

Any Banach lattice satisfying (L) is called an abstract L-space or an AL-space. Thus the dual of an AM-space is an AL-space. It is quite easy to verify that on the other hand the dual of an AL-space is an AM-space with unit, the unit being the uniquely determined linear functional that coincides with the norm on the positive cone. Putting this together, one gets that the second dual of an AM-space E is an AM-space with unit. If E already has a unit u , then u is also the unit of E'' , so that the ideal of E'' generated by E is all of E'' . By contrast, if E is an AL-space, then E is an ideal (even a band) in E'' . Infinite-dimensional AL- or AM-spaces are never reflexive.

The importance of AL- and AM-spaces in the general theory of Banach lattices is due to the fact that these spaces have very special concrete representations as function lattices and that, on the other hand, any general Banach lattice E is in a very intimate way connected to certain families of AL- and AM-spaces canonically associated with E . Let us first discuss the natural representations of AM- and AL-spaces.

If E is an AM-space with unit u , then the set K of lattice homomorphisms (cf. Section 6) from E into \mathbb{R} taking the value 1 on u is a non-empty, compact subset of the weak dual of E and the natural evaluation map from E into \mathbb{R}^K maps E isometrically onto the continuous real-valued functions on K . This is the Kakutani-Krein Representation Theorem, which is an order-theoretic counterpart to the Gelfand Representation Theorem in the theory of commutative C^* -algebras. If E is an AM-space without unit, then the second dual of E has a unit and thus gives a representation of E as a closed sublattice of a space $C(K)$. If E is an AL-space, then the representation of the dual of E as a space $C(K)$ leads to an interpretation of the elements of E' as Radon measures on K . If E_+ has a quasi-interior point h , then in this interpretation E consists exactly of the measures absolutely continuous with respect to (the measure corresponding to) h , thus by the Radon-Nikodym Theorem $E = L^1(K, h)$. In general, a similar argument leads to a representation of E as a space $L^1(X, \mu)$ constructed over a locally compact space X .

If E is an arbitrary Banach lattice, $f \in E_+$, then the ideal I generated by f in E (which is the union of the positive multiples of the interval $[-f, f]$) can be made into an AM-space with unit f