In addition, $(\lambda-A)$ is surjective: For $g \in E$ there exists $\hat{f} \in E_/$ such that $(\lambda-A_/)\hat{f} = \hat{g}$, i.e. there exists $h \in N$ such that $(\lambda-A)f - g = h = (\lambda-A)k$ for some $k \in D(A_|)$.
Therefore we obtain $(\lambda-A)(f-k) = g$.

(iii) The integral representation of the resolvent for $\lambda > \omega(T)$ (see A-I, Prop.1.11) shows that $R(\lambda,A)N \subset N$. By the power series expansion for holomorphic functions this extends to all $\lambda \in \rho_+(A)$. Therefore the restriction $R(\lambda,A)_|$ coincides with the resolvent $R(\lambda,A_|)$. On the other hand $R(\lambda,A)_/$ is well defined on $E_/$ and satisfies

$$R(\lambda,A)_/(f+N) = R(\lambda,A)f + N$$

(use again the integral representation). This proves that

$$R(\lambda,A)_/ = R(\lambda,A_/) .$$

□

<u>Corollary</u> 4.3. Under the above assumptions take a point μ in the closure of $\rho_+(A)$. Then

(i) $\mu \in \sigma(A)$ if and only if $\mu \in \sigma(A_|)$ or $\mu \in \sigma(A_/)$.

(ii) μ is a pole of $R(\cdot,A)$ if and only if μ is a pole of $R(\cdot,A_|)$ and of $R(\cdot,A_/)$. In that case,

$$\max(k_|,k_/) \leq k \leq k_| + k_/$$

for the respective pole orders.

<u>Proof</u>. (i) follows from Prop.4.2, inclusions (ii) and (iii).
(ii) By the previous assertion we may assume that for some $\delta > 0$ the pointed disc

$$\{\lambda \in \mathbb{C} : 0 < |\lambda-\mu| < \delta\}$$

is contained in $\rho(A) \cap \rho(A_|) \cap \rho(A_/)$. Call U_n the coefficients of the Laurent expansion of $R(\cdot,A)$. Since N is $R(\lambda,A)$-invariant for $\lambda \in \rho_+(A)$ the same holds for each U_n . With the obvious notations we have

$$R(\lambda,A) = \sum U_n(\lambda-\mu)^n , \quad R(\lambda,A)_| = \sum U_{n|}(\lambda-\mu)^n \quad \text{and} \quad R(\lambda,A)_/ = \sum U_{n/}(\lambda-\mu)^n$$

which shows $\max(k_|,k_/) \leq k$. If $R(\cdot,A)_|$ has a pole in μ of order ℓ , then $U_{-(\ell+1)|} = 0$, i.e. $U_{-(\ell+1)}N = \{0\}$. Similarly it follows that $U_{-(m+1)}E \subset N$ if $R(.,A)_/$ has a pole in μ of order m .