ordering (given by $\phi \leq \psi$ if and only if $\phi(f) \leq \psi(f)$ for all $f \in E_+$) under which $E^{\#}$ is an order complete vector lattice. In particular, positive part, negative part and absolute value exist for any order bounded functional on E , the absolute value of $\phi \in E^{\#}$ being given by

$$|\phi|(f) = \sup\{\phi(h):|h|\leq f\} \qquad (f \in E_+) .$$

As a consequence, one has $|\phi(f)| \leq |\phi|(|f|)$ for all f in E whenever ϕ is order bounded, and $|\phi(f)| \leq \phi(|f|)$ if and only if ϕ is positive. An order bounded linear functional ϕ is called <u>order-continuous</u> (<u>σ-order-continuous</u>) if both positive and negative part of ϕ have the property that they transform any decreasing net (any decreasing sequence) with infimum 0 into a net (sequence) converging to 0 in $\mathbb{R}$. The order-continuous (σ-order-continuous) functionals form a band in $E^{\#}$. In general, a vector lattice E need not admit any non-zero order-bounded linear functional. However, if E is a normed lattice, then any continuous functional is order-bounded, and if E is a Banach lattice then one has coincidence between $E^{\#}$ and E'. Still, order-continuous functionals $\neq 0$ need not exist on a Banach lattice. Situations where every order-bounded functional is order-continuous will be briefly discussed in Section 5.

If E is a Banach lattice, then the dual norm on $E' = E^{\#}$ is a lattice norm, hence E' is an order-complete Banach lattice under the natural norm and order. The evaluation map from E into the second dual E'' is a lattice homomorphism (for the definition see Section 6) into the band of order-continuous functionals on E'. In particular, every dual Banach lattice E admits sufficiently many order-continuous functionals to separate the points of E .

4. <u>AM- AND AL-SPACES</u>

If the norm on a Banach lattice E satisfies

(M) $\qquad \|\sup(f,g)\| = \sup(\|f\|,\|g\|)$ for $f , g \in E_+$

then E is called an abstract M-space or an AM-space. If in addition the unit ball of E contains a largest element u , then u must be an order unit of E and E is then called an (AM)-space with unit. Condition (M) in E implies that in the dual of E one has

(L) $\qquad \|f + g\| = \|f\| + \|g\|$ for $f , g \geq 0$.