Then $(1-(\lambda_o-A)BR(\lambda_o,A)R(\lambda,A)))^{-1} = (1 - SR(\lambda,A))$ is invertible. Consequently, also $(1-BR(\lambda,A))^{-1}$ exists (since $\sigma(TR(\lambda_o,A)) \setminus \{0\} = \sigma(R(\lambda_o,A)T) \setminus \{0\}$, $T = (\lambda_o-A)BR(\lambda,A))$.

Let $C = (A-\lambda)B(A-\lambda)^{-1} \in L(E)$. Then $A + C$ is the generator of a strongly continuous semigroup by Theorem 1.29. We show that $A + B$ is similar to $A + C$. In fact, let $U = (1-BR(\lambda,A))$. Then U is an isomorphism on E such that $U(D(A)) = D(A)$.

Moreover, $U(A+C)U^{-1} = U(A-\lambda+C)U^{-1} + \lambda = U[(A-\lambda - (A-\lambda)BR(\lambda,A)]U^{-1} + \lambda$ $= U(A-\lambda)[1-BR(\lambda,A)]U^{-1} + \lambda = U(A-\lambda) + \lambda = A-\lambda+B+\lambda = A+B$.

□

Corollary 1.32. Keeping the hypotheses and notations of Theorem 1.31 denote by $(S(t))_{t\geq 0}$ the semigroup generated by $A+B$. If $(T(t))_{t\geq 0}$ is norm continuous or compact or holomorphic, then $(S(t))_{t\geq 0}$ has the corresponding properties. If B is compact as an operator on $D(A)$ endowed with the graph norm and if $(T(t))_{t\geq 0}$ is eventually norm continuous then so is $(S(t))_{t\geq 0}$.

Proof. This follows from Theorem 1.30 since $(US(t)U^{-1})_{t\geq 0}$ has $A+C$ as generator.

□

Domains of Uniqueness

Given a semigroup $(T(t))_{t\geq 0}$ frequently it is frequently difficult to determine the precise domain of its generator A. So it is important to know which (possibly strict) subspaces of $D(A)$ determine the semigroup uniquely. This can be formulated more precisely in the following way. Let D_o be a subspace of $D(A)$ and consider the restriction A_o of A to D_o. Under which condition on D_o is A the only extension of A_o which is a generator? One obvious condition is that D_o is a core. [In fact, in that case, A is the closure of A_o. Since every generator B extending A_o is closed, it follows that $A \subset B$ and hence $A = B$ since $\rho(A) \cap \rho(B) \neq \emptyset$].

We now show that cores are the only domains of uniqueness.

Theorem 1.33. Let A be the generator of a semigroup and D_o a subspace of $D(A)$. Consider the restriction A_o of A to D_o. If D_o is not a core of A, then there exists an infinite number of extensions of A_o which are generators.