Finally, in Section 5 we show that $(T(t))_{t\geq 0}$ is a lattice semigroup (i.e., $|T(t)f| = T(t)|f|$ for all $t \geq 0$, $f \in E$) if and only if A satisfies <u>Kato's equality</u>. This parallels the case when $E = C_o(X)$, but if E has order continuous norm the strong form of Kato's equality can be considered (in particular, $f \in D(A)$ implies $|f| \in D(A)$ if A is the generator of such a semigroup).

1. POSITIVE CONTRACTION SEMIGROUPS AND BOUNDED GENERATORS

In this section we first characterize generators of positive contraction semigroups on a real Banach lattice E.
For that we use the results developed in A-II,Section 2 for the canonical half-norm $N^+ : E \to \mathbb{R}$ given by

$$(1.1) \qquad N^+(f) = \|f^+\| \qquad (f \in E) .$$

<u>Remark</u>. It is easy to see that $N^+(f) = \inf\{\|f+g\| : g \in E_+\} = \operatorname{dist}(-f, E_+)$ (cf. A-II,Rem.2.8).

It is obvious that N^+ is a strict half-norm (see A-II,(2.12)).
The subdifferential of N^+ is given by

$$(1.2) \quad dN^+(f) = \{\phi \in E'_+ : \|\phi\| \leq 1 , \langle f,\phi\rangle = \|f^+\|\}$$

(this follows from the definition, see A-II,(2.5)).

<u>Examples</u> 1.1. a) Let $E = C_o(X)$ (X locally compact). Let $f \in E$. There exists $x \in X$ such that $f(x) = \|f^+\|_\infty$. Then $\delta_x \in dN^+(f)$.

b) Let $E = L^p(X,\Sigma,\mu)$, where (X,Σ,μ) is a σ-finite measure space and $1 < p < \infty$. Let $f \in E$ satisfy $f^+ \neq 0$. Let

$$\phi(x) = \begin{cases} c\cdot f(x)^{p-1} & \text{if } f(x) > 0 \\ 0 & \text{if } f(x) \leq 0 \end{cases}$$

where $c > 0$ is such that $\int f(x)\phi(x)dx = \|f^+\|$.
Then $dN^+(f) = \{\phi\}$.

c) Let $E = L^1(X,\Sigma,\mu)$, where (X,Σ,μ) is a σ-finite measure space, and $f \in E$. Let $\phi \in L^\infty(X,\Sigma,\mu)_+$. Then $\phi \in dN^+(f)$ if and only if

$\phi(x) = 1$ if $f(x) > 0$,
$0 \leq \phi(x) \leq 1$ if $f(x) = 0$ and
$\phi(x) = 0$ if $f(x) < 0$.