the closed linear span of all eigenvalues of A pertaining to the eigenvalues in $i\mathbb{R}$. Moreover, the dual group of K can be identified with the subgroup of $i\mathbb{R}$ generated by $P\sigma(A) \cap i\mathbb{R}$. We call Q the <u>semigroup projection associated with</u> T. On the other hand, T is always strongly ergodic with projection P onto $\mathrm{Fix}(T)$. Obviously, the relation

$$0 \leqq P \leqq Q \leqq \mathrm{Id}$$

holds, where the order relation is defined by the inclusion of the range spaces.

There are two extreme cases: First $Q = \mathrm{Id}$ and $\mathrm{rank}(P) = 1$. This corresponds to the Halmos-von Neumann Theorem in commutative ergodic theory and is discussed, at least for irreducible semigroups, in [Olesen-Pedersen-Takesaki (1980)]. Second, $\mathrm{Id} > Q = P$, in particular $\mathrm{rank}(P) = 1$. This latter case will be investigated in detail for $M = B(H)$, the W*-algebra of all bounded linear operators on a Hilbert space H. But we first need some preparations.

<u>Theorem</u> 3.5. Let T be an identity preserving semigroup of Schwarz type on the predual of a W*-algbra M and suppose there exists a faithful family of T-invariant states on M. Let N be the $\sigma(M,M_*)$-closed linear span of all eigenvectors of A' pertaining to the eigenvalues in $i\mathbb{R}$. If Q is the semigroup projection associated with T the following holds:

(a) The adjoint of Q is a faithful normal conditional expectation from M onto the W*-subalgebra N.

(b) The restriction of T' to N can be embedded into a $\sigma(M,M_*)$-continuous, one-parameter group of *-automorphisms.

(c) If, in addition, T is irreducible and if ϕ is the normal state generating the fixed space of T, then $\phi_{|N}$ is a faithful normal trace.

<u>Proof</u>. Consider $H := P\sigma(A) \cap i\mathbb{R}$ which is not empty by assumptions. From Proposition 3.1 it follows that T is relatively compact in the weak operator topology. Let K be the semigroup kernel of $\bar{T} \subseteq L_w(M_*)$ and Q the unit of K. Recall that $Q\psi_\eta = \psi_\eta$ for all $\psi_\eta \in M_*$ such that $A\psi_\eta = \eta\psi_\eta$ $(\eta \in H)$. Let U be the family of all