In case $G = C(K)$ these conditions are equivalent to the following:

(iv) if $(f_n) \subset C(K)$ is a bounded sequence then (Rf_n) has a subsequence which converges pointwise to a continuous function.

(b) If R is weakly compact then it maps weakly convergent sequences into norm convergent sequences. In particular, the square of a weakly compact operator $T : C(K) \to C(K)$ is a compact operator.

<u>Proof</u>.(a) (i)→(ii) follows from the following characterization of weakly compact operators (see e.g.,II.Prop.9.4 of Schaefer (1974)):

An operator is weakly compact if and only if its second adjoint maps the bidual into the original space.

(ii)→(iii) is trivial and it remains to show that (iii) implies (i): On the Borel field $\mathcal{B}$ we define m by $m(C) := R''(\chi_C)$. Then m is a G-valued additive set function. For $y' \in G'$ we have $y' \circ m = R'y' \in M(K)$. Hence for every $y' \in G'$ $y' \circ m$ is a countable additive set function, i.e., m is weakly countably additive. By Pettis' Theorem (see IV.Thm.10.1 in Dunford-Schwartz (1958)) we have that m is countably additive with respect to the norm. In particular, for a sequence U_n of mutually disjoint Borel sets we have $\lim_{n\to\infty} \|m(U_n)\| = 0$. It follows that $\lim_{n\to\infty} y' \circ m(U_n) = 0$ uniformly for $y' \in G'$, $\|y'\| \leq 1$. Now condition (iii) of Prop.2.3 shows that $\{R'y' : y' \in G' , \|y'\| \leq 1\}$ is relatively weakly compact, i.e., R' is weakly compact. Thus R is weakly compact as well.

In case $G = C(K)$ the equivalence of (i) and (iv) is a consequence of two results: First, Eberlein's Theorem states that for the weak topology in any Banach space compactness and sequential compactness are equivalent. Second, Lebesgue's Dominated Convergence Theorem assures that a sequence $(f_n) \subset C(K)$ converges weakly to $f \in C(K)$ if and only if it is bounded and $f_n(x) \to f(x)$ for every $x \in K$.

(b) Suppose (f_n) is a sequence in $C(K)$ which converges to 0 for the weak topology. Since R is weakly compact the same is true for the adjoint R', i.e., $\{R'y' : y' \in G' , \|y'\| \leq 1\}$ is relatively weakly compact in $M(K)$. From Prop.2.3 (i)→(ii) we obtain that $\langle Rf_n, y'\rangle = \langle f_n, R'y'\rangle \to 0$ as $n \to \infty$ uniformly for $y' \in G'$, $\|y'\| \leq 1$. That is $\lim_{n\to\infty} \|Rf_n\| = 0$.

The final assertion follows from the first and the characterization of weakly compact operators stated in (iv) of (a) .

□