A band in a normed vector lattice is necessarily closed. By contrast, an ideal need not be closed, but the closure of an ideal is again an ideal. The situation where every closed ideal is a band will be briefly discussed in Section 5.

2. ORDER UNITS, WEAK ORDER UNITS, QUASI-INTERIOR POINTS

An element u in the positive cone of a vector lattice E is called an order unit, if the ideal generated by u is all of E. If the band generated by u is all of E (which is equivalent to $\{u\}^d = 0$ whenever E is archimedean, hence in particular if E is a normed vector lattice) then u is called a weak order unit of E. If E is a Banach lattice, then any order unit in E is an interior point of the positive cone E_+, and conversely any interior point of E_+ must be an order unit of E. Every space $C(K)$ has order units (namely, the strictly positive functions), and conversely by the Kakutani-Krein Representation Theorem (see Section 4) every Banach lattice with an order unit is isomorphic to a space $C(K)$. If an element u in the positive cone of a Banach lattice E has the property that the closed ideal generated by u is all of E, then u is called a quasi-interior point of E_+. Quasi-interior points of the positive cone exist, e.g., in any separable Banach lattice. If $E = C(K)$, then the quasi-interior points and the interior points of E_+ coincide, while the weak order units of E are the (positive) functions vanishing on a nowhere dense subset of K. If E is a space $L^p(\mu)$ with σ-finite μ and $1 \leq p < \infty$, then the weak order units and the quasi-interior points of E_+ coincide with the functions strictly positive μ-a.e., while E_+ does not contain any interior point.

3. LINEAR FORMS AND DUALITY

A linear functional ϕ on a vector lattice E is called

order-bounded, if ϕ is bounded on order intervals of E,
positive, if $\phi(f) \geq 0$ for all $f \geq 0$,
strictly positive, if $\phi(f) > 0$ for all $f > 0$.

Any positive linear functional is order bounded, and the positive functionals form a proper convex cone with vertex 0 in the linear space $E^{\#}$ of all order bounded functionals, thus defining a natural