

Below we identify  $(M_*)^\wedge$  via  $J$  with this translation invariant subspace. From the construction the following is obvious: If  $T$  is an identity preserving Schwarz map with preadjoint  $T_* \in L(M_*)$ , then  $\hat{T}$  is an identity preserving Schwarz map on  $\hat{M}$  such that  $(T_*)^\wedge = \hat{T}'|_{(M_*)^\wedge}$ .

Theorem 2.5. Let  $T$  be an identity preserving semigroup of Schwarz type with generator  $A$  on the predual of a  $W^*$ -algebra  $M$ . If  $T$  is uniformly ergodic with finite dimensional fixed space, then every  $\gamma \in \sigma(A) \cap i\mathbb{R}$  is a pole of the resolvent  $R(\cdot, A)$  and  $\dim \ker(\gamma - A) \leq \dim \text{Fix}(T)$ .

Proof. Let  $D = \{\lambda \in \mathbb{C} : \text{Re}(\lambda) > 0\}$  and  $R$  the  $M_*$ -valued pseudo-resolvent of Schwarz type induced by  $R(\cdot, A)$  on  $D$ . Then

$$P = \lim_{\mu \downarrow 0} \mu R(\mu)$$

exists in the uniform operator topology and  $\text{rank}(P) = \dim \text{Fix}(T) < \infty$ . From this we obtain  $\text{rank}(P) = \text{rank}(\hat{P}) < \infty$  where  $\hat{P}$  is the canonical extension of  $P$  onto  $(M_*)^\wedge$ . Since  $\hat{P} = \lim_{\mu \downarrow 0} \mu R(\mu)^\wedge$  it follows that

$$\dim \text{Fix}((\lambda - i\alpha)\hat{R}(\lambda)) \leq \text{rank}(\hat{P}) < \infty$$

(Proposition 2.1) for all  $\alpha \in \mathbb{R}$ . Therefore the assertion follows from Lemma 2.2.

□

The consequences of this result for the asymptotic behavior of one-parameter semigroups will be discussed in D-IV, Section 4.

#### NOTES.

Section 1. The Perron-Frobenius theory for a single positive operator on a non-commutative operator algebra is worked out in Albeverio-Høegh-Krohn (1978) and Groh (1981). The limitations of the theory (in the continuous as in the discrete case)