which means that $0 \in \rho({}^A|I_o)$. Since ${}^A|I_o$ generates a positive semigroup and $R(\lambda, {}^A|I_o) = {}^{R(\lambda,A)}|I_o$ is positive for $\lambda > 0$ it follows from Cor.1.3. that $s(A_o) = s({}^A|I_o) < 0$.

□

One can check the different steps of the proof by studying the following example. Consider the following matrix as generator on $\mathbb{C}^4$.

$$\begin{pmatrix} -1 & a & b & c \\ 0 & 0 & d & e \\ 0 & 0 & 0 & f \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \text{where } a,b,c,d,e,f \geq 0 .$$

The result is summarized in the following diagram ($e_j := (\delta_{jk})$) :

	pole order	I_o	I_1	I_2	I_3
$d>0,\ f>0$	3	$\langle e_1\rangle$	$\langle e_1,e_2\rangle$	$\langle e_1,e_2,e_3\rangle$	$\mathbb{C}^4$
$d=0,\ f>0,$	2	$\langle e_1\rangle$	$\langle e_1,e_2,e_3\rangle$	$\mathbb{C}^4$	
$d=0,\ f=0,\ e>0$	2	$\langle e_1\rangle$	$\langle e_1,e_2,e_3\rangle$	$\mathbb{C}^4$	
$d>0,\ f=0,\ e>0$	2	$\langle e_1\rangle$	$\langle e_1,e_2\rangle$	$\mathbb{C}^4$	
$d>0,\ f=0,\ e=0$	2	$\langle e_1\rangle$	$\langle e_1,e_2,e_4\rangle$	$\mathbb{C}^4$	

This example also shows that the operators Q_{k-1} , ... , Q_1 are not necessarily positive (e.g. $a>0$, $b=c=0$, $d=e=f=2$) . A more general (and more interesting) example is the following:
Suppose that A_i $(i = 1,\dots,n)$ are generators of positive semigroups on $C_o(X)$ such that $s(A_i) = 0$ is a first order pole of the resolvent. And let A_{ij} $(1 \leq i < j \leq n)$ be positive bounded operators on $C_o(X)$.

$$\text{Then} \quad A := \begin{pmatrix} A_1 & A_{12} & \cdots & A_{1n} \\ 0 & A_2 & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_n \end{pmatrix}$$

is the generator of a positive semigroup on $C_o(X,\mathbb{C}^n) \cong C_o(X) \times C_o(X) \times \dots \times C_o(X)$, and $s(A) = 0$ is a pole of the resolvent of order k where $1 \leq k \leq n$.

<u>Theorem</u> 2.9. Suppose A is the generator of a positive semigroup on $C_o(X)$ such that every point of $\sigma_b(A)$ is a pole of the resolvent. Then $P\sigma_b(A) = \sigma_b(A)$ is cyclic.