CHAPTER A-II

CHARACTERIZATION OF SEMIGROUPS ON BANACH SPACES

In this chapter two different problems are treated:

1) to characterize generators of strongly continuous semigroups;

2) to characterize various properties of strongly continuous semigroups in terms of their generators.

In Section 1 the first problem is solved by finding conditions on the Cauchy problem associated with A and also by finding conditions on the resolvent of A. The second problem is treated for a hierarchy of smoothness properties of the semigroup.

Contraction semigroups are considered in Section 2. Here, the first problem has a simple and extremely useful solution: A densely defined operator A is generator of a contraction semigroup if and only if A is dissipative and satisfies a range condition.
Our approach is quite general. We do not only consider contractions with respect to the norm but also with respect to "half-norms". This will allow us to obtain results on positive contraction semigroups simultaneously by choosing a suitable half-norm (cf. C-II,Sec.1).

The last section contains a surprising result: on certain Banach spaces (e.g., L^∞) only bounded operators are generators of strongly continuous semigroups.