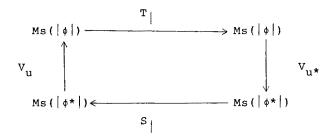
Remark 1.3. Take S and T as in Lemma 1.2 (b) . If V_{u^*} (resp. V_{u}) is the map $(x \to xu^*)$ (resp. $(x \to xu)$) on M, then V_{u^*} is a continuous bijection from $Ms(|\phi|)$ onto $Ms(|\phi^*|)$ with inverse V_{u^*} (because $V_{u^*} \circ V_{u^*} = Id_{Ms}(|\phi|)$ and $V_{u^*} \circ V_{u^*} = Id_{Ms}(|\phi^*|)$. Let $x \in M$. From T(xu) = S(x)u we obtain $T(xu)u^* = S(x)uu^*$. In particular, if $Ms(|\phi^*|)$ is S-invariant, then

$$(V_{u*} \circ T \circ V_u)(x) = T(xu)u* = S(x)$$
.

for every $x \in Ms(|\phi^*|)$. Let $T_{||}$ (resp. $S_{||}$) be the restriction of T to $Ms(|\phi|)$ (resp. of S to $Ms(|\phi^*|)$). Then the following diagram is commutative :



In particular, $\sigma(S_{\parallel}) = \sigma(T_{\parallel})$. Therefore we may deduce spectral properties of S_{\parallel} from T_{\parallel} and vice versa. More concrete applications of Lemma 1.2. will follow.

We now investigate the fixed space $Fix(R) := Fix(\lambda R(\lambda))$, $\lambda \in D$, of a pseudo-resolvent R with values in the predual of a W*-algebra M.

<u>Proposition</u> 1.4. Let R be a pseudo-resolvent on D = $\{\lambda \in \mathbb{C} : \operatorname{Re}(\lambda) > 0\}$ with values in the predual M_{*} of a W*-algebra M and suppose R to be identity preserving and of Schwarz type.

- (a) If $\alpha \in \mathbb{R}$ and $\psi \in M_*$ such that $(\gamma i\alpha)R(\gamma)\psi = \psi$ for some $\gamma \in D$, then $\lambda R(\lambda) |\psi| = |\psi|$ and $\lambda R(\lambda) |\psi^*| = |\psi^*|$ for all $\lambda \in D$.
- (b) Fix(R) is invariant under the involution in M_{\star} . If $\psi \in Fix(R)$ is self adjoint, then the positive part ψ^+ and the negative part ψ^- of ψ are elements of Fix(R) .