Here $\tilde{h}$ denotes the continuous extension of $h/|h|$ to βX .
Defining $T_1 := M_{\tilde{h}}^{-1}\tilde{R}M_{\tilde{h}}$ we have by (2.9)

(2.10) $T_1 1 = M_{\tilde{h}}^{-1}\tilde{R}\tilde{h} = 1$ and

(2.11) $|T_1 f| = |M_{\tilde{h}}^{-1}\tilde{R}M_{\tilde{h}}f| = |\tilde{R}M_{\tilde{h}}f| \leq \tilde{T}|M_{\tilde{h}}f| = \tilde{T}|f|$ for all f .

Hence we have $\|T_1\| \leq \|\tilde{T}\| = 1$ (by (2.11), (2.9), (2.1)). Then it follows from Lemma 2.1 that T_1 is a positive operator. Thus (2.11) implies that $0 \leq T_1 \leq \tilde{T}$ and therefore $\|\tilde{T} - T_1\| = \|(\tilde{T} - T_1)1\| = 0$ (by (2.10), (2.9), (2.1)).

□

We are now able to prove a result which in some sense is the key to cyclicity results for the spectrum. These general results will be proved by reducing the problem in such a way that the following theorem can be applied.

Theorem 2.4.(a) Let T be a positive linear operator on $C_o(X)$, let $h \in C_o(X)$ and $\lambda \in \mathbb{C}$, $|\lambda| = 1$. If $Th = \lambda h$ and $T|h| = |h|$, then we have $Th^{[n]} = \lambda^n h^{[n]}$ for every $n \in \mathbb{Z}$ (cf. (2.4)).
If h does not have zeros in X , then $\lambda T = S_h^{-1}TS_h$.

(b) Suppose A is the generator of a positive semigroup, $h \in C_o(X)$, $\alpha,\beta \in \mathbb{R}$ such that $Ah = (\alpha+i\beta)h$ and $A|h| = \alpha|h|$. Then we have $Ah^{[n]} = (\alpha+in\beta)h^{[n]}$ for every $n \in \mathbb{Z}$.
If h does not have zeros then $S_h D(A) = D(A)$ and $i\beta + A = S_h^{-1}AS_h$.

Proof.(a) The closed principal ideal $\overline{E_h}$ which is canonically isomorphic to $C_o(X_1)$ with $X_1 = \{x \in X : h(x) \neq 0\}$, is T-invariant. We give an object a tilde when we consider it as an element of $\overline{E_h} \cong C_o(X_1)$. Defining $\tilde{R} := \bar{\lambda}\tilde{T}$, then $\tilde{T}$, $\tilde{R}$, $\tilde{h}$ satisfy (2.8) , hence we have

(2.12) $\tilde{T} = S_{\tilde{h}}^{-1}\circ\tilde{R}\circ S_{\tilde{h}} = \bar{\lambda}\cdot S_{\tilde{h}}^{-1}\circ\tilde{T}\circ S_{\tilde{h}}$

which by iteration yields

(2.13) $\tilde{T} = \bar{\lambda}^n\cdot S_{\tilde{h}}^{-n}\circ\tilde{T}\circ S_{\tilde{h}}^n$ for all $n \in \mathbb{Z}$.

It follows that
$\tilde{T}\tilde{h}^{[n]} = \tilde{T}\circ S_{\tilde{h}}^n|\tilde{h}| = \lambda^n\cdot S_{\tilde{h}}^n\circ\tilde{T}|\tilde{h}| = \lambda^n\cdot S_{\tilde{h}}^n\tilde{h} = \lambda^n\cdot\tilde{h}^{[n]}$
(see (2.7) and (2.12)), which is precisely $Th^{[n]} = \lambda^n h^{[n]}$ for all $n \in \mathbb{Z}$. If h does not have zeros, then $\overline{E_h} = E$, hence $T = \tilde{T}$, $h = \tilde{h}$ and the remaining assertion follows from (2.12).