$$\lim_{s\to\infty} C(s)''(\psi) = \sum_{i=1}^{k} \phi_i(\psi)x_i \in M_*$$

for every $\psi \in M'$. Hence $\mathrm{Fix}(\mathcal{T}'') \subseteq M_*$ which proves (c).

If (c) is fulfilled then $\mathrm{Fix}(\mathcal{T}) = \mathrm{Fix}(\mathcal{T}'')$. Therefore the fixed space of $\mathcal{T}'$ separates the points of $\mathrm{Fix}(\mathcal{T}'')$ which shows that $\mathcal{T}'$ is strongly ergodic on M ([Krengel (1985), Chap.2, Thm.1.4]). If (b) holds then

$$P = \lim_{\mu\downarrow o} \mu R(\mu, A')$$

exists in the strong operator topology, where A' is the generator of $\mathcal{T}'$. Therefore $\dim \mathrm{Fix}(\mu R(\mu)^\wedge) < \infty$ in some ultrapower of M (Theorem 4.4.(b)). It follows from D-III, Proposition 2.3 that 0 is a pole of the resolvent of $R(.,A)$. Therefore $\mathcal{T}$ is uniformly ergodic.

□

NOTES.

Section 1. The stability concepts appearing in Theorem 1.7 coincide not only for positive semigroups on C*-algebras but on any order unit Banach space. We refer to Batty-Robinson (1984) for this more general setting and to B-IV, Section 1 for the analogous results on $C_o(X)$.

Section 2. Theorem 2.2 generalizes the Liapunov stability theorem from the matrix algebra $B(\mathbb{C}^n)$ to arbitrary W*-algebras. For the algebra $B(H)$ it is due to Mil'stein (1975) and in the general form to Groh-Neubrander (1981).

Section 3. From the many papers dealing more or less explicitely with the asymptotic behavior of semigroups on operator algebras we quote Frigerio-Verri (1982) and Watanabe (1982). The background for our ergodic theorems (Prop.3.3 and Prop. 3.4) can be found best in Krengel (1985). The "automatic" convergence theorem for an irreducible W*-dynamical system on $B(H)$ stated in Corollary 3.9 is the continuous version of a result in Groh (1984c). Finally, the characterization of convergence towards a periodic semigroup through spectral properties of the generator (Thm. 3.11) is due to Nagel (1984) in the commutative, i.e. $L^2(\mu)$, case (see also C-IV, Thm.2.14).

Section 4. Again we refer to Krengel (1985) for the (uniform) ergodic theory for a single operator or a one-parameter semigroup on a Banach space. The characterization given in Corollary 4.5 for positive semigroups on W*-algebras is based on a sophisticated use of ultrapower techniques and has its discrete forerunners in Lotz (1981) and Groh (1984b).