(c) For $\alpha > s(A)$, $r > \tau$, $f \in E_+$ we have

$$\int_0^r e^{-\alpha s}\|T(s)f\|\,ds = \int_0^\tau e^{-\alpha s}\|T(s)f\|\,ds + e^{-\alpha\tau}\int_0^{r-\tau} e^{-\alpha s}\|T(\tau)T(s)f\|\,ds \leq$$
$$\leq \int_0^\tau e^{-\alpha s}\|T(s)f\|\,ds + e^{-\alpha\tau}\int_0^{r-\tau} e^{-\alpha s}\langle T(s)f,\phi\rangle\,ds \leq$$
$$\leq \int_0^\tau e^{-\alpha s}\|T(s)f\|\,ds + \|R(\alpha,A)f\|$$

(the last inequality follows from C-III,Thm.1.2).
Now Datko's Theorem (A-IV,Thm.1.11) implies $\omega(A) < \alpha$

□

For L^p-spaces, $p \neq 1$, 2 , ∞ , it is not known whether spectral- and growth bound of an arbitrary positive semigroup coincide. Using interpolation techniques and Thm.1.1 one can treat some special cases. Before doing this we have to recall some facts on interpolation. For details we refer to [Dunford-Schwartz (1958), VI.10] or [Reed-Simon (1975) , IX.4.].
Let (X,Σ,μ) be a σ-finite measure space, $1 \leq p < q < \infty$ and suppose that $T_o : L^p(\mu) \cap L^q(\mu) \to L^p(\mu) \cap L^q(\mu)$ is a linear operator which satisfies $\|T_o f\|_p \leq c_p\|f\|_p$ and $\|T_o f\|_q \leq c_q\|f\|_q$. Then for every $r \in [p,q]$, T_o has a (unique) continuous extension $T_r : L^r(\mu) \to L^r(\mu)$. Moreover ,

(1.1) $u \to \log\|T_{1/u}\|$ is a convex function on the interval $[\frac{1}{q},\frac{1}{p}]$.

Applying this result to the powers T_r^n $(n \in \mathbb{N})$ and using the fact that the pointwise limit of convex functions is convex, we obtain that the logarithm of the spectral radius is convex, i.e.,

(1.2) $u \to \log(r(T_{1/u})) = \lim_{n\to\infty} \frac{1}{n}\log\|T_{1/u}^n\|$ is convex on $[\frac{1}{q},\frac{1}{p}]$.

In the following we suppose that for every $r \in [p,q]$ we have a strongly continuous semigroup $(T_r(t))_{t\geq 0}$ on $L^r(\mu)$ such that

(1.3) $T_r(t)|_{L^r \cap L^s} = T_s(t)|_{L^r \cap L^s}$ for all $r,s \in [p,q]$, $t \geq 0$.

Let A_r be the generator of $(T_r(t))$, $\omega(r)$ its type and $s(r)$ the spectral bound of A_r . In this situation we have the following corollary of Thm.1.1.

<u>Corollary</u> 1.2. Suppose that the semigroups $(T_r(t))_{t\geq 0}$ are positive.
(a) In case $p < 2 < q$ and $\omega(r)$ independent of $r \in [p,q]$, one has $s(r) = \omega(r)$ for all $r \in [p,q]$.
(b) If $p = 1$, $q \geq 2$ and $s(r)$ independent of $r \in [p,q]$ then $s(r) = \omega(r)$ for $r \in [1,2]$.