

Section 1. Theorem 1.1 was stated by Karlin (1959), but a complete proof is given in Derndinger (1980). Proposition 1.5 is taken from Greiner (1982) and Theorem 1.6 is (implicitly) contained in Derndinger-Nagel (1979). A generalization to (non-lattice) ordered Banach spaces can be found in Sec.2.4 of Batty-Robinson (1984).

Section 2. Lemma 2.3 dates back to Rota (see Schaefer (1974)). Our approach follows Greiner (1981). The notion '(imaginary) additively cyclic' was introduced by Derndinger (1980) (and Schaefer (1980) respectively). Derndinger proves some cyclicity results for the boundary spectrum. A result similar to Proposition 2.7 is given in Sec.7.4 of Davies (1980). Lemma 2.8 in combination with C-III, Lemma 3.13 can be used to characterize semigroups whose spectral bound is a pole of finite algebraic multiplicity (see C-III, (3.19)). The hypothesis of Theorem 2.9 can be weakened, one only needs that $s(A)$ is a pole of the resolvent (see C-III, Cor.2.12). Further results on the cyclicity of the boundary spectrum will be given in Chapter C-III. In particular we refer to C-III, Thms.2.10, 3.11 and 3.13. The dichotomy stated in (2.19) is probably the most interesting consequence of cyclicity results. It has far reaching consequences on the asymptotic behavior of positive semigroups. Example 2.13 is due to Davies (unpublished note). Example 2.14(b) will be discussed in more detail and more generality in Section 3 of Chapter B-IV. We return to Remark 2.15(b) in Section 2 of B-IV.

Section 3. The concept of irreducibility as defined in 3.1 is closely related to various other notions: In topological dynamics flows inducing irreducible semigroups are called 'minimal flows' (cf. Example 3.4(a)). Moreover, 'ergodicity' and 'unique ergodicity' are closely related to irreducibility (see Cornfeld-Fomin-Sinai (1982) or Krengel (1985)). Irreducible semigroups are discussed to some extent in Davies (1980). E.g. he proves a special case of Theorem 3.6. Proposition 3.3 will be generalized in C-III, Prop.3.3. Assertion (a) of Proposition 3.5 was proven by Schaefer (1983) while Theorem 3.6 is taken from Greiner (1982). Elliptic operators (more general than Example 3.10(b)) as generators on spaces of continuous functions, were investigated by many people, e.g. Bony-Courrège-Priouret (1968), Kuhn (1985), Roth (1976)&(1978) and Stewart (1974).

Section 4. Theorem 4.1 is due to Derndinger (1984). The spectrum of semigroups of Markov lattice homomorphisms is investigated by Derndinger-Nagel (1979). In particular they prove Theorem 4.4 for Markov semigroups. Earlier results are due to Scarpellini (1974). We indicated briefly in Example 4.6 that there is a relationship between spectral properties of lattice semigroups and differentiable dynamics. For more details we refer to Chicone-Swanson (1981) and Sacker-Sell (1978). E.g., the 'annular hull theorem' is a special case of Theorem 4.11(b). The general result 4.11 was proven by Arendt-Greiner (1984).