

whenever one side exists and give rise to the following definitions:

$$\begin{aligned}\sup(f, -f) &= |f| \text{ is called the } \underline{\text{absolute value of } f} \\ \sup(f, 0) &= f^+ \text{ is called the } \underline{\text{positive part of } f} \\ \sup(-f, 0) &= f^- \text{ is called the } \underline{\text{negative part of } f} .\end{aligned}$$

Note that the negative part of f is positive.

We call two elements f, g of a vector lattice orthogonal or lattice disjoint and write $f \perp g$, if $\inf(|f|, |g|) = 0$. Apart from this, the above definitions allow us to formulate the axiom of compatibility between norm and order requested in a Banach lattice in the following short way:

$$(LN) \quad |f| \leq |g| \text{ implies } \|f\| \leq \|g\| .$$

A norm on a vector lattice is called a lattice norm, if it satisfies (LN), and with these notations we can now give the definition of a Banach lattice as follows: A Banach lattice is a Banach space E endowed with an ordering \leq such that (E, \leq) is a vector lattice and the norm on E is a lattice norm. By a normed vector lattice we understand a vector lattice endowed with a lattice norm.

There is a number of elementary, but very important formulas valid in any vector lattice, such as

$$\begin{aligned}f &= f^+ - f^- & |f + g| &\leq |f| + |g| \\ |f| &= f^+ + f^- & f + g &= \sup(f, g) + \inf(f, g) \\ \text{etc.} & & &\end{aligned}$$

Let us note in passing the following consequences:

- (i) The lattice operations $(f, g) \mapsto \sup(f, g)$ and $(f, g) \mapsto \inf(f, g)$ and the mappings $f \mapsto f^+$, $f \mapsto f^-$, $f \mapsto |f|$ are uniformly continuous.
- (ii) The positive cone is closed.
- (iii) Order intervals, i.e. sets of the form

$$[f, g] = \{ h \in E : f \leq h \leq g \}$$
 are closed and bounded.

Instead of dwelling upon a detailed discussion of the above equalities and inequalities let us rather formulate the following principle,