Our first result yields an estimate for the dimension of the eigenspaces pertaining to eigenvalues of a pseudo-resolvent.

<u>Proposition</u> 2.1. Let R be an identity preserving pseudo-resolvent of Schwarz type on $D = \{\lambda\in\mathbb{C} : \mathrm{Re}(\lambda) > 0\}$ with values in the predual of a W*-algebra M . If $\mathrm{Fix}(\lambda R(\lambda))$ is finite dimensional for some $\lambda\in D$, then

$$\dim \mathrm{Fix}((\gamma - i\alpha)R(\gamma)) \leq \dim \mathrm{Fix}(\lambda R(\lambda))$$

for all $\gamma\in D$ and $\alpha\in\mathbb{R}$.

<u>Proof</u>. By D-IV, Remark 3.2.c we may assume without loss of generality that there exists a faithful family of R-invariant normal states on M . In particular the fixed space N of the adjoint pseudo-resolvent R' is a W*-subalgebra of M with $1\in N$ (by Lemma 1.1.b). Since N is finite dimensional there exist a natural number n and a set $P := \{p_1, .., p_n\}$ of minimal, mutually orthogonal projections in N such that $\sum_{k=1}^{n} p_k = 1$. These projections are also mutually orthogonal in M with sum 1 .

Let R_j be the $\sigma(M,M_*)$-closed right ideal p_jM and L_j the closed left invariant subspace M_*p_j $(1\leq j\leq n)$. The map $\mu R(\mu)'$, $\mu\in\mathbb{R}_+$ is an identity preserving Schwarz map . From Lemma 1.1.b we therefore obtain that for all $x\in N$ and $y\in M$,

$$\mu R(\mu)'(xy) = x(\mu R'(\mu)y) .$$

In particular, R_j , resp., L_j are invariant under R' , respectively, R . Furthermore, if $\psi\in L_j$ with polar decomposition $\psi= u|\psi|$, then $u^*u = s(|\psi|) \leq p_j$. Consequently, $|\psi|\in L_j$. Let now $\alpha\in\mathbb{R}$ and suppose that there exists $\psi_\alpha\in L_j$ of norm 1, $\psi_\alpha = u_\alpha|\psi_\alpha|$, such that

$$\psi_\alpha\in\mathrm{Fix}((\lambda - i\alpha)R(\lambda)) , \lambda\in D .$$

Since $\lambda R(\lambda)|\psi_\alpha| = |\psi_\alpha|$ (Proposition 1.4), we obtain

$$\mu R(\mu)'(1-s(|\psi_\alpha|)) \leq (1-s(|\psi_\alpha|) , \mu\in\mathbb{R}_+.$$

From the existence of a faithful family of R-invariant normal states and since R' is identity preserving it follows that

$$\mu R(\mu)'s(|\psi_\alpha|) = s(|\psi_\alpha|) .$$