positive linear. From (3.5) and (3.6) one deduces that
$(KR(\lambda,A)f)(x,v) = \int_0^t \int_0^t k(x,v,x',v')f(x',v')\,dx'dv'$
where the kernel k is given by $k(x,v,x',v') := \kappa(x,v,v')r_\lambda(x,x',v')$ (cf. (3.5), (3.7)). Using this representation of $KR(\lambda,A)$ it follows that K is A-compact. Moreover for λ sufficiently large one has $R(\lambda,A-M) = R(\lambda,A)(1 - MR(\lambda,A))^{-1}$ which shows that K is also (A-M)-compact. In order to apply Thm.3.14 one needs $s(B) > s(A-M)$ (see Prop.3.17) which is difficult to verify. In case the function σ is continuous one can state a sufficient condition as follows:

There exist $r \in \mathbb{R}$ and $g \in L^1([0,1]\times[-1,1])$, $g > 0$ such that $r < \inf\{\sigma(x,0) : x \in [0,1]\}$ and $Bg \geq -rg$.

The additional assumption made in the second part of Prop.3.17 is not satisfied in this example. Nevertheless one can show that $s(B)$ is strictly dominant in this situation (provided that $s(B) > s(A)$). For details we refer to Greiner (1984d) or Voigt (1985) where the linear transport equation in higher dimensional spaces is discussed.

4. SEMIGROUPS OF LATTICE HOMOMORPHISMS

In Section 2 we proved that the boundary spectrum of certain positive semigroups is a cyclic set. For semigroups of lattice homomorphisms much more can be said: The whole spectrum is an imaginary additively cyclic subset of $\mathbb{C}$ (cf. Thm.4.2). This result can be used to derive cyclicity results for the eigenvalues in the boundary spectrum of positive semigroups (cf. Cor.4.3). In the last part of this section we discuss a spectral decomposition of positive groups (cf. Thm.4.10).

Lemma 4.1. Suppose that $(T(t))_{t\geq 0}$ is a semigroup of lattice homomorphisms on a Banach lattice E with generator A.
In case $i\alpha \in R\sigma(A)$, $\alpha \in \mathbb{R}$, then one of the following assertions is true:

(a) $i\alpha\mathbb{Z} \subset R\sigma(A)$;

(b) $\{\lambda \in \mathbb{C} : \operatorname{Re}\lambda < 0\} \subset R\sigma(A)$.

Proof. There exists $\phi \in E'$, $\phi \neq 0$ such that $T(t)'\phi = e^{i\alpha t}\phi$ $(t \geq 0)$. Then we have $|\phi| = |T(t)'\phi| \leq T(t)'|\phi|$ $(t \geq 0)$.
If we fix $r > \omega(T)$ and define $\psi := rR(r,A)'|\phi|$, we have

$$(4.1)\quad T(t)'\psi \leq e^{rt}\psi\ ,\ T(t)'\psi \geq \psi\ (t \geq 0)\ \text{ and }\ |\phi| \leq \psi\ .$$