The semigroup of *-automorphisms $(x \to u_t x u_t^*)$ on $M_2(\mathbb{C})$ is identity preserving and of Schwarz type but the spectrum of its generator is $\{0, \lambda, \lambda^*\}$ hence is not additively cyclic.

It turns out that, in order to obtain a non commutative analogue of the Perron-Frobenius theorems, one has to consider semigroups which are irreducible. Recall that a semigroup S of positive operators on an ordered Banach space (E,E_+) is called <u>irreducible</u> if no closed face of E_+, different from $\{0\}$ and E_+, is invariant under S. Here a face F in E is a subcone of E_+ such that the conditions $0 \leqq x \leqq y$, $x \in E$, $y \in F$ imply $x \in F$ (compare Definitions 3.1 in B-III and C-III).

In the context of W*-algebras M we call a semigroup S of positive maps on M <u>weak*-irreducible</u>, if no $\sigma(M,M_*)$-closed face of M_+ is S-invariant. Since the norm closed faces of M_* and the $\sigma(M,M_*)$-closed faces of M are related by formation of polars with respect to the dual system $\langle M,M_* \rangle$ (see [Pedersen (1979), Theorem 3.6.11 and Theorem 3.10.7.]) a semigroup S is (norm) irreducible on M_* if and only if its adjoint semigroup is weak*-irreducible.

<u>Theorem</u> 1.10. Let $\mathcal{T}$ be an irreducible, identity preserving semigroup of Schwarz type with generator A on the predual of a W*-algebra and suppose $P\sigma(A) \cap i\mathbb{R} \neq \emptyset$.

(a) The fixed space of $\mathcal{T}$ is one dimensional and spanned by a faithful normal state.

(b) $P\sigma(A) \cap i\mathbb{R}$ is an additive subgroup of $i\mathbb{R}$,

$$\sigma(A) = \sigma(A) + (P\sigma(A) \cap i\mathbb{R})$$

and every eigenvalue in $i\mathbb{R}$ is simple.

(a)* The fixed space of the adjoint weak*-semigroup $\mathcal{T}'$ is one-dimensional.

(b)* $P\sigma(A') \cap i\mathbb{R} = P\sigma(A) \cap i\mathbb{R}$ for the generator A' of the adjoint semigroup, and every $\gamma \in P\sigma(A') \cap i\mathbb{R}$ is simple.

<u>Proof</u>. Since $P\sigma(A) \cap i\mathbb{R} \neq \emptyset$ there exists $\psi \in \mathrm{Fix}(\mathcal{T})_+$ of norm one (Corollary 1.5). If $F := \{x \in M_+ : \psi(x) = 0\}$ then F is a $\sigma(M,M_*)$-closed, $\mathcal{T}'$-invariant face in M, hence $F = \{0\}$. Therefore every $0 \neq \psi \in \mathrm{Fix}(\mathcal{T})_+$ is faithful. Let ψ_1, $\psi_2 \in \mathrm{Fix}(\mathcal{T})_+$ be states such that