that $\langle\phi, f\rangle = 0$ for all $\phi \in S$. Consequently, $f = 0$.

□

<u>Remark</u>. Using the Fourier transform one can show that the semigroups in example c) and d) are given by

$$(1.6) \qquad (T(t)f)(x) = (4\pi t)^{-n/2} \int_{\mathbb{R}^n} \exp(-(x-y)^2/4t)\, f(y)\, dy$$

$(f \in E)$, where $z^2 := \sum_{i=1}^{n} z_i^2 \qquad (z \in \mathbb{R}^n)$.

e) The following example is the analog of a) for higher dimension. Let $\Omega \subset \mathbb{R}^n$ be a bounded open and connected set and $E = C_o(\Omega)$. We assume that the Dirichlet problem

$$(1.7) \qquad \begin{aligned} u(x) - \Delta u(x) &= 0 \qquad && (x \in \Omega) \\ u(x) &= b(x) && (x \in \partial\Omega) \end{aligned}$$

has a solution $u \in C^2(\Omega) \cap C(\bar{\Omega})$ for every $b \in C(\partial\Omega)$. For example, this is the case if the boundary $\partial\Omega$ is C^2 (see [Gilbarg-Trudinger (1977), Thm. 6.13]).

Let A be given by $Af = \Delta f$ on

$D(A) = \{f \in C^2(\Omega) \cap C_o(\Omega) : \Delta f \in C_o(\Omega)\}$.

Then A is closable and the closure of A is the generator of a positive contraction semigroup.

<u>Proof</u>. $D(A)$ is clearly dense in E . Moreover, one can show as in c) that A is dispersive. It remains to prove that $(Id - A)D(A)$ is dense in E . The space $C_c^\infty(\Omega)$ of all infinitely differentiabel functions on Ω with compact support contained in Ω is dense in E . Let $g \in C_c^\infty(\Omega)$. We show that there exists $f \in D(A)$ satisfying $(Id - A)f = g$. Let $\bar{g} : \mathbb{R}^n \to \mathbb{R}$ be given by $\bar{g}(x) = g(x)$ if $x \in \Omega$ and 0 if $x \notin \Omega$. Then $\bar{g} \in S(\mathbb{R}^n)$. By (1.3) there exists $\bar{f} \in S(\mathbb{R}^n)$ such that $\bar{f} - \Delta\bar{f} = \bar{g}$. Consider the function $b \in C(\partial\Omega)$ given by $b(x) = \bar{f}(x)$ for all $x \in \partial\Omega$. Then by our hypothesis there exists $u \in C(\bar{\Omega}) \cap C^2(\Omega)$ satisfying (1.7). Let $f(x) = \bar{f}(x) - u(x)$ $(x \in \bar{\Omega})$. Then $f \in C^2(\Omega) \cap C_o(\Omega)$ and $(f - \Delta f)(x) = g(x) \quad (x \in \bar{\Omega})$. Thus $\Delta f = f - g$ vanishes on $\partial\Omega$. Hence $f \in D(A)$ and $f - Af = g$.

□

f) Let $\Omega \subset \mathbb{R}^n$ be as in e) and $E = L^p(\Omega)$. Define $Af = \Delta f$ on $D(A) = \{f \in C^2(\Omega) \cap C_o(\Omega) : \Delta f \in C_o(\Omega)\}$. Then A is closable and the closure of A is the generator of a positive contraction semigroup on E .