is strictly less than zero (by Thm.1.1(a)). Moreover, the residue corresponding to the resolvent of T_k, we denote it P_k, is the restriction of P to J_k.
P_k is strictly positive and $P(J_k) = \text{span}\{e_k\}$. To show that T_k is irreducible we consider an invariant ideal I. Then we have $R(\lambda,A_k)I \subset I$ for $\lambda > 0$ hence $P_k = \lim_{\lambda\to 0}\lambda R(\lambda,A_k)$ leaves I invariant. If $I \neq \{0\}$ then $P_k I \neq \{0\}$ since P_k is strictly positive. Then $e_k \in P_k J \subset I$ which implies that $J_k \subset I$.

□

Combining the lemma with Prop.2.11 one obtains the following:
If $s(A)$ is a pole of finite algebraic multiplicity then there exists a finite chain of T-invariant ideals $I_{-1} := \{0\} \subset I_o \subset \ldots \subset I_N := E$ ($N \in \mathbb{N}$) such that the following is true:

(3.19) For the semigroup T_n on I_n/I_{n-1} $(0 \leq n \leq N)$ which is induced by T we have either $s(A_n) = s(A)$ and T_n is irreducible or $s(A_n) < s(A)$.

The following theorem is an immediate consequence of (3.19), Thm.3.12 and A-III,Prop.4.2.

<u>Theorem</u> 3.14. Let T be a positive semigroup on a Banach lattice with generator A. If $s(A)$ is a pole of finite algebraic multiplicity then $\sigma_b(A)$ is a finite union of discrete subgroups (i.e., $\sigma_b(A) = s(A) + \cup_{k=1}^{N} i\alpha_k\mathbb{Z}$ with $\alpha_k \in \mathbb{R}$).
Moreover, σ_b contains only poles of finite algebraic multiplicity.

Here the assumption that the multiplicity of $s(A)$ is finite is essential as can be seen from the following example.

<u>Example</u> 3.15. Consider $X := [0,1] \times V$, $V := \{v \in \mathbb{R} : v_1 < |v| < v_2\}$ $(0 < v_1 < v_2 < \infty)$. The flow in the phase space X which describes the free motion in the interval $[0,1]$ with velocities in V assuming that the particles are reflected at the endpoints generates a positive semigroup on $L^p(X,\mu)$ (μ the Lebesgue measure). For the spectrum of the generator A one obtains $\sigma(A) = \{i\gamma : n\gamma_1 \leq |\gamma| \leq n\gamma_2$ for some $n \in \mathbb{N}_o\}$ with $\gamma_1 := \pi v_1^{-1}$, $\gamma_2 := \pi v_2^{-1}$. Moreover, 0 is a first order pole of the resolvent, obviously the only pole in $\sigma_b(A) = \sigma(A)$. These statements can be verified by calculating the resolvent explicitely. This can be done using the integral representation. The semi-