CHAPTER C-III

SPECTRAL THEORY OF POSITIVE SEMIGROUPS ON BANACH LATTICES

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In Chapter B-III we have shown that positive semigroups on spaces $C_o(X)$ possess several interesting spectral properties. Now we are going to extend many of the results obtained there to the more general setting of Banach lattices. We will improve some of the results of B-III considerably and give the complete proof of B-III,Thm.4.1.

Throughout this chapter we will assume that $E \neq \{0\}$ is a complex Banach lattice.

1. THE SPECTRAL BOUND

The fact that the spectral bound of a positive semigroup is always contained in the spectrum (provided that the spectrum is non-empty) is also true in the general setting of Banach lattices. The proof given in B-III,Thm.1.1 for spaces $C_o(X)$ works in the general case too. Another proof is given below (cf. Cor.1.4). Furthermore, Cor.1.3 and Prop.1.5 of B-III are true in the setting of Banach lattices and their proofs can be carried over to the general case. For the sake of completeness we summarize these results in the following theorem.

Theorem 1.1. Let A be the generator of a positive semigroup $(T(t))_{t\geq 0}$ on a Banach lattice E .

(a) $s(A) \in \sigma(A)$ unless $\sigma(A) = \emptyset$.

(b) For $\lambda_o \in \rho(A)$ we have:
$R(\lambda_o,A)$ is positive if and only if $\lambda_o > s(A)$.
In this case $r(R(\lambda_o,A)) = (\lambda_o - s(A))^{-1}$.

(c) If $T(1)$ has a positive fixed vector h_o , then $\ker A$ contains a positive element h such that $h_o \in \overline{E_{|h|}}$.