suitable constants $\delta > 0$, $M \geq 1$ we have

(2.6) $\|e^{-rt} \cdot T(t) - P\| \leq M \cdot e^{-\delta t}$ for all $t \geq 0$.

(b) In case $(T(t))_{t \geq 0}$ is irreducible and $\omega(T) \geq 0$ there exist a strictly positive function $h \in C_o(X)$ and a strictly positive bounded measure $\nu \in M(X)$ such that for suitable constants $\delta > 0$, $M \geq 1$ one has

(2.7) $\|\exp(-\omega(T)t) \cdot T(t) - \nu \otimes h\| \leq M \cdot e^{-\delta t}$ for all $t \geq 0$.

In both cases (a) and (b) the estimates (2.3) for $\|T(t)f\|$ hold true (in case (a) one has to replace $|\int f d\nu|$ by $\|Pf\|$) .

Proof. (a) By B-III,Cor.2.11 we know that $s(A)$ is a strictly dominant eigenvalue of A . By Thm.2.10 both $s := s(A)$ and r are poles of the resolvent. Moreover, there exists a positive measure ν such that $A'\nu = s\nu$. Denoting the strictly positive eigenfunction corresponding to r by h we have $\langle h,\nu\rangle > 0$. Hence $s\langle h,\nu\rangle = \langle h,A'\nu\rangle = \langle Ah,\nu\rangle = r\langle h,\nu\rangle$ implies $r = s$. By B-III,Rem.2.15 we know that s is a first order pole of the resolvent. Since s is strictly dominant (2.6) follows from (2.5).
Assertion (b) can be proved in the same way as Cor.2.2 . We omit the details.

□

Cor.2.11 can be used to describe the asymptotic behavior as $t \to \infty$ of certain semigroups if only the generators are known. We explain this by discussing a concrete example.

Example 2.12. Let $X := [0,\infty)$ and define on $E := C_o(X)$ the operator A as follows

$Af := -f' + mf$ with domain $D(A)$ given by

(2.8) $D(A) := \{f \in C_o(X) : f$ is differentiable, $f' \in C_o(X)$ and $f'(0) = \alpha f(0) - \int_0^\infty f(x)\, d\nu(x) \}$.

Here α is a real number, ν is a bounded positive Borel measure with $\nu(\{0\}) = 0$ and m is a continuous function on X such that $m(\infty) := \lim_{x\to\infty} m(x)$ exists. It is not difficult to see that A generates a positive semigroup. Moreover, one can show that it is quasi-compact if (and only if) $m(\infty) < 0$. In order to find eigen-