In order to show uniqueness, assume that u is a solution of (ACP) with initial value 0. We have to show that $u \equiv 0$.
Let $v(t) = \int_0^t u(s)ds$. Then $v(t) \in D(A)$ and $Av(t) = \int_0^t Au(s)ds = \int_0^t \dot{u}(s)ds = u(t) \in D(A)$. Consequently, $v(t) \in D(A^2)$ for all $t \geq 0$. Moreover, $\dot{v}(t) = u(t) = Av(t)$ and $\frac{d}{dt} Av(t) = Au(t) = A\dot{v}(t) = A^2 v(t)$. Thus $v \in C^1([0,\infty),E_1)$ and $\dot{v}(t) = A_1 v(t)$. Since $v(0) = 0$, it follows that $v \equiv 0$. Thus $u \equiv v \equiv 0$.

□

If (ACP) has a unique solution for every initial value in $D(A)$, then A is the generator of a strongly continuous semigroup only if some additional assumptions on the solutions (continuous dependence from the initial value) or on A ($\rho(A) \neq \emptyset$) are made.

Corollary 1.2. Let A be a closed operator. Consider the following existence and uniqueness condition.

(EU) For every $f \in D(A)$ there exists a unique solution $u(\cdot,f) \in C^1([0,\infty),E)$ of the Cauchy problem associated with A having the initial value $u(0,f) = f$.

The following assertions are equivalent.

(i) A is the generator of a strongly continuous semigroup.

(ii) A satisfies (EU) and $\rho(A) \neq \emptyset$.

(iii) A satisfies (EU) and for every $\mu \in \mathbb{R}$ there exists $\lambda > \mu$ such that $(\lambda - A)D(A) = E$.

(iv) A satisfies (EU), has dense domain and for every sequence (f_n) in $D(A)$ satisfying $\lim_{n\to\infty} f_n = 0$ one has $\lim_{n\to\infty} u(t,f_n) = 0$ uniformly in $t \in [0,1]$.

Proof. It is clear that (i) implies the remaining assertions. So assume that A satisfy (EU). Then by Theorem 1.1., A_1 is a generator. If there exists $\lambda \in \rho(A)$, then $(\lambda - A)$ is an isomorphism from E_1 onto E and A is similar to A_1 via this isomorphism [since $D(A_1) = \{(\lambda-A)^{-1}f : f \in D(A)\}$ and $Af = (\lambda-A)A_1(\lambda-A)^{-1}f$ for all $f \in D(A)$, see A-I,3.0]. Thus A is a generator on E and we have shown that (ii) implies (i).
If (iii) holds, then there exists $\lambda > s(A_1)$ such that $(\lambda-A)D(A) = E$. We show that $(\lambda-A)$ is injective. Then $\lambda \in \rho(A)$ since A is closed. Assume that $Af = \lambda f$ for some $f \in D(A)$.
Then $f \in D(A^2) = D(A_1)$, and so $f = 0$ since $\lambda \in \rho(A_1)$. This proves that (iii) implies (ii).