to the domain of the generator. Moreover, $D(A) \otimes D(B)$ is dense in $E \tilde{\otimes}_\alpha F$ and invariant under $(S(t) \otimes T(t))_{t \geq 0}$, hence it is a core of $A \otimes Id + Id \otimes B$ by Prop.1.9.ii .

$\square$

3.8. <u>The Product of Commuting Semigroups</u>

Let $(S(t))_{t \geq 0}$ and $(T(t))_{t \geq 0}$ be semigroups with generators A and B, respectively on some Banach space E. It is not difficult to see that the following assertions are equivalent.

(i) $S(t)T(t) = S(t)T(t)$ for all $t \geq 0$.

(ii) $R(\mu,A)R(\mu,B) = R(\mu,B)R(\mu,A)$ for some $\mu \in \rho(A) \cap \rho(B)$.

(iii) $R(\mu,A)R(\mu,B) = R(\mu,B)R(\mu,A)$ for all $\mu \in \rho(A) \cap \rho(B)$.

In that case $U(t) = S(t)T(t)$ $(t \geq 0)$ defines a semigroup $(U(t))_{t \geq 0}$. Using Prop.1.9(ii) one easily shows that $D_o := D(A) \cap D(B)$ is a core for its generator C and $Cf = Af + Bf$ for all $f \in D_o$.

<u>NOTES</u>.

For a more complete information on semigroup theory we refer the reader to Hille-Phillips (1957), to the recent monographs by Davies (1980), Goldstein (1985a) and Pazy (1983), to the survey article by Krein-Khazan (1985) and to the bibliography by Goldstein (1985b).