(necessarily contractive) semigroup. At first we present some consequences of p-dissipativity.

<u>Theorem</u> 2.7. Let A be a p-dissipative operator. If $D(A)$ is dense, then A is strictly p-dissipative.

<u>Proof</u>. Let $f \in D(A)$, $\phi \in dp(f)$. Then for every $t > 0$ and $g \in D(A)$ we have

$$\begin{aligned}
\langle Af,\phi\rangle &= 1/t\ (\langle f + tAf,\phi\rangle - \langle f,\phi\rangle) \leq 1/t(p(f + tAf) - p(f)) \\
&\leq 1/t(p(f + tg) + tp(Af - g) - p(f)) \\
&\leq 1/t(p((Id - tA)(f + tg)) + tp(Af - g) - p(f)) \qquad \text{(by (2.7))} \\
&\leq 1/t(p(f) + tp(g - Af) + t^2p(-Ag) + tp(Af - g) - p(f)) \\
&\leq 1/t\ (2tc\|g - Af\| + t^2c\|Ag\|) \qquad \text{(by (2.3))} \\
&= 2c\|g - Af\| + tc\|Ag\| .
\end{aligned}$$

Letting $t \to 0$ we obtain $\langle Af,\phi\rangle \leq 2c\|g - Af\|$ for all $g \in D(A)$. Since $D(A)$ is dense in E, this implies that $\langle Af,\phi\rangle \leq 0$

□

We now impose stronger conditions on p . A continuous sublinear function $p : E \to \mathbb{R}$ is called <u>half-norm</u> if

(2.11) $\quad p(f) + p(-f) > 0 \quad$ whenever $f \neq 0$;

and p is called a <u>strict half-norm</u> if in addition there exists some constant $d > 0$ such that

(2.12) $\quad p(f) + p(-f) \geq d\|f\| \quad$ for all $f \in E$.

If p is a half-norm, then

(2.13) $\quad \|f\|_p = p(f) + p(-f) \quad (f \in E)$

defines a norm on E which is equivalent to the given norm if and only if p is strict.

<u>Remark</u> 2.8. Every half-norm p induces a closed proper cone $E_p :=$ $\{f \in E : p(-f) \leq 0\}$ on E . Any p-contractive operator T on E leaves the cone E_p invariant (i.e. T is positive for the corresponding ordering).
Conversely, given a closed proper cone E_+ on E , then $p(f) :=$ $\mathrm{dist}\,(-f,E_+) = \inf\,\{\|f + g\| : g \in E_+\}$ defines a half-norm on E such that $E_+ = E_p$. This half-norm is called the <u>canonical</u> <u>half-norm</u> on the ordered Banach space (E,E_+) . The canonical half-norm is strict if and only if the cone E_+ is <u>normal</u> (this is equivalent to the fact that for every $\phi \in E'$ there exist positive linear forms ϕ_1 and ϕ_2 on E such that $\phi = \phi_1 - \phi_2$ (see [Batty-Robinson (1984)] and [Schaefer (1966), Chap.V]).