differentiable on (a,b) and $m(x)\frac{\partial}{\partial x}\phi(t,x) = m(\phi(t,x))$ for all $x \in (a,b)$. Let $f \in D_o(\delta_m) = D(\delta_m) \cap C^1$, $t \in \mathbb{R}$. Then $T(t)f = f\circ\phi_t$ is differentiable and so in $D_o(\delta_m)$.

Conversely, assume that δ_m is generator of a group $(T(t))_{t\in\mathbb{R}}$ on $C_o(a,b)$. Since δ_m is a derivation, there exists a continuous flow $(\phi_t)_{t\in\mathbb{R}}$ on (a,b) such that $T(t)f = f\circ\phi_t$ for all $f \in C_o(a,b)$, $t \in \mathbb{R}$. In order to show that m is admissible let $a \leq c < d \leq b$ such that $m(x) \neq 0$ for all $x \in (c,d)$ and $m(c) = 0$ or $a = c = -\infty$ and $m(d) = 0$ or $d = b = \infty$.
If $a < c$ then $m(c) = 0$; consequently $(\delta_m f)(c) = 0$ for all $f \in D(\delta_m)$. Thus $(T(t)f)(c) = f(c)$ for all $f \in D(\delta_m)$ and $t \in \mathbb{R}$. This shows that $\phi(t,c) = c$ for all $t \in \mathbb{R}$. Consequently $\phi_t(a,c) \subset (a,c)$ for all $t \in \mathbb{R}$. Similary $\phi_t(d,b) \subset (d,b)$ for all $t \in \mathbb{R}$. Thus the space $E_o := \{f \in C_o(a,b) : f \text{ vanishes off } (c,d)\}$ is invariant under the group $(T(t))_{t\in\mathbb{R}}$. We denote the group restricted to E_o by $(T_o(t))_{t\in\mathbb{R}}$ and by A_o its generator. Then $D(A_o) = \{f \in E_o \cap D(\delta_m) : \delta_m f \in E_o\}$. Identifying E_o with $C_o(c,d)$ we obtain $A_o = \delta_{m'}$, where m' denotes the restriction of m to (c,d). So it follows from Prop. 3.18 that m' is admissible.

□

<u>Remark</u> 3.19. If ϕ is a flow on (a,b), a point $x \in (a,b)$ is called <u>stationary</u> if $\phi(t,x) = x$ for all $t \in \mathbb{R}$. Let δ be the generator of the group $(T(t))_{t\in\mathbb{R}}$ associated with ϕ. Then $x \in (a,b)$ is a stationary point if and only if $(\delta f)(x) = 0$ for all $f \in D(\delta)$. If m is an admissible function on (a,b) then we have seen that $x \in (a,b)$ is a stationary point of the flow associated with δ_m if and only if $m(x) = 0$. This does no longer hold for functions which are not admissible as the following example shows.

<u>Example</u> 3.20. Consider the flow $\phi(t,x) = (x^{1/3} + t)^3$ on $\mathbb{R}$ and the group $(T(t))_{t\in\mathbb{R}}$ induced by this flow on $C_o(\mathbb{R})$. One can easily see that the generator δ of $(T(t))_{t\in\mathbb{R}}$ is the following operator. Let $m(x) = 3x^{2/3}$. Then $(\delta f)(x) = m(x)f'(x)$ for $x \neq 0$ and
$D(\delta) = \{ f \in C_o(\mathbb{R}) : f$ is differentiable in $x \neq 0$ and $m(x)f'(x)$ has a continuous extension in $C_o(\mathbb{R})\}$. However the function m is not admissible. And in fact $m(0) = 0$ but 0 is not a stationary point of ϕ. In particular, there exists a function $f \in D(\delta)$ such that $(\delta f)(0) \neq 0$.

Next we describe an arbitrary continuous flow on an open interval.