$q'(x) = 0$ a.e. Then $V : C_o(\mathbb{R}) \to C_o(\mathbb{R})$ given by $Vf = f\circ q$ is an algebra isomorphism. Let A be the generator of the translation group on $C_o(\mathbb{R})$ and $\delta = V^{-1}AV$. Then $D(\delta) = \{ f \in C_o(\mathbb{R}) : Vf \in D(A)\} = \{f \in C_o(\mathbb{R}) : f\circ q \in C^1(\mathbb{R}) , (f\circ q)' \in C_o(\mathbb{R}) \}$.
Let $f \in C^1(\mathbb{R}) \cap D(\delta)$. If $f \not\equiv 0$, then f is not constant. Hence there exists $x_o \in \mathbb{R}$ such that $f'(x_o) \not\equiv 0$. Then f has a continuously differentiable inverse in some open neighbourhood of x_o. Since $f\circ q \in C^1(\mathbb{R})$, it follows that q is continuously differentiable in some neighborhood of $q^{-1}(x_o)$. This is a contradiction since $q'(y) = 0$ a.e.

Theorem 3.23. Let δ be the generator of an automorphism group on $C_o((a,b))$, where $-\infty \leq a < b \leq \infty$. The following assertions are equivalent.

(i) There exists a continuous admissible function $m : (a,b) \to \mathbb{R}$ such that $\delta = \delta_m$.

(ii) $C_c^1(a,b) \subset D(\delta)$ and $D_o(\delta) = \{ f \in D(\delta) : f$ is differentiable $\}$ is a core of δ.

Proof. We have already pointed out that (i) implies (ii).
So assume that (ii) holds. Let $(T(t))_{t\in\mathbb{R}}$ be the group generated by δ and ϕ the continuous flow associated with the group. We can assume that ϕ is of the form given in Prop. 3.21.
Let $n \in J$. We show that $r_n^{-1} : \mathbb{R} \to (a_n,b_n)$ is continuously differentiable. Let $x_o \in (a_n,b_n)$. There exists $f \in C_c^1(a,b)$ such that $f(x) = x$ in a neighborhood of x_o. Then $r_n^{-1}(r_n(x_o) + t)$ $= f(\phi(t,x_o)) = (T(t)f)(x_o)$ for all t in some neighborhood of 0. Since $f \in D(\delta)$ it follows that the function $t \to r_n^{-1}(r_n(x_o) + t)$ is continuously differentiable in some neighborhood of 0 and so r_n^{-1} is continuously differentiable in $r_n(x_o)$. Since $r_n : (a_n,b_n) \to \mathbb{R}$ is surjective this proves the claim.
Next we show $(r_n^{-1})'(t) \not\equiv 0$ for all $t \in \mathbb{R}$. In fact, let $x_o \in (a_n,b_n)$ and assume that $(r_n^{-1})'(r_n(x_o)) = 0$. Then for all $f \in D_o(\delta)$ one has
$(\delta f)(x_o) = \frac{\partial}{\partial t}\big|_{t=0} f(r_n^{-1}(r_n(x_o) + t)) = f'(x_o)(r_n^{-1})'(r_n(x_o)) = 0$.
Since $D_o(\delta)$ is a core of δ this implies that $\phi(t,x_o) = x_o$ for all $t \in \mathbb{R}$. Hence $x_o \in K$, a contradiction. It follows that $r_n : (a_n,b_n) \to \mathbb{R}$ is a C^1-diffeomorphism for all $n \in J$.