

A linear operator A on E is called p-dissipative if for all $f \in D(A)$ there exists $\phi \in dp(f)$ such that $\operatorname{Re}\langle Af, \phi \rangle \leq 0$.

The arguments given above show that also in the situation considered here A is p-dissipative if and only if

$$p((1-tA)f) \geq p(f)$$

for all $f \in D(A)$, $t \geq 0$.

The results of this section carry over if they are appropriately modified. We explicitly state the most important result for the case when p is the norm. A linear operator A is simply called dissipative if it is N-dissipative where $N(f) = \|f\|$ ($f \in E$).

Theorem 2.13 (Lumer-Phillips). Let A be a densely defined operator on a complex Banach space E . The following assertions are equivalent.

- (i) A is closable and the closure of A is the generator of a contraction semigroup.
- (ii) A is dissipative and $(\lambda - A)$ has dense range for some $\lambda > 0$.

3. SEMIGROUPS ON L^∞ AND H^∞

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In this section we shall prove that on L^∞ , on $H^\infty(D)$, and on some other classical Banach spaces every strongly continuous semigroup of operators is uniformly continuous.

Lemma 3.1. Let $T = (T(t))_{t \geq 0}$ be a one-parameter semigroup of operators on a Banach space E . Suppose that $s = \limsup_{t \rightarrow 0} \|T(t) - \operatorname{Id}\|$ is finite. If $\lim_{t \rightarrow 0} \|(T(t) - \operatorname{Id})^2\| = 0$, then T is uniformly continuous.

Proof. The identity $2(T(t) - \operatorname{Id}) = T(2t) - \operatorname{Id} - (T(t) - \operatorname{Id})^2$ shows