invariant under $(T(t))$, then $D(A)\cap C_c(X)$ is invariant as well. It is dense because the elements of the form $\int_0^r T(s)f\,ds$, $f \in C_c(X)$, $r > 0$, belong to $C_c(X)$ and to $D(A)$. Hence $D(A)\cap C_c(X)$ is a core (by A-I,Cor.1.34).

□

Prop.4.11 can be used to prove that flows corresponding to certain ordinary differential equations on $\mathbb{R}^n$ generate strongly continuous semigroups on $L^p(\mathbb{R}^n)$ (where $\mathbb{R}^n$ is equipped with the Lebesgue measure). One has to impose conditions on the corresponding vector field. Note that for continuous flows condition (4.12) is automatically satisfied because for a compact $K \subset X$ the set $\phi_t^{-1}(K) = \phi_{-t}(K)$ is compact as the continuous image of a compact set.

Example 4.12. Let $F : \mathbb{R}^n \to \mathbb{R}^n$ be a C^1-vector field and assume that the derivative DF is uniformly bounded on $\mathbb{R}^n$. Then the ordinary differential equation $y' = F(y)$ possesses a global flow $\phi : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ which is C^1 . Moreover, we have

$$(4.13)\qquad \|D\phi_t(x)\| \le e^{M|t|} \quad \text{for all } x \in \mathbb{R}^n,\ t \in \mathbb{R},\ \text{where}$$
$$M := \sup\{\|DF(x)\| : x \in \mathbb{R}^n\} .$$

All these properties were proven in Ex.3.15 of B-II.
We will show that ϕ satisfies condition (ii) of Prop.4.11(a). Hence it induces a strongly continuous (semi-)group of lattice homomorphisms on $L(\mathbb{R}^n)$ $(1 \le p < \infty)$ via $T(t)f = f\circ\phi_t$.
This is done using the change of variables formula as follows:
Let U be an open subset of $\mathbb{R}^n$, then $\phi_t^{-1}(U) = \phi_{-t}(U) =: U(-t)$. If λ denotes the Lebesgue measure then

$$(4.14)\qquad \begin{aligned} &\lambda(\phi_t^{-1}(U)) = \int_{U(-t)} 1\,dx = \int_U 1\circ\phi_{-t}(x)\cdot|\det D\phi_{-t}(x)|\,dx = \\ &\int_U |\det D\phi_{-t}(x)|\,dx \le \int_U e^{nM|t|}\,dx = e^{nM|t|}\cdot\lambda(U) .\end{aligned}$$

Here we used (4.13) and the fact that the determinant of an $n\times n$-matrix B satisfies $|\det B| \le \|B\|^n$.

In general, existence of a global flow does not ensure that one can associate a semigroup of bounded linear operators on $L^p(\mathbb{R}^n)$, even if the vector field is C^∞ . For example the differential equation $y' = \sin(y^2)$ does not induce a semigroup on $L^p(\mathbb{R})$.
There is another important class of differential equations which do induce semigroups of lattice homomorphisms on L^p-spaces: Hamiltonian differential equations. In fact, Liouville's Theorem states that the