In the following we write $E \otimes_\alpha F$ for the tensor product of E and F endowed - if applicable - with one of the norms π, ε, h just defined. In each case one has $\|f \otimes g\| = \|f\|\|g\|$ for $f \in E$, $g \in F$. By $E \tilde{\otimes}_\alpha F$ we mean the completion of $E \otimes_\alpha F$. Moreover we recall how examples 1 and 2 above fit into this pattern:

$$L^1(\mu \otimes \nu) = L^1(\mu) \tilde{\otimes}_\pi L^1(\nu) , \quad L^2(\mu \otimes \nu) = L^2(\mu) \tilde{\otimes}_h L^2(\nu) ,$$

$$C(X \otimes Y) = C(X) \tilde{\otimes}_\varepsilon C(Y) .$$

Finally we point out that for any $S \in L(E)$, $T \in L(F)$, the mapping

$$\textstyle\sum_{i=1}^n f_i \otimes g_i \to \sum_{i=1}^n Sf_i \otimes Tg_i$$

defined on $E \otimes F$ is linear and continuous on $E \otimes_\alpha F$, hence has a continuous extension to $E \tilde{\otimes}_\alpha F$. This operator, as well as its continuous extension, will be denoted by $S \otimes T$ and satisfies $\|S \otimes T\| = \|S\|\|T\|$. The notation $A \otimes B$ will also be used in the obvious way if A and B are not necessarily bounded operators on E and F. We are now ready to consider semigroups induced on tensor product.

<u>Proposition</u>. Let $(S(t))_{t\geq 0}$ and $(T(t))_{t\geq 0}$ be strongly continuous semigroups on Banach spaces E, F, and let A, B be their generators. Then the family

$$(S(t) \otimes T(t))_{t\geq 0}$$

is a strongly continuous semigroup on $E \tilde{\otimes}_\alpha F$.
The closure of

$$A \otimes Id + Id \otimes B ,$$

defined on the core $D(A) \otimes D(B)$ is its generator.

<u>Proof</u>. It is immediately verified that $(S(t)\otimes T(t))_{t\geq 0}$ is in fact a semigroup of operators on $E \tilde{\otimes}_\alpha F$. The strong continuity need only be verified at $t = 0$ and on elements of the form $u = f \otimes g \in E \otimes F$. This verification being straightforward, there remains to show that the generator of $(S(t)\otimes T(t))_{t\geq 0}$ is obtained as the closure of $(A \otimes Id + Id \otimes B, D(A) \otimes D(B))$. To this end, let $f \in D(A)$ and $g \in D(B)$. Then $\lim_{h\to 0} \frac{1}{h}(T(h) \otimes S(h)(f \otimes g) - f \otimes g)$

$$= \lim_{h\to 0} \frac{1}{h}(T(h)f \otimes (S(h)g-g) + (T(h)f-f) \otimes g)$$

$$= (f \otimes Bg) + (Af \otimes g) .$$

Since the elements of the form $f \otimes g$, $f \in D(A)$, $g \in D(B)$, generate the linear subspace $D(A) \otimes D(B)$ of $E \tilde{\otimes}_\alpha F$, this subspace belongs