Grothendieck space, where D is the open unit disc $\{z : |z| < 1\}$. If O is a finitely connected domain and H^∞ does not only contain the constant functions, then $H^\infty(O)$ is isomorphic to a finite direct sum of copies of $H^\infty(D)$. (Note that $H^\infty(D)$ is isomorphic to $\{f \in H^\infty(D) : f(0) = 0\}$ via the map $f \to zf$. Then use [Grothendieck (1953),p.77 and Prop.4.4.1]). Hence $H^\infty(O)$ is a Grothendieck space with the Dunford-Pettis property.

□

Final Remark. It follows from Theorem 3.6 that on L^∞ the infinitesimal generator of a strongly continuous semigroup is necessarily bounded. It is not obvious that on $L^\infty([0,1])$ there exist closed densely defined unbounded operators. To see this let A be a closed densely defined unbounded operator form ℓ^2 into $L^\infty([0,1])$ with domain D (such operators can easily be constructed). By the Khintchine inequality, the map $R: (a_n) \to \sum a_n r_n$ where r_n denotes the n^{th} Rademacher function, from ℓ^2 into $L^1([0,1])$ is a topological isomorphism. Hence $T = R'$ maps $L^\infty([0,1])$ onto ℓ^2. Banach's homomorphism theorem implies that $T^{-1}(D)$ is dense in $L^\infty([0,1])$ and that AT is a closed densely defined unbounded operator on $L^\infty([0,1])$ with domain $T^{-1}(D)$. This solves a problem raised by R.Kaufman.

H. Porta and the author have shown that if a Banach space E has an infinite dimensional separable quotient space and F is an infinite dimensional Banach space then there always exists a closed densely defined unbounded operator from E into F.

NOTES.

Section 1. The abstract Cauchy problem is treated systematically in the monographs of Krein (1971) and Fattorini (1983). We refer to these books for more details and historical notes. One implication of Theorem 1.1 is proved in Krein (1971) (Thm.2.11).

The Hille-Yosida theorem has been proved independently by Hille (1948) and Yosida (1948) for contraction semigroups. The extension to arbitrary strongly continuous semigroups is independently due to Feller (1953), Miyadera (1952) and Phillips (1953). Thus our terminology is slightly incorrect, some authors refer to the general version as the Hille-Yosida-Phillips theorem which is slightly more correct. Holomorphic semigroups belong to the standard material of the theory of one-parameter semigroups. Our Theorem 1.14 deviates from the usual presentation since the condition on the resolvent is merely required on a half-plane.

Differentiable semigroups are treated in detail in the book of Pazy (1983) who discovered Theorem 1.17 and 1.18 .