PART C

POSITIVE SEMIGROUPS ON BANACH LATTICES

CHAPTER C-I

B A S I C R E S U L T S O N B A N A C H L A T T I C E S A N D P O S I T I V E O P E R A T O R S

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This introductory chapter is intended to give a brief exposition of those results on Banach lattices and ordered Banach spaces which are indispensable for an understanding of the subsequent chapters. We do not want to give proofs of the results we are going to present, since these can easily be found in the literature (e.g., in Schaefer 1974). We rather want to give the reader who is unfamiliar with these results or with the terminology used in this book the necessary information for an intelligent reading of the main discussions. Since relatively few general results on ordered Banach spaces are needed, we will primarily talk about Banach lattices. The scalar field will be $\mathbb{R}$ except for the last three sections, where complex Banach lattices will be discussed.

The notion of a Banach lattice was devised to get a common abstract setting within which one could talk about phenomena related to positivity that had previously been studied in various types of spaces of real-valued functions, such as the spaces $C(K)$ of continuous functions on a compact topological space K, the Lebesgue spaces $L^1(\mu)$ or more generally the spaces $L^p(\mu)$ constructed over a measure space (X,Σ,μ) for $1 \leq p \leq \infty$. Thus it is a good idea to think of such spaces first in order to get a feeling for the concrete meaning of the abstract notions we are going to introduce. Later we will see that the connections between abstract Banach lattices and the "concrete" function lattices $C(K)$ and $L^1(\mu)$ are closer than one might think at first. We will use without further explanation the terms order relation (ordering), ordered set, majorant, minorant, supremum, infimum.