(2.1) $p(f+g) \leq p(f) + p(g)$ $(f, g \in E)$

(2.2) $p(\lambda f) = \lambda p(f)$ $(f \in E, \lambda \geq 0)$.

The continuity of $p$ implies that there exists a constant $c > 0$ such that

(2.3) $|p(f)| \leq c\|f\|$ $(f \in E)$.

Moreover, it follows from (2.1) and (2.2) that

(2.4) $p(f) + p(-f) \geq p(0) = 0$ $(f \in E)$.

A bounded operator $T$ on $E$ is called p-contractive if $p(Tf) \leq p(f)$ for all $f \in E$. Similarly, a semigroup $(T(t))_{t\geq 0}$ is called p-contractive if $T(t)$ is p-contractive for all $t \geq 0$.
Of course, the most important case we have in mind in this section is the case when $p$ is the norm function $N$ given by $N(f) = \|f\|$ $(f \in E)$. An N-contractive operator is just a contraction in the usual sense.

Remark. However in Chapter B-II and C-II it will be important to dispose of a variety of sublinear functionals other than $N$. For example, we will consider $N^+$ on $C[0,1]$ given by $N^+(f) = \sup_{x\in[0,1]} f(x)$. Then a bounded operator $T$ is $N^+$-contractive if and only if $T$ is positive and $\|T\| \leq 1$.

We first want to solve the following problem. Given the generator $A$ of a semigroup $(T(t))_{t\geq 0}$ find a condition on $A$ which is equivalent to $T(t)$ being p-contractive for all $t \geq 0$.

The subdifferential $dp$ of $p$ in $f$ is defined by

(2.5) $$dp(f) = \{\phi \in E' : \langle g,\phi\rangle \leq p(g) \text{ for all } g \in E, \ \langle f,\phi\rangle = p(f)\}.$$

It follows from the Hahn-Banach theorem that $dp(f) \neq \emptyset$ for all $f \in E$.

Definition 2.1. An operator $A$ on $E$ is called p-dissipative if for all $f \in D(A)$ there exists $\phi \in dp(f)$ such that $\langle Af,\phi\rangle \leq 0$; $A$ is called strictly p-dissipative if for all $f \in D(A)$ the inequality $\langle Af,\phi\rangle \leq 0$ holds for all $\phi \in dp(f)$.