<u>Proof</u>. We only consider $1 \leq p < 2$. The assertion for $p > 2$ then follows by duality while $p = 2$ was treated in Prop.2.13.
At first we observe that without loss of generality we may assume that μ is a probability measure and that $T(t)1 = 1$ for every $t \geq 0$. In fact, the assumptions imply that $T(t)h = h$ for some $h >> 0$, $\|h\|_p = 1$. We consider the measure ν which has the density h^p with respect to μ . Then ν is a probability measure and $M : L^p(\nu) \to L^p(\mu)$; defined by $Mh := hf$, is an isometric lattice isomorphism of $L^p(\nu)$ onto $L^p(\mu)$. The semigroup defined by $\tilde{T}(t) := M^{-1}T(t)M$ possesses the same properties as $(T(t))$ and satisfies $\tilde{T}(t)1 = 1$ for $t \geq 0$.
Now the properties $T(t)1 = 1$, $T(t) \geq 0$ imply that $L^\infty(\mu)$ is an invariant subspace for every operator $T(t)$ which is contractive with respect to the L^∞-norm. The Riesz Convexity Theorem [Dunford-Schwartz (1958), VI.10.11] then implies that by restricting the semigroup $(T(t))$ to $L^q(\mu)$ $(p < q < \infty)$ we obtain a strongly continuous semigroup $(T_q(t))_{t\geq 0}$ on $L^q(\mu)$ such that $\|T_q(t)\| \leq \|T(t)\|^{p/q}$ for $t \geq 0$, $q \geq p$.
Let A_q be the generator of $(T_q(t))$. In order to apply Prop.2.13 we have to show that 0 is a pole of the resolvent of A_2 . Denoting the residue of $R(.,A)$ at 0 by P then $P = h\otimes 1$ for a suitable $h \in (L^p(\mu))'$. Since $(L^p(\mu))' \subset (L^2(\mu))'$, P can also be considered as bounded operator on $L^2(\mu)$. We denote it by P_2. From $AP = PA = 0$ it follows that

$$(R(1,A)(Id-P))^n = R(1,A)^n - P \quad (n \in \mathbb{N}) \quad \text{and}$$
$$(R(1,A_2)(Id-P_2))^n = R(1,A_2)^n - P_2 \quad (n \in \mathbb{N}) \ .$$

The Riesz Convexity Theorem yields the following estimate for the operator norm:

$$\begin{aligned}\|R(1,A_2)^n - P_2\| &\leq \|R(1,A)^n - P\|^{2/p}\|R(1,A)^n_\infty - P_\infty\|^{1-2/p}\\ &\leq \|R(1,A)^n - P\|^{2/p}(1 + \|P_\infty\|)^{1-2/p} \ .\end{aligned}$$

Since 0 is a pole with residue P , the spectral radius of the operator $R(1,A)(1-P)$ is less than 1 . Thus for the right hand side of the inequality tends to 0 as $n \to \infty$. It follows that $r_{ess}(R(1,A_2)) < 1$, hence 1 is a pole of the resolvent of $R(1,A_2)$, or equivalently, 0 is a pole of $R(.,A_2)$ (see A-III,Prop.2.5).
Now we can apply Prop.2.13 and obtain a projection Q such that $\lim_{t\to\infty}\|T(t)f - R_\tau(t)\circ Qf\|_2 = 0$ for every $f \in L^2(\mu)$. On order intervals of $L^\infty(\mu)$ both, L^p- and L^2-norm induce the same topology (see