eigenvectors of A' pertaining to the eigenvalues in H. Then U is closed with respect to the multiplication in M and the formation of adjoints. Thus N is a W*-subalgebra of M [Sakai (1971), Corollary 1.7.9.] and $T_o(t)' := T(t)'|_N$ is multiplicative (for this see D-III, Lemma 1.1).

Since $Q \in \mathcal{T}^- \subseteq \mathcal{L}(_w(M_*)$ there exists an ultrafilter $\mathcal{U}$ on $\mathbb{R}_+$ such that $\lim_{\mathcal{U}} \langle T(t)\psi, x\rangle = \langle Q\psi, x\rangle$ for all $x \in M$ and $\psi \in M_*$. If $\eta \in H$ and $\psi_\eta \in M_*$ such that $A\psi_\eta = \eta\psi_\eta$, then for all $x \in M$:

$$\langle \psi_\eta, x\rangle = \langle Q\psi_\eta, x\rangle = \lim_{\mathcal{U}} \langle T(t)\psi_\eta, x\rangle = (\lim_{\mathcal{U}} e^{\eta t})\langle \psi_\eta, x\rangle ,$$

hence $\lim_{\mathcal{U}} e^{\eta t} = 1$. From this it follows that for all $\psi \in M_*$ we have

$$\langle \psi, Q'(u_\eta)\rangle = \lim_{\mathcal{U}} \langle \psi, T(t)'u_\eta\rangle =$$

$$= (\lim_{\mathcal{U}} e^{\eta t})\langle \psi, u_\eta\rangle = \langle \psi, u_\eta\rangle .$$

Hence $N \subseteq Q'(M)$.

For γ in the dual group of K and $x \in M$ we define x_γ by

$$\psi(x_\gamma) := \int_K \langle S\psi, x\rangle\langle S, \gamma\rangle^* \, dm(S) \qquad (\psi \in M_*^+) .$$

Then $x_\gamma \in M$ and $T(t)'x_\gamma = \langle QT(t), \gamma\rangle x_\gamma$. Therefore $x_\gamma \in N$. Thus the inclusion $Q'M \subseteq N$ is proved if we can show that $Q'M$ belongs to the $\sigma(M,M_*)$-closed linear span of $\{x_\gamma : \gamma \in K , x \in M\}$. For this it is enough to show that every linear form $\psi \in M_*$ such that $\psi(x_\gamma) = 0$ for all $\gamma \in K$ satisfies $\psi(Qx) = 0$ for all $x \in M$. But if $\psi(x_\gamma) = 0$ then

$$\int_K \langle S\psi, x\rangle\langle S, \gamma\rangle^* \, dm(S) = 0 , \ \gamma \in K .$$

Since the map $(S \to \psi(Sx))$ is continuous on K and since the elements of K form a complete orthonormal basis in $L^2(K,dm)$, we obtain $\psi(Sx) = 0$ for all $S \in K$, in particular $\psi(Qx) = 0$ as desired.

Since the range of Q' is a W*-subalgebra of M it follows from [Takesaki (1979), Theorem III.3.4] that Q' is a completely positive, normal conditional expectation. Q' is faithful, i.e. $\ker(Q') \cap M_+ = \{0\}$ since $Q\phi = \phi$ for the faithful linear form ϕ.