PART A

ONE-PARAMETER SEMIGROUPS ON BANACH SPACES

CHAPTER A-I

BASIC RESULTS ON SEMIGROUPS

ON BANACH SPACES

by
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Since the basic theory of one-parameter semigroups can be found in several excellent books (e.g. Davies (1980), Goldstein (1985a), Pazy (1983) or Hille-Phillips (1957)) we do not want to give a self-contained introduction to this subject here. It may however be useful to fix our notation, to collect briefly some important definitions and results (Section 1), to present a list of standard examples (Section 2) and to discuss standard constructions of new semigroups from a given one (Section 3).

In the entire chapter we denote by E a (real or) complex Banach space and consider one - parameter semigroups of bounded linear operators $T(t)$ on E. By this we understand a subset $\{T(t) : t \in \mathbb{R}_+\}$ of $L(E)$, usually written as $(T(t))_{t \geq 0}$, such that

$T(0) = Id$,

$T(s+t) = T(s) \cdot T(t)$ for all s , $t \in \mathbb{R}_+$.

In more abstract terms this means that the map t → T(t) is a homomorphism from the additive semigroup $(\mathbb{R}_+, +)$ into the multiplicative semigroup $(L(E), \cdot)$. Similarly, a one-parameter group $(T(t))_{t \in \mathbb{R}}$ will be a homomorphic image of the group $(\mathbb{R}, +)$ in $(L(E), \cdot)$.

1. STANDARD DEFINITIONS AND RESULTS

We consider a one-parameter semigroup $(T(t))_{t \geq 0}$ on a Banach space E and observe that the domain $\mathbb{R}_+$ and the range $L(E)$ of the (semi-