3. LINEAR OPERATORS

A linear mapping T from $C_o(X,\mathbb{R})$ into $C_o(Y,\mathbb{R})$ is called

> positive (notation: $T \geq 0$), if Tf is a positive function whenever f is positive,
> a lattice homomorphism if $|Tf| = T|f|$ for all f ,
> a Markov-operator if X and Y are compact and T is a positive operator mapping 1_X to 1_Y .

In the case of complex scalars T can be decomposed into real and imaginary parts. We call T positive in this situation if the imaginary part of T is $= 0$ and the real part is positive. The terms "Markov operator" and "lattice homomorphism" are defined formally in the same way as above. Note that a complex lattice homomorphism is necessarily positive, and that the complexification of a real lattice homomorphism is a complex lattice homomorphism. Positive Operators are always continuous.

Since the adjoint of a Markov operator T maps positive normalized measures into positive normalized measures while the adjoint of an algebra homomorphism (lattice homomorphism) maps point measures into (multiples of) point measures, the adjoint of a Markov lattice homomorphism as well as the adjoint of an algebra homomorphism induces a continuous map ϕ from Y (viewed as a subset of the weak dual $C(Y)'$) into X (viewed as a subset of $C(X)'$). This mapping ϕ determines T in a natural and unique way, so that the following are equivalent assertions on a linear mapping T from a space $C(X)$ into a space $C(Y)$:

(a) T is a Markov lattice homomorphism
(b) T is a Markov algebra homomorphism
(c) There exists a continuous map ϕ from Y into X such $Tf = f\circ\phi$ for all $f \in C(X)$.

If T is in addition bijective, then the mapping ϕ in (c) is a homeomorhism from Y onto X . This characterization of homomorphisms carries over mutatis mutandis to situations where the conditions on X , Y or T are less restrictive. For later reference we explicitly state:

(i) Let K be compact. Then $T \in L(C(K))$ is a lattice homomorphism if and only if there is a mapping ϕ from K into K and a function