The next result which is an immediate consequence of Thm.2.1 and Prop.2.4 is motivated by the theory of Markov processes. For a Markov operator (see B-I,Sec.3) condition (ii) of Prop.2.4(a) is called the <u>strong Feller property</u> .

<u>Theorem</u> 2.5. Let $(T(t))_{t\geq 0}$ be semigroup of Markov operators on $C(K)$, K compact, such that one operator $T(t_o)$ has the strong Feller property. Then there exists a positive projection P of finite rank such that $\|T(t) - P\| \leq M\cdot e^{-\delta t}$ for suitable constants $\delta>0$, $M\geq 1$.

<u>Proof</u>. By Prop.2.4(a) $T(t_o)$ is weakly compact. Thus by Prop.2.4(b) $T(2t_o)$ is compact, i.e., $(T(t))_{t\geq 0}$ is eventually compact. Moreover, by B-III,Cor.2.11 $s(A) = 0$ is strictly dominant and a first order pole of the resolvent by B-II,Rem.2.15(a). The assertion now follows easily from Thm.2.1.

□

We close the discussion of eventually compact semigroups by describing a situation where Thm.2.5 can be applied. A more detailed description of the relation between Markov processes and positive semigroups on $C(K)$ is given in Chap.2 of van Casteren (1985).

<u>Example</u> 2.6. Let K be a compact space and $\{P_t : t > 0\}$ be a Markov transition function on K which satisfies the strong Feller property and which is stochastically continuous. That is, every P_t is a real-valued function defined on the product $K \times \mathcal{B}$ where $\mathcal{B}$ denotes the Borel field on K , such that

(a) for $x \in K$ and $t > 0$ fixed, $P_t(x,.)$ is probability measure;
(b) for $C \in \mathcal{B}$ and $t > 0$ fixed, $P_t(.,C)$ is a continuous function;
(c) $P_{t+s}(x,C) = \int_K P_s(y,C)P_t(x,dy)$ for all $s,t > 0$, $x \in K$, $C \in \mathcal{B}$;
(d) $\lim_{t\downarrow 0} P_t(x,U) = 1$ for every open set U containing x .

Condition (b) is the strong Feller property, (c) is the Chapman-Kolmogorov equation and (d) expresses stochastic continuity.
Given $\{P_t\}$ as above one defines for $f \in C(K)$, $x \in K$ and $t > 0$

(2.4) $$(T(t)f)(x) := \int_K f(y)P_t(x,dy) .$$

Then it is not difficult to verify that $T(t)f \in C(K)$, that $T(t)$ is a Markov operator on $C(K)$ and that $(T(t))_{t\geq 0}$ - with $T(0) = Id$ - is a one-parameter semigroup. In fact, the first assertion is a consequence of (a) and (b), the second follows from (a) and the semigroup property is implied by the Chapman-Kolmogorov equation.