actly: B is a band in E if B is an ideal in E and $\sup M$ is contained in B whenever M is contained in B and has a supremum in E. Since the notions of sublattice, ideal, band are invariant under the formation of arbitrary intersections there exists, for any subset B of E, a uniquely determined smallest sublattice (ideal, band) of E containing B: the <u>sublattice (ideal, band) generated by B</u>.

If we denote by B^d the set $\{h \in E: \inf(|h|,|f|) = 0 \text{ for all } f \in B\}$, then B^d is a band for any subset B of E, and $(B^d)^d = B^{dd}$ is a band containing B. If E is a normed vector lattice (more generally, if E is archimedean ordered, see e.g. Schaefer (1974)), then B^{dd} is the band generated by B.

If two ideals I, J of a vector lattice E have trivial intersection $\{0\}$, then I and J are <u>lattice disjoint</u>, i.e. $I \subset J^d$. Thus if E is the direct sum of two ideals I, J then one has automatically $I = J^d$ and $J = I^d$, hence $I = I^{dd}$ and $J = J^{dd}$ must be bands in this situation. In general, an ideal I need not have a complementary ideal J, even if $I = I^{dd}$ is a band. This amounts to the same as saying that even if $I = I^{dd}$ (which is always true if I is a band in a normed vector lattice) one need not necessarily have $E = I + I^d$. An ideal I is called a <u>projection band</u> if it does have a complementary ideal, and in this case the projection of E onto I with kernel I^d is called the <u>band projection</u> belonging to I. An example of a band which is not a projection band is furnished by the subspace of $C([0,1])$ consisting of the functions vanishing on $[0,1/2]$. The <u>Riesz Decomposition Theorem</u> asserts that in an order complete vector lattice every band is a projection band. As a consequence, if E is order complete and B is an arbitrary subset of E, then E is the direct sum of the complementary bands B^d and B^{dd}. This Theorem, which is quite easy to prove, is widely used in Analysis and gives an abstract background to, e.g., the decomposition of a measure into atomic and diffuse parts (the atomic measures being those contained in the band generated by the point measures, the diffuse measures those disjoint to the latter) or, more specifically, to the well-known decomposition of a measure on $[a,b]$ into an atomic part and a diffuse part, which latter can in turn be decomposed into the sum of a measure which is absolutely continuous (which means, contained in the band generated by Lebesgue measure) and a singular measure (which means, a diffuse measure disjoint to Lebesgue measure).