when we can show that $H_{2\varepsilon} \subseteq R\sigma(A_/)$. In case (c) one has $K_s = K_\infty$ for some $s < \infty$, which implies $T(s)|I_\infty = 0$. Hence we have $\sigma(A|I_\infty) = \emptyset$ and therefore $\sigma(A) = \sigma(A/I_\infty)$ by A-III,Prop.4.2 .
Henceforth we will assume that $K = K_\infty$, that is, ϕ is surjective (cf. Lemma 4.2(b)).
We choose $\tau > 0$ such that (4.5) is true. Since ϕ is surjective, for every $f \in C(K)$ there is a $x_f \in K$ such that $\|f\| = \|f(\phi(\tau,x_f))\|$ and we obtain for $\lambda \in H_{2\varepsilon}$, $\lambda = \alpha + i\beta$, $\alpha,\beta \in \mathbb{R}$:

$$(4.9) \quad \begin{aligned} \|(e^{\lambda\tau} - T(\tau))f\| &\geq |h_\tau(x_f)f(\phi(\tau,x_f)) - e^{\lambda\tau}f(x_f)| \\ &\geq h_\tau(x_f)\|f\| - e^{\alpha\tau}|f(x_f)| \\ &\geq e^{(\alpha+\varepsilon)\tau}\|f\| - e^{\alpha\tau}\|f\| \\ &= e^{\alpha\tau}(e^{\varepsilon\tau} - 1)\ \|f\| . \end{aligned}$$

It follows that the disc $D := \{\lambda \in \mathbb{C} : |\lambda| < \exp(\underline{c}(h,\phi)-2\varepsilon)\}$ has an empty intersection with $A\sigma(T(\tau))$ and therefore $H_{2\varepsilon} \cap A\sigma(A) = \emptyset$ by A-III,6.2 . Since every boundary point of the spectrum is an approximate eigenvalue (A-III,Prop.2.2(i)) we have the following alternative:

(4.10) Either $D \subseteq \rho(T(\tau))$ and $H_{2\varepsilon} \subseteq \rho(A)$ or else
$D \subseteq R\sigma(T(\tau))$ and $H_{2\varepsilon} \subseteq R\sigma(A)$.

It is not difficult to see that $0 \in \rho(T(\tau))$ whenever ϕ_t is bijective and that 0 is an eigenvalue of $T(\tau)'$ if ϕ_τ is not injective. Since we assumed that that ϕ is surjective, assertions (b) and (c) of the theorem are immediate consequences of (4.10).

□

The examples 4.3(a), (b) and (c) respectively are prototypes of the three different cases considered in Thm.4.4 . Ex.4.3(c) also shows that there are semigroups whose spectrum is contained in a right half-plane, although they cannot be embedded in a group (compare Cor.4.5 below!). Ex.4.3(d) shows that (a) and (b) do not exclude each other.

<u>Corollary</u> 4.5. If ϕ is injective or surjective, then the following assertions are equivalent:

(i) A is the generator of a strongly continuous group.

(ii) $\sigma(A)$ is contained in a right half-plane.