<u>Theorem</u> 1.1. Every strongly continuous one-parameter semigroup of Schwarz type on a properly infinite W*-algebra M is uniformly continuous.

<u>Proof</u>. Let $\mathcal{T} = (T(t)_*)_{t \geq 0}$ be strongly continuous on M and suppose $\mathcal{T}$ not to be uniformly continuous. Then there exists a sequence $(T_n) \subset \mathcal{T}$ and $\varepsilon > 0$ such that $\|T_n - \mathrm{Id}\| \geq \varepsilon$ but $T_n \to \mathrm{Id}$ in the strong operator topology. We claim that for every sequence (p_k) of mutually orthogonal projections and all bounded sequences (x_k) in M

$$\lim_n \|(T_n - \mathrm{Id})(p_k x_k p_k)\| = 0$$

uniformly in $k \in \mathbb{N}$. This follows from an application of the Lemma of Phillips and the fact that the sequence $(p_k x_k p_k)$ is summable in the $s^*(M,M_*)$-topology (compare Elliot (1972)).

Let (p_k) be a sequence of mutually orthogonal projections in M such that every p_k is equivalent to 1 via some $u_k \in M$ [Sakai (1971), 2.2].
Without loss of generality we may assume $\|(T_n - \mathrm{Id})(u_n)\| \leq n^{-1}$ since the semigroup $\mathcal{T}$ is strongly continuous. Thus we obtained the following:

(1) $\lim_n \|(T_n - \mathrm{Id})(p_k x_k p_k)\| = 0$ uniformly in $k \in \mathbb{N}$ for every bounded sequence (x_k) in M.

(2) Every projection p_k is equivalent to 1 via some $u_k \in M$.

(3) $\|(T_n - \mathrm{Id})u_n\| \leq n^{-1}$ for all $n \in \mathbb{N}$.

For the following construction see A-I,3.6 and D-II,Sec.2.

Let $\hat{M}$ be an ultrapower of M, let $p := (p_k)^\wedge \in \hat{M}$, $T := (T_n)^\wedge \in \mathcal{L}(\hat{M})$ and $u := (u_k)^\wedge \in \hat{M}$. Then T is identity preserving and of Schwarz type on $\hat{M}$. Since $u^*u = p$ and $uu^* = 1$, it follows $pu^* = u^*$ and $(uu^*)x(uu^*) = x$ for all $x \in \hat{M}$. Finally, $T(pxp) = pxp$ for all $x \in \hat{M}$, which follows from (1), and $T(u^*) = T(pu^*) = pu^* = u^*$ and $T(u) = u$, which follows from (3). Using the Schwarz inequality we obtain

$$T(uu^*) = T(1) \leq 1 = uu^* = T(u)T(u)^* .$$