CHAPTER B-IV

ASYMPTOTICS OF POSITIVE SEMIGROUPS ON $C_0(X)$

In the following chapter we will examine the asymptotic behavior of positive semigroups on spaces of continuous functions.
The first section is devoted to the various "growth constants" defined in Chapter A-IV and to their coincidence for positive semigroups.
In the second section we treat the asymptotic behavior of positive semigroups which do not differ "too much" from compact semigroups. Properties such as eventual compactness or quasi-compactness allow to describe the long term behavior of the semigroup by using the results from A-III and B-III on the spectrum of the generator.
In the last section we investigate differential delay equations by semigroup methods. In particular, we characterize the spectral bound of the solution semigroups thereby finding simple conditions for stability. Numerous examples conclude the chapter.

1. STABILITY OF POSITIVE SEMIGROUPS ON $C_0(X)$

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In Chapter A-IV we have seen that the long term behavior of a semigroup $(T(t))_{t\geq 0}$ is strongly connected with the existence (and growth) of the resolvent of the generator A in a right halfplane. In particular, the exponential growth of certain semigroups is determined solely by the location of the spectrum (see A-IV,(1.7) and (1.8)). In these cases spectral bound $s(A)$ and growth bound $\omega(A)$ coincide and the equality

$$s(A) = \omega_1(A) = \omega(A) \tag{1.1}$$

holds.