CHAPTER B-III

SPECTRAL THEORY OF POSITIVE SEMIGROUPS ON $C_o(X)$

by
Günther Greiner

It is known that for a single operator $T \in L(C_o(X))$ the positivity of T has influence on the spectrum of T, mainly on the peripheral spectrum, i.e. the part of the spectrum containing all spectral values of maximal absolute value. This part of the spectrum is of interest because it determines the asymptotic behavior of the iterates T^n for large $n \in \mathbb{N}$. The spectral properties indicated above were first proved by Perron (1907) and Frobenius (1909) for positive square matrices, i.e. for positive operators on the Banach lattice $\mathbb{C}^n$. Later these results were extended to the infinite dimensional setting; important contributions are due to Jentzsch, Karlin, Krein, Krasnoselski'i, Lotz, Rota, Rutman, Schaefer and others (see Chapt.V of Schaefer (1974)).

In this chapter we investigate the spectrum $\sigma(A)$ of the generator A of a positive semigroup $\mathcal{T} = (T(t))_{t \geq 0}$ on the Banach space $C_o(X)$. Throughout this chapter we assume that $C_o(X)$ is the space of all complex-valued functions on the locally compact space X. In case we restrict to compact spaces we write K instead of X.

1. THE SPECTRAL BOUND

One of the basic results on the spectrum of a positive operator is the fact that its spectral radius is an element of the spectrum (see V.Prop.4.1 of Schaefer (1974)). We begin the investigation of the spectrum of positive semigroups with the analogous result. To that purpose we recall that the spectral bound $s(A)$ of a generator A is defined as the least upper bound of the real parts of all spectral values (cf. A-III,(1.2)).