Definition 3.16. A function $m : (a,b) \to \mathbb{R}$ is admissible if it is continuous and the following holds.
Whenever $a \leq c < d \leq b$ such that $m(x) \neq 0$ for $x \in (c,d)$ and $m(c) = 0$ or $c = a = -\infty$ and $m(d) = 0$ or $d = b = +\infty$, then $\int_c^z 1/|m(x)|\, dx = \int_z^d 1/|m(x)|\, dx = \infty$ for $z \in (c,d)$.

Note: If m is admissible and $a > -\infty$, then $m(a) = 0$; similary, if $b < \infty$, then $m(b) = 0$. Moreover every Lipschitz continuous function is admissible.

Theorem 3.17. Let $m : (a,b) \to \mathbb{R}$ be a continuous function. The operator δ_m is generator of an automorphism group on $C_o(a,b)$ if and only if m is admissible.
In that case $D_o(\delta_m) := \{ f \in D(\delta_m) : f \text{ is differentiable on } (a,b) \}$ is a core of δ_m.
Additional properties. If m is admissible, then the flow ϕ defining the group generated by δ_m can be described explicitely:
The set $\{ x \in (a,b) : m(x) \neq 0 \}$ is the union of a finite or countable number of disjoint intervals (a_n,b_n) $(n \in J)$. Let $c_n \in (a_n,b_n)$ and $q_n(x) := \int_{c_n}^x 1/m(y)\,dy$ $(x \in (a_n,b_n)\ ,\ n \in J)$.
Since m is admissible, q_n is a homeomorphism from (a_n,b_n) onto $\mathbb{R}$. Now the flow ϕ is defined by

$$(3.22)\qquad \phi(t,x) = \begin{cases} x & \text{if } m(x) = 0 \\ \\ q_n^{-1}(q_n(x)+t) & \text{if } x \in (a_n,b_n) \end{cases}$$

for all $t \in \mathbb{R}$.

We first prove a special case of Theorem 3.17.

Proposition 3.18. Suppose that $m(x) \neq 0$ for all $x \in (a,b)$. Then δ_m is the generator of a group on $C_o(a,b)$ if and only if m is admissible. In that case the group generated by δ_m is similar to the translation group on $C_o(\mathbb{R})$.

Proof. Let $q \in C^1(a,b)$ such that $q'(x) = 1/m(x)$ for all $x \in (a,b)$. Then q is a C^1-diffeomorphism from (a,b) onto an interval (a',b'). By $Vf = f\circ q$ one defines an isomorphism from $C_o(a',b')$ onto $C_o(a,b)$. Let B on $C_o(a',b')$ be given by
$B = V^{-1}\delta_m V$. Then $D(B) = \{ g \in C_o(a',b') : g\circ q \in D(\delta_m) \}$
$= \{ g \in C_o(a',b') \cap C^1(a',b') : g'\circ q = m\cdot(g\circ q)' \in C_o(a,b) \}$
$= \{ g \in C_o(a',b') \cap C^1(a',b') : g' \in C_o(a',b') \}$ and
$Bg = V^{-1}\delta_m V = V^{-1}(m(g\circ q)') = V^{-1}(g'\circ q) = g'$.