An operator A on E is called [strictly] dispersive if A is [strictly] N^+-dissipative; that is, for every $f \in D(A)$ one has $\langle Af,\phi\rangle \leq 0$ for some [resp., all] $\phi \in dN^+(f)$ (see A-II,Sec.2). Generators of positive contraction semigroups are characterized by the following theorem which is due to Phillips (1962).

Theorem 1.2. Let A be a densely defined operator on a real Banach lattice E. The following assertions are equivalent.

(i) A is the generator of a positive contraction semigroup.

(ii) A is dispersive and $(\lambda-A)$ is surjective for some $\lambda > 0$.

Frequently an operator is known explicitly only on a core. In that case one can use the following result.

Corollary 1.3. Let A be a densely defined dispersive operator on a real Banach lattice E. If $(\lambda - A)D(A)$ is dense in E for some $\lambda > 0$, then A is closable and the closure $\bar{A}$ of A is the generator of a positive contraction semigroup.

Theorem 1.2 and Corollary 1.3 immediately follow from A-II,Thm.2.11 and A-II,Cor.2.12 if one observes the following.

Lemma 1.4. A bounded linear operator T on a Banach lattice E is a positive contraction if and only if $\|(Tf)^+\| \leq \|f^+\|$ for all $f \in E$ (i.e., if T is N^+-contractive).

Proof of the lemma. If T is a positive contraction, then $0 \leq (Tf)^+ \leq Tf^+$ and so $N^+(Tf) \leq \|Tf^+\| \leq \|f^+\| = N^+(f)$ for all $f \in E$. Conversely, assume that T is an N^+-contraction. Let $f \geq 0$. Then $\|(Tf)^-\| = N^+(T(-f)) \leq N^+(-f) = \|f^-\| = 0$. Hence $(Tf)^- = 0$; i.e., $Tf \geq 0$. We have proved that T is positive. In particular, $|Tf| \leq T|f|$ for all $f \in E$. Hence $\|Tf\| = \||Tf|\| \leq \|T|f|\| = N^+(T|f|) \leq N^+(|f|) = \|f\|$ for all $f \in E$. So T is a contraction.

□

Examples 1.5. a) Consider the second derivative with Dirichlet boundary condition on $E = C_o(0,1)$; i.e., we let $Af = f''$ with domain $D(A) = \{f \in C^2[0,1] : f(0) = f(1) = f''(0) = f''(1) = 0\}$.
A is dispersive. In fact, let $f \in D(A)$. Then there exists $x \in (0,1)$ such that $f(x) = \sup_{y\in[0,1]} f(y) = \|f^+\|_\infty$. Thus $\delta_x \in dN^+(f)$. But $\langle Af,\delta_x\rangle = f''(x) \leq 0$ since f has a maximum in x.
Let $g \in E$. Define $f_o(x) = 1/2\ [e^x \int_x^1 e^{-y}g(y)dy - e^{-x} \int_x^1 e^y g(y)dy]$.