Then Ax is given as the continuous extension of $(*)$. We shortly write $Ax = xB + B^*x$.

In the next theorem we give some equivalent conditions for the uniform exponential stability of an implemented semigroup . As we shall see, the operator equality

$$yB + B^*y = -x \quad (x,y \in M_+)$$

is necessary and sufficient, which is in complete analogy to the classical Liapunov stability result.

<u>Theorem</u> 2.2. Let M be a W*-algebra on a Hilbert space H and let $\mathcal{T} = (T(t))_{t\geq 0}$ be a weak*-semigroup on M with generator A implemented by the semigroup $(U(t))_{t\geq 0}$ on H with generator B . Then the following assertions are equivalent.

(a) $\omega(\mathcal{T}) = s(A) < 0$.

(b) The semigroup $(U(t))_{t\geq 0}$ is uniformly exponentially stable.

(c) There exists $0 \leq x \in D(A)$ such that $Ax = -1$.

(d) There exists $0 \leq x \in D(A)$ such that $x(D(B)) \subseteq D(B^*)$ and $xB + B^*x = -1$.

(e) For every $0 \leq x \in D(A)$ there exists $0 \leq y \in D(A)$ such that $Ay = -x$.

(f) For every $0 \leq x \in D(A)$ there exists $0 \leq y \in D(A)$ such that $y(D(B)) \subseteq D(B^*)$ and $yB + B^*y = -x$.

(g) $\int_0^\infty \|U(s)\xi\|^2 ds$ exists for all $\xi \in H$.

(h) $\int_0^\infty ((T(s)x)\xi | \zeta) ds$ exists for all $\xi, \zeta \in H$ and all $x \in M$.

<u>Proof</u>. The equivalence of (a) and (b) follows from Remark 2.1.(a) whereas (c) and (d), resp., (e) and (f) are equivalent by the Remark 2.1.(c).

(a) $\to$ (c): Since $s(A) < 0$ the resolvent $R(0,A)$ exists and is a positive map on M . Therefore $R(0,A)1 \in D(A)_+$ or $Ax = -1$ for some $x \in D(A)_+$.