are equivalent. Moreover, these statements hold if and only if

$$s(A) = \sup\{\operatorname{Re}\lambda : \lambda \in \sigma(A)\} < 0 ,$$

see A-III,Cor.1.2 .

As already discussed in Chapter A-III the situation is far more difficult in the infinite dimensional case. Here, and for unbounded generators, we have to distinguish between strong and generalized (mild) solutions of $\dot{u}(t) = Au(t)$ and between various notions of stability. Recall that for $f \in D(A)$ the function $T(\cdot)f$ is a <u>strong solution</u> of (ACP) (see A-II,Cor.1.2.); for arbitrary $f \in E$ the function $T(\cdot)f$ is called a <u>generalized</u> or <u>mild solution</u> of (ACP). Next we introduce several constants characterizing the growth of the solutions of (ACP).

<u>Definition</u> 1.1 ($1^{st}$ part). Let $A$ be the generator of a strongly continuous semigroup $(T(t))_{t\geq 0}$ on a Banach space $E$ .
Then

(i) $\omega(f) := \inf\{w : \|T(t)f\| \leq Me^{wt}$ for some $M$ and every $t \geq 0\}$ is called the (exponential) <u>growth bound of $T(\cdot)f$</u> .

(ii) $\omega_1(A) := \sup\{\omega(f) : f \in D(A)\}$ is called the (exponential) <u>growth bound of the solutions of the Cauchy problem</u> $\dot{u}(t) = Au(t)$ .

(iii) $\omega(A) = \sup\{\omega(f) : f \in E\}$ is called the (exponential) <u>growth bound of the mild solutions of the Cauchy problem</u> $\dot{u}(t) = Au(t)$ .

Note that, by the Principle of Uniform Boundedness, $\sup\{\omega(f) : f \in E\} = \inf\{\omega : \|T(t)\| \leq Me^{\omega t}$ for some $M$ and every $t\geq 0\}$ . Hence $\omega(A)$ coincides with the growth bound of the semigroup $(T(t))_{t\geq 0}$ as defined in A-I,1.3. With the constants defined above we obtain the following stability concepts.

<u>Definition</u> 1.1 ($2^{nd}$ part). The semigroup $(T(t))_{t\geq 0}$ is called

(iv) <u>uniformly exponentially stable</u> if $\omega(A) < 0$ ;

(v) <u>exponentially stable</u> if $\omega_1(A) < 0$ ;

(vi) <u>uniformly stable</u> if $\|T(t)f\| \to 0$ (as $t\to\infty$) for every $f \in E$ ;

(vii) <u>stable</u> if $\|T(t)f\| \to 0$ (as $t \to \infty$) for every $f \in D(A)$ .