

Define  $m : (a,b) \rightarrow \mathbb{R}$  by

$$m(x) = \begin{cases} 0 & \text{if } x \in K \\ 1/q_n'(x) & \text{if } x \in (a_n, b_n) \end{cases}$$

Then  $m$  is continuous and admissible and the flow  $\psi$  coincides with the flow constructed from  $m$  in Theorem 3.17. Hence  $\delta_m$  is the generator of the group  $(S(t))_{t \in \mathbb{R}}$  given by  $S(t)f = f \circ \psi_t = f \circ \beta^{-1} \circ \phi_t \circ \beta = VT(t)V^{-1}f$ , where  $V$  is the isomorphism on  $C_0(a,b)$  given by  $Vf = f \circ \beta$ . Consequently,  $\delta = V^{-1}\delta_m V$ .

□

Now we are able to describe arbitrary generators of positive groups on  $C_0(a,b)$ .

**Theorem 3.25.** Let  $-\infty \leq a < b \leq \infty$ . An operator  $A$  generates a positive group on  $C_0(a,b)$  if and only if there exist

- a lattice isomorphism  $V$  on  $C_0(a,b)$ ,
- an admissible function  $m$  on  $(a,b)$ ,
- a bounded continuous function  $h : (a,b) \rightarrow \mathbb{R}$  such that

$$(3.24) \quad A = V^{-1}\delta_m V + h.$$

**Proof.** Let  $A$  be the generator of a positive group on  $C_0(a,b)$ . By Theorem 3.14 there exist a continuous bounded function  $p : (a,b) \rightarrow \mathbb{R}$  such that  $\inf_{x \in (a,b)} p(x) > 0$  and  $h \in C^b(a,b)$  and the generator  $\delta$  of an automorphism group such that  $A = M\delta M^{-1} + h$  where  $M \in L(C_0(a,b))$  is given by  $Mf = p \cdot f$ . By Theorem 3.24 there exist an admissible continuous function  $m : (a,b) \rightarrow \mathbb{R}$  and a lattice isomorphism  $U \in L(C_0(a,b))$  such that  $\delta = U\delta_m U^{-1}$ . Setting  $V = MU$  we obtain  $A = V\delta_m V^{-1} + h$ .

□

Finally we consider compact intervals. Let  $-\infty \leq a < b \leq \infty$  and  $\phi$  be a continuous flow on  $[a,b]$ . Then it is easy to see that

$\phi(a,t) = a$  and  $\phi(b,t) = b$  for all  $t \in \mathbb{R}$ . So the restriction  $\phi_0$  of  $\phi$  to  $(a,b)$  is a continuous flow on  $(a,b)$ .

Conversely, if  $\phi_0$  is a continuous flow on  $(a,b)$  the extension  $\phi_0$  to  $\phi : \mathbb{R} \times [a,b] \rightarrow [a,b]$  by setting  $\phi(t,a) = a$ ;  $\phi(t,b) = b$  for all  $t \in \mathbb{R}$  defines a continuous flow on  $[a,b]$ . This consideration allows us to extend easily the preceding results to the space  $C[a,b]$ . Let  $m : (a,b) \rightarrow \mathbb{R}$  be a continuous function. We define the operator  $\tilde{\delta}_m$  on  $C[a,b]$  by  $\tilde{\delta}_m f = g$  such that