We call (RCP) an <u>abstract retarded Cauchy problem</u>.

A function $u \in C([-1,\infty),F)$ is a <u>solution</u> of (RCP), if

(a) u is right-sided differentiable at 0 and continuously differentiable for $t > 0$,

(b) $u(t) \in D(B)$ for $t \geq 0$,

(c) (RCP) is satisfied for $t \geq 0$.

To (RCP) we associate the following operator A on the Banach space E. Let A be the differential operator

$$\begin{aligned} &Af := f' \\ (3.1) \quad &D(A) := \{f \in C^1([-1,0],F) : f(0) \in D(B) \, , \, f'(0) = Bf(0) + \Phi f\}. \end{aligned}$$

First we show that A is a generator on E.

<u>Theorem</u> 3.1. The operator A defined in (3.1) is the generator of a strongly continuous semigroup $(T(t))_{t\geq 0}$ on E satisfying the "translation property"

$$\text{(T)} \qquad T(t)f(s) = \begin{cases} f(t+s) & \text{if } t+s \leq 0 \\ T(t+s)f(0) & \text{if } t+s > 0 \end{cases} \, , \; f \in E.$$

<u>Proof</u>. We argue as in B-III, Example 2.14.(b) and consider the operator $A_o f := f'$ on the domain

$D(A_o) := \{f \in C^1([-1,0],F) : f(0) \in D(B) \, , \, f'(0) = Bf(0)\}$.

If $(S(t))_{t\geq 0}$ is the semigroup on F generated by B, then A_o generates the semigroup $(T_o(t))_{t\geq 0}$ given by

$$T_o(t)f(s) = \begin{cases} f(t+s) & \text{if } t+s \leq 0 \\ S(t+s)f(0) & \text{if } t+s > 0 \end{cases} \, , \; f \in E.$$

For $\lambda > w$ define the map $S_\lambda \in L(E)$ by $S_\lambda f := f - \varepsilon_\lambda \otimes R(\lambda,B)\Phi f$ where $\varepsilon_\lambda(s) = e^{\lambda s}$ and $(h\otimes x)(s) := h(s)\cdot x$ for $h \in C[-1,0]$, $x \in F$ and $s \in [-1,0]$. Since $\|R(\lambda,B)\| \leq (\lambda-w)^{-1}$ it follows that S_λ is invertible for $\lambda > \|\Phi\| + w$ and that $\|S_\lambda^{-1}\| \leq (\lambda-w)\cdot(\lambda-\|\Phi\|-w)^{-1}$. Moreover, S_λ induces a bijection from $D(A)$ onto $D(A_o)$ such that

$$\begin{aligned} &\lambda - A = (\lambda - A_o)S_\lambda \\ (3.2) \quad &R(\lambda,A) = S_\lambda^{-1}R(\lambda,A_o). \end{aligned}$$

Proceeding as in the example mentioned above we obtain

$$\begin{aligned} \|R(\lambda,A)\| &\leq (\lambda - w)\cdot(\lambda - \|\Phi\| - w)^{-1}\cdot(\lambda - w)^{-1} \\ &\leq (\lambda - \|\Phi\| - w)^{-1} . \end{aligned}$$

Thus A is a generator by A-II, Thm.1.7.