

## CHAPTER C-IV

### A S Y M P T O T I C S

#### O F P O S I T I V E S E M I G R O U P S

#### O N B A N A C H L A T T I C E S

In this chapter we describe the long term behavior of positive semigroups and discuss some concrete examples in more detail.

The first section is devoted to the stability of positive semigroups, and we give sufficient and necessary conditions which ensure that the semigroup (and the solution of the abstract Cauchy problem, respectively) converges to zero as  $t \rightarrow \infty$ . It is shown that for positive semigroups stability is determined fairly well by spectral properties of the generator.

In the second section we describe conditions ensuring convergence of the semigroup (as  $t \rightarrow \infty$ ) to an equilibrium point or to a periodic solution. Again we are interested in spectral conditions ensuring such a behavior.

In the final section a series of examples is discussed in more detail. In particular we consider semigroups related to retarded equations and discuss existence of solutions, spectral properties and asymptotic behavior. Most of the examples are motivated by biological models.