Example 2.14.(a) (One-dimensional Schrödinger operator).
Let $X = \mathbb{R}$, $E = C_o(X)$ and $V : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $\inf V(x) > -\infty$.
If we define

$$(2.22)\qquad \begin{aligned}(Af)(x) &:= f''(x) - V(x)f(x) ,\\ D(A) &:= \{f \in C_o(X) : f \in C^2 , Af \in C_o(X)\},\end{aligned}$$

then A is the generator of a positive semigroup.
In case $\lim_{|x|\to\infty} V(x) = \infty$, A has compact resolvent. Then by Cor.2.10 there exists a dominant real eigenvalue with corresponding positive eigenfunction. Actually, the eigenfunction is strictly positive. (In fact, if $f \in C^2$, $f \geq 0$ and $f(x_o) = 0$ for some x_o, then $f'(x_o) = 0$. Therefore the uniqueness theorem for ordinary differential equations implies that f is identically zero).

(b) (A retarded linear differential equation).
Consider $E = C[-1,0]$ and define A_m, A_o as follows:

$$(2.23)\qquad A_m f := f' , \quad f \in D(A_m) = C^1[-1,0]$$

$$(2.24)\qquad A_o f := f' , \quad f \in D(A_o) = \{f \in C^1[-1,0] : f'(0) = 0\} .$$

A_o generates a contraction semigroup $(T_o(t))_{t\geq 0}$ which is given by

$$(2.25)\qquad (T_o(t)f)(x) = \begin{cases} f(x+t) & \text{if } x+t \leq 0 ,\\ f(0) & \text{if } x+t \geq 0 .\end{cases}$$

This semigroup is positive, eventually norm continuous ($T_o(t) = \delta_o \otimes 1$ for $t \geq 1$) and has compact resolvent. Given a linear functional Ψ on $C[-1,0]$, we consider

$$(2.26)\qquad A_\Psi := A_m|D(A_\Psi) \quad \text{with} \quad D(A_\Psi) := \{f \in C^1[-1,0] : f'(0) = \langle f,\Psi\rangle\}.$$

Denoting the exponential function $x \to e^{\lambda x}$ by e_λ, we have for real λ and $\lambda > \|\Psi\|$:

$$(2.27)\qquad \begin{aligned}&Id - 1/\lambda\cdot\Psi\otimes e_\lambda \text{ is a bijection of } D(A_\Psi) \text{ onto } D(A_o) \text{ and}\\ &\lambda - A_\Psi = (\lambda - A_o)(Id - 1/\lambda\cdot\Psi\otimes e_\lambda) .\end{aligned}$$

Using the Neumann series expansion of $(Id - 1/\lambda\cdot\Psi\otimes e_\lambda)^{-1}$ one obtains the following estimate:

$$(2.28)\qquad \|(Id - 1/\lambda\cdot\Psi\otimes e_\lambda)^{-1}\| \leq \lambda/(\lambda - \|\Psi\|) \qquad \text{if } \lambda > \|\Psi\| .$$