groups out of a given semigroup $(T(t))_{t\geq 0}$ on a Banach space E. Let V be an isomorphism from E onto E. Then $S(t) := VT(t)V^{-1}$, $t \geq 0$, defines a strongly continuous semigroup. If A is the generator of $(T(t))_{t\geq 0}$ then

$$B := VAV^{-1} \text{ with domain } D(B) := \{f \in E : V^{-1}f \in D(A)\}$$

is the generator of $(S(t))_{t\geq 0}$.

3.1. The Rescaled Semigroup

For fixed $\lambda \in \mathbb{C}$ and $\alpha > 0$ the operators

$$S(t) := \exp(\lambda t)T(\alpha t)$$

yield a new semigroup having generator

$$B := \alpha A + \lambda \mathrm{Id} \quad \text{with} \quad D(B) = D(A) .$$

This 'rescaled semigroup' enjoys most of the properties of the original semigroup and the same is true for the corresponding generators. However, by using this procedure certain constants associated with $(T(t))_{t\geq 0}$ and A can be normalized. For example, by this rescaling we may in many cases suppose without loss of generality that the growth bound ω is zero.

Another application is the following: For $\lambda \in \mathbb{C}$ and $S(t) := \exp(-\lambda t)T(t)$ the formulas (1.3) and (1.4) yield:

$$\begin{aligned} &e^{-\lambda t}T(t)f - f = (A-\lambda) \int_0^t e^{-\lambda s}T(s)f\, ds \\ (3.1) \qquad &\text{or} \\ &(e^{\lambda t}-T(t))f = (\lambda-A) \int_0^t e^{\lambda(t-s)}T(s)f\, ds \qquad \text{for } f \in E, \end{aligned}$$

and

$$\begin{aligned} &e^{-\lambda t}T(t)f - f = \int_0^t e^{-\lambda s}T(s)(A-\lambda)f\, ds \\ (3.2) \qquad &\text{or} \\ &(e^{\lambda t} - T(t))f = \int_0^t e^{\lambda(t-s)}T(s)(\lambda-A)f\, ds \qquad \text{for } f \in D(A) . \end{aligned}$$

3.2. The Subspace Semigroup

Assume F to be a closed $(T(t))$-invariant or, equivalently, $R(\lambda,A)$-invariant ($\lambda \in \mathbb{C}$, $\mathrm{Re}\lambda > \omega$) subspace of E. Then the semigroup $(T(t)_|)_{t\geq 0}$ of all restrictions $T(t)_| := T(t)_{|F}$ is strongly continuous on F. If $(A,D(A))$ denotes the generator of $(T(t))_{t\geq 0}$ it follows from the $(T(t))$-invariance and closedness of F that A maps $D(A) \cap F$ into F. Therefore

$A_| := A_{|D(A)\cap F}$ with domain $D(A_|) := D(A) \cap F$ is the generator of $(T(t)_|)$.