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## One-parameter Semigroups of Positive Operators

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## **Contents**

List of Symbols

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$E_{\mathbb{R}}, E_{\mathbb{C}}$	real, complex Banach lattice
$E_{+}$	positive cone of an ordered vector space
E'	dual Banach space
$E^*$	semigroup dual
$E_F^{\mathcal{T}}$	$\mathcal{F}$ -product of $E$ with respect to the semigroup $\mathcal{T}$
$E_{\mathcal{F}}$	$\mathcal{F}$ –product of $E$
$E_f$	see C-I, 4
$(E, \varphi)$	see C-I, 4
$E \otimes F$	tensor product
$\mathcal{L}(E)$	bounded linear operators on E
$\mathcal{Z}(E)$	center of E
$E_n$	n-th Sobolev space
$\mathcal{B}(H)$	W*-algebra of all bounded linear operators on <i>H</i>
S(M)	state space of a C*-algebra <i>M</i>
$M_{+}$	positive cone of the C*-algebra M
$M_*$	predual of a W*-algebra M
$M^{sa}$	self-adjoint part of a C*-algebra
$M_n$	$C^*$ -algebra of all $n \times n$ -matrices
AC	absolutely continuous functions
BV	functions of bounded variation
K	compact topological space
X	locally compact topological space
C(K), C(K, E)	continuous functions (with values in $E$ )
$C_c(X)$	continuous functions with compact support
$C_0(X)$	continuous functions vanishing in infinity
$C^b(X)$	bounded continuous functions
$C_{ru}(X)$	uniformly continuous functions
$C^n, C^{(n)}$	continuous differentiable functions ( <i>n</i> -times)
$C_c^{\infty}(\mathbb{R}^n)$	infinitely differentiable functions with compact support
$L^p(\mu)$	p-integrable functions

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S(\mathbb{R}^n)
                                  Schwartz space
M(X)
                                  regular Borel measures
                                  bounded regular Borel measures
M_b(X)
                                  (one-parameter) semigroup
\mathcal{T} = (T(t))_{t \ge 0}
T_{\parallel}
                                  subspace (reduced) semigroup
T_{/}
                                  quotient semigroup
Fix(\mathcal{T})
                                  fixed space of \mathcal{T}
                                  generator of a C_0-semigroup
\boldsymbol{A}
A'
                                  adjoint operator of A
A^*
                                  adjoint generator
\sigma(A)
                                  spectrum of A
\varrho(A)
                                  resolvent set of A
\sigma_{ess}(A)
                                  essential spectrum of A
\sigma_b(A)
                                  boundary spectrum of A
P_{\sigma}(A)
                                  point spectrum of A
P_{\sigma_b}(A)
                                  boundary point spectrum
A_0(A)
                                  approximate point spectrum of A
R_{\sigma}(A)
                                  residual spectrum ç
\omega; \omega(A); \omega(\mathcal{T}); \omega(T(t))
                                  growth bound
s(A)
                                  spectral bound
\omega_I(A)
                                  growth bound of the solution of the (ACP)
\omega(f)
                                  growth bound of T(\cdot) f
                                  spectral radius of A
r(A)
\omega_{ess}(A)
                                  essential growth bound of A
r_{ess}(T)
                                  essential spectral radius of A
R(\lambda, A)
                                  resolvent operator of A
I^{d}, \{I^{d}\}_{d=1}^{dd}
                                  orthogonal band of I (of I^d)
                                  infimum
Λ
                                  supremum
|T|
                                  modulus of a regular operator
\hat{f}, \check{f}
                                  Fourier (inverse Fourier) transformation
dp(f)
                                  subdifferential of p in f
                                  subdifferential of the norm in f
dN(f)
dN^+(f)
                                  subdifferential of the canonical half-norm in f
im(T)
                                  range of T
ker(T)
                                  null-space of T
\operatorname{Im} z
                                  imaginary part of z
                                  real part of z
\text{Re } z
                                  see C-I, 7
Re(f), Im(f)
Re T, Im T
                                  see C-I, 7
                                  complex conjugate of f
S_f
                                  signum operator with respect to f
sign(f)
                                  signum of f see C-II, 2.2
f^{[n]}
                                  see B-III,2.2; C-III,2.1
|f|
                                  absolute value of f
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CONTENTS 3

$f^+$	positive part of $f$
$f^-$	negative part of $f$
Id	identity operator
$M_{p}$	multiplication operator
1	function identically 1
$\mathbb{1}_B$	characteristic function of the set $XB$

 $\delta_x$ Dirac measure in x

trace tr

 $\operatorname{span} M$ linear subspace generated by Msector in the complex plane  $S(\alpha)$ (ACP)abstract Cauchy problem positive minimum principle (*P*)

(P')see B-II,1.21

Kato's (equality) inequality (K)(RCP)retarded Cauchy problem

(RE)retarded equation (T)translation property