Schaefer (1974), V.8.3), hence $\lim_{t\to\infty} \|T(t)f - R_\tau(t)\circ Qf\|_p = 0$ for every $f \in L^\infty(\mu)$. Since $(T(t))$ is bounded we finally obtain convergence in the L^p-norm for every $f \in L^p(\mu)$.

□

We give an example for the situation described in Thm.2.14. The equation we consider describes the division of a cell population. For details we refer to Diekmann-Heijmans-Thieme (1984).

Example 2.15. Let $E = L^1([\frac{1}{4},1],wdx)$, where the density w is a continuous positive function on $[\frac{1}{4},1]$, vanishes at $x = 1$ and is strictly positive in $[\frac{1}{4},1)$.
We consider the operator $C = A + B$ where A is defined by $(Af)(x) := -xf'(x)$ on the domain $D(A) := \{f \in AC : f(\frac{1}{4}) = 0\}$ and B is defined by

$$Bf(x) := \begin{cases} k(x)f(2x) & \text{if } x \le \frac{1}{2}, \\ 0 & \text{if } x > \frac{1}{2}. \end{cases}$$

Here k is a positive continuous function on $[\frac{1}{4},1]$ satisfying

(2.13) $k(x) > 0$ for $\frac{1}{4} < x < \frac{1}{2}$ and $\int_{1/4}^{1/2} \frac{k(y)}{y}\, dy = 1$.

In the following we show that under these hypotheses and for suitable w the semigroup generated by C fulfills the assertions of Thm.2.14.
The operator A generates the nilpotent semigroup $(T(t))$ defined by

$$(T(t)f)(x) = \begin{cases} f(e^{-t}x) & \text{if } e^{-t}x \ge \frac{1}{4}, \\ 0 & \text{otherwise}. \end{cases}$$

We have $(R(\lambda,A)f)(x) = x^{-\lambda}\int_{1/4}^{x} y^{\lambda-1}f(y)\, dy$ $(f \in E,\ x \in [\frac{1}{4},1])$. It follows that A has compact resolvent. Since B is bounded and positive, C is the generator of a positive semigroup $(S(t))$ having compact resolvent as well. Using C-III,Prop.3.3 one can show that $(S(t))$ is irreducible. Indeed, the non-trivial $(T(t))$-ideals are of the form $I_s = \{f \in E : f \text{ vanishes on } [\frac{1}{4},s]\}$ with s satisfying $\frac{1}{4} < s < 1$. Since none of these ideals is invariant under B, the semigroup $(S(t))$ is irreducible.
A suitable choice of the weight function w ensures that $(S(t))$ is bounded. Take

$$(2.14)\qquad w(x) := \begin{cases} \frac{1}{x} & \text{for } x \le \frac{1}{2}, \\ \frac{1}{x}\cdot\{1 - \int_{1/4}^{x/2} \frac{k(y)}{y}\, dy\} & \text{for } x \ge \frac{1}{2}. \end{cases}$$