Proof. In view of C-III,Thm.1.2 we have for $\phi \in E'_+$:
$\lim_{t\to\infty} \langle\int_0^t T(s)f\,ds,\phi\rangle = \sup_{t>0} \int_0^t \langle T(s)f,\phi\rangle\,ds =$
$= \sup_{t>0}\sup_{\lambda>0} \int_0^t e^{-\lambda s}\langle T(s)f,\phi\rangle\,ds = \sup_{\lambda>0}\sup_{t>0} \int_0^t e^{-\lambda s}\langle T(s)f,\phi\rangle\,ds =$
$= \sup_{\lambda>0} \langle R(\lambda,A)f,\phi\rangle = \lim_{\lambda\downarrow 0} \langle R(\lambda,A)f,\phi\rangle$.
Thus either both limits exist with respect to $\sigma(E,E')$-topology or none. Since both nets are monotonically increasing, the assertion follows from Dini's Theorem (see Schaefer (1974), II.Thm.5.9).

□

Proposition 1.9. Let A be the generator of a positive, bounded semigroup $(T(t))_{t\geq 0}$ on a Banach lattice E . If there is a subset $D \subset E_+$ which is total in E such that $\lim_{\lambda\to 0+} R(\lambda,A)f$ exists for every $f \in D$, then $(T(t))_{t\geq 0}$ is uniformly stable.

Proof. By Lemma 1.8 $\int_0^\infty T(t)f\,dt$ exists for every f in the linear hull of D . But D is total, $(T(t))_{t\geq 0}$ is bounded and hence, by A-IV,Thm.1.16, uniformly stable.

□

Remark 1.10. If A is the generator of a positive semigroup, then for every $n \in \mathbb{N}$, $D(A^n)_+$ and $D_+^\infty = (\bigcap_{n=0}^\infty D(A^n))_+$ are total subsets of E . This follows from $f \in D(A^n)$, $f = R(\lambda,A)^n g = R(\lambda,A)^n(g_1 - g_2)$ $= f_1 - f_2$ where f_1 , $f_2 \in D(A^n)_+$ and Thm.1.43 in Davies (1980).

In the rest of this section we discuss the long term behavior of the solutions of the inhomogeneous equation

$$(1.6) \qquad \dot{u}(t) = Au(t) + F(t) \ , \ u(0) = u_0 \in D(A)$$

where the forcing term $F(t)$ converges to some $f_0 \in E$ as $t \to \infty$. In case that A generates a positive semigroup the assumption '$\omega(A) < 0$' , which is needed to prove the next proposition for arbitrary generators (see [Pazy (1983),Thm.4.4.4]), can be replaced by the 'stability' of the semigroup. We recall that some important generators as, for example, the Laplacian on $L^p(\mathbb{R}^n)$, $1 < p < \infty$, generate positive, stable semigroups which are not uniformly exponentially stable. Therefore, the weakening of the assumptions on A mentioned above - i.e., replacing '$\omega(A) < 0$' by 'positive and stable' - widens the class of equations (1.6) for which the following stability result is applicable. For additional results of this kind see Neubrander (1985b).