

A global flow exists for example if F is globally Lipschitz continuous or if F is uniformly bounded. In case $\{x \in \mathbb{R}^n : (x|F(x)) > 0\}$ is bounded in \mathbb{R}^n a global semiflow always exists (see [Deimling (1977), Sec.5.2]).

(b) We do not assume that ϕ_0 is globally defined. Instead we consider a bounded domain $\Omega \subset \mathbb{R}^n$ with smooth boundary $\partial\Omega$ such that $(F(x)|v(x)) > 0$ for every $x \in \partial\Omega$. Here $v(x)$ denotes the outward normal vector.

Then for $x \in \bar{\Omega}$ we have $\underline{t}_x = -\infty$. Moreover, either $\phi_0(t, x) \in \Omega$ for all $t \geq 0$ or else there exists a unique s_x with $0 \leq s_x < \bar{t}_x$ such that $\phi_0(s_x, x) \in \partial\Omega$. In the first case we write $s_x := \infty$. Then we define $\phi : \mathbb{R}_+ \times \bar{\Omega} \rightarrow \bar{\Omega}$ as follows:

$$\phi(t, x) := \begin{cases} \phi_0(t, x) & \text{if } 0 \leq t < s_x \\ \phi_0(s_x, x) & \text{if } t \geq s_x \end{cases}$$

ϕ is a continuous semiflow on the compact set $K := \bar{\Omega}$. We have $K_\infty = K$ and $\phi|_{K_\infty}$ is not injective.

In case F is differentiable, the generator of the corresponding semigroup is the closure of the operator A_2 defined by $A_2 f := (F|\text{grad } f)$, $D(A_2) := \{f \in C^1(\bar{\Omega}) : (F|\text{grad } f) = 0 \text{ on } \partial\Omega\}$.

(c) We consider Ω as in (b) and assume that $(F(x)|v(x)) \leq 0$ for every $x \in \partial\Omega$. Then for every $x \in \bar{\Omega}$ we have $\bar{t}_x = \infty$.

Thus $\phi := \phi_0|_{\mathbb{R}_+ \times \bar{\Omega}}$ is a continuous semiflow on $K := \bar{\Omega}$.

If $(F(x)|v(x)) < 0$ for some $x \in \partial\Omega$ we have $K_t \subsetneq K_s$ whenever $t > s$ and $\phi|_{K_\infty}$ is injective. For a differentiable vector field F the generator of the corresponding semigroup is the closure of A_3 defined as follows: $A_3 f := (F|\text{grad } f)$, $D(A_3) := C^1(\bar{\Omega})$.

We conclude the discussion of semi-flows associated with ordinary differential equations by remarking that the ideas of (b) and (c) can be combined to obtain semigroups for more general subsets Ω .

We continue the discussion of the spectrum of semigroups of lattice homomorphisms on $C(K)$ given by (4.1). Thm.4.4 gives a good description of the part which is contained in $\{\lambda \in \mathbb{C} : \text{Re } \lambda < \underline{c}(h, \phi)\}$.

It is easy to see that the half-plane $\{\lambda \in \mathbb{C} : \text{Re } \lambda > \bar{c}(h, \phi)\}$ is always a subset of the resolvent set (see Prop.4.8(a) below). The description of the remaining part $\{\lambda \in \sigma(A) : \underline{c}(h, \phi) \leq \text{Re } \lambda \leq \bar{c}(h, \phi)\}$ is more difficult. First we discuss some examples and then give a partial answer to this problem (see Prop.4.8(b)-(e)).