Since algebraic manipulation with these latter functions is governed by the formal rules of a tensor product, it is customary to identify (for example) the function

$$(x,y) \to f(x)g(y)$$

with the tensor product $f \otimes g$ and to consider limits of linear combinations of such functions as elements of a completed tensor product. To be more precise, we briefly present the most important examples for this situation.

Examples. 1. Let (X,Σ,μ) and (Y,Ω,ν) be measure spaces. Identifying for $f_i \in L^p(\mu)$, $g_i \in L^p(\nu)$ the elements $\sum_{i=1}^n f_i \otimes g_i$ of the tensor product

$$L^p(\mu) \otimes L^p(\nu)$$

with the (class of $\mu \times \nu$-a.e.-defined) functions

$$(x,y) \to \sum_{i=1}^n f_i(x)g_i(y) ,$$

$L^p(\mu) \otimes L^p(\nu)$ becomes a dense subspace of $L^p(X\times Y,\Sigma\times\Omega,\mu\times\nu)$ for $1 \leq p < \infty$.

2. Similarly, let X,Y be compact spaces. Then

$$C(X) \otimes C(Y)$$

becomes a dense subspace of $C(X\times Y)$ by identifying, for $f \in C(X)$ and $g \in C(Y)$, $f \otimes g$ with the function

$$(x,y) \to f(x)g(y) .$$

We do not intend to go into a deeper investigation of the quite sophisticated problems related to normed tensor products of general Banach spaces, but will rather confine ourselves to the discussion of certain special cases. These will always be related to one of the following standard methods to define a norm on the tensor product of two Banach spaces E , F :

Let $u := \sum_{i=1}^n f_i \otimes g_i$ be an element of $E \otimes F$. Then

(i) $\|u\|_\pi := \inf\{\sum_{i=1}^m \|h_i\|\|k_i\| : u = \sum_{i=1}^m h_i \otimes k_i , h_i \in E , k_i \in F\}$ defines the "greatest cross norm π" on $E \otimes F$.

(ii) $\|u\|_\varepsilon := \sup\{\langle u,\phi \otimes \psi\rangle : \phi \in E' , \psi \in F' , \|\phi\| , \|\psi\| \leq 1\}$ defines the "least cross norm ε" on $E \times F$. Here, $\langle u,\phi \otimes \psi\rangle$ denotes the canonical bilinear form on $(E \otimes F) \times (E' \otimes F')$, i.e. $\langle\sum_{i=1}^n f_i \otimes g_i,\phi \otimes \psi\rangle = \sum_{i=1}^n \langle f_i,\phi\rangle\langle g_i,\psi\rangle$.

(iii) if E and F are Hilbert spaces, $\|u\|_h = (u|u)_h^{1/2}$, where the scalar product $(\cdot|\cdot)_h$ is defined as in (ii), defines the "Hilbert norm h" on $E \otimes F$.