The preceding considerations remain true if we consider an (arbitrary) finite time delay τ where $0 < \tau < \infty$. Clearly, (RCP) can be treated as an differential equation with corresponding generator A (see (3.1) for the definition) in $C([-\tau,0],F)$ (instead of $C([-1,0],F)$).

Example 3.10. In order to illustrate the consequences of Cor.3.8 we consider the Cauchy problem

$$(3.7)\qquad \begin{aligned} \dot{u}(t) &= Bu(t) + Su(t-\tau)\ , \quad t \geq 0\ , \\ u(t) &= \psi(t)\ , \quad -\tau \leq t \leq 0 \ (0 < \tau < \infty)\ , \ \psi \in E\ , \end{aligned}$$

where B is the generator of a positive semigroup on F, $\sigma(B) \neq \emptyset$ and $S \in L(F)$ is positive.
Using the above terminology, we have $\Phi f = S(f(-\tau))$ for all $f \in E$, hence $\Phi_o = S$. By Cor.3.8 the solution semigroup corresponding to the retarded differential equation (3.7) is exponentially stable if and only if the semigroup generated by $B + S$ is exponentially stable.
But the semigroup generated by $B + S$ is the solution semigroup of the "undelayed" Cauchy problem

$$(3.8)\qquad \begin{aligned} \dot{u}(t) &= Bu(t) + Su(t)\ , \quad t \geq 0\ , \\ u(0) &= x\ , \qquad\qquad x \in F. \end{aligned}$$

More precisely, we obtain the following corollary.

Corollary. The solution of (3.7) is exponentially stable for every $\tau > 0$ if and only if the solution of (3.8) is exponentially stable.

In other words, the corollary states that for this "positive-type" delay equations ($(S(t))_{t\geq 0}$ and Φ positive) exponential stability is independent of the delay (see [Kerscher (1986)] for a detailed analysis of this phenomenon).
This is a rather untypical behavior since even a scalar valued delay differential equation may be stable for "small" delays but unstable for "large" delays.
We give an example and show how a stable Cauchy problem with non-positive solutions (see the remark following Prop.3.5) can be destabilized by an increase of the time lag τ.
Let $0 < \tau < \infty$ and $p,q \in \mathbb{R}$ and consider the following (RCP):

$$(3.9)_\tau\qquad \begin{aligned} \dot{u}(t) &= pu(t) + qu(t-\tau)\ , \quad t \geq 0\ , \\ u(t) &= \Psi(t)\ , \qquad -\tau \leq t \leq 0\ , \ \Psi \in C[-\tau,0]\ . \end{aligned}$$