$\phi_o := P_*\phi$ and let $\{p_i : i\in\Delta\}$ be an increasing net of projections of finite rank in $B(H)$ with strong limit 1. Since the set $K := \{\phi_t : t\geq 0\}$ is relatively compact in the $\sigma(B(H)_*, B(H))$-topology, there exists for every $\delta>0$ an index $i_o\in\Delta$ such that

$$\|(1 - p_i)\psi(1 - p_i)\| \leq \delta$$

for every $\psi\in K$ and $i\geq i_o$ [Takesaki (1979), Theorem III.5.4.(vi)]. In particular

$$|\psi(1 - p_i)| \leq \delta \quad , \ \psi\in K,\ i(o)\leq i \ .$$

Let $p := p(i(o))$. Then for all x in the unit ball of M it follows that

$$\begin{aligned}
&|(\phi_t - \phi_o)(x)| \leq \\
&\leq |(\phi_t - \phi_o)(pxp)| + |(\phi_t - \phi_o)((1-p)xp)| + \\
&\quad + |(\phi_t - \phi_o)(x(1-p))| \leq \\
&\leq |(\phi_t - \phi_o)(pxp)| + 4\sqrt{\delta} \ .
\end{aligned}$$

Since the W*-algebra $pB(H)p$ is finite dimensional, there exists $U\in\mathcal{U}$ such that

$$\|(\phi_t - \phi_o)_{|pB(H)p}\| \leq \delta$$

for all $t\in U$. Consequently

$$\|(\phi_t - \phi_o)\| \leq (\delta + 4\sqrt{\delta})$$

for all $t\in U$. Therefore $\lim_{\mathcal{U}} T(t)_*\phi = P_*\phi$ in the strong operator topology. Since the positive cone of $B(H)_*$ is generating, the assertion is proved.

□

For irreducible W*-dynamical systems on $B(H)$ the above properties always hold.

<u>Theorem</u> 3.8. Let $(B(H),\Phi,\mathcal{T})$ be an irreducible W*-dynamical system. Then

$$P\sigma(A) \cap i\mathbb{R} = \{0\} \ .$$