Thus $s(|\psi_\alpha|) \leq p_j$ and even $s(|\psi_\alpha|) = p_j$ by the minimality property of p_j. On the other hand, $\psi_\alpha^* \in \mathrm{Fix}((\lambda + i\alpha)R(\lambda))$. As above we obtain

$$\mu R(\mu)' s(|\psi_\alpha^*|) = s(|\psi_\alpha^*|) .$$

Consequently, the closed left ideals $Ms(|\psi_\alpha^*|)$ and $Ms(|\psi_\alpha|)$ are R'-invariant.

Next fix $\mu \in \mathbb{R}_+$, let $S := (\mu - i\alpha)R(\mu)'$ and $T = \mu R(\mu)'$. Then $(Sx)(Sx)^* \leq T(xx^*)$, $S_*(\psi_\alpha^*) = \psi_\alpha^*$, $T_*(|\psi_\alpha^*|) = |\psi_\alpha^*|$, and T is an identity preserving Schwarz map. Since $s(|\psi_\alpha^*|)M$ is T-invariant, the assumptions of Lemma 1.2 are fulfilled and we obtain for every $x \in M$

$$S(x)u_\alpha^* = T(xu_\alpha^*) .$$

Since the closed left ideal Mp_j is S-invariant it follows

$$S(x) = T(xu_\alpha^*)u_\alpha \quad , \ x \in Mp_j,$$

(see Remark 1.3). Since u_α does not depend on $\mu \in \mathbb{R}_+$ we obtain for all $\mu \in \mathbb{R}_+$

$$\mu R(\mu + i\alpha)'x = \mu R(\mu)'(xu_\alpha^*)u_\alpha .$$

Consequently, the holomorphic functions $(\mu \to \mu R(\mu)'(xu_\alpha)u_\alpha^*)$ and $(\mu \to \mu R(\mu+i\alpha)'x)$ coincide on $\mathbb{R}_+$ from which we conclude

$$\lambda R(\lambda+i\alpha)'x = \lambda R(\lambda)'(xu_\alpha^*)u_\alpha$$

for every $\lambda \in D$ and all $x \in Mp_j$. Since the map $(y \to yu_\alpha)$ is a continuous bijection from $M(u_\alpha u_\alpha^*)$ onto Mp_j and its inverse is the map $(y \to yu_\alpha^*)$, we can deduce that

$$\dim \mathrm{Fix}((\lambda - i\alpha)R(\lambda)'|Mp_j) = \dim \mathrm{Fix}(\lambda R(\lambda)')|M(u_\alpha u_\alpha^*) \leq$$

$$\leq \dim \mathrm{Fix}(R') .$$

Since $\oplus_{j=1}^n Mp_j = M$ and $\oplus_{j=1}^n L_j = M_*$ we obtain

$$\dim \mathrm{Fix}((\lambda - i\alpha)R(\lambda)')) = \dim \mathrm{Fix}(\lambda R(\lambda)') =$$

$$= \dim \mathrm{Fix}(\lambda R(\lambda))$$