3. IRREDUCIBLE SEMIGROUPS

The concept of irreducibility is very natural in the general setting of Banach lattices. However, some of the (equivalent) assertions stated in B-III,Def.3.1 do not ma.e sense here, others need a slightly different formulation.

Definition 3.1. A positive semigroup $(T(t))_{t\geq 0}$ on a Banach lattice E with generator A is called irreducible if one of the following (mutually equivalent) conditions is satified:

(i) There is no $(T(t))$-invariant closed ideal except $\{0\}$ and E .

(ii) Given $f \in E$, $\phi \in E'$ such that $f > 0$, $\phi > 0$ then $\langle T(t_o)f,\phi\rangle > 0$ for some $t_o \geq 0$.

(iii) For arbitrary $f,g \in E_+$, $f > 0$, $g > 0$ there exists t_o such that $\inf\{T(t_o)f,g\} > 0$.

(iv) For some (every) $\lambda > s(A)$ there is no closed ideal other than $\{0\}$ or E which is invariant under $R(\lambda,A)$.

(v) For some (every) $\lambda > s(A)$ we have: $R(\lambda,A)f$ is a quasi-interior point of E_+ whenever $f > 0$.

Equivalence of the five conditions above is obtained by a slight modification of the arguments given in B-III,Def.3.1 . Since there are no difficulties we omit a detailed proof. Obviously, a semigroup is irreducible if one single operator $T(t_o)$ is irreducible. In general the converse does not hold (see p.65 in Greiner (1982)). The situation is different when holomorphic semigroups are considered. Then an even stronger assertion holds: In fact irreducibility of a holomorhic semigroup implies that every single operator maps the positive elements onto quasi-interior points. This is the second statement of the following theorem (see also B-III,Rem.3.2).

Theorem 3.2.(a) If $(T(t))_{t\geq 0}$ is an irreducible semigroup then every operator $T(t)$ is strictly positive.
I.e., given $f > 0$, $t \geq 0$, then $T(t)f > 0$.

(b) Suppose $(T(t))_{t\geq 0}$ is a holomorphic positive semigroup.
If $(T(t))$ is irreducible then $T(t)f$ is a quasi-interior point of E_+ whenever $f > 0$ and $t > 0$. Equivalently, given $f \in E$, $\phi \in E'$ such that $f > 0$, $\phi > 0$, then $\langle T(t)f,\phi\rangle > 0$ for all $t > 0$.