(c) → (e): Let $x \in D(A)_+$ such that $Ax = -1$. Then

$$T(t)x - x = \int_0^t T(s)Axds = -\int_0^t T(s)1ds \quad (t \geq 0) ,$$

hence

$$0 \leq \int_0^t T(s)1ds \leq x \quad (t \in \mathbb{R}_+) .$$

Since the family $(\int_0^t T(s)1ds)_{t \geq 0}$ is increasing and bounded,

$$\lim_{t\to\infty} \int_0^t T(s)1ds$$

exists in the weak operator topology on $B(H)$. Since on bounded sets of M the weak operator topology is equivalent to the $\sigma(M,M_*)$-topology, [Sakai (1971), 1.15.2.] , for every $\phi \in M_*$ the integral $\int_0^\infty \phi(T(s)1)ds$ exists. Take $x \in M_+$ and $\phi \in M_*{}^+$. Then $x \leq \|x\|1$ and therefore

$$\phi(T(s)x) \leq \|x\|\phi(T(s)1) \quad (s \in \mathbb{R}_+).$$

Hence $\int_0^\infty \phi(T(s)x)ds$ exists. Since the positive cones of M and M_* are generating, $\int_0^\infty \phi(T(s)x)ds$ exists for every $x \in M$ and $\phi \in M_*$. Therefore $R(0,A)$ exists and is positive which proves (e).

(c) → (g) From the last paragraph we obtain that for all $\xi \in H$

$$\int_0^\infty \|U(s)\|^2 ds = \int_0^\infty (T(s)1\xi \,|\, \xi)ds$$

exists.

(g) → (h): It follows from the polarization identity that the integral

$$\int_0^\infty (U(s)\xi \,|\, U(s)\zeta)ds$$

exists for all $\xi,\zeta \in H$. Using [Takesaki (1979), Theorem III.4.2 and Theorem II.2.6] we conclude as in the implication from (c) to (e) that for all $\xi,\zeta \in H$ the integral

$$\int_0^\infty (((T(s)x)\xi \,|\, \zeta)ds \quad (x \in M)$$

is finite.