($Y := X\times[-1,1]$) is given by $\operatorname{supp} h = [\alpha,1]\times[0,1] \cup [0,\beta]\times[-1,0]$. Since we assumed that I_Y is $R(\lambda,A)$-invariant we have $h \in I_Y$, i.e., $\operatorname{supp} h \subset Y = X\times[-1,1]$. Obviously, this is true only if $Y = [0,1]\times[-1,1]$ or $I_Y = E$.
A weaker condition than (3.7) entailing irreducibility is the following.

(3.8) There exists $\delta > 0$ such that κ is strictly positive on the sets $[0,\delta]\times[-1,1]$ and $[1-\delta,1]\times[-1,1]$.

For details we refer to Greiner (1984d).

In the following proposition we list some properties which are consequences of irreducibility. This extends B-III,Prop.3.5 to the setting of Banach lattices. The first assertion of the latter proposition is no longer true in the general setting (see Ex.3.6 and Thm.3.7).

Proposition 3.5. Suppose A is the generator of an irreducible, positive semigroup on a Banach lattice E . Then the following assertions are true:

(a) Every positive eigenvector of A is a quasi-interior point.
(b) Every positive eigenvector of A' is strictly positive.
(c) If $\ker(s(A) - A')$ contains a positive element, then $\dim\bigl(\ker(s(A) - A)\bigr) \leq 1$.
(d) If $s(A)$ is a pole of the resolvent, then it has algebraic (and geometric) multiplicity 1 .
The corresponding residue has the form $P = \phi\otimes u$, where $\phi \in E'$ is a positive eigenvector of A' , $u \in E$ is a positive eigenvector of A and $\langle u,\phi\rangle = 1$.

Proof. To prove (a) , (b) and (d) one can proceed as in the case $C_o(X)$ (see B-III,Prop.3.5). We only prove (c) and assume $s(A) = 0$. By assumption and by assertion (a) there exists $\phi \gg 0$ such that $T(t)'\phi = \phi$ $(t\geq 0)$. Given $f \in \ker A$ then $T(t)f = f$ hence $|f| = |T(t)f| \leq T(t)|f|$. Since ϕ is strictly positive and $\langle |f|,\phi\rangle \leq \langle T(t)|f|,\phi\rangle = \langle |f|,\phi\rangle$ it follows that $|f| = T(t)|f|$. We have shown that $\ker A$ is a sublattice. Then for $f \in \ker A$, f real, i.e., $f = \bar{f}$, we have that f^+ and f^- are elements of $\ker A$. Hence the principal ideals generated by f^+ and f^- are $\mathcal{T}$-invariant. Since these ideals are orthogonal the irreducibility of $\mathcal{T}$ implies that either f^+ or f^- is zero.