Introduction to Statistical Modeling

Outliers

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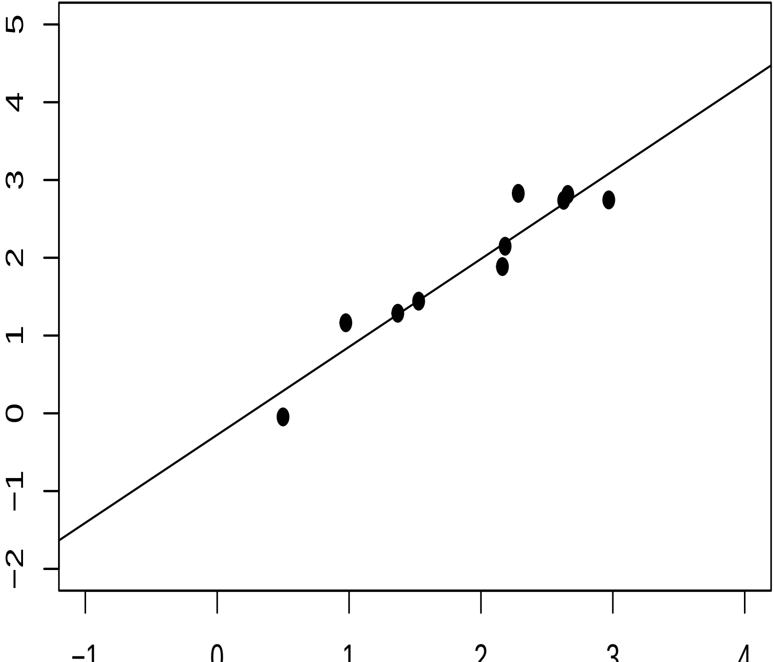
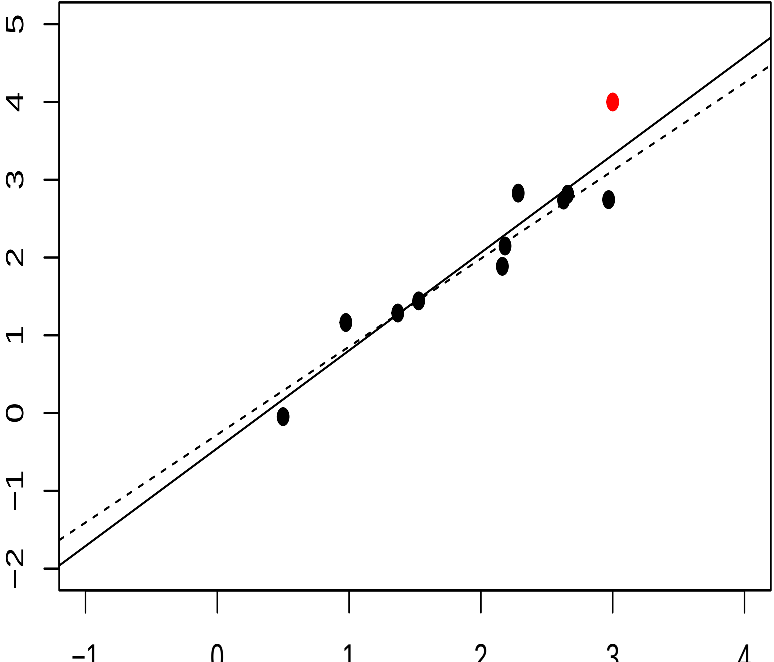
Outliers / Influential observations

- Dataset often contains extreme values for outcome Y and/or predictors X
- These **can** influence regression line strongly (but don't have to)

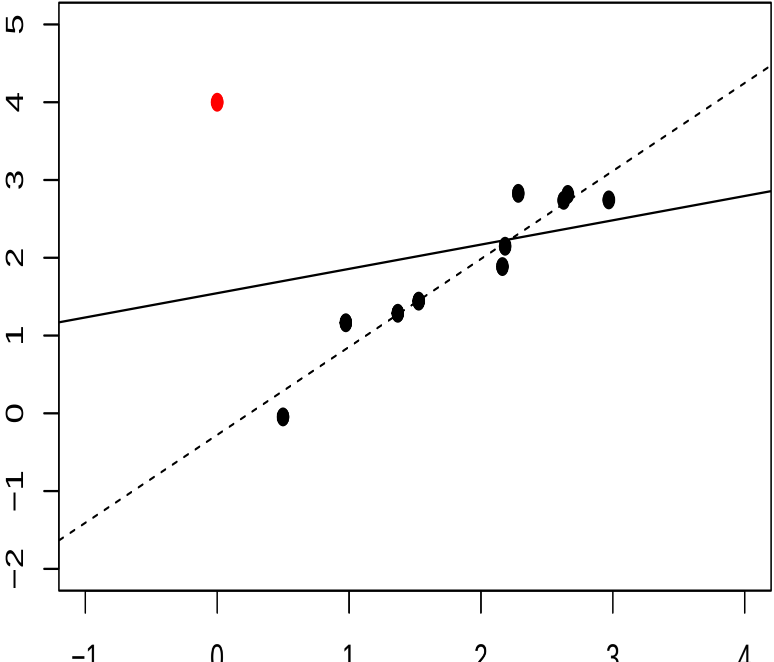
Influence of influential observations



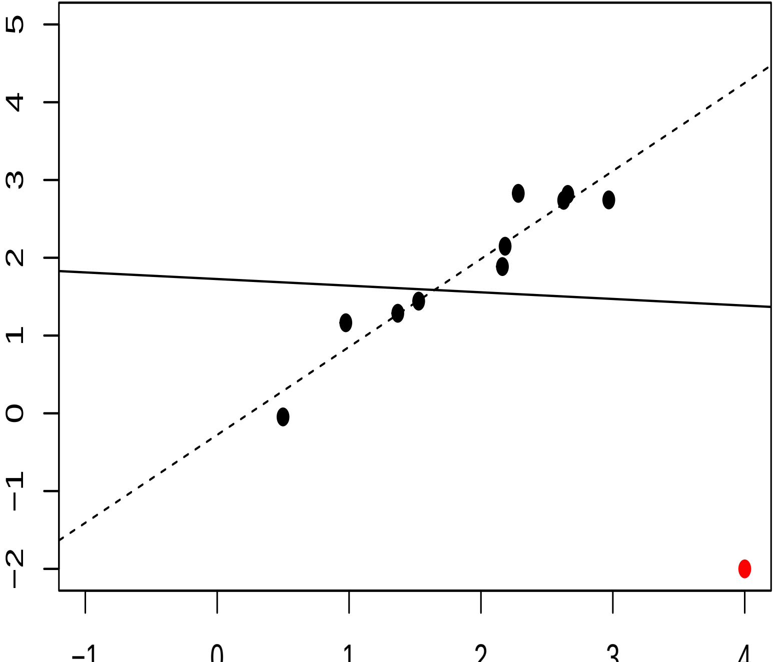
Influence of influential observations



Influence of influential observations



Influence of influential observations



Tracking influential observations

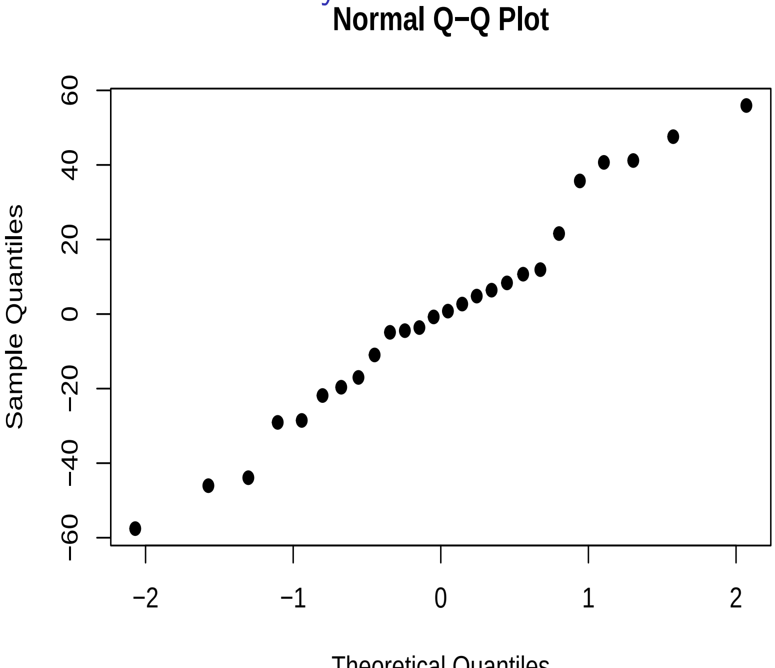
Residuals:

- Indicate how far outcome deviates from regression line
- Normally distributed with mean 0 and variance $\sigma^2 = MSE$.

Hence, can be used to identify extreme outcomes:

- 95% of residuals expected in interval $[-2\sigma, 2\sigma]$
- Observations where residual is much larger are probably outliers.

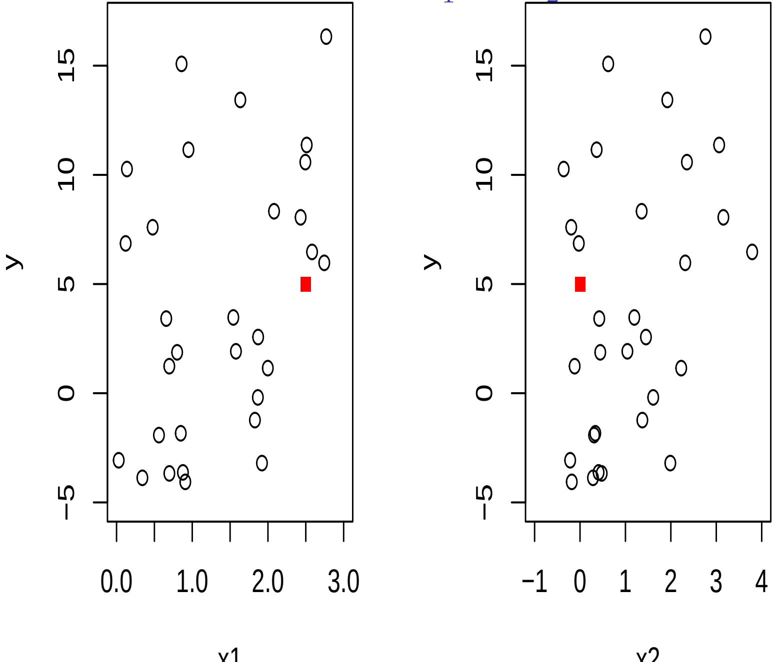
Exteme outcomes in analysis larches?



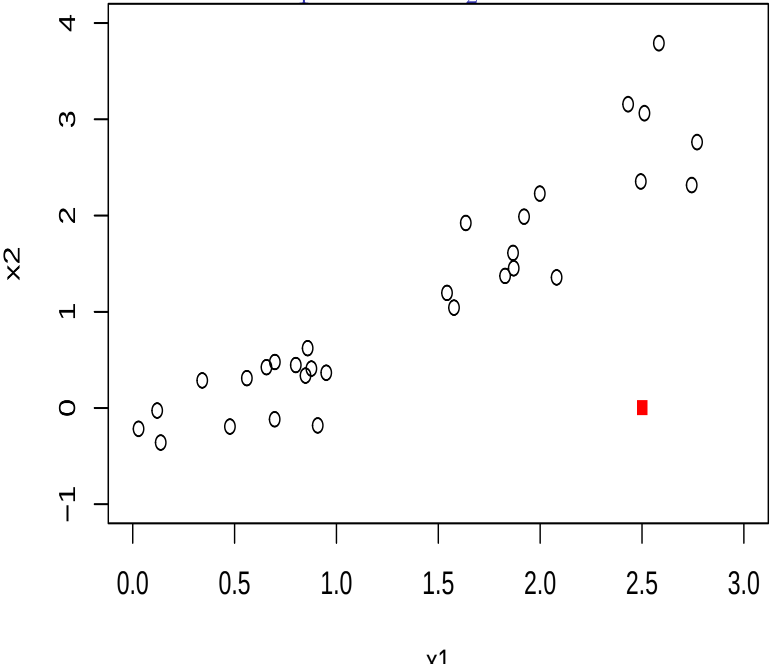
Tracking influential observations

- **Scatterplots** of outcome in function of predictors can be used to identify extreme outcomes and predictors
- When multiple predictors, these plots have serious shortcomings

Multivariate outliers: Y versus X_1 or X_2



Multivariate outliers: X_1 versus X_2



Leverage

- Diagnostic measure to identify influential predictor-observations
- Data point has high leverage if it has “extreme” predictor values (low or high)

Mathematically:

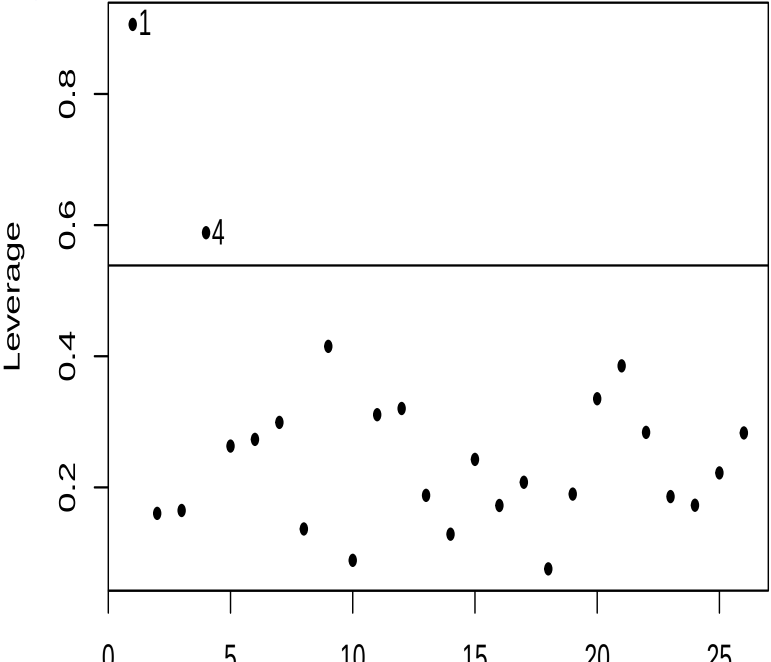
- Weighted distance between predictor for observation i and mean predictor.
- How much the ith observed value affects the ith fitted value:

$$h_{ii} = \frac{\partial \hat{y}_i}{\partial y_i}.$$

Interpretation of leverage

- If high leverage for i^{th} observation, then
  - it has predictor values that deviate strongly from the mean
  - it **possibly** has large influence on regression coefficients and predictions
- Leverage is on average p/n with p number of unknown parameters
- **Extreme leverage**: larger than $2p/n$

Leverage in analysis of larches



Cook's distance

- Diagnostic measure for influence of i^{th} observation on all predictions / estimated coefficients.
- Cook's distance for i^{th} observation is obtained by comparing each prediction $\widehat{Y}_j$ with prediction $\widehat{Y}_{j(i)}$ that would be obtained **if i^{th} observation was deleted**:

$$D_i = \frac{\sum_{j=1}^{n} (\hat{Y}_j - \hat{Y}_{j(i)})^2}{p \cdot \mathrm{MSE}}$$

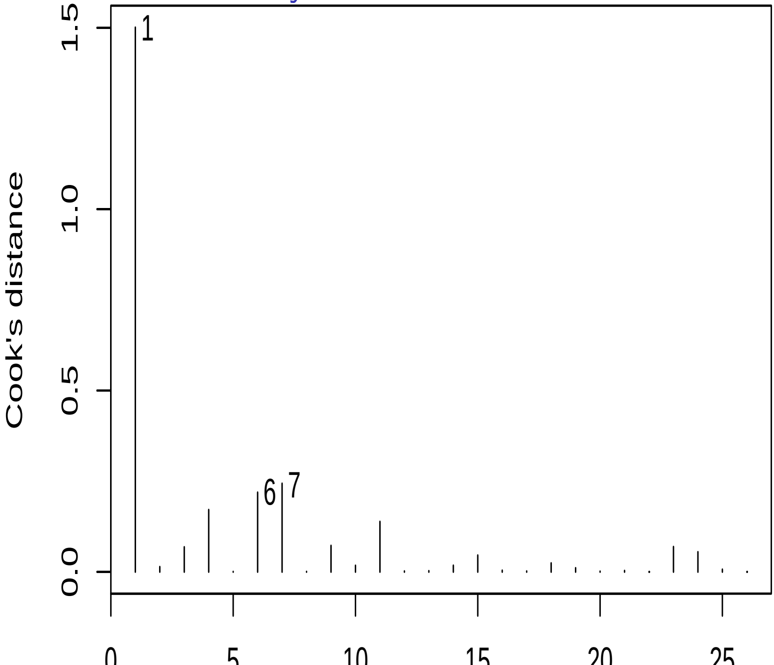
Interpretation Cook's distance

- If Cook's distance D_i large, then i^{th} observation has large influence on predictions and coefficients
- **Extreme Cook's distance**: exceeds 50% percentile of $F_{p,n-p}$-distribution

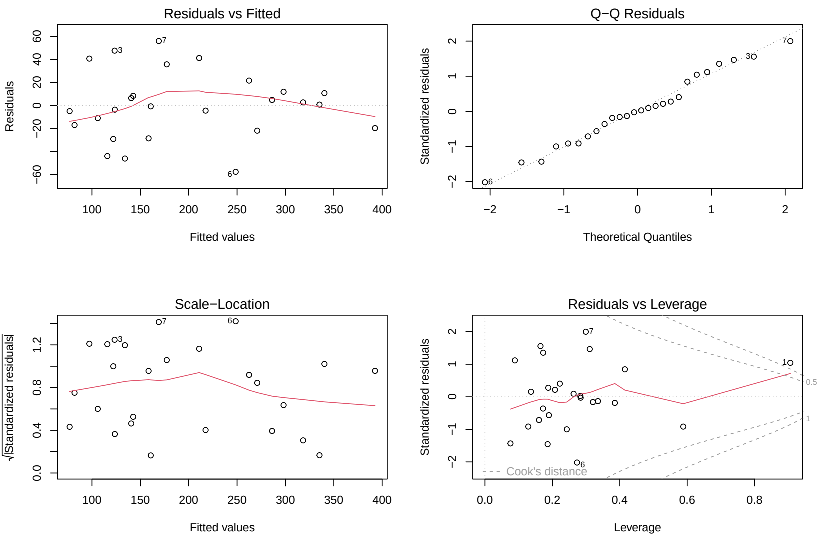
Example:

- In analysis of larches is $p = 7, n = 26$ and the 50% percentile of $F_{p,n-p}$-distribution 0.94
- Cook's distance of first observation is 1.5 and corresponds to 77% percentile
- Conclusion: first observation has large influence on estimated regression coefficients

Cook's distance in analysis of larches



Analysis of larches: residual plots



DFBETAs

On what coefficient(s) will first observation have large influence?

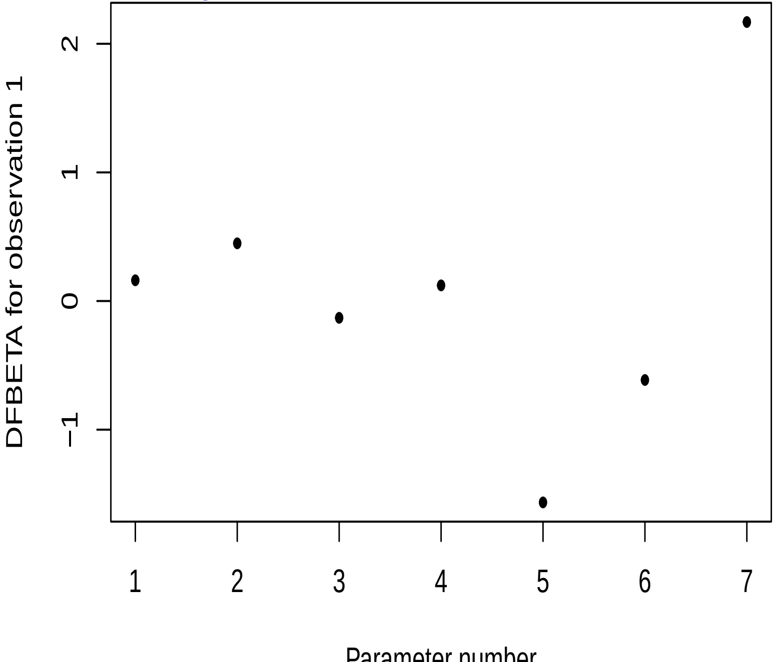
- Diagnostic measure for influence of i^{th} observation **on each regression coefficient separately**
- DFBETAs for i^{th} observation and j^{th} coefficient is obtained by comparing j^{th} coefficient $\hat{\beta}_j$ with coefficient $\hat{\beta}_{j(i)}$ from model **if i^{th} observation would have been deleted**

$$\mathrm{DFBETA}_{j(i)} = \frac{\hat{\beta}_j - \hat{\beta}_{j(i)}}{\mathrm{SD}(\hat{\beta}_j)}$$

Interpretation DFBETAs

- Sign indicates if deleting observation i causes an increase (DFBETA< 0) or decrease (DFBETA> 0) in each coefficient
- **Extreme DFBETAs**: exceeds 1 in small to moderate datasets, and $2/\sqrt{n}$ in large datasets

DFBETAs in analysis of larches



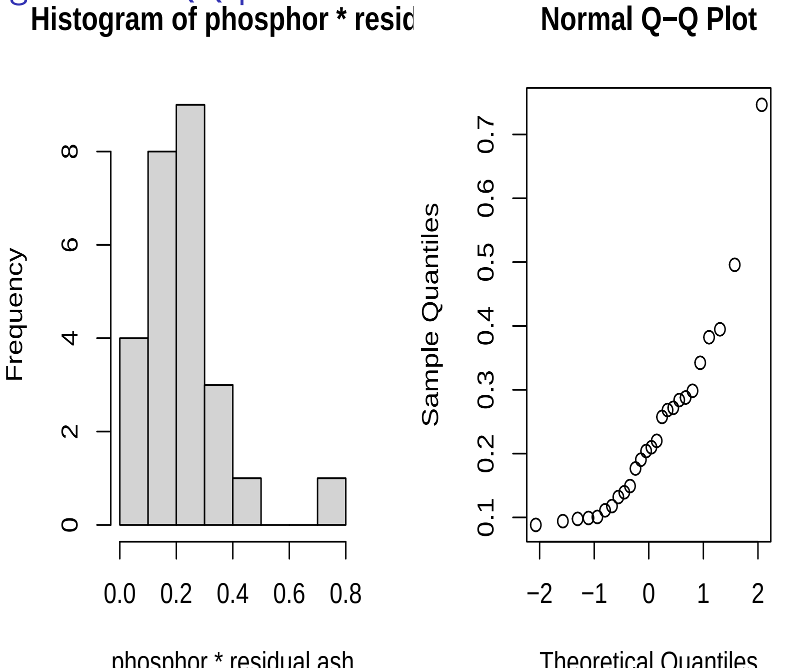
DFBETAs in analysis of larches

First observation has **large influence on interaction between phosphorus and residual ash**:

- current coefficient is -598.08 (SE 290.02);
- DFBETA is 2.17;
- after deletion of first observation, interaction between phosphorus and residual ash will be around

$$-598.08 - 2.17 \times 290.02 = -1227.42$$

Histogram and QQ-plot of interaction



Analysis of larches after deletion 1^{st} observation

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	101.8433	170.89955	0.5959248	0.558645291
nitrogen2	-194.2135	105.36608	-1.8432260	0.081826133
phosphor2	-911.1361	686.55995	-1.3271035	0.201063465
potassium2	132.9597	41.41191	3.2106631	0.004847492
residu2	332.9542	159.95466	2.0815538	0.051930532
nitrogen2:phosphor2	1179.3050	429.71839	2.7443671	0.013331564
phosphor2:residu2	-1225.6303	665.87615	-1.8406279	0.082222531

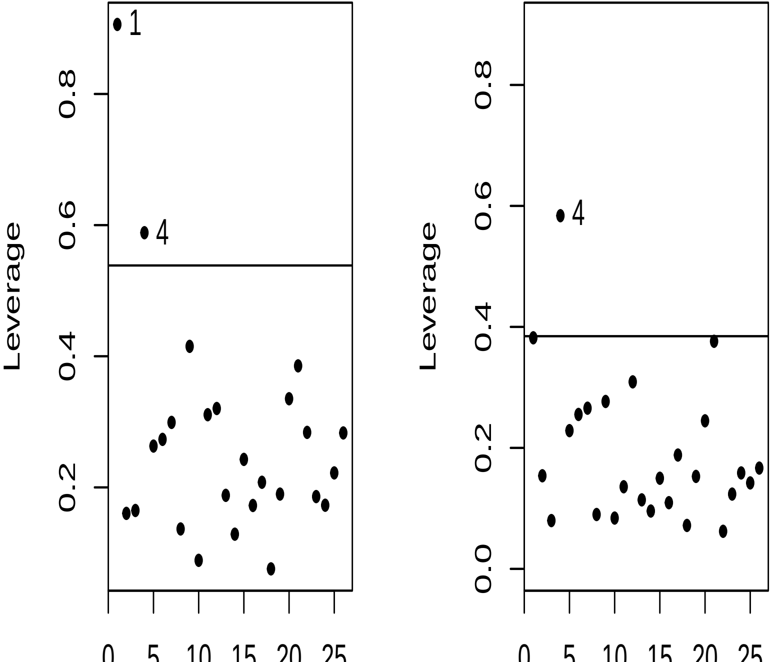
Analysis of larches after deletion interaction

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	134.47244	182.5540	0.7366172	0.469908260
nitrogen	-66.35461	94.9178	-0.6990744	0.492556067
phosphor	-1024.58992	736.3183	-1.3915041	0.179357766
potassium	128.83662	44.2072	2.9143806	0.008574138
residu	23.51194	36.0929	0.6514284	0.522185637
nitrogen:phosphor	661.49644	370.7815	1.7840600	0.089594772

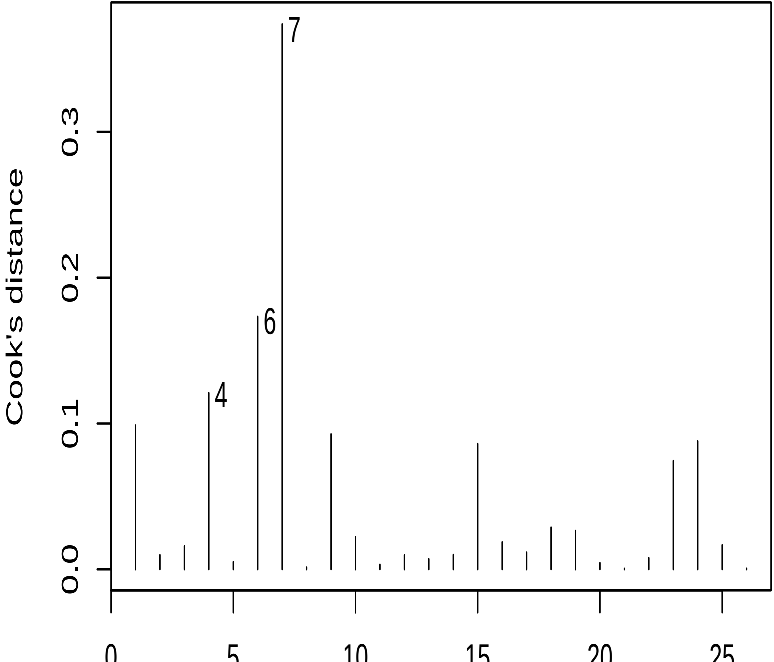
Analysis of larches: final model

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	160.66283	175.61424	0.9148622	0.370649894
nitrogen	-76.49677	92.34000	-0.8284250	0.416746264
phosphor	-1120.70470	711.42841	-1.5752881	0.130135986
potassium	138.06170	41.29966	3.3429260	0.003084272
nitrogen:phosphor	724.38231	353.05353	2.0517634	0.052870451

Final analysis: leverage



Final analysis: Cook's distance



Final analysis: residual plots