closed subsets of $\partial\Omega$. On $E = L^p(\Omega)$ $(1 \leq p < \infty)$ we consider a differential operator $L_{p,o}$ which is defined as follows:

(2.23) $L_{p,o}f := \sum_{i,j=1}^n a_{ij}\, f''_{ij} + \sum_{i=1}^n b_i\, f'_i + cf$, with domain
$D(L_{p,o}) := \{f \in C^2(\bar{\Omega}) : f(x) = 0 \text{ for } x \in \Gamma_o \text{ and}$
$\partial f/\partial\nu\,(x) + \gamma(x)f(x) = 0 \text{ for } x \in \Gamma_1\}$

Here f'_i stands for $\partial f/\partial x_i$, thus $f''_{ij} = \partial^2 f/\partial x_i \partial x_j$. We assume that $L_{p,o}$ is elliptic and that all coefficients are real-valued satisfying $a_{ij} \in C^2(\bar{\Omega})$, $b_i \in C^1(\bar{\Omega})$, $\gamma \in C^1(\bar{\Omega})$, $c \in C^1(\bar{\Omega})$.
Then $L_{p,o}$ is closable and its closure L_p is the generator of a holomorphic semigroup of positive operators. Moreover, the resolvent is compact. Thus Cor.2.13 is applicable and one obtains that $s(A)$ is strictly dominant provided that $\sigma(A) \neq \emptyset$. Using the results of Section 3 one can show that $\sigma(A) \neq \emptyset$ and that $s(A)$ is an algebraically simple eigenvalue (see Thm.3.7 and Prop.3.5).

We conclude with some remarks.

Remarks 2.15.(a) In the proof of Thm.2.10 we did not use the assumption that R is the resolvent of a semigroup. In fact one can state this theorem for closed operators having positive resolvent. In this case one has to assume that $\{(\lambda-s(A))R(\lambda,A) : s(A) < \lambda < s(A)+1\}$ is bounded in $\mathcal{L}(E)$.
One can go even further and consider positive pseudo-resolvents $\{R(\lambda)\}_{\lambda\in D}$. This is also possible with respect to Cor.2.12.

(b) If $s(A)$ is a pole, then the criteria stated in B-III,Rem.2.15(a) for $s(A)$ to be a first order pole are valid in the setting of arbitrary Banach lattices as well. In particular, one has a first order pole provided that $\ker(s(A) - A)$ contains a quasi-interior point or in case that $\ker(s(A) - A')$ contains a strictly positive linear form.

(c) It is not difficult to give examples of semigroups whose resolvent does not grow slowly or cannot be reduced by a finite chain of invariant ideals as described after Cor.2.12 . E.g., one can take a bounded positive operator A which is not nilpotent and satifies $\sigma(A) = \{0\}$. However, the following question is still unanswered:

(2.23) Does every positive semigroup have cyclic boundary spectrum?