(b) This can be deduced easily from (a) as follows:
If $Ah = (\alpha+i\beta)h$, $A|h| = \alpha|h|$, then we have by A-III,Cor.6.4 :
$e^{-\alpha t}T(t)h = e^{i\beta t}h$ and $e^{-\alpha t}T(t)|h| = |h|$ for every $t \geq 0$.
Hence by (a) $e^{-\alpha t}T(t)h^{[n]} = e^{in\beta t}h^{[n]}$ $(t \geq 0, n \in \mathbb{Z})$, which is equivalent to $Ah^{[n]} = (\alpha+in\beta)h^{[n]}$. If h does not have zeros, then $e^{-\alpha t}T(t) = e^{-\alpha t}e^{i\beta t}S_h^{-1}T(t)S_h$ for every $t \geq 0$ which is equivalent to the final statement of (b).

□

Before we state a first cyclicity result we give the definition and illustrate it by some examples.

Definition 2.5. A subset $M \subset \mathbb{C}$ is called imaginary additively cyclic (or simply cyclic), if it satisfies the following condition: $\alpha + i\beta \in M$, $\alpha,\beta \in \mathbb{R}$ implies that $\alpha + ik\beta \in M$ for every $k \in \mathbb{Z}$.

Every subset of $\mathbb{R}$ is cyclic. On the other hand, if M is cyclic and $M \not\subset \mathbb{R}$, then M has to be unbounded.
For a subset M of $i\mathbb{R}$ we give the following equivalent conditions:
(i) M is imaginary additively cyclic;
(ii) M is the union of (additive) subgroups of $i\mathbb{R}$;
(iii) $M = \cup_{\alpha \in S} i\alpha\mathbb{Z}$ for some set $S \subset \mathbb{R}$.

Here are some concrete cyclic subsets of $i\mathbb{R}$:

$M_1 = \{0\}$, $M_2 = i\mathbb{R}$, $M_3 = i\alpha\mathbb{Z}$ $(\alpha > 0)$,
$M_4 = i\alpha\mathbb{Z} + i\beta\mathbb{Z} = \{in\alpha + im\beta : n,m \in \mathbb{Z}\}$ $(\alpha,\beta \in \mathbb{R})$,
$M_5 = \{0\} \cup \{i\lambda : \lambda \in \mathbb{R}, |\lambda| \geq 1\}$,
$M_6 = \cup_{n=0}^{\infty}\{i\lambda : \lambda \in \mathbb{R}, n\alpha \leq |\lambda| \leq n\beta\}$ $(0 < \alpha \leq \beta, \alpha,\beta \in \mathbb{R})$.

In the following we consider the boundary spectrum of several semigroups. The letter M_i refers to the sets just defined.

Examples 2.6.(a) For the Laplacian Δ on $\mathbb{R}^n$ or the second derivative on $[0,1]$ with Neumann boundary conditions the boundary spectrum is M_1.

(b) The first derivative on $\mathbb{R}$ or $\mathbb{R}_+$ is an example where the boundary spectrum of the generator is M_2.

(c) The rotation semigroup on $C(\Gamma)$ (see A-III,Ex.5.6) with period $2\pi/\alpha$ has boundary spectrum M_3.