

theory, see Hille-Phillips (1957) and Widder (1946). Theorem 1.3. (ii) is a semi-group version of Theorem 1.2.7 and 1.2.8 in Doetsch (1950). The lemma in the proof of Thm.1.3 is taken from Mil'stein (1975). Theorem 1.4 and Corollary 1.5 can be found in Neubrander (1985a). Example 1.6 follows Remark 2 in Zabczyk (1975). Statement (1.8) is sometimes called the 'spectrum determined growth assumption', see, for example, Triggiani (1975b). Theorem 1.9 is due to Slemrod (1976). The proof given here is based on a much sharper version of the inversion formula for the Laplace transform, than the one given by Hille-Phillips (1957), p.349. Using Widder (1946), p.66 or Doetsch (1950), p.212 one can show the following theorem (see Neubrander (1984b)).

Theorem. Let A be the generator of a strongly continuous semigroup $(T(t))_{t \geq 0}$ on a Banach space E . For every $f \in D(A)$ and $p > \omega_1(A)$ we have

$$T(t)f = \frac{1}{2\pi i} \int_{p-i\infty}^{p+i\infty} e^{\mu t} R(\mu, A) f \, d\mu.$$

The equivalence of the statements (1.12), (1.13) and ' $\omega(A) < 0$ ' were observed by many authors, see for example, Balakrishnan (1976), p.178 or Benchimol (1978). Theorem 1.11 is due to Datko (1970) and Delfour (1974); for a proof see Pazy (1983), p. 116. Theorems 1.13 and 1.16 can be found in Neubrander (1985b) and Corollary 1.14 is due to Komatsu (1969). An example of an unstable semigroup generator A with $\operatorname{Re} \mu < 0$ for all $\mu \in \sigma(A)$ is given in Datko (1983).

Section 2. For a discussion of well-posedness of inhomogeneous Cauchy problems we refer to Goldstein (1985a), p.83 and Pazy (1983), p.105. Further results on the asymptotic behavior of the solutions of the inhomogeneous problem can be found in Rao-Hengartner (1974), Zaidman (1979), Pazy (1983), and Neubrander (1985b). Results similar to Lemma 2.2 and Theorem 2.4 are due to Prüß (1984). For a discussion of the asymptotic behavior of the solutions of $u'(t) = A(t)u(t) + F(t)$ see Datko (1972) and Pazy (1983), p.172.