Title:

Experimental and computational study of the large angle pendulum

Research Question:

How does the period T of a swinging pendulum vary with large amplitudes?

Subject:

Physics

Word Count:

3852

Session:

May 2024

Personal code:

kqb412

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Abstract

In the physics curriculum the pendulum is traditionally studied in the small angle approximation leading to the theory of the harmonic oscillator. Here we study the large angle pendulum using the following approaches.

1. Experimentally by building a pendulum setup allowing to precisely measure the period of the oscillation. We find that the large angle pendulum period deviates substantially from the small angle approximation period. Moreover, we find an excellent agreement between our measurements and our computer simulation up to opening angles of 165 degrees.
2. Traditional mathematical derivations of the equation of motion using, I) Newton's laws, and II) Conservation of energy.
3. By creating a computer simulation based on the equation of motion and compare the results with the experimental values. As an additional check, the simulation results are compared with a mathematical analysis that follows from approach 1. above.

Introduction

In the physics syllabus we study various types of harmonic oscillators. One of the oscillators we study is the simple pendulum. It consists of a point mass suspended by a massless unstretchable string (Young et al., 2003). Harmonic oscillators have been studied for countless years and has a very rich history. This started when the Italian scientist, Galileo Galilei began experimenting with them and making extraordinary observations regarding the pendulum's period. He recognized the movement of the pendulum through the swinging motion of a lamp in a Pisa cathedral with his own pulse rate. Nevertheless, it was the Dutch mathematician/scientist Huygens who made a revolutionary invention in the 17th century, the pendulum clock

(“Pendulum | Definition, Formula, & Types | Britannica,” 2023). There are real world applications as well, such as the swinging motion of monkeys. Monkeys swing from branch to branch and have an incredible technique when doing so. Including hanging onto the branches with their feet and letting themselves drop to the bottom to ensure that they have maximum kinetic energy so when they let go of the branch, they harness the maximum amount of energy from the swinging motion.

Ever since, the pendulums have been crucial in regulating movement of clocks because of their constant period. During the HL Physics course, we study the pendulum in the so-called small angle approximation and derive the period T.

$$T_0 = 2\pi\sqrt{\frac{L}{g}}$$

Note that T_0 is independent of the initial opening angle under the conditions of small amplitude oscillations. The remarkable aspect of this formula is that it does not include the the initial opening angle of the pendulum (θ_0). This independence is due to the approximation made in the small-angle approximation, where the sine and tangent of the angle is approximated to be equal to the angle itself when measured in radians. I questioned why larger amplitudes weren’t in the syllabus and it opened a new realm of investigation and potentially an extended essay topic. Which was then I started to wonder what would happen to period T when the pendulum is released at not only amplitudes slightly above the small-angle approximation but all the way up to 180° eventually leading me to my research question.

We will challenge the simulation values (T_0) of the small angle approximation. By plotting the period T vs θ_0 (max amplitude) to derive the real period value of angles beyond the small-angle approximation will show that deviations are substantial. Then will delve into the methodology to gather data including a detailed analysis of independent, dependent, and controlled variables. Subsequently, followed by an analysis and conclusions drawn based on the collected data.

Mathematical Analysis

We will present two derivations for the equations describing the motion of large angle pendulums, one fully derived, and the other literature derivation is indicated briefly. As we will show that these equations are equivalent, each shows an interesting aspect of the problem.

There are **assumptions** made when deriving the time period for small angles including:

1) There is negligible air resistance or any other form of damping force.

2) The mass of the string or rod supporting the pendulum is small.

Later, we will soften this assumption somewhat by including the mass of the rod through reducing the length of the pendulum to the center of gravity of rod/pendulum bob.

3) Pendulum swings in a perfect plane without bending or compression of the string.

Ia. Derivation of equation of motion using Newton's laws

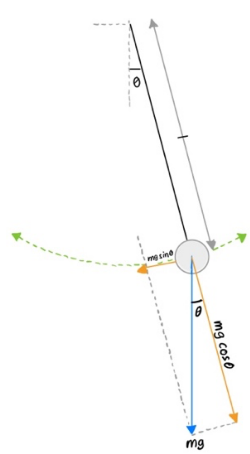


Figure 1: The pendulum has a mass m and length L, g is the gravitational constant. This figure shows a simple pendulum being analysed to its components. The path of the pendulum is of an arc of a circle shown in the green line. The blue line represents the gravitational force acting on the bob and the yellow arrows are the forces becoming components parallel and perpendicular to the bob's motion.

Now Newton's 2nd law of motion states:

$$\sum F = ma$$

where, F is the net forces on the object, m is its mass, a is its acceleration. Applying Newton's equations to the radial axis only, we get:

$$F = ma = -mgsin\theta \text{ or } a = -gsin\theta$$

Note that the g is independent of mass. For the movement along the arc, we have s = Lθ, then,

$$v = \frac{ds}{dt} = L\frac{d\theta}{dt} \quad \text{and a} = \mathrm{L}\frac{d^2\theta}{dt^2}$$

Therefore,

$$\mathrm{L}\,\frac{d^2\theta}{dt^2} = -gsin\theta \quad \text{or} \quad \frac{d^2\theta}{dt^2} + \frac{g}{L}sin\theta = 0 \qquad \text{(Eq. 1)}$$

We note that in the small angle approximation of sinθ ≈ θ, it simplifies the equation (T_0) leading to the theory of the harmonic oscillator.

Ib. Derivation of equation of motion based on conservation of energy

It is possible to derive an equation of motion using energy conservation. That is, gravitational potential caused by the height of the pendulum bob is converted into kinetic energy as the bob moves down and inversely, when the bob moves up again.

The equation is:

$$\frac{d\theta}{dt} = \sqrt{\frac{2g}{L}\,(\cos\theta - \cos\theta_0\,)} \qquad (Eq.2)$$

It gives the velocity in terms of the location and includes an integration constant related to the initial displacement (θ_0). Note that differentiating Eq.2 again will give us Eq.1 above. Eq.2 will permit us to perform an independent check on our calculations based on Eq.1 (Wikipedia Contributors, 2023).

IIa. Calculation of the period T with a computer simulation using $\frac{d^2\theta}{dt^2} + \frac{g}{L} sin\theta = 0$

We write a computer program simulating the large angle pendulum using equation 1 above. This procedure is akin to solving differential equation 1 numerically as shown for example in the (Chang Wathall et al., 2019). As far as we know Eq.1 has no simple mathematical solution. Therefore, we take recourse to a numerical solution by making a computer simulation of the pendulum movement.

The procedure for the simulation is as follows:

- We simulate the fall of the pendulum from $\theta = \theta_0$ to $\theta = 0$. This corresponds to a time interval of $\frac{T}{4}$
- We define ω as $\frac{d\theta}{dt}$. Equation X then becomes $\frac{d\omega}{dt} + \frac{g}{L} \sin\theta = 0$. The discrete version of this equation is $\Delta\omega = -\frac{g}{L} \sin\theta \cdot \Delta t$

- $\frac{d\theta}{dt}$ obviously changes continuously with time. For the sake of the algorithm, we assume that $\frac{d\theta}{dt}$ is constant over a small-time interval: Δt. Using the computer program, we can choose Δt as small as we have to
- In the first-time interval Δt, the increase in velocity $\Delta\omega_1 = -\frac{g}{L}\sin\theta \cdot \Delta t$
- Now θ moves to position $\theta_1 = \theta_0 + \Delta\omega_1 \cdot \Delta t$
- As from θ_1, the pendulum picks up a velocity $= -\frac{g}{L}\sin\theta_1 \cdot \Delta t$
- The total velocity from Δt_1 to Δt_2 becomes $\Delta\omega_2 = \Delta\omega_1 - \frac{g}{L}\sin\theta_1 \cdot \Delta t$
- The position after 2Δt, is $\theta_2 = \theta_1 + \Delta\omega_2 \cdot \Delta t$
- After each step, the algorithm calculates the new angular velocity based on the previous one and angle and updates θ accordingly.
- For a certain N, we will reach a vertical position $\theta_n = 0$, then stop the algorithm and $N_{steps} \cdot \Delta t = \frac{T}{4}$

The above algorithm can be treated perfectly in the loop structure such as in the Python programming language. Note how remarkably concise the code is. This is basically since θ_1, θ_2… and $\Delta\omega_1$, $\Delta\omega_2$ … can be collapsed into one variable that is updated and overwritten through each iteration of the loop (Online Python Compiler (Interpreter), 2023).

```python
import math

L = 0.485 #The length of the pendulum in meters
g = 9.81 #Gravity constant
Theta0 = math.pi / 2 #Initial angle of the pendulum
Nstep = 100 #Step size for algorithm
DelT = math.pi * 2 * math.sqrt(L / g) / 4 / Nstep #Reasonable estimate for Delta Time using the period of the small angle pendulum

DelOmega = 0 #Delta Omega, the incremental increase in velocity over time Delta T
Theta = Theta0
Omega = 0 #Total angular velocity
T = 0 #Time Period

for i in range(1,2 * Nstep):
    T = T + DelT
    DelOmega = -g / L * math.sin(Theta) * DelT
    Omega = Omega + DelOmega
    Theta = Theta + Omega * DelT
    if Theta <= 0:
        break

T_small_angle = math.pi * 2 * math.sqrt(L / g)
T_energy_derivation = 4 * T
print(f'Using small angle approximation, period T is {round(T_small_angle, 3)}')
print(f'Using the derivation based on conversation of energy, period T is {round(T_energy_derivation, 3)}')
```

Figure 2: The Python code created to implement the algorithm as described above, permitting calculation of the period T as a function of the opening angle θ_0

IIb. Calculation of period T using $\frac{d\theta}{dt} = \sqrt{\frac{2g}{L}\left(\cos\theta - \cos\theta_0\right)}$

We only briefly summarize the literature result (Wikipedia Contributors, 2023). Noting that the time for the pendulum to move from θ_0 to θ is $\frac{T}{4}$.Using (Eq.2) we find:

$$T = 4\sqrt{\frac{L}{2g}} \int_0^{\theta^0} \frac{d\theta}{\sqrt{(\cos\theta - \cos\theta^0)}} \qquad (Eq.3)$$

Using (Eq.3), T can be expressed as:

$$T = 4\sqrt{\frac{L}{g}} \int_0^{\frac{\pi}{2}} \frac{du}{\sqrt{1 - k^2 \sin^2 u}} \quad with\ k = sin\frac{\theta_0}{2}$$

This is a so-called elliptical integral. It can be shown that:

$$T = K(k)\frac{2}{\pi} * T_0$$

$$with \qquad K(k) = \frac{\pi}{2}\left(1 + \frac{1^2}{2^2}k^2 + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2}k^4 + \frac{1^2 \cdot 3^2 \cdot 5^2}{2^2 \cdot 4^2 \cdot 6^2}k^6 + \cdots\right) \qquad (Eq. 4)$$

Equation 4 will permit us to do some independent checks on the numerical simulation results.

IIc. Numerical comparison of computer simulation and elliptical integral calculation of T

θ / Degrees	T (Simulation) (sec) ± 0.001s	T (Elliptical functions) (sec) Up until k^2 order ± 0.001	T (Elliptical functions) (sec) Up until k^8 order ± 0.001	Percentage difference of simulation and elliptical functions (k^2 order)	Percentage difference of simulation and elliptical functions (k^8 order)
15°	1.397	1.403	1.403	0%	0%
30°	1.425	1.420	1.421	2%	0%
45°	1.453	1.448	1.453	4%	0%
60°	1.495	1.484	1.499	7%	1%
75°	1.565	1.526	1.562	11%	3%
90°	1.649	1.572	1.644	17%	5%
105°	1.760	1.617	1.745	23%	9%
120°	1.914	1.659	1.860	31%	15%
135°	2.138	1.695	1.979	42%	26%

150°	2.459	1.723	2.084	55%	43%
165°	2.793	1.740	2.157	67%	60%

Table 1: The comparison between the measurement of period T using elliptical functions and using the simulation results. We calculate the percentage difference between the two to show as we increase the order of the k^2 term, the precision of our data improves.

Delving deeper into the infinite series (Eq.4), we consider higher orders of k^2. From table 1, we observe a correlation enhancement in the accuracy of our time period T values compared to the simulation results which we use as a comparison. The comparison of the various calculations makes us very confident that the computer simulation is correct and from now on in this essay, we will use the computer simulation for comparison with the experimental data. As it is easier to calculate and covers a wider range of opening angles more accurately.

Methodology

I. Set up and data acquisition methodology.

We will describe here the experimental setup and how we built it.

We are using the following components:

- 1 Weighted plate (0.500 kg) serving as the pendulum bob. We choose a heavy weight such as to minimize the influence of the weight of the rod and, minimize the influence of air friction (which would add a term $\frac{constant}{M} \cdot v^2$ to Eq.1)
- 1 threaded metal rod (diameter 3mm), (weight 0.024kg)
- 2 washers (0.023kg)

- 2 flange coupling connectors (0.020kg)
- 2 wooden plates (1.00m x 0.5m)
- 2 blocks of wood
- 1 metal rod (diameter 6mm) to suspend the pendulum rod.
- 1 carpet to damp vibrations of the setup caused by the moving pendulum.

There will be a figure right after each step to visualize what I am doing. The method is as follows:

1) Using the materials above and a few screws, we can build a rod with a mass attached to it. The figure to the right is what we are using in the pendulum as our bob.

2) Afterwards, we are going to attach 2 wooden plates to each other and drill a hole so we can later attach a rod for the pendulum to swing by. Ultimately, having enough space to analyze the pendulum's motion.

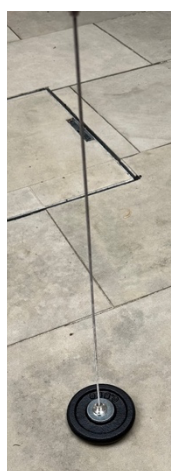


Figure 3: Pendulum assembly featuring weight bob, washers, and rod

3) We set this up against a wall and use a self-levelling crossline laser to get a view of the x-axis and y-axis and draw on the line to get the perfect horizontal and vertical line.

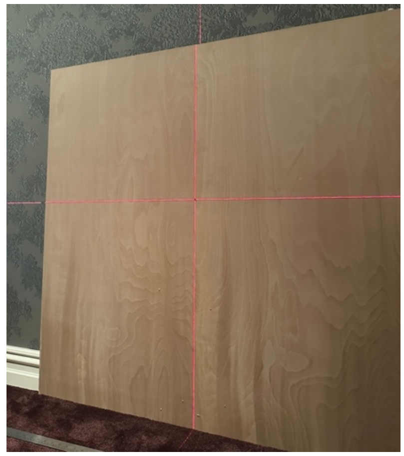


Figure 4: Laser background for precise horizontal and vertical alignment.

4) Using the length of the horizontal line, we use trigonometry to calculate the angles and draw lines to indicate initial θ_0: 15, 30, 45, 60, 75, 90, 105, 120, 135, 150 and 165

Figure 5: Wooden board labeled at angles 15-165 degrees.

5) Then place these 2 plates of wood facing towards us and use a Digital Level Box to make sure that it is completely horizontal and not have a slight tilt which can impact the reading of angles.

Figure 6: Pendulum secured with two wooden blocks for stability on a wooden board

6) After sliding the metal rod inside the hole in the middle we fix the whole setup to the wall to further enhance the stability.

7) After attaching the pendulum to the rod, we put two flange coupling connectors to tighten these to the rod to ensure that the rod won't swing away from the rod.

   We check again with the self levelling cross line laser that the setup is properly aligned.

   Then we screw the setup to the wall.

8) We use an iPad to make videos of the experiment. Angles vary from 15 degrees to 165 degrees. We use LoggerPro's video analysis tool and calculate the exact readings of the period for every single angle mentioned in Step (4)

9) All the readings of the period are then inputted into a Spreadsheet where we can later graphically represent it and analyze it.

We only take one series of measurement per angle θ_0 .We believe we do not gain precision by repeating the measurement many times because the time measurement error (0.001 sec) is very small. Moreover, we have video recordings that can precisely diagnose problems if they should occur.

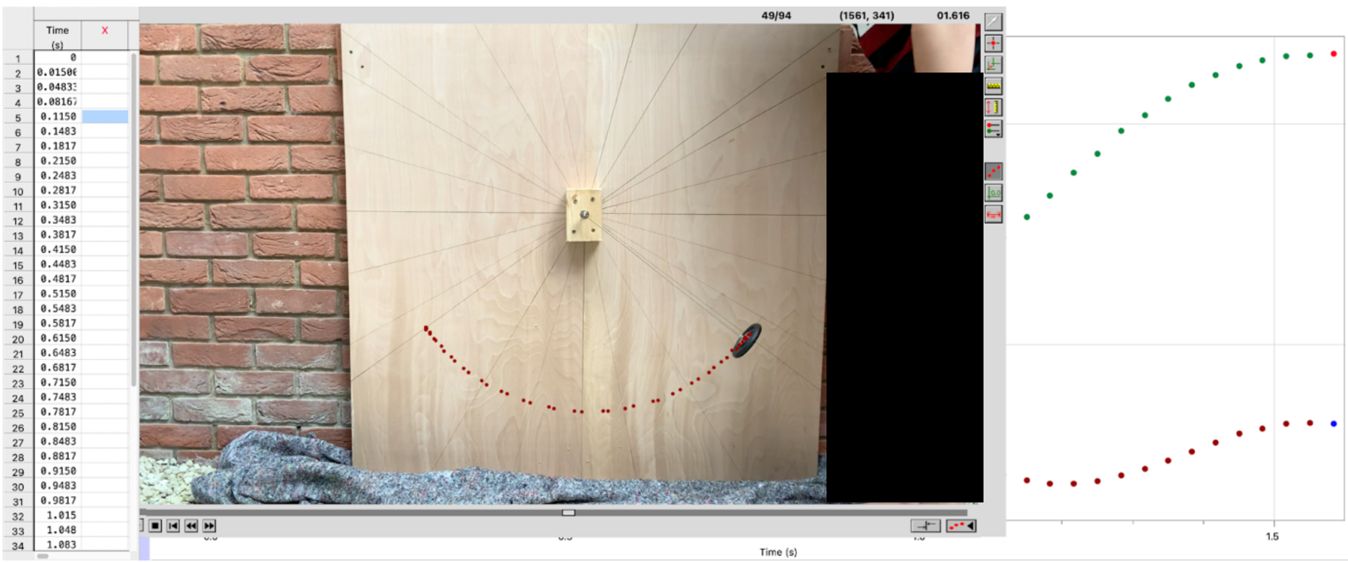


Figure 7: LoggerPro Video analysis

LoggerPro is a data-collection and analysis software that we are using to derive the period T. The pendulum had an initial position of θ_0= 45°. Using the software, we can time this precisely. We can also very precisely time when the rod passes the vertical line corresponding to θ is 0 by adding a series of points throughout the video and when one full period of points is added. We look at the Tables to the left and see how many seconds it took to derive one whole period (Logger Pro® 3 - Vernier, 2019).

II. Safety Precautions

To ensure maximum amount of safety through the experiment, several safety precautions must be heeded. My main tool was a drill to make holes in the wood/wall and to fasten screws. Additionally, sharp objects such as screws or even the edges of the wooden plates can be dangerous. To avoid this, I used personal protective equipment (PPE) like safety goggles, gloves to protect myself against potential hazards. It's always important to adhere to safety precautions to further minimize risks and accidents during the experiment. I took care not to look directly in the self-levelling cross laser used to align the setup. The weighted plate at the end of the rod is of iron and weight of 0.5kg. Its top speed is about 4.7 m/s. During the experiment, care needs to be taken that the experimenter stays out of its trajectory.

III. Variables

a) Independent variables

The initial angles of the pendulum vary from 15 degrees ($\frac{\pi}{12}$ radians) with increments of 15 degrees ($\frac{\pi}{12}$ radians) up to 165 degrees ($\frac{11\pi}{12}$ radians).

b) Dependent variables

The period T of the pendulum measured in seconds.

c) Controlled variables

- Mass of the pendulum bob and rod

The mass of the pendulum bob includes the mass of the couplings and washers used to fix the weight to the rod. The mass for the bob is 0.541kg and the mass of the rod is 0.024kg.

- Length of pendulum

The length of the rod remains '0.485m' throughout the whole experiment. The length of the string plays an important role in the period T. The effective length of the pendulum is not simply the length of the rod (0.500m). The threaded rod is fixed to a metal rod with an eye nut. This increases the pendulum's length by 0.8cm. Analyzing the center of gravity in the pendulum is important. The following diagram will explain how the final length L comes about. To calculate the center of gravity of the pendulum, we first assume that the mass of the rod is at $\frac{L}{2}$. At x meters from the bottom the center of mass of the weighted plate is situated. M_{rod} is the mass of the rod, M_w is the mass of the bob, L is the length of the rod.

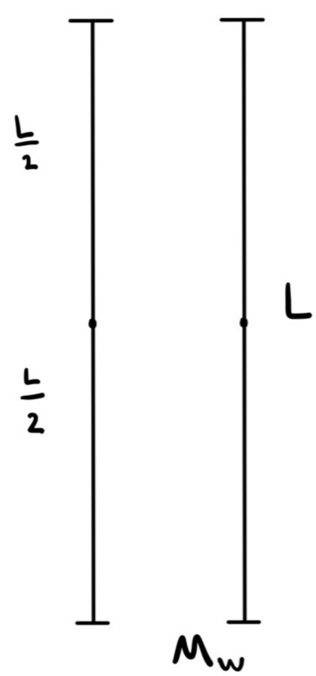


Figure 8: Pendulum with a length and center of gravity showing mass of weight and mass of rod.

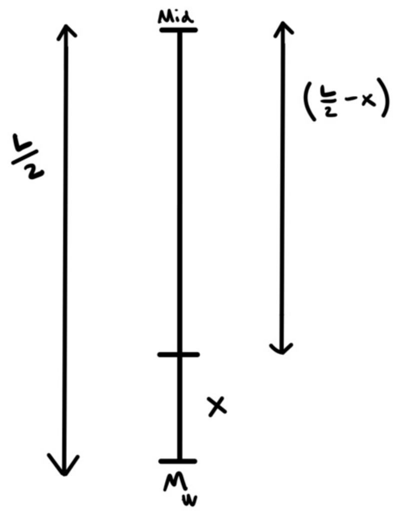


Figure 9: Determining the Center of Mass in a Pendulum. For the calculation of the COM, it is assumed to be situated at $\frac{L}{2}$.

The center of mass of the pendulum is determined by the following equation:

$$M_{rod}\left(\frac{L}{2}-x\right) = M_w\,(x)$$

We calculate x to be 1.05cm, therefore we take the effective length of the pendulum to be 0.485m. This will also be the length L used in the computer simulation.

Experimental Results

The results for the period T are shown in Table 2. Table 2 also shows the numerical data as calculated by the algorithm later described below. The simulation is our main simulation reference for T. The simulation allows us to acknowledge the limitations, assumptions, and potential errors in the physical experiment. These assumptions help us identify what will cause the errors because certain factors in the experiment such as air friction, friction between the rod and pendulum, measurement errors can influence the obtained period values. The findings will further benefit my understanding on the factors that influence the period which has many reallife implications in different fields such as engineering and mechanics. A simple visual inspection shows that the agreement between our measurement and the simulation expectation are good up to ± 0.01 sec for small angles and ± 0.02 for larger angles.

I. Data collection (Experimental)

θ / Degrees ± 1	Period T (Trial #1) (sec) ±0.001	Period T (Trial #2) (sec) ±0.001	Period T (Trial #3) (sec) ±0.001	Period T (Trial #4) (sec) ±0.001	Period T (Trial #5) (sec) ±0.001	Period T (Average) (sec)	ΔT (sec) $\frac{max - \min}{2}$
15°	1.387	1.389	1.395	1.415	1.414	1.400	0.014
30°	1.436	1.432	1.440	1.433	1.432	1.435	0.004
45°	1.465	1.474	1.453	1.489	1.459	1.468	0.018
60°	1.499	1.498	1.501	1.502	1.505	1.501	0.003
75°	1.567	1.563	1.571	1.569	1.570	1.568	0.004
90°	1.667	1.668	1.669	1.670	1.666	1.668	0.002
105°	1.765	1.769	1.770	1.771	1.765	1.768	0.003
120°	1.936	1.937	1.938	1.939	1.935	1.937	0.002
135°	2.164	2.166	2.167	2.163	2.165	2.165	0.002
150°	2.433	2.436	2.435	2.439	2.432	2.435	0.004
165°	2.838	2.837	2.838	2.835	2.837	2.837	0.002

<u>Table 2:</u> This table shows the raw measurements collected through my experiment of the period T in seconds.

II. Data analysis

θ / Degrees	T average (Experimental) (sec)	T (Simulation) (sec) ±0.001	Percentage difference of experimental and small angle approximation T_0
15°	1.400	1.397	0%
30°	1.435	1.425	2%
45°	1.468	1.453	4%
60°	1.501	1.495	7%
75°	1.568	1.565	11%
90°	1.668	1.649	17%
105°	1.768	1.760	23%
120°	1.937	1.914	31%
135°	2.165	2.138	42%
150°	2.435	2.459	55%
165°	2.837	2.793	67%

<u>Table 3:</u> This table shows the measurement of time period T jointly with its simulation

expectation based on the computer simulation. The data seemed to agree very well with the computer simulation values.

III. Sources of Error

a) Precise measurement of the length of the pendulum was not straightforward because we needed to determine the center of gravity of the rod, screws, washers, and the weighted plate. Moreover, there was a 1-2 mm slack between the rod and its fixing on the axis. All together we believe that 0.5cm is a reasonable estimate of this error. Using error propagation: $\Delta T = \frac{\max - \min}{2}$, we find the error ΔT.

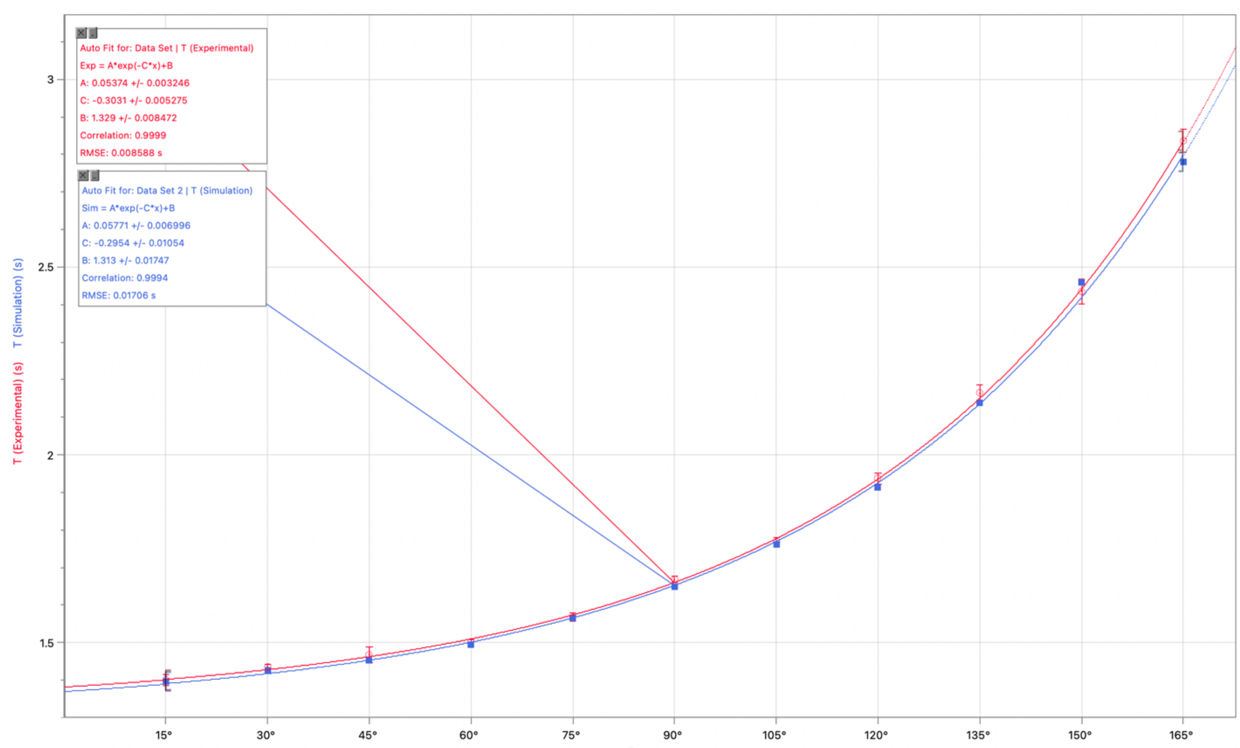
b) Error on the measurement of the initial position θ_0 of the pendulum. We believe that 1 degree is a reasonable estimate for this error. I run the simulation for the initial angle θ_0 and θ_0 + 1 degree and we take the difference between these two numbers to be the error on T. I note here that for very large angles our simulation runs into problems. For example, it cannot distinguish between 165° and 166° no matter how small we choose the step size.

c) The time measurement error using LoggerPro is less than 0.001 seconds and is a neglible error for this analysis.

θ / Degrees	ΔT from error propagation (sec)	ΔT due to $\Delta\theta_0$ (sec)	Total ΔT (sec)
15°	0.014	0.001	0.015
30°	0.004	0.003	0.007

45°	0.018	0.003	0.021
60°	0.003	0.004	0.007
75°	0.004	0.006	0.010
90°	0.002	0.007	0.009
105°	0.003	0.010	0.013
120°	0.002	0.013	0.015
135°	0.002	0.018	0.021
150°	0.004	0.028	0.032
165°	0.002	0.028	0.030

<u>Table 4:</u> Errors on T versus initial angles. The errors for ΔT due to ΔL and ΔT due to $\Delta\theta_0$ are added in quadrature as both sources of error are independent.



<u>Graph 1:</u> *This graph shows the relationship between the measurements for period T of the large angle pendulum, obtained through LoggerPro video analysis, versus the initial angles in degrees ranging from 15° to 165°. An algorithm was used to draw a smooth line through the data points - they are just there to guide the eyes. The blue data points represent the expected simulation value for the period obtained by the simulation and the red data points represent the experimental data for T. The error bars are determined in Table 4 above. The period for the small angle approximation is* $T_0 = 2\pi\sqrt{\frac{L}{g}}$ *= 1.397 seconds. For 45°, 90°, 165° the real period is respectively about 3°, 5° larger. Recall that theoretically the graph extends to 180° would show an asymptote at* θ_0 *= 180°.*

Using LoggerPro, we can input error bars to each data point to show the uncertainty with the measured values. Just from observing the graph, we can see that the simulation data points (blue) and the experimental data points (red) are in very close agreement across all initial angles indicating excellent precision.

V. Statistical evaluation of experimental results

Just eyeballing graph 1 leads us to believe that the agreement between data and prediction is very good. We would like to make some statistical statements with respect to Graph 1. Our discussion is based on reference (Lyons, 1989) & (Lecture 3: Hypothesis Testing and Model-Fitting, n.d.). Normally, the expected values depend on some parameters that can be fitted to the data. This is not the case here, the expected Ts are “absolute”. There are no parameters to be fitted. Regression analysis involves minimization of the so-called $\chi 2$ statistic. The chi-squared test is a statistical method used to determine if there is a significant difference between the expected values and the observed values. The $\chi 2$ with errors is defined is as:

$$\chi 2 = \sum \left(\frac{O_i - E_i}{\sigma_i} \right)^2 \qquad \text{(Eq. 5)}$$

where O_i is the observed period for data point θ_{0i}, E_i is the expected value of T (θ_{0i}) and σ_i is the experimental error on O_i. We show in the Table 5 below the $\chi 2$ calculation and the contribution of each data point to the total $\chi 2$.

θ / Degrees	Chi squared with errors
15°	0.0
30°	2.0
45°	0.5
60°	0.9
75°	0.1
90°	4.8
105°	0.4
120°	2.4
135°	1.7
150°	0.5
165°	2.2

Total $\chi 2$	15.7

<u>Table 5:</u> *Contribution to the chi squared statistic for each data point and total chi squared using Eq.5. The expected T and the measured T values come from Table 3. The errors on the measurement come from Table 4.*

We find $\chi 2$ to be 15.7. But what to expect for $\chi 2$? From the Eq. 5, we see that when each data point contributes on average 1 to the total of $\chi 2$ the fit is reasonable. We allow each datapoint on average to be one σ_i off its Expected Value. Indeed, the expected value of the chi-squared distribution = N and its Variance ($\chi 2$) = 2N. N = 11 is the number of data points (strictly N is the number of degrees of freedom, in our case this is the number of data points because there are no parameters to fit). For our case, the observations (N = 11, standard deviation = $\sqrt{22}$ = 4.7) fit the expectation if $\chi 2$ is in the range of 11 ± 4.7. We note that our measured $\chi 2$ of 15.7 fits its statistical expectation enforcing on us the conclusion that the data and our simulation model are in excellent agreement.

Conclusions

In conclusion, exploring my research question: <u>How does the period T of a swinging pendulum vary with large amplitudes?</u> The research shows compelling evidence for the fact that the time T of a simple pendulum increases significantly for large amplitudes, challenging the limitations small angle approximation The essay also proved the small angle approximation occurs up to angles of 15 to 30 degrees. I was intrigued by the so-called small angle approximations being independent of the opening angle, this essay provided a more precise analysis and measurements leading to dramatic deviations of T_0. For example, an opening angle of 90-degree, the period T is 17% longer than predicted by the small angle approximation. At an initial angle of 165 degrees, the period doubles compared to the approximation. For the initial angles up to about 30 degrees, the small angle approximation remains accurate within a few percent error.

The experimental methodology, involving a detailed pendulum setup along with computer simulations based on the pendulum's equation of motion, $\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$, played a pivotal role. These simulations not only were crucial in predicting period T for various amplitudes but also provided a reliable check against experimental measurements, achieving a excellent agreement for angles up to 165 degrees.

The research acknowledges certain limitations, such as the precise measurements of the pendulum's length and initial angle which could influence the results. However, these aspects along with the need for more extensive data collection give opportunities for future research potentially employing advanced technologies or alternative methods for even more accurate results.

The study of the large angle pendulum, seemingly straightforward system gives intricate dynamics which challenge classical assumptions. The computational and experimental results reinforce our understanding of pendulum motion at large amplitudes but also highlights the pendulum's significance in physics and its applications encouraging further exploration in the field.

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