is the predual of the W*-algebra $\hat{M}''$ and R' is identity preserving (since $R''1 = R1 = 1$) and of Schwarz type (because $\mu R''(\mu) = (\mu R(\mu))''$ is a Schwarz map for all $\mu \in \mathbb{R}_+$) we may apply Lemma 4.3.

Suppose that the fixed space of the canonical extension of R' to some ultrapower of M' is infinite dimensional. Thus we may choose a sequence of states (ϕ_k) in M' and a sequence (z_k) in $(M'')_1^+$ satisfying (a) - (b) of Lemma 4.3. Remark (3) above implies that no subsequence of (ϕ_k) can converge in the $\sigma(M',M'')$-topology.

(a) If ϕ is a $\sigma(M',M)$-accumulation point of (ϕ_k) , then $\phi \in \mathrm{Fix}(R')$. Since $\mathrm{Fix}(R')$ is finite dimensional, the set of accumulation points of the sequence (ϕ_k) is metrizable in the $\sigma(M',M)$-topology. Hence there exists a sequence $(k(n))$ of natural numbers, such that $\sigma(M',M)\text{-}\lim_n \phi_{k(n)} = \phi$. Consequently, $\phi = \sigma(M',M'')\text{-}\lim_n \phi_{k(n)}$ by Remark (1) above. But this leads to a contradiction , which proves (a).

(b) Since $\dim \mathrm{Fix}(R) = \dim \mathrm{Fix}(R') = \mathrm{rank}(P) < \infty$, (b) follows from (a).

(c) Suppose that the fixed space of R' is infinite dimensional. Since $\mathrm{Fix}(R') \subseteq M_*$ there exists a sequence of states (ψ_n) in $\mathrm{Fix}(R')$ with mutually orthogonal support projections in M (Lemma 4.1). Since every $\sigma(M',M)$-accumulation point of the ψ_n's belongs to $\mathrm{Fix}(R')$, hence is normal, the sequence (ψ_n) is relatively $\sigma(M_*,M)$-compact. By Eberleins theorem, we may assume that this sequence is weakly convergent. By the orthogonality of the $s(\psi_n)$'s this sequence converges to zero in the $s^*(M,M_*)$-topology, hence $\lim_n \psi_k(s(\psi_n)) = 0$ uniformly in $k \in \mathbb{N}$, a contradiction. Consequently $\dim \mathrm{Fix}(R) < \infty$ and (c) is proved.

(d) We prove $\dim \mathrm{Fix}(R') = 1$ and apply (a) once again. Useful for this is the following observation : If ψ is a faithful state on M then the normal part is faithful too. Indeed, if $0 \neq x \in M$ such that $\psi^{(n)}(x) = 0$ choose a projection $0 \neq p \in M$ such that $\psi^{(n)}(p) = \psi^{(s)}(p) = 0$ (use [Takesaki (1979), Theorem III.3.8]), hence $\psi(p) = 0$ which conflicts with the faithfulness of ψ .

If $2 \leq \dim \mathrm{Fix}(R')$ there are states ψ_1 and ψ_2 in $\mathrm{Fix}(R')$ such that the corresponding support projections are orthogonal in M'' (Remark 4.2). Since every R'-invariant state ψ is faithful on M , $\psi_i^{(n)} \neq 0$ (otherwise the norm closed face $\{\psi(x) = 0 \colon x \in M_+\}$ would