The heuristic idea is the following. Let $(T(t))_{t\geq 0}$ be a lattice semigroup with generator $A$. Let $f \in D(A)$ and assume that the modulus function $\Theta$ given by $\Theta(g) = |g|$ is differentiable in $f$ (in some sense which has to be made precise). Then one expects that a chain rule holds so that $\Theta(T(t)f) = |T(t)|$ is differentiable at $t = 0$. Since $|T(t)f| = T(t)|f|$, this implies $|f| \in D(A)$ and $A|f| = d/dt_{|t=0}\ \Theta(T(t)f) = D_{Af}\ \Theta(f)\ d/dt_{|t=0}\ T(t)f = (D_{Af}\Theta(f)\ Af)$ (where the precise meaning of $(D_{Af}\Theta(f))Af$ depends on the chain rule which we will have to establish). So we obtain an identity for the generator $A$ which corresponds exactly to the lattice property $|T(t)f| = T(t)|f|$ of the semigroup. We will see in C-II, Sec. 5 that in a Banach lattice with order continuous norm the above argument is rigorous (for all $f \in D(A)$). On $C_o(X)$ we have to use a weak form of the argument and $|f| \in D(A)$ only holds for special $f \in D(A)$ (see Cor. 2.8).

We start by investigating differentiability of the modulus and by establishing a chain rule. For later use we formulate the following definition and proposition for a general Banach space $G$ even though only $G = \mathbb{C}$ will be considered in this section.

<u>Definition</u> 2.2. Let $G$ be a Banach space and $\Theta : G \to G$ a mapping. Let $f \in G$, $u \in G$. Then $\Theta$ is called right-sided Gateaux differentiable in $f$ in direction $u$ if

$$(2.3) \quad D_u\Theta(f) := \lim_{t\downarrow o} 1/t\ (\Theta(f+tu) - \Theta(f)) \quad \text{exists.}$$

The mapping $\Theta$ is right-sided Gateaux differentiable in $f$ if $D_u\Theta(f)$ exists for all directions $u \in G$; and if $\Theta$ is right-sided Gateaux-differentiable in every point $f \in G$, then we call $\Theta$ right-sided Gateaux differentiable.

<u>Proposition</u> 2.3 (chain rule). Let $G$ be a Banach space and $k : \mathbb{R} \to G$ be right-sided differentiable in $a \in \mathbb{R}$ (with right derivative $k'(a)$). Suppose that $\Theta : G \to G$ is a Lipschitz continuous mapping. If $\Theta$ is right-sided Gateaux-differentiable in $k(a)$ in the direction of $k'(a)$, then $\Theta \circ k : \mathbb{R} \to G$ is right-sided differentiable in $a$ and has a right derivative

$$(2.4) \qquad (\Theta \circ k)'(a) = D_{k'(a)}\Theta(k(a)) .$$