is periodic of period τ it follows that 0 is a pole of the resolvent of its generator B with residuum $P = 1 \otimes 1$ and $\{\frac{2\pi i}{\tau} k : k \in \mathbb{Z}\} = \sigma(B)$. Thus $\mathcal{R}$ is irreducible and uniformly ergodic on $L^1(\Gamma,m)$ (see A-II, Section 5).

Now let $\mathcal{T}$ be a semigroup on M_* . It is called <u>partially periodic</u>, if there exists a projection $Q \in L(M_*)$ reducing $\mathcal{T}$ such that $Q(M_*) \cong L^1(\Gamma,m)$ and $\mathcal{T}_{|\mathrm{im}(Q)}$ is conjugate to a periodic semigroup on $L^1(\Gamma,m)$. In the main result we present a non commutative version of [Nagel (1984)] showing that certain dynamical systems converge to partially periodic semigroups.

<u>Proposition</u> 3.10. Let $\mathcal{T}$ be an irreducible, identity preserving semigroup of Schwarz type with generator A on the predual of a W*-algebra M . If $\mathcal{T}$ is uniformly ergodic, then $\sigma(A) \cap i\mathbb{R} = P\sigma(A) \cap i\mathbb{R} = i\alpha\mathbb{Z}$ for some $\alpha \in \mathbb{R}$. If additionally $\sigma(A) \cap i\mathbb{R} \neq \{0\}$, there exists a strictly positive projection Q on M_* which is identity preserving and completely positive such that:

(a) Q reduces $\mathcal{T}$ and $Q(M_*) \cong L^1(\Gamma)$, Γ being the one dimensional torus.

(b) The restriction $\mathcal{T}_o$ of $\mathcal{T}$ to $\mathrm{im}(Q)$ is irreducible and conjugate to a rotation semigroup of period $\tau = \frac{2\pi}{\alpha}$ on Γ .

(c) The spectral bound $s(A_{|\ker(Q)})$ is strictly smaller than 0 .

<u>Proof</u>. By D-III,Thm.1.11 and D-III,Thm.2.5 it follows that

$$\sigma(A) \cap i\mathbb{R} = P\sigma(A) \cap i\mathbb{R} = i\alpha\mathbb{Z}$$

for some $\alpha \in \mathbb{R}$. Suppose $\alpha \neq 0$. Since $\sigma(A) + i\alpha\mathbb{Z} = \sigma(A)$ and since every $\eta \in i\alpha\mathbb{Z}$ is isolated, it follows that there exists $\delta > 0$ such that

$$\sigma(A) \setminus i\alpha\mathbb{Z} \subseteq \{\lambda \in \mathbb{C} : \mathrm{Re}(\lambda) \leq \delta\} .$$

Let $\{u_\alpha^k : k \in \mathbb{Z}\}$ be a family of unitary eigenvectors of A' pertaining to the eigenvalues in $i\mathbb{R}$. Then $Q'(M)$ is a commutative W*-algebra. Let $\tau := \frac{2\pi}{\alpha}$. Then $T(\tau)'u_\alpha^k = u_\alpha^k$, hence $\mathcal{T}_{|\mathrm{im}(Q)}$ is periodic. From the Halmos-von Neumann theorem (see [Schaefer (1974),