

that any increasing norm-bounded net be convergent. This condition is satisfied if and only if any one of the following equivalent assertions holds:

- (a) E is (under evaluation) a band in E'' .
- (b) E is weakly sequentially complete.
- (c) Every order-continuous linear form on E' belongs to E .
- (d) No closed sublattice of E is isomorphic to c_0 .

The most important examples of non-reflexive Banach lattices with this property are the AL-spaces.

6. POSITIVE OPERATORS, LATTICE HOMOMORPHISMS

A linear mapping T from an ordered Banach space E into an ordered Banach space F is called positive (notation: $T \geq 0$) if $Tf \in F_+$ for all $f \in E_+$; T is called strictly positive if $T \geq 0$ and $\{f \in E: T|f| = 0\} = \{0\}$. The set of all positive linear mappings is a convex cone in the space $L(E, F)$ of all linear mappings from E into F defining the natural ordering of $L(E, F)$. The linear subspace of $L(E, F)$ generated by the positive maps (i.e. the space of linear maps that can be written as differences of positive maps) is denoted by $L^r(E, F)$ and its elements are called regular mappings. If E and F are Banach lattices, then any regular mapping from E into F is continuous, but $L^r(E, F)$ is in general a proper subspace of the space $L(E, F)$ of all continuous linear mappings. One has coincidence of $L^r(E, F)$ and $L(E, F)$ e.g. when $E = F$ is an order complete AM-space with unit or an AL-space. At any rate, if F is order complete, then $L^r(E, F)$ under the natural ordering is an order-complete vector lattice, and a Banach lattice under the norm

$$T \mapsto \|T\|_r = \| |T| \|,$$

the right hand side denoting the operator norm of the absolute value of T . The absolute value of $T \in L^r(E, F)$, if it exists, is given by

$$|T|(f) = \sup\{Th : |h| \leq f\} \quad (f \in E_+).$$

Thus T is positive if and only if $|Tf| \leq T|f|$ holds for any f in E .