(d) E has order continuous norm and either $T(t_o)$ or $R(\lambda_o,A)$ is a kernel operator for some $t_o \geqq 0$ ($\lambda_o \in \rho(A)$). (For a precise definition of a kernel operator we refer to Sec.IV.9 of Schaefer (1974) or Chap.13 of Zaanen (1983)).

(e) E is reflexive and there exist $t_o > 0$, $h \in E_+$ such that $T(t_o)E \subset E_h$;

Proof. (a) is proved in B-III,Prop.3.5(a).
Assertion (b)-(f) will be proved utilizing the analoguous results for a single operator. In view of A-III,Prop.2.5 we have to show that $r(R(\lambda,A)) > 0$ for some $\lambda \in \rho(A)$. Moreover, from A-I,(3.1) we deduce
$T(t)R(\lambda,A) = e^{\lambda t}R(\lambda,A) - e^{\lambda t}\int_0^t e^{-\lambda s}T(s)ds \leqq e^{\lambda t}R(\lambda,A) \quad (t \geqq 0 \ , \ \lambda > s(A)).$
Since the spectral radius is an isotone function on the cone of positive operators, it is enough to show that

(3.12) $r(T(t)R(\lambda,A)) > 0$ for some $t \geqq 0$, $\lambda > s(A)$.

Using Thm.3.2(a) it is easy to see that $T(t)R(\lambda,A) = R(\lambda,A)T(t)$ is irreducible.
The assertions (b),(d) and (e) now follow from the corresponding results for a single operator as presented in Sect.V.6 of Schaefer (1974) (see Prop.6.1, Thm.6.5 Cor. and Thm.6.5 l.c.). (c) follows from the recent result of de Pagter (1986) which ensures that every positive operator on a Banach lattice which is compact and irreducible has positive spectral radius.

□

The theorem can be used to prove that elliptic operators as described in Ex.2.14 have non-empty spectrum. It is shown in Amann (1983) that these operators have compact resolvent and generate irreducible semigroups. Thus the assumption of (c) is satisfied.

Concerning the eigenvalues of an irreducible semigroup which are contained in $\sigma_b(A)$ all statements established for spaces $C_o(X)$ in B-III,Thm.3.6 are true in the setting of Banach lattices. The proof can be translated without difficulties and will be omitted (see also [Greiner (1982), Thm.2.6]).

Theorem 3.8. Suppose $\mathcal{T}$ is an irreducible semigroup on the Banach lattice E and let A be its generator. Assume that $s(A) = 0$ and that there exists a positive linear form $\psi \in D(A')$ with $A'\psi \leqq 0$.