

which allows us to verify any of them and to invent, prove or disprove new ones whenever necessary:

Any general formula relating a finite number of "variables" to each other by means of lattice operations and/or linear operations is valid in any Banach lattice as soon as it is valid in the real number system.

In fact, we are going to see below that any Banach lattice  $E$  is, as a vector lattice, "locally" of type  $C(X)$ , more exactly: Given any finite number  $x_1, \dots, x_n$  of elements in  $E$  there is a compact topological space  $X$  and a vector sublattice  $J$  of  $E$  which is isomorphic to  $C(X)$  and contains  $x_1, \dots, x_n$  (see Section 4). The above principle is an easy consequence of this: In a space  $C(X)$  it is clear that a formula of the type considered need only be verified pointwise, i.e. in  $\mathbb{R}$ .

The reader may now be prepared to follow a concise presentation of the most basic facts on Banach lattices.

### 1. SUBLATTICES, IDEALS, BANDS

The notion of a vector sublattice of a vector lattice  $E$  is self-explanatory, but it should be pointed out that a vector subspace  $F$  of  $E$  which is a vector lattice for the ordering induced by  $E$  need not be a vector sublattice of  $E$  (formation of suprema may differ in  $E$  and in  $F$ ), and that a vector sublattice need not contain (or may lead to different) infinite suprema and infima. The following are necessary and sufficient conditions on a vector subspace  $G$  of  $E$  to be a vector sublattice:

- (i)  $|h| \in G$  for all  $h \in G$
- (ii)  $h^+ \in G$  for all  $h \in G$
- (iii)  $h^- \in G$  for all  $h \in G$ .

A subset  $B$  of a vector lattice is called solid if  $f \in B$ ,  $|g| \leq |f|$  implies  $g \in B$ . Thus a norm on a vector lattice is a lattice norm if and only if its unit ball is solid. A solid linear subspace is called an ideal. Ideals are automatically vector sublattices since  $|\sup(f, g)| \leq |f| + |g|$ . On the other hand, a vector sublattice  $F$  is an ideal in  $E$  if  $g \in F$  and  $0 \leq f \leq g$  imply  $f \in F$ . A band in a vector lattice  $E$  is an ideal which contains arbitrary suprema, more ex-