

and 7, for a more detailed discussion and to recall some basic notations here: If μ is a linear form on $C_0(X, \mathbb{R})$ then

$\mu \geq 0$ means $\mu(f) \geq 0$ for all $f \geq 0$; μ is then called positive (positivity automatically implies continuity),
 $\mu > 0$ means that $\mu \geq 0$, but μ does not vanish identically,
 $\mu \gg 0$ means that $\mu(f) > 0$ for any $f > 0$; μ is then called strictly positive.

If μ is a linear form on $C_0(X, \mathbb{C})$, then μ can be written uniquely as $\mu = \mu_1 + i\mu_2$ where μ_1 and μ_2 map $C_0(X, \mathbb{R})$ into \mathbb{R} (decomposition into real and imaginary parts). We call μ positive (strictly positive) and use the above notations if $\mu_2 = 0$ and μ_1 is positive (strictly positive). We point out that strictly positive linear forms need not exist on $C_0(X)$, but if X is separable then a strictly positive linear form is obtained by summing a suitable sequence of point measures.

The coincidence of the notions of a closed algebraic and a closed lattice ideal in $C_0(X)$ has of course its effect on the algebraic and the lattice theoretic notions of a homomorphism. The case of a homomorphism into another space $C_0(Y)$ will be discussed below. As for homomorphisms into the scalar field, we have essentially coincidence between the algebraic and the order theoretic meaning of this word, more exactly: A linear form $\mu \neq 0$ on $C_0(X)$ is a lattice homomorphism if and only if μ is, up to normalization, an algebra homomorphism (algebra homomorphisms $\neq 0$ must necessarily have norm 1). Since the algebra homomorphisms $\neq 0$ on $C_0(X)$ are known to be the point measures (denoted by δ_t) on X and since on the other hand μ is a lattice homomorphism if and only if $|\mu(f)|$ equals $\mu(|f|)$ for all f , it follows that this latter condition on μ is equivalent to $\mu = \alpha \delta_t$ for a suitable t in X and a positive real number α . This can be summarized by saying that X can be canonically identified, via the map $t \rightarrow \delta_t$, with the subset of the dual $C_0(X)'$ consisting of the non-zero algebra homomorphisms, which is also the set of all normalized lattice homomorphisms. This identification is a topological isomorphism with respect to the weak*-topology of $C_0(X)'$.