restrict ourselves to positive groups (or equivalently semigroups of lattice isomorphisms). And in fact, it is not difficult to show by an example that the corresponding result is wrong for lattice semigroups in general.

A mapping $\phi : \mathbb{R} \times X \to X$ is called a <u>flow</u> on X if the partial maps $\phi_t : X \to X$ given by $\phi_t(x) = \phi(t,x)$ are continuous and satisfy

$$\phi_o(x) = x \qquad (x \in X) \tag{3.8}$$

$$\phi_s \circ \phi_t = \phi_{s+t} \qquad (s,t \in \mathbb{R}) \ . \tag{3.9}$$

It follows from the definition that each ϕ_t is a homeomorphism on X and $\phi_{-t} = (\phi_t)^{-1}$.
A flow ϕ is called <u>continuous</u> if it is continuous with respect to the product topology on $\mathbb{R} \times X$.
Given a flow ϕ a family $(h_t)_{t\in\mathbb{R}} \subset C^b(X)$ is called a <u>cocycle</u> of ϕ if

$$h_o = 1 \tag{3.10}$$

$$h_{t+s} = h_t \cdot (h_s \circ \phi_t) \qquad (s,t \in \mathbb{R}) \ . \tag{3.11}$$

It follows from (3.10) and (3.11) that $h_t(x) \neq 0$ for all $x \in X$ and $1/h_t(x) = h_{-t}(\phi_t(x))$ $(t \in \mathbb{R})$. The cocycle is called continuous if the mapping $(t,x) \to h_t(x)$ from $\mathbb{R} \times X$ into $\mathbb{R}$ is continuous with respect to the product topology on $\mathbb{R} \times X$.
Let ϕ be a flow and $(h_t)_{t\in\mathbb{R}}$ a cocycle of ϕ . Then

$$T(t)f = h_t \cdot f \circ \phi_t \tag{3.12}$$

defines a bounded operator $T(t)$ on $C_o(X)$ $(t \in \mathbb{R})$. Clearly $T(s+t) = T(s)T(t)$ for all $s,t \in \mathbb{R}$.

<u>Proposition</u> 3.8. Let $\phi : \mathbb{R} \times X \to X$ be a flow and $(h_t)_{t\in\mathbb{R}}$ a cocycle of ϕ . If for every $x \in X$ the mappings $t \to \phi_t(x)$ and $t \to h_t(x)$ are continuous, then (3.12) defines a strongly continuous group.

<u>Proof</u>. We first note that $\|T(t)\|$ is bounded on compact intervals of $\mathbb{R}$. This follows from [Hille-Phillips (1957), 7.4.1] since $q(t) = \log\|T(t)\|$ defines a subadditive, measurable function from $\mathbb{R}$ into $\mathbb{R}$. [In fact, $\|T(t)\| = \sup_{x\in X} |h_t(x)|$ for $t \in \mathbb{R}$. So it follows from the assumption that $t \to \|T(t)\|$ is lower semicontinuous and hence measurable]. If $f \in C_o(X)$, then by hypothesis the function