Thm. III.7.11]) it follows that $\mathcal{T}_{|\mathrm{im}(Q)}$ is conjugate to the rotation semigroup of period τ on $L^1(\Gamma,m)$.

□

Using this proposition we obtain

<u>Theorem</u> 3.11. Let $\mathcal{T} = (T(t))_{t\geq 0}$ be a uniformly ergodic, identity preserving semigroup of Schwarz type on the predual of a W*-algebra M and suppose $\sigma(A) \cap i\mathbb{R} \neq \{0\}$. Then there exists a partially periodic, identity preserving semigroup $\mathcal{S} = (S(t))_{t\geq 0}$ of Schwarz type on M_* such that

$$\lim_{t\to\infty} (T(t) - S(t)) = 0$$

in the strong operator topology.

<u>Proof</u>. Let ϕ be the normal state on M generating the fixed space of $\mathcal{T}$. Let $\mathcal{S} = (S(t))_{t\geq 0}$ where $S(t) := T(t)\circ Q$ and Q is as in 2.6. Obviously, $\mathcal{S}$ is partially periodic and $\phi\in\mathrm{Fix}(\mathcal{S})$. Let H_ϕ be the GNS-Hilbert space pertaining to ϕ . Since ϕ is fixed under $\mathcal{T}$, $\mathcal{S}$ and Q these objects have a canonical extension to H_ϕ (in the following denoted by the same symbols). If $H_o := \ker(Q) \subseteq H_\phi$ then it is easy to see that H_o is invariant under the extension to H_ϕ of the multiplication maps we defined in D-III, Remark 1.3. Consequently, using the results in Groh-Kümmerer (1982) it follows that there exists $c\in\mathbb{R}$ such that for all γ near 0 and all $\beta\in\mathbb{R}$:

$$\| R(\gamma + i\beta, A_o) \| \leq c \qquad (*)$$

where $A_o := A_{|\ker(Q)}$ (the norm taken in $L(H_\phi)$). Using the result in A-III,Cor.7.11 it follows that

$$\lim_{t\to\infty} \|T(t)|H_o\| = 0 .$$

Since the $s(M,M_*)$-topology on the unit ball of M is nothing else than the restriction of the norm topology on H_ϕ , we obtain

$$s(M,M_*)\text{-}\lim_{t\to\infty} (T(t)' - S(t)')(x) = 0$$

uniformly on M_1 . From this the assertion follows.

□