necessary. [For example, let $A \in L(E)$ such that $(e^{tA})_{t\geq 0}$ is positive and let $B = -A$. Then $A + B$ generates a positive semigroup, but $(e^{tB})_{t\geq 0}$ is positive only if $A \in Z(E)$.] The situation is different when A generates a lattice semigroup.

Theorem 5.18. Let E be a real Banach lattice with order continuous norm and A be the generator of a lattice semigroup. Let $B \in L(E)$. The semigroup generated by $A + B$ is positive if and only if $(e^{tB})_{t\geq 0}$ is positive. The semigroup generated by $A + B$ is a lattice semigroup if and only if $B \in Z(E)$.

Proof. Assume that $A + B$ generates a positive semigroup. Let $f \in D(A)_+$, $\phi \in E'_+$ such that $\langle f,\phi\rangle = 0$. Since A is local, it follows that $\langle Af,\phi\rangle = 0$. But $\langle (A+B)f,\phi\rangle \geq 0$ by Prop.1.7. Hence $\langle Bf,\phi\rangle \geq 0$. We have shown that $B_{|D(A)}$ satisfies the positive minimum principle (Def.1.6). Since $D(A)$ is a sublattice of E (by Cor.5.9), it follows from Thm.1.8 that $(e^{tB})_{t\geq 0}$ is positive.
By Cor.5.9 the operator $A + B$ generates a lattice semigroup if and only if $A + B$ is local. Since A is local, this is equivalent to $B_{|D(A)}$ being local. By Lemma 5.17 this is true if and only if $B \in Z(E)$.

□

NOTES.

Section 1. The notion of dispersiveness is due to Phillips (1962) who uses a semi-scalar product instead of the subdifferential of the canonical half-norm. Our approach follows Arendt-Chernoff-Kato (1982). Bounded generators of positive semigroups on a special class of ordered Banach spaces (which includes Banach lattices and C*-algebras) were characterized by the positive minimum principle by Evans and Hanche-Olsen (1979). The equivalence of (i) and (iv) in Theorem 1.10 is due to Nagel-Uhlig (1981). Theorem 1.8 has been obtained independently by Arendt (1984a) and van Casteren (1984).

Section 2. The classical distributional Kato's inequality for the Laplacian is due to Kato (1973). It is a most elegant tool to prove essential selfadjointness of Schrödinger operators with domain $C_c^\infty(\mathbb{R}^n)$ (cf. Example 4.7).
The relation between Kato's inequality and positivity of $e^{t\Delta}$ was first pointed out by Simon (1977). A criterion for a formnegative operator on a space L^2 to generate a positive semigroup is given by Beurling-Deny (1958), see also Reed-Simon (1978), Vol. IV, Sec.XIII.12. It was a conjecture of Nagel that some abstract version of