satisfying $\| e_n \otimes e_n \| \leq 1$ and

$$(e_n \otimes e_n)(e_m \otimes e_m) = \delta_{n,m}(e_n \otimes e_n) \quad \text{for} \quad n \in \mathbb{Z} .$$

For $t > 0$ we define

$$T(t) := \sum_{n\in\mathbb{Z}} \exp(-\pi^2 n^2 t) \cdot e_n \otimes e_n$$

$$= e_0 \otimes e_0 + 2\sum_{n=1}^{\infty} \exp(-\pi^2 n^2 t) \cdot e_n \otimes e_n$$

or

$$T(t)f(x) = \int_0^1 k_t(x,y) f(x) dy$$

$$\text{where } k_t(x,y) = 1 + 2\sum_{n=1}^{\infty} \exp(-\pi^2 n^2 t) \cos\pi nx \cos\pi ny .$$

The Jacobi identity

$$w_t(x) := 1/(4\pi t)^{\frac{1}{2}} \sum_{m\in\mathbb{Z}} \exp(-(x+2m)^2/4t)$$

$$= \frac{1}{2} + \sum_{n\in\mathbb{N}} \exp(-\pi^2 n^2 t) \cos\pi nx$$

and trigonometric relations show that

$$k_t(x,y) = w_t(x+y) + w_t(x-y)$$

which is a positive function on $[0,1]^2$. Therefore $T(t)$ is a bounded operator on $C[0,1]$ with

$$\|T(t)\| = \|T(t)1\| = \sup_{x\in[0,1]} \int_0^1 k_t(x,y) dy = 1 .$$

From the behavior of $T(t)$ on the dense subspace E_o it follows that $(T(t))_{t\geq 0}$ with $T(0) = Id$ is a strongly continuous semigroup on E and its generator A coincides with B on E_o . Finally we observe that E_o is a core for $(A,D(A))$ by Prop.1.9(ii).

Consequently $(T(t))_{t\geq 0}$ is the semigroup generated by the closure of the second derivative with domain $D(B)$.

2.8. <u>n-dimensional Diffusion Semigroup</u>

On $E = L^p(\mathbb{R}^n)$, $1 \leq p < \infty$, the operators

$$T(t)f(x) := (4\pi t)^{-n/2} \int_{\mathbb{R}^n} \exp(-|x-y|^2/4t) f(y) dy$$

$$:= \mu_t * f(x)$$

for $x \in \mathbb{R}^n$, $t > 0$ and $\mu_t(x) := (4\pi t)^{-n/2} \exp(-|x|^2/4t)$ form a strongly continuous semigroup:

In fact the integral exists for every $f \in L^p(\mathbb{R}^n)$, since μ_t is an element of the Schwartz space $\mathcal{S}(\mathbb{R}^n)$ of all rapidly decreasing smooth functions on $\mathbb{R}^n$.

Moreover,

$$\|T(t)f\|_p \leq \|\mu_t\|_1 \|f\|_p = \|f\|_p$$

by Young's inequality [Reed-Simon (1975), p.28] , hence $\|T(t)\| \leq 1$ for every $t > 0$. Next we observe that $\mathcal{S}(\mathbb{R}^n)$ is dense in E and invariant under each $T(t)$. Therefore we can apply the Fourier trans-