$$(3.25)\quad g(x) = \begin{cases} m(x)f'(x) & \text{if } m(x) \neq 0 \\ 0 & \text{if } m(x) = 0 \end{cases}$$

for all $x \in (a,b)$ and $D(\tilde{\delta}_m) = \{ f \in C[a,b] : f$ is differentiable in $x \in (a,b)$ whenever $m(x) \neq 0$ and there exists a (necessarily unique) $g \in C[a,b]$ such that (3.25) holds $\}$.

Theorem 3.26. Let m be a continuous function on (a,b) . The operator $\tilde{\delta}_m$ is generator of an automorphism group on $C[a,b]$ if and only if m is admissible.

Proof. If $\tilde{\delta}_m$ generates an automorphism group $(T(t))_{t\in\mathbb{R}}$ then by the remark above $T(t)C_o(a,b) = C_o(a,b)$ $(t \in \mathbb{R})$. The generator of the restricted group has the domain
$\{ f \in C_o(a,b) \cap D(\tilde{\delta}_m) : \tilde{\delta}_m f \in C_o(a,b) \} = D(\delta_m)$. Hence δ_m is a generator and so m is admissible by Theorem 3.17. Conversely, if m is admissible, then δ_m generates a group on $C_o(a,b)$ given by a flow ϕ_o on (a,b) . Extending ϕ_o to $[a,b]$ as above one obtains a continuous flow ϕ on $[a,b]$ which defines a group $(T(t))_{t\in\mathbb{R}}$. It is easy to verify, that the generator of this group is $\tilde{\delta}_m$.

□

Theorem 3.27. Let δ be the generator of an automorphism group on $C[a,b]$. Then there exists an admissible function $m : (a,b) \to \mathbb{R}$ and an algebra isomorphism V from $C[a,b]$ onto $C[a,b]$ such that $\delta = V^{-1}\tilde{\delta}_m V$.

Proof. The restriction δ_o of δ to $C_o(a,b)$ is the generator of an automorphism group. Thus by Theorem 3.24 there exists a continuous admissible function $m : (a,b) \to \mathbb{R}$ and an algebra isomorphism V_o from $C_o(a,b)$ onto $C_o(a,b)$ such that $\delta_o = V_o^{-1}\delta_m V_o$. Let V be the unique algebra isomorphism on $C[a,b]$ which extends V_o . Then it is easy to see that $\delta = V^{-1}\tilde{\delta}_m V$.

□

Theorem 3.28. An operator A on $C[a,b]$ is generator of a positive group on $C[a,b]$ if and only if there exist
- a lattice isomorphism V on $C[a,b]$
- an admissible function $m : (a,b) \to \mathbb{R}$
- and a function $h \in C[a,b]$ such that $A = V^{-1}\tilde{\delta}_m V + h$.

The proof follows from Theorem 3.14 via Theorem 3.27 in the same way as Theorem 3.25 (via Theorem 3.24).