Now we are going to prove the main result of this section. As in the proof of Thm.2.10 we will utilize pseudo-resolvents on a suitable $\mathcal{F}$-product of the Banach lattice. To simplify the proof we isolate two lemmas.

Lemma 3.10. Let $\mathcal{F}$ be a filter on $\mathbb{N}$ which is finer than the Frechet filter and let $E_{\mathcal{F}}$ be the $\mathcal{F}$-product of the Banach lattice E. Given $R \in L(E)$ and denoting its canonical extension to $E_{\mathcal{F}}$ by $R_{\mathcal{F}}$ the following is true:
If $\alpha \in A\sigma(R)\setminus P\sigma(R)$ then $\ker(\alpha - R_{\mathcal{F}})$ is infinite dimensional.

Proof: Let $(f_n)_{n\geq 1}$ be a normalized approximate eigenvector of R corresponding to α. Since every accumulation point of (f_n) is an eigenvector of R, the assumption $\alpha \notin P\sigma(A)$ implies that (f_n) does not have any accumulation points. Then there exist an $\varepsilon > 0$ and a subsequence (g_n) of (f_n) such that

(3.14) $\|g_n - g_m\| \geq \varepsilon$ whenever $n \neq m$.

Obviously, (g_n) is a normalized approximate eigenvector of R and so is every subsequence of (g_n). In particular for $k \in \mathbb{N}$ the sequence $(g_{n+k})_{n\geq 1}$ is a normalized approximate eigenvector of R. Then the elements $\hat{g}^k \in E_{\mathcal{F}}$ given by $\hat{g}^k := ((g_{n+k})_{n\geq 1} + c_{\mathcal{F}}(E))$ are normalized eigenvectors of $R_{\mathcal{F}}$ corresponding to α. As a consequence of (3.14) we obtain
$\|\hat{g}^k - \hat{g}^m\| = \mathcal{F}\text{-}\lim\sup\|g_{n+k} - g_{n+m}\| \geq \varepsilon$ provided that $k \neq m$.
This shows that the unit ball in $\ker(\alpha - R_{\mathcal{F}})$ is not relatively compact, hence $\ker(\alpha - R_{\mathcal{F}})$ has to be infinite dimensional.

□

Lemma 3.11. Let E be a Banach lattice and let M, L be two linear subspaces of E.
Assume that $f \in M$ implies $|f| \in L$, then $\dim L \geq \dim M$.

Proof. Let $\{g_1, g_2, \ldots, g_m\}$ $(m\geq 1)$ be any (finite) subset of M which is linearly independent. For $u := \sum_{n=1}^{m} |g_n|$ all vectors g_n are contained in the principal ideal E_u which (by the Kakutani-Krein Theorem) is isomorphic to a space $C(K)$. Considering $g_1, g_2, \ldots, g_m$ as continuous functions on K, there exist points $x_1, x_2, \ldots, x_m \in K$ and functions $h_1, h_2, \ldots, h_m \in \operatorname{span}\{g_1, g_2, \ldots, g_m\}$ such that $h_i(x_j) = \delta_{ij}$. Then $|h_i|(x_j) = \delta_{ij}$ hence $\{|h_j| : 1 \leq j \leq m\}$