Since $C + \|C\| \cdot Id \geq 0$ by Thm.1.11, $C$ is regular. Let $C = C_o + N$ where $C_o \in Z(E)^d$ and $N \in Z(E)$. Since $C \geq ReB$ by what we just proved, it follows that $N \geq ReM$.
Let $f \in E_+$, $\phi \in E'_+$ satisfy $<f,\phi> = 0$. Then for all $\alpha \in \mathbb{R}$,
$<Re(e^{i\alpha}B)f,\phi> = \lim_{t\downarrow 0} 1/t<Re(e^{i\alpha}e^{tB})f,\phi> \leq \lim_{t\downarrow 0} 1/t<e^{tC}f,\phi> = <Cf,\phi>$.
Thus $C - Re(e^{i\alpha}B)$ satisfies the positive minimum principle (P) for all $\alpha \in \mathbb{R}$. It follows from Thm. 1.11 that $C - Re(e^{i\alpha}B) + (\|C\|+\|B\|)Id \geq 0$ for all $\alpha \in \mathbb{R}$. Applying the band projection onto $Z(E)^d$ on both sides of this inequality one obtains that
$|B_o| = \sup_{\alpha\in\mathbb{R}} Re(e^{i\alpha}B) \leq C_o$ (since $|T| = \sup_{\alpha\in\mathbb{R}} Re(e^{i\alpha}T)$ for all $T \in L^r(E)$, see C-I,Sec.7).
We have proved that $ReM \leq N$ and $|B_o| \leq C_o$. This implies that
$Re((\operatorname{sign} \bar{f})Bf) = Re((\operatorname{sign} \bar{f})B_o f) + (ReM)|f| \leq C_o|f| + N|f| = C|f|$ for all $f \in E$. It follows from Thm.4.2 that $(e^{tB})_{t\geq 0}$ is dominated by $(e^{tC})_{t\geq 0}$.

□

Remark. The proof of Thm. 4.17 shows that any semigroup dominating a semigroup whose generator is bounded and regular has a bounded generator as well.

Example 4.19. Let $E = \ell^p$ $(1 \leq p < \infty)$ or $c_o$ and $B \in L^r(E)$ be given by the matrix $(b_{ij})$. The generator $A$ of the modulus semigroup of $(e^{tB})_{t\geq 0}$ is given by the matrix $(a_{ij})$ where $a_{ij} = |b_{ij}|$ when $i \neq j$ and $a_{ii} = Re\, b_{ii}$.

A related question is under which condition a semigroup $(S(t))_{t\geq 0}$ is dominated by some positive semigroup. Of course, a necessary condition is that every $S(t)$ is a regular operator. On an AL-space this condition is automatically satisfied. But Kipnis (1974) gives an example of a strongly continuous semigroup on $\ell^1$ which is not dominated. On the other hand, it has been independently shown by Kipnis (1974) and Kubokawe (1975) that every contraction semigroup on an $L^1$-space possesses a modulus semigroup (which is contractive as well).