Lecture 1

Rapid course in special relativity; Key ideas:

Principle of relativity [TW 3.1, 3.2, 3.3 (1.3)]

Inertial (Free Float) Frames [TW 2.2, 2.3, 2.4 (1.2, 1.4); MTW Fig. 1.7]

Isotropy of Space [TW (1.3), 3.12]

$\boxed{\text{Note}}$:

[TW 3.1, 3.2, 3.3 (1.3)] means sections 3.1, 3.2, 3.3 in the 2nd (NEW!) edition of E. Taylor & J.A. Wheeler, Spacetime Physics (1992)

The (1.3) refers to section (1.3) in the 1st (OLD) edition (1966)

Outline of Lecture 1

1. The inductive structure of this course

2. Rapid Course in Special Relativity

   (1) P. of R.
   (2) Free float ("inertial") frames
   (3) Isotropy of space

Next Lecture
   (4) Invariance of the Interval
   (5) Relativity of simultaneity.

The purpose of this course is to develop and then perform a conceptual integration – a marriage – between multilinear algebra and multivariable calculus on steroids. The result will be modern differential geometry, an expression of moving frames codified by means of Cartan's two universal structural equations.

Special relativity is the source, the precondition for this conceptual integration – Cartan's two structural equations is the result.

Thus, the inductive structure of Math 5756 is as follows

Moving Frames:
Cartan's Structural Equations

Multilinear algebra
+
Multivariable calculus on steroids

Special Relativity

Please note that the conceptual basis, the root - the fountainhead - of modern differential geometry is observation: special relativity, i.e physics.

Relativity is composed of (1) linear spacetime geometry, which is Special Relativity and (2) curvilinear spacetime geometry, which is General Relativity. We shall develop both of them. First we consider linear spacetime geometry.

Thus we start with
A RAPID COURSE IN SPECIAL RELATIVITY

The key ideas of special relativity are:

	OLD Red	NEW Blue
(1) the Principle of Relativity	T-W 1.3	3.1-3.3
(2) Static Inertial (Free Float) Frame	MTW Fig. 1.7; T-W 1.2, 1.4	2.2-3.4
(3) Isotropy of Space in every Frame.		3.12
(4) Invariance of the Interval	1.5	1.2, 3.6-3.8
(5) Relativity of Simultaneity.		3.4

I) What is the Principle of Relativity, how did we arrive at it, and where did it come from?

1. The P. of R. is a generalization which, in its positive form, is stated as follows:

"All laws of physics are the same in every inertial reference frame."

More on that later

2. The P. of R. was arrived at by induction

Induction is the process of inferring generalizations from particular instances.

The Structure of Inductive Reasoning is explicated in Chap 1 of "The Logical Leap: Induction in Physics" by David Harriman.

3. The P. of R. comes from
(i) observational evidence plus
(ii) the relevant conceptual framework.

II.)

For our context the conceptual framework consists of the laws of physics:

1. Newton's three laws of motion
2. Lorentz's law of motion for a charge in an e. m. field.
3. Maxwell's laws of electromagnetism
4. Thermodynamics, etc.

III.) The observational evidence is consist of on a comparison between the outcomes of experiments in different inertial frames.

More explicitly, consider
two experiments in two different inertial reference frames.
Suppose that these experiments are the same, i.e. one has

1. identical instructions
2. same experimental setup
3. same procedure
4. same data set
5. same data reduction

Then, within experimental errors(!) one will observe the same result.
From this particular pair of experiments one infers the following generalization

In different inertial frames
the same experiments yield
within experimental errors
the same observed results.

This finding is the same regardless of whether the two experiments involve

Newtonian mechanics
electricity and magnetism
thermodynamics or
any combination of these
laws of physics.

Laws of Physics: 1. Newton's 3 laws of motion;
3. Maxwell's electrodynamics
2. Lorentz's Law of motion for a charge in an e.m. field

The finding, therefore, applies to all laws, and one has:

"All laws of physics are the same in every inertial reference frame"

Thus a) the form of these laws is the same in every inertial frame

and b) the numerical values of the physical constants which these laws contain are the same w.r.t. every inertial frame.

e, m, c, $\hbar$

Restated negatively one has:

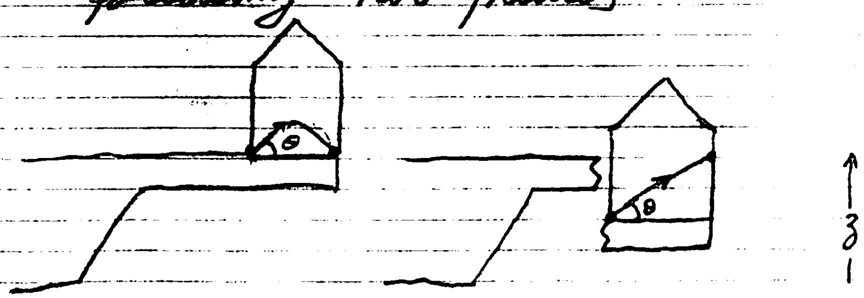
"The laws of physics cannot provide a way of distinguishing one inertial reference frame from another."

Each of these statements is called the Principle of Relativity.

Inertial Frames

The relativity principle is a statement about the behavior of things in different inertial reference frames. How can one tell such frames from non-inertial frames? Within the framework of classical (i.e. non-quantum) mechanics the answer can be given by examining the measured infinitely sharp trajectories of free particles.

Compare the recorded trajectories in the following two frames



Non-inertial frame — Inertial ("free float") frame

Inertial frame of reference $\Rightarrow$ Newton's 1st law of motion

" $\Leftarrow$ "

SKIP

(As an aside, we shall see that these two pictures illustrate what Einstein called the "happiest thought" of his life)

(2) An inertial frame is defined by Newton's first law of motion (free particles remain in a state of rest or in a state of uniform straightline motion ~~remain in such states~~); in other words, a frame is said to be inertial (or "free float") to the extent that all free particles in it comply with Newton's first law of motion.

(2) Inertial Frame: Definition. (continued)

More precisely, one has the following

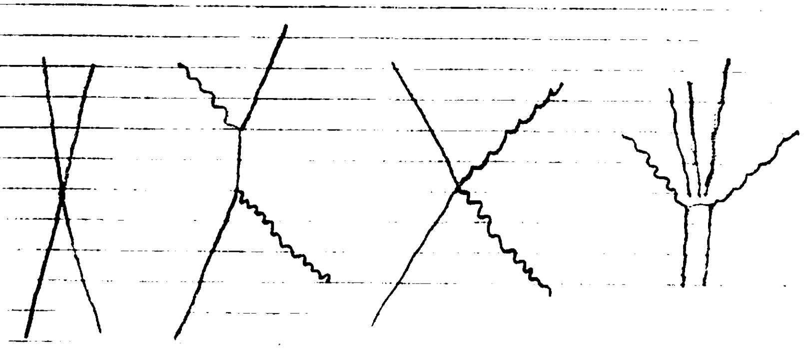
Definition (Inertial Frame)

Given: i) a region of space and an interval of time

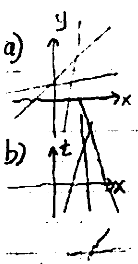
ii) any set of free floating particles in this spacetime region

Then: A free float (= inertial) frame →

(cont'd on next page)



is that region of spacetime as coordinatized by a latticework of clocks and measuring rods in such a way that — *within some specified accuracy* — the free particles travel



a) along straight lines

b) with constant velocity

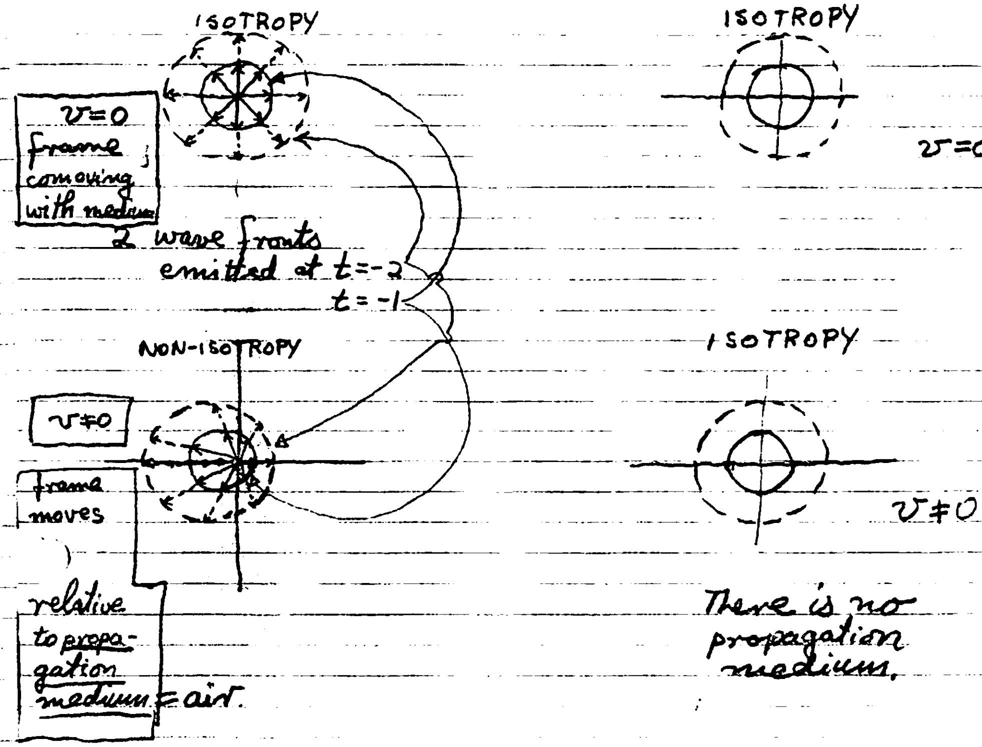
for *each* and *every* particle in that region of spacetime.

~~Insert (3)~~

(3) *Isotropy of Space*

One of the most counterintuitive manifestations of the Principle of Relativity is the *isotropy* of light propagating in different inertial reference frames. To appreciate this manifestation compare the *propagation* of sound waves with electromagnetic waves in different inertial frames.

Sound vs. Light



Propagation of sound wave relative to a moving ($v \neq 0$) open railroad car.

Propagation of electromagnetic waves

In a frame moving with respect to the "medium" ($v \neq 0$) sound waves propagate *non-isotropically* ("different speed in different directions") while e.m. waves still propagate *isotropically* (same speed in all directions)

Thus we have a far reaching result:

$\boxed{\text{Isotropy of space is frame independent}}$

This principle is contained in the Maxwell field equations and it also expresses the result of the Michelson-Morley experiment. (for an interesting discussion see Ex. 3.12 in T-W)

Isotropy of all inertial frames says nothing about the numerical value of the speed of light. The result of the Kennedy-Thorndike experiment (see problem 3.13 in T-W for an interesting discussion) says that even the magnitude of the velocity of light is frame independent.