Therefore only a finite number of spectral projections P_n are distinct from 0 and we have the following characterization.

<u>Corollary</u> 5.5. Let $\mathcal{T} = (T(t))_{t\geq 0}$ be a semigroup with bounded generator on some Banach space E. This semigroup has period τ/k for some $k \in \mathbb{N}$ if and only if there exist finitely many pairwise orthogonal projections P_n, $-m \leq n \leq m$, $P_{-m} \neq 0$ or $P_m \neq 0$, such that

(i) $\sum_{-m}^{+m} P_n = Id$,

(ii) $T(t) = \sum_{-m}^{+m} \exp(2\pi int/\tau) P_n$,

(iii) $A = \sum_{-m}^{+m} (2\pi in/\tau) P_n$.

<u>Example</u> 5.6. From A-I,2.5 we recall briefly the rotation group $R_\tau(t) f(z) := f(\exp(2\pi int/\tau)\cdot z)$ on $E = C(\Gamma)$, resp. $E = L^p(\Gamma,m)$ for $1 \leq p < \infty$. The spectrum of the generator

$$Af(z) = (2\pi i/\tau) z\cdot f'(z)$$

is

$$\sigma(A) = (2\pi i/\tau)\cdot\mathbb{Z} .$$

The eigenfunctions $\varepsilon_n(z) := z^n$ yield the projections

$$P_n = (1/2\pi i)\cdot \varepsilon_{-(n+1)} \otimes \varepsilon_n \text{ , i.e.}$$

$$P_n f(z) = (1/2\pi i)\cdot(\textstyle\int_\Gamma f(w) w^{-(n+1)} dw)\cdot z^n .$$

It is left as an exercise to compute the norms of $Q_m := \sum_{-m}^{+m} P_n$ in $L^p(\Gamma)$ for various p and then check the assertions of Theorem 5.4. Clearly, this proves some classical convergence theorems for Fourier series (compare Davies (1980), Chap.8.1).

6. <u>SPECTRAL MAPPING THEOREMS</u>

We now return to the question posed in the introduction to this chapter: In which form and under which conditions is it true that the spectrum $\sigma(T(t))$ of the semigroup operators is obtained - via the exponential map - from the spectrum $\sigma(A)$ of the generator, or briefly

$$\sigma(T(t)) = \exp(t\sigma(A)) \text{ ?}$$

This and similar statements will be called <u>spectral mapping theorems</u> for the semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ and its generator A .