Proof. If D_o is not dense in $D(A)$ with respect to the graph norm, then there exists a non-zero linear form ϕ on $D(A)$ which is continuous for the graph norm such that $\phi(f) = 0$ for all $f \in D_o$. Let $u \in D(A)$ and $B : D(A) \to D(A)$ be given by $Bf = \phi(f)u$ for all $f \in D(A)$. Then B is continuous for the graph norm. So by Theorem 1.31 the operator $A+B$ with domain $D(A)$ is a generator. Clearly, $A+B \neq A$ if $u \neq 0$ but $Af+Bf = Af$ for all $f \in D_o$. It is obvious that an infinite number of generators can be constructed in that way.

□

Corollary 1.34. Let $(T(t))_{t\geq 0}$ be a strongly continuous semigroup with generator A. Let D_o be a dense subspace of E. Assume that $D_o \subset D(A)$ and $T(t)D_o \subset D_o$ for all $t \geq 0$. Then D_o is a core.

Proof. Let $(S(t))_{t\geq 0}$ be a semigroup with generator B such that $B_{|D_o} = A_{|D_o}$. Let $f \in D_o$. Then $u(t) := T(t)f$ satisfies $u(0) = f$ and $\dot{u}(t) = AT(t)f = BT(t)f = Bu(t)$ $(t\geq 0)$. Since $v(t) = S(t)f$ $(t\geq 0)$ also is a solution of the Cauchy problem defined by B with initial value f it follows that $S(t)f = T(t)f$ $(t\geq 0)$. Since D_o is dense in E, it follows that $S(t) = T(t)$ $(t\geq 0)$.

□

2. CONTRACTION SEMIGROUPS AND DISSIPATIVE OPERATORS

by
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The Hille-Yosida theorem gives a characterization of generators in terms of the resolvent of the operator. However, given an operator A, frequently it is difficult to compute the resolvent (and its powers). So it is desirable to find conditions more immanent on A. This is possible for generators of contraction semigroups. For later purposes (see B-II and C-II) it will be useful not only to consider semigroups which are contractive with respect to the norm but to consider more general sublinear functionals than the norm as well.

So our setting is the following. By E we denote a real Banach space throughout, and $p : E \to \mathbb{R}$ is a continuous sublinear function; i.e., p satisfies