But $\lim_{n\to\infty} (m_n - m)u = 0$ since $u \in D(m)$. Thus $g \in D(\bar{B})$ and $(\lambda - \bar{B})g = f$. We have shown that $E_u \subset (\lambda - \bar{B})D(\bar{B})$. Hence $(\lambda - \bar{B})D(\bar{B})$ is dense in E .

□

Example 4.11. If in the situation explained before Theorem 4.9 $D(A) \subset L^\infty(X,\mu)$ and $m \in L^p(X,\mu)$, then the hypotheses of Theorem 4.9 are satisfied.

Now we want to indicate how the results of this section look like for $C_o(X)$. In fact, most of them carry over with a different interpretation of "sign" but the same proofs. We want to state the analogs of Theorem 4.2 and Theorem 4.3 explicitly. Here we use the notation of B-II,Sec.2.

Theorem 4.12. Let $E = C_o(X)$ where X is locally compact. Let $(T(t))_{t\geq 0}$ be a strongly continuous positive semigroup with generator A and $(S(t))_{t\geq 0}$ a semigroup with generator B . The following assertions are equivalent.

(i) $|S(t)f| \leq T(t)|f|$ for all $f \in E$, $t > 0$.

(ii) $\mathrm{Re}\langle(\mathrm{sign}\, \bar{f})Bf,\phi\rangle \leq \langle|f|,A'\phi\rangle$ for all $f \in D(B)$, $\phi \in D(A')_+$.

Recall that by definition
$\mathrm{Re}\,\langle(\mathrm{sign}\,\bar{f})Bf,\phi\rangle = \int [(\mathrm{sign}\,\overline{f(x)})\cdot(Bf)(x)]\, d\phi(x)$ where $\mathrm{sign}\, f(x) = f(x)/|f(x)|$ if $f(x) \neq 0$ and $\mathrm{sign}\, 0 = 0$.

Theorem 4.13. Let $E = C_o(X)$ (X locally compact) and let $(T(t))_{t\geq 0}$ be a positive semigroup on E with generator A . Let B be a densely defined operator such that

$$\mathrm{Re}\,\langle(\mathrm{sign}\,\bar{f})Bf,\phi\rangle \leq \langle|f|,A'\phi\rangle \quad \text{for all } f \in D(B)\ ,\ \phi \in D(A')_+ . \tag{4.12}$$

Then B is closable. Moreover, if $(\lambda - B)D(B)$ is dense in E for some $\lambda > \max\{0,s(A)\}$, then $\bar{B}$ (the closure of B) generates a semigroup which is dominated by $(T(t))_{t\geq 0}$.

Example 4.14. Let $E := C([-1,0],\mathbb{C})$, $\alpha \in \mathbb{R}$, $\beta \in \mathbb{C}$, $\mu \in M[-1,0]_+$ and $\nu \in M[-1,0]$ such that $\mu(\{0\}) = \nu(\{0\}) = 0$. Then the operator A given by $Af = f'$ on $D(A) = \{f \in C^1([-1,0],\mathbb{C}) : f'(0) = \alpha f(0) + \langle f,\mu\rangle\}$ generates a strongly continuous positive semigroup $(T(t))_{t\geq 0}$ (see B-II,Example 1.22).