A-II,(1.8). Since $\delta 1 = 0$ one has $T_o(t)1 = 1$ for all $t \geq 0$ and the equivalence of (i) and (ii) follows with the help of Theorem 3.4. Now assume that (i) and (ii) hold. Let

$$(S(t)f)(x) = \exp\left(\int_0^t h(\phi(s,x))ds\right) \cdot f(\phi(t,x))$$

for all $x \in K$, $f \in C(K)$, $t \geq 0$. Then one easily shows that $(S(t))_{t \geq 0}$ is a strongly continuous semigroup. Denote by B its generator. For $f \in D(\delta)$, $\frac{d}{dt}\big|_{t=0} S(t)f = h \cdot f + \delta f$. Hence $\delta + h \subset B$. Since $\delta + h$ also is a generator, it follows that $\delta + h = B$.

□

Theorem 3.6. An operator A is generator of a lattice semigroup on $C(K)$ if and only if there exists a derivation δ which is a generator, a function $h \in C(K)$ and a strictly positive function $p \in C(K)$ such that

(3.7) $\quad A = M\delta M^{-1} + h$

where $M \in \mathcal{L}(C(K))$ is given by $Mf = p \cdot f$.

Proof. In order to show the non-trivial implication assume that A generates a lattice semigroup. Since $D(A)$ is dense in $C(K)$, there exists $0 \ll p \in D(A)$. Let $h(x) = (Ap)(x)/p(x)$. The operator given by $Mf = f \cdot p$ is a lattice isomorphism. Thus $\delta := M^{-1}(A - h)M$ generates a lattice semigroup. Since $M1 = p \in D(A)$ one has $1 \in D(\delta)$ and $\delta 1 = M^{-1}(A - h)p = 0$. Thus δ is the generator of a semigroup of algebra homomorphisms, hence a derivation by Theorem 3.4.

□

At the end of this section we will show that any derivation on $C[0,1]$ which generates a group is similar to a differential operator of first order. This in connection with Theorem 3.6. will enable us to describe all generators of positive groups as perturbations of a differential operator.

In Section 1 we had obtained a very simple condition describing generators of positive semigroups on $C(K)$ by the positive minimum principle and a range condition. This result yields a characterization of generators of automorphism groups by "locality" and a range condition. By an automorphism we understand an algebra isomorphism of $C(K)$ onto itself.