<u>Example</u> 1.16. Let $A = (a_{ij})$ be an $n\times n$-matrix such that $a_{ij} \geq 0$ whenever $i \neq j$ (see Example 1.4b). Then by Corollary 1.13, $s(A) = \inf\{\lambda \in \mathbb{R} : Au \leq \lambda u$ for some strictly positive $u\} = \inf_{u>>0} \inf\{\lambda \in \mathbb{R} : Au \leq \lambda u\} = \inf\{\sup_i \sum_{j=1}^{n} a_{ij}\, u_j/u_i : u >> 0\}$. This formula is due to Collatz (1942) (see also [Schaefer (1974), Chap, Exercise 20] and Wielandt (1950)).

<u>Corollary</u> 1.17. Let $(T(t))_{t\geq 0}$ be a strongly continuous positive semigroup on $C(K)$. Then $T(t)u >> 0$ for all $u >> 0$, $t \geq 0$.

<u>Proof</u>. Denote by A the generator of $(T(t))_{t\geq 0}$. Then by the proof of Thm. 1.13 there exist $u >> 0$ and $\lambda \in \mathbb{R}$ such that $A - \lambda$ is p_u-dissipative. This implies that $p_u(T(t)f) \leq e^{\lambda t} p_u(f)$. Observing that $f >> 0$ if and only if $p_u(-f) < 0$ the claim follows.

□

<u>Remark</u> 1.18. Corollaries 1.14 and 1.17 do not hold on $C_o(X)$. For example, let $X = [0,1)$ and

$$(T(t)f)(x) = \begin{cases} f(x+t) & \text{if } x+t \leq 1 \\ 0 & \text{if } x+t > 0 . \end{cases}$$

Then $(T(t))_{t\geq 0}$ is a positive semigroup on $C_o(X)$ and $T(t) = 0$ for all $t \geq 1$. The generator A of $(T(t))_{t\geq 0}$ has empty spectrum, so that (1.7) is violated. However, it is still true that $s(A) = \omega(A)$ for generators of positive semigroups on $C_o(X)$ (see B-IV,Thm.1.4).

<u>Remark</u> 1.19. So far, the results of this section do not depend on the lattice structure of $C(K)$. They also hold on an ordered Banach space E with normal cone E_+ which has non-empty interior. We refer to Arendt-Chernoff-Kato (1982) and to Batty-Robinson (1984) for this more general setting.

Next we apply Theorem. 1.13 to prove a result on the multiplicative perturbation of a generator A which is due to Dorroh (1966) in the case when A is dissipative.

<u>Theorem</u> 1.20. Let A be the generator of a positive semigroup on $C(K)$ and $m \in C(K)$ be strictly positive. Then the operator $m\cdot A$ given by $(m\cdot A)f = m\cdot(Af)$ on the domain $D(m\cdot A) = D(A)$ is the generator of a positive semigroup. Moreover,

$$(1.10) \qquad \|m^{-1}\|_\infty^{-1}\, \omega(A) \leq \omega(m\cdot A) \leq \|m\|_\infty\, \omega(A) .$$