<u>Proof</u>. We can assume that $\|m\|_\infty \leq 1$ (in fact, if $B := (m/\|m\|_\infty)\cdot A$ is the generator of a positive semigroup, then by A-I,3.1 $m\cdot A = \|m\|_\infty B$ also generates a positive semigroup). The assertion of the theorem holds for A if and only if it is valid for $A - w$ ($w \in \mathbb{R}$). So by the proof of Thm. 1.13 we can assume that there exists $0 \ll u \in C(K)$ such that A is p_u-dissipative. We first show,

(1.11) if B is a p_u-dissipative operator and $0 \ll q \in C(K)$, then $q\cdot B$ is p_u-dissipative.

Let $f \in D(q\cdot B) = D(B)$. There exists $x \in K$ such that $\phi_x \in dp_u(f)$ (by (1.6)). Hence $\langle Bf,\phi_x\rangle \leq 0$. Consequently, $\langle q\cdot Bf,\phi_x\rangle = q(x)$ $\langle Bf,\phi_x\rangle \leq 0$.

Next we show,

(1.12) if B is the generator of a p_u-contraction semigroup and $1 \geq q \in C(K)_+$ is such that $\|1 - q\|_\infty < 1/2$, then $q\cdot B$ generates a p_u-contraction semigroup.

Because of (1.11) we only have to show that $(I - q\cdot B)$ is surjective. Note that $1 \in \rho(B)$. We have $(Id - q\cdot B) = (Id - B - (q-1)B) = (Id - (q-1)BR(1,B))(Id - B)$. Thus it suffices to show that $Id - (q-1)BR(1,B)$ is invertible. The norm $\|f\|_u = \max\{p_u(f), p_u(-f)\} = \sup_{x\in K} |f(x)|/u(x)$ is equivalent to the supremum norm. Denote by $\|T\|_u$ the operator norm corresponding to $\|\ \|_u$ ($T \in \mathcal{L}(E)$). Then it is enough to show that $\|(q-1)BR(1,B)\|_u = \|(q-1)(R(1,B) - I)\|_u < 1$.
For $r \in C(K)_+$ denote by M_r the multiplication operator given by $M_r f = r\cdot f$. Then $\|M_r\|_u = \sup\{\|r\cdot f\|_u : \|f\|_u \leq 1\} = \sup\{\sup_{x\in K} r(x)|f(x)|/u(x) : \|f\|_u \leq 1\} \leq \|r\|_\infty$. Since B is p_u-dissipative we have $\|R(1,B)\|_u \leq 1$ (by A-II,Lemma 2.10). This gives
$\|(q-1)(R(1,B) - I)\|_u \leq \|M_{(1-q)}\|_u(\|R(1,B)\|_u + 1) \leq 2\|1 - q\|_\infty < 1$.
The proof of (1.12) is complete.

There exists $k \in \mathbb{N}$ such that $\|1 - m^{1/k}\|_\infty < 1/2$. Applying now (1.12) succesively to $B = m^{1/k}\cdot A$ and $q = m^{1/k}$ $(1 = 1, \ldots, k-1)$ we obtain that $m\cdot A$ generates a p_u-contraction semigroup (which in particular is positive).
Finally we show (1.10) to hold.
Let $0 \ll u \in D(A) = D(m\cdot A)$ and $Au \leq \lambda u$. Then $m\cdot Au \leq \|m\|_\infty \lambda u$. So (1.8) implies that $\omega(m\cdot A) \leq \|m\|_\infty\ \omega(A)$. This is one part of (1.10).