Then $f_o \in C^2[0,1]$ and $f_o - f_o'' = g$. There exist $a, b \in \mathbb{R}$ such that $f(x) = f_o(x) + ae^x + be^{-x}$ defines a function $f \in C^2[0,1]$ satisfying $f(0) = f(1) = 0$. Since $f - f'' = f_o - f_o'' = g$ this implies that $f \in D(A)$ and $f - Af = g$. We have shown that $(Id - A)$ is surjective. It follows from Thm.1.2 that A is the generator of a positive contraction semigroup.

b) Let $E = L^p[0,1]$ $(1 \leq p < \infty)$ and A be given by $Af = f''$ on $D(A) = \{f \in E : f \in C^1[0,1]\ ,\ f' \in AC[0,1]\ ,\ f'' \in L^p[0,1]\ ,\ f(0) = f(1) = 0\}$. Then A is the generator of a positive contraction semigroup.

<u>Proof</u>. A is dispersive. In fact, let $f \in D(A)$. Since the set $M = \{x \in (0,1) : f(x) > 0\}$ is open, there exists a countable set of disjoint intervals (a_n,b_n) such that $M = \bigcup_{n\in\mathbb{N}} (a_n,b_n)$.

First case: $p > 1$.

Let $\phi \in dN^+(f)$. Then there exists $c \geq 0$ such that $\phi(x) = c\, f(x)^{p-1}$ for all $x \in M$ and and $\phi(x) = 0$ if $f(x) \leq 0$ (see Ex. 1.1b). Thus integration by parts yields

$$\begin{aligned}\langle Af,\phi\rangle &= \sum_n \int_{a_n}^{b_n} f''(x)\phi(x)\,dx \\ &= -c \sum_n \int_{a_n}^{b_n} f'(x)f'(x)(p-1)f(x)^{p-2}\,dx \\ &\leq 0 .\end{aligned}$$

Second case: $p = 1$.

Let $\phi(x) = 1$ for $x \in M$ and $\phi(x) = 0$ for $x \notin M$. Then $\phi \in dN^+(f)$ and

$$\langle Af,\phi\rangle = \sum_n \int_{a_n}^{b_n} f''(x)\,dx = \sum_n (f'(b_n) - f'(a_n)) \leq 0$$

since $f'(b_n) \leq 0$ and $f'(a_n) \geq 0$ for all n.

We have shown that A is dispersive. As in a) one shows that $(Id - A)$ is surjective. Now the claim follows from Thm.1.2.

□

c) Consider $E = C_o(\mathbb{R}^n)$. Let $D(A) = \mathcal{S}(\mathbb{R}^n)$ (the Schwartz space of all infinitely differentiable rapidly decreasing functions) and $Af = \Delta f$ $(f \in D(A))$. Then A is closable and the closure of A generates a positive contraction semigroup on E.

<u>Remark</u>. In addition one can show that the closure $\bar{A}$ of A is given by $\bar{A}f = \Delta f$ with domain $D(\bar{A}) = \{f \in E : \Delta f \in E\}$ where for