PART B

POSITIVE SEMIGROUPS ON $C_o(X)$

CHAPTER B-I

BASIC RESULTS ON SPACES $C_o(X)$

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This part of the book is devoted to a study of one-parameter semigroups of operators on spaces of continuous functions of type $C_o(X)$, spaces which are Banach lattices of a very special kind. We treat this case separately since we feel that an intermingling with the abstract Banach lattice situation considered in Part C would have made it difficult to appreciate the easy accessibility and the pilot function of methods and results available here. In this chapter we want to fix the notation we are going to use and to collect some basic facts about the spaces we are considering.

If X is a locally compact topological space, then $C_o(X)$ denotes the space of all continuous complex-valued functions on X which vanish at infinity, endowed with the supremum-norm. If X is compact, then any continuous function on X "vanishes at infinity" and $C_o(X)$ is the space of all continuous functions on X. We often write $C(X)$ instead of $C_o(X)$ in this situation. We sometimes study real-valued functions and write the corresponding real spaces as $C_o(X,\mathbb{R})$ and $C(X,\mathbb{R})$, and the notations $C_o(X,\mathbb{C})$ and $C(X,\mathbb{C})$ are used if there is the possibility of confusion between both cases.

1. ALGEBRAIC AND ORDER-STRUCTURE; IDEALS AND QUOTIENTS

Any space $C_o(X)$ is a commutative C^*-algebra under the sup-norm and the pointwise multiplication, and by the Gelfand Representation Theorem any commutative C^*-algebra can, on the other hand, be canonically represented as an algebra $C_o(X)$ on a suitable locally compact space X. The algebraic notions of ideal, quotient, homomorphism are