Example 4.7.(a) Consider the flow on $[-\frac{\pi}{2},\frac{\pi}{2}]$ defined by
$\phi(t,x) := \arctan(\tan x - t)$, $x \in [-\frac{\pi}{2},\frac{\pi}{2}]$, $t \in \mathbb{R}$
(it belongs to the differential equation $y' = -\cos^2 y)$, and a continuous function $h : [-\frac{\pi}{2},\frac{\pi}{2}] \to \mathbb{R}$ with $h(-\frac{\pi}{2}) \leq h(\frac{\pi}{2})$. Then we have $\underline{c}(h,\phi) = h(-\frac{\pi}{2})$ and $\bar{c}(h,\phi) = h(\frac{\pi}{2})$. The spectrum of the corresponding semigroup is given by $\sigma(A) = \{\lambda \in \mathbb{C} : h(-\frac{\pi}{2}) \leq \operatorname{Re}\lambda \leq h(\frac{\pi}{2})\}$.

(b) Consider $K = \{z \in \mathbb{C} : 1 \leq |z| \leq 2\} = \{r\cdot e^{i\omega} : \omega \in \mathbb{R} , 1 \leq r \leq 2\}$ and a continuous function $\kappa : [1,2] \to \mathbb{R}_+$.
Let $\bar{\phi}$ be the flow on K governed by the differential equation $\dot{\omega} = \kappa(r)$, $\dot{r} = 0$ (hence $\bar{\phi}(t,r\cdot e^{i\omega}) = r\cdot e^{i(\omega+\kappa(r)t)})$.
For a continuous function $h : K \to \mathbb{R}$ let $h\hat{\ }(r) := \frac{1}{2\pi}\cdot\int_0^{2\pi} h(r\cdot e^{it})\,dt$ $(1 \leq r \leq 2)$. The spectrum of the semigroup corresponding to ϕ and h (cf. (4.1)) is given by
$\sigma(A) = \{h\hat{\ }(r) + ik\kappa(r) : k \in \mathbb{Z} , 1 \leq r \leq 2\}^- \cup \{h(z) : \kappa(|z|) = 0\}$.

Proposition 4.8. Suppose the semigroup $(T(t))_{t\geq 0}$ on $C(K)$ is given by (4.1) and let $\underline{c}(h,\phi)$, $\bar{c}(h,\phi)$ be defined as in (4.4). Then the following assertions hold:

(a) $\{\lambda \in \mathbb{C} : \operatorname{Re}\lambda > \bar{c}(h,\phi)\} \subset \rho(A)$;

(b) $\bar{c}(h,\phi)$ and $\underline{c}(h,\phi)$ are spectral values ;

(c) If $\phi(t,x_o) = x_o$ for every $t \geq 0$, then $h(x_o) \in R\sigma(A)$;

(d) Assume x_o has a finite orbit (i.e., $\phi(\mathbb{R}_+,x_o) = \phi([0,T],x_o)$ for some $T < \infty$) and $\tau := \inf\{t > 0 : \phi(T+t,x_o) = \phi(T,x_o)\} > 0$, then $h\hat{\ }(x_o) + \frac{2\pi}{\tau}i\mathbb{Z} \subset R\sigma(A)$ where $h\hat{\ }(x_o) := 1/\tau\int_T^{T+\tau} h(\phi(s,x_o))ds$.

(e) If x_o has an infinite orbit and $h\hat{\ } := \lim_{t\to\infty} h(\phi(t,x_o))$ exists, then $h\hat{\ } + i\mathbb{R} \subset \sigma(A)$.

Proof.(a) and (b): A look at (4.4) shows that $\bar{c}_t(h,\phi) = 1/t\cdot\log\|T(t)\|$ hence $\bar{c}(h,\phi) = \omega(A)$ (cf. A-I,(1.1)). Consequently, we have $\{\lambda \in \mathbb{C} : \operatorname{Re}\lambda > \bar{c}(h,\phi)\} \subset \rho(A)$ and $\bar{c}(h,\phi) \in \sigma(A)$ by Thm.1.6. To prove $\underline{c}(h,\phi) \in \sigma(A)$, we can assume by Thm.4.4 that $K_\infty = K_s$ for some s and that ${}^{\phi}|K_\infty$ is injective. It is easy to see that $\underline{c}(h,\phi) = \underline{c}({}^{h}|K_\infty, {}^{\phi}|K_\infty)$, moreover, we have $\sigma({}^{A}|I_\infty) = \emptyset$ hence $\sigma(A) = \sigma(A/I_\infty)$ by A-III,Prop.4.2 . This shows that we also can assume that $K = K_\infty$, i.e., ϕ is bijective or A is the generator of a group. Now the assertion follows from
$\underline{c}(h,\phi) = \underline{c}(h,\phi^{-1}) = -\bar{c}(-h,\phi^{-1}) = -s(-A)$.

(c) and (d): One can check easily that in case of (c) the Dirac functional δ_{x_o} is an eigenvector of A' corresponding to $h(x_o)$.