that $D(A)$ is an ideal. Assume that $\inf\{|h|,|f|\} = 0$. Denote by P the band projection onto $\{|h|\}^{dd}$. Then $PAf = APf = A0 = 0$. Thus $Af \in \{|h|\}^{d}$. We have proved that A is local.

□

Corollary 5.14. A multiplication semigroup $(T(t))_{t\geq 0}$ on a complex Banach lattice E with order continuous norm is positive if and only if its generator A is real; i.e., $f \in D(A)$ implies $\bar{f} \in D(A)$ and $A\bar{f} = \overline{(Af)}$.

Proof. The condition is equivalent to $T(t)E_{\mathbb{R}} \subset E_{\mathbb{R}}$ $(t \geq 0)$ (cf. Rem. 3.1), so it is clearly necessary. Conversely, if A is real, then denote by $(T_{\mathbb{R}}(t))_{t\geq 0}$ the restriction semigroup on $E_{\mathbb{R}}$ and by $A_{\mathbb{R}}$ its generator. Then $A_{\mathbb{R}}$ is local (since A is local) and $D(A_{\mathbb{R}})$ is a sublattice of $E_{\mathbb{R}}$. Thus $(T_{\mathbb{R}}(t))_{t\geq 0}$ is a lattice semigroup (and so positive) by Cor. 5.9.

□

The class of bounded operators which generate a lattice semigroup is very restricted.

Proposition 5.15. Let E be a real or complex Banach lattice and $A \in \mathcal{L}(E)$. The following assertions are equivalent.

(i) $A \in \mathcal{Z}(E)$.

(ii) e^{tA} is disjointness preserving for all $t \geq 0$.

(iii) $e^{tA} \in \mathcal{Z}(E)$ for all $t \in \mathbb{R}$.

Moreover, if $A \in \mathcal{Z}(E)$ is real, then $e^{tA} \geq 0$ for all $t \in \mathbb{R}$.

Proof. Since $\mathcal{Z}(E)$ is a closed subalgebra of $\mathcal{L}(E)$ (see C-I, Sec. 9), it is clear that (i) implies (iii). Assertion (ii) follows trivially from (iii). If (ii) holds, then A is local by Prop. 5.4. Hence $A \in \mathcal{Z}(E)$.

The last assertion follows from the fact that $\mathcal{Z}(E)$ is isomorphic to a space $C(K)$ as a Banach lattice and a Banach algebra.

□

Proposition. 5.16. Let E be a complex Banach lattice.

Every strongly continuous group $(T(t))_{t\geq 0}$ of real operators contained in $\mathcal{Z}(E)$ has a bounded generator.