5. SEMIGROUPS OF DISJOINTNESS PRESERVING OPERATORS

In this section we consider a special case of domination. Recall from C-I,Sec.6 that a linear operator S on E is called lattice homomorphism if

(5.1) $\quad |Sf| = S|f| \quad$ for all $f \in E$.

An operator $S \in L(E)$ is called disjointness preserving if

(5.2) $\quad f \perp g$ implies $Sf \perp Sg$ for all $f,g \in E$.

Note that an operator S is a lattice homomorphism if and only if S is positive and disjointness preserving.

In the following we will consider disjointness preserving semigroups (by this we mean semigroups of disjointness preserving operators) and lattice semigroups (i.e., semigroups of lattice homomorphisms). For example, the semigroup $(T_d(t))_{t\geq 0}$ defined in Section 3 is disjointness preserving for all $d \in \mathbb{R}$ and a lattice semigroup if $d \geq 0$.

Proposition 5.1. A bounded operator S on a complex Banach lattice E is disjointness preserving if and only if there exists a linear operator $|S|$ on E such that

(5.3) $\quad |Sf| = |S||f| \quad (f \in E)$.

In that case the operator $|S|$ is uniquely determined by (5.3). $|S|$ is a lattice homomorphism and the modulus of S (i.e., one has $|S| \leq T$ for all $T \in L(E)$ such that $|Sf| \leq T|f| \quad (f \in E)$).

For the proof of the proposition we refer to Arendt (1983) and de Pagter (1985).

Proposition 5.2. Let $(S(t))_{t\geq 0}$ be a disjointness preserving semigroup. Let $T(t) = |S(t)| \quad (t \geq 0)$. Then $(T(t))_{t\geq 0}$ is a strongly continuous semigroup.

Proof. Let $0 \leq s,t$ and $f \in E_+$. Then by (5.1), $T(s)T(t)f = T(s)|S(t)f| = |S(s)S(t)f| = |S(s+t)f| = T(s+t)f$. Since $\operatorname{span} E_+ = E$, it follows that $(T(t))_{t\geq 0}$ is a semigroup. Moreover, for $f \in E_+$, $\lim_{t\to 0} T(t)f = \lim_{t\to 0} |S(t)f| = |f| = f$. This implies that $(T(t))_{t\geq 0}$ is strongly continuous.

□