<u>Lemma</u> 1.2. Let X be a locally compact space, $x \in X$ and μ a regular bounded Borel measure on X such that $\mu(\{x\}) = 0$. Then $\mu \geq 0$ if and only if $\langle f,\mu\rangle \geq 0$ for all $f \in C_o(X)_+$ satisfying $f(x) = 0$.

We omit the easy proof.

<u>Theorem</u> 1.3. Let X be locally compact and A be a bounded operator on $C_o(X)$. The following assertions are equivalent.

(i) $e^{tA} \geq 0 \quad (t \geq 0)$.

(ii) For $0 \leq f \in C_o(X)$ and $x \in X$, $f(x) = 0$ implies $(Af)(x) \geq 0$.

(iii) $A + \|A\| Id \geq 0$.

<u>Proof</u>. (i) implies (ii). Let $f \in C_o(X)_+$ and $x \in X$ such that $f(x) = 0$. Then

$$\begin{aligned}(Af)(x) &= \lim_{t\to 0} 1/t\,((e^{tA}f(x) - f(x))\\ &= \lim_{t\to 0} 1/t\,((e^{tA}f(x)) \geq 0 .\end{aligned}$$

(ii) implies (iii). Let $x \in X$. We have to show that $(Af)(x) + \|A\|f(x) \geq 0$ for all $f \in C_o(X)$. Let $A'\delta_x = \mu + c\delta_x$ where $\mu \in M(X)$ such that $\mu(\{x\}) = 0$ and $c \in \mathbb{R}$. We claim that $\mu \geq o$. Let $0 \leq f \in C_o(X)$ such that $f(x) = 0$. Then $\langle f,\mu\rangle = \langle f,A'\delta_x\rangle = (Af)(x) \geq 0$ by (ii). Thus $\mu \geq 0$ by Lemma 1.1. Moreover, $|c| = \|c\delta_x\| \leq \|c\delta_x + \mu\| = \|A'\delta_x\| \leq \|A\|$. Hence, for $f \in C_o(X)_+$, $(Af)(x) + \|A\|f(x) = \langle f, A'\delta_x + \|A\|\delta_x\rangle = \langle f, \mu + (c+\|A\|)\delta_x\rangle \geq 0$. This shows (ii) to hold.
(iii) implies (i). We have $e^{tA} = e^{-t\|A\|}\, e^{t(A+\|A\|)} \geq e^{-t\|A\|}\, Id$ for all $t \geq 0$.

□

<u>Example</u> 1.4. a) Let B be a positive operator on $C_o(X)$ and $m : X \to \mathbb{R}$ be a continuous and bounded mapping. Let $Af = Bf - m\cdot f \quad (f \in C_o(X))$. Then $e^{tA} \geq 0$ for all $t \geq 0$.
b) Let A be a nxn - matrix. Then $e^{tA} \geq 0$ for all $t \geq 0$ if and only if $a_{ij} \geq 0$ for $i \neq j$. This is the linear version of Kamke's theorem (see Kamke (1932)).

Now we come to the actual subject of this section, the characterization of strongly continuous positive semigroups on $C(K)$. Here K