Applied Master Project

Portfolio Insurance: OBPI vs CPPI

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Abstract

This paper evaluates and compares Option-Based Portfolio Insurance (OBPI) and Constant Proportion Portfolio Insurance (CPPI) strategies applied to Bitcoin. Using both historical backtests and Monte Carlo simulations under various price dynamics—including stochastic volatility and jump diffusion—we assess the risk-return profiles of each strategy across diverse market conditions. While CPPI offers greater upside potential, it exhibits higher vulnerability to gap risk and extreme drawdowns. OBPI provides more stable downside protection but limits upside capture. Extensions such as Maximum-Drawdown CPPI and Leveraged OBPI are also analyzed. The findings highlight critical trade-offs between return amplification and capital protection, with implications for strategy selection in volatile asset classes.

Contents

1 Introduction

The cryptocurrency market has seen a remarkable surge in mainstream interest in recent years, evolving from a niche technology into a global financial phenomenon. Bitcoin, as the pioneering and most prominent digital asset, has captured the attention of institutional investors, retail traders, and even traditional financial institutions, reflecting its increasing adoption across diverse sectors. This growing traction is driven by the promise of high returns and the potential to diversify investment portfolios. However, the crypto market's infamous volatility—with Bitcoin often experiencing sharp price swings—presents significant risks that can unsettle risk-averse investors. This is where portfolio insurance strategies come into play, offering a practical solution to balance opportunity and stability.

They offer a solution by enabling participation in upside potential while limiting downside exposure. Among these, Option-Based Portfolio Insurance (OBPI) and Constant Proportion Portfolio Insurance (CPPI) are two prominent approaches. OBPI replicates the payoff of a protective put option, while CPPI dynamically adjusts exposure to risky and risk-free assets based on a pre-specified floor.

In this paper, we conduct a comparative analysis of OBPI and CPPI applied to Bitcoin investments. We first perform historical backtests over varying market regimes between 2016 and 2023. To complement the empirical analysis, we implement Monte Carlo simulations using Geometric Brownian Motion, Stochastic Volatility, and Merton Jump Diffusion model. Strategy extensions addressing identified limitations, such as Maximum-Drawdown CPPI and Leveraged OBPI, are also examined. Our objective is to assess the performance, robustness, and practical relevance of these strategies under real-world and simulated conditions, offering insights for investors navigating high-volatility environments.

We conclude by presenting our insights on the most suitable strategy for navigating the volatility of the cryptocurrency market, based on the results of our analysis.

2 Programming and Backtesting Portfolio Insurance

2.1 Strategy Descriptions

> Cadle et al. *(2022)* define: "A portfolio insurance strategy is a dynamic hedging process that provides the investor with the potential to limit downside risk while allowing participation on the upside so as to maximize the terminal value of a portfolio over a given investment horizon."

Portfolio insurance creates a convex payoff, offering upside potential while limiting downside risk. Methods such as Option-Based Portfolio Insurance (OBPI) and Constant Proportion Portfolio Insurance (CPPI) adjust allocations between risky and safe assets to maintain a minimum value. These strategies are ideal for cautious investors seeking exposure to volatile assets like Bitcoin, balancing growth with risk mitigation.

This analysis explores a strategy where, if the portfolio value drops below the guaranteed floor due to gap risk—the risk of a sudden and significant drop in asset prices that occurs between trading intervals, often caused by unexpected market events or extreme volatility, which can cause the portfolio to breach the guaranteed floor before protective measures can be activated—the remaining capital is shifted to a risk-free asset to accrue interest. If the value recovers before maturity, reinvestment in risky assets resumes; otherwise, the final value may fall short of the guarantee due to severe market downturns.

2.1.1 Option-Based Portfolio Insurance

The Option-Based Portfolio Insurance (OBPI) strategy, first introduced by Leland and Rubinstein *(1976)*, seeks to safeguard a portfolio from significant losses by purchasing a put option, thereby limiting downside risk. However, in practice, implementing OBPI can be difficult due to market frictions, such as illiquidity or the unavailability of suitable options.

To address these challenges, Dynamic OBPI offers a practical alternative by adjusting allocations between risky and risk-free assets to replicate a put option. As Hull *(2018)* notes, this involves maintaining a delta-equivalent position in the underlying asset. The strategy rebalances dynamically—reducing risk exposure in downturns and increasing it in rallies—aiming to ensure a minimum terminal portfolio value.
The allocation to the risky asset at each rebalancing point is determined using the following formula:

$$w_t = \frac{(1 + \Delta_{P,t}) \times S_t}{P_t + S_t}$$

Where:

- $\Delta_{P,t}$ is the delta of the put option at time t, calculated using the Black-Scholes model,
- S_t is the price of the risky asset at time t,
- P_t is the price of the put option at time t, also derived from the Black-Scholes formula.

OBPI ensures a minimum value of K plus upside potential by holding a put option with strike price K on the underlying asset S. The initial portfolio value, $S + P(S, K)$, where $P(S, K)$ is the Black-Scholes price of the put, may differ from the initial investment. To set a floor of $z\%$ of initial capital, the strike price K is determined by solving the equation:

$$K = z \times (S + P(S, K))$$

2.1.2 Constant Proportion Portfolio Insurance

The Constant Proportion Portfolio Insurance (CPPI) strategy, introduced by Black and Jones *(1987)*, aims to protect a portfolio by maintaining a minimum value (the floor) while allowing for potential gains. The strategy adjusts the mix of the risky asset and the safe asset based on the cushion $C = V - F$, the difference between the portfolio value V and the floor F. The multiplier m controls the proportion of the cushion allocated to the risky asset, with higher values increasing exposure to the riskier asset.

During market upswings, the cushion grows, and more funds are invested in the risky asset. Conversely, in downturns, the cushion shrinks, and funds are shifted to the safe asset. If the portfolio value falls below the floor ($V < F$), the strategy reallocates all funds to the safe asset. This makes the strategy more sensitive to gap risk, especially in volatile markets or when rebalancing occurs infrequently, as large price movements can cause the portfolio to breach the floor before adjustments can be made.
The risky asset allocation is given by:

$$w_t = m \times C_t \quad \text{or} \quad w_t = \max\left[0,\ (V_t - F) \times m\right]$$

where m governs the risk-return profile of the strategy. The inverse of the multiplier $\frac{1}{m}$ is often viewed as the maximum drawdown the strategy can withstand before crossing the floor between two rebalancing dates.

2.2 Data and Calibration

The dataset does not account for transaction costs, assuming frictionless trading. This means that portfolio rebalancing does not incur slippage or fees, which could affect real-world performance.

2.2.1 Risky asset

The chosen risky asset for this analysis is Bitcoin, which acts as a proxy for the broader cryptocurrency market. The dataset comprises daily closing prices sourced from Yahoo

Finance (ticker: BTC-USD), starting from September 2014 to February 2025, offering a historical span of just over a decade.

2.2.2 Risk-free asset

The 13-week U.S. Treasury Bill (Yahoo Finance ticker: ˆIRX) serves as a proxy for the risk-free asset. The choice of this instrument is justified by its low credit risk and short duration, making it a standard benchmark for risk-free investments.
As a note, while Bitcoin prices are available daily throughout the year, risk-free rate data is not. To address this, we applied a linear interpolation method to estimate the missing values.

2.2.3 Volatility

To model the conditional volatility of Bitcoin returns, we rely on the EGARCH(1,1) specification introduced by Nelson *(1991)*. Unlike the standard GARCH model, EGARCH allows for asymmetric volatility responses and models the logarithm of the conditional variance, ensuring non-negativity without the need for parameter constraints.

Formally, the EGARCH(1,1) model is expressed as:

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \alpha \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} \tag{1}$$

The estimation results *(Appendix: Figure 25)* highlight two key parameters shaping Bitcoin return volatility. The volatility persistence parameter ($\beta = 0.9696$) indicates strong volatility clustering, where high volatility tends to persist. The magnitude effect parameter ($\alpha = 0.2228$) shows that large price movements lead to significant future volatility adjustments. Both parameters are statistically significant (p-values < 0.001), confirming the model's robustness.
Additionally, the Generalized Error Distribution's shape parameter ($\nu = 1.010$) suggests heavy tails in the return distribution, typical of cryptocurrency markets, reflecting Bitcoin's extreme price fluctuations. The GED is used to better capture these fat tails and excess kurtosis, which standard normal distributions often fail to model accurately.
Conversely, the asymmetry term ($\gamma = -0.0023$) is not significant (p-value $= 0.863$), implying no substantial difference in volatility responses to negative versus positive shocks during the sample period.
Overall, the EGARCH(1,1) model fits the data well, demonstrating its capability for reliable volatility forecasting during market instability.

2.3 Analysis

To gain a deeper understanding of how portfolio insurance strategies behave across various market environments, five distinct backtests were conducted. Each corresponds to a specific market regime: Bearish, Bullish, *Flat (Appendix: Section 6.1.2)*, Low volatility and High volatility. All backtests span a one-year period to ensure consistency with the following simulation framework.

Due to Bitcoin's significant volatility, we opted for daily rebalancing in our main analysis, with additional results for weekly rebalancing included in the *Appendix*.

The CPPI strategy was implemented with a multiplier of 3, balancing capital protection and market exposure. As noted in *Section 2.1.2*, the inverse multiplier $\frac{1}{3} \approx 0.33$ represents approximately the maximum drawdown tolerated between rebalancing dates—roughly matching Bitcoin's largest one-day drop of -37%.

During bear markets, both CPPI and OBPI strategies converge to their protection floors, growing at the risk-free rate. CPPI reduces risky exposure as the cushion erodes, while OBPI's dynamic hedge moves toward a full risk-free allocation as the synthetic put becomes deeply in-the-money. Empirical backtesting showed that a CPPI strategy with an 80% floor and a dynamic OBPI with a 100% strike provide comparable downside protection, resulting in similar capital preservation despite differing initial risk profiles. This configuration aligns with the OBPI framework outlined in *Section 2.1.1*.

2.3.1 Bear market

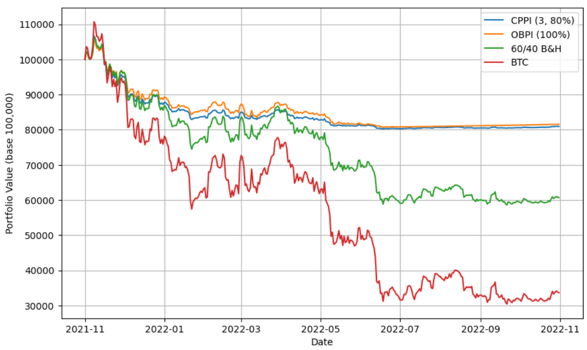

Figure 1: Performance Comparison of CPPI, OBPI, Buy-and-Hold Strategies, and Bitcoin in Bear Market Conditions (Nov 2021 – Oct 2022)

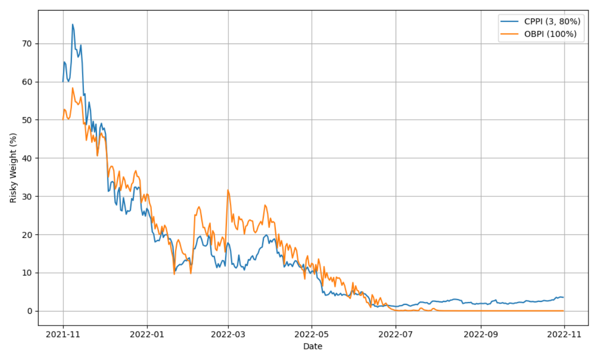

Figure 2: Evolution of CPPI and OBPI Weights in Bear Market Conditions (Nov 2021 - Oct 2022)

During the bearish phase, both CPPI and OBPI performed similarly, hitting their protection floors with returns of -19.05% and -18.44%, and maximum drawdowns of -24.74%

and -23.54%, respectively. However, OBPI showed slightly higher volatility (12.77% vs. 12.65%) and turnover (422.63% vs. 320.20%).

Both CPPI and OBPI aim to limit drawdowns but respond differently to market declines. CPPI reduces risky asset exposure quickly as the cushion erodes, while OBPI adjusts more gradually through the delta of a static put option *(Figure 2)*. Rebalancing frequency (daily vs. weekly) has minimal impact on performance or risk metrics, suggesting little advantage to more frequent adjustments during the studied period *(Appendix: Table 5)*.

Between July and November, a period during which Bitcoin prices swung between $40,000 and $30,000 after a sharp decline, CPPI and OBPI displayed notably different allocation behaviors. CPPI maintained a slightly higher average exposure to the risky asset. Since its allocation adjusts dynamically based on the cushion level, CPPI increased its risky asset holdings during brief market recoveries within this volatile range. In contrast, OBPI kept its exposure close to zero, as the delta and pricing of its embedded put option led to a more cautious stance amid the prevailing downward trend. This divergence highlights CPPI's greater reactivity to market movements, whereas OBPI adopted a steadier, more protective approach.

2.3.2 Bull market

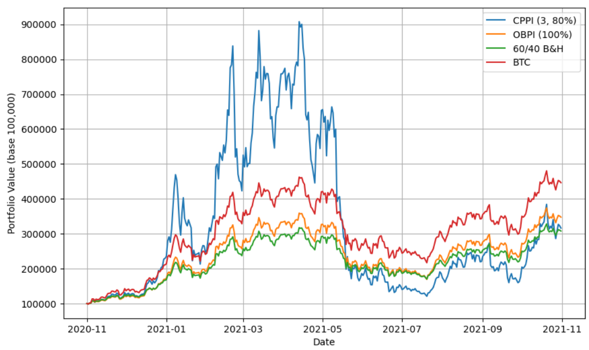

Figure 3: Performance Comparison of CPPI, OBPI, Buy-and-Hold Strategies, and Bitcoin in Bull Market Conditions (Nov 2020 - Oct 2021)

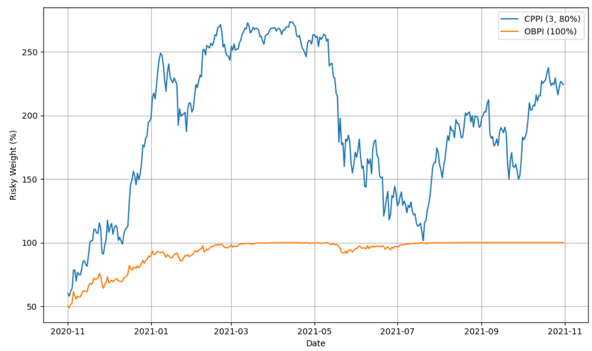

Figure 4: Evolution of CPPI and OBPI Weights in Bull Market Conditions (Nov 2020 - Oct 2021)

During the bull market, OBPI outperforms CPPI with a higher Sharpe ratio (3.23 vs. 1.31), stronger downside protection, and lower volatility (76.67% vs. 164.68%). While CPPI shows strong early gains, peaking over $800,000, it suffers heavy drawdowns (-86.68%) due to high leverage. OBPI, meanwhile, maintains stability with a smaller drawdown (-52.90%) while capturing much of the upside.

This period highlights the significant path dependency of the CPPI strategy, making it highly volatile. Although it outperformed for much of the period, CPPI ultimately ended as the worst performer in terms of return, as it was particularly sensitive to the Bitcoin correction between May and July, during which the strategy maintained high leverage. This sharp market decline rapidly eroded the cushion, forcing substantial de-risking and locking in losses. In contrast, OBPI was less sensitive to short-term market fluctuations.

2.3.3 Low volatility market

To define a low-volatility period, we identified the one-year rolling window with the lowest volatility using the EGARCH model, which corresponded to the timeframe from January 2016 to December 2016.

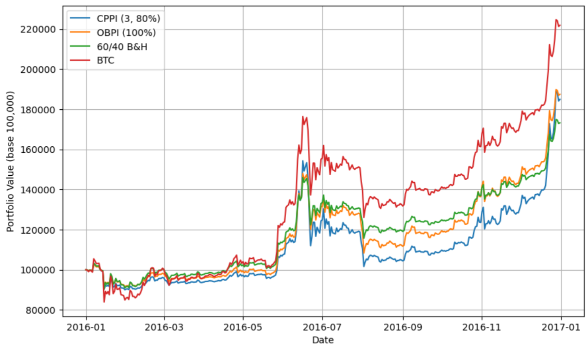

Figure 5: Performance Comparison of CPPI, OBPI, Buy-and-Hold Strategies, and Bitcoin in Low-Volatility Market Conditions (Jan 2016 - Dec 2016)

Figure 6: Evolution of CPPI and OBPI Weights in Low-Volatility Market Conditions (Jan 2016 - Dec 2016)

In this low volatility environment, both CPPI and OBPI performed well, delivering returns above 80% with relatively contained drawdowns. OBPI achieved a Sharpe ratio of 2.44 and CPPI followed closely with 1.95, reflecting favorable conditions for steady exposure to risky assets. Risk allocations remained moderate and relatively stable for both strategies throughout the period, allowing for smooth capital growth without breaching floors. CPPI ended with a max drawdown of -34.15%, slightly higher than OBPI's -26.81%, but both remained within acceptable risk limits.

2.3.4 High volatility market

Similarly to the approach used for the low-volatility period, the EGARCH model identified the highest one-year rolling window volatility during the period from May 2017 to April 2018.

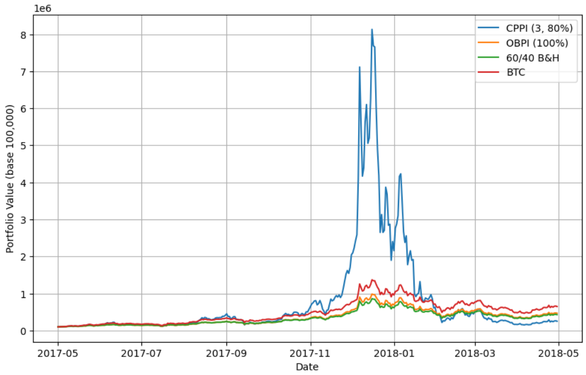

Figure 7: Performance Comparison of CPPI, OBPI, Buy-and-Hold Strategies, and Bitcoin in High-Volatility Market Conditions (May 2017 - Apr 2018)

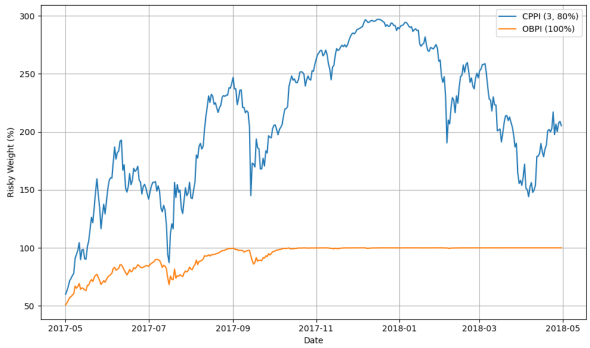

Figure 8: Evolution of CPPI and OBPI Weights in High-Volatility Market Conditions (May 2017 – Apr 2018)

During this highly volatile period, OBPI significantly outperformed CPPI. OBPI achieved a return of 367.93% with a Sharpe ratio of 3.64, while CPPI delivered a much lower return of 153.30% and a Sharpe ratio of 0.62. The poor performance of CPPI was largely due to excessive exposure, which reached nearly 300% *(Figure 8)*. This overexposure amplifies losses during sharp market corrections, leading to a maximum drawdown of -98.11%. In contrast, OBPI maintained a steady 100% exposure, limiting its drawdown to -65.99%, even in the midst of the turbulence. Additionally, CPPI's extremely high turnover rate of 2198.58% reflects the aggressive rebalancing needed to manage leverage under volatile conditions *(Appendix: Table 9)*.

This period highlights, even more so than the previous Bull Market, the vulnerability of CPPI in highly volatile markets. The leverage used by CPPI not only magnifies losses but also risks missing out on locking in significant gains. While OBPI is not immune to drawdowns, it offers a more robust performance profile, with better downside control and superior risk-adjusted returns.

2.4 Rebalancing Tolerance

A common limitation in the literature on dynamic investment strategies is the frequent omission of transaction costs, which can significantly reduce realized performance. To illustrate their potential impact, we calculated portfolio turnover for each strategy.

Drawing inspiration from volatility trading practices—where move-based rebalancing is often favored or combined with calendar-based methods—we implemented tolerance-based rebalancing to evaluate its impact on both performance and cost-efficiency.

In CPPI, we applied a target multiple-bandwidth approach: the risky asset allocation, determined by the product of the multiplier and the cushion, is updated only when its weight deviates by more than a specified tolerance. For OBPI, we used a delta-bandwidth rule, triggering rebalancing when the option delta changes beyond a specified threshold. Empirical testing suggested that a 5% threshold under daily monitoring offers an effective balance, significantly lowering turnover while maintaining overall strategy performance.

Market Regimes	Bear Market	Bull Market	Flat Market	Low Volatility Market	High Volatility Market
Calendar-based rebalancing					
CPPI Turnover	320.20%	2057.67%	863.01%	910.97%	2198.58%
OBPI Turnover	422.63%	255.47%	1035.52%	434.82%	269.02%
Move-based rebalancing					
CPPI Turnover	158.32%	1720.74%	581.15%	603.40%	1864.03%
OBPI Turnover	144.41%	91.96%	709.49%	227.63%	140.55%

Table 1: Turnover across different market regimes using a rebalancing tolerance of 5%.

The implementation of a move-based rebalancing mechanism with a 5% threshold significantly reduces portfolio turnover and associated transaction costs, while largely maintaining performance. Under Bear Market conditions, this adjustment even improves the Sharpe ratio *(Appendix: Table 10)*. Performance analysis in *Appendix Section 6.1.4* shows that higher rebalancing thresholds (10% and 25%) lead to minimal deviations from the baseline strategy in terms of risk-adjusted returns. In some cases, performance improves, while in others, it declines—reflecting the path-dependent nature of strategies, particularly CPPI. Nevertheless, the move-based approach remains effective in lowering turnover and thereby mitigating the impact of transaction costs. However, although higher thresholds further reduce turnover, they also increase exposure to path dependency, with effects that vary depending on market conditions.

3 Simulating Portfolio Insurance

3.1 Data and Calibration

In the subsequent Section, we analyze the performance of the three strategies—OBPI, CPPI, and Buy and Hold—within a simulated framework. To thoroughly evaluate their effectiveness, we utilize Monte Carlo simulations to simulate the strategies' outcomes across diverse market scenarios. For the baseline analysis, the simulations are structured with the following parameters: 1,000 iterations to ensure statistical reliability, a one-year time horizon, and a daily rebalancing frequency. Once again, given Bitcoin's volatility, we opt for daily rebalancing to better align with real-world conditions and provide a more meaningful analysis. The initial portfolio value is established at $100,000, reflecting the starting investment in this study. Additionally, we establish the initial price as the Bitcoin closing price on the last day of February 2025 (February 28th), recorded at $84,373.007812, which serves as the starting point for our simulations.

3.1.1 OBPI and CPPI Parameter Calibration

Defining a protection floor is essential for both strategies. In OBPI, this can be set either as the strategy floor or the option's strike price, with their relationship outlined in *Section 2.1.1*. Focusing on downside risk mitigation, we calibrated these parameters accordingly, with iterative simulations indicating that a CPPI floor of 80% and an OBPI strike price of 90% provided the most balanced comparison. In contrast, for the historical backtesting component, we selected OBPI at 100% alongside CPPI at 80%. The divergence arises from differences in key inputs to the OBPI option pricing model, particularly volatility and the risk-free rate. In simulations, volatility was estimated using a Generalized Error Distribution, while the constant volatility input in the model failed to capture fat tails. The historical analysis used time-varying rates, while the simulations relied on a constant risk-free rate. These differences likely caused variations in put prices and deltas, leading to discrepancies in the strategy calibration between the two approaches.

Additionally, the CPPI strategy requires the selection of a multiplier value. As previously noted, a higher multiplier increases the risk of breaching the floor. To maintain consistency with the historical analysis and to leverage the strengths of the CPPI strategy while managing its risks, we choose a multiplier of 3. However, the choice of multiplier is a significant topic in itself, and we will explore its impact in more detail in a later Section.

3.1.2 Returns Distribution

To simulate Bitcoin prices using Geometric Brownian Motion (GBM), we first assess whether returns follow a normal distribution. The Q-Q plot *(Figure 9)* and the histogram *(Figure 10)* show deviations from normality—particularly heavy tails—indicating excess kurtosis and a higher likelihood of extreme returns.

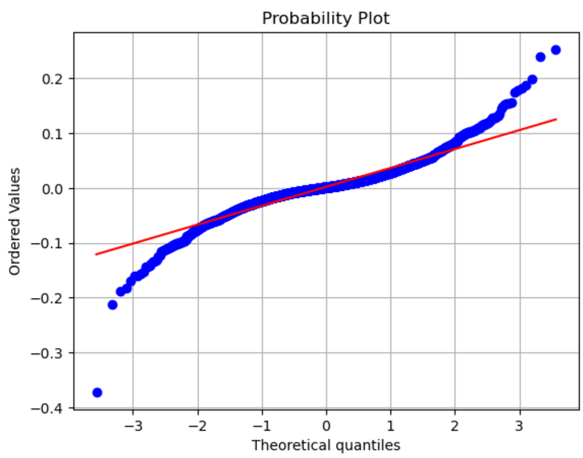

Figure 9: Q-Q Plot of Bitcoin Returns Against a Normal Distribution

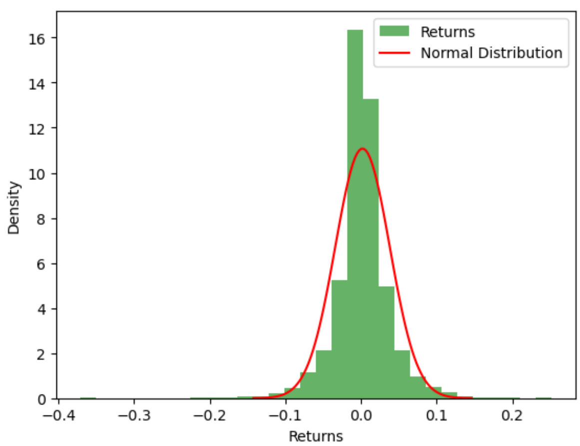

Figure 10: Histogram of Bitcoin Returns with Normal Curve

Our analysis confirms this, with a skewness of -0.11 (slight left skew) and excess kurtosis of 7.6. The Jarque-Bera test (p-value = 0) rejects the null of normality, as expected for Bitcoin. While skewness is minor, the heavy tails suggest a leptokurtic distribution. Despite this, we initially model returns using GBM with constant volatility based on historical data, and later revisit this assumption.

3.2 Analysis

For simplicity and consistency across subsequent simulations, we assume a constant risk-free rate of 4%. This value approximates the level observed in February 2025, and given the one-year investment horizon, it is reasonable to expect it to remain relatively stable.

The estimation of expected returns poses a more substantial methodological challenge. Merton *(1980)* observed that increasing the frequency of return sampling fails to improve the precision of expected return estimates, leaving them fundamentally imprecise due to their dependence on only the initial and final price observations. Drawing on Pastor *(2001)*, who posited that averaging historical returns offers an unbiased estimator of future expected returns, we compute the expected return as the annualized average of Bitcoin's daily returns over the period from September 17, 2014, to February 28, 2025, resulting in a value of 73.80%.

Additionally, we require an estimate for return volatility. In the preliminary analysis, we employ a constant volatility estimate, derived as the historical standard deviation of returns over the same period used for expected returns, resulting in 69.31%.

3.2.1 Simulation Results Under Geometric Brownian Motion with Constant Volatility

In our initial analysis, we model Bitcoin's price dynamics with Geometric Brownian Motion (GBM), employing a constant volatility derived from historical Bitcoin return data. Under this framework, the price $S(t)$ follows the stochastic differential equation:

$$dS(t) = \mu S(t)\, dt + \sigma S(t)\, dW(t)$$

Applying Itô's lemma to the logarithm of the price yields the discrete-time solution:

$$S(t + \delta t) = S(t) \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) \delta t + \sigma \varepsilon \sqrt{\delta t}\right]$$

Where:

- $S(t)$: Asset price at time t,

- μ: Drift term, representing the expected return,
- σ: Volatility of returns,
- δt: Small time increment,
- ε: Standard normal random variable, $\varepsilon \sim \mathcal{N}(0, 1)$.

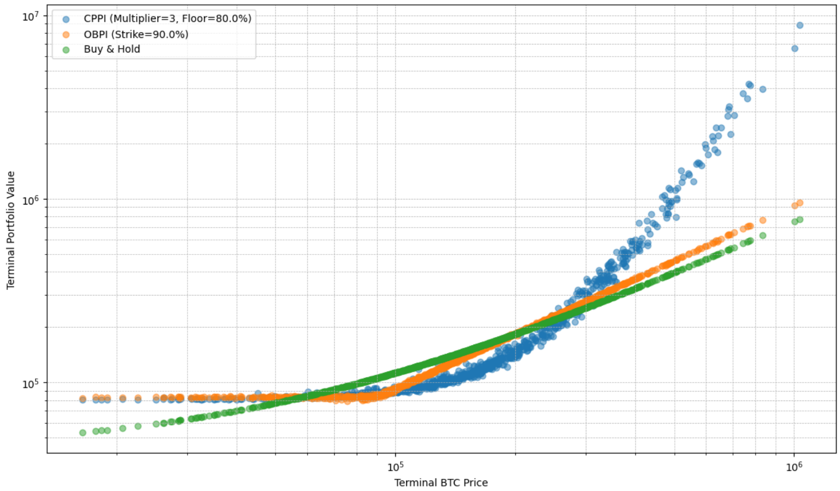

Figure 11: Performance Comparison of CPPI, OBPI, and Buy-and-Hold Strategies via Monte Carlo Simulations under GBM with Constant Volatility

For subsequent graphs, we use a logarithmic scale on both the x-axis (Terminal BTC Price) and y-axis (Terminal Portfolio Value) to enhance visualization, particularly for extreme Bitcoin prices, ensuring clear representation and easy comparison of outliers.

Performance Metrics	**CPPI (M=3, F=80%)**	**OBPI (Strike=90%)**	**Buy and Hold**
Path-level metrics (Avg. Across All Paths)			
Return	150.36%	73.95%	70.99%
Volatility	64.21%	45.74%	45.18%
Tracking Error	44.66%	29.84%	24.87%
Sharpe Ratio	2.28	1.53	1.48
Sortino Ratio	2.63	2.21	2.52
VaR 5%	-5.72%	-4.00%	-3.70%
Expected Shortfall 5%	-9.19%	-5.42%	-4.73%
Skewness	-0.07	0.02	0.08
Excess Kurtosis	2.78	1.55	0.12
Turnover	1447.50%	601.57%	0.00%
Max Drawdown	-42.77%	-32.77%	-32.70%
Information Ratio	-0.01	-0.04	-0.05
Distribution metrics (Cross-Sectional)			
Median Return	4.74%	27.83%	40.63%
Maximum Return	8753.39%	852.06%	676.34%
Minimum Return	-19.62%	-20.77%	-46.75%
Volatility of Return Distribution	544.84%	121.51%	98.04%
Var 5%	-18.22%	-17.70%	-24.31%
Expected Shortfall 5%	-18.87%	-18.33%	-34.10%
Skewness	8.19	2.18	1.94
Excess Kurtosis	93.17	6.15	5.14
Worst Maximum Drawdown	-91.71%	-73.42%	-75.76%

Table 2: Simulations Performance Metrics: Brownian Motion and Constant Volatility

CPPI delivers a substantially higher average return of 150.36%, nearly double that of OBPI's 73.95%. However, this comes at the cost of noticeably greater volatility (64.21% vs. 45.74%) and a higher tracking error. The increased variability reflects CPPI's more aggressive positioning, particularly in bull markets where its convex payoff structure allows it to capitalize more effectively on upward trends.
Risk-adjusted metrics reveal a more nuanced picture. CPPI exhibits a higher Sharpe ratio (2.28 vs. 1.53), suggesting superior compensation per unit of risk, but its Sortino ratio only marginally improves upon OBPI (2.63 vs. 2.21), indicating that much of the additional volatility is indeed downside risk. The tail risk measures reinforce this point: CPPI faces deeper potential losses, with a 5% Value-at-Risk of -5.72% and an Expected Shortfall of -9.19%, both worse than OBPI's -4.00% and -5.42%, respectively.

The distributional metrics highlight CPPI's extreme behavior under certain paths. While its maximum return far exceeds that of OBPI (8753.39% vs. 852.06%), its worst-case drawdowns are also significantly more severe. The cross-sectional skewness and excess kurtosis of CPPI (8.19 and 93.17) point to a highly asymmetric and fat-tailed return distribution, implying that a small subset of scenarios drive the majority of its performance. This is further reflected in the median return, which is much lower for CPPI (4.74%) than OBPI (27.83%), suggesting that while CPPI has higher upside in select scenarios, most outcomes are considerably less favorable.

Finally, turnover provides insight into implementation costs and strategy dynamics. CPPI's turnover is more than double that of OBPI (1447.50% vs. 601.57%), highlighting its highly path-dependent nature and the intensive rebalancing it requires. This not only introduces additional transaction costs but also greater operational complexity. Altogether, CPPI represents a higher-risk, higher-reward approach compared to the more stable and balanced profile of OBPI.

3.2.2 Simulation Results Under Geometric Brownian Motion with Stochastic Volatility

Next, we apply the EGARCH model to simulate Bitcoin's conditional volatility. To better capture the heavy-tailed nature of return distributions, we also use a Generalized Error Distribution (GED). This enhances the model's flexibility in reflecting extreme events and excess kurtosis, thereby improving its ability to represent the higher probability of large price swings often seen in Bitcoin returns.

To integrate this time-varying volatility estimate, we modify the price equation as follows:

$$S(t+\delta t) = S(t)\exp\left[\left(\mu - \frac{\sigma(t)^2}{2}\right)\delta t + \sigma(t)\varepsilon\sqrt{\delta t}\right]$$

Where $\sigma(t)$ represents the time-varying volatility of returns.

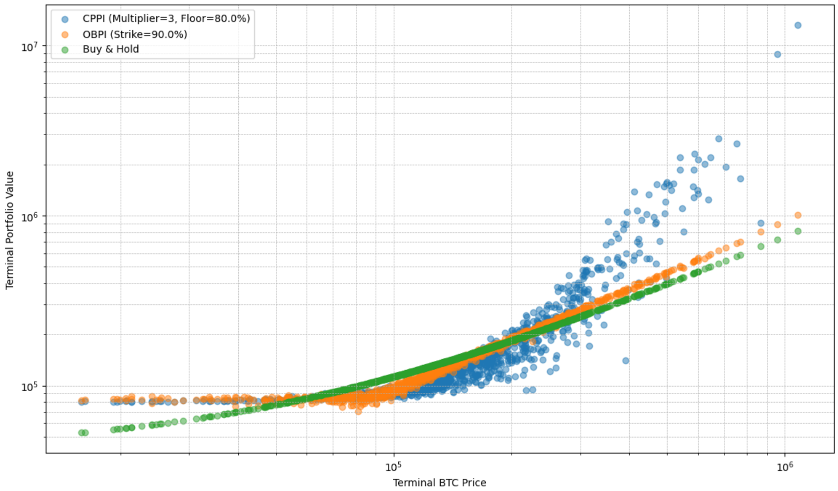

Figure 12: Performance Comparison of CPPI, OBPI, and Buy-and-Hold Strategies via Monte Carlo Simulations under GBM with Time-Varying Volatility

Performance Metrics	**CPPI (M=3, F=80%)**	**OBPI (Strike=90%)**	**Buy & Hold**
Path-Level Metrics (Avg. Across All Paths)			
Return	128.00%	67.73%	65.86%
Volatility	61.56%	43.61%	42.28%
Tracking Error	39.66%	26.40%	22.85%
Sharpe Ratio	2.01	1.46	1.46
Sortino Ratio	3.37	2.52	2.58
VaR 5%	-4.92%	-3.48%	-3.36%
Expected Shortfall 5%	-7.25%	-4.93%	-4.57%
Skewness	-0.14	0.01	0.09
Excess Kurtosis	4.28	3.06	1.50
Turnover	1358.36%	546.49%	0.00%
Max Drawdown	-41.74%	-31.43%	-30.67%
Information Ratio	-0.03	-0.07	-0.07
Distribution Metrics (Cross-Sectional)			
Median Return	12.29%	36.85%	44.61%
Max Return	13088.70%	913.25%	709.63%
Min Return	-19.90%	-29.33%	-47.09%
Volatility of Return Distribution	575.32%	106.00%	85.50%
VaR 5%	-17.86%	-17.93%	-22.46%
Expected Shortfall 5%	-18.83%	-20.56%	-33.93%
Skewness	15.48	2.58	2.29
Excess Kurtosis	309.35	10.24	8.55
Worst Max Drawdown	-95.60%	-69.51%	-67.34%

Table 3: Simulations Performance Metrics: Brownian Motion and Stochastic Volatility

In this updated simulation using Brownian motion with time-varying volatility, the key distinctions between the CPPI and OBPI strategies remain consistent, though the magnitude of their differences is even more pronounced. CPPI again delivers significantly higher average returns at 128.00%, nearly double the 67.73% achieved by OBPI. This strong outperformance is largely driven by CPPI's convex exposure to market rallies, which allows it to capitalize more aggressively on upward price trends.

Risk-adjusted metrics further highlight CPPI's advantage, with a notably higher Sharpe ratio of 2.01 compared to OBPI's 1.46. The gap is even more apparent in the Sortino ratio, where CPPI reaches 3.37, indicating better downside-risk-adjusted performance. However, this enhanced reward comes with increased risk: CPPI's volatility is higher (61.56% vs. 43.61%), and it experiences deeper tail losses, as reflected by its more negative Value-at-Risk (-4.92%) and Expected Shortfall (-7.25%).

Looking at distribution metrics, CPPI continues to demonstrate highly skewed and fat-tailed behavior. Its maximum return reaches an extraordinary 13,088.70%, compared to OBPI's 913.25%, underscoring CPPI's ability to generate massive gains in extreme bull runs. Yet this comes with considerable dispersion—its return distribution is over five times as volatile (575.32% vs. 106.00%), and the strategy shows extreme skewness (15.48) and excess kurtosis (309.35), reflecting the rare but explosive nature of some paths. In contrast, OBPI's distribution is still positively skewed and leptokurtic, but far more stable and contained.
The difference in typical outcomes is also clear in the median return, which is higher for OBPI at 36.85% versus 12.29% for CPPI. This suggests that while CPPI thrives in select paths, its average performance is heavily influenced by a few extreme cases, whereas OBPI offers more consistent returns across simulations. CPPI also continues to exhibit significantly higher turnover (1,358.36% vs. 546.49%), emphasizing the large transaction volumes required and the resulting operational burden.

Drawdown statistics again reinforce the higher risk of CPPI. Its worst-case maximum drawdown hits -95.60%, a severe loss far beyond OBPI's -69.51%, signaling the potential for significant capital erosion under adverse conditions. These differences collectively reinforce the notion that CPPI is a high-convexity, high-volatility strategy that thrives in bullish markets with sustained momentum, while OBPI provides a more balanced and defensively structured exposure, better suited to environments with moderate gains and higher uncertainty.

3.2.3 Simulation Results Under Merton Jump Diffusion Model

Lastly, to more accurately capture market dynamics and assess gap risk, we introduce a Poisson jump process featuring random downward jumps. Consequently, we model the asset price as a combination of Brownian motion with drift and an additional jump component. The resulting process is expressed as:

$$S(t+\delta t) = S(t)\exp\left[\left(\mu - \lambda k - \frac{\sigma^2}{2}\right)\delta t + \sigma\varepsilon\sqrt{\delta t}\right]\exp\left[m\left(\mu_j - \frac{\sigma_j^2}{2}\right) + \sigma_j\sum_{i=0}^{m}\varepsilon_i\right]$$

Where:

- λ: Jump intensity (average number of jumps per year),
- $k = e^{\mu_j + \frac{1}{2}\sigma_j^2} - 1$: Expected relative jump size,
- μ_j: Mean of the jump size distribution,
- σ_j: Standard deviation of the jump size,
- m: Number of jumps during δt, drawn from Poisson($\lambda \delta t$),
- ε, $\varepsilon_i \sim \mathcal{N}(0, 1)$: Standard normal random variables.

To estimate the jump parameters of Bitcoin's return distribution, we employed a two-step filtering approach. First, we fitted an EGARCH(1,1) model to Bitcoin's daily log returns to capture time-varying volatility and asymmetric effects. Next, we applied a thresholding method to the standardized residuals—representing returns adjusted for conditional heteroskedasticity—to identify jumps as observations exceeding three standard deviations from the mean. Based on these detected jumps, we estimated the annual jump intensity ($\lambda = 5.5$) by scaling the number of jumps to an annual basis, and computed the mean ($\mu_j = -0.65\%$) and standard deviation ($\sigma_j = 15\%$) of the jump sizes directly from the sample of identified jumps. This procedure enables a data-driven calibration of the jump parameters, capturing the inherent fat tails and volatility dynamics of Bitcoin returns.

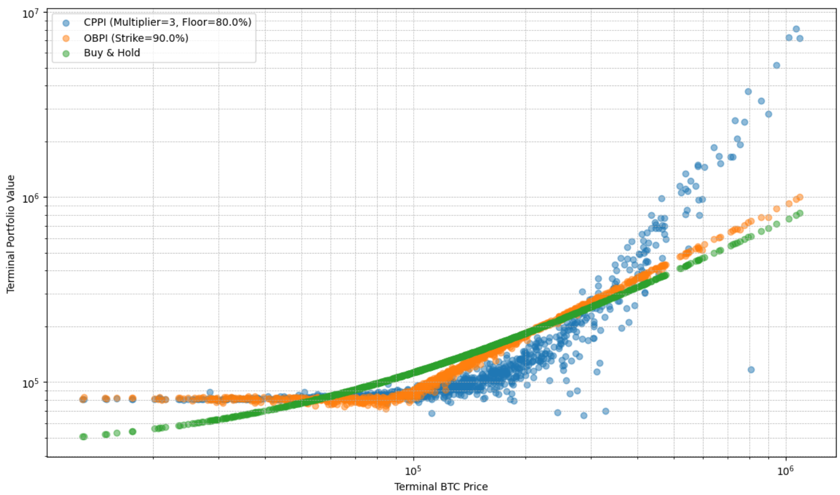

Figure 13: Performance Comparison of CPPI, OBPI, and Buy-and-Hold Strategies via Monte Carlo Simulations under Merton Jump Diffusion Model

Performance Metrics	CPPI (M=3, F=80%)	OBPI (Strike=90%)	Buy & Hold
Path-Level Metrics (Avg. Across All Paths)			
Return	99.27%	58.38%	59.20%
Volatility	61.83%	47.94%	49.31%
Tracking Error	53.38%	37.15%	29.32%
Sharpe Ratio	1.54	1.13	1.12
Sortino Ratio	2.50	1.91	1.95
VaR 5%	-4.69%	-3.66%	-3.70%
Expected Shortfall 5%	-7.22%	-5.34%	-5.19%
Skewness	-0.81	-0.25	0.13
Excess Kurtosis	17.94	11.09	6.82
Turnover	1318.31%	613.19%	0.00%
Max Drawdown	-42.93%	-35.24%	-36.35%
Information Ratio	-0.03	-0.06	-0.05
Distribution Metrics (Cross-Sectional)			
Median Return	-7.94%	12.02%	30.20%
Max Return	80.13%	9.00%	7.18%
Min Return	-105.07%	-27.97%	-49.19%
Volatility of Return Distribution	523.85%	122.01%	100.61%
VaR 5%	-19.26%	-21.46%	-34.00%
Expected Shortfall 5%	-22.41%	-23.28%	-39.51%
Skewness	10.34	2.84	2.49
Kurtosis	129.32	10.70	8.80
Worst Max Drawdown	-100.30%	-73.92%	-68.02%

Table 4: Simulations Performance Metrics: Merton Jump Diffusion model

In this simulation incorporating the Merton Jump-Diffusion model, the fundamental distinctions between the CPPI and OBPI strategies remain, but the introduction of jumps significantly magnifies their differences, particularly in terms of downside risk and return distribution characteristics. CPPI still achieves the highest average return at 99.27%, outperforming OBPI (58.38%) and Buy & Hold (59.20%). This relative outperformance continues to be primarily driven by CPPI's convex exposure to strong bullish trends. However, the introduction of jumps has a clear adverse impact on CPPI's performance metrics.
The presence of sudden price jumps exposes CPPI's structural vulnerability to gap risk, leading to a deterioration in its path-level risk-adjusted measures. Although CPPI maintains a superior Sharpe ratio (1.54) and Sortino ratio (2.50) compared to OBPI (1.13 and 1.91 respectively), these figures are meaningfully lower than in previous simulations without jumps, where CPPI displayed much stronger risk-adjusted performance. In particular, the more negative Value-at-Risk at 5% (-4.69%) and Expected Shortfall (-7.22%) illustrate how gap events exacerbate CPPI's downside risk relative to smoother market dynamics.

At the distributional level, the impact of jumps becomes even more pronounced. CPPI's return distribution shows extreme dispersion, with a volatility of 523.85% compared to 122.01% for OBPI. Skewness (10.34) and excess kurtosis (129.32) are high, revealing that CPPI's average return is heavily dependent on a small subset of highly favorable outcomes, while the majority of paths experience weaker performance or substantial losses. Critically, the median return for CPPI falls sharply into negative territory at

-7.94%, indicating that, across most simulations, CPPI fails to deliver positive returns. This contrasts with positive medians for OBPI (12.02%) and Buy & Hold (30.20%), underscoring CPPI's heightened fragility when sudden adverse movements occur.
Moreover, CPPI experiences the largest average maximum drawdown (-42.93%) at the path level among the three strategies, and in extreme scenarios, it incurs massive losses (-100.30%) as a result of leverage. This dramatic deterioration, compared to previous results under pure Brownian dynamics, clearly demonstrates that CPPI's floor protection mechanism becomes ineffective in the presence of downward jumps, where the portfolio cannot deleverage rapidly enough to prevent breaching the guaranteed floor.
In contrast, OBPI is far less affected by the introduction of jumps. Although its risk metrics deteriorate slightly compared to smooth diffusion settings, its structure—anchored by a pre-purchased put option—provides more effective gap risk protection. As a result, OBPI maintains relatively stable volatility, smaller tail losses, and a more symmetric return distribution compared to CPPI.

Overall, these findings illustrate that while CPPI benefits significantly in trending, low-volatility environments, its performance is severely compromised under jump risk, highlighting the critical importance of modeling discontinuities when evaluating dynamic portfolio insurance strategies. Conversely, OBPI demonstrates greater robustness to jumps, albeit with lower upside potential in extreme bull markets.

3.3 Robustness Analysis

To evaluate the robustness of our simulations, we model Bitcoin price movements using Geometric Brownian Motion (GBM) with constant volatility (consistent with prior settings) while adjusting key parameters like rebalancing frequency, volatility, or floor level.

3.3.1 Impact of Rebalancing Frequency

To examine the effect of rebalancing frequency, we apply CPPI and OBPI strategies with varying frequencies—365 (daily), 52 (weekly), 12 (monthly), 4 (quarterly), and 2 (semi-annual)—while keeping other parameters constant. Specifically, we simulate over a 1-year period across 1,000 paths, with CPPI set at m = 3 and floor = 80%, and OBPI at a strike = 90%.

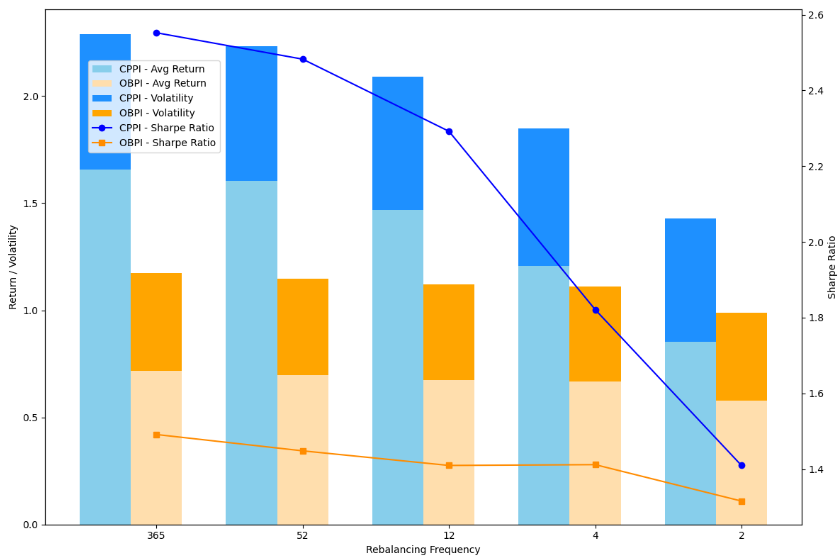

Figure 14: Impact of Rebalancing Frequency on the Performance of CPPI and OBPI Strategies

In the following graphs, the 'Return / Volatility' axis doesn't indicate a ratio—it simply shows stacked bars: the lighter bar for return and the darker bar for volatility, with no overlap.

Despite differences in scale, both strategies follow a consistent trend: as rebalancing frequency decreases, the Sharpe ratio drops. This decline is more pronounced for CPPI, while it is relatively milder for OBPI. Both strategies depend on dynamic risk allocation, adjusting the balance between risky and risk-free assets based on market conditions. When rebalancing is less frequent, such as on a weekly rather than daily basis, the portfolio is less responsive to market movements. In rising markets, this delay prevents the portfolio from increasing its exposure to risky assets quickly enough, missing potential gains. Conversely, in falling markets, infrequent rebalancing delays the reduction of risky exposure, leading to larger losses. This results in lower returns and higher volatility. However, while more frequent rebalancing could enhance performance, it comes with the trade-off of higher turnover and, consequently, higher transaction costs.

3.3.2 Impact of Volatility Level

To investigate the impact of volatility, we implement CPPI and OBPI strategies with different volatility levels—200%, 150%, 100%, 50%, and 25%—while maintaining all other parameters unchanged.

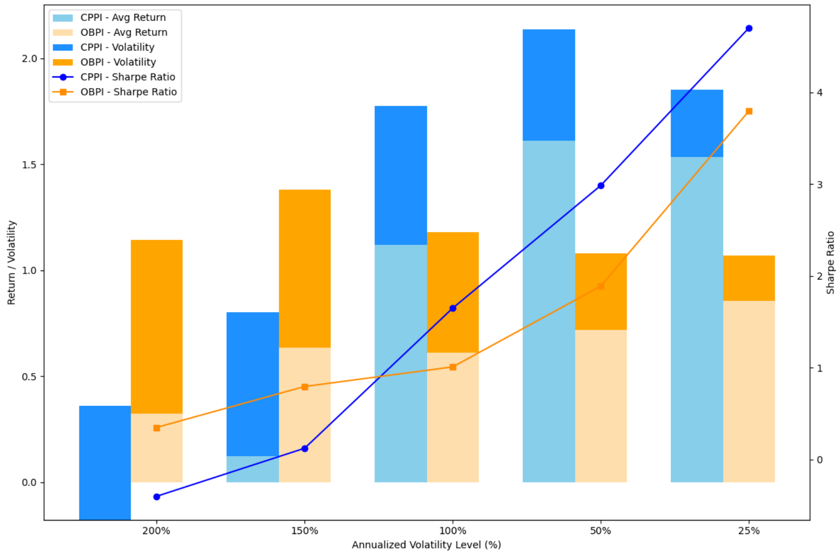

Figure 15: Impact of Volatility Level on the Performance of CPPI and OBPI Strategies

The Sharpe ratio behavior of CPPI and OBPI follows similar patterns in response to changes in market volatility. Both strategies benefit from a decrease in market volatility, with CPPI gaining even more due to its leverage mechanism. However, as volatility increases, both strategies are adversely affected, but a notable difference emerges. OBPI's Sharpe ratio, although small, remains positive even when market volatility reaches 200%. In contrast, CPPI is more significantly impacted by high volatility, with negative Sharpe ratio at 200% volatility level. This is due to the leverage embedded in CPPI, which makes it more susceptible to large market downturns in highly volatile environments, resulting in negative returns.

These differences emphasize the distinct behaviors of the two strategies under varying volatility levels. CPPI performs well in low-volatility conditions, benefiting from its higher convexity during strong market rallies, but it is highly vulnerable in periods of high volatility. On the other hand, OBPI sacrifices some performance in favorable market conditions but is less vulnerable to volatility shocks, offering more consistent returns.

3.3.3 Impact of Floor Level

To explore the effects of the protection level, we apply CPPI and OBPI strategies with varying levels—50%, 60%, 70%, 80%, 90%, and 100%—for the CPPI floor and the OBPI put option strike price, while keeping other parameters unchanged.

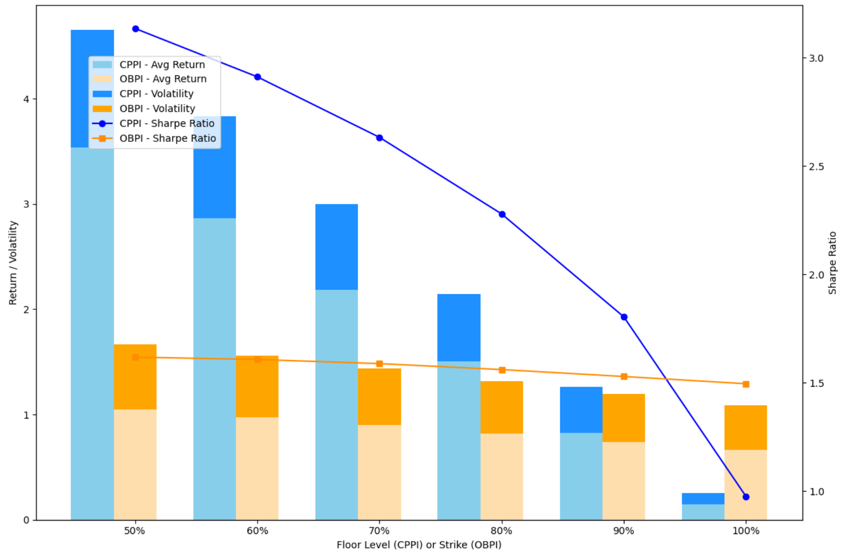

Figure 16: Impact of Floor Level on the Performance of CPPI and OBPI Strategies

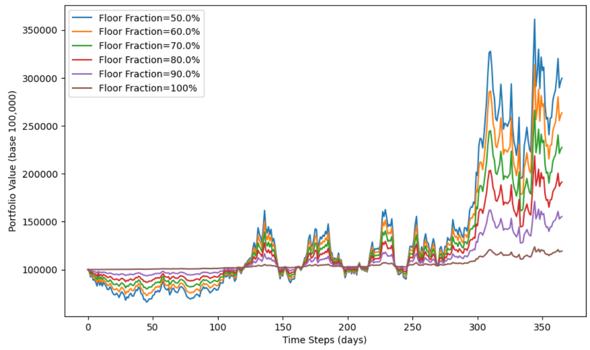

Figure 17: Impact of CPPI Floor Level on Portfolio Value During a Bull Market Scenario

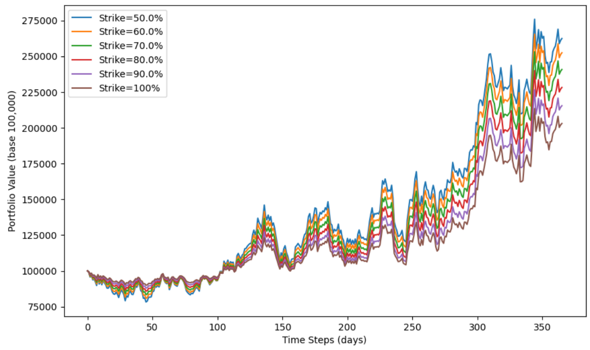

Figure 18: Impact of OBPI Strike Price on Portfolio Value During a Bull Market Scenario

Similar to the effect of rebalancing frequency, we observe that both strategies follow a consistent trend: as the level of protection increases — indicated by the floor level in CPPI and the strike price in OBPI — the Sharpe ratio decreases, despite differences in magnitude. This outcome also ties closely to the behavior of the simulated Bitcoin price paths, which predominantly display an upward trend with high volatility. In such an environment, increasing protection raises the likelihood of the portfolio interacting with its barrier. For CPPI, a higher floor reduces the cushion, prompting earlier and more frequent shifts towards conservative allocations, which limits the strategy's ability to capitalise on upward market movements. Similarly, in OBPI, a higher strike price demands a greater initial outlay for protection, reducing the allocation to the risky asset and constraining potential gains. This aligns with findings by Bertrand and Prigent *(2004)*, who demonstrated that the expected returns of CPPI and OBPI decrease as the insured percentage or strike rises.

While higher protection would be beneficial in bearish conditions, preserving capital and stabilising returns, it proves counterproductive here, where most simulated paths show rising Bitcoin prices. This dynamic is well illustrated in *Figures 17 and 18*, where, along an upward-trending path, both strategies follow the market but diverge in responsiveness. CPPI exhibits greater convexity, lagging during flat periods but accelerating sharply in bullish spikes, whereas OBPI progresses more steadily due to its linear participation rate. Notably, the performance gap between high and low protection levels is wider in CPPI than in OBPI. For instance, CPPI portfolios with a 100% floor remain nearly flat, while those with a 50% floor nearly triple in value, explaining the steep decline in Sharpe ratios as the floor rises. In contrast, OBPI portfolios show a more moderate difference between strikes, resulting in a flatter Sharpe ratio curve.

3.3.4 Impact of Multiplier (CPPI)

To assess the impact of the multiplier, we implement the CPPI strategy with varying multiplier values—1, 2, 3, 4, 5, and 6—while keeping other parameters unchanged.

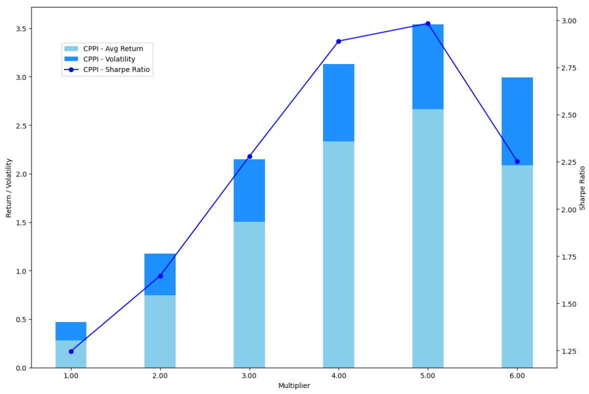

Figure 19: Impact of Multiplier on the Performance of CPPI Strategy

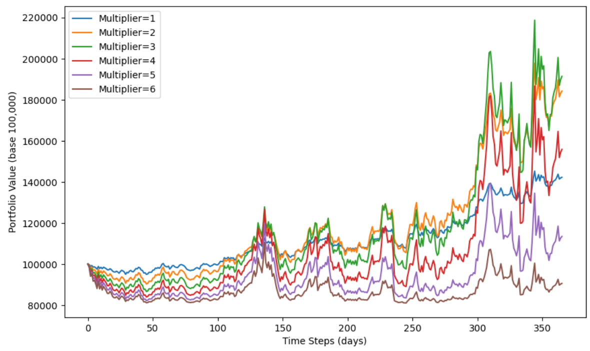

Figure 20: Impact of CPPI Multiplier Values on Portfolio Value During a Bull Market Scenario

In our main analysis, a multiplier of 3 was selected to strike a balance between capturing upside potential and limiting downside risk, based on Bitcoin's historical return profile. However, under the specific assumptions and simulation framework employed, multipliers of 4 or 5 would have delivered higher average risk-adjusted returns. As shown by Bertrand and Prigent *(2004)*, increasing the multiplier enhances the convexity of the CPPI payoff function, while also amplifying the volatility of the return distribution *(Appendix: Figure 28)*. That said, this improvement in Sharpe ratio comes at a cost: higher multipliers also amplify downside risk, as reflected by increases in Value at Risk, Expected Shortfall, and Maximum Drawdown *(Appendix: Table 21)*. A higher multiplier makes the CPPI strategy more sensitive to Bitcoin's price movements. As illustrated in the same bull market scenario *(Figure 20)*, performance improves with the multiplier only up to a certain point—beyond a multiplier of 3, the effect reverses. Due to CPPI's path dependency, higher multipliers magnify the impact of intermediate drawdowns, which can lead to premature de-risking and reduced overall performance.

Furthermore, Bertrand and Prigent *(2004)* showed that for any given strike, there exists a unique multiplier that equates the expected return of CPPI and OBPI. In our results, a multiplier of 2 yields an average return of approximately 75%, closely matching the return of OBPI with a 90% strike. While this could have served as a suitable basis for direct comparison between the two strategies, we retained the multiplier of 3 to maintain consistency with the historical analysis and to explore the greater convexity properties of CPPI, while still preserving a comparable level of downside protection.

4 Extensions to Conventional PI Strategies

Over time, several extensions to conventional portfolio insurance strategies have been proposed to address their limitations and enhance performance under real-world market conditions. Many of these modifications involve dynamic adjustments to the exposure to risky asset, such as the implementation of drawdown-based floors, leverage constraints, or time-varying multipliers. These enhancements aim to improve downside protection, reduce exposure to adverse market moves, or better capture upside potential depending on the market environment. However, such adjustments may also introduce trade-offs, potentially limiting performance in strongly trending markets or increasing sensitivity to market reversals.

In this Section, we analyze the market conditions under which these extensions are likely to lead to performance improvements or deteriorations compared to their conventional counterparts. Evidence is provided through Monte Carlo simulations conducted under the same assumptions as outlined in *Section 3*.

4.1 CPPI Maximum Drawdown

The conventional CPPI strategy is particularly vulnerable in highly volatile markets, where uncontrolled leverage buildup during downturns can lead to substantial losses. To reinforce downside protection, we consider a drawdown-aware variant of CPPI, inspired by the work of Estep and Kritzman *(1988)*. Rather than relying on a fixed floor throughout the investment horizon, this extension dynamically adjusts the floor upward based on the portfolio's historical peak value. By preserving a minimum fraction of accumulated wealth, the strategy aims to explicitly limit losses from the highest achieved value, thereby mitigating the risks associated with excessive exposure during adverse market phases characterized by sharp reversals or prolonged volatility.

This extension proves particularly beneficial in environments where markets are volatile, mean-reverting, or prone to sudden corrections, as the dynamic adjustment mechanism allows the strategy to lock in gains and limit drawdowns more effectively than the classical CPPI. However, under strongly trending bull markets, where prices exhibit sustained upward momentum without significant reversals, the upward adjustment of the floor may constrain risk-taking too early. This can result in a lower overall participation in the rally and hence, underperformance relative to the conventional CPPI, which would otherwise maintain a higher exposure to risky assets throughout the trend.

At each time step t, the peak value is calculated as:

$$\text{Peak}_t = \max(\text{Peak}_{t-1}, P_t)$$

where P_t is the portfolio value at time t.
To ensure the drawdown remains below a chosen threshold of $1 - \delta$, we define a dynamic floor F_t that evolves as:

$$F_t = \delta \cdot \text{Peak}_t$$

To remain consistent with the previous analysis, we set $\delta = 80\%$, which establishes a dynamic floor at 80% of the historical peak at each time step, thereby ensuring that the maximum drawdown does not exceed 20%.

This dynamic floor provides profit lock-in by increasing the minimum value as the portfolio grows. Consequently, risk exposure is automatically reduced after periods of strong performance, since the cushion between portfolio value and floor narrows. While this limits upside potential, it also prevents large reversals in value, which is particularly valuable in volatile markets or for investors with low risk tolerance.

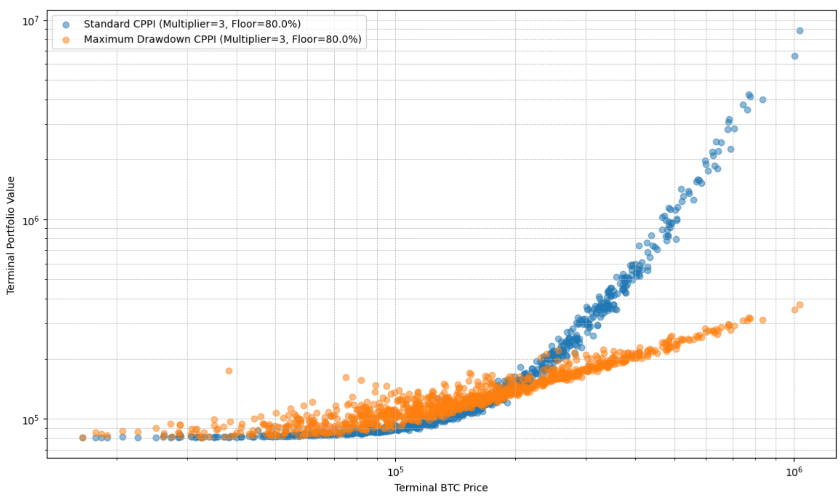

Figure 21: Comparison of Original CPPI and Maximum Drawdown CPPI Performances in Monte Carlo Simulations over 1 year investment horizon

As illustrated in *Figure 21*, the Max Drawdown CPPI strategy offers a more balanced risk-return profile compared to the original standard CPPI. While the standard CPPI delivers a higher average annualized return of 150.36%, significantly outperforming the 35.60% of Max Drawdown CPPI, it does so at the cost of much greater volatility (64.21% vs. 25.56%) and a deeper average maximum drawdown of -42.77%, making it highly vulnerable to sharp market downturns *(Appendix: Table 22)*.
The Max Drawdown CPPI strategy, however, significantly reduces downside risk by implementing a dynamic floor mechanism. It achieves a worst-case maximum drawdown of -19.79%, in line with the 20% loss tolerance set by the 80% floor parameter, compared to the extreme -91.71% observed under the original CPPI. This feature provides stronger protection during market declines.
Although Max Drawdown CPPI results in lower Sharpe (1.24 vs. 2.28) and Sortino (2.15 vs. 3.89) ratios than the standard CPPI, it delivers a more stable and predictable return profile. The strategy significantly reduces tail risk, with a Value-at-Risk (5%) of -2.07% and an Expected Shortfall of -2.86%, versus -5.26% and -7.34% for the standard approach. Moreover, the Max Drawdown CPPI strategy has a higher median return, closer to the mean, indicating less return dispersion (544.84% vs. 46.52%) and more symmetry in the return distribution (skewness: 1.47 vs. 8.19; excess kurtosis: 2.44 vs. 93.17).
Max Drawdown CPPI also reduces portfolio turnover (821% vs. 1448%), which could lower transaction costs in practical applications. Overall, the strategy offers more robust downside protection, reduced volatility, and greater resilience, making it a more suitable option for risk-averse investors who prioritize stability over maximum return.

4.2 Leveraged OBPI

The conventional OBPI strategy inherently sacrifices part of the upside potential to guarantee downside protection through the embedded put option. To enhance the strategy's performance in bullish environments, a leveraged OBPI variant can be considered, wherein the allocation to the risky asset is amplified using a leverage factor $\lambda > 1$. Specifically, this approach involves scaling the portfolio's exposure beyond the standard delta of the option, thereby increasing sensitivity to favorable price movements. In the analysis that follows, we implement a leverage factor of 1.5, meaning the portfolio targets 150% of the risky asset's delta exposure.

This extension is particularly advantageous in markets characterized by strong and persistent upward trends, where the amplified exposure allows the portfolio to capture a greater portion of the rally and close the performance gap with more aggressive strategies like CPPI. However, in bear markets or periods of heightened volatility, the leveraged OBPI may underperform relative to its conventional counterpart. The increased exposure exacerbates losses when the risky asset declines, and the embedded protection becomes less effective, as the higher initial allocation leaves less room for error before the put option activation is triggered. Consequently, while leverage enhances return potential in favorable conditions, it also magnifies risks under adverse market regimes.

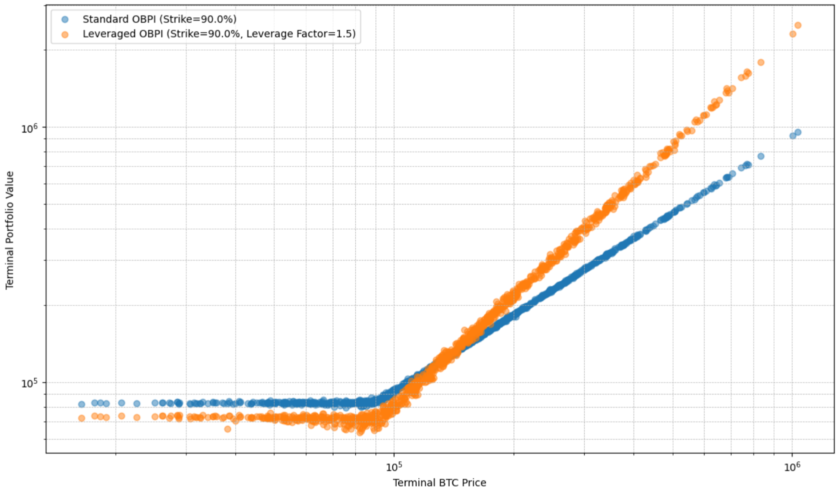

Figure 22: Comparison of Original OBPI and Leveraged OBPI (1.5x) Performances in Monte Carlo Simulations over 1 year investment horizon

The leveraged OBPI strategy achieves higher terminal portfolio values in bullish markets, resulting in a significantly higher return of 132.53%, compared to 73.95% for the standard OBPI. However, this performance boost comes at the cost of increased volatility (68.61% vs. 45.74%) and weaker downside risk measures *(Appendix: Table 23)*. Specifically, the Value-at-Risk (5%) and Expected Shortfall deteriorate to -5.63% and -7.42%, respectively, while the average maximum drawdown widens markedly (-45.93% vs. -32.77%).

This strategy amplifies exposure to the risky asset, leading to improved return potential, with both median (30.32% vs. 27.83%) and average return (132.53% vs. 73.95%) outperforming the standard approach. Despite the higher maximum return (2403.65% vs. 852.06%), it significantly amplifies return variability (265.30% vs. 121.51%) and weakens capital protection, as evidenced by a worst drawdown of -87.30% compared to -73.42%. Additionally, the leveraged strategy's risk-adjusted returns are improved, with a higher Sharpe ratio (1.87 vs. 1.53) and Sortino ratio (3.28 vs. 2.68), but this comes with a substantial increase in turnover (902.35% vs. 601.57%) due to the leverage factor ($601.57\% \times 1.5 = 902.35\%$). This higher turnover could lead to increased transaction costs in practice, partially offsetting the performance gains.

These findings underscore the trade-off between enhanced returns and the structural vulnerability of leverage. While the strategy boosts return potential, it also heightens downside risks, particularly during market downturns, due to its amplified exposure and lack of adaptive risk controls.

4.3 CPPI vs OBPI Extensions Comparison

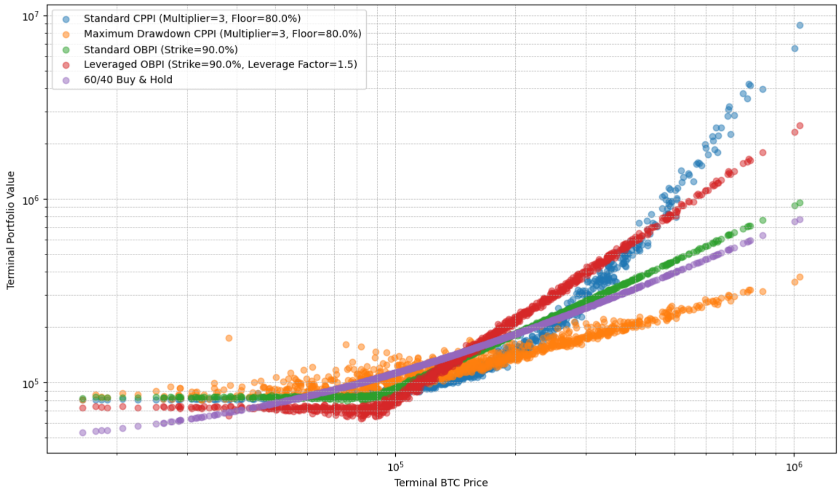

Figure 23: Comparison of Original CPPI and OBPI and their respective extensions Performances in Monte Carlo Simulations over 1 year investment horizon

By plotting the terminal values of the standard CPPI and OBPI strategies alongside their respective extensions, we offer a focused comparison of their payoff structures across varying terminal Bitcoin prices. This visual framework allows for a direct, side-by-side evaluation of how each strategy—and its modified version—performs at maturity under consistent assumptions. Although the figure abstracts from the inherently path-dependent nature of these strategies and does not reflect intra-horizon dynamics such as volatility or drawdowns, it nonetheless captures the cumulative effects of these features on terminal outcomes. The impact of modifications—such as dynamic floors in CPPI and leveraged exposures in OBPI—is evident in the shape, slope, and dispersion of the terminal payoff profiles. This terminal-value analysis provides a concise yet informative perspective on how structural adjustments influence end-of-horizon results, aiding investors in assessing their practical implications across different market conditions.

5 Conclusion

One practical method to assess the performance of a strategy under realistic conditions is through backtesting. Consequently, the first Section of this paper presents the backtesting results of the CPPI and dynamic OBPI strategies, applied to historical Bitcoin returns across various market regimes.

Our analysis shows that both CPPI and OBPI strategies tend to converge toward their protection floors during major bear markets. CPPI reduces exposure to risky asset more quickly than OBPI, maintaining greater sensitivity to rebounds—provided it has not dropped too far below the floor. In bull markets, CPPI seeks to exploit convexity through

leverage, but this makes it highly vulnerable to sharp drawdowns. OBPI, by contrast, follows a steadier, less volatile growth path. Due to their pro-cyclicality, both strategies underperform in trendless markets, often producing similar returns.
Volatility regimes significantly affect performance: low volatility generally benefits both strategies, while high volatility harms CPPI more, given its leveraged sensitivity to price swings. This often results in greater return volatility for CPPI. Overall, both strategies exhibit considerable path dependency, with CPPI being particularly sensitive to market dynamics. Acknowledging the limitations of historical backtesting, we extended our analysis with Monte Carlo simulations.

Drawing on a panel of statistical indicators derived from 1,000 Monte Carlo simulations, we find that the CPPI strategy tends to generate higher returns relative to OBPI, albeit at the expense of greater return volatility and diminished downside protection. The superior performance of CPPI is largely attributable to a small subset of extreme positive outcomes during periods of pronounced Bitcoin rallies, where leveraged exposure significantly amplifies gains. This dynamic is further evidenced by the skewness and excess kurtosis of CPPI's return distribution, which reveal a heavy-tailed profile and an asymmetric dependence on rare, large positive shocks. Furthermore, the relatively low median return, compared to the mean, reinforces that CPPI's outperformance is not representative of the typical investment path.
When extending the modeling framework to incorporate discontinuous price jumps, modeled through the Merton Jump-Diffusion process, the limitations of CPPI become even more apparent. Under this specification, CPPI's median return turns negative, indicating that gap risk materially impairs the strategy's performance across the majority of simulated paths. The heightened exposure to sudden market dislocations severely deteriorates CPPI's risk-adjusted returns and further weakens its downside protection. By contrast, OBPI demonstrates greater robustness to changes in market dynamics. Despite maintaining the valuation of the embedded put option under the classical Black-Scholes assumptions, OBPI's return distribution remains relatively stable under both stochastic volatility and jump conditions. This resilience highlights OBPI's superior adaptability across different volatility regimes and market structures, contrasting with the more fragile and path-dependent nature of CPPI.

We further conducted robustness checks by analyzing how different parameter values affect the final performance of each strategy. Our results indicate that both strategies benefit from increased rebalancing frequency, which improves their responsiveness to market changes and thus their Sharpe ratios. However, as market volatility increases, Sharpe ratios decrease for both strategies, with CPPI being more sensitive—showing negative Sharpe ratio at high volatility level. Portfolio insurance strategies are inherently trend-following and path-dependent. Both strategies benefit from more flexible protection levels (i.e., lower floors or strikes), partly due to the simulated Bitcoin price dynamics. The CPPI strategy achieves its highest Sharpe ratios with a multiplier of 4 or 5; however, higher multipliers come at the expense of reduced downside protection.

Finally, we examined two extensions of standard portfolio insurance strategies. First, CPPI with Maximum Drawdown protection, which employs a rising dynamic floor to lock in gains and limit drawdowns. Second, the Leveraged OBPI strategy, applying a leverage factor $\lambda = 1.5$ to scale risky asset exposure based on option delta, aiming to capture CPPI-like returns during strong market rallies.
A comparative analysis of the standard strategies and their extensions highlighted their respective strengths, weaknesses, and suitability for different investor profiles. Therefore, the choice of the most appropriate strategy for structuring a capital-protected product depends on the historical dynamics of the underlying asset, current market conditions, expectations over the investment horizon and the investor's risk appetite.

As noted in the introduction, Bitcoin and the broader cryptocurrency market continue to attract increasing interest from investors, driven by rapid price appreciations. This growing interest enhances market liquidity and contributes to lower volatility as the market matures. In 2024, Bitcoin prices surged, driven by trends in systematic investing through ETFs and favorable political stances from the Trump administration.

Although our analysis focused exclusively on Bitcoin, we recommend, where feasible, the use of a broader cryptocurrency index to mitigate the idiosyncratic risk associated with investing in a single asset. While such an approach may reduce average returns—since some coins will inevitably underperform—it can also lower overall portfolio volatility through diversification and partial compensation among assets. Despite Bitcoin's dominant influence in the crypto market, which results in high correlations with other coins, the use of an index still provides a meaningful reduction in asset-specific risk. In this context, Liu and Tsyvinski *(2021)* introduced the Cryptocurrency Market Index (CMKT), which aggregates value-weighted returns of all cryptocurrencies with market capitalizations above \$1 million (1,707 coins in total) over the period from January 1, 2011, to December 31, 2018. Similar to how the S&P 500 offers diversified exposure to equities, the CMKT serves as a useful benchmark for capturing broad crypto market performance while reducing exposure to individual asset volatility. For the purposes of this paper, and given Bitcoin's strong correlation with the broader crypto market, we maintain our focus on Bitcoin, under the simplifying assumption that it carries no idiosyncratic risk.

Historically, Bitcoin has exhibited episodes of substantial returns alongside pronounced volatility. Although the market shows signs of maturation and a gradual decline in volatility, Bitcoin remains significantly more volatile than most equity indices or individual stocks. As a result, gap risk continues to be a key consideration when selecting an appropriate portfolio insurance strategy. The investor's choice among strategies will depend largely on their individual risk tolerance, particularly with respect to the desired level of capital protection. Regarding the multiplier in the CPPI framework, our analysis suggests that a multiplier greater than 3 should be avoided: beyond this threshold, the additional convexity becomes marginal, while the dispersion in final returns increases significantly *(Appendix: Figure 28)*.

When selecting among standard strategies and their respective extensions, given a specified protection floor and CPPI multiplier, the optimal choice depends fundamentally on the investor's market expectations. CPPI offers several advantages over dynamic OBPI, including greater flexibility, conceptual simplicity, and ease of implementation. Specifically, in environments where an explosive rally is anticipated, traditional CPPI is likely to perform best. For a strong but less extreme rally, Leveraged OBPI presents a more balanced risk-return profile. In cases forecasting moderate appreciation, standard OBPI may be the most appropriate choice. Alternatively, if the potential for upside is limited and the primary focus is on minimizing drawdowns, the CPPI Maximum Drawdown variant offers a well-balanced solution. However, if a steep and prolonged market decline is anticipated, none of the portfolio insurance strategies discussed would be appropriate.
It is important to note that CPPI is more sensitive to market volatility than OBPI, as higher volatility increases the likelihood of breaching the floor and triggering capital reallocation to the safe asset. This makes volatility dynamics—particularly its mean-reverting nature—an important consideration when selecting both the investment horizon and entry point. Given that most investors are risk-averse and tend to experience losses more acutely than equivalent gains, we recommend OBPI-based strategies for Bitcoin exposure. Depending on individual risk appetite, either the standard or leveraged OBPI can be used, with protection levels tailored accordingly.

Figure 23 provides a unified visualization of all strategies and their extensions, allowing for a direct comparison of their performance profiles under a common set of assumptions. It is also worth noting that numerous other extensions exist beyond those discussed, including Dynamic CPPI, which adjusts the multiplier according to prevailing market conditions, and OBPI structures incorporating exotic options, such as barrier or Asian options, to tailor protection and participation more finely.

Like a well-fitted suit, Portfolio Insurance strategies can be tailored to match any investor's goals and risk appetite, whatever the market may bring.

6 Appendix

6.1 Backtesting

6.1.1 Volatility Modeling

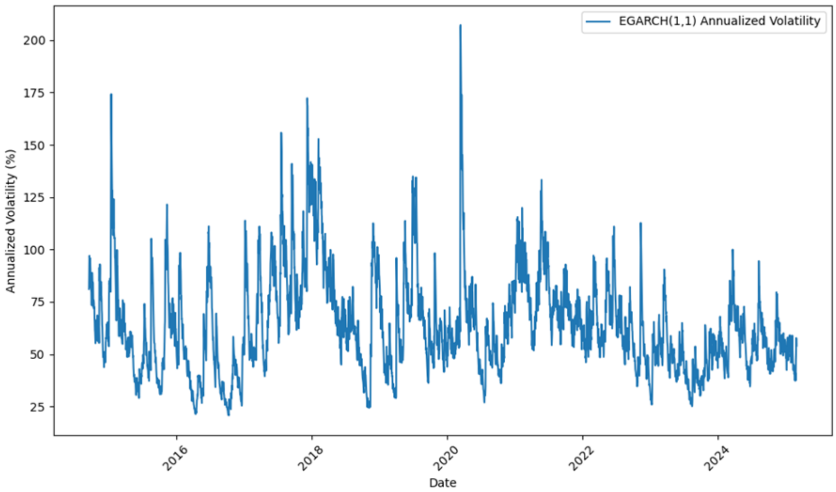

Figure 24: Annualized Volatility EGARCH(1,1)

```
                     Constant Mean - EGARCH Model Results
==========================================================================================
Dep. Variable:                        BTC-USD   R-squared:                         0.000
Mean Model:                     Constant Mean   Adj. R-squared:                    0.000
Vol Model:                             EGARCH   Log-Likelihood:                 -9503.93
Distribution:     Generalized Error Distribution   AIC:                          19019.9
Method:                    Maximum Likelihood   BIC:                             19057.3
                                                No. Observations:                   3817
Date:                        Mon, Apr 14 2025   Df Residuals:                       3816
Time:                                11:02:05   Df Model:                              1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.1024  3.089e-03     33.163 3.621e-241 [9.639e-02,  0.108]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
----------------------------------------------------------------------------
omega          0.0889  2.208e-02      4.027  5.650e-05    [4.564e-02,  0.132]
alpha[1]       0.2228  2.665e-02      8.360  6.259e-17      [  0.171,  0.275]
gamma[1]  -2.2885e-03  1.330e-02     -0.172      0.863 [-2.836e-02,2.379e-02]
beta[1]        0.9696  8.224e-03    117.901      0.000      [  0.954,  0.986]
                              Distribution
========================================================================
...
nu             1.0100  3.994e-02     25.287 4.428e-141 [  0.932,  1.088]
========================================================================

Covariance estimator: robust
```

Figure 25: EGARCH Model Results

6.1.2 Flat market

Given Bitcoin's high volatility, a flat period does not resemble the typical sideways markets seen in other asset classes. Therefore, we selected a timeframe in which the price both started and ended at approximately the same level, while remaining within a relatively narrow range throughout.

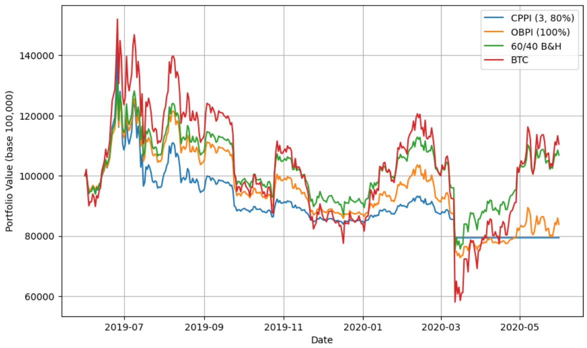

Figure 26: Performance Comparison of CPPI, OBPI, Buy-and-Hold Strategies, and Bitcoin in Flat Market Conditions (Jun 2019 - May 2020)

Figure 27: Evolution of CPPI and OBPI Weights in Flat Market Conditions (Jun 2019 - May 2020)

In a flat market, both CPPI and OBPI underperformed buy-and-hold, posting returns of -20.61% and -16.18%, respectively, versus +6.83% for the 60/40 portfolio. This was expected, as portfolio insurance strategies tend to lag in sideways markets.

While the entire period remained relatively flat, from July to March, there was a downward trend, leading to a gradual de-risking for both strategies. On March 12th, the crypto market experienced a flash crash, with Bitcoin losing about 37% of its value in a single day. Unlike the bear market period studied, where CPPI maintained some risky exposure due to price swings, the Bitcoin price recovery in this instance did not trigger a re-investment. Once CPPI fully divested from the risky asset, the only way to bring the portfolio value back above the floor was through the risk-free asset, and the remaining time in the period did not allow for reinvestment in Bitcoin.

Similarly, OBPI exhibited a breach of the protection floor. This violation occurred due to the high gamma sensitivity of the put option near the money in this environment, where price fluctuations led to a lag in portfolio adjustment. However, despite the volatility of the risky allocation on the day of the crash (reaching 0), the strategy reinvested in Bitcoin and benefited from the subsequent rebound, recovering from the floor violation and ultimately outperforming the CPPI strategy.

While weekly rebalancing might have prevented the drawdown, daily rebalancing yielded a moderate enhancement in risk-adjusted performance for CPPI but had little impact on OBPI.

6.1.3 Calendar-Based Rebalancing

Performance Metrics	CPPI (m=3, f=80%)		OBPI (strike=100%)		Buy and Hold		Bitcoin	
	Daily	Weekly	Daily	Weekly	Daily	Weekly	Daily	Weekly
Return	-19.05%	-19.02%	-18.44%	-16.54%	-39.32%	-39.93%	-66.40%	-67.41%
Volatility	12.65%	11.19%	12.77%	11.47%	28.82%	24.89%	63.67%	55.04%
Sharpe Ratio	-1.61	-1.82	-1.55	-1.55	-1.41	-1.66	-1.06	-1.25
Sortino Ratio	-1.66	-1.64	-1.49	-1.47	-1.84	-2.22	-1.43	-1.73
VaR (5%)	-0.89%	-2.75%	-1.05%	-3.06%	-2.96%	-7.09%	-5.99%	-12.62%
Expected Shortfall (5%)	-2.00%	-5.20%	-1.98%	-4.81%	-3.86%	-8.30%	-8.21%	-17.61%
Skewness	-0.72	-2.37	-0.69	-1.39	-0.18	-0.27	-0.17	-0.31
Excess Kurtosis	12.51	7.82	7.56	3.99	3.06	-0.06	3.21	0.03
Turnover	320.20%	117.63%	422.63%	139.73%	0.00%	0.00%	-	-
Max Drawdown	-24.74%	-21.33%	-23.54%	-18.72%	-44.95%	-42.96%	-72.55%	-71.28%
Information Ratio	1.22	1.58	1.24	1.66	1.20	1.58	-	-
Tracking Error	56.22%	48.13%	55.64%	47.65%	35.74%	30.66%	-	-

Table 5: Performance Metrics: Bear Market (2021-11 to 2022-10)

Performance Metrics	CPPI (m=3, f=80%)		OBPI (strike=100%)		Buy and Hold		Bitcoin	
	Daily	Weekly	Daily	Weekly	Daily	Weekly	Daily	Weekly
Return	216.26%	138.30%	247.79%	240.49%	207.84%	207.84%	346.37%	346.37%
Volatility	164.68%	168.46%	76.67%	76.65%	65.54%	66.16%	81.47%	82.13%
Sharpe Ratio	1.31	0.82	3.23	3.14	3.17	3.14	4.25	4.22
Sortino Ratio	1.86	1.05	4.90	4.58	4.82	4.60	6.62	6.56
VaR (5%)	-13.42%	-41.04%	-5.89%	-16.13%	-5.10%	-13.93%	-6.22%	-16.16%
Expected Shortfall (5%)	-19.29%	-53.72%	-8.51%	-22.07%	-7.27%	-19.03%	-8.90%	-22.31%
Skewness	-0.00	-0.42	0.02	-0.47	0.01	-0.47	0.07	-0.34
Excess Kurtosis	2.98	1.11	1.65	0.16	1.58	0.16	1.45	-0.01
Turnover	2057.67%	880.24%	255.47%	91.52%	0.00%	0.00%	-	-
Max Drawdown	-86.68%	-86.59%	-52.90%	-47.01%	-46.37%	-40.95%	-53.06%	-47.19%
Information Ratio	0.78	0.64	-2.97	-2.75	-2.92	-2.90	-	-
Tracking Error	88.45%	91.95%	9.69%	11.45%	16.77%	16.97%	-	-

Table 6: Performance Metrics: Bull Market (2020-11 to 2021-10)

Performance Metrics	CPPI (m=3, f=80%)		OBPI (strike=100%)		Buy and Hold		Bitcoin	
	Daily	Weekly	Daily	Weekly	Daily	Weekly	Daily	Weekly
Return	-20.61%	-20.01%	-16.18%	-13.33%	6.83%	5.46%	10.47%	8.21%
Volatility	41.47%	30.61%	41.27%	35.66%	48.47%	42.89%	81.43%	72.23%
Sharpe Ratio	-0.53	-0.70	-0.42	-0.41	0.11	0.10	0.11	0.10
Sortino Ratio	-0.52	-0.84	-0.49	-0.51	0.13	0.12	0.13	0.12
VaR (5%)	-2.45%	-8.14%	-3.13%	-8.94%	-3.20%	-9.49%	-5.71%	-15.57%
Expected Shortfall (5%)	-5.86%	-10.15%	-5.64%	-11.94%	-5.96%	-14.09%	-9.67%	-23.62%
Skewness	-1.51	-0.17	-1.17	-0.60	-1.54	-0.70	-1.46	-0.72
Excess Kurtosis	21.56	1.88	8.58	0.91	15.40	1.46	17.35	1.70
Turnover	863.01%	339.36%	1035.52%	388.98%	0.00%	0.00%	-	-
Max Drawdown	-44.32%	-31.32%	-44.08%	-35.17%	-42.58%	-34.98%	-61.81%	-53.21%
Information Ratio	-1.00	-1.05	-1.19	-1.10	-0.79	-0.69	-	-
Tracking Error	59.36%	49.58%	45.42%	38.63%	33.46%	29.23%	-	-

Table 7: Performance Metrics: Flat Market (2019-06 to 2020-05)

Performance Metrics	CPPI (m=3, f=80%)		OBPI (strike=100%)		Buy and Hold		Bitcoin	
	Daily	Weekly	Daily	Weekly	Daily	Weekly	Daily	Weekly
Return	84.95%	69.80%	87.47%	80.67%	73.25%	65.16%	121.89%	108.41%
Volatility	43.51%	45.72%	35.69%	35.59%	31.59%	31.46%	48.08%	47.71%
Sharpe Ratio	1.95	1.52	2.44	2.26	2.31	2.06	2.53	2.27
Sortino Ratio	2.19	1.70	2.89	3.21	2.72	2.79	2.98	3.14
VaR (5%)	-2.36%	-5.05%	-2.05%	-4.99%	-1.88%	-4.41%	-2.73%	-7.22%
Expected Shortfall (5%)	-5.55%	-13.57%	-4.44%	-9.25%	-3.90%	-8.90%	-5.86%	-13.58%
Skewness	-0.19	-0.74	-0.14	0.08	-0.27	-0.10	-0.28	0.00
Excess Kurtosis	10.98	6.71	7.84	2.41	8.17	2.28	8.94	2.01
Turnover	910.97%	408.14%	434.82%	217.88%	0.00%	0.00%	-	-
Max Drawdown	-34.15%	-31.95%	-26.81%	-22.77%	-20.71%	-18.39%	-28.56%	-25.31%
Information Ratio	-1.14	-1.20	-1.31	-1.19	-1.87	-1.82	-	-
Tracking Error	17.79%	17.83%	16.87%	16.35%	16.81%	16.41%	-	-

Table 8: Performance Metrics: Low Volatility Market (2016-01 to 2016-12)

Performance Metrics	CPPI (m=3, f=80%)		OBPI (strike=100%)		Buy and Hold		Bitcoin	
	Daily	Weekly	Daily	Weekly	Daily	Weekly	Daily	Weekly
Return	153.30%	-2.15%	367.93%	320.05%	330.50%	294.43%	550.01%	489.91%
Volatility	245.90%	237.13%	100.63%	113.22%	91.43%	102.25%	106.63%	121.65%
Sharpe Ratio	0.62	-0.01	3.64	2.82	3.60	2.87 5.15	4.02	
Sortino Ratio	0.89	-0.02	5.63	5.67	5.58	5.60	8.29	8.60
VaR (5%)	-19.18%	-47.58%	-7.87%	-19.55%	-7.14%	-17.22%	-7.91%	-21.52%
Expected Shortfall (5%)	-27.76%	-65.85%	-10.77%	-25.54%	-9.78%	-23.45%	-10.98%	-26.80%
Skewness	0.41	0.08	0.23	-0.08	0.29	-0.13	0.32	-0.04
Excess Kurtosis	4.53	1.41	2.32	-0.42	2.50	-0.39	2.20	-0.47
Turnover	2198.58%	1169.04%	269.02%	129.60%	0.00%	0.00%	-	-
Max Drawdown	-98.11%	-97.32%	-65.99%	-64.22%	-62.86%	-60.81%	-65.96%	-64.24%
Information Ratio	1.04	0.48	-2.97	-2.32	-3.05	-2.62	-	-
Tracking Error	146.36%	138.82%	13.14%	19.04%	18.42%	23.64%	-	-

Table 9: Performance Metrics: High Volatility Market (2017-05 to 2018-04)

6.1.4 Move-Based Rebalancing

	CPPI (m=3, f=80%)					OBPI (strike=100%)				
Rebalancing Thresholds	**1%**	**2.5%**	**5%**	**10%**	**25%**	**1%**	**2.5%**	**5%**	**10%**	**25%**
Return	-19.05%	-18.85%	-18.99%	-19.00%	-19.09%	-18.61%	-18.27%	-18.20%	-18.18%	-25.45%
Volatility	12.71%	12.55%	12.72%	13.48%	13.01%	12.80%	12.79%	13.20%	14.09%	18.50%
Sharpe Ratio	-1.60	-1.61	-1.59	-1.50	-1.57	-1.55	-1.53	-1.48	-1.38	-1.45
Sortino Ratio	-1.65	-1.61	-1.65	-1.61	-1.46	-1.54	-1.64	-1.44	-1.51	-1.81
VaR (5%)	-0.92%	-0.91%	-0.95%	-1.10%	-0.87%	-1.06%	-1.08%	-1.18%	-1.28%	-1.77%
Expected Shortfall (5%)	-2.01%	-2.00%	-2.00%	-2.09%	-2.15%	-1.99%	-1.99%	-2.04%	-2.14%	-2.63%
Skewness	-0.69	-0.74	-0.70	-0.82	-1.85	-0.68	-0.70	-0.71	-0.58	-0.43
Excess Kurtosis	12.22	12.68	12.09	10.08	18.39	7.47	7.47	7.22	7.82	2.97
Turnover	268.36%	206.27%	158.32%	84.12%	57.69%	383.21%	277.61%	144.41%	78.80%	26.96%
Max Drawdown	-24.73%	-24.80%	-24.89%	-25.05%	-24.92%	-23.67%	-23.46%	-23.20%	-23.30%	-31.63%
Information Ratio	1.22	1.22	1.22	1.26	1.18	1.24	1.26	1.26	1.29	1.31
Tracking Error	56.07%	56.43%	56.09%	54.58%	58.11%	55.51%	55.22%	55.01%	54.09%	46.64%

Table 10: Performance Metrics: Bear Market (2021-11 to 2022-10) across Rebalancing Thresholds (Daily Observations)

Rebalancing Thresholds	CPPI (m=3, f=80%) 1%	2.5%	5%	10%	25%	OBPI (strike=100%) 1%	2.5%	5%	10%	25%
Return	217.59%	219.95%	201.92%	202.31%	242.76%	247.29%	248.54%	237.25%	237.88%	204.08%
Volatility	164.80%	165.03%	162.70%	161.83%	160.56%	76.43%	75.90%	75.89%	73.38%	61.19%
Sharpe Ratio	1.32	1.33	1.24	1.25	1.51	3.23	3.27	3.13	3.24	3.33
Sortino Ratio	1.87	1.89	1.75	1.76	2.14	4.90	4.97	4.72	4.93	5.13
VaR (5%)	-13.42%	-13.33%	-13.13%	-13.41%	-12.79%	-5.89%	-5.87%	-5.88%	-5.60%	-4.72%
Expected Shortfall (5%)	-19.29%	-19.31%	-19.13%	-19.11%	-18.81%	-8.49%	-8.42%	-8.48%	-8.15%	-6.76%
Skewness	-0.00	0.01	-0.00	-0.02	-0.00	0.03	0.03	0.02	0.06	0.06
Excess Kurtosis	2.97	2.98	3.11	3.07	2.86	1.67	1.65	1.75	1.78	1.57
Turnover	2027.07%	1901.32%	1720.74%	1361.97%	653.19%	206.53%	140.40%	91.96%	44.15%	25.86%
Max Drawdown	-86.67%	-86.65%	-86.36%	-86.05%	-85.53%	-52.65%	-52.01%	-52.68%	-50.66%	-42.50%
Information Ratio	0.79	0.80	0.71	0.70	0.84	-3.01	-2.95	-3.23	-3.01	-2.54
Tracking Error	88.55%	88.73%	86.82%	86.08%	84.21%	9.69%	9.89%	10.06%	11.35%	20.83%

Table 11: Performance Metrics: Bull Market (2020-11 to 2021-10) across Rebalancing Thresholds (Daily Observations)

Rebalancing Thresholds	CPPI (m=3, f=80%) 1%	2.5%	5%	10%	25%	OBPI (strike=100%) 1%	2.5%	5%	10%	25%
Return	-20.66%	-21.01%	-21.16%	-20.44%	-5.64%	-16.37%	-17.03%	-17.38%	-13.01%	-11.77%
Volatility	41.54%	41.18%	41.54%	39.26%	42.45%	41.39%	41.49%	41.47%	41.87%	39.77%
Sharpe Ratio	-0.53	-0.54	-0.54	-0.56	-0.16	-0.43	-0.44	-0.45	-0.34	-0.33
Sortino Ratio	-0.52	-0.53	-0.53	-0.54	-0.17	-0.49	-0.51	-0.52	-0.40	-0.36
VaR (5%)	-2.50%	-2.54%	-2.51%	-2.21%	-2.38%	-3.13%	-3.12%	-3.17%	-3.08%	-2.69%
Expected Shortfall (5%)	-5.88%	-5.84%	-5.91%	-5.58%	-5.95%	-5.66%	-5.68%	-5.68%	-5.68%	-5.20%
Skewness	-1.51	-1.53	-1.57	-1.58	-1.57	-1.21	-1.28	-1.26	-1.21	-2.54
Excess Kurtosis	21.44	21.80	21.56	23.14	16.55	8.86	9.48	9.31	9.42	25.35
Turnover	822.45%	737.89%	581.15%	477.60%	248.71%	989.80%	871.22%	709.49%	467.68%	241.71%
Max Drawdown	-44.35%	-44.41%	-44.69%	-42.84%	-41.63%	-44.15%	-44.34%	-44.44%	-41.75%	-40.66%
Information Ratio	-1.00	-1.01	-1.02	-0.98	-0.82	-1.20	-1.22	-1.23	-1.13	-1.11
Tracking Error	59.19%	59.45%	58.74%	61.35%	50.70%	45.31%	45.12%	45.11%	44.41%	44.46%

Table 12: Performance Metrics: Flat Market (2019-06 to 2020-05) across Rebalancing Thresholds (Daily Observations)

Rebalancing Thresholds	CPPI (m=3, f=80%) 1%	2.5%	5%	10%	25%	OBPI (strike=100%) 1%	2.5%	5%	10%	25%
Return	83.78%	83.02%	84.30%	67.44%	68.80%	86.80%	86.35%	85.60%	78.43%	79.11%
Volatility	43.25%	43.13%	43.67%	39.95%	37.46%	35.64%	35.55%	35.41%	34.78%	32.19%
Sharpe Ratio	1.93	1.92	1.92	1.68	1.83	2.43	2.42	2.41	2.25	2.45
Sortino Ratio	2.17	2.15	2.15	1.82	1.99	2.87	2.86	2.83	2.56	2.89
VaR (5%)	-2.34%	-2.35%	-2.37%	-2.04%	-2.09%	-2.03%	-2.03%	-2.06%	-2.00%	-1.91%
Expected Shortfall (5%)	-5.52%	-5.50%	-5.58%	-5.19%	-4.87%	-4.44%	-4.42%	-4.40%	-4.40%	-3.96%
Skewness	-0.20	-0.21	-0.27	-0.51	-0.55	-0.16	-0.14	-0.17	-0.42	-0.21
Excess Kurtosis	11.01	11.02	11.21	12.18	10.52	7.85	8.01	8.43	8.90	7.90
Turnover	848.14%	753.13%	603.40%	462.63%	256.57%	389.37%	305.88%	227.63%	156.44%	26.42%
Max Drawdown	-33.97%	-33.75%	-34.41%	-33.05%	-30.09%	-26.85%	-26.78%	-26.79%	-27.77%	-22.21%
Information Ratio	-1.19	-1.22	-1.16	-1.75	-1.64	-1.33	-1.36	-1.38	-1.59	-1.58
Tracking Error	17.76%	17.66%	17.75%	18.09%	19.46%	16.87%	16.78%	16.82%	17.22%	17.64%

Table 13: Performance Metrics: Low Volatility Market (2016-01 to 2016-12) across Rebalancing Thresholds (Daily Observations)

Rebalancing Thresholds	CPPI (m=3, f=80%) 1%	2.5%	5%	10%	25%	OBPI (strike=100%) 1%	2.5%	5%	10%	25%
Return	153.29%	149.26%	154.69%	152.06%	106.86%	370.51%	366.00%	361.01%	360.35%	309.20%
Volatility	245.93%	245.21%	245.72%	242.70%	235.15%	100.09%	98.76%	98.32%	96.19%	80.72%
Sharpe Ratio	0.62	0.60	0.62	0.62	0.45	3.69	3.69	3.66	3.73	3.82
Sortino Ratio	0.89	0.87	0.90	0.89	0.62	5.71	5.72	5.66	5.81	6.09
VaR (5%)	-19.18%	-19.19%	-19.21%	-19.04%	-17.82%	-7.85%	-7.71%	-7.65%	-7.45%	-6.04%
Expected Shortfall (5%)	-27.77%	-27.73%	-27.76%	-27.42%	-27.40%	-10.70%	-10.56%	-10.50%	-10.21%	-8.38%
Skewness	0.41	0.41	0.39	0.39	0.28	0.24	0.23	0.23	0.26	0.34
Excess Kurtosis	4.52	4.54	4.38	4.45	4.42	2.31	2.28	2.35	2.31	2.37
Turnover	2172.04%	2103.77%	1864.03%	1504.21%	1063.55%	233.42%	194.25%	140.55%	70.39%	25.53%
Max Drawdown	-98.11%	-98.10%	-98.09%	-97.93%	-97.94%	-65.61%	-64.85%	-64.48%	-63.31%	-54.26%
Information Ratio	1.04	1.02	1.05	1.00	0.79	-2.97	-3.08	-3.20	-3.15	-2.66
Tracking Error	146.37%	145.76%	146.08%	143.26%	137.03%	13.12%	13.38%	13.34%	14.27%	26.47%

Table 14: Performance Metrics: High Volatility Market (2017-05 to 2018-04) across Rebalancing Thresholds (Daily Observations)

6.2 Simulating

6.2.1 Rebalancing Frequency

Performance Metrics	Daily (365)	Weekly (52)	Monthly (12)	Quarterly (4)	Semi-Annual (2)
Path-level metrics (Avg. Across All Paths)					
Return	165.70%	160.31%	146.71%	120.78%	85.22%
Volatility	63.35%	62.96%	62.26%	64.12%	57.62%
Tracking Error	44.68%	45.61%	47.03%	46.79%	43.50%
Sharpe Ratio	2.55	2.48	2.29	1.82	1.41
Sortino Ratio	4.35	4.48	4.75	5.78	7.21
VaR 5%	-5.23%	-12.30%	-19.34%	-13.96%	7.19%
Expected Shortfall 5%	-7.25%	-17.29%	-27.32%	-17.83%	4.31%
Skewness	-0.08	-0.20	-0.32	-0.16	0.00
Excess Kurtosis	2.82	2.51	1.82	0.72	-2.00
Turnover	1429.46%	539.96%	259.24%	155.78%	109.88%
Max Drawdown	-42.41%	-39.16%	-32.32%	-18.29%	-7.43%
Information Ratio	-0.03	-0.08	-0.16	-0.24	-1.15
Distribution metrics (Cross-Sectional)					
Median Return	4.50%	2.36%	0.06%	14.05%	32.44%
Maximum Return	28282.11%	58963.74%	39505.48%	7892.25%	2771.43%
Minimum Return	-19.70%	-19.77%	-490.78%	-355.32%	-264.18%
Volatility of Return Distribution	931.46%	1072.87%	819.14%	341.98%	167.08%
VaR 5%	-18.39%	-18.52%	-22.46%	-34.99%	-32.89%
Expected Shortfall 5%	-18.92%	-18.99%	-41.42%	-68.87%	-55.01%
Skewness	18.57	36.22	26.53	7.75	4.00
Excess Kurtosis	461.38	1845.78	1112.18	106.05	29.99
Worst Maximum Drawdown	-94.37%	-98.55%	-121.90%	-155.11%	-161.19%

Table 15: CPPI (m=3, f=80%) Strategy - Impact of Rebalancing Frequency on Performance Metrics

Performance Metrics	Daily (365)	Weekly (52)	Monthly (12)	Quarterly (4)	Semi-Annual (2)
Path-level metrics (Avg. Across All Paths)					
Return	71.88%	69.57%	67.28%	66.67%	57.80%
Volatility	45.52%	45.28%	44.89%	44.39%	40.89%
Tracking Error	30.13%	31.14%	33.83%	38.89%	43.18%
Sharpe Ratio	1.49	1.45	1.41	1.41	1.32
Sortino Ratio	2.61	2.75	3.20	4.87	7.12
VaR 5%	-3.75%	-8.77%	-13.38%	-8.94%	6.01%
Expected Shortfall 5%	-4.93%	-11.47%	-17.27%	-11.42%	3.96%
Skewness	0.01	0.03	0.01	-0.02	0.00
Excess Kurtosis	1.64	1.35	0.79	0.28	-2.00
Turnover	604.70%	231.97%	117.63%	74.50%	58.33%
Max Drawdown	-32.68%	-29.40%	-22.97%	-12.03%	-4.67%
Information Ratio	-0.07	-0.17	-0.31	-0.44	-1.30
Distribution metrics (Cross-Sectional)					
Median Return	29.31%	27.37%	27.44%	28.34%	28.40%
Maximum Return	1292.94%	1739.56%	2165.61%	1488.99%	1156.72%
Minimum Return	-23.10%	-28.57%	-39.80%	-51.73%	-55.21%
VaR 5%	-17.82%	-18.84%	-20.22%	-20.16%	-21.71%
Expected Shortfall 5%	-18.44%	-20.56%	-23.40%	-24.51%	-27.29%
Skewness	2.86	3.03	3.68	3.05	2.77
Excess Kurtosis	13.28	16.38	29.77	17.04	13.15
Worst Maximum Drawdown	-73.55%	-71.49%	-69.86%	-72.26%	-75.34%

Table 16: OBPI (strike=90%) Strategy - Impact of Rebalancing Frequency on Performance Metrics

6.2.2 Volatility Level

Performance Metrics	Vol = 200%	Vol = 150%	Vol = 100%	Vol = 50%	Vol = 25%
Path-level metrics (Avg. Across All Paths)					
Return	-17.66%	12.28%	112.03%	161.14%	153.30%
Volatility	53.87%	67.97%	65.52%	52.62%	31.75%
Tracking Error	185.66%	130.45%	75.91%	27.86%	12.32%
Sharpe Ratio	-0.40	0.12	1.65	2.99	4.70
Sortino Ratio	-0.59	0.19	2.69	5.20	8.69
VaR 5%	-3.12%	-5.04%	-5.37%	-4.29%	-2.44%
Expected Shortfall 5%	-7.05%	-8.67%	-8.00%	-5.84%	-3.32%
Skewness	-1.97	-0.92	-0.37	-0.00	0.08
Excess Kurtosis	39.87	20.25	7.69	1.51	1.08
Turnover	867.69%	1257.37%	1438.72%	1235.66%	737.34%
Max Drawdown	-39.18%	-45.26%	-44.92%	-34.59%	-17.03%
Information Ratio	-0.02	-0.03	-0.03	-0.03	0.03
Distribution metrics (Cross-Sectional)					
Median Return	-19.69%	-19.29%	-16.06%	35.44%	110.46%
Maximum Return	422.25%	10582.79%	21496.57%	14455.50%	1465.28%
Minimum Return	-57.67%	-19.95%	-19.87%	-17.97%	-2.14%
Volatility of Return Distribution	18.07%	417.21%	1000.53%	583.18%	154.21%
Var 5%	-19.94%	-19.88%	-19.59%	-13.11%	22.11%
Expected Shortfall 5%	-20.71%	-19.90%	-19.71%	-15.19%	13.77%
Skewness	17.60	20.08	14.38	16.34	3.55
Excess Kurtosis	378.33	454.40	251.97	369.50	19.66
Worst Maximum Drawdown	-99.79%	-99.89%	-97.34%	-84.85%	-51.60%

Table 17: CPPI (m=3, f=80%) Strategy - Impact of Volatility Level on Performance Metrics

Performance Metrics	Vol = 200%	Vol = 150%	Vol = 100%	Vol = 50%	Vol = 25%
Path-level metrics (Avg. Across All Paths)					
Return	32.52%	63.37%	61.26%	71.95%	85.54%
Volatility	81.79%	74.67%	56.76%	35.98%	21.47%
Tracking Error	138.50%	89.89%	52.79%	18.84%	6.26%
Sharpe Ratio	0.35	0.80	1.01	1.89	3.80
Sortino Ratio	0.59	1.36	1.73	3.35	7.14
VaR 5%	-6.81%	-6.22%	-4.73%	-2.92%	-1.66%
Expected Shortfall 5%	-9.48%	-8.38%	-6.31%	-3.85%	-2.16%
Skewness	-0.04	-0.01	-0.04	0.04	0.08
Excess Kurtosis	5.84	3.90	2.75	1.26	0.53
Turnover	676.09%	690.84%	672.96%	551.58%	322.67%
Max Drawdown	-57.69%	-53.17%	-42.41%	-24.48%	-11.27%
Information Ratio	-0.03	-0.04	-0.05	-0.08	-0.11
Distribution metrics (Cross-Sectional)					
Median Return	-37.00%	-31.24%	-10.94%	45.74%	79.42%
Maximum Return	3507.28%	3981.58%	2020.20%	895.07%	308.53%
Minimum Return	-46.62%	-40.88%	-29.97%	-17.28%	-5.64%
Volatility of Return Distribution	269.90%	278.44%	184.54%	91.01%	46.74%
VaR 5%	-40.05%	-34.26%	-25.81%	-12.67%	21.52%
Expected Shortfall 5%	-41.36%	-35.37%	-26.75%	-13.39%	11.45%
Skewness	7.01	6.94	4.93	2.28	1.01
Excess Kurtosis	61.53	71.95	33.38	9.67	2.01
Worst Maximum Drawdown	-95.12%	-89.19%	-81.30%	-58.18%	-30.05%

Table 18: OBPI (strike=90%) Strategy - Impact of Volatility Level on Performance Metrics

6.2.3 Floor (or Strike) Level

Performance Metrics	Floor = 50%	Floor = 60%	Floor = 70%	Floor = 80%	Floor = 90%	Floor = 100%
Path-level metrics (Avg. Across All Paths)						
Return	353.91%	286.06%	218.21%	150.36%	82.51%	14.66%
Volatility	111.62%	96.87%	81.36%	64.21%	43.50%	10.92%
Tracking Error	58.86%	51.40%	46.32%	44.66%	48.08%	62.81%
Sharpe Ratio	3.13	2.91	2.63	2.28	1.80	0.98
Sortino Ratio	5.48	5.05	4.54	3.89	3.04	1.57
VaR 5%	-9.18%	-7.96%	-6.68%	-5.26%	-3.54%	-0.82%
Expected Shortfall 5%	-12.13%	-10.68%	-9.12%	-7.34%	-5.11%	-1.37%
Skewness	-0.02	-0.03	-0.05	-0.07	-0.08	-0.10
Excess Kurtosis	1.52	1.89	2.30	2.78	3.41	6.65
Turnover	1763.62%	1741.04%	1642.42%	1447.50%	1099.65%	305.52%
Max Drawdown	-64.95%	-58.84%	-51.67%	-42.77%	-30.69%	-8.67%
Information Ratio	0.01	0.00	-0.02	-0.03	-0.05	-0.06
Distribution metrics (Cross-Sectional)						
Median Return	2.88%	3.34%	3.94%	4.74%	5.35%	4.74%
Maximum Return	21225.12%	17067.87%	12910.63%	8753.39%	4596.14%	438.90%
Minimum Return	-49.64%	-39.62%	-29.62%	-19.62%	-9.62%	0.39%
Volatility of Return Distribution	1318.55%	1060.63%	802.72%	544.84%	287.10%	32.79%
VaR 5%	-47.42%	-37.66%	-27.92%	-18.22%	-8.58%	0.91%
Expected Shortfall 5%	-48.60%	-38.67%	-28.76%	-18.87%	-9.01%	0.71%
Skewness	8.25	8.24	8.22	8.19	8.08	6.27
Excess Kurtosis	94.44	94.23	93.87	93.17	91.14	53.75
Worst Max Drawdown	-97.39%	-96.15%	-94.17%	-91.71%	-86.70%	-66.20%

Table 19: CPPI (m=3) Strategy - Impact of Floor Level on Performance Metrics

Performance Metrics	Strike = 50%	Strike = 60%	Strike = 70%	Strike = 80%	Strike = 90%	Strike = 100%
Path-level metrics (Avg. Across All Paths)						
Return	104.48%	97.54%	89.82%	81.78%	73.95%	66.62%
Volatility	62.08%	58.18%	54.02%	49.82%	45.74%	41.86%
Tracking Error	10.31%	15.26%	20.32%	25.24%	29.84%	34.07%
Sharpe Ratio	1.62	1.61	1.59	1.56	1.53	1.50
Sortino Ratio	2.91	2.88	2.83	2.76	2.68	2.61
VaR 5%	-5.04%	-4.74%	-4.42%	-4.08%	-3.75%	-3.43%
Expected Shortfall 5%	-6.34%	-6.01%	-5.66%	-5.30%	-4.94%	-4.59%
Skewness	0.08	0.07	0.05	0.04	0.02	0.00
Excess Kurtosis	0.25	0.46	0.75	1.11	1.55	2.03
Turnover	305.25%	411.49%	496.03%	559.74%	601.57%	623.99%
Max Drawdown	-42.49%	-40.34%	-37.89%	-35.33%	-32.77%	-30.29%
Information Ratio	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07
Distribution metrics (Cross-Sectional)						
Median Return	55.51%	49.35%	42.31%	35.20%	27.83%	20.47%
Maximum Return	1057.52%	1013.23%	962.06%	907.43%	852.06%	797.92%
Minimum Return	-45.54%	-37.74%	-30.44%	-26.05%	-20.77%	-15.86%
Volatility of Return Distribution	153.33%	146.63%	138.75%	130.26%	121.51%	112.81%
VaR 5%	-43.55%	-35.40%	-28.41%	-22.54%	-17.70%	-13.86%
Expected Shortfall 5%	-44.27%	-35.93%	-29.00%	-23.10%	-18.33%	-14.43%
Skewness	1.98	2.02	2.06	2.11	2.18	2.25
Excess Kurtosis	5.30	5.43	5.62	5.85	6.15	6.51
Worst Max Drawdown	-85.33%	-82.33%	-79.32%	-76.34%	-73.42%	-70.52%

Table 20: OBPI Strategy - Impact of Strike Level on Performance Metrics

6.2.4 Multiplier

Performance Metrics	Multiplier = 1	Multiplier = 2	Multiplier = 3	Multiplier = 4	Multiplier = 5	Multiplier = 6
Path-level metrics (Avg. Across All Paths)						
Return	27.87%	74.67%	150.36%	233.35%	266.18%	208.39%
Volatility	19.15%	42.94%	64.21%	79.37%	87.88%	90.72%
Tracking Error	50.98%	36.52%	44.66%	60.43%	73.79%	82.53%
Sharpe Ratio	1.25	1.65	2.28	2.89	2.98	2.25
Sortino Ratio	2.22	2.86	3.89	4.84	4.90	3.62
VaR 5%	-1.55%	-3.50%	-5.26%	-6.49%	-7.09%	-7.09%
Expected Shortfall 5%	-2.02%	-4.75%	-7.34%	-9.35%	-10.65%	-11.28%
Skewness	0.06	0.01	-0.07	-0.20	-0.38	-0.61
Excess Kurtosis	0.57	1.43	2.78	4.96	8.16	12.54
Turnover	196.68%	737.15%	1447.50%	2172.18%	2801.92%	3273.55%
Max Drawdown	-14.80%	-31.26%	-42.77%	-48.89%	-51.31%	-51.63%
Information Ratio	-0.06	-0.05	-0.03	-0.02	-0.02	-0.03
Distribution metrics (Cross-Sectional)						
Median Return	17.30%	17.32%	4.74%	-8.76%	-15.55%	-18.03%
Maximum Return	241.87%	1875.90%	8753.39%	25574.96%	46017.92%	48992.32%
Minimum Return	-14.51%	-18.71%	-19.62%	-19.82%	-19.88%	-19.92%
Volatility of Return Distribution	34.77%	165.23%	544.84%	1272.18%	1992.76%	1979.02%
VaR 5%	-6.29%	-14.61%	-18.22%	-19.34%	-19.69%	-19.83%
Expected Shortfall 5%	-9.78%	-16.60%	-18.87%	-19.53%	-19.76%	-19.86%
Skewness	1.92	4.64	8.19	11.87	15.24	17.59
Excess Kurtosis	5.06	31.09	93.17	188.94	300.99	387.76
Worst Max Drawdown	-44.45%	-76.36%	-91.71%	-97.12%	-98.99%	-99.65%

Table 21: CPPI (f=80%) Strategy - Impact of Multiplier on Performance Metrics

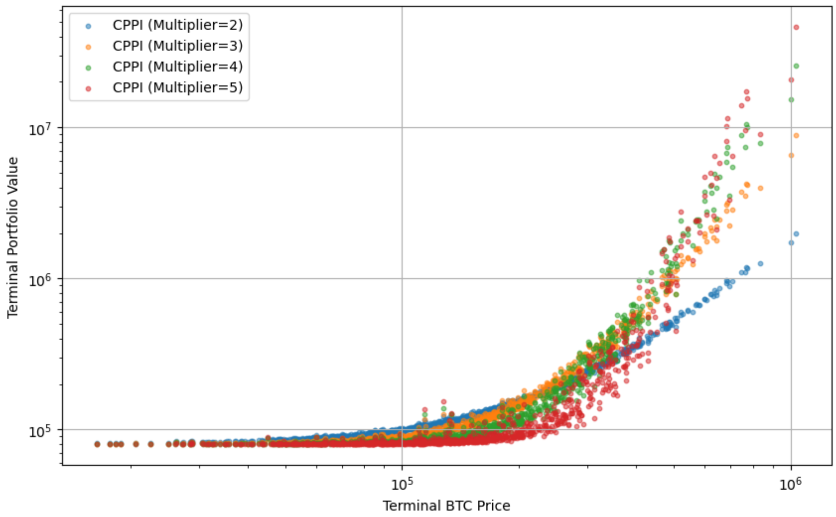

Figure 28: Effect of Multiplier on CPPI Payoff Convexity

6.3 Extensions

6.3.1 Maximum Drawdown CPPI

Performance Metrics	Max Drawdown CPPI	Standard CPPI
Path-level metrics (Avg. Across All Paths)		
Return	35.60%	150.36%
Volatility	25.56%	64.21%
Tracking Error	47.13%	44.66%
Sharpe Ratio	1.24	2.28
Sortino Ratio	2.15	3.89
VaR 5%	-2.07%	-5.26%
Expected Shortfall 5%	-2.86%	-7.34%
Skewness	0.09	-0.07
Excess Kurtosis	2.63	2.78
Max Drawdown	-17.09%	-42.77%
Information Ratio	-0.06	-0.03
Turnover	821.10%	1447.50%
Distribution metrics (Cross-Sectional)		
Median Return	22.19%	4.74%
Maximum Return	274.82%	8753.39%
Minimum Return	-19.40%	-19.62%
Volatility of Return Distribution	46.52%	544.84%
VaR 5%	-14.44%	-18.22%
Expected Shortfall 5%	-16.56%	-18.87%
Skewness	1.47	8.19
Excess Kurtosis	2.44	93.17
Worst Max Drawdown	-19.79%	-91.71%

Table 22: Comparison of Max Drawdown CPPI (m=3, f=80%) and Standard CPPI (m=3, f=80%) Strategy Metrics

6.3.2 Leveraged OBPI

Performance Metrics	Leveraged OBPI	Standard OBPI
Path-level metrics (Avg. Across All Paths)		
Return	132.53%	73.95%
Volatility	68.61%	45.74%
Tracking Error	29.49%	29.84%
Sharpe Ratio	1.87	1.53
Sortino Ratio	3.28	2.68
VaR 5%	-5.63%	-3.75%
Expected Shortfall 5%	-7.42%	-4.94%
Skewness	0.02	0.02
Excess Kurtosis	1.55	1.55
Max Drawdown	-45.93%	-32.77%
Information Ratio	-0.01	-0.07
Turnover	902.35%	601.57%
Distribution metrics (Cross-Sectional)		
Median Return	30.32%	27.83%
Maximum Return	2403.65%	852.06%
Minimum Return	-36.60%	-20.77%
Volatility of Return Distribution	265.30%	121.51%
VaR 5%	-29.92%	-17.70%
Expected Shortfall 5%	-32.26%	-18.33%
Skewness	3.29	2.18
Excess Kurtosis	15.26	6.15
Worst Max Drawdown	-87.30%	-73.42%

Table 23: Comparison of Leveraged OBPI (strike=90%) and Standard OBPI (strike=90%) Strategy Metrics

6.4 Code

The code for the standard CPPI and OBPI functions used in this paper is provided below. The complete code utilized throughout the study can be accessed at the following link: `https://github.com/ugoescato/Applied-Master-Project`.

Standard CPPI

```
def cppi_strategy(risky_asset, riskless_asset, multipliers,
    floor_fractions, initial_portfolio_value):
    """
    Parameters:
        risky_asset (array-like): Prices of the risky asset (e.g
            ., BTC).
        riskless_asset (array-like): Prices of the risk-free
            asset (e.g., T-Bill).
```

```
    multipliers (array-like): Array of multipliers.
    floor_fractions (array-like): Array of floor fractions of
        the initial portfolio value.
    initial_portfolio_value (float): Initial value of the
        portfolio.

Returns:
    dict: A dictionary containing DataFrames for each (
        multiplier, floor_fraction) combination.
"""
# Convert into arrays
risky_asset = np.asarray(risky_asset)
riskless_asset = np.asarray(riskless_asset)

# Ensure same length
if len(risky_asset) != len(riskless_asset):
    raise ValueError("The lengths of risky_asset and
        riskless_asset must be the same.")

dates = btc_daily.loc[start_date:end_date].index

results = {}

for multiplier in multipliers:
    for floor_fraction in floor_fractions:

        floor = initial_portfolio_value * floor_fraction
        cushion = max(initial_portfolio_value - floor, 0) /
            initial_portfolio_value

        risky_w = max(multiplier * cushion, 0)
        riskless_w = 1 - risky_w

        risky_weights = [risky_w]
        riskless_weights = [riskless_w]
        basket_values = [initial_portfolio_value]

        for i in range(1, len(risky_asset)):
            risky_return = (risky_asset[i] - risky_asset[i -
                1]) / risky_asset[i - 1]
```

```
                riskless_return = ((1 + riskless_asset[i] / 100)
                    ** (1 / 365)) - 1

                portfolio_return = risky_w * risky_return +
                    riskless_w * riskless_return
                basket_value = basket_values[i - 1] * (1 +
                    portfolio_return)

                cushion = max(basket_value - floor, 0) /
                    basket_value
                risky_w = multiplier * cushion
                risky_w = max(risky_w, 0)
                riskless_w = 1 - risky_w

                risky_weights.append(risky_w)
                riskless_weights.append(riskless_w)
                basket_values.append(basket_value)

            cppi_df = pd.DataFrame({
                'Basket Value': basket_values,
                'Risky Weights': risky_weights,
                'Risk-Free Weights': riskless_weights
            }, index=dates)

            results[(multiplier, floor_fraction)] = cppi_df

    return results
```

Listing 1: CPPI Strategy Function

Standard OBPI

```
def obpi_strategy(risky_asset, riskless_rate, floor_fractions,
    initial_portfolio_value, time_to_maturity, sigma):
    """
    Parameters:
        risky_asset (array-like or pd.Series): Prices of the
            risky asset (e.g., BTC).
        riskless_rate (array-like or pd.Series): Annualized risk-
            free rates (e.g., T-Bill).
        floor_fractions (list): List of floor fractions to test.
```

```
        initial_portfolio_value (float): Initial value of the
            portfolio.
        time_to_maturity (float): Time to maturity in years.
        sigma (float): Volatility of the risky asset (annualized)
            .

    Returns:
        dict: Dictionary containing DataFrames for each
            floor_fraction.
    """

    def black_scholes_put(S, K, T, r, sigma):
        d1 = (np.log(S / K) + (r + 0.5 * sigma ** 2) * T) / (
            sigma * np.sqrt(T))
        d2 = d1 - sigma * np.sqrt(T)
        put_price = K * np.exp(-r * T) * norm.cdf(-d2) - S * norm
            .cdf(-d1)
        put_delta = -norm.cdf(-d1)
        return put_price, put_delta

    if isinstance(risky_asset, pd.Series):
        risky_asset = risky_asset.values
    if isinstance(riskless_rate, pd.Series):
        riskless_rate = riskless_rate.values
    if len(risky_asset) != len(riskless_rate):
        raise ValueError("The lengths of risky_asset and
            riskless_rate must be the same.")

    results = {}
    dt = 1 / 365
    dates = btc_daily.loc[start_date:end_date].index

    for floor_fraction in floor_fractions:
        floor = initial_portfolio_value * floor_fraction

        put_price, put_delta = black_scholes_put(
            risky_asset[0], risky_asset[0]*floor_fraction,
                time_to_maturity, riskless_rate[0] / 100, sigma.
                iloc[0]
        )
```

```
risky_w = ((1 + put_delta) * risky_asset[0]) / (
    risky_asset[0] + put_price)
risky_w = max(0, min(1, risky_w))
riskless_w = 1 - risky_w

risky_weights = [risky_w]
riskless_weights = [riskless_w]
basket_values = [initial_portfolio_value]
put_prices = [put_price]
put_deltas = [put_delta]

for i in range(1, len(risky_asset)):
    remaining_ttm = time_to_maturity - i * dt

    risky_return = (risky_asset[i] - risky_asset[i - 1])
        / risky_asset[i - 1]
    riskless_return = ((1 + riskless_rate[i] / 100) ** (1
         / 365)) - 1
    portfolio_return = risky_w * risky_return +
        riskless_w * riskless_return
    basket_value = basket_values[i-1] * (1 +
        portfolio_return)

    put_price, put_delta = black_scholes_put(
        risky_asset[i], risky_asset[0]*floor_fraction,
            remaining_ttm, riskless_rate[i] / 100, sigma.
            iloc[i]
    )

    risky_w = ((1 + put_delta) * risky_asset[i]) / (
        risky_asset[i] + put_price)
    risky_w = max(0, min(1, risky_w))
    riskless_w = 1 - risky_w

    risky_weights.append(risky_w)
    riskless_weights.append(riskless_w)
    basket_values.append(basket_value)
    put_prices.append(put_price)
    put_deltas.append(put_delta)

obpi_df = pd.DataFrame({
```

```
            'Basket Value': basket_values,
            'Risky Weights': risky_weights,
            'Risk-Free Weights': riskless_weights,
            'Put Price': put_prices,
            'Put Delta': put_deltas
        }, index=dates)

        results[floor_fraction] = obpi_df

    return results
```

Listing 2: OBPI Strategy Function

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8 Disclaimer

This paper has been rephrased with the assistance of artificial intelligence (AI) tools to improve clarity and readability.