$$D(A) = \{f \in E : f \text{ absolutely continuous, } f' \in E\} ,$$
$$Af(z) = (2\pi i/\tau)\cdot z\cdot f'(z) .$$

An isomorphic copy of the group $(R_1(t))_{t\in\mathbb{R}}$ is obtained if we consider $E = \{f \in C[0,1] : f(0) = f(1)\}$, resp. $E = L^p([0,1])$ and the group of 'periodic translations'

$$T(t)f(x) := f(y) \qquad \text{for } y \in [0,1] ,\ y = x+t \bmod 1$$

with generator

$$D(A) := \{f \in E : f \text{ absolutely continuous, } f' \in E\} ,$$
$$Af := f' .$$

2.6. Nilpotent Translation Semigroups

Take $E = L^p([0,\tau],m)$ for $1 \leq p < \infty$ and define

$$T(t)f(x) := \begin{cases} f(x+t) & \text{if } x+t \leq \tau \\ 0 & \text{otherwise} . \end{cases}$$

Then $(T(t))_{t\geq 0}$ is a semigroup satisfying $T(t) = 0$ for $t \geq \tau$. Its generator is still the first derivative $A = \frac{d}{dx}$, but its domain is $D(A) = \{f \in E : f \text{ absolutely continuous, } f' \in E ,\ f(\tau) = 0\}$. In fact, if $f \in D(A)$ then f is absolutely continuous with $f' \in E$. By Prop.1.6.i it follows that $T(t)f$ is absolutely continuous and hence $f(\tau) = 0$.

2.7. One-dimensional Diffusion Semigroup

For the second derivative

$$Bf(x) := \frac{d^2}{dx^2} f(x) = f''(x)$$

we take the domain

$$D(B) := \{f \in C^2[0,1] : f'(0) = f'(1) = 0\}$$

in the Banach space $E = C[0,1]$. Then $D(B)$ is dense in $C[0,1]$, but closed for the graph norm. Obviously, each function

$$e_n(x) := \cos \pi nx \quad , \quad n \in \mathbb{Z} ,$$

is contained in $D(B)$ and an eigenfunction of B pertaining to the eigenvalue $\lambda_n := -\pi^2 n^2$. The linear hull

$$\operatorname{span} \{e_n : n \in \mathbb{Z}\} =: E_0$$

forms a subalgebra of $D(B)$ which by the Stone-Weierstrass theorem is dense in E.

We now use e_n to define bounded linear operators

$$e_n \otimes e_n : f \to \left(\int_0^1 f(x)e_n(x)\,dx \right) e_n = \langle f,e_n\rangle e_n$$