tive elements the equation (1.3) holds for every $\Phi \in C_o(X)^*$, $\rho \in C_o(X)''$. This means that the net $(\int_0^r T(t)^*\Phi\,dt)_{r>0}$ converges weakly to $R(0,A^*)\Phi$. But for positive Φ the net is monotone and therefore strongly convergent by Dini's Theorem (see Schaefer (1974), II.Thm.5.9). Hence $R(0,A^*)\Phi = \int_0^\infty T(t)^*\Phi\,dt$ for every $\Phi \in C_o(X)^*$.
Now we make use of the special character of the space $C_o(X)$. For positive functions f_1, $f_2 \in C_o(X)$ we have $\sup(\|f_1\|,\|f_2\|) = \|\sup(f_1,f_2)\|$. Let μ_1, $\mu_2 \in C_o(X)'_+$ and $\varepsilon > 0$. Then there are positive elements f, g in the unit ball of $C_o(X)$ such that $\langle f,\mu\rangle \geq \|\mu\| - \varepsilon$ and $\langle g,\mu_2\rangle \geq \|\mu_2\| - \varepsilon$. For $h := \sup(f,g)$ we obtain $\|h\| \leq 1$ and
$\|\mu_1 + \mu_2\| \geq \langle h,\mu_1 + \mu_2\rangle \geq \langle f,\mu_1\rangle + \langle g,\mu_2\rangle \geq \|\mu_1\| + \|\mu_2\| - 2\varepsilon$.
Hence $\|\mu_1 + \mu_2\| = \|\mu_1\| + \|\mu_2\|$ for μ_1, $\mu_2 \in C_o(X)'_+$ (see also C-I). Approximating the integral by Riemann sums one obtains
$\|\int_0^r T(t)^*\mu\,dt\| = \int_0^r \|T(t)^*\mu\|\,dt$ for $\mu \in C_o(X)^*_+$, $r > 0$ and therefore, for $r \to \infty$, $\|R(0,A^*)\mu\| = \|\int_0^\infty T(t)^*\mu\,dt\| = \int_0^\infty \|T(t)^*\mu\|\,dt$ ($\mu \in C_o(X)^*_+$). Given $\mu \in C_o(X)'$ there is a sequence $\mu_n \in C_o(X)^*_+$ converging $\sigma(E',E)$ to $|\mu|$ (Lemma 1.3). From $|\langle f,T(t)'\mu\rangle| \leq \langle T(t)|f|,|\mu|\rangle = \lim_{n\to\infty} \langle T(t)|f|,\mu_n\rangle$ we conclude $|\langle f,T(t)'\mu\rangle| \leq \liminf_{n\to\infty}\|f\|\;\|T(t)^*\mu_n\|$ and therefore $\|T(t)'\mu\| \leq \liminf_{n\to\infty}\|T(t)^*\mu_n\|$ ($t \geq 0$). Applying Fatou's Lemma we obtain
$\int_0^\infty \|T(t)'\mu\|\,dt \leq \int_0^\infty (\liminf \|T(t)'\mu\|)\,dt \leq$
$\liminf \int_0^\infty \|T(t)'\mu\|\,dt = \liminf \|R(0,A^*)\mu_n\| \leq \|R(0,A^*)\|\cdot\liminf \|\mu_n\| < \infty$.
(observe that $t \to \|T(t)'\mu\| = \sup\{\langle T(t)f,\mu\rangle : \|f\| \leq 1\}$ is lower semi-continuous and hence measurable). Using A-IV,Thm.1.10 we obtain $\omega(A^*) < 0$. But $\omega(A) = \omega(A^*)$ by A-III,4.4(iii), which contradicts $\omega(A) = 0$.

□

<u>Remark</u> 1.5. If $(T(t))$ is a positive semigroup on an α-directed ordered Banach space E (see Asimow-Ellis (1980),p.39), then the dual of E admits a reversion of the triangle inequality;
i.e. $\sum\|\mu_i\| \leq \alpha\|\sum\mu_i\|$ for $\mu_i \in E'_+$, and Theorem 1.4 remains valid (see Batty-Davies (1983)). The proof given above may be used with almost no modification.

At this point we close the discussion of the stability of positive semigroups on $C_o(X)$ and refer to Section 1 of C-IV and D-IV respectively, where the stability of positive semigroups on arbitrary Banach lattices and on C^*-algebras will be treated.