Remark 4.2. (a) If $\dim \operatorname{Fix}(R) \geq 2$ then it follows from the Jordan decomposition of self adjoint linear functionals, that there are at least two states in $\operatorname{Fix}(R)$ which have orthogonal support (compare the proof of D-III, Theorem 1.10.(a)).

(b) If R is a pseudo-resolvent with values in a W*-algebra such that $\operatorname{Fix}(R')$ is contained in M_*, then it follows from the proof of D-III, Lemma 1.2 that there exists a sequence of normal states in $\operatorname{Fix}(R')$ whith orthogonal supports in M.

Lemma 4.3. Let R be an identity preserving pseudo-resolvent of Schwarz type on $D = \{\lambda \in \mathbb{C} : \operatorname{Re}(\lambda) > 0\}$ with values in the predual of a W*-algebra M. If the fixed space of the canonical extension $\hat{R}$ of R to some ultrapower of M_* is infinite dimensional, then there exists a sequence (z_n) in M_1^+ and a sequence of states (ϕ_n) in M_* such that:

(a) $\lim_n z_n = 0$ in the $s^*(M,M_*)$-topology.

(b) $\lim_n \|(Id - \lambda R(\lambda))\phi_n\| = 0$ for all $\lambda \in D$.

(c) $\phi_n(z_n) \geq \frac{1}{2}$ for all $n \in \mathbb{N}$.

Proof. Let $(M_*)^\wedge$ be the ultrapower of M_* with respect to some free ultrafilter $\mathcal{U}$ on $\mathbb{N}$. Since $(M_*)^\wedge$ is the predual of a W*-subalgebra of $\hat{M}''$ (see D-III, Remark 2.4.(b)), there exists a sequence of states $(\hat{\psi}_n)$ in $\operatorname{Fix}(\hat{R})$ such that the corresponding support projections are mutually orthogonal in $\hat{M}''$ (Lemma 4.1). For every $n \in \mathbb{N}$ let $(\psi_{n,k}) \in \hat{\psi}_n$ be a representing sequence of states, let

$$\phi := \sum_{n,k} 2^{-(n+k+1)} \psi_{n,k}$$

and let

$$p := \sup\{s(\psi_{n,k}) : n,k=1,..\}$$

in M. Then ϕ is a normal state on M which is faithful on the W*-algebra M_p. Since

$$1 = \langle \psi_{n,k}, s(\psi_{n,k})\rangle = \psi_{n,k}(p) \qquad (n,k \in \mathbb{N})$$

it follows $\hat{\psi}_n(\hat{p}) = 1$ where $\hat{p}$ is the canonical image of p in $\hat{M}$.