PART D

POSITIVE SEMIGROUPS ON C* - AND W*-ALGEBRAS

by
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CHAPTER D-I

BASIC RESULTS ON SEMIGROUPS AND OPERATOR ALGEBRAS

This is not a systematic introduction into the theory of strongly continuous semigroups on C*- and W*-algebras. For that we refer to Bratteli-Robinson (1979), Davies (1976) and the survey article of Oseledets (1984). We only prepare for the subsequent chapters on spectral theory and asymptotics by fixing the notations and introducing some standard constructions.

1. NOTATIONS

1. By M we shall denote a C*-algebra with unit 1. $M^{sa} := \{x \in M : x^*=x\}$ is the self-adjoint part of M and $M_+ := \{ x^*x : x \in M \}$ the positive cone in M. If M' is the dual of M, then $M'_+ := \{\psi \in M' : \psi(x) \geq 0, x \in M_+ \}$ is a weak*-closed generating cone in M'. $S(M) := \{\psi \in M'_+ : \psi(1) = 1 \}$ is called the state space of M. For the theory of C*-algebras and related notions we refer to [Pedersen (1979)]. M is called a W*-algebra, if there exists a Banach space M_*, such that its dual $(M_*)'$ is (isomorphic to) M. We call M_* the predual of M and $\psi \in M_*$ a normal linear functional. It is known that M_* is unique [Sakai (1971), 1.13.3.]. For further properties of M_* we refer to [Takesaki (1979), Chapter III].

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