Then we have

(2.14) $|h| = |e^{i\alpha t}h| = |T(t)h| \leq T(t)|h|$ or $T(t)|h| - |h| \geq 0$,

(2.15) $\langle T(t)|h| - |h|, \Psi\rangle = \langle |h|, T(t)'\Psi\rangle - \langle |h|, \Psi\rangle = 0$.

Since Ψ is strictly positive, (2.14) and (2.15) imply that $T(t)|h| = |h|$ for $t \geq 0$ or equivalently $A|h| = 0$.
Now Thm.2.4 implies that $Ah^{[n]} = in\alpha h^{[n]}$ $(n \in \mathbb{Z})$.

□

Concerning the hypothesis $T(t_o)'\phi = \exp(s(A)t_o)\cdot\phi >> 0$ we recall that in case X is compact there are always positive linear forms such that $T(t)'\phi = e^{s(A)t}\phi$ (see Thm.1.6). If the semigroup is irreducible, then one also has $\phi >> 0$ (see Sec.3 below).

In a second result we consider semigroups having compact resolvent. An important step of the proof is isolated as a lemma. Before stating it we recall that given a closed ideal $I \subset C_o(X)$ then I as well as $C_o(X)/I$ are spaces of continuous functions on a locally compact space vanishing at infinity. More precisely, if $I = \{f \in C_o(X) : f_{|M} = 0\}$ for a suitable closed subset $M \subset X$, then $I \cong C_o(X\backslash M)$ and $C_o(X)/I \cong C_o(M)$ (cf. B-I). It follows that given another closed ideal $J = \{f \in C_o(X) : f_{|N} = 0\}$ such that $I \subset J$ i.e., $N \subset M$, then J/I can be identified with $C_o(M\backslash N)$. We do not use this concrete representation of J/I . However, this shows that we stay within our setting of Banach spaces of continuous functions on locally compact spaces.

<u>Lemma</u> 2.8. Suppose A is the generator of a positive semigroup $\mathcal{T}$ such that the spectral bound $s(A)$ is a pole of the resolvent of order k . Then there is a sequence

(2.16) $I_{-1} := \{0\} \subset I_0 \subsetneqq I_1 \subsetneqq \ldots \subsetneqq I_k := E$

of $\mathcal{T}$-invariant closed ideals with the following properties:
Denoting by A_n $(n = 0,1,\ldots,k)$ the generator of the semigroup on I_n/I_{n-1} which is induced by $(T(t))$ we have :

(a) $s(A_0) < s(A)$;

(b) If $n \geq 1$ then $s(A_n) = s(A)$ is a first order pole of the resolvent $R(.,A_n)$. The corresponding residue is a strictly positive operator.