diction $\phi(1) = 0$. Hence $-\infty < \omega$ and $\exp(\omega t) \in P\sigma(T(t)')$ for every $t \in \mathbb{R}_+$. Thus $\omega \in \sigma(A)$ or $\omega = s(A)$.

□

Remark 1.2. (a) If we consider the nilpotent translation semigroup on the C*-algebra $C_o([0,1))$ then $\sigma(A) = \emptyset$ and $\omega = -\infty$. This shows that the existence of a unit is essential.

(b) '$s(A) = \omega$' still holds for positive semigroups on commutative C*-algebras without unit (see B-IV, Rem.1.2.b).

Theorem 1.3. Let M be a W*-algebra with predual M_* and let $(T(t))_{t \geq 0}$ be a positive semigroup on M_* . Then $s(A) = \omega$.

Proof. For all $\lambda > s(A)$ and $\phi \in M_*$

$$R(\lambda,A)\phi = \int_0^\infty e^{-\lambda t} T(s)\phi ds$$

which follows as in C-III, Section 1 or [Greiner-Voigt-Wolff (1981), Theorem 3]. Since $\|\phi\| = \phi(1)$ for every $\phi \in M_*^+$ and since the norm is additive on the positive cone of M_* the integral

$$\int_0^\infty e^{\lambda t} \|T(s)\phi\| ds$$

exists for all $\phi \in M_*$ and all $\lambda > s(A)$. From this the assumption follows by A-IV,Thm.1.11.

□

Corollary 1.4. Let M be a C*-algebra and $(T(t))_{t \geq 0}$ a positive semigroup on M' . Then $s(A) = \omega$ holds.

This follows from the fact that the bidual of a C*-algebra is a W*-algebra (see [Takesaki (1979), Theorem III.2.4.]).

Remark 1.5. A simple modification of A-III, Example 1.4 (take c_o instead of ℓ^2) shows that Theorem 1.3 is no longer true for non-positive semigroups (for details see [Groh-Neubrander (1981), Beispiel 2.5]).

While the growth bound ω characterizes uniform exponential stability of the semigroup there are other (and weaker) stability concepts (cf. A-IV, Section 1).