Trigonometry (1/5): Introduction and Overview

Introduction to Engineering Mathematics

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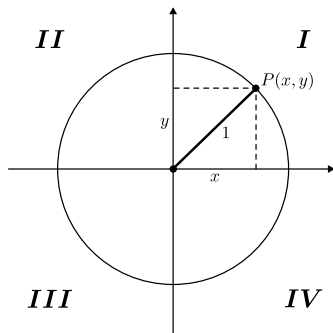
Contents

- ① Angles and points on the unit circle
- ② Trigonometric functions as coordinates
- ③ Basic trigonometric identities
- ④ Graphs of trigonometric functions

Angles and points on the unit circle

The unit circle

- The circle of radius 1 in the xy -plane, centered on the origin.
- Equation: $x^2 + y^2 = 1$
- Four quadrants: I , II , III , IV

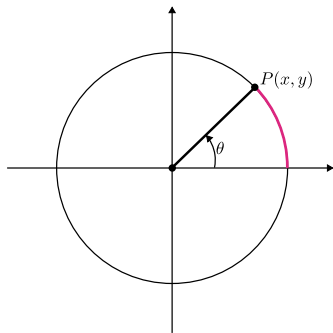


Example

If $P(\sqrt{3}/2, y)$ is a point on the unit circle, find the value of y .

Angles and points on the unit circle

- Each point $P(x, y)$ defines an angle θ measured from the positive x -axis in counterclockwise direction.
- Angles measured in degrees or radians.
 - Value of θ in radians: length of arc subtended by θ (length of the red segment)



Converting between angles and radians

General formula to convert between degrees and radians:

$$\text{degrees} \begin{array}{c} \xrightarrow{\times \frac{\pi}{180}} \\ \xleftarrow{\times \frac{180}{\pi}} \end{array} \text{radians}$$

| | Degrees | Radians |
|----------------|-------------|---------|
| Full circle | 360° | 2π |
| Half circle | 180° | π |
| Quarter circle | 90° | $\pi/2$ |



Negative angles

Measured from the positive x -axis, in clockwise direction.

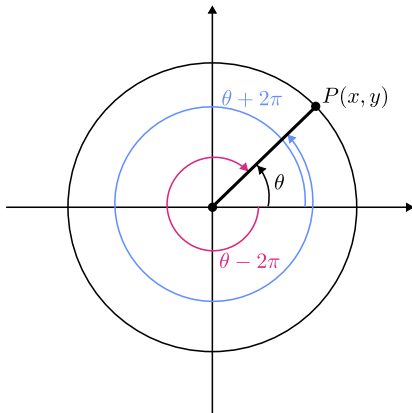


Adding 2π to an angle

- Point P is determined by the angle θ .
- P stays same when adding $\pm 2\pi$ to θ .

\Rightarrow All angles $\theta + 2k\pi$ with $k \in \mathbb{Z}$ give the same point P .

Principal angle: θ such that $-\pi < \theta \leq \pi$.



Trigonometric functions as coordinates

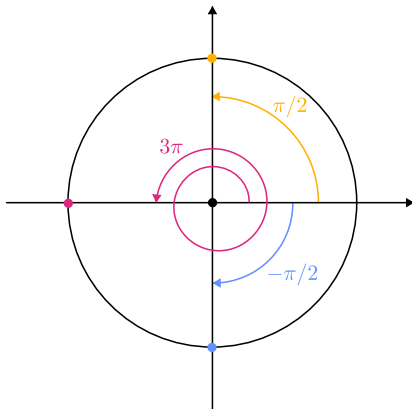
Finding the coordinates of a point

Given an angle θ , find the coordinates of $P(x, y)$.

① $\theta = \pi/2$

② $\theta = 3\pi$

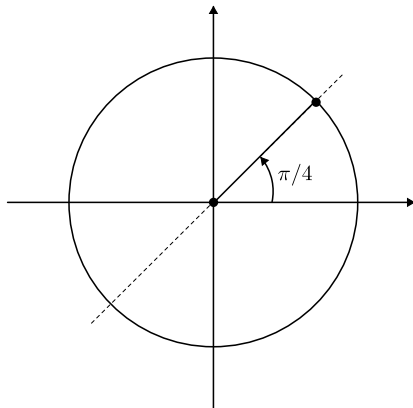
③ $\theta = -\pi/2$



Finding the coordinates of a point

Slightly more involved case:

④ $\theta = \pi/4$



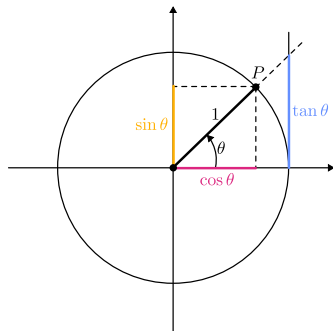
Important angles

| Angle | x -coordinate | y -coordinate |
|---------|-----------------|-----------------|
| 0 | 1 | 0 |
| $\pi/6$ | $\sqrt{3}/2$ | $1/2$ |
| $\pi/4$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ |
| $\pi/3$ | $1/2$ | $\sqrt{3}/2$ |
| $\pi/2$ | 0 | 1 |
| π | -1 | 0 |
| 2π | 1 | 0 |

Trigonometric functions as coordinates

Let θ be an angle with point $P(x, y)$.

| Name | Notation | Definition |
|-----------|---------------|-----------------------------------|
| Cosine | $\cos \theta$ | x |
| Sine | $\sin \theta$ | y |
| Tangent | $\tan \theta$ | $\frac{\sin \theta}{\cos \theta}$ |
| Cotangent | $\cot \theta$ | $\frac{\cos \theta}{\sin \theta}$ |
| Cosecant | $\csc \theta$ | $\frac{1}{\sin \theta}$ |
| Secant | $\sec \theta$ | $\frac{1}{\cos \theta}$ |



Example

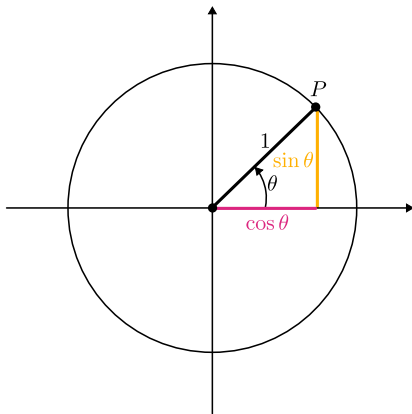
Given that $\theta = \frac{\pi}{6}$, find the values of all 6 trigonometric functions.

Basic trigonometric identities

Fundamental identity

- $P(x, y)$ is on the unit circle: $x^2 + y^2 = 1$
- Put $x = \cos \theta$ and $y = \sin \theta$ to obtain the **fundamental identity**:

$$\cos^2 \theta + \sin^2 \theta = 1$$



Aside: notation

Be very careful when you see $\sin^k \theta$.

- Positive exponent (**power**):

$$\sin^k \theta = (\sin \theta)^k.$$

- Negative exponent -1 (**inverse function**):

$$\sin^{-1} y = \arcsin y.$$

Fundamental identity: consequences

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Example

Suppose $\cos \theta = -\frac{4}{5}$ and θ is in quadrant III. Find $\sin \theta$ and $\tan \theta$.

Periodicity of sine and cosine

- Sine and cosine are 2π -periodic:

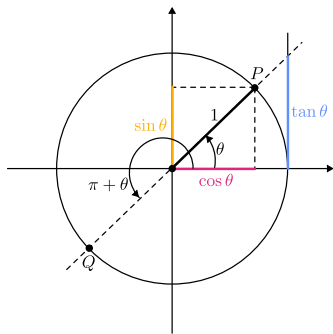
$$\sin(\theta \pm 2\pi) = \sin \theta$$

$$\cos(\theta \pm 2\pi) = \cos \theta$$

- The tangent is π -periodic:

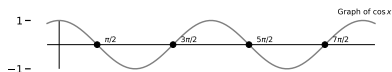
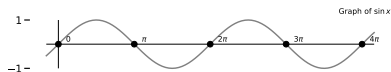
$$\tan(\theta \pm \pi) = \tan \theta$$

Example: Compute $\tan\left(\frac{8093\pi}{4}\right)$



Graphs of trigonometric functions

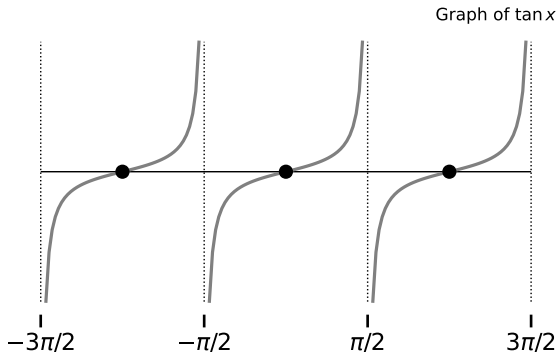
Graphs of sine/cosine



The sine and cosine:

- Are defined for every real number.
- Oscillate between -1 and $+1$.
- Repeat themselves every 2π radians (**fundamental period**).

Graph of the tangent function



The tangent function:

- Is defined for every real number, **except multiples of $\pi/2$** .
- Can take on arbitrary values.
- Repeats itself every π radians (fundamental period).

Example

Find the fundamental period of $\sin(2x)$.

