

(the set in brackets is an order bounded subset of  $E_{\mathbb{R}}$ ). But for this we just have to observe that the set  $M = \{\operatorname{Re}(e^{i\theta}h) : 0 \leq \theta \leq 2\pi\}$  is contained and order bounded in the ideal generated in  $E_{\mathbb{R}}$  by  $|f| + |g|$ , which in turn is by the Kakutani-Krein Representation Theorem isomorphic to a space  $C_{\mathbb{R}}(X)$  under the pointwise ordering. Now the pointwise supremum of  $M$  in  $\mathbb{R}^X$  is readily seen to be a continuous function (namely, the modulus of the complex valued continuous function corresponding to  $f + ig$ ) so that  $M$  has a supremum in  $C_{\mathbb{R}}(X) = (E_{\mathbb{R}})_{|f|+|g|}$ .

Since the mapping  $f \mapsto |f|$  now has a meaning in  $E$ , the definition of an ideal can be extended formally unchanged to the complex situation. We observe that  $|f+ig| = |f-ig| \leq |f|+|g|$  implies that any ideal  $J$  in a complex Banach lattice is conjugation invariant and itself the complexification of the ideal  $J \cap E_{\mathbb{R}}$  of the real part of  $E$ . Suffice it now to say that the meaning of most of the terms introduced for real Banach lattices above can be extended to the complex situation under retention (*mutatis mutandis*) of the corresponding results valid in the real case by either using the complex modulus or else, if the formation of suprema or infima is involved, by relating them to real parts. For example  $f \in E$  is called positive if  $f = |f|$  which means that  $f$  is a positive element of  $E_{\mathbb{R}}$ ,  $E$  is called order complete if  $E_{\mathbb{R}}$  is order complete, and an ideal  $J$  is called a band if the real part of  $J$  is a band. We refer to Chapter II, Section 11 of Schaefer (1974) for a detailed discussion of this and restrict ourselves to a short discussion of linear mappings.

Let  $E$  and  $F$  be complex Banach lattices with real parts  $E_{\mathbb{R}}$  and  $F_{\mathbb{R}}$ . Then a linear mapping  $T$  from  $E$  into  $F$  is determined by its restriction  $T_0$  to  $E_{\mathbb{R}}$ , and  $T_0$  can be written in the form  $T_0 = T_1 + iT_2$  with real-linear mappings  $T_j$  from  $E_{\mathbb{R}}$  into  $F_{\mathbb{R}}$ . Thus  $L(E, F)$  is the complexification of the real linear space  $L(E_{\mathbb{R}}, F_{\mathbb{R}})$ . With the above notation,  $T$  is called real if  $T_2 = 0$ , positive if  $T$  is real and  $T_1$  is positive, and a lattice homomorphism if  $T$  is real and  $T_1$  is a lattice homomorphism. Lattice homomorphisms are characterized by the equality  $|Th| = T|h|$  as in the real case.