Combining Thm.2.10 and Prop.2.11 one obtains the following generalization of B-III,Thm.2.9. In contrast with the former result we do not assume that every point of $\sigma_b(A)$ is a pole.

Corollary 2.12. If A is the generator of a positive semigroup such that $s(A)$ is a pole of the resolvent then $\sigma_b(A)$ is cyclic.

Proof. Considering the sequence of ideals as described in Prop.2.11 and the corresponding generators A_n $(0 \leq n \leq k)$, then we have by A-III,Prop.4.2 $\sigma_b(A) = \cup_{n=1}^{k} \sigma_b(A_n)$.
By Thm.2.10 each set $\sigma_b(A_n)$ is cyclic hence so is $\sigma_b(A)$.

□

The proof of the preceding corollary indicates a possible generalization of Thm.2.10. Instead of assuming that the resolvent grows slowly one merely needs that there exist a finite chain of closed $\mathcal{T}$-invariant ideals as described in (2.22) such that the semigroups induced on the corresponding quotient spaces have slowly growing resolvents.

Knowing that $\sigma_b(A)$ is cyclic one has the alternative (cf. B-III, (2.19)):

Either $\sigma_b(A) = \{s(A)\}$ or else $\sigma_b(A)$ is an unbounded set.

Obviously one can use this fact to prove the existence of a dominant spectral value (cf. the explanation preceding B-III,Cor.2.11). We give a typical example.

Corollary 2.13. Let A be the generator of a positive, eventually norm-continuous semigroup. Suppose either that the resolvent grows slowly or that $s(A)$ is a pole of the resolvent. Then $s(A)$ is a dominant spectral value.

Proof. The boundary spectrum $\sigma_b(A)$ is cyclic (Thm.2.10 and Cor.2.12 resp.) and compact (A-II,Thm.1.20). Hence $\sigma_b(A) = \{s(A)\}$.

□

A situation in which Cor.2.13 can be applied is described in the following example. For more details and proofs we refer to Amann (1983)

Example 2.14. Let Ω be a bounded domain in $\mathbb{R}^n$ of class C^2 . Assume that $\partial\Omega = \Gamma_o \cup \Gamma_1$ where Γ_o and Γ_1 are disjoint open and