Proof. There exists $L \geq 0$ such that $\|\Theta(x) - \Theta(y)\| \leq L\|x - y\|$ for all $x,y \in G$. Then

$\lim_{t\downarrow o} \|1/t\ (\Theta(k(a+t)) - \Theta(k(a))) - D_{k'(a)}\Theta(k(a))\| \leq$
$\text{lim-sup}_{t\downarrow o} \|1/t\ (\Theta(k(a+t)) - \Theta(k(a)+tk'(a))\| +$
$\quad \text{lim-sup}_{t\downarrow o} \|1/t\ [\Theta(k(a)+tk'(a)) - \Theta(k(a)) - D_{k'(a)}\Theta(k(a))]\| \leq$
$\text{lim-sup}_{t\downarrow o} L\cdot\|1/t\ (k(a+t) - k(a)-tk'(a))\| + 0 = 0$.

□

For $z \in \mathbb{C}$ we let

$$(2.5)\qquad \operatorname{sign} z = \begin{cases} z/|z| & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 . \end{cases}$$

Lemma 2.4. The function $\Theta : \mathbb{C} \to \mathbb{C}$ given by $\Theta(z) = |z|$ is right-sided Gateaux differentiable and

$$(2.6)\qquad D_u\Theta(z) = \begin{cases} \operatorname{Re}[(\operatorname{sign} \bar{z})\cdot u] & \text{if } z \neq 0 \\ |u| & \text{if } z = 0 . \end{cases}$$

Proof. If $z = 0$, relation (2.6) is obvious from the definiton. Let $z = (x_o + iy_o) \neq 0$. We identify $\mathbb{C}$ and $\mathbb{R}^2$. Then
$\Theta(x,y) = (x^2 + y^2)^{1/2}$ is differentiable in z and
$D_u\Theta(z) = (\operatorname{grad} \Theta(x_o,y_o)|u) = 1/|z|\ ((x_o,y_o)|(u_1,u_2)) =$
$1/|z|\ (x_ou_1 + y_ou_2) = 1/|z|\ \operatorname{Re}((x_o-iy_o)\cdot(u_1+iu_2)) = \operatorname{Re}[(\operatorname{sign} \bar{z})\cdot u]$,
where $u = u_1 + iu_2 = (u_1,u_2) \in \mathbb{C} = \mathbb{R}^2$ and $(v|u)$ denotes the canonical scalar product of $v,u \in \mathbb{R}^2$.

□

Let $f, g \in C_o(X)$. We denote by $(\widehat{\operatorname{sign}}\ f)(g)$ the bounded Borel function given by

$$(2.7)\qquad [(\widehat{\operatorname{sign}}\ f)(g)](x) = \begin{cases} (\operatorname{sign} f(x))\cdot g(x) & \text{if } f(x) \neq 0 \\ |g(x)| & \text{if } f(x) = 0 . \end{cases}$$

Similarly, $(\operatorname{sign} f)(g)$ is defined by

$$(2.8)\qquad [(\operatorname{sign} f)(g)](x) = (\operatorname{sign} f(x))\cdot g(x) .$$

We identify the dual space of $C_o(X)$ with $M(X)$, the space of all bounded regular Borel measures on X . We extend the duality by setting