Since $s^*(M_p,(M_p)_*) = s^*(M,M_*)|M_p$ (a) follows immediately from (i)''. Using the resolvent equation for R it is easy to see that (ii)'' implies

$$\lim_n \|(Id - \lambda R(\lambda))\phi_n\| = 0$$

for all $\lambda \in D$ and the proof is complete.

□

Without further comments we will make use of the following facts in the rest of this section :

(1) A sequence (ϕ_n) in M'_+ converges in the $\sigma(M',M)$-topology if and only if it converges in $\sigma(M',M'')$-topology [Akeman-Dodds-Gamlen (1972)].

(2) We can decompose $\phi \in M'_+$ into its normal and singular part $\phi = \phi^{(n)} + \phi^{(s)}$, $0 \leq \phi^{(n)} \in M_*$, $0 \leq \phi^{(s)} \in M_*^{\perp}$ and $\|\phi\| = \|\phi^{(n)}\| + \|\phi^{(s)}\|$ [Takesaki (1979), Theorem III.2.14].

(3) If (ϕ_n) is a sequence in M_* which converges to zero in the $\sigma(M_*,M)$-topology and if (x_n) is a sequence in M which converges to zero in the $s^*(M,M_*)$-topology, then $\lim_n \phi_k(x_n) = 0$ uniformly in $k \in \mathbb{N}$ [Takesaki (1979), Lemma III.5.5].

<u>Theorem</u> **4.4.** Let R be an identity preserving pseudo-resolvent on $D = \{\lambda \in \mathbb{C} : Re(\lambda) > 0\}$ with values in a W*-algebra M which is of Schwarz type and let R' its adjoint pseudo-resolvent. Any one of the following conditions implies $\dim Fix(\hat{R}) < \infty$ in some ultrapower of M .

(a) The fixed space of R' is finite dimensional.

(b) $\lim_{\mu \downarrow 0} \mu R(\mu) = P$ exists in the strong operator topology and $rank(P) < \infty$.

(c) The fixed space of R' is contained in M_* .

(d) Every map $\mu R(\mu)$, $\mu \in \mathbb{R}_+$, is irreducible on M .

<u>Proof</u>. Suppose that the dimension of the fixed space of $(R'')^\wedge$ in some ultrapower $(M')^\wedge$ of M' is infinite dimensional. Since $(M')^\wedge$