is E, and the spectra of the restrictions satisfy

$\sigma(A|I_\mu) = \sigma(A)\cap\{\lambda \in \mathbb{C} : \mathrm{Re}\,\lambda < \mu\}$,

$\sigma(A|J_\mu) = \sigma(A)\cap\{\lambda \in \mathbb{C} : \mathrm{Re}\,\lambda > \mu\}$.

Proof. At first we consider I_μ. Obviously it is a closed subset. From Lemma 4.7 we deduce that it is a lattice ideal. Moreover, I_μ is $R(\mu,A)$-invariant and $(T(t))_{t\in\mathbb{R}}$-invariant as well (use Lemma 4.7 again).
Since $-A$ is the generator of the positive group $(T(-t))_{t\in\mathbb{R}}$ and $J_\mu = \{f \in E : R(-\mu,-A)|f| \geq 0\}$, J_μ has the same properties.
If $f \in I_\mu\cap J_\mu$ then $R(\mu,A)|f| = 0$ hence $f = 0$ which shows that $I_\mu\cap J_\mu = \{0\}$. On the other hand, decomposing $0 \leq h = h_1 + h_2 \in E_+$ according to Lemma 4.6 , then assertion (b) of this lemma implies that $h_1 \in I_\mu$, while assertion (c) ensures that $h_2 \in J_\mu$. Since the positive cone E_+ is generating we have $E = I_\mu\oplus J_\mu$ and the first part of the theorem is proved.
Since I_μ is $R(\mu,A)$-invariant we have $\mu \in \rho(A|I_\mu)$ and $R(\mu,A|I_\mu) = R(\mu,A)|I_\mu \geq 0$. C-III,Thm.1.1(b) then implies $\sigma(A|I_\mu) \subset \{\lambda \in \mathbb{C} : \mathrm{Re}\,\lambda < \mu\}$. The same argument applied to $-A$ and $-\mu$ yields $\sigma(A|J_\mu) \subset \{\lambda \in \mathbb{C} : \mathrm{Re}\,\lambda > \mu\}$. Now the assertion follows from A-III,Prop.4.2.

□

The spectral projections corresponding to the spectral decomposition described above have the expected representation as an integral 'around' the spectral sets, see Corollary 3 in Greiner (1984c).

Corollary 4.9. Assume that the assumptions of the theorem are satisfied, $\mu \in \rho(A)\cap\mathbb{R}$, $\beta > s(A)$, $\alpha < -s(-A)$. If we denote the projections corresponding to the decomposition $E = I_\mu\oplus J_\mu$ by P_μ and Q_μ (i.e., $P_\mu E = \ker Q_\mu = I_\mu$, $Q_\mu E = \ker P_\mu = J_\mu$), then for $f \in D(A^2)$ we have

$$\begin{aligned} P_\mu f &= \frac{1}{2\pi}\cdot\int_{-\infty}^{\infty} R(\mu+i\tau,A)f\,d\tau - \frac{1}{2\pi}\cdot\int_{-\infty}^{\infty} R(\alpha+i\tau,A)f\,d\tau ,\\ Q_\mu f &= \frac{1}{2\pi}\cdot\int_{-\infty}^{\infty} R(\beta+i\tau,A)f\,d\tau - \frac{1}{2\pi}\cdot\int_{-\infty}^{\infty} R(\mu+i\tau,A)f\,d\tau . \end{aligned} \tag{4.10}$$

(The integrals appearing in (4.10) are improper Riemann integrals.)

We mention another consequence of Thm.4.8. Like Prop.4.5 it is a spectral mapping theorem for the real part of the spectrum.