

semigroup  $(T_1(t))$  on  $C(K_1)$  corresponding to  $\phi_1$  and  $h$  leaves the ideal  $I := \{f \in C(K_1) : f|_{K_0} = 0\}$  invariant and induces via restriction a semigroup  $(T(t))$  on  $I = C_0(X)$ , where  $X = K_1 \setminus K_0$ . In this case one can construct semi-flows associated with  $(T(t))$  on  $XU\{\infty\}$  or on  $\bar{X}$  (closure in  $K_1$ ), but in general one cannot find a corresponding multiplicator which is defined on one of these compactifications.

The situation is much nicer when groups of lattice homomorphisms instead of semigroups are considered. In this case there is an analogue of (4.1) (cf. B-II, Thm.3.14) and the spectrum can be described completely. For more details and the proof of the following result we refer to Arendt-Greiner (1984).

**Theorem 4.11.** Suppose  $X$  is a locally compact space and  $(T(t))_{t \in \mathbb{R}}$  is a group of lattice homomorphisms governed by the flow  $\phi$  and the multiplicator  $h$ . Then we have:

- (a)  $\sigma(A) = \sigma_1 \cup \sigma_2 \cup \sigma_3$  where the sets  $\sigma_i$  are defined as follows  
 $\sigma_1 := \{h^\wedge(x) + i \cdot 2\pi k / \tau_x : x \in X, 0 < \tau_x < \infty\}^-$ ,  
 $\sigma_2 := \{h(x) : x \in X, \tau_x = 0\}^-$ ,  $\sigma_3 := \{\lambda \in \mathbb{C} : \lambda + i\mathbb{R} \subseteq \sigma(A)\}$
- (b)  $\sigma(T(t)) = \overline{\exp(t\sigma(A))}$  for every  $t \geq 0$ .
- (c) Every isolated point of  $\sigma(A)$  is a first order pole of the resolvent.

#### NOTES.

Spectral theory for a single positive operator is an essential cornerstone for spectral theory of positive one-parameter semigroups. Many of the results we have presented in this chapter have analogues in the discrete case (i.e. for a single operator). This relation may serve as a guide. However, only in few cases can the continuous version be deduced directly from its discrete analogue. Therefore we will not try to trace back the roots of every result to the discrete version. Instead we refer to Schaefer (1974) and the notes and references given there.

Many of the results we have presented in this chapter can be extended (more or less easily) to the more general setting of Banach lattices, which include the very important examples of  $L^p$ -spaces. Others are typical for  $C(X)$  and allow no extension. We will discuss this fact in more detail in Chapter C-III. The more general setting considered there also allows us to prove results for  $C(X)$  which we could not obtain staying within the framework of spaces of continuous functions.