3. A SEMIGROUP APPROACH TO RETARDED DIFFERENTIAL EQUATIONS

by
Annette Grabosch and Ulrich Moustakas

The aim of this section is to put into evidence the connection between retarded differential equations and one-parameter semigroups. Special emphasis will lie, as the general theme of this chapter suggests, on positive solutions of such equations and on their asymptotic behavior. Scalar examples were already considered in B-III,Ex.2.14 , B-II,Ex. 1.21 , B-II,Ex.1.23, B-II,Ex.2.11 and B-IV,Ex.2.12 . In this section we will treat retarded differential equations, also called "delay differential equations", with values in arbitrary Banach spaces. A slight modification of the methods used in the scalar case will work in this setting, too. The main question is whether or how a time delay affects the qualitative behavior of the solution of an abstract Cauchy problem. In particular we will show in Thm.3.7, resp. Cor.3.8 that under certain positivity assumptions the delay has no influence on the stability.

Let F be a Banach space, let $E = C([-1,0],F)$ be the Banach space of all continuous functions on $[-1,0]$ with values in F normed by the supremum norm, and let Φ be a bounded linear operator from E into F. For $u \in C([-1,\infty),F)$ and $t \geq 0$ we define the function $u_t \in E$ by $u_t(s) := u(t+s)$ for all $s \in [-1,0]$. This is the "history" segment of u with length 1 starting at $t-1$. Furthermore, let B be the generator of a strongly continuous semigroup on F such that $B - w$ generates a contraction semigroup for some $w \in \mathbb{R}_+$. This additional condition can always be satisfied by renorming the Banach space F (see e.g. [Goldstein (1985a),Thm.2.13]).

Using this framework throughout this section it should be mentioned that in general $E = C([-1,0],F)$ is not a space of type $C(K)$ or even $C_o(X)$. Nevertheless, the formal appearence justifies a treatment in this chapter. Moreover, if $F = C(M)$ (M compact) it is well-known that E is isomorphic to $C([-1,0] \times M)$ and thus is a space of type $C(K)$ (K compact) as well.

With the above notations we consider

$$\text{(RCP)} \qquad \begin{aligned} \dot{u}(t) &= Bu(t) + \Phi(u_t) \ , \quad t \geq 0 \ , \\ u_o &= g \in E \ . \end{aligned}$$