If R is a pseudo-resolvent on $D = \{\lambda \in \mathbb{C} : \mathrm{Re}(\lambda) > 0\}$ with values in a C^*- or W^*-algebra, then R is called of <u>Schwarz type</u> if

$$(R(\lambda)x)(R(\lambda)x)^* \leq (\mathrm{Re}\lambda)^{-1} R(\mathrm{Re}\lambda)xx^*$$

for all $\lambda \in D$ and $x \in M$. R is called <u>identity preserving</u> if $\lambda R(\lambda)1 = 1$ for all $\lambda \in D$.

For examples and properties of a pseudo-resolvent see C-III, 2.5. We state what will be used without further reference.

(a) If $\alpha \in \mathbb{C}$ and $x \in E$ such that $(\alpha - \lambda)R(\lambda)x = x$ for some $\lambda \in D$, then $(\alpha - \mu)R(\mu)x = x$ for all $\mu \in D$ (use the "resolvent equation").

(b) If F is a closed subspace of E such that $R(\lambda)F \subseteq F$ for some $\lambda \in D$, then $R(\mu)F \subseteq F$ for all μ in a neighbourhood of λ. This follows from the fact that for all $\mu \in D$ near λ the pseudo-resolvent in μ is given by

$$R(\mu) = \sum_n (\lambda - \mu)^n R(\lambda)^{n+1} .$$

<u>Definition</u> 2.4. We call a semigroup $\mathcal{T}$ on the predual M_* of a W^*-algebra M <u>identity preserving and of Schwarz type</u>, if its adjoint weak*-semigroup has these properties. Likewise, a pseudo-resolvent R on $D = \{\lambda \in \mathbb{C} : \mathrm{Re}(\lambda) > 0\}$ with values in M_* is called <u>identity preserving and of Schwarz type</u>, if R' has these properties.

Since for a semigroup of contractions on a Banach space

$$\mathrm{Fix}(\mathcal{T}) = \bigcap_{t \geq 0} \ker(Id - T(t)) =$$

$$= \ker(Id - \lambda R(\lambda,A)) = \mathrm{Fix}(\lambda R(\lambda,A))$$

for all $\lambda \in \mathbb{C}$ with $\mathrm{Re}(\lambda) > 0$, it follows that a semigroup of contractions on M is identity preserving if and only if the (pseudo)-resolvent on $D = \{\lambda \in \mathbb{C} : \mathrm{Re}(\lambda) > 0\}$ given by

$$R(\lambda) := R(\lambda,A)_{|D}$$

is identity preserving. By Corollary 2.2 an analogous statement holds for 'Schwarz type'.