Finally we prove the group property of G . Let $\mathcal{V}$ be an ultrafilter on $\mathbb{R}$ such that $\lim_{\mathcal{V}} T_o(t) = Id$ in the strong operator topology. For positive $s \in \mathbb{R}$ let $S := \lim_{\mathcal{V}} T(t-s)$. Then $ST_o(s) = T_o(s)S = Id$, hence $T_o(s)^{-1}$ exists in G for all $s \in \mathbb{R}_+$. From this it follows that G is a group.

□

Remark 1.13. (a) Let $\kappa : \mathbb{R} \to G$ be given by

$$\kappa(t) = \begin{cases} T_o(t) & \text{if } 0 \leq t , \\ T_o(t)^{-1} & \text{if } t \leq 0 . \end{cases}$$

Then κ is a continuous homomorphism with dense range, i.e. (G,κ) is solenoidal (see [Hewitt-Ross (1963)]).

(b) The compact group G and the discret group $P\sigma(A) \cap i\mathbb{R}$ are dual in the sense of locally compact Abelian groups.

(c) Let (G,κ) be a solenoidal compact group and let $N_* = L^1(G)$. Then the induced lattice semigroup $\mathcal{T} = (\kappa(t))_{t \geq 0}$ fulfils the assertions of Theorem 1.10. For example, if G is the dual of $\mathbb{R}_d$, then $P\sigma(A) \cap i\mathbb{R} = i\mathbb{R}$. Since the fixed space of $\kappa(t)$ is given by

$$\mathrm{Fix}(\kappa(t)) = (\mathrm{span} \bigcup_{k \in \mathbb{Z}} \ker(\tfrac{2\pi i k}{t} - A))^{--} ,$$

no $T(t) \in \mathcal{T}$ is irreducible.

(d) If $\mathcal{T}$ is the irreducible semigroup of Schwarz type on the predual of $B(H)$ given in [Evans (1977)] then $P\sigma(A) \cap i\mathbb{R} = \emptyset$.

2. SPECTRAL PROPERTIES OF UNIFORMLY ERGODIC SEMIGROUPS

The aim of this section is the study of spectral properties of semigroups which are uniformly ergodic, identity preserving and of Schwarz type. For the basic theory of uniformly ergodic semigroups on Banach spaces we refer to Dunford-Schwartz (1958).