Remark 1.8. If we consider the translation semigroup $(T(t))_{t\geq 0}$ on $C_0(\mathbb{R}_+)$, then $\|T(t)\| = 1$, hence $s(A) = 1$, but $(T(t))_{t\geq 0}$ is uniformly stable. The same holds for the translation semigroup on $L^1(\mathbb{R}_+)$. Thus Theorem 1.7 is not true for semigroups on C*-algebras with unit or on preduals of W*-algebras. For the discussion of the commutative situation we refer to B-IV, Section 1.

2. STABILITY OF IMPLEMENTED SEMIGROUPS

Let H be a Hilbert space, $\mathcal{U} = (U(t))_{t\geq 0}$ a strongly continuous semigroup on H with generator B and $M \subseteq B(H)$ be a W*-algebra, where $B(H)$ is the W*-algebra of all bounded linear operators on H. Suppose $U(t)MU(t)^* \subseteq M$. Then one can define a weak*-continuous semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ on M by $T(t)x := U(t)xU(t)^*$ $(t\in\mathbb{R}_+,\ x\in M)$. We call $\mathcal{T}$ an implemented semigroup. Every map $T(t)$ of an implemented semigroup is weak*-continuous and n-positive for every $n\in\mathbb{N}$.

Remarks 2.1. (a) Because of

$$\|T(t)\| = \|T(t)1\| = \|U(t)U(t)^*\| = \|U(t)\|^2$$

it follows that $\omega(\mathcal{T}) = 2\omega(\mathcal{U})$.

(b) If $(T(t))_{t\geq 0}$ is an implemented semigroup, then the preadjoint semigroup is strongly continuous on M_*. Therefore $s(A) = \omega$ for $(T(t))_{t\geq 0}$ by Theorem 1.3.

(c) Since $(U(t))_{t\geq 0}$ is a (strongly continuous) semigroup the same is true for the adjoint semigroup $(U(t)^*)_{t\geq 0}$ and its generator is given by B^*. In analogy to [Bratteli-Robinson (1979), 3.2.55] the following assertions for $x\in M$ are equivalent:

(i) $x\in D(A)$.

(ii) For $\xi\in D(B)$ it follows $x\xi\in D(B^*)$ and the linear mapping

$$(*) \qquad (\xi \to x(B\xi)+B^*(x\xi)) : D(B) \to H$$

has a continuous extension to H.