2. THE BOUNDARY SPECTRUM

In Chapter B-III we have seen that under suitable assumptions the boundary spectrum $\sigma_b(A)$, which consists of all spectral values with maximal real part, is a cyclic set (cf. B-III,Def.2.5). In the main theorem of this section we prove a result which is more general and which is true for arbitrary Banach lattices.
We first want to extend some of the notions used in B-III to the more general setting considered here. If E is a Banach lattice and $f,g \in E$ such that $g \in E_{|f|}$, then $(\text{sign } f)g$ is well-defined (cf. Sec.8 of C-I). Thus the following definition makes sense:

Definition 2.1. If E is a Banach lattice then for $f \in E$, $n \in \mathbb{Z}$ we define $f^{[n]}$ recursively as follows:

$$(2.1)\qquad \begin{aligned} f^{[0]} &:= |f| \\ f^{[n]} &:= (\text{sign } f) f^{[n-1]} && \text{if } n > 0 \\ f^{[n]} &:= (\text{sign } \bar{f}) f^{[n+1]} && \text{if } n < 0 . \end{aligned}$$

Obviously, for $E = C_o(X)$ this amounts to the same as B-III,Def.2.2. Moreover, in case E is an L^p-space, then $f^{[n]}$ is the function given by

$$(2.2)\qquad f^{[n]}(x) = \begin{cases} \big(f(x)/|f(x)|\big)^{n-1} f(x) & \text{if } f(x) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

The following properties are immediate consequences of Def.2.1 :

$$(2.3)\qquad f^{[0]} = |f| \ , \ f^{[1]} = f \ , \ f^{[-1]} = \bar{f} \ , \ |f^{[n]}| = |f| \quad (n \in \mathbb{Z})$$

$$(2.4)\qquad f^{[n]} = (\text{sign } f) f^{[n-1]} = (\text{sign } \bar{f}) f^{[n+1]} \quad \text{for all } n \in \mathbb{Z} ;$$

$$(2.5)\qquad (\alpha f)^{[n]} = \alpha(\alpha/|\alpha|)^{n-1} f^{[n]} \quad \text{for } n \in \mathbb{Z} \ , \ \alpha \in \mathbb{C} \ , \ \alpha \neq 0 .$$

Next we show that B-III,Thm.2.4 is true for arbitrary Banach lattices. For defintion and simple properties of the signum operator S_h see C-I,Sec.8.

Theorem 2.2. Let $(T(t))_{t\geq 0}$ be a positive semigroup on a Banach lattice E with generator A and suppose that for $h \in E$, $\alpha,\beta \in \mathbb{R}$ we have

$$(2.6)\qquad Ah = (\alpha+i\beta)h \quad , \quad A|h| = \alpha|h| .$$