faithful on $p_r M p_r$ and consists of $\mathcal{T}^{(r)}$-invariant elements, it follows that:

(i) $P\sigma(A) \cap i\mathbb{R} = P\sigma(A^{(r)}) \cap i\mathbb{R}$.

(ii) $\ker(i\alpha - A) \subset p_r M_* p_r$ for all $\alpha \in \mathbb{R}$.

(iii) The semigroup $\mathcal{T}^{(r)}$ is relatively compact in the weak operator topology and therefore strongly ergodic.

(c) Similarly, let R be an identity preserving pseudo-resolvent with values in M_* on $D = \{\lambda \in \mathbb{C} : \mathrm{Re}(\lambda) > 0\}$ which is of Schwarz type. It follows as in (b) that $\mathrm{Fix}((\lambda - i\alpha)R(\lambda))$ is contained in $p_r M_* p_r$ for all $\lambda \in D$ and $\alpha \in \mathbb{R}$, where p_r is the associated recurrent projection.

We now give a characterization of strong ergodicity of semigroups which are identity preserving and of Schwarz type. For this we need that the Cesàro means $C(s)$, where

$$C(s)x = \frac{1}{s}\int_0^s T(t)x\,dt \qquad (x \in M\ ,\ 0 < s \in \mathbb{R})$$

are Schwarz maps. We omit the simple calculation (compare D-I,Thm.2.1).

<u>Proposition</u> 3.3. Let $\mathcal{T}$ be an identity preserving semigroup of Schwarz type on the predual of a W*-algebra M . Then the following assertions are equivalent:

(a) $\mathcal{T}$ is strongly ergodic on M_* .

(b) $\sigma(M,M_*)\text{-}\lim_{s\to\infty} C(s)'p_r = 1$.

(c) $s^*(M,M_*)\text{-}\lim_{s\to\infty} C(s)'p_r = 1$.

<u>Proof</u>. Suppose that (a) holds. Since $\mathrm{Fix}(\mathcal{T})$ separates $\mathrm{Fix}(\mathcal{T}')$ (see [Krengel (1985), Chap.2,Thm.1.4]), the fixed space of $\mathcal{T}'$ is non trivial, hence $p_r \neq 0$. Let $0 \leq \psi \in M_*$, then

$$\psi_o := \lim_{s\to\infty} C(s)\psi \in \mathrm{Fix}(\mathcal{T})$$

and $s(\psi_o) \leq p_r$.