Let $(p_n)_{n\in\mathbb{N}}$ be a decreasing sequence of projections in M such that $\inf_n p_n = 0$. Then $\lim_n \phi(p_n) = 0$ for every $\phi\in\Phi$. Since

$$(T(t)p_n)^2 \leq T(t)p_n \ , \ t\in\mathbb{R}_+ \ ,$$

we obtain by a classical inequality of Kadison that

$$0 \leq \phi((T(t)p_n)^2) \leq \phi(T(t)p_n) \leq \phi(p_n) \ ,$$

hence $\lim_n \phi(T(t)p_n) = 0$ uniformly in $t\in\mathbb{R}_+$. Since the family Φ is faithful on M , it follows from [Takesaki (1979), Proposition III.5.3] that $(T(t)p_n)$ converges to zero in the $s(M,M_*)$-topology uniformly in $t\in\mathbb{R}_+$. Since this topology is finer than the weak*topology on M we obtain the relative compactness of $\mathcal{T}$ which implies the strong ergodicity.

□

Let $\mathcal{T}$ be an identity preserving semigroup of Schwarz type on the predual of a W*-algebra M . We call

$$p_r := \sup\{s(|\phi|) \ : \ \phi\in\mathrm{Fix}(\mathcal{T})\}$$

the <u>recurrent projection</u> associated with $\mathcal{T}$. For a motivation of this definition compare, e.g., [Davies (1976), Section 6.3].

Since $T(t)|\phi| = |\phi|$ for all $\phi\in\mathrm{Fix}(\mathcal{T})$ (D-III, Cor. 1.5) we obtain $T(t)'p_r \geq p_r$ (see D-I,Sec.3.(c)) . Let $\mathcal{T}^{(r)}$ be the reduced semigroup on $p_rM_*p_r$ with generator $A^{(r)}$. Then $\mathcal{T}^{(r)}$ is identity preserving and of Schwarz type. Similarly, if R is a pseudo-resolvent on $D = \{\lambda\in\mathbb{C} : \mathrm{Re}(\lambda) > 0\}$ with values in M_* such that R is identity preserving and of Schwarz type, then the recurrent projecton associated with R is defined using $\mathrm{Fix}(R)$.

<u>Remark</u> 3.2. (a) Let $\phi\in M_*$ and $\alpha\in\mathbb{R}$ such that

$$(\mu - i\alpha)R(\mu)\phi = \phi \quad \text{for some} \quad \mu\in\mathbb{R}_+ \ .$$

Since $s(|\phi|)$ and $s(|\phi^*|)$ are majorized by p_r (D-III,Prop.1.4) it follows that ϕ and ϕ^* are in $p_rM_*p_r$.

(b) From (a) and the observation that the family $\{|\phi| \ : \ \phi\in\mathrm{Fix}(\mathcal{T})\}$ is