Proof. Let $u \in D(A)$ be strictly positive. Then there exists $\lambda \in \mathbb{R}$ such that $Au \leq \lambda u$. The operator $B = A - \lambda$ satisfies (P) as well and $Bu \leq 0$. Then by Prop. 1.10, B is p_u-dissipative. Hence B is closable and the closure $\bar{B}$ of B is p_u-dissipative as well (by A-II Prop. 2.9). Then by Prop. 1.10 $\bar{B}$ satisfies (P). Thus A is closable and its closure $\bar{A} = \bar{B} + \lambda$ satisfies (P) as well.

□

Corollary 1.12. Let $A : C(K) \to C(K)$ be linear. If A satisfies (P) then A is bounded and $A + \|A\| Id \geq 0$.

Proof. It follows from Corollary 1.11 that A is closed. Hence A is bounded. Since A satisfies (P), it follows from Thm. 1.3 that $A + \|A\| Id \geq 0$.

□

The next result is a strengthened form of Theorem 2.6. It is somewhat similar to the Lumer-Phillips theorem (A-II, Thm. 2.13). Note that, however, in contrast with the condition of dissipativity, $A + w$ satisfies (P) for any $w \in \mathbb{R}$ whenever (P) holds for A. Thus (P) is not a "metric" condition; that is, it does not imply any norm estimate for the semigroup. We also point out that, if $(T(t))_{t\geq 0}$ is a positive semigroup on $C(K)$, then in general none of the semigroups $(e^{-wt}T(t))_{t\geq 0}$ $(w \in \mathbb{R})$ is contractive (see Batty-Davies (1983) or Derndinger (1983)).

Theorem 1.13. Let A be a densely defined operator on $C(K)$ which satisfies (P). Then

$$\lambda_o := \inf \{\lambda \in \mathbb{R} : Au \leq \lambda u \text{ for some } 0 \ll u \in D(A)\} < \infty .$$

(a) If $(\lambda - A)D(A)$ is dense for some $\lambda > \lambda_o$, then A is closable and the closure $\bar{A}$ of A is the generator of a positive semigroup.
(b) If $\lambda - A$ is surjective for some $\lambda > \lambda_o$, then A is the generator of a positive semigroup.

Proof. It follows from Prop.1.10 that $\lambda_o < \infty$.
Assume that $(\lambda - A)D(A)$ is dense, where $\lambda > \lambda_o$. Let $\lambda_o < \mu < \lambda$ and $B = A - \mu$. Then B satisfies (P) and $Bu \leq 0$ for some strictly positive $u \in D(B) = D(A)$. Thus B is p_u-dissipative by Prop.1.10. Moreover, $((\lambda-\mu) - B)D(B)$ is dense. Thus by A-II,Cor.2.12 the closure $\bar{B}$ of B generates a p_u-contraction semigroup. Hence the