with positive operators S_j . Such a decomposition of S_o exists since the order completeness of E implies the order completeness of $E_{|f|} = C(K)$ and since every continuous linear operator on a space C(K) is necessarily order-bounded.

9. THE CENTER OF $\mathcal{L}(E)$

We give a short description of a special, but important class of operators.
Let E be a (complex) Banach lattice. For $T \in \mathcal{L}(E)$ the following conditions are equivalent:

(a) $f \perp g$ implies $Tf \perp g$ $(f, g \in E)$
(b) $\pm T \leq \|T\| Id$
(c) $TJ \subset J$ for every ideal J in E .

If E is countably order complete, then this is equivalent to:

(d) $TJ \subset J$ for ervery projection band J in E .

The last assertion also means that T commutes with every band projection.
The set of all $T \in \mathcal{L}(E)$ satisfying the above conditions is called the <u>center</u> of $\mathcal{L}(E)$ and denoted $\mathcal{Z}(E)$. $\mathcal{Z}(E)$ is under the natural ordering, the operator norm and the composition product isomorphic to a Banach lattice algebra C(K) (K compact). The following examples may illustrate the situation and explain why the term "multilication operator" is often used for operators in $\mathcal{Z}(E)$.
(i) If $E = L^p(X,\Sigma,\mu)$ $(1 \leq p \leq \infty)$ with σ-finite μ , then $\mathcal{Z}(E)$ is isomorphic to $L^\infty(\mu)$ via the natural identification of a function $f \in L^\infty(\mu)$ with the multiplication operator $g \to f \cdot g$ on E .
(ii) If X is locally compact, $E = C_o(X)$ then similarly $\mathcal{Z}(E) \cong C^b(X)$ via the identification of $f \in C^b(X)$ with the mapping

$$g \to f \cdot g \qquad (g \in C_o(X)).$$