Now observe that m is admissible if and only if $a' = -\infty$ and $b' = \infty$.

If $a' = -\infty$ and $b' = \infty$, then B is the generator of the translation group on $C_o(\mathbb{R})$. Hence also δ_m is the generator of a group $(T(t))_{t\in\mathbb{R}}$ on $C_o(a,b)$.

Conversely, assume that B generates a group $(T(t))_{t\in\mathbb{R}}$. Assume that $a' > -\infty$. Then $C_o(a',b')$ is a closed subspace of $C_o[a',b')$. Let

$$(T_1(t)f)(x) = \begin{cases} f(x+t) & \text{for } x+t < b' \\ 0 & \text{for } x+t \geqq b' \end{cases}$$

for all $f \in C_o[a',b')$, $x \in [a',b')$, $t \geqq 0$. Then $(T_1(t))_{t\geqq 0}$ is a semigroup on $C_o[a',b')$ with generator B_1 given by $B_1 f = f'$ with domain $D(B_1) = \{ f \in C_o[a',b') \cap C^1(a',b) : \lim_{x\to b'} f'(x) = 0 \}$. If we consider B as an operator on $C_o[a',b')$, then $B \subset B_1$. Let $f \in D(B)$. Then $u(t) := T(t)f \in D(B) \subset D(B_1)$ for all $t \geqq 0$; and $\dot{u}(t) = Bu(t) = B_1u(t)$; $u(0) = f$. It follows from A-I,Thm.1.7. (or A-II,Cor1.2.) that $T_1(t)f = u(t)$. Hence $T_1(t)f \in C_o(a',b')$ for all $t \geqq 0$ and $f \in D(B)$. This is impossible since $a' > -\infty$. Similary one shows that $b' = \infty$.

□

Proof of Theorem 3.17. Suppose that m is admissible. It is easy to see that (3.22) then defines a continuous flow on (a,b). Moreover, for every $x \in (a,b)$ the function $\phi(\cdot,x)$ is differentiable and satisfies

(3.23) $\frac{\partial}{\partial t}\phi(t,x) = m(\phi(t,x)) \quad (x \in (a,b)\ ,\ t \in \mathbb{R})$.

Denote by $(T(t))_{t\in\mathbb{R}}$ the group on $C_o(a,b)$ given by $T(t)f = f\circ\phi_t$ ($t \in \mathbb{R}$, $f \in C_o(a,b)$) and let A be its generator. Take $g \in C_o(a,b)$ and $f = R(1,A)g$. Then $f(x) = \int_0^\infty e^{-t}g(\phi(t,x))dt$, $x \in (a,b)$. If $m(x) = 0$ then $f(x) = \int_0^\infty e^{-t}g(x)dt = g(x)$. If $x \in (a_n,b_n)$ $(n \in J)$, then $f(x) = \int_0^\infty e^{-t}g(q_n^{-1}(q_n(x) + t))dt = e^{q_n(x)} \int_{q_n(x)}^\infty e^{-s}g(q_n^{-1}(s))ds$.

Thus f is differentiable in x and $f'(x) = (1/m(x))(f(x) - g(x))$. Consequently $f \in D(\delta_m)$ and $\delta_m f = f - g$. This shows that $A \subset \delta_m$. In order to show the converse inclusion, let $f \in D(\delta_m)$ and $g = f - \delta_m(f) \in C_o(a,b)$. Then $R(1,A)g(x) = f(x)$ if $m(x) = 0$ and $R(1,A)g(x) = \int_0^\infty e^{-t}f(\phi(t,x))dt - \int_0^\infty e^{-t}m(\phi(t,x))f'(\phi(t,x))dt$
$= \int_0^\infty e^{-t}f(\phi(t,x))dt - \int_0^\infty e^{-t}\frac{\partial}{\partial t}f(\phi(t,x))dt$ (by (3.23))
$= f(x)$ by integrating by parts. This shows that $f = R(1,A)g \in D(A)$ and that δ_m is the generator of the group $(T(t))_{t\in\mathbb{R}}$.

Finally we show that $T(t)D_o(\delta_m) \subset D_o(\delta_m)$, which implies that $D_o(\delta_m)$ is a core (by A-II,Cor 1.34). Let $t \in \mathbb{R}$. The function $\phi_t(\cdot)$ is