values and eigenfunctions one has to solve the ordinary differential equation $f' = mf - \lambda f$. Any solution has (up to a constant) the following form

$$(2.9) \quad g_\lambda(x) = \exp\{\int_0^x (m(y) - \lambda)\,dy\} = e^{-\lambda x}\cdot\exp\{\int_0^x m(y)\,dy\} .$$

We assume that $m(\infty) < 0$ and $r \geq 0$. Then g_r is differentiable with g_r , $g_r' \in C_o(X)$. Thus $g_r \in D(A)$ if and only if $g_r'(0) = \alpha g(0) - \int_0^\infty g_r(y)\,d\nu(y)$. Inserting (2.9) this condition becomes $m(0) - r = \alpha - \int_0^\infty e^{-ry}\cdot\exp\{\int_0^y m(z)\,dz\}d\nu(y)$.

By monotonicity this equation has a unique solution $r \geq 0$ if and only if

$$(2.10) \quad m(0) + \int_0^\infty \exp\{\int_0^y m(z)\,dz\}d\nu(y) \geq \alpha .$$

In case α , ν and m satisfy (2.10) and $m(\infty) < 0$ then g_r is a strictly positive eigenfunction of A corresponding to $r \geq 0$. Thus all assumptions of Cor.2.11(a) are satisfied. In addition, the semigroup is irreducible if (and only if) the support of ν is an unbounded subset of $[0,\infty)$.

Similar examples will be discussed in B-IV,Sec.3 and C-IV,Sec.3 .
We finally give a criterion for quasi-compactness of positive semigroups on spaces $C(K)$. It is based on a criterion given by Doeblin for operators on spaces of bounded measurable functions and can be easily deduced from [Lotz (1981), Prop.3].

<u>Proposition</u> 2.13. Let $\mathcal{T} = (T(t))_{t\geq 0}$ be a semigroup of Markov operators on $C(K)$, K compact satisfying the following condition.

(2.12) There exist $t_o > 0$, $0 < \mu \in M(K)$ and $\gamma \in \mathbb{R}$, $0 < \gamma < 1$ such that $T(t_o)f - \mu(f)1_K \leq \gamma\cdot 1_K$ for all $0 \leq f \leq 1_K$.

Then $\mathcal{T}$ is quasi-compact.