I

HL Physics Internal Assessment

Candidate code: kqb412

Effects of mass on terminal velocity

Introduction

Terminal velocity is a concept studied in the topic 'Mechanics' in IB Physics, particularly when examining the behavior of objects in free fall. Terminal velocity is the maximum velocity attainable by an object as it falls through a fluid, on my case, air. Terminal velocity is marked by the precise moment when the gravitational force (mg) is exactly balanced by the drag force exerted by air resistance, where further acceleration is negated, and the object descends at a constant velocity (Tsokos, 2014, p. 51). We can easily imagine how adding a mass to a balloon will make it drop faster, or that velocity increases depending on the mass. But to what extent? The aim of this experiment is to calculate the terminal velocity of a balloon when dropped from a consistent height while systematically varying the masses attached to it. That is, studying the relationship between mass and terminal velocity in a medium (air). It is anticipated that such an experiment is challenging, a perfect vertical fall is required to ensure absolute accuracy yet factors such as unintended rotational motions of the balloon affect this. Despite the challenges, the experiment is designed to provide insights into the practical application of free-falling objects leading to my research question: **How does the terminal velocity of a balloon change as its mass is incrementally increased by 1 gram at a time, given an initial mass of 1.27 grams and a radius of 8 cm, when falling from a height of 5.3m under the influence of gravity and air resistance?**

Background Information

We first show formula for the drag force is derived. The drag force equation is:

$$F_d = \frac{1}{2}\rho A C_d \mathrm{v}^2 \qquad \text{Eq.1}$$

where ρ is the air density in room temperature, A is the cross-sectional area of the balloon, C_d is the drag coefficient and v is the velocity of the balloon

It was first derived by Rayleigh using a complex dimensional analysis. There is no simple mathematical derivation for this formula, and it is essentially an empirical relation with complications like turbulence, viscosity, shape, absorbed and empirically determined constant C_d. (Walker, Halliday, and Resnick, 2020, p. 110). We present here a different, simpler derivation.

Let A be the maximum cross-sectional area of the falling balloon.

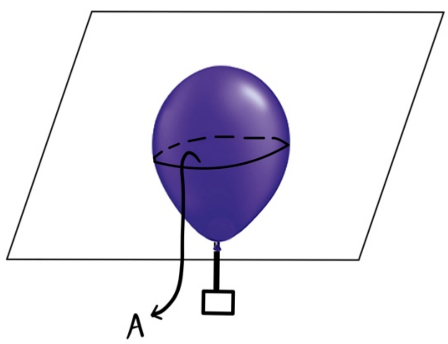

Figure 1: Balloon falling through a plane illustrating the maximum cross sectional area

Next, we consider the drag force F_d caused by an object with area A moving through a fluid with velocity v:

$$F_d \propto P_d \cdot A$$

where P_d is the dynamic pressure that the fluid experiences

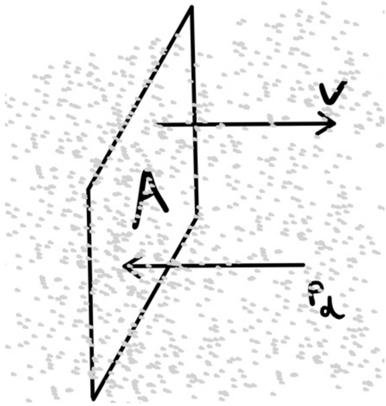

Figure 2: Molecules hitting the area A with velocity v, and transfer of their momentum resulting in a pressure / force on the area.

The pressure P on A is the same for the object in rest and the air column moving with a velocity v towards the object.

There is the classical argument of momentum transfer of molecules to a surface area A:

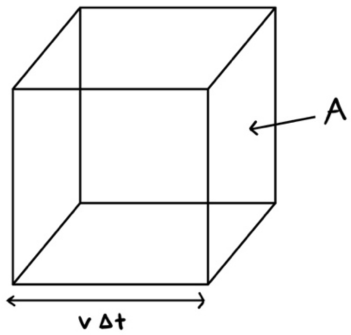

Figure 3: Pressure on a surface due to air column movement

$$F_d = A \cdot P_d$$

Where F_d is the drag force, A is the cross-sectional area of the surface, P_d is the dynamic pressure.

$$\frac{\Delta p}{\Delta t} = A \cdot P_d$$

$$\frac{2mv}{\Delta t} = A \cdot P_d$$

The object's mass is determined by its density and volume:

$$m = \rho V$$

According to Figure 3, the volume is expressed as:

$$m = \rho \cdot (v\Delta t) \cdot A$$

Therefore,

$$\frac{2\rho \cdot (v\Delta t) \cdot A \cdot v}{\Delta t} = A \cdot P_d$$

$$2\rho v^2 = P_d$$

My argument is somewhat simplistic, but it gives the core result that the drag force F_d is proportional to ρ, v^2, and A. The main takeaway is that there is no precise derivation, it is ultimately an empirical relation, the purpose of this is to make the formula plausible. It would also be good to note that experimentally there is a drag coefficient C_d added, finally resulting in Eq.1 (Walker, Halliday, and Resnick, 2020, p. 110). I now proceed to derive an expression for the (terminal) velocity. Defining down as positive, the net force on the object is the following:

$$F_{net} = ma$$

$$W - F_d = F_{net}$$

$$mg - \frac{1}{2}\rho A C_d v_t^2 = ma \qquad \text{Eq.2}$$

At equilibrium I know, the net force is zero ($F_{net} = 0$). Therefore,

$$mg - \frac{1}{2}\rho A C_d v_t^2 = 0$$

$$v_t = \sqrt{\frac{2mg}{pAC_d}} \qquad \text{Eq.3}$$

Note that $v_t^2 \propto m$

In my case specifically, when the balloon was released, it initially ascended slightly indicating that air resistance (AR_{up}) was greater than the force of gravity (mg) due to the balloon's surface area relative to its small mass. This ascent continued to carry on until the balloon encountered an opposing air current, exerting another resistive downward force (AR_{down}), which, in turn, decelerated the balloon's ascent. The additional force combined with gravity then creates a greater total downward force on the balloon. Once the deceleration stops, the forces of air resistance and gravity reach equilibrium (AR = mg) because air resistance is now back up. Subsequently, the balloon began its descent at constant velocity indicating that it achieved constant velocity almost immediately after being dropped from 5.3m. Although constant speed is not indicative of the terminal velocity typically described in physics (due to the balloon's interaction with air currents and light masses). It offers the closest approximation to terminal velocity within a school environment. The observation of the balloon deviates from the common phenomenon of objects accelerating until they reach to a terminal velocity. Going forward, I will refer it to as constant velocity for clarity.

Hypothesis

My hypothesis for this experiment is that my balloon's terminal velocity (V_t) will be increasing as mass increases based on the principle that in free fall, an object accelerates downwards until air resistance equals to the gravitational force. As mentioned before, this isn't the case for my experiment. AR = mg straight from the beginning, my hypothesis will be based that as this air resistance is proportional to v^2. Expecting that for each incremental addition of mass, a corresponding increase in velocity should happen. The experiment will test this hypothesis by using measuring the constant velocity for each mass and compare the results to the expected values based on the gravitational constant.

Methodology

Variables

Independent Variable	How it is changed
Mass of balloon	The total mass of the balloon undergoing freefall is composed of the combined mass of the balloon, the rope attached and 'n' number of cardboard disks.

Dependent Variable	How it is measured
Constant velocity of balloon (leading to the determination of drag force and air resistance effects)	Constant velocity is determined by tracking the descent of the balloon in a video using LoggerPro software. A displacement versus time graph is generated automatically from these tracked points. The slope of the line connecting the last five data points before the balloon achieves steady descent (indicating constant velocity) is calculated. This is particularly tied to minimizing the impact of any initial wobbling or unsteady behaviour that the balloon might exhibit as it starts its descent.

Controlled Variable	How it is changed
Initial Height at which the balloon is dropped from	The balloon will be released height of 5.3m. The height is determined through LoggerPro video analysis, a meter stick is included within the video as a reference scale (defined within LoggerPro as exactly one meter in length). LoggerPro interprets the tracked data points of the balloon's displacement. Consequently, when the software calculates a displacement of 5.3 meters, it is recognized as the balloon's peak height.
Size/Shape of Balloon	The same balloon of 1.27 grams will be used to ensure that the diameter (16cm) and volume is consistent. By conducting the experiment in a 1-hour span, effects of any changes to the temperature or pressure within the room are minimized.

Precautions	
Experiment must be conducted indoors	Conducting the experiment indoors ensures a controlled environment where variables like wind and temperature are relatively constant. The same indoor location will be used for all trials to ensure no external factors influence the results.

Video Analysis Point Selection	During video analysis, each point is meticulously plotted by <u>marking the bottom</u> of the balloon throughout its fall to reduce the potential error.
Manner in which the balloon is dropped	The balloon will be dropped without initial velocity to ensure that the only force acting on the balloon is gravity and I'm the only person releasing the balloon in the same manner for each trial to maintain consistency.
Camera Location	The camera will be placed at a fix location to ensure a consistent distance and angle from the drop point for each trial. The camera will be secured on a stable tripod. It is also marked to ensure it cannot be accidentally moved or re-angled during the experiment.

Material List

Balloon 1.27g ± 0.01g
Rope 1.06g ± 0.01g
5 Cardboard disks ≈ 1.00g each ± 0.01g
Meter Stick ± 0.01m
iPad
Electronic Balance

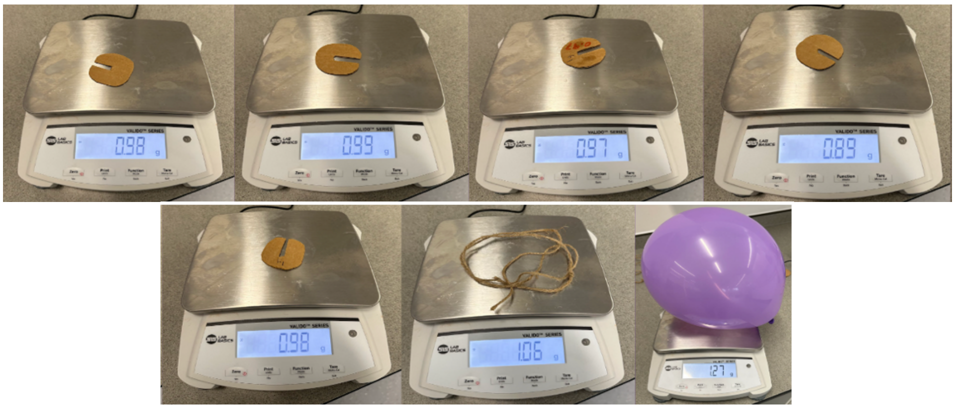

Figure 4: List of Apparatus Used in Balloon Drop Experiment

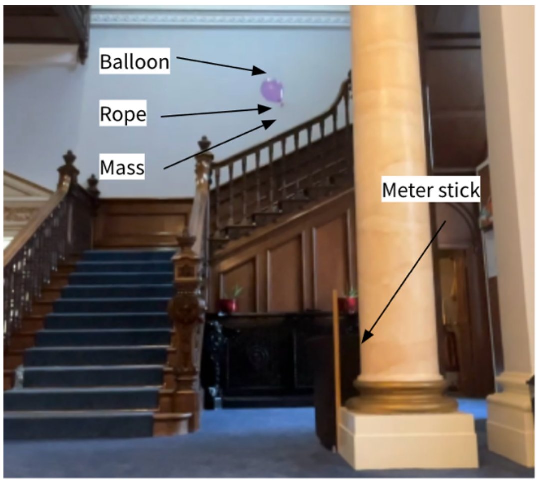

Figure 5: Visual representation of my experiment

Procedure

1. Inflate one balloon to a relatively normal size (Radius: 8cm) (Seen in Figure 4)
2. Attach a rope to the bottom of the balloon securely.

3. For the first mass trial, attach one cardboard disk to the bottom of the balloon.
4. Position a meter stick vertically and secure it to a surface to ensure it is perpendicular to the ground.
5. Set up a tripod and affix an iPad to it, ensuring it is in a fixed position to record the experiment.
6. Mark the location of the iPad setup to ensure consistency in camera angle and position for all trials.
7. Start the video recording on the iPad.
8. Walk up the staircase carrying the balloon to the predetermined height for drop consistency.
9. Release the same balloon from the same height for every trial, allowing it to fall freely to the ground.
10. Stop recording once the balloon has landed.
11. Save the video footage after each drop.
12. Repeat steps 1-12 for each additional mass, attaching an additional cardboard disk for each new set of trials, up to five masses.

Procedure: Video Analysis

1. Upload the recorded video footage to LoggerPro.
2. Use the ruler function to set the meter stick in the video as the scale for one meter.
3. Establish the origin in the software at the initial position of the balloon drop (Approximately 5.3m)
4. Select the point analysis function.
5. Begin selecting points at the bottom of the balloon, doing so for every video frame to track the entire descent.
6. Repeat the same process for each of the three trials for each mass increment.

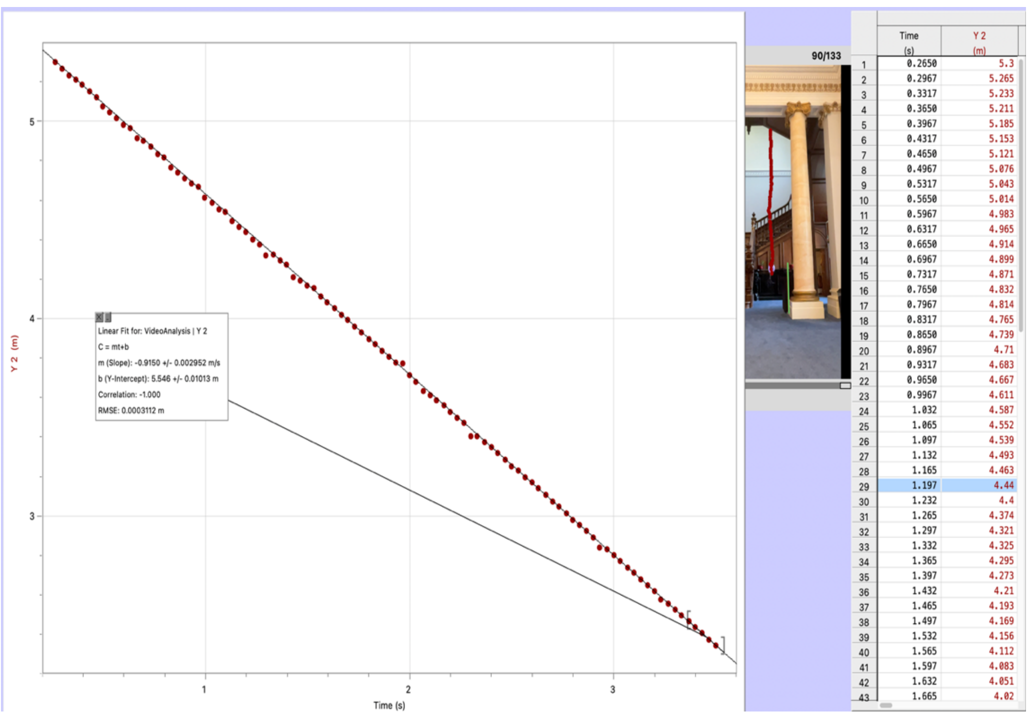

Figure 6: Video analysis on the experiment using LoggerPro.

NOTE: Once the points are fully selected, I plot the displacement against time. For consistency, I'll be taking the slope of the last 5 points as that is what I determine to be my velocity for the balloon. For the sake of the graph, I've chosen the values of Mass 1/Trial 1 (0.003kg).

Safety Issues/Risk Assessment

There are no major safety concerns however I understand the safety risks and have taken measures to prevent accidents during the balloon drop, such as securing the setup and keeping the drop area clear.

Data collection

Table 1: Raw qualitative measurements

Mass (kg) ± 0.001	Velocity (ms^{-1}) ± 0.001		
	Trial 1	Trial 2	Trial 3
0.003	0.915	0.891	0.895
0.013	2.040	2.034	2.014
0.023	2.886	2.763	2.749
0.033	3.339	3.318	3.351
0.042	3.812	3.781	3.789

Data processing

Table 2: Uncertainties in Velocity Measurements

		Absolute Uncertainty	Percentage Uncertainty of v		Percentage Uncertainty of v^2	Absolute Uncertainty of v^2
Mass (kg) ± 0.001	Average v ms^{-1} ± 0.001	Δv ms^{-1}	$\frac{\Delta v}{v} \cdot 100$	Average v^2 m^2s^{-2}	$2 \cdot \frac{\Delta v}{v} \cdot 100$	Δv^2 m^2s^{-2}
0.003	0.900	0.012	1.333	0.81	2.666	0.02
0.013	2.029	0.013	0.641	4.12	1.281	0.05
0.023	2.799	0.069	2.447	7.84	4.894	0.39
0.033	3.336	0.017	0.495	11.13	0.989	0.11
0.042	3.794	0.015	0.409	14.39	0.817	0.12

Sample calculations

Consider using trial 1 for processed sample calculations. We will calculate the absolute uncertainty of v, percentage uncertainty of v, percentage uncertainty of v^2 and absolute uncertainty of v^2 in that specific order.

1. $\Delta v = \frac{v_{\max} - v_{min}}{2} = \frac{0.915 - 0.891}{2} = 0.012$

2. $\frac{\Delta v}{v} \cdot 100 = \frac{0.012}{0.900} \cdot 100 = 1.333$

3. $2\frac{\Delta v}{v} \cdot 100 = 2 \cdot \frac{0.012}{0.900} \cdot 100 = 2.667$

4. $\frac{2\frac{\Delta v}{v}\cdot 100}{100}\cdot v^2 = \frac{2\frac{0.012}{0.900}\cdot 100}{100}\cdot 0.811 = 0.022$

Graph

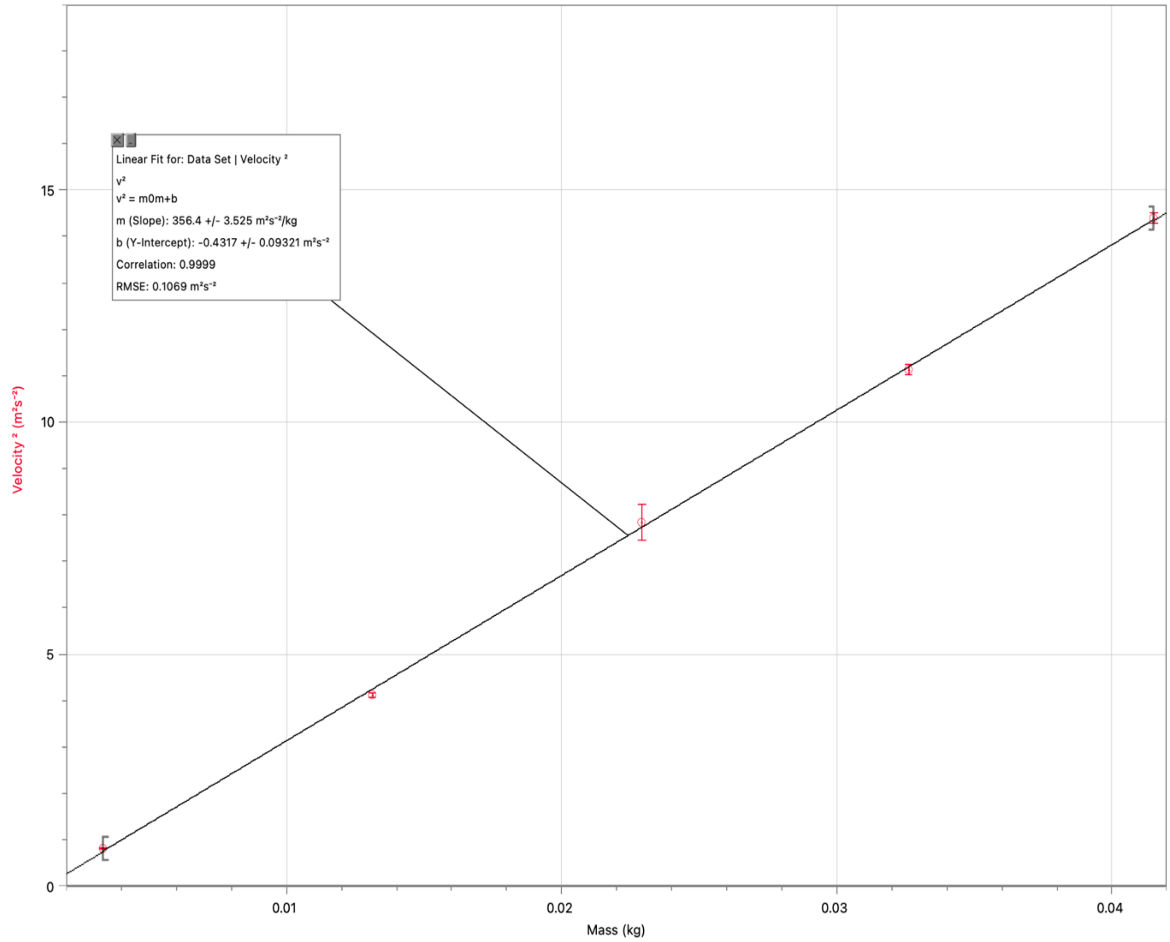

Figure 7: Graphical representation of processed data

Calculation of g in experiment		
$g = \frac{slope\cdot\rho AC}{2}$	$= \frac{356.4\cdot 1.204\cdot 4\pi 0.08^2\cdot 0.47}{2} \pm \frac{3.525}{356.4}$	$= (8.1 \pm 0.1)\ ms^{-2}$

NOTE: It is good to note that there was no need for a max-min graph as the error bars are small.

There are several **assumptions** to be made. One of them being that we take the density of air to be in room temperature (1.204 kg/m^3) and that our drag coefficient is equivalent to of a spherical balloon as both shapes are extremely similar.

In graph 1 above, we plot v^2 against mass. The gradient of the line represents $\frac{2g}{\rho AC}$, where g is the acceleration due to gravity, ρ is the density of air in room temperature, A is the cross-sectional area of the balloon and C_d is the constant related to the drag coefficient. I'll be taking the drag coefficient to be the same as a sphere as the balloon has a relatively same shape compared to a sphere, errors from the C_d will be considered (At a Glance, n.d.). From the slope of our graph, we determine the experimental value of gravity to be:

$$(8.1 \pm 0.1)\ \text{ms}^{-2}$$

This figure is then compared to the standard value of 9.81ms^{-2}, resulting in an error margin of approximately 17%. While this error seems substantial, it is likely attributable to the weaknesses of the experimental setup that we used, mentioned in the Evaluation. The quoted error here is basically only the statistical error. Larger systematical errors are expected as discussed below.

Evaluation

Strengths	Explanation
Conducting many repeats	I conduct three trials for each mass strengthening the reliability of the data. Multiple trials allow for a more accurate average to be calculated and can help identify any outliers in the data
Video Analysis	LoggerPro allows for precise tracking of the balloon's descent enabling frame-by-frame analysis enhancing the accuracy of measurements of displacement with time, crucial for determining the constant velocity.
Line of best fit	The trend line has a strong correlation coefficient (r = 0.9993) indicating a very strong linear relationship between velocity squared and mass. However, the 17% error from the expected value of 9.81 $ms^{-2,}$ indicating potential systematic errors.
Time of experiment	Completing the experiment within an hour reduces the potential for room temperature which could affect the size of the balloon and thus the experimental results. This controlled variable adds strength to my experiment as it ensures that conditions remain constant.

Limitations / Explanation / Improvement

Angular misalignment leads to errors in tracking the balloon's position over time, especially if the fall is not perfectly vertical. It can skew the data when tracking the balloon's descent affecting my constant velocity. Utilizing a digital angle gauge with a magnetic base on top of iPad to record ensures that the camera's alignment is vertically true.

My experiment uses small mass increments of approximately 1 gram which might be within the margin of error for measuring constant velocity. Selecting a broader spectrum of masses for the experiment provides the chance to achieve the traditional terminal velocity where the gravitational force (mg) initially exceeds air resistance (AR), thus reducing that initial ascent phase which I encountered. Additionally, conducting preliminary tests with various number of weights to identify the range that produces measurable constant velocity helping me determine the appropriate masses to use.

The shape of the balloon is essential as it impacts the projected cross-sectional area, a crucial variable in Eq.1. Any deformation from the balloon's expected spherical/oval shape due to friction changes can affect the drag coefficient, another crucial variable in Eq.1. I used the C_d of 0.47 for the calculation of the measured value of gravity (At a Glance, n.d.). I note that C_d differs vastly for different shapes. (E.g. A half-sphere has a C_d of 0.42. The resulting g would be 12% different!). Such a change makes it difficult to video analyse as a consistent path of the balloon's descent is needed. The unexpected ascent is potentially influenced by the cross-sectional area of the balloon as well; any deformation in the shape may produce an air resistance upwards when released. Also, a deformed balloon might move erratically making it difficult to select precise data points affecting my constant velocity. Using balloons made of more rigid materials are less prone to air friction, therefore less prone to the swinging and ensuring a consistent descent. The data could be more accurate by determining a more appropriate value for C_d.

The results of the experiments are very precise, the y-intercepts are consistent with 0 and there are small error bars (-0.04 ± 0.09 ms^{-2}). Even though, the y-intercept and error bars suggest a precise experiment, it can be ascertained as inaccurate as it is above the 5% experimental error acceptable in physics.

The setup involving a balloon attached to a mass via a rope is susceptible to dynamic changes during its free fall. If the rope is not rigid, it can bend, and if the mass is not secured properly, it can shift. These changes can alter the centre of gravity of the entire mass system. In a stable descent, the centre of gravity should follow a straight, vertical path. If this shifts, the balloon might start to exhibit complex movements in the plane causing spinning, wobbling making the process for tracking the balloon's complex. The unpredictable movement introduces additional forces which would skew the results and lead to inaccuracies in the calculation not only for constant velocity, but the coefficient of drag. To mitigate these, a stiffer connecting material that doesn't easily bend or flex like a lightweight rod or a rigid tube helping me maintain the alignment of the mass with the balloon ensuring the centre of gravity to be stable. Securing the mass to the balloon with a firmer attachment will prevent it from shifting its position relative to the balloon. A stable centre of gravity will minimize complex movements and ensure that the balloon falls straight down simplifying video analysis and interpretation of data.

Rotational motion occurs when the balloon and its attached masses do not descend straight down but instead spin around a vertical axis. The spinning motion complicates the drag force as Eq.1 assumes that the object is moving linearly through the air. However, if the balloon starts to rotate, this introduces aerodynamic forces Eq.1 doesn't account for. The rotation can disrupt the airflow around the balloon leading to an inconsistent drag force. Instead of a smooth flow, the air may become more turbulent. Turbulent flow has a higher drag coefficient than laminar flow meaning actual drag force could be extremely different than what is calculated using a model that assumes laminar flow. Additionally, the rotation could change the effective surface area perpendicular to the direction of motion further altering the drag force. To prevent rotational motion and ensure a straight descent. The balloon should be setup so that its centre of gravity is aligned with its geometric centre as any asymmetry in mass distribution can cause the balloon to spin. Using computer simulations offers a approach to understanding the rotational motion. Creating a virtual model of the balloon and its attached mass, the simulation can represent the experimental setup and can then calculate the effects of air resistance, gravity, and inertia on the balloon's movement.

Firstly, the manual release of the balloon can introduce error in my controlled variable. If the balloon isn't released from the same exact height can lead to variations in V_t and the balloon may have more or less time to reach constant velocity. There is a human error when walking up to 5.3m and dropping the balloon the same place at the exact same time. Even though, I've marked the exact height at which the balloon should be released from. This could be further improved by using a release mechanism that ensures the balloon starts from the same height every time.

Motion blur occurs when the balloon recorded moves quickly relative to the frame rate of the iPad. In LoggerPro, the iPad's frame rate compared to the balloon's rapid descent can appear blurred making it challenging to pinpoint the exact position of the balloon frame by frame when using point analysis. This lack of clarity introduces errors in V_t. Using a better camera with a higher frame rate can address this issue. High-speed cameras can capture more frames per second, significantly reducing motion blur and this increased clarity allows LoggerPro to track the balloon's position with greater precision leading to more accurate data.

All the above imperfections in the experiment are difficult to quantify as this would require more extensive studies and experiments. An exception to this is that, as I have seen, drag coefficient differs very vastly as a function of the object's shape. Here however, further studied would be required on what the appropriate C_d is for the balloon.

Extension

For further investigation, it would be interesting to test different sizes and shapes of the balloon to understand how the drag coefficient of an object can influence terminal velocity. Moreover, investigating how various masses travelling through different fluids such as water affect terminal velocity.

Reflecting on the constraints of a school environment, although providing a foundational but limited setting for experimentation. A more advanced facility would allow for broader exploration potentially offering precision instruments and controlled conditions for examining the effects of the balloon's shape. Access to wind tunnels and high-speed photography would refine my understanding of drag forces and terminal velocities. This ideal setting would give deeper insights into fluid dynamics and material properties.

Conclusion

The aim of this investigation was to determine how varying masses affect the constant velocity of a balloon when dropped from a consistent height. Through a series of trials, I varied the mass attached to a balloon and measured its descent using LoggerPro. The main findings revealed a clear linear relationship between the mass and its constant velocity as hypothesized, that is $v^2 \propto m$. As the mass increase, constant velocity also increased clearly validating through the linear relationship in Figure 7 which is consistent with the theoretical understanding that a greater force of air resistance is needed to balance the gravitational force for heavier objects.

However, the value of gravity determined from the slope of this graph was 8.1 ± 0.1 ms^{-2} which deviates from the standard constant of 9.81ms^{-2} by approximately 17% suggesting the presence of systematic errors attributed. Moreover, a significant further analysis and experimentation would be required to pin down these sources of systematic errors.

Bibliography

Tsokos, K.A. (2014). *Physics for the IB Diploma*. 6th ed. Cambridge: Cambridge University Press.

Walker, J., Halliday, D. and Resnick, R. (2020). *Halliday & Resnick's principles of physics*. Hoboken, Nj Wiley.

at a glance. (n.d.). https://www.hoddereducation.com/media/Documents/magazine-extras/Physics/PhysRev%2029_2/PhysicsReview29_2_poster.pdf