3. A SEMIGROUP APPROACH TO RETARDED EQUATIONS

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As indicated by the above title of this section there is a close relationship to B-IV,Section 3. First, the considered Cauchy problems are "similar" to (RCP). Second, there again is a correspondence to a class of semigroups generated by the first derivative.
Instead of the differential equation in (RCP) we will study equations of the form

$$\text{(RE)}\qquad \begin{aligned} u(t) &= \Phi(u_t)\ ,\ t \geq 0\ ,\\ u_o &= g\ .\end{aligned}$$

We use the following setting: Let F be a Banach space, consider $E := L^1([-1,0],F)$ and take $\Phi \in \mathcal{L}(E,F)$. For $u \in L^1_{loc}([-1,\infty),F)$ we denote by $u_t \in E$ the function given by $u_t(s) := u(t+s)$, $t \geq 0$, $s \in [-1,0]$.

By a <u>solution</u> of (RE) with initial function $g \in E$ we understand a function $u \in L^1{}_{loc}([-1,\infty),F)$ which satisfies equation (RE).
(RE) is called <u>well-posed</u> if for each $g \in E$ there exists exactly one solution.
<u>Remarks</u>. 1. The equation

$$\begin{aligned} u(t) &= Bu(t) + \Phi(u_t)\ ,\ t \geq 0\ ,\\ u_o &= g\ ,\end{aligned}$$

(where B is the generator of a bounded semigroup on F) is in better analogy to the retarded Cauchy problem of B-IV,Sec.3 and seems to be more general than the one introduced above, but can be reduced to an equation of the type (RE). In fact, since $1 \in \rho(B)$ we have

$$u(t) = R(1,B)\Phi(u_t)\ .$$

Clearly, this equation is of the previous type (with a different "delay functional").

2. The choice of "L^1-functions" instead of "C-functions" (as in the case of (RCP)) enforces the solutions of (RE) to yield a strongly continuous semigroup of operators (on the space E of initial functions) as in B-IV,Section 3.