Hence from Lemma 1.2.a it follows

$$\mu R(\mu)'(xv_\alpha^*) = ((\mu - i\alpha)R(\mu)'x)v_\alpha^*$$

for all $x \in M$. Let ψ_α be the normal linear functional $(x \to \psi(xv_\alpha^*))$ on M and note that $\psi_\alpha(v_\alpha) \neq 0$. Then

$$\langle x, (\mu - i\alpha)R(\mu)\psi_\alpha\rangle = \langle((\mu - i\alpha)R(\mu)'x)v_\alpha^*, \psi\rangle =$$

$$\langle \mu R(\mu)'(xv_\alpha^*), \psi\rangle = \psi(xv_\alpha^*) = \psi_\alpha(x)$$

for all $x \in M$. Consequently $i\alpha \in P\sigma(A)$ and

$$\dim \ker(i\alpha - A') \leq \dim \ker(i\alpha - A) ,$$

which proves the assertion.

□

<u>Remark</u> 1.9. From the above proof we obtain the following: If $0 \neq \psi_\alpha \in \ker(i\alpha - A)$ with polar decomposition $\psi_\alpha = u_\alpha|\psi_\alpha|$ $(\alpha \in \mathbb{R})$ then $A'u_\alpha = i\alpha u_\alpha$. Conversely, if $0 \neq v_\alpha \in \ker(i\alpha - A')$, then there exists $\psi \in \Psi$ such that $\psi(v_\alpha v_\alpha^*) \neq 0$ and the normal linear form

$$\psi_\alpha := (x \to \psi(xv_\alpha^*))$$

is an eigenvector of A pertaining to the eigenvalue $i\alpha$.

If $\mathcal{T}$ is a C_o-semigroup of Markov operators on a commutative C*-algebra with generator A , it has been shown in B-III, that the boundary spectrum $\sigma(A) \cap i\mathbb{R}$ of its generator is additively cyclic. This is no longer true in the non commutative case:

For $0 \neq \lambda \in i\mathbb{R}$ and $t \in \mathbb{R}$ let

$$u_t := \begin{pmatrix} 1 & 0 \\ 0 & e^{\lambda t} \end{pmatrix} \in M_2(\mathbb{C}) .$$