are three different cases. Before we state the general result (Thm.4.4) we give some typical examples.

Examples 4.3.(a) Consider on $K = [0,\infty]$ the semiflow defined by $\phi(t,x) := x + t$ $(\infty + t = \infty)$. Then we have $K_t = [t,\infty]$ and $K_\infty = \{\infty\}$. The spectrum of the corresponding semigroup $T(t)f = f\circ\phi_t$ is given by $\sigma(A) = A\sigma(A) = \{\lambda \in \mathbb{C} : \operatorname{Re}\lambda \leq 0\}$.

(b) Consider again $K = [0,\infty]$ and define a semiflow by

$$\phi(t,x) := \begin{cases} x - t & \text{if } x \geq t \\ 0 & \text{if } x < t \end{cases} \qquad (\infty - t = \infty).$$

Then we have $K_t = K$ for all t, hence $K_\infty = K$ and $\sigma(A) = \{\lambda \in \mathbb{C} : \operatorname{Re}\lambda \leq 0\}$, $R\sigma(A) = \{\lambda \in \mathbb{C} : \operatorname{Re}\lambda < 0\} \cup \{0\}$.

(c) Consider on $K_1 := [-1,\infty)$ the equivalence relation $\sim$ defined by " $x \sim y$ if ond only if $x,y \geq 0$ and $x - y \in \mathbb{Z}$ ". The semiflow ϕ_1 on K_1 given by $\phi_1(t,x) = x + t$ induces a semiflow ϕ on the quotient space $K := K_{1/\sim}$. We have for $0 < t < 1$: $K \supsetneqq K_t \supsetneqq K_\infty$ $(K_\infty = [0,1]/_\sim \cong \Gamma)$. The spectrum of the corresponding semigroup on K is given by $\sigma(A) = 2\pi i\mathbb{Z}$.

(d) Consider on $K = [-1,1]$ the flow ϕ given by

$$\phi(t,x) := \begin{cases} -1 & \text{if } x < 0 \text{ and } t > -\frac{x+1}{x} \\ \frac{x}{1+tx} & \text{otherwise}. \end{cases}$$

Then we have $K_t = [-1,\frac{1}{1+t}]$, $K_\infty = [-1,0]$ and $\sigma(A) = \{\lambda \in \mathbb{C} : \operatorname{Re}\lambda \leq 0\}$, $\{\lambda \in \mathbb{C} : \operatorname{Re}\lambda < 0\} \subsetneqq A\sigma(A)\cap R\sigma(A)$.

Further examples related to ordinary differential equations on $\mathbb{R}^n$ will be given after we have stated and proved the general result:

Theorem 4.4. Suppose $\mathcal{T}$ is a semigroup of lattice homomorphisms given by (4.1) with generator A. Considering $H := \{\lambda \in \mathbb{C} : \operatorname{Re}\lambda < \underline{c}(h,\phi)\}$, where $\underline{c}(h,\phi)$ is given by (4.4), we have:

(a) If $K_t \neq K_\infty$ for every $t < \infty$, then $H \subseteq A\sigma(A)$.

(b) If $\phi|K_\infty$ is not injective, then $H \subseteq R\sigma(A)$.

(c) If $K_s = K_\infty$ for some $s < \infty$ and $\phi|K_\infty$ is injective, then $H \cap \sigma(A) = \emptyset$.

Proof. For $\varepsilon > 0$ we define $H_\varepsilon = \{\lambda \in \mathbb{C} : \operatorname{Re}\lambda < \underline{c}(h,\phi) - \varepsilon\}$. Obviously it is enough to prove assertion (a),(b) and (c) respectively for $H_{2\varepsilon}$, ε arbitrary, instead of H.