$T \in L(E,F)$ is called a <u>lattice homomorphism</u> if $|Tf| = T|f|$ holds for all $f \in E$. Lattice homomorphisms are alternatively characterized by any one of the following conditions:

(a) $(Tf)^+ = T(f^+)$ $(f \in E)$

(a') $(Tf)^- = T(f^-)$ $(f \in E)$

(b) $T(f \vee g) = Tf \vee Tg$ $(f \in E)$

(b') $T(f \wedge g) = Tf \wedge Tg$ $(f \in E)$

(c) $T(f^+) \wedge T(f^-) = 0$ $(f \in E)$.

The kernel of a lattice homomorphism is an ideal. If T is bijective, then T is a lattice homomorphism if and only if T and T^{-1} are positive.

7. COMPLEX BANACH LATTICES

Although the notion of a Banach lattice is intrinsically related to the real number system, it is possible and often desirable to extend discussions to complexifications of Banach lattices in such a way that the order-related terms introduced in the real situation essentially retain their meaning. Thus we define a <u>complex Banach lattice</u> E to be the complexification of a real Banach lattice $E_{\mathbb{R}}$ in the sense that

$$E = E_{\mathbb{R}} \oplus iE_{\mathbb{R}}$$

which means more exactly $E = E_{\mathbb{R}} \times E_{\mathbb{R}}$ with scalar multiplication $(\alpha+i\beta)(x,y) = (\alpha x-\beta y, \beta x+\alpha y)$. $E_{\mathbb{R}}$ will sometimes be called the underlying real Banach lattice or the <u>real part of E</u> . The classical complex Banach spaces $C(X)$, $L^p(\mu)$ are complex Banach lattices in this sense, the underlying real Banach lattices being the corresponding (real) subspaces of real-valued functions. We want to extend the formation of absolute values, which is a priori defined only in the real part of E , in such a way that in the classical situation $E = C(X)$ or $E = L^p(\mu)$ the usual absolute value of a function is obtained. This is in fact possible by putting, for $h = f + ig$ in E

$$|h| = \sup\{ \operatorname{Re}(e^{i\theta}h) : 0 \le \theta \le 2\pi \} ,$$

the only problem with this definition being the existence of the right hand side without the assumption of order-completeness on $E_{\mathbb{R}}$