Here $Bf := -f'$ on $D(B) = \{f \in L^1[\alpha,1] : f \in AC[\alpha,1],\ f(\alpha) = 0\}$ and $L : F \to F$ is defined by

$$Lf(x) = \begin{cases} k(x)f(2x-r) & \text{if } x \in [\alpha, 1/2(r+1)]\ , \\ 0 & \text{if } x \in (1/2(r+1), 1]\ , \end{cases}$$

where $k \in C[\alpha,1]$.

It is easy to verify that L is positive and bounded, and that B is the generator of the positive semigroup $(S(t))_{t\geq 0}$ defined by

$$[S(t)f](x) = \begin{cases} f(x-t) & \text{if } x-t \geq \alpha \\ 0 & \text{if } x-t < \alpha \end{cases} \qquad (x \in [\alpha,1]).$$

Furthermore B has compact resolvent. Define $\Phi f := L(f(-r))$ for $f \in E$ such that (3.12) can be written as retarded Cauchy problem (RCP).

As before (see Formula (3.3)) Φ_λ is defined by $\Phi_\lambda x := \Phi(\varepsilon_\lambda \otimes x)$ for $x \in F$. Gyllenberg and Heijmans have shown that $s(B + \Phi_\lambda) > -\infty$. Thus we can apply Thm.3.7 and obtain that $s(A) = \lambda$ if and only if $\lambda = s(B + \Phi_\lambda)$.

NOTES.

Section 1. The coincidence of spectral bound and growth bound for positive semigroups on $C(K)$ was first observed by Derendinger (1980) and then generalized to $C_0(X)$ and non-commutative C^*-algebras by Batty-Davies (1982) and Groh-Neubrander (1981). The stability theorem 1.1 is a continuous version of a result of Choquet-Foias (see Schaefer (1974), V.8.8).

Section 2. For the Riesz-Schauder Theory of compact operators we refer to Dunford-Schwartz (1958), Sec.VII.4 and Pietsch (1978), Sec.26 . Theorem 1.1 seems to be folklore. Prop 2.3 is due to Grothendieck (1953) and can be found in Sec. II.9 of Schaefer (1974). Proposition 2.4 is due to Dieudonné (see §3 of Grothendieck (1953) and Schaefer (1974), II.Exc. 27). The notion 'strong Feller property' used in Theorem 2.5 is due to Girsanov (see Dynkin (1965)) while the theorem itself was proven by Davies (1982). It is well known that there is a close relationship between Markov processes and Markov semigroups. A description of this relation more detailed than Example 2.6 can be found e.g. in Dynkin (1965), in Chap.2 of van Casteren (1985) or in Chap.7 of Lamperti (1977). The notion "quasi-compact" for a single operator dates back to Eberlein (1949) (see also Yosida-Kakutani (1941) and Sec.26.4 of Pietsch (1978)). Quasi-compactness for strongly continuous semigroups and its relationship to uniform ergodicity is investigated in Lin (1975). Proposition 2.9 is due to Voigt (1980), a special case was proven by Vidav (1970). Corollaries 2.2 and 2.11 can be found in Greiner (1984). The criterion stated in (2.12) is known as 'Doeblin's condition' (see e.g. Yosida-Kakutani (1941)). It is sufficient and