$f \in C_o(\mathbb{R}^n)$ the expression Δf is understood in the sense of distributions. Moreover, the space $C_c^\infty(\mathbb{R}^n)$ (of all infinitely differentiable functions with compact support) is a core of $\bar{A}$ (cf. d).

Proof. A is dispersive. In fact, let $f \in D(A)$. If $f^+ = 0$, then $\phi := 0 \in dN^+(f)$. So assume that $f^+ \neq 0$. Then there exists $x \in \mathbb{R}^n$ such that $f(x) = \|f\|_\infty = \sup\{f(y) : y \in \mathbb{R}^n\}$. Thus $\delta_x \in dN^+(f)$. Since f has a maximum in x it follows that $\langle Af, \delta_x\rangle = (\Delta f)(x) = \mathrm{tr}(\partial^2 f/\partial x_i \partial x_j)(x) \leq 0$. Moreover,

(1.3) $(Id - \Delta)$ is an isomorphism from $S(\mathbb{R}^n)$ onto $S(\mathbb{R}^n)$.

In fact, the Fourier transform $f \to \hat{f}$ is a bijection from $S(\mathbb{R}^n)$ onto $S(\mathbb{R}^n)$.
But $[(Id - \Delta)f]\hat{} = M\hat{f}$ where $(Mg)(y) = (1 + \sum_{i=1}^n y_i^2)g(y)$ $(g \in S(\mathbb{R}^n))$. It follows from (1.3) that $(Id - A)D(A)$ is dense in E. So the claim follows from Cor.1.3.

□

d) Let $E = L^p(\mathbb{R}^n)$ $(1 \leq p < \infty)$ and A be given by $Af = \Delta f$ with domain $D(A) = \{f \in L^p(\mathbb{R}^n) : \Delta f \in L^p(\mathbb{R}^n)\}$ where for $f \in L^p(\mathbb{R}^n)$ the expression Δf is understood in the sense of distributions. Then A is the generator of a positive contraction semigroup. Moreover, the space $C_c^\infty(\mathbb{R}^n)$ is a core of A.

Proof. It is easy to see that A is closed. Let A_o denote the restriction of A to $S := S(\mathbb{R}^n)$. Then $A_o f = \Delta f$ in the classical sense for all $f \in S$. One can show in an analogous way as in b) that A_o is dispersive. Moreover, it follows from (1.3) that $(Id - A_o)D(A_o)$ is dense. Hence by Cor. 1.3 the closure $\bar{A}_o$ of A_o is the generator of a positive contraction semigroup.
By construction one has $\bar{A}_o \subset A$. We prove that $\bar{A}_o = A$. For that it is enough to show that

(1.4) $(Id - A)$ is injective.

In fact, since the restriction $(Id - \bar{A}_o)$ of $(Id - A)$ is bijective from $D(\bar{A}_o)$ onto E it follows from (1.4) that $D(\bar{A}_o) = D(A)$. So let us show (1.4). Assume that there is $f \in E$ such that $f - Af = 0$. Let $\phi \in C_c^\infty(\mathbb{R}^n)$. Then

(1.5) $\langle \phi - \Delta\phi, f\rangle = 0$.

Since $C_c^\infty(\mathbb{R}^n)$ is dense in S for the topology of S, it follows from (1.3) that $(Id - \Delta)C_c^\infty(\mathbb{R}^n)$ is dense in S. Hence (1.5) implies