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MATHEMATICS AND APPLIED MATHEMATICS

Flow Field Reconstruction from Sparse Data using Physics-Informed Neural Networks

2nd Report of the Practical Task

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1. **What task do you solve?**

The main task of this project is to employ Physics-Informed Neural Networks (PINNs) to reconstruct the flow field of blood from limited, noisy data. This is particularly valuable in situations where obtaining high-quality blood flow measurements is challenging due to low Signal-to-Noise Ratio (SNR). The project‘s goal is to combine readily available noisy data with the governing physical laws, enabling the reconstruction of both the flow field and the boundary shape using PINNs.

2. **What data/images do you analyze?**

This project primarily utilizes low-resolution (LR) synthetic fluid flow data, obtained by finite volume (FV)-based numerical simulations. The LR data is characterized by limited spatial resolution, sparsity, and noise due to limited computational or experimental resources. Low spatial resolution measurements are replicated by creating a coarse mesh. The geometry of a blood vessel is prepared and the corresponding mesh is generated using an open-source platform SALOME. The flow field of a fluid is obtained using OpenFOAM, an open-source software package for Computational Fluid Dynamics (CFD) simulations. The calculated flow field is visualized using a multi-platform data analysis and visualization application ParaView. The LR velocity field is corrupted by an independent and identically distributed (i.i.d.) Gaussian noise factor. These transformations are performed to mimic the LR data obtained with the Magnetic Resonance Velocimetry (MRV) technique.

Additionally, high-resolution (HR) flow field data is generated using the same methods except with a dense mesh. This HR data will be used to evaluate the effectiveness of the method used for flow field reconstruction.

3. **What computing resources do you use?**

For this project, I use Google Colaboratory, a cloud-based computing environment. The platform offers access to a T4 Graphics Processing Unit (GPU) with 16 GB of GDDR6 RAM.

4. **What neural network architecture and its hyperparameters do you use?**

The neural network architecture used in this project consists of several convolutional decoders. Each decoder has three hidden convolutional layers designed to extract features from the input data. The input layer (LR noisy flow field data) is up-sampled to the target resolution using bicubic interpolation and then passed through the convolutional layers. The physics-informed part is implemented by incorporating knowledge of the governing physical equations (Navier-Stokes equations) and boundary conditions into the training process of the neural network. This approach ensures that the HR flow fields generated by the network follow the laws of fluid dynamics, and the network is capable of denoising and upscaling the LR data without the need for HR labels. This approach reduces the reliance on HR labels, making it suitable for scenarios, where such labels may be unavailable or challenging to obtain. The hyperparameters of the sub-CNN are as follows:

- Learning rate: 10^-3
- Number of hidden layers: 3
- Number of hidden channels: [16, 32, 16]
- Optimizer: Adam
- Padding size: 2
- Strides: 1
- Kernel size: 5x5
- Non-linearity: ReLU

These hyperparameter values were not yet put to the test.

5. **What research do you plan to do in the next stage?**

My next research step involves developing a script to automatically generate LR velocity field data. Currently, I can only create this data manually, so I aim to automate the process for more efficient research.

Another priority is to develop a complete data preprocessing pipeline. This includes tasks such as extracting the LR data out of the OpenFOAM configuration file, loading it to the working environment, introducing noise to the dataset, and partitioning the dataset into train, validation and test sets.

One more future step involves the development of a custom loss function that leverages the PyTorch autograd functionality. This custom loss function will be designed to incorporate the loss, based on the Navier-Stokes equations, into the model.

6. **Other aspects you think are important.**

A standard CNN can not work with irregular shapes, i.e. the input must be rectangular. A non-linear coordinate transformation could be employed to handle irregular domains. The transformation would map the irregular physical domain to a regular rectangular shape, allowing the use of CNN operations on non-rectangular domains.

To mimic the LR data obtained from the flow MR imaging even more accurately, the five-point balanced phase-contrast method could be employed. It encodes the velocity field into the phase space, adds different levels of Gaussian noise to it, and then maps the data back to a synthetic noisy MR flow field via the inverse five-point transformation. The Gaussian noise imposed in the phase space will become highly non-Gaussian once it is mapped back to the physical velocity space.

Another aspect under consideration is the potential use of the DeepXDE framework, which offers customization for spatio-temporal problem domains and boundary conditions. Depending on the project's needs, it could enhance the approach.