This theorem can be proved in the same way as Theorem 3.8.

Remark. If X is separable, then there exist strictly positive measures on $C_o(X)$. In that case the analogue of Corollary 3.9 holds as well.

Now we want to discuss the results obtained so far.

As a first example we consider the first derivative with boundary conditions on $E = L^p[0,1]$ $(1 \leq p < \infty)$. By $AC[0,1]$ we denote the space of all absolutely continuous functions on $[0,1]$. Let A_{max} be given by

$$D(A_{max}) = \{f \in AC[0,1] : f' \in L^p[0,1]\}$$
$$A_{max}f = f' \quad (f \in D(A_{max})) .$$

The following lemma is easy to prove.

Lemma 3.14. Let $f \in AC[0,1]$. Then $|f| \in AC[0,1]$ and $|f|' = (\text{sign } f)\cdot f'$ (a.e.).

As a consequence of the lemma, $D(A_{max})$ is a sublattice of E and

$$(3.7) \qquad (\text{sign } f)A_{max}f = A_{max}|f| \quad (f \in D(A_{max})) .$$

For $\lambda > 0$ one has

$$(3.8) \qquad \ker(\lambda - A_{max}) = \mathbb{R}\cdot e_\lambda \quad \text{where } e_\lambda(x) = e^{\lambda x} .$$

Hence A_{max} is not a generator. We impose the following boundary conditions.

Let $d \in \mathbb{R}$. Consider the restriction A_d of A_{max} to the domain

$$D(A_d) = \{f \in D(A_{max}) : f(1) = df(0)\} .$$

Then A_d is the generator of the semigroup $(T_d(t))_{t\geq 0}$ given by

$$(3.9) \qquad T_d(t)f(x) = d^n\cdot f(x+t-n) \quad \text{if } x+t \in [n, n+1) \quad (n \in \mathbb{N}) .$$

This is not difficult to prove. Actually (3.9) defines a group if $d \neq 0$ and if we let $t \in \mathbb{R}$, $n \in \mathbb{Z}$. For $d = 0$ one obtains the nilpotent shift semigroup on E . It follows from (3.9) that the semigroup $(T_d(t))_{t\geq 0}$ is positive if and only if $d \geq 0$.

Let us fix $d < 0$. Let $A = A_d$ and $T(t) = T_d(t)$ for $t \geq 0$. Then