Proof. (a) This is an immediate consequence of Thm.4.4 and Prop.4.8 .
(b) The first assertion follows form Thm.4.4 and Thm.4.1 . Moreover, as in the proof of Thm.4.4(b) and (c) we can assume without loss of generality that $K = K_\infty$, hence ϕ is bijective. If there is no upper bound for the length of the orbits, then $\sigma(A) = i\mathbb{R}$ by assertions (d) and (e) of Prop.4.8
Now we assume that the lengths of the orbits are bounded by T . Because ϕ is bijective, for every $x \in K$ there exists a $r = r_x$ with $T/2 \leq r \leq T$ such that
$\phi(t,x) = \phi(t+r,x) = \phi(t+2r,x) = \ldots = \phi(t+kr,x)$ $(t \in \mathbb{R}_+ , k \in \mathbb{N})$.
Therefore we have for $\lambda \in \mathbb{C}$, $\operatorname{Re}\lambda > 0$, $f \in C(K)$, $x \in K$:

$$
\begin{aligned}
(4.15)\qquad (R(\lambda,A)f)(x) &= \int_0^\infty e^{-\lambda t} f(\phi(t,x))\, dt = \\
&= \sum_{k=0}^{\infty} \exp(-\lambda kr)\cdot\int_{kr}^{(k+1)r} \exp(-\lambda(t-kr)\cdot f(\phi(t-kr,x))\, dt \\
&= (1 - e^{-\lambda r})^{-1}\cdot\int_0^r \exp(-\lambda t) f(\phi(t,x))\, dt .
\end{aligned}
$$

If $0 < \beta < 2\pi/T$, then the assumption $T/2 \leq r \leq T$ implies that there exists a neighborhood U of $\lambda_o := i\beta$ such that the functions $\lambda \mapsto (1 - \exp(-\lambda r_x))^{-1}$ are uniformly bounded on U , by M say. Then (4.15) implies that $\|R(\lambda,A)f\| \leq M(\int_0^r |e^{-\lambda t}|dt)\|f\|$ for $\lambda \in U$, $\operatorname{Re}\lambda > 0$, therefore $\lambda_o = i\beta \in \rho(A)$.

□

Remark 4.10. In case $\sigma(A) \subsetneqq i\mathbb{R}$, then $\phi|K_\infty$ is bijective and has only finite orbits. Therefore every point $x \in K_\infty$ has a well-defined period $\tau_x := \inf\{\tau > 0 : \phi(\tau,x) = x\}$. A more detailed analysis yields the following description of $\sigma(A)$:

$$(4.16)\qquad \sigma(A) = \overline{\{i\cdot 2\pi k/\tau_x : k \in \mathbb{Z} , x \in K_\infty , \tau_x > 0\}} \cup \{0\} .$$

The inclusion "$\subseteq$" can be derived from Thm.4.11 which is stated below. The reverse inclusion follows from Prop.4.8(d) .

In our detailed discussion of the spectrum of lattice homomorphisms we restricted ourselves to the case where the space K is compact. The main reason is that there is no description as given in (4.1) of the semigroups for locally compact spaces X . In general, it is difficult to define a semiflow on X because points may tend to infinity in a finite time. But even if one can find a flow on a suitable compactification of C , it may be impossible to find a multiplicator. This can be seen by studying the following example:
Suppose ϕ_1 is a semiflow on a compact space K_1 and K_o is a closed ϕ_1-invariant subset, h a continuous function on K_1 . The