Let $w > \omega(A)$, λ_1 . Then it follows from (4.9) that $\|(\lambda - w)^n R(\lambda,\bar{B})^n\| \leq \|(\lambda - w)^n R(\lambda,A)^n\|$ for all $\lambda > w$, $n \in \mathbb{N}$. So by the Hille-Yosida theorem, $\bar{B}$ is the generator of a semigroup $(S(t))_{t\geq 0}$. Finally, the domination of $(S(t))_{t\geq 0}$ by $(T(t))_{t\geq 0}$ follows from (4.8) and Prop.4.1.

□

Example 4.4. a) Let E be a σ-order complete complex Banach lattice and $(T(t))_{t\geq 0}$ be a positive semigroup with generator A . Let $M \in Z(E)$ (the center of E (see C-I,Sec.9). For example, if $E = L^p(X,\mu)$ (where (X,μ) is a σ-finite measure space and $1 \leq p \leq \infty$) then M is the multiplication operator defined by a function in $L^\infty(X,\mu)$.
Let $B = A + M$. Then B generates a semigroup $(S(t))_{t\geq 0}$. Assume that $\operatorname{Re}M \leq 0$. Let $f \in D(B)$ and $\phi \in D(A')_+$. Then

$$\begin{aligned} \operatorname{Re}\langle(\operatorname{sign}\bar{f})Bf,\phi\rangle &= \operatorname{Re}\langle(\operatorname{sign}\bar{f})Af,\phi\rangle + \operatorname{Re}\langle(\operatorname{sign}\bar{f})Mf,\phi\rangle \\ &= \operatorname{Re}\langle(\operatorname{sign}\bar{f})Af,\phi\rangle + \operatorname{Re}\langle M|f|,\phi\rangle \\ &\leq \langle|f|,A'\phi\rangle \ . \end{aligned}$$

Thus, by Theorem 4.2, $(S(t))_{t\geq 0}$ is dominated by $(T(t))_{t\geq 0}$.

b) Let E be an order complete complex Banach lattice and B be a regular bounded operator on E . Then B can be written as $B = B_o + M$ where $M \in Z(E)$ and $B_o \in L^r(E)$ such that $\inf\{|B_o|,\mathrm{Id}\} = 0$. Let $A = |B_o| + \operatorname{Re}M$. Then the semigroup $(e^{tB})_{t\geq 0}$ is dominated by $(e^{tA})_{t\geq 0}$.
In fact, let $f \in E$. Then $\operatorname{Re}[(\operatorname{sign}\bar{f})Bf] = \operatorname{Re}[(\operatorname{sign}\bar{f})B_of] + \operatorname{Re}M\cdot|f| \leq |B_o||f| + \operatorname{Re}M\cdot|f| = A|f|$. This implies condition (ii) in Thm. 4.2.

Domination and positivity are characterized simultaneously as follows.

Proposition 4.5. Let E be a σ-order complete real Banach lattice. Let $(T(t))_{t\geq 0}$ be a positive semigroup with generator A and let $(S(t))_{t\geq 0}$ be a semigroup with generator B . The following are equivalent.

(i) $0 \leq S(t) \leq T(t)$ for all $t \geq 0$.
(ii) $\langle P_{(f^+)}Bf,\phi\rangle \leq \langle f^+,A'\phi\rangle$ for all $f \in D(B)$, $\phi \in D(A')_+$.
(iii) $\langle P_{(f^+)}Bf,\phi\rangle \leq \langle f^+,A'\phi\rangle$ for all $f \in D_o$, $\phi \in D(A')_+$, where D_o is a core of B .

Remark 4.6. Condition (ii) implies (4.4) (cf. Remark 3.12).