2. A map $T \in L(M)$ is called <u>positive</u> (in symbols $T \geq 0$) if $T(M_+) \subseteq M_+$. $T \in L(M)$ is called <u>n-positive</u> ($n \in \mathbb{N}$) if $T \otimes Id_n$ is positive from $M \otimes M_n$ in $M \otimes M_n$, where Id_n is the identity map on the C*-algebra M_n of all n×n-matrices. Obviously, every n-positive map is positive. We call $T \in L(M)$ a <u>Schwarz map</u> if T satisfies the inequality

$$T(x)T(x)^* \leq T(xx^*) \quad , \; x \in M \; .$$

Note that such T is necessarily a contraction. It is well known that every n-positive contraction, $n \geq 2$ and that every positive contraction on a commutative C*-algebra is a Schwarz map [Takesaki (1979), Corollary IV. 3.8.]. As we shall see, the Schwarz inequality is crucial for our investigations.

3. If M is a C*-algebra we assume $\mathcal{T} = (T(t))_{t \geq 0}$ to be a <u>strongly continuous semigroup</u> (abbreviated <u>semigroup</u>) while on W*-algebras we consider <u>weak*-semigroups</u>, i.e. the mapping $(t \to T(t)x)$ is continuous from $\mathbb{R}_+$ into $(M, \sigma(M,M_*))$, M_* the predual of M, and every $T(t) \in \mathcal{T}$ is $\sigma(M,M_*)$-continuous. Note that the preadjoint semigroup

$$\mathcal{T}_* = \{ \, T(t)_* \; : \; T(t) \in \mathcal{T} \, \}$$

is weakly, hence strongly continuous on M_* (see e.g., Davies (1980), Prop.1.23). We call $\mathcal{T}$ <u>identity preserving</u> if $T(t)1 = 1$ and of <u>Schwarz type</u> if every $T(t) \in \mathcal{T}$ is a Schwarz map.

For the notations concerning one-parameter semigroups we refer to Part A. In addition we recommend to compare the results of this section of the book with the corresponding results for commutative C*-algebras, i.e. for $C_o(X)$, $C(K)$ and $L^\infty(\mu)$ (see Part B).

2. A FUNDAMENTAL INEQUALITY FOR THE RESOLVENT

If $\mathcal{T} = (T(t))_{t \geq 0}$ is a strongly continuous semigroup of Schwarz maps on a C*-algebra M (resp. a weak*-semigroup of Schwarz type on a W*-algebra M) with generator A, then the spectral bound $s(A) \leq 0$. Then for $\lambda \in \mathbb{C}$, $\mathrm{Re}(\lambda) > 0$, there exists a representation for the resolvent $R(\lambda,A)$ given by the formula

$$R(\lambda,A)x = \int_0^\infty e^{-\lambda t} \, T(t)x dt \quad , \quad x \in M$$

where the integral exists in the norm topology.