<u>Remark</u> 3.1. Let $(S(t))_{t\geq 0}$ be a semigroup on a complex Banach lattice E with generator A. Then $S(t)E_{\mathbb{R}} \subset E_{\mathbb{R}}$ for all $t \geq 0$ if and only if

(3.1) $f \in D(A)$ implies $\bar{f} \in D(A)$ and $A\bar{f} = (Af)^{-}$.

In that case the generator $A_{\mathbb{R}}$ of the restriction semigroup on $E_{\mathbb{R}}$ is given by $A_{\mathbb{R}}f = Af$ on $D(A_{\mathbb{R}}) = D(A) \cap E_{\mathbb{R}}$.

We will see below that for generators of a strongly continuous semigroup Kato's inequality alone is not sufficient to ensure the positivity of the semigroup. So we introduce another condition.

<u>Definition</u> 3.2. A subset M' of E' is called <u>strictly positive</u> if for every $f \in E_+$, $\langle f,\phi\rangle = 0$ for all $\phi \in M'$ implies $f = 0$. Accordingly, an element ϕ of E'_+ is called <u>strictly positive</u> if the set $\{\phi\}$ is strictly positive.

<u>Example</u> 3.3. Let $E = L^p(X,\mu)$ $(1 \leq p < \infty)$, where (X,μ) is a σ-finite measure space. Then $\phi \in E' = L^q(X,\mu)$ (where $1/p + 1/q = 1$) is strictly positive if and only if $\phi(x) > 0$ μ-a.e. Note that strictly positive elements of E' always exist in this case.

<u>Definition</u> 3.4. Let B be an operator on a Banach lattice F and let $u \in F$. Then u is called a <u>positive subeigenvector</u> of B if

a) $0 < u \in D(B)$ and

b) $Bu \leq \lambda u$ for some $\lambda \in \mathbb{R}$.

<u>Proposition</u> 3.5. Let $(T(t))_{t\geq 0}$ be a positive semigroup on a real Banach lattice with generator A . Then there exists a strictly positive set M' of subeigenvectors of A' (the adjoint of the generator A). Moreover, if there exist strictly positive linear forms on E , then there exists a strictly positive subeigenvector of A'.

<u>Proof</u>. Let $\lambda > 0$ be such that $R(\lambda,A) = (\lambda - A)^{-1}$ exists and such that $R(\lambda,A) \geq 0$. Let $N' \subset E'_+$ be strictly positive. Then $M' := \{R(\lambda,A)'\psi : \psi \in N'\} \subset D(A')_+$. We show that M' is strictly positive. Indeed, let $f \in E_+$ such that $\langle f,\phi\rangle = 0$ for all $\phi \in M'$. Then $\langle R(\lambda,A)f,\psi\rangle = 0$ for all $\psi \in N'$. Hence $R(\lambda,A)f = 0$ since N' is strictly positive. Consequently, $f = 0$. The set M' consists of