semigroup property. Conditions for strong continuity are given in the following lemma.

Lemma 3.2. The following assertions are equivalent:

(i) The mapping $\phi : \mathbb{R}_+ \times K \to K$ is continuous (where $\mathbb{R} \times K$ carries the product topology).

(ii) The mapping ϕ is separately continuous.

(iii) $(T(t))_{t\geq 0}$ is a strongly continuous semigroup on $C(K)$.

Proof. (i) trivially implies (ii).
If (ii) holds, then $t \to T(t)f$ is weakly continuous for every $f \in C(K)$ (by the theorem of dominated convergence). This implies strong continuity (see for example [Davies (1980); Prop. 1.23]).
It remains to show that (iii) implies (i). Because of (3.1) it suffices to show that the restriction ϕ_o of ϕ to $[0,1] \times K$ is continuous. By hypothesis, the mapping $W : f \to (t \to T(t)f)$ from $C(K)$ into $C([0,1],C(K))$ is continuous. Identifying $C([0,1],C(K))$ canonically with $C([0,1] \times K)$ the mapping W obtains the form $f \to f\circ\phi_o$. Since W is continuous, ϕ_o is continuous as well.

□

A semiflow is called continuous if it satisfies the equivalent conditions of Lemma 3.2.

Definition 3.3. An operator δ on $C(K)$ is called derivation if $D(\delta)$ is a subalgebra of $C(K)$ such that

(3.4) $\delta(f\cdot g) = (\delta f)g + f(\delta g)$ for all $f,g \in D(\delta)$.

(3.5) $1 \in D(\delta)$

Note that (3.4) implies $\delta 1 = 0$.
A lattice semigroup $(T(t))_{t\geq 0}$ on $C(K)$ is called Markovian if $T(t)1 = 1$ for all $t \geq 0$.

Theorem 3.4. Let $(T(t))_{t\geq 0}$ be a semigroup on $C(K)$ with generator A . The following assertions are equivalent.

(i) $(T(t))_{t\geq 0}$ is a Markovian lattice semigroup.

(ii) $T(t)$ is an algebra homomorphism for every $t \geq 0$.

(iii) There exists a continuous semiflow ϕ on K such that $T(t)f = f\circ\phi_t \quad (t \geq 0)$.

(iv) A is a derivation.

Proof. (i) and (ii) are equivalent by the remark at the beginning of this section.