In the following we will describe the spectrum of the semigroup given by (4.1) in terms of ϕ and h . At first we have to fix some notation. Let K , ϕ , h be as in (4.1).

(4.2) $K_t := \phi_t(K)$ $(t < \infty)$, $K_\infty := \bigcap_{t<\infty} K_t$.

Some properties of the sets K_t are listed in the following lemma. The proof is not very difficult and is left as an exception.

<u>Lemma</u> 4.2. Every K_t $(0 \leq t \leq \infty)$ is a non-empty closed subset of K which is invariant under the semiflow ϕ . Moreover, the following assertions are true:

(a) For $s > t$ we have $K_s \subset K_t$. In case that $K_s = K_t$ then $K_t = K_\infty$.

(b) $\phi_t(K_\infty) = K_\infty$ for all $t \geq 0$.

(c) If one partial mapping ϕ_t , $t > 0$, is injective (surjective), then all mappings ϕ_s are injective (surjective).

We call a <u>semiflow ϕ injective (surjective)</u> if one and hence all of the partial mappings ϕ_t are injective (surjective). Studying the spectrum of the semigroup given by (4.1) we divide the complex plane into three sets:

(4.3) $\{\lambda \in \mathbb{C} : \operatorname{Re}\lambda < \underline{c}(h,\phi)\}$

$\{\lambda \in \mathbb{C} : \underline{c}(h,\phi) \leq \operatorname{Re}\lambda \leq \bar{c}(h,\phi)\}$

$\{\lambda \in \mathbb{C} : \bar{c}(h,\phi) < \operatorname{Re}\lambda\}$.

The quantities $\underline{c}(h,\phi)$ and $\bar{c}(h,\phi)$ are defined as follows:

(4.4) $\bar{c}(h,\phi) := \lim_{t\to\infty} \bar{c}_t(h,\phi) = \inf_{t>0} \bar{c}_t(h,\phi)$ where

$\bar{c}_t(h,\phi) := \sup_{x\in K} \{1/t\cdot\int_0^t h(\phi(s,x))\, ds\}$ $(t > 0)$.

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It is easy to see that $\bar{c}_t(h,\phi) = 1/t\cdot\log\|T(t)\|$, hence in the definition of $\bar{c}(h,\phi)$, both the limit and the infimum exist and coincide with the growth bound (see A-I,(1.1)). Furthermore, $\underline{c}_t(h,\phi) = -\bar{c}_t(-h,\phi)$. Therefore, $\underline{c}(h,\phi)$ is well defined too.

First we will describe the part of $\sigma(A)$ which is contained in the left half-plane determined by $\underline{c}(h,\phi)$. It turns out that either the whole half-plane is contained in $\sigma(A)$ or it has empty intersection with $\sigma(A)$. This depends only on properties of ϕ . Essentially there