This follows from Prop.3.8 and Prop.3.9.

Example 3.11. Let ϕ be a continuous flow on X .
a) Let p be a continuous function on X such that $\inf_{x\in X} p(x) > 0$ and $\sup_{x\in X} p(x) < \infty$. Then

$$(3.13) \qquad p_t := p/p\circ\phi_t \quad (t \in \mathbb{R})$$

defines a continuous cocycle of ϕ .

b) For $h \in C^b(X)$ define

$$(3.14) \qquad h_t(x) := \exp(\int_0^t h(\phi(s,x))ds) \ .$$

Then $(h_t)_{t\in\mathbb{R}}$ is a continuous cocycle of ϕ (compare (3.6)).

Cocycles as defined by (3.13) are always globally bounded. In general this is false for cocycles of the second type. On the other hand, a cocycle described by (3.14) is differentiable with respect to t . This is not satisfied by cocycles of the first type in general. Thus neither (3.13) nor (3.14) gives a description of arbitrary cocycles. However every positive cocyle is a product of cocycles of the form (3.13) and (3.14). More precisely, we have the following lemma.

Lemma 3.12. Let ϕ be a continuous flow on X and $(k_t)_{t\in\mathbb{R}} \subset C^b(X)_+$ a continuous cocycle of ϕ . Then there exist $p \in C^b(X)$ satisfying $\inf_{x\in X} p(x) > 0$ and $h \in C^b(X)$ such that

$$(3.15) \qquad k_t(x) = (p(x)/p(\phi(t,x))\cdot\exp(\int_0^t h(\phi(s,x))ds$$

for all $t \in \mathbb{R}$, $x \in X$.

Proof. We first note that there exist constants $M,w \geq 1$ such that

$$(3.16) \qquad (Me^{(w-1)|t|})^{-1} \leq k_t(x) \leq Me^{(w-1)|t|} \quad \text{for all } t \in \mathbb{R} \ , \ x \in X \ .$$

In fact, let $(T(t))_{t\in\mathbb{R}}$ be the group given by $T(t)f = k_t\cdot f\circ\phi_t$ $(t \in \mathbb{R}$, $f \in C_o(X))$. Then there exist constants $M,w \geq 1$ such that

$$(3.17) \qquad \|T(t)\| \leq Me^{(w-1)|t|}$$

for all $t \in \mathbb{R}$. Since $\|T(t)\| = \sup_{x\in X} k_t(x)$ the right inequality of