Example 5.3. Let $d \in \mathbb{C}$ and $S(t) = T_d(t)$ be given by (3.9). Then $|T_d(t)| = T_{|d|}(t)$ $(t \geq 0)$.

Proposition 5.4. Let B be the generator of a disjointness preserving semigroup $(S(t))_{t\geq 0}$ on a Banach lattice E. Then B is local; i.e.

(5.4) $f \perp g$ implies $Bf \perp g$ for all $f \in D(B)$, $g \in E$.

Proof. Let $f \in D(B)$ and $g \in E$ such that $\inf\{|f|,|g|\} = 0$. Then

$$
\begin{aligned}
|1/t(S(t)f - f)| \wedge |g| &\leq |1/tS(t)f| \wedge |g| + 1/t|f| \wedge |g| \\
&= 1/t\ |S(t)f| \wedge |g| \\
&\leq 1/t\ |S(t)f| \wedge |S(t)g - g| + (1/t|S(t)f|) \wedge |S(t)g| \\
&= 1/t\ |S(t)f| \wedge |S(t)g - g| \\
&\leq |S(t)g - g| .
\end{aligned}
$$

Letting $t \to \infty$ one obtains $|Bf| \wedge |g| = 0$.

□

We now describe the relation between the generator of a disjointness preserving semigroup and the generator of the modulus semigroup.

Theorem 5.5. Assume that E is a complex Banach lattice with order continuous norm. Let $(S(t))_{t\geq 0}$ be a semigroup with generator B. The following assertions are equivalent.

(i) $(S(t))_{t\geq 0}$ is disjointness preserving.

(ii) There exists a semigroup $(T(t))_{t\geq 0}$ with generator A such that

(5.5) $f \in D(B)$ implies $|f| \in D(A)$ and $\mathrm{Re}((\hat{\mathrm{sign}}\ \bar{f})Bf) = A|f|$.

Moreover, if these equivalent conditions are satisfied, then $T(t) = |S(t)|$ $(t \geq 0)$.

Remark. By B-II,Lemma 2.9 the relation (5.5) is equivalent to $\langle \mathrm{Re}((\hat{\mathrm{sign}}\ \bar{f})Bf),\phi\rangle = \langle |f|,A'\phi\rangle$ $(f \in D(B),\ \phi \in D(A'))$.

b) It is remarkable that, in contrast with the situation considered in Theorem 3.8, here condition (ii) implies the positivity of $(T(t))_{t\geq 0}$ without further assumptions.

The basic idea of the proof of Theorem 5.5 is to differentiate the equation $|S(t)f| = T(t)|f|$ (where $T(t) = |S(t)|$, cf. (5.3)). For