By definition, a Banach lattice is a Banach space $(E,\| \ \|)$ which is endowed with an order relation, usually written $\leq$, such that $(E,\leq)$ is a lattice and the ordering is compatible with the Banach space structure of E. We are going to elaborate this in more detail now.

The axioms of compatibility between the linear structure of E and the order are as follows:

(LO_1) $f \leq g$ implies $f + h \leq g + h$ for all f, g, h in E.
(LO_2) $f \geq 0$ implies $\lambda f \geq 0$ for all f in E and $\lambda \geq 0$.

Any (real) vector space with an ordering satisfying (LO_1) and (LO_2) is called an <u>ordered vector space</u>. The property expressed in (LO_1) is sometimes called translation invariance and implies that the ordering of an ordered vector space E is completely determined by the <u>positive part</u> $E_+ = \{f \in E: f \geq 0\}$ of E. In fact, one has $f \leq g$ if and only if $g - f \in E_+$. (LO_1) together with (LO_2) furthermore imply that the positive part of E is a convex set and a cone with vertex 0 (often called the <u>positive cone</u> of E). It is easily verified that conversely any proper convex cone C with vertex 0 in E is the positive part of E with respect to a uniquely determined compatible ordering.

An ordered vector space E is called a <u>vector lattice</u> if any two elements f, g in E have a supremum, which is denoted by $\sup(f,g)$ or by $f \vee g$, and an infimum, denoted by $\inf(f,g)$ or by $f \wedge g$. It is obvious that the existence of, e.g., the supremum of any two elements in an ordered vector space implies the existence of the supremum of any finite number of elements in E and, since $f \leq g$ is equivalent to $-g \leq -f$ this automatically implies the existence of finite infima. However, suprema (infima) of infinite majorized subsets need not exist in a vector lattice. If they do, then the vector lattice is called <u>order complete</u> (<u>countably order complete</u> or <u>σ-order complete</u> if suprema of countable majorized subsets exist). At any rate, the binary relations sup and inf in a vector lattice automatically satisfy the (infinite) distributive laws

$$\inf(\sup_\alpha f_\alpha, h) = \sup_\alpha(\inf(f_\alpha, h))$$
$$\sup(\inf_\alpha f_\alpha, h) = \inf_\alpha(\sup(f_\alpha, h))$$