Assume that (ii) holds. Then there exists a continuous mapping $\phi_t: K \to K$ such that $T(t)f = f\circ\phi_t$ for all $f \in C(K)$ (see B-I,Sec.3). The semigroup property implies that $(\phi_t)_{t\geq 0}$ is a continuous semiflow. This shows (iii) to hold.
If (iii) holds, then $T(t)1 = 1$ for all $t \geq 0$ hence $1 \in D(A)$ and $A1 = 0$. Let $f,g \in D(A)$. Then $d/dt|_{t=0}\, T(t)(f\cdot g) = d/dt|_{t=0}\,(T(t)f)\cdot(T(t)g) = (Af)\cdot g + f\cdot(Ag)$. Thus $f\cdot g \in D(A)$ and (3.4) holds. Hence A is a derivation.
Finally assume that (iv) holds. We prove (ii), i.e., we have to show that $T(t)(f\cdot g) = T(t)f\cdot T(t)g$ for $t > 0$. Since $D(A)$ is a dense subalgebra, we can assume that $f,g \in D(A)$. Define $\eta: [0,t] \to C(K)$ by $\eta(s) := T(t-s)[T(s)f\cdot T(s)g]$. Then $\eta(0) = T(t)(f\cdot g)$ and $\eta(t) = T(t)f\cdot T(t)g$. Since A is a derivation, $\eta'(s) = 0$ for $s \in [0,t]$. Hence $\eta(0) = \eta(t)$. This shows (ii) to hold.

□

If δ is the generator of a semigroup $(T(t))_{t\geq 0}$ given by $T(t)f = f\circ\phi_t$, then we call ϕ given by $\phi(t,x) = \phi_t(x)$ the semiflow associated with $(T(t))_{t\geq 0}$ (or associated with δ). We now can describe the generator of any lattice semigroup as a perturbation of a derivation. If 1 is in the domain of the generator, an additive perturbation (by a multiplication operator) suffices; in general a similarity transformation has to be applied in addition. This is the assertion of the following two theorems.

<u>Theorem</u> 3.5. Let A be a generator of a semigroup $(T(t))_{t\geq 0}$ on $C(K)$. Suppose that $1 \in D(A)$. Then the following assertions are equivalent.

(i) $(T(t))_{t\geq 0}$ is a lattice semigroup.
(ii) There exist a derivation δ (generating a semigroup of algebra homomorphisms) and a multiplier $h \in C(K)$ such that $A = \delta + h$ (i.e., $D(A) = D(\delta)$ and $Af = \delta f + h\cdot f$ for $f \in D(A)$).

Moreover, if (ii) holds, then $(T(t))_{t\geq 0}$ is given by

$$(3.6)\qquad (T(t)f)(x) = \exp\left(\int_0^t h(\phi(s,x))ds\right)\cdot f(\phi(t,x))$$

where ϕ is the semiflow associated with δ.

<u>Proof</u>. Let $h = A1$ and $\delta = A - h$. Then the semigroup $(T_o(t))_{t\geq 0}$ generated by δ is a lattice semigroup if and only if $(T(t))_{t\geq 0}$ is a lattice semigroup [since $T_o(t)f = \lim_{n\to\infty} (e^{-t/n\cdot h}\cdot T(\frac{t}{n}))^n f$ and $T(t)f = \lim_{n\to\infty} (e^{t/n\cdot h}\cdot T_o(\frac{t}{n}))^n f$ for all $t \geq 0$, $f \in C(K)$ by