After the abstract characterization in Section 2 we show that every continuous semiflow on X together with a cocycle defines a lattice semigroup in a canonical way, and on $C(K)$, every lattice semigroup can be so represented. This furnishes a wide class of examples. Furthermore, positive one-parameter groups on $C_o(X)$ (which form a particular type of lattice semigroups) are discussed. Their generators are similar to a derivation perturbated by a multiplication operator (Section 3).

1. Generators of Positive Semigroups on $C(K)$.

Let X be a locally compact space. Throughout this section we denote by $C_o(X)$ the space of all real-valued continuous functions on $C_o(X)$ which vanish in infinity. Recall that a semigroup $(T(t))_{t\geq 0}$ on $C_o(X)$ is called positive if $T(t) \geq 0$ for all $t \geq 0$. It is easy to describe the positivity of $(T(t))_{t\geq 0}$ in terms of the resolvent $R(\lambda,A)$ of its generator A because of the close relation between these two objects. In fact, the resolvent is expressed by the semigroup by

(1.1) $$R(\lambda,A) = \int_o^\infty e^{-\lambda t}\, T(t)\, dt \qquad (\lambda > \omega(A));$$

and conversely, the semigroup by the resolvent via the formula

(1.2) $$T(t) = \lim_{n\to\infty} (n/tR(n/t,A))^n \quad \text{strongly}$$

(cf. A-II,Prop.1.10). So we obtain the following.

Proposition 1.1. Let $(T(t))_{t\geq 0}$ be a semigroup with generator A. The semigroup is positive if and only if $R(\lambda,A) \geq 0$ for all sufficiently large real λ.

It is more difficult and more interesting to characterize the positivity of the semigroup by intrinsic conditions on the generator. This is the purpose of this section. As a first orientation we consider bounded generators. We need the following lemma.