Proof. Once it is shown that both functions $u \to s(1/u)$ and $u \to \omega(1/u)$ are convex on $[\frac{1}{q},\frac{1}{p}]$, the assertion follows from Thm.1.1 and the relation $s(r) \leq \omega(r)$ for every r. Since $\omega(u) = \log r(T_u(1))$ (see A-III,(1.4)), (1.2) implies that $u \to \omega(1/u)$ is a convex function. By C-III,Thm.1.1 we have $r(R(k,A_u)) = (k-s(u))^{-1}$ for $k \in \mathbb{N}$ sufficiently large. The assumption (1.3) implies that $R(\lambda,A_r)_{|L^r \cap L^s} = R(\lambda,A_s)_{|L^r \cap L^s}$ for $r, s \in [p,q]$ and $\lambda \in \mathbb{C}$ with Re λ large enough. Hence by (1.2) $u \to \log[r(R(k,A_{1/u}))]$ is a convex function for large $k \in \mathbb{N}$. We have

$$\log[(1-\tfrac{1}{k}s(1/u))^{-k}] = k\cdot\log k + k\cdot\log[k-s(1/u)]^{-1} =$$
$$= k\cdot\log k + k\cdot\log[r(R(k,A_{1/u})]^{-1},$$

hence all the functions $u \to \log[(1-\frac{1}{k}s(1/u))^{-k}]$, $k \in \mathbb{N}$, are convex. It follows that $u \to s(1/u) = \lim_{k\to\infty}(\log[(1-\frac{1}{k}s(1/u))^{-k}])$ is convex as well.

□

One can apply the corollary to Schrödinger operators on the spaces $L^p(\mathbb{R}^n)$, i.e., operators $A = \Delta + V$ where Δ is the Laplacian and V is a multiplication operator, see Simon (1982) for details. In Thm. B.5.1 (l.c.) it is shown that for certain potentials V the type is independent of $p \in [1,\infty)$. Thus the assumptions of (a) are satisfied. Part (b) can be applied if $q > 2$ and if A_1 has compact resolvent. Then all operators A_r, $1 \leq r < q$ have compact resolvent and therefore their spectra coincide. In particular, $s(A_r)$ is independent of $r \in [1,q)$.

As shown in A-IV,Ex.1.2(2), the equality $s(A) = \omega(A)$ may not hold for positive semigroups on arbitrary Banach lattices. However, the knowledge of $s(A)$ is still sufficient to determine the growth bound $\omega_1(A)$ of the strong solutions of the abstract Cauchy problem. In fact, combining Theorems 1.1 and 1.2 of C-III with Theorem 1.4 of A-IV we obtain the following fundamental result for the stability of positive semigroups.

Theorem 1.3. Let A be the generator of a positive semigroup $(T(t))_{t\geq 0}$ on a Banach lattice. Then $s(A) = \omega_1(A) \in \sigma(A)$.

Recalling the definition of $\omega_1(A)$ (see A-IV,Def.1.1) and the fact that $s(A)$ is always an element of $\sigma(A)$, we can reformulate the statement of Thm.1.3 as follows.