$t \to S(t)\tilde{g}$ is not differentiable in $t' \in \mathbb{R}_+$.
Assume that there exists a solution of (RCP). By the preceding considerations

$u(t) = S(t)g(0) + \int_0^t S(t-s)\Phi(u_s)\,ds = S(t)\tilde{g} + \int_0^t S(t-s)\Phi(u_s)\,ds.$

Thus u is not differentiable in t' and we have a contradiction.

<u>Corollary</u> 3.3. Keep the above notation and let F be finite dimensional. Then the solution semigroup $(T(t))_{t\geq 0}$ in E corresponding to (RCP) is compact for each $t > 1$ and therefore is eventually norm continuous.

<u>Proof</u>. Let $t > 1$. By the translation property (T) we have $T(t)f(s) = T(t+s)f(0)$. Whenever $t + s > 0$ then Rem.2 following Cor.3.2 shows that $(T(t)f)(s) = (T(t+s)f)(0) = u(t+s)$ is differentiable with respect to $s \in [-1,0]$ for each $f \in E$.
Since $t > 1$ we thus have $T(t)f \in C^1([-1,0],F)$ for all $f \in E$. The closed graph theorem yields the continuity of $T(t)$ from E into C^1. Hence $T(t)$ maps the unit ball of E into a bounded set of $C^1([-1,0],F)$. Again we use the assertion that $\dim F < \infty$ and obtain by the theorem of Arzela-Ascoli that every bounded set of $C^1([-1,0],F)$ is relatively compact in E. Thus $T(t)$ is compact for each $t > 1$.

□

The assertion of Cor.3.3 remains true if $(S(t))_{t\geq 0}$ is a compact semigroup on a (not necessarily finite dimensional) Banach space F (see [Travis-Webb (1974)]).

In order to describe the asymptotic behavior of the solutions of (RCP) it is enough to examine the corresponding semigroup $(T(t))_{t\geq 0}$ on E. Indeed, Cor.3.2 shows that the solutions u are given by $u(t) = T(t)g(0)$ for all $t > 0$ and thus the long term behavior of u can be deduced from that one of $(T(t))_{t\geq 0}$. Our approach is based on the characterization of the stability of the semigroup $(T(t))_{t\geq 0}$ by the location of the spectrum $\sigma(A)$ of the generator A as developed in A-IV,Sec.1, B-IV,Sec.1 and C-IV,Sec.1 .

We define, for $\lambda \in \mathbb{C}$, operators $\Phi_\lambda \in L(F)$ by

$$(3.3) \qquad \Phi_\lambda x := \Phi(\varepsilon_\lambda \otimes x) \;,\; x \in F \;.$$

Since Φ_λ is bounded the operator $B + \Phi_\lambda$ is a generator on F. The spectrum of A can now be characterized in terms of the spectrum of the operators $B + \Phi_\lambda$.