If $\lambda > \alpha + \|\mu\|$, then $\lambda - \alpha - \int_{-1}^{0} e^{\lambda x} d\mu(x) \neq 0$ and there exists exactly one $c \in \mathbb{R}$ satisfying (2.8). We have shown that $(\lambda - A)$ is bijective for $\lambda > \alpha + \|\mu\|$.
By Thm.1.13, it follows from a),b) and c) that A generates a positive semigroup.

□

Let us mention in addition that it follows from a) in the proof that $(\alpha + \|\mu\|, \infty) \subset \rho(A)$, since A is closed. By Remark 1.7 we thus have

$$s(A) \leq \alpha + \|\mu\| . \tag{2.9}$$

<u>Example</u> 1.23. Let $E = C([-1,0],\mathbb{R}^n)$. Then $u \in E$ is given by $u = (u_1,\ldots,u_n)$ where $u_i \in C[-1,0]$ $(i=1,\ldots,n)$. Let A be defined by $Au = u' = (u_1',\ldots,u_n')$ with domain $D(A) = \{u \in C^1([-1,0],\mathbb{R}^n) : u'(0) = Lu\}$.
Here L is defined by

$$Lu = \begin{pmatrix} L_{11}u_1 + \ldots + L_{1n}u_n \\ \vdots \\ L_{n1}u_1 + \ldots + L_{nn}u_n \end{pmatrix}$$

where $L_{ij} \in C[-1,0]'$ $(1 \leq i,j \leq n)$.
Let $L_{ii} = c_i\delta_o + \mu_i$ with $\mu_i(\{0\}) = 0$ $(i = 1,\ldots,n)$.
Then A generates a positive semigroup if and only if

$$L_{ij} \geq 0 \quad \text{for } i \neq j \text{ and } \mu_i \geq 0 \quad (i,j = 1,\ldots,n) .$$

This can be proved in a similar way as the claim in Example 1.21 (see Arendt (1984a)).

<u>Example</u> 1.24. Let A on $C[0,1]$ be given by $Af = f''$ with domain $D(A) = \{f \in C^2[0,1] : f'(0) + \alpha f(0) = 0 , f'(1) + \beta f(1) = 0\}$, where $\alpha,\beta \in \mathbb{R}$. Then A is the generator of positive semigroup.

<u>Proof</u>. The operator A satisfies (P). In fact, let $0 \leq f \in D(A)$ and $f(a) = 0$ where $a \in [0,1]$. If $a \in (0,1)$ then $f''(a) \geq 0$ since f has a minimum in a . If $a = 0$, then $f'(0) = f'(0) + \alpha f(0) = 0$ since $f \in D(A)$. Consequently, $f(x) = \int_o^x (x-y)f''(y)dy \geq 0$ for all $x \geq 0$. This implies $f''(0) \geq 0$. The argument for $a = 1$ is analoguous.