<u>Theorem</u> 3.7. Let A be a densely defined operator on $C(K)$. The following assertions are equivalent.

(i) A is the generator of an automorphism group.

(ii) $1 \in D(A)$ and $A1 = 0$; $(\pm 1 - A)D(A) = C(K)$ and A is <u>local</u>, in the sense that for $0 \leq f \in D(A)$, $f(x) = 0$ implies $(Af)(x) = 0 \quad (x \in K)$.

<u>Proof</u>. An invertible operator T such that $T \geq 0$ and $T^{-1} \geq 0$ is an automorphism if and only if $T1 = 1$. Hence A is the generator of an automorphism group if and only if A and $-A$ generate a positive group, $1 \in D(A)$ and $A1 = 0$. Thus Theorem 3.7. follows from Theorem 1.13.

□

<u>Remark</u>. It is remarkable that from locality, the range condition and $1 \in D(A)$, $A1 = 0$ it follows that $D(A)$ actually is a subalgebra of $C(K)$ and A is a derivation. The "order-theoretical" property of locality is in some aspects stronger than the algebraic property of being a derivation. For example a local, densely defined operator is closable (by Prop.1.11); but there exist derivations on $C[0,1]$ which are not closable (see Bratteli-Robinson (1975)).

<u>Remark</u> (an excursion to C^*-algebras).
Theorem 3.7 also holds for non-commutative C^*-algebras. More precisely: Let $\mathcal{A}$ be a C^*-algebra with unit 1 and let $\mathcal{A}_h$ be the real Banach space of all hermitian elements in $\mathcal{A}$. Then $\mathcal{A}_h$ is a real ordered Banach space and 1 is an interior point of $(\mathcal{A}_h)_+$. Let A be a densely defined operator on $\mathcal{A}_h$.
Then A is the generator of an automorphism group if and only if $1 \in D(A)$ and $A1 = 0$; $(\pm 1 - A)(D(A)) = \mathcal{A}_h$ and A is <u>local</u> in the sense that for $0 \leq x \in D(A)$, $0 \leq \phi \in (\mathcal{A}_h)'$, $\phi(x) = 0$ implies $\phi(Ax) = 0$.
The proof of Theorem 3.7 can be carried over to this case if one notices the following. A strongly continuous group $T(t)_{t \in \mathbb{R}}$ on $\mathcal{A}_h$ is an automorphism group if and only if it is positive and $T(t)1 = 1$ for all $t \in \mathbb{R}$ [see Bratteli-Robinson (1979), Cor. 3.2.21].

Now we let X be a locally compact space and consider positive groups on $C_o(X) = C_o(X,\mathbb{R})$, the space of all continuous real-valued functions on X which vanish at infinity. Our aim is to describe their generators as perturbations of generators of automorphism groups; i.e., we will extend Theorem 3.6 by allowing X to be noncompact but