Let us first recall some facts on normal linear functionals. If ϕ is a normal linear functional on a W*-algebra M then there exists a partial isometry $u \in M$ and a positive linear functional $|\phi| \in M_*$ such that

$$\phi(x) = |\phi|(xu) =: (u|\phi|)(x) \quad , \; x \in M$$

$$u^*u = s(|\phi|) ,$$

where $s(|\phi|)$ denotes the support projection of $|\phi|$ in M. We refer to this as the <u>polar decomposition</u> of ϕ [Takesaki (1979), Theorem III.4.2]. In addition, $|\phi|$ is uniquely determined by the following two conditions [Takesaki (1979), Proposition III.4.6]:

$$(*) \qquad \begin{aligned} &\|\phi\| = \|\,|\phi|\,\| , \\ &|\phi(x)|^2 \leq |\phi|(xx^*) \quad (x \in M) . \end{aligned}$$

For the polar decomposition of ϕ^*, where $\phi^*(x) = \phi(x^*)^*$, we obtain

$$\phi^* = u^*|\phi^*| \; , \; |\phi^*| = u|\phi|u^* \; \text{ and } \; uu^* = s(|\phi^*|) .$$

It is easy to see that $u^* \in s(|\phi|)M$.

If Ψ is a subset of the state space of a C*-algebra M, then Ψ is called <u>faithful</u> if $0 \leq x \in M$ and $\psi(x) = 0$ for all $\psi \in \Psi$ implies $x = 0$. Ψ is called <u>subinvariant</u> for a positive map $T \in L(M)$ (resp., positive semigroup $\mathcal{T}$) if $T'\psi \leq \psi$ for all $\psi \in \Psi$ (resp., $T(t)'\psi \leq \psi$ for all $T(t) \in \mathcal{T}$ and $\psi \in \Psi$). Recall that for every positive map $T \in L(M)$ there exists a state ϕ on M such that $T'\phi = r(T)\phi$ [Groh (1981), Theorem 2.1], where $r(T)$ denotes the spectral radius of T .

Let us start our investigation with two lemmas. Recall that $\mathrm{Fix}(T)$ is the fixed space of T , i.e. the set $\{ x \in M : Tx = x \}$.

<u>Lemma</u> 1.1. Suppose M to be a C*-algebra and $T \in L(M)$ an identity preserving Schwarz map.

(a) Let $b: M \times M \to M$ be a sesquilinear map such that for all $z \in M$ $b(z,z) \geq 0$. Then $b(x,x) = 0$ for some $x \in M$ if and only if $b(x,y) = 0$ and $b(y,x) = 0$ for all $y \in M$.