$f(z) = (e^{-i\alpha t}T(t_n)f)(z) = e^{-i\alpha t}f(\phi(t_n,z)) \to 0$. Thus $f = 0$.
Proof of (iii): At first we point out that for $f \in C_o(X)$ such that f vanishes on the unit circle Γ , we have $\lim_{t\to\infty} \|T(t)f\| = 0$.
Assume that μ is a bounded Borel measure such that $T(t)'\mu = e^{i\alpha t}\mu$ for every $t \geq 0$ and some $\alpha \in \mathbb{R}$. Then $<f,\mu> = e^{i\alpha t}<f,T(t)'\mu> = e^{i\alpha t}<T(t)f,\mu> \to 0$ for every $f \in C_o(X)$ with $f_{|\Gamma} = 0$. It follows that the support of μ is contained in Γ . Since $\lim_{t\to\infty} \phi(t,z) = 1$ for every $z \in \Gamma$, we obtain for arbitrary $f \in C_o(X)$:
$(T(t)f)(z) \to 0$ μ-a.e.. Lebesgue's Dominated Convergence Theorem implies $<f,\mu> = e^{-i\alpha t}<T(t)f,\mu> \to 0$ as $t \to \infty$ for every $f \in C_o(X)$.
Thus $\mu = 0$.

Now we are going to prove the main result of this section. At first we note that the positive part of the domain of the adjoint operator is sufficiently large. In fact, we know that $\lambda R(\lambda,A) \to Id$ strongly as $\lambda \to \infty$. It follows that $\lambda^2 R(\lambda,A)^2 \to Id$ strongly, hence $\lambda^2 R(\lambda,A)'^2 \to Id$ with respect to $\sigma(E',E)$-topology. If A generates a positive semigroup then $\lambda^2 R(\lambda,A)'^2\mu \in D(A^*) := D(A^*)\cap E'_+$ for $\mu \in E'_+$. (Note that $R(\lambda,A)'E' \subset D(A') \subset E^*$, thus $R(\lambda,A)'^2E' \subset R(\lambda,A)'E^* = D(A^*)$.)
We summarize these observations in the following lemma.

Lemma 1.3. Let A be the generator of a positive semigroup on a Banach lattice E . Then every $\mu \in E'_+$ is the $\sigma(E',E)$-limit of elements in $D(A^*)_+$; i.e., $\overline{D(A^*)_+}^{\sigma(E',E)} = E'_+$.

Theorem 1.4 Let A be the generator of a positive semigroup on $C_o(X)$. Then

$$s(A) = \omega_1(A) = \omega(A) \in \sigma(A)$$

Proof. Rescaling the semigroup we may assume $\omega(A) = 0$. (In case $\omega(A) = -\infty$, then $\sigma(A) = \emptyset$ hence $s(A) = -\infty$)
Suppose $0 \notin \sigma(A) = \sigma(A^*)$. Then, by the holomorphy of the resolvent and by A-II,Prop.1.11
$R(0,A^*)\Phi = \sum_{n=0}^\infty R(1,A^*)^{n+1}\Phi = \sum_{n=0}^\infty \int_0^\infty \frac{1}{n!} t^n e^{-t} T(t)^*\Phi \, dt$ for every $\Phi \in C_o(X)^*$. If $0 \leq \Phi \in C_o(X)^*$ and $0 \leq \rho \in C_o(X)''$ we can interchange integration and summation by the Monotone Convergence Theorem; i.e.

$$(1.3) \qquad \begin{aligned} <R(0,A^*)\Phi,\rho> &= \sum_{n=0}^\infty \int_0^\infty \frac{1}{n!} t^n e^{-t} <T(t)^*\Phi,\rho> \, dt \\ &= \int_0^\infty <T(t)^*\Phi,\rho> \, dt . \end{aligned}$$

Since every element of $C_o(X)^*$ and $C_o(X)''$ is the difference of posi-