

NOTES.

Section 1. Coincidence of spectral and growth bounds for L^1 -spaces was proven by Derndinger (1980). For L^2 -spaces the result is due to Greiner-Nagel (1983). For the result on AM-spaces we refer to Remark 1.1 of B-IV and the corresponding notes. Interpolation techniques in order to obtain results on arbitrary L^p -spaces were used by Voigt (1985). He proved Corollary 1.2(a). Theorem 1.3 as well as Propositions 1.6, 1.7, 1.9 are taken from Neubrander (1985a). For a comprehensive discussion of the coincidence of the spectral bound $s(A)$ with other growth bounds of positive semigroups on ordered Banach spaces, see Klein (1984). Similar results for finite dimensional (non-lattice) ordered spaces can be found in Stern (1982). For general results on convergence of the solutions of the inhomogeneous Cauchy problem we refer to Pazy (1983) and the references therein.

Section 2. For quasi-compact semigroups (as considered in Theorem 2.1) we refer to the notes of B-IV, Sec.2. Example 2.3 is discussed in more detail in Webb (1984) and Greiner (1984). Further examples of this type are considered in Section 3. It was Lotz (1986) who observed that Doeblin's condition is sufficient for quasi-compactness in reflexive L^p -spaces. (Obviously this is false in L^1 -spaces since in this case the identity operator satisfies Doeblin's condition.)

The 0-2-Law for certain bounded operators on L^1 was first established by Ornstein and Sucheston. A special case of the 0-2-Law for one-parameter semigroups (Theorem 2.6) was proven by Winkler (1972) while the general result and its corollaries can be found in Greiner (1982). Corollary 2.11 remains true when the assumption ' $T(t)$ is a kernel operator' is replaced by ' $T(t)_0$ is an irreducible Harris operator' (see Lin (1983)).

It is well-known that semigroups play an important role in probability theory (see e.g. Dynkin (1965), Feller (1952) and Hille-Phillips (1957)). A more detailed discussion than the one in Example 2.8 is given in Chapter 2 of van Casteren (1985). Convergence to periodic solutions is investigated in Kerscher-Nagel (1984) and Nagel (1984) where Proposition 2.13 is proved. They proved Proposition 2.13. The equation considered in Example 2.15 describes a linear model for cell division with exponential growth of individual cells. The occurring phenomena are conjectured by Diekmann et al. (1984).

Section 3. One of the starting points in the study of retarded equations was the book of Bellmann-Cooke (1963) on differential-difference equations. Initiated by Hale's semigroup approach (see B-IV, Sec.3) to retarded differential equations, Dyson-Villella-Bressan (1979), Villella-Bressan (1985) and Webb (1977) used such methods to investigate retarded equations. These similarly apply to Volterra equations [Miller (1974), Webb (1977)], and to age-dependent population equations [Prüß (1981), Webb (1984), (1985a)]. Recently, the aspect of positivity has led to statements on the asymptotic behavior of solutions of retarded equations. In this context the investigation of population equations by Greiner (1984), Heijmans (1985a) and Webb (1985b) should be mentioned.