Remark 1.3. Take S and T as in Lemma 1.2 (b) . If V_{u^*} (resp. V_u) is the map $(x \to xu^*)$ (resp. $(x \to xu)$) on M , then V_{u^*} is a continuous bijection from $Ms(|\phi|)$ onto $Ms(|\phi^*|)$ with inverse V_u (because $V_u \circ V_u{}^* = Id_{Ms(|\phi|)}$ and $V_u{}^* \circ V_u = Id_{Ms(|\phi^*|)}$). Let $x \in M$. From $T(xu) = S(x)u$ we obtain $T(xu)u^* = S(x)uu^*$. In particular, if $Ms(|\phi^*|)$ is S-invariant, then

$$(V_{u^*} \circ T \circ V_u)(x) = T(xu)u^* = S(x) \quad .$$

for every $x \in Ms(|\phi^*|)$. Let $T_|$ (resp. $S_|$) be the restriction of T to $Ms(|\phi|)$ (resp. of S to $Ms(|\phi^*|)$) . Then the following diagram is commutative :

$$\begin{tikzcd} Ms(|\phi|) \arrow[rr, "T_|"] & & Ms(|\phi|) \arrow[d, "V_{u^*}"] \\ Ms(|\phi^*|) \arrow[u, "V_u"] & & Ms(|\phi^*|) \arrow[ll, "S_|"] \end{tikzcd}$$

In particular, $\sigma(S_|) = \sigma(T_|)$. Therefore we may deduce spectral properties of $S_|$ from $T_|$ and vice versa. More concrete applications of Lemma 1.2. will follow.

We now investigate the fixed space $Fix(R) := Fix(\lambda R(\lambda))$, $\lambda \in D$, of a pseudo-resolvent R with values in the predual of a W*-algebra M .

Proposition 1.4. Let R be a pseudo-resolvent on $D = \{\lambda \in \mathbb{C} : Re(\lambda) > 0\}$ with values in the predual M_* of a W*-algebra M and suppose R to be identity preserving and of Schwarz type.

(a) If $\alpha \in \mathbb{R}$ and $\psi \in M_*$ such that $(\gamma - i\alpha)R(\gamma)\psi = \psi$ for some $\gamma \in D$, then $\lambda R(\lambda)|\psi| = |\psi|$ and $\lambda R(\lambda)|\psi^*| = |\psi^*|$ for all $\lambda \in D$.

(b) $Fix(R)$ is invariant under the involution in M_* . If $\psi \in Fix(R)$ is self adjoint, then the positive part ψ^+ and the negative part ψ^- of ψ are elements of $Fix(R)$.