is everywhere defined and therefore bounded (use Prop.1.9(i)). In general the precise extent of the domain $D(A)$ is essential for the characterization of the generator. But since the domain is canonically associated to the generator of a semigroup we shall write in most cases A instead of $(A,D(A))$.
As a first result we collect some information on the domain of the generator.

<u>Proposition</u> 1.6. For the generator A of a semigroup $(T(t))_{t\geq 0}$ on a Banach space E the following assertions hold:

(i) If $f \in D(A)$ then $T(t)f \in D(A)$ for every $t \geq 0$.

(ii) The map $t \to T(t)f$ is differentiable on $\mathbb{R}_+$ if and only if $f \in D(A)$. In that case one has

(1.2) $\frac{d}{dt} T(t)f = AT(t)f = T(t)Af.$

(iii) For every $f \in E$ and $t > 0$ the element $\int_0^t T(s)fds$ belongs to $D(A)$ and one has

(1.3) $A\int_0^t T(s)f\,ds = T(t)f - f.$

(iv) If $f \in D(A)$ then

(1.4) $\int_0^t T(s)Af\,ds = T(t)f - f.$

(v) The domain $D(A)$ is dense in E.

The identity (1.2) is of great importance and shows how semigroups are related to certain Cauchy problems. We state this explicitely in the following theorem.

<u>Theorem</u> 1.7. Let $(A,D(A))$ be the generator of a strongly continuous semigroup $(T(t))_{t\geq o}$ on the Banach space E. Then the '<u>abstract Cauchy problem</u>'

(ACP) $\frac{d}{dt}\xi(t) = A\xi(t)\ ,\ \xi(0) = f_o\ ,$

has a unique solution $\xi : \mathbb{R}_+ \to D(A)$ in $C^1(\mathbb{R}_+,E)$ for every $f_o \in D(A)$. In fact, this solution is given by $\xi(t) := T(t)f_o$.