It remains to show that $\mu - A$ is surjective for large real μ. Let $g \in C[0,1]$. Let $\lambda > 0$ and $k = 1/2\lambda\, [e^{\lambda x} \int_x^1 e^{-\lambda y} g(y)\,dy - e^{-\lambda x} \int_x^1 e^{\lambda y} g(y)\,dy]$. Then $k \in C^2[0,1]$ and $\lambda^2 k - k'' = g$. Let $h = ae^{\lambda x} + be^{-\lambda x}$, where $a,b \in \mathbb{R}$. Then $h \in C^2[0,1]$ and $\lambda^2 h - h'' = 0$. Let $f = k + h$. Then $\lambda^2 f - f'' = g$. The condition that $f \in D(A)$ leads to two linear equations in a and b, and it is easy to see that they have a solution $(a,b) \in \mathbb{R}^2$ if $(\lambda+\alpha)(\beta-\lambda) + (\lambda-\alpha)(\lambda+\beta)\exp(\lambda^2) \neq 0$. Thus there exists a solution if λ is large enough, and $(\lambda^2 - A)$ is surjective.

□

2. Lattice Semigroups on $C_o(X)$

Throughout this section X denotes a locally compact space and $C_o(X,\mathbb{R})$ (resp., $C_o(X,\mathbb{C})$) the space of all real-valued (resp., complex-valued) continuous functions on X which vanish at infinity. If we do not want to specify the field we simply write $C_o(X)$.
Recall from B-I,Sec.3 that a linear bounded operator T on $C_o(X)$ is positive if and only if

(2.1) $\quad |Tf| \leqq T|f| \quad$ for all $f \in C_o(X)$.

The operator T is a lattice homomorphism if and only if in (2.1) equality holds; i.e.,

(2.2) $\quad |Tf| = T|f| \quad$ for all $f \in C_o(X)$.

Remark 2.1. If T is a lattice homomorphism on $C_o(X,\mathbb{C})$, then T leaves $C_o(X,\mathbb{R})$ invariant and the restriction $T_{\mathbb{R}}$ of T to $C_o(X,\mathbb{R})$ is a lattice homomorphism. Conversely, the linear extension T of a lattice homomorphism $T_{\mathbb{R}}$ on $C_o(X,\mathbb{R})$ to $C_o(X,\mathbb{C})$ is a lattice homomorphism (see B-I,Sec.3).

A semigroup $(T(t))_{t\geqq 0}$ is called lattice semigroup if $T(t)$ is a lattice homomorphism for all $t \geqq 0$. In Section 3 we will give a concrete representation of lattice-semigroups which shows that there is a large variety of examples. This section is devoted to the characterization of lattice semigroups in terms of their generators.