The infinite dimensional problem of determining $s(A)$ is therefore reduced to solving a real one-dimensional equation.

The differential equation (3.13) may be interpreted as follows. Consider n populations and let r be the maximal age of an individual. Further let $u_i(t,\alpha)$ denote the density of the number of members of the population i with respect to age α at time t. The birth-rate is denoted by β. The first equation expresses the process of growing old. The second equation defines the initial state at time zero and the last equation describes mutual dependences of birth from individuals of the n populations.

Example 3.13. Take $F := L^1(\Omega)$ where $\Omega \subset \mathbb{R}^2$ is bounded and take $E := L^1([0,r],F) = L^1([0,r] \times \Omega)$ for some $r \in \mathbb{R}_+$. Furthermore, let $\Phi = \beta \otimes B$ where $0 < \beta \in L^\infty[0,r]$ and $B \in \mathcal{L}(F)$ is an integral operator with positive bounded kernel k.
The corresponding Cauchy problem is the following.

$$\frac{\partial}{\partial t}u(t,\alpha,x) = -\frac{\partial}{\partial\alpha}u(t,\alpha,x)$$

with initial condition

$$u(0,\alpha,x) = v(\alpha,x) \tag{3.14}$$

and boundary condition

$$u(t,0,x) = \int_0^r \beta(\alpha)\cdot\int_\Omega k(x,y)\cdot u(t,\alpha,y)\,dy\,d\alpha \quad \text{for } t\in\mathbb{R}_+,\ x\in\Omega,\ \alpha\in[0,r].$$

From Thm.3.1 we conclude that for every $v \in D(A)$ there exists a solution $u \in E$. The boundedness of the integral kernel k yields weak compactness of B (see [Schaefer (1974),Sec.II.5] and thus compactness of B^2 by the Dunford-Pettis-Property of $L^1(\Omega)$ (see [Schaefer (1974),Chap.II,Thm.9.9]).

Thus we are in the situation of Ex.3.9 and from Formula (3.10) we obtain the equivalence

$$s(A) \lesseqgtr \lambda \quad \text{if and only if} \quad \int_0^r \beta(\alpha)e^{-\lambda\alpha}\,d\alpha \cdot s(B) \lesseqgtr 1\ .$$

Again we can find a biological interpretation. Let $u(t,\alpha,x)$ denote the density of the number of individuals in a given population with respect to age α and position x at time t. As in Ex.3.12 the first equation in (3.14) corresponds to the aging process. The second equation fixes the initial state of the population and the last equation describes the dependence of newborns on the birth rate β and the distribution of the population over the "area" Ω.