Chapter B-II

CHARACTERIZATION OF POSITIVE SEMIGROUPS ON $C_o(X)$

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It lies in the very nature of the theory of one-parameter semigroups that frequently an operator A is known to be a generator but the semigroup is not known explicitly. Thus, since the semigroup is uniquely determined by the generator, it is a central task in the theory to express properties of the semigroup in terms of its generator. In this chapter we do this for two properties. We characterize generators of positive semigroups and generators of lattice semigroups.

In Section 1 we consider a semigroup $(T(t))_{t\geq 0}$ on the real space $C(K)$ (K compact). It is shown that the semigroup consists of positive operators if and only if its generator satisfies a positive minimum principle (P). Even without assuming a priori that A is a generator the positive minimum principle has strong consequences. Together with a range condition it implies that A is a generator (of a positive semigroup). Moreover, we show that for a densely defined operator A to be the generator of a positive semigroup it is already sufficient that the resolvent $R(\lambda,A)$ of A exists and is positive for all sufficiently large real λ. For all these results it is essential to assume that K is compact. Concerning the characterization of positive semigroups on $C_o(X)$ (X locally compact, non-compact) we follow a completely different line and will treat this case in the context of general Banach lattices in Chapter C-II.

A special class of positive semigroups are lattice semigroups; i.e., semigroups of lattice homomorphisms. We show in Section 2 that $(T(t))_{t\geq 0}$ is a lattice semigroup if and only if its generator A satisfies an identity (K), the so-called Kato's equality (Theorem 2.5). We refer to Chapter C-II for a discussion of this identity and classical analogs for the Laplacian due to Kato (1973).