by endowing it with the gauge function p_f of $[-f,f]$. We denote (I,p_f) by E_f. On the other hand, if f' is a positive linear functional on E, then the mapping $q_{f'}: f \to \langle |f|, f' \rangle$ is a semi-norm on E. The kernel J of $q_{f'}$ is an ideal in E, and the completion of $^E/_J$ with respect to the norm canonically derived from $q_{f'}$ becomes an AL-space which we denote by (E,x'). A good illustration for these constructions is the following: If $E = C(K)$ and if μ is a positive linear form (Radon measure) on E, then (E,μ) is just $L^1(K,\mu)$; if $E = L^p(\mu)$ $(1 \leq p < \infty$, μ finite) then $E_{1_X} = L^\infty(\mu)$.

5. SPECIAL CONNECTIONS BETWEEN NORM AND ORDER

If an increasing net $(x_\alpha)_{\alpha \in A}$ in a normed vector lattice is convergent, then its limit must be the supremum: this is a consequence of the closedness of the positive cone. On the other hand, if $\{x_\alpha : \alpha \in A\}$ has a supremum, the net $(x_\alpha)_{\alpha \in A}$ need not converge. A Banach lattice is said to have order-continuous norm (σ-order-continuous norm) if any increasing net (sequence) which has a supremum is automatically convergent. This is of course equivalent to saying that any decreasing net (sequence) with an infimum is convergent, and since infimum and limit must coincide, the order continuity (σ-order continuity) of the norm in a Banach lattice is also equivalent to the following:

Any decreasing net (sequence) with infimum 0 converges to 0.

A Banach lattice with order-continuous norm must be order complete, but σ-order-continuity of the norm need not imply order completeness. At any rate, one has the following characterization:

A Banach lattice E has order-continuous norm if and only if any one of the following equivalent assertions holds:

(a) E is σ-order complete and has σ-order-continuous norm.
(b) Every order interval in E is weakly compact.
(c) E is (under evaluation) an ideal in E''.
(d) Every continuous linear form on E is order continuous.
(e) Every closed ideal in E is a projection band.

An even more stringent condition than order-continuity of the norm is