weak form (i.e.; we replace stable by weakly stable and 'lim' by 'weak-lim'). The proofs require only some obvious modifications.
If A has a compact resolvent or if A is the generator of a bounded holomorphic semigroup, then weak stability implies stability. In general, this is no longer true; e.g., the translation semigroup on $L^2(\mathbb{R})$ is weakly uniformly stable but not stable (see also B-IV,Ex.1.2).

2. STABILITY: INHOMOGENEOUS CASE

Using the results of the first section, we now investigate the long term behavior of the solutions of the inhomogeneous initial value problem

(2.1) $$\dot{u}(t) = Au(t) + F(t) \quad , \quad u(0) = f$$

where A is the generator of a strongly continuous semigroup on a Banach space E and $F(\cdot)$ is a locally integrable function from $\mathbb{R}_+$ into E called <u>forcing term</u>. A function $u(\cdot)$ is called a <u>(strong) solution</u> of (2.1) if $u(\cdot) : \mathbb{R}_+ \to D(A)$, $u(\cdot) \in C^1(\mathbb{R}_+,E)$ and (2.1) is satisfied for $t \geq 0$.
The assumption that A is the generator of a semigroup $(T(t))_{t\geq 0}$ yields the uniqueness of the solution of (2.1). If $u(\cdot)$ is a solution of (2.1), then the function $v(s) := T(t-s)u(s)$, $0 \leq s \leq t$, is differentiable and $v'(s) = T(t-s)F(s)$. But $F(\cdot)$ is locally integrable, and by $\int_0^t T(t-s)F(s)\,ds = v(t) - v(0) = u(t) - T(t)f$ we see that the solution $u(t)$ of (2.1) is given by

(2.2) $$u(t) = T(t)f + \int_0^t T(t-s)F(s)\,ds \quad .$$

<u>Example</u>. Let $(T(t))_{t\geq 0}$ be not eventually differentiable. Then there exists $g \in E$ such that $t \to T(t)g$ is not differentiable on $(0,\infty)$. The initial value problem $\dot{u}(t) = Au(t) + T(t)g$, $u(0) = 0$ has no (strong) solution $u(\cdot)$ because otherwise
$$u(t) = \int_0^t T(t-s)T(s)g\,ds = tT(t)g$$
has to be differentiable on $\mathbb{R}_+$.

Whenever the expression (2.2) makes sense we call it a <u>generalized (or mild) solution</u> of (2.1). If $F(\cdot)$ is continuous and $f \in D(A)$, then the generalized solution of (2.1) is a strong solution if and only if $v(t) := \int_0^t T(t-s)F(s)\,ds$ is differentiable (see Pazy (1983) Chap.4,