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# One-parameter Semigroups of Positive Operators

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# List of Symbols

|                                  |   |
|----------------------------------|---|
| $E_{\mathbb{R}}, E_{\mathbb{C}}$ | real, complex Banach lattice  |
| $E_+$                            | positive cone of an ordered vector space                                  |
| $E'$                             | dual Banach space   |
| $E^*$                            | semigroup dual  |
| $E_{\mathcal{T}}$                | $\mathcal{F}$ -product of $E$ with respect to the semigroup $\mathcal{T}$ |
| $E_{\mathcal{T}}$                | $\mathcal{F}$ -product of $E$   |
| $E_f$                            | see C-I, 4  |
| $(E, \varphi)$                   | see C-I, 4  |
| $E \otimes F$                    | tensor product  |
| $\mathcal{L}(E)$                 | bounded linear operators on $E$   |
| $\mathcal{Z}(E)$                 | center of $E$   |
| $E_n$                            | $n$ -th Sobolev space   |
| $\mathcal{B}(H)$                 | $W^*$ -algebra of all bounded linear operators on $H$                     |
| $S(M)$                           | state space of a $C^*$ -algebra $M$                                       |
| $M_+$                            | positive cone of the $C^*$ -algebra $M$                                   |
| $M_*$                            | predual of a $W^*$ -algebra $M$   |
| $M^{sa}$                         | self-adjoint part of a $C^*$ -algebra                                     |
| $M_n$                            | $C^*$ -algebra of all $n \times n$ -matrices                              |
| $AC$                             | absolutely continuous functions   |
| $BV$                             | functions of bounded variation  |
| $K$                              | compact topological space   |
| $X$                              | locally compact topological space   |
| $C(K), C(K, E)$                  | continuous functions (with values in $E$ )                                |
| $C_c(X)$                         | continuous functions with compact support                                 |
| $C_0(X)$                         | continuous functions vanishing in infinity                                |
| $C^b(X)$                         | bounded continuous functions  |
| $C_{ru}(X)$                      | uniformly continuous functions  |
| $C^n, C^{(n)}$                   | continuous differentiable functions ( $n$ -times)                         |
| $C_c^\infty(\mathbb{R}^n)$       | infinitely differentiable functions with compact support                  |
| $L^p(\mu)$                       | $p$ -integrable functions   |

|  |   |
|--|---|
| $S(\mathbb{R}^n)$                                      | Schwartz space                                    |
| $M(X)$   | regular Borel measures                            |
| $M_b(X)$   | bounded regular Borel measures                    |
| $\mathcal{T} = (T(t))_{t \geq 0}$                      | (one-parameter) semigroup                         |
| $T $   | subspace (reduced) semigroup                      |
| $T/$   | quotient semigroup                                |
| $\text{Fix}(\mathcal{T})$                              | fixed space of $\mathcal{T}$                      |
| $A$  | generator of a $C_0$ -semigroup                   |
| $A'$   | adjoint operator of $A$                           |
| $A^*$  | adjoint generator                                 |
| $\sigma(A)$  | spectrum of $A$                                   |
| $\varrho(A)$   | resolvent set of $A$                              |
| $\sigma_{ess}(A)$                                      | essential spectrum of $A$                         |
| $\sigma_b(A)$  | boundary spectrum of $A$                          |
| $P_\sigma(A)$  | point spectrum of $A$                             |
| $P_{\sigma_b}(A)$                                      | boundary point spectrum                           |
| $A_0(A)$   | approximate point spectrum of $A$                 |
| $R_\sigma(A)$  | residual spectrum $\mathfrak{c}$                  |
| $\omega; \omega(A); \omega(\mathcal{T}); \omega(T(t))$ | growth bound                                      |
| $s(A)$   | spectral bound                                    |
| $\omega_I(A)$  | growth bound of the solution of the (ACP)         |
| $\omega(f)$  | growth bound of $T(\cdot)f$                       |
| $r(A)$   | spectral radius of $A$                            |
| $\omega_{ess}(A)$                                      | essential growth bound of $A$                     |
| $r_{ess}(T)$   | essential spectral radius of $A$                  |
| $R(\lambda, A)$  | resolvent operator of $A$                         |
| $I^d, \{I^d\}_{d=1}^{dd}$                              | orthogonal band of $I$ (of $I^d$ )                |
| $\wedge$   | infimum   |
| $\vee$   | supremum  |
| $ T $  | modulus of a regular operator                     |
| $\hat{f}, \check{f}$                                   | Fourier (inverse Fourier) transformation          |
| $dp(f)$  | subdifferential of $p$ in $f$                     |
| $dN(f)$  | subdifferential of the norm in $f$                |
| $dN^+(f)$  | subdifferential of the canonical half-norm in $f$ |
| $\text{im}(T)$   | range of $T$                                      |
| $\ker(T)$  | null-space of $T$                                 |
| $\text{Im } z$   | imaginary part of $z$                             |
| $\text{Re } z$   | real part of $z$                                  |
| $\text{Re}(f), \text{Im}(f)$                           | see C-I, 7  |
| $\text{Re } T, \text{Im } T$                           | see C-I, 7  |
| $\bar{f}$  | complex conjugate of $f$                          |
| $S_f$  | signum operator with respect to $f$               |
| $\text{sign}(f)$                                       | signum of $f$ see C-II, 2.2                       |
| $f^{[n]}$  | see B-III, 2.2 ; C-III, 2.1                       |
| $ f $  | absolute value of $f$                             |

|                  |   |
|------------------|---|
| $f^+$            | positive part of $f$                    |
| $f^-$            | negative part of $f$                    |
| $\text{Id}$      | identity operator                       |
| $M_P$            | multiplication operator                 |
| $\mathbb{1}$     | function identically 1                  |
| $\mathbb{1}_B$   | characteristic function of the set $XB$ |
| $\delta_x$       | Dirac measure in $x$                    |
| $\text{tr}$      | trace                                   |
| $\text{span } M$ | linear subspace generated by $M$        |
| $S(\alpha)$      | sector in the complex plane             |
| $(ACP)$          | abstract Cauchy problem                 |
| $(P)$            | positive minimum principle              |
| $(P')$           | see B-II,1.21                           |
| $(K)$            | Kato's (equality) inequality            |
| $(RCP)$          | retarded Cauchy problem                 |
| $(RE)$           | retarded equation                       |
| $(T)$            | translation property                    |