(3.16) follows. Moreover, $k_{-t} = 1/k_t \circ \phi_{-t}$. Hence $\|T(-t)\|$ $= \sup_{x\in X} 1/k_t(x) = [\inf_{x\in X} k_t(x)]^{-1}$. So (3.17) also implies the first inequality in (3.16).
Now we define p and h by

$$p(x) := \int_0^\infty e^{-ws}k_s(x)ds\ ,\ h(x) = w - 1/p(x) \quad (x \in X)\ .$$

Then p is a continuous function and we have $(M(2w-1))^{-1} =$
$= \int_0^\infty e^{-ws}(Me^{(w-1)s})^{-1}ds \le p(x) \le \int_0^\infty e^{-ws}Me^{(w-1)s}ds = M$ for all $x \in X$. In particular, it follows that $h \in C^b(X)$.
For all $x \in X$, $t \in \mathbb{R}$ we have
$k_t(x)\cdot p(\phi(t,x)) = \int_0^\infty e^{-ws}k_{t+s}(x)ds = e^{wt}\int_t^\infty e^{-ws}k_s(x)ds.$
Now fix $x \in X$ and define $f : \mathbb{R} \to \mathbb{R}$ by
$f(t) := k_t(x)p(\phi(t,x))/p(x) = [e^{wt}/p(x)]\cdot\int_t^\infty e^{-ws}k_s(x)\,ds$.
The function f is differentiable and satisfies the following differential equation $f'(t) = wf(t) - k_t(x)/p(x) = h(\phi(t,x))f(t)$. Moreover $f(0) = 1$. Hence $f(t) = \exp(\int_0^t h(\phi(s,x))ds)$ for every $t \in \mathbb{R}$. This is (3.15).

□

As before we call a group $(T_o(t))_{t\in\mathbb{R}}$ on $C_o(X)$ an <u>automorphism group</u> if each $T_o(t)$ is an algebra isomorphism on $C_o(X)$.
Analogously an operator δ on $C_o(X)$ is called a <u>derivation</u> if $D(\delta)$ is a subalgebra of $C_o(X)$ and $\delta(f\cdot g) = (\delta f)\cdot g + f\cdot(\delta g)$ for all $f,g \in D(\delta)$.

<u>Proposition</u> 3.13. Let $(T_o(t))_{t\in\mathbb{R}}$ be a group on $C_o(X)$. The following assertions are equivalent.
(i) $(T_o(t))_{t\in\mathbb{R}}$ is an automorphism group.
(ii) There exists a continuous flow ϕ on X such that
$T_o(t)f = f\circ\phi_t \quad (f \in C_o(X)\ ,\ t \in \mathbb{R})$.
(iii) The generator of $(T_o(t))_{t\in\mathbb{R}}$ is a derivation.

<u>Proof</u>. Every automorphism group is positive. So by Prop. 3.9 it is defined via (3.12) by some continuous flow and cocycle. It is easy to see that the cocycle is identically 1 . Thus (i) implies (ii). One shows as in Theorem 3.4 that (ii) implies (iii) and (iii) implies (i).

If $(T_o(t))_{t\in\mathbb{R}}$ is an automorphism group with generator δ we call ϕ in (ii) of Prop.3.13 <u>the flow associated with</u> $(T_o(t))_{t\in\mathbb{R}}$ (or associated with δ).
Now we can show that every generator of a positive group is a perturbation of a derivation.