and the assertion follows from Lemma 1.6.

□

Before going on let us recall the basic facts of the ultrapower $\hat{E}$ of a Banach space E with respect to some free ultrafilter $\mathcal{U}$ on $\mathbb{N}$ (compare A-I,3.6). If $\ell^\infty(E)$ is the Banach space of all bounded functions on $\mathbb{N}$ with values in E, then

$$c_{\mathcal{U}}(E) := \{(x_n) \in \ell^\infty(E) : \lim_{\mathcal{U}} \|x_n\| = 0\}$$

is a closed subspace of $\ell^\infty(E)$ and equal to the kernel of the seminorm

$$\|(x_n)\| := \lim_{\mathcal{U}} \|x_n\| , \quad (x_n) \in \ell^\infty(E) .$$

By the ultrapower $\hat{E}$ we understand the quotient space $\ell^\infty(E)/c_{\mathcal{U}}(E)$ with norm

$$\|\hat{x}\| = \lim_{\mathcal{U}} \|x_n\| , \quad (x_n) \in \hat{x} \in \hat{E} .$$

Moreover, for a bounded linear operator $T \in L(E)$, we denote by $\hat{T}$ the well defined operator $\hat{T}\hat{x} := (Tx_n) + c_{\mathcal{U}}(E)$, $(x_n) \in \hat{x}$. It is clear by virtue of $(x \to (x, x, \ldots) + c_{\mathcal{U}}(E))$ that each $x \in E$ defines an element $\hat{x} \in \hat{E}$. This isometric embedding as well as the operator map $(T \to \hat{T})$ are called canonical. In particular, if $R: (D \to L(E))$ is a pseudo-resolvent, then

$$\hat{R} := (\lambda \to R(\lambda)\hat{\ }) : D \to L(\hat{E}) ,$$

is a pseudo-resolvent, too. Recall that the approximative point spectrum $A\sigma(T)$ is equal to the point spectrum $P\sigma(\hat{T})$ (see, e.g., [Schaefer (1974), Chapter V, §1]). This construction gives us the possibility to characterize uniformly ergodic semigroups with finite dimensional fixed space.

<u>Lemma</u> 2.2. Let R be a pseudo-resolvent on $D = \{\lambda \in \mathbb{C}: \mathrm{Re}(\lambda) > 0\}$ such that $\|\mu R(\mu+i\alpha)\| \leq 1$ for all $(\mu,\alpha) \in \mathbb{R}_+ \times \mathbb{R}$ and suppose

$$0 < \dim \mathrm{Fix}((\lambda - i\alpha)\hat{R}(\lambda)) < \infty \quad \text{for some } \lambda \in D , \ \alpha \in \mathbb{R}$$

and the canonical extension $\hat{R}$ on some ultrapower $\hat{E}$.