substitution $\tilde{z} := (\lambda_o - z)^{-1}$ yields a path $\tilde{\Gamma}$ around $(\lambda_o-\alpha)^{-1}$ and we obtain

$$P = \frac{1}{2\pi i} \int_{\tilde{\Gamma}} R(\tilde{z}, R(\lambda_o, A))\, d\tilde{z} ,$$

which is the residue of $R(., R(\lambda_o, A))$ at $(\lambda_o-\alpha)^{-1}$. The final assertion on the pole order follows from the identities

$$V_{-n} = ((\lambda_o-\alpha)^{-1} R(\lambda_o, A))^{n-1} U_{-n} , \quad n \in \mathbb{N} ,$$

where U_n, resp. V_n stand for the n-th coefficient in the Laurent series of $R(.,A)$, resp. $R(.,R(\lambda_o,A))$ at α, resp. $(\lambda_o-\alpha)^{-1}$. This has already been proved for $n = 1$ and follows for $n > 1$ by induction, using the relations

$$U_{-n-1} = (A - \alpha) U_{-n} \quad \text{and} \quad V_{-n-1} = (R(\lambda_o, A) - (\lambda_o-\alpha)^{-1}) V_{-n} .$$

□

3. SPECTRAL DECOMPOSITION

In the next two sections we develop some important techniques for our further investigation of semigroups and their generators. Even though these methods are well known (compare, e.g. Section VII.3 of Dunford-Schwartz (1958)) or rather technical, it is useful to present them in a coherent way.

Our interest in this section is the following : Let E be a Banach space and $\mathcal{T} = (T(t))_{t \geq 0}$ a strongly continuous semigroup with generator A. Suppose that the spectrum $\sigma(A)$ splits into the disjoint union of two closed subsets σ_1 and σ_2. Does there exist a corresponding decomposition of the space E and the semigroup $\mathcal{T}$?

In the following definition we explain what we understand by "corresponding decomposition".

Definition 3.1. Assume that $\sigma(A)$ is the disjoint union

$$\sigma(A) = \sigma_1 \cup \sigma_2$$

of two non-empty closed subsets σ_1, σ_2. A decomposition

$$E = E_1 \oplus E_2$$

of E into the direct sum of two non-trivial closed $\mathcal{T}$-invariant subspaces is called a spectral decomposition corresponding to $\sigma_1 \cup \sigma_2$ if the spectrum $\sigma(A_i)$ of the generator A_i of $\mathcal{T}_i := (T(t)_{|E_i})_{t\geq 0}$ coincides with σ_i for $i = 1 , 2$.