strictly positive we have $\langle h,w\rangle > 0$ hence $s(A)\langle h,w\rangle = \langle h,A'w\rangle = \langle Ah,w\rangle = 0$ which implies $s(A) = 0$.
Consequently the semigroup generated by A satisfies all the assumptions of Thm.2.1 provided that μ and β satisfy (2.3). (The boundedness of the semigroup follows from Rem.2.2(c)). It is not difficult to see that (up to a constant) h is the unique eigenfunction of A corresponding to 0 . Thus the projection P has the form $P = v\otimes h$ for a suitable positive $v \in L^{\infty}([0,\infty))$.
For more general generators of the type (2.2) we refer to C-IV, Section 3.

Clearly, quasi-compactness was essential in the above example as well as in Thm.2.1. For spaces $C_o(X)$ we proved in B-IV,Thm.2.12 that Doeblin's condition is sufficient for quasi-compactness. Actually this is true in L^p-spaces with $1 < p < \infty$ as well. We quote the result from Lotz (1986).

<u>Proposition</u> 2.4. Let $(T(t))_{t\geq 0}$ be a bounded positive semigroup on $E = L^p(\mu)$, $1 < p < \infty$.
Assume that there exist $t_o \geq 0$, $\phi \in E'_+$, $b < 1$ such that

(2.4) $\|T(t_o)f\| \leq \langle f,\phi\rangle + b\|f\|$ for all $f \geq 0$.

Then $(T(t))_{t\geq 0}$ is quasi-compact.

In the following result we replace quasi-compactness by eventual norm-continuity of the semigroup.

<u>Theorem</u> 2.5. Let $\mathcal{T} = (T(t))_{t\geq 0}$ be a bounded, eventually norm-continuous positive semigroup with gererator A on a reflexive Banach lattice E . Then $Pf := \lim_{t\to\infty} T(t)f$ exists for every $f \in E$. P is a positive projection onto the fixed space $\mathrm{Fix}(\mathcal{T}) = \ker A$.

<u>Proof</u>. In view of Thm.1.5 it suffices to consider the case $s(A) = 0 \in P\sigma(A)$. We define $F := \{f \in E : \lim_{t\to\infty} T(t)f \text{ exists}\}$. F is closed since $(T(t))_{t\geq 0}$ is bounded and obviously $\ker A \subset F$. Since $\sigma(A)\cap i\mathbb{R}$ is cyclic and bounded (see C-III,Thm.2.10 and A-II,Thm.1.20 resp.) we have $\sigma(A)\cap i\mathbb{R} = \{0\}$. Since the spectral mapping theorem holds (cf. A-III,Thm.6.6) we conclude $\sigma(T(t))\cap\Gamma = \{1\}$ for all $t \geq 0$. Then (1.4) implies $\lim_{n\to\infty}\|T(n) - T(n+1)\| = 0$ hence $\lim_{t\to\infty}\|T(t) - T(t+1)\| = 0$. Take $f = g - T(1)g$. Then $\|T(t)f\| = \|T(t)g - T(t+1)g\| \leq \|T(t) - T(t+1)\|\|g\|$ implies $\lim_{t\to\infty} T(t)f = 0$.
Thus