

A compact Hausdorff space is called an F-space if the closures of two disjoint open F_σ -sets are disjoint and is called a Stonean (res., σ -Stonean) space if the closure of every open set (res., open F_σ -set) is open. Every σ -Stonean space is an F-space.

Theorem 3.6. Every strongly continuous semigroups of operators on one of the following Banach spaces is uniformly continuous:

- 1) $C(K)$, where K is a compact F-space.
- 2) $L^\infty(S, \Sigma, \mu)$ for any measure space (S, Σ, μ) .
- 3) The Banach space $B(S, \Sigma)$ of bounded Σ -measurable functions on S if Σ is a σ -algebra of subsets of S .
- 4) The Banach space $H(0)$ of bounded continuous solutions of

$$\sum_{1 \leq i \leq n} (\partial^2 f / \partial x_i^2) = 0$$
 on an open subset O of \mathbb{R}^n .
- 5) The Banach space $W(0)$ of bounded continuous solutions of

$$\sum_{1 \leq i \leq n} (\partial^2 f / \partial x_i^2) = (\partial f / \partial x_{n+1})$$
 on an open subset O of \mathbb{R}^{n+1} .
- 6) The Banach space $H^\infty(0)$ of bounded analytic functions on a finitely connected domain O of the complex plane.

Proof. By Theorem 3.5 it suffices to show that the space listed above are Grothendieck spaces with the Dunford-Pettis property.

1) If K is compact, then $C(K)$ has the Dunford-Pettis property [Grothendieck (1953) Théorème 4]. If K is a compact F-space, then $C(K)$ is a Grothendieck space [Seever (1958) Theorem 2.5]; the special cases for Stonean and σ -Stonean spaces are due to [Grothendieck (1953), Théorème 9] and [Ando (1961)] respectively.

2) and 3) It is well known that every σ -order complete AM-space with unit is isometric to a space $C(K)$ where K is a compact σ -Stonean space. Obviously, the spaces under 2) and 3) are σ -order complete AM-spaces with unit and therefore by 1) Grothendieck spaces with the Dunford-Pettis property.

4) and 5) These spaces are order complete vector lattices. This follows from [Bauer (1966) pp.18-22, Standardbeispiele 1 and 2 p.55]. Since these spaces contain the constant functions on O they are complete for the supremum-norm. Indeed if (f_n) is a Cauchy-sequence for this norm, it is easily seen that (f_n) converges in norm to $\inf_n \sup_k (f_k : n < k)$. Therefore these spaces are σ -order complete AM-spaces with unit and so as before Grothendieck spaces with the Dunford-Pettis property.

6) Bourgain [(1980), Cor. 3] proves that $H^\infty(D)$ has the Dunford-Pettis property and in [(1984), Proposition III.1], that $H^\infty(D)$ is a