Hence $0 \leq \lambda Pf < -PAPf$. This implies that $Pf \neq 0$ and $\lambda\|Pf\| \leq \|PAPf\| \leq \|A\|\cdot\|Pf\|$. Consequently $\lambda \leq \|A\|$.

□

Remark 1.12. It follows from the proof of Theorem 1.11 that on a σ -order complete Banach lattice condition (1.9) is equivalent to the positivity of the semigroup $(e^{tA})_{t\geq 0}$.

Examples 1.13. Let $E = \ell^p$ $(1 \leq p \leq \infty)$ or $E = c_o$.

a) An operator $A \in L(E)$ can be canonically represented by a matrix (a_{ij}) . It follows from Thm. 1.11 that $e^{tA} \geq 0$ for all $t \geq 0$ if and only if $a_{ij} \geq 0$ whenever $i \neq j$.

b) Let A be the generator of a strongly continuous contraction semigroup $(T(t))_{t\geq 0}$ on E . Suppose that the space c_{oo} of all sequences which vanish off a finite set is a core of A . Let $(a_{nm})_{m\in\mathbb{N}} = (Ae_n)$ where $e_n = (\delta_{nm})_{m\in\mathbb{N}}$ denotes the n^{th} unit vector. Then it follows from Thm.1.8 that the semigroup is positive if and only if $a_{nm} \geq 0$ whenever $n \neq m$.

2. KATO's INEQUALITY

A strongly continuous semigroup on $C(K)$ (K compact) or a norm continuous semigroup on an arbitrary Banach lattice is positive if and only if its generator A satisfies the positive minimum principle (P). However, we will see that in general (P) is not sufficient for the positivity of the semigroup. One reason seems to be that (P) involves merely positive elements in $D(A)$ but $D(A)_+$ can be small if the semigroup is not positive (cf. Remark 3.16).

Our aim in this section is to find a different condition on the generator which is necessary for the positivity of the semigroup.

We recall from Chapter C-I,Sec.8 definition and properties of the signum operator.

Proposition 2.1. Let E be a σ-order complete (real or complex) Banach lattice. For every $f \in E$ there exists a unique linear operator (sign f) on E which satisfies