be non trivial and $\mu R(\mu)$-invariant). The support projections of the $\psi_i^{(n)}$'s in M'' are orthogonal (since $\psi_i^{(n)} \leq \psi_i$) and different from zero. Let (z_γ) be a net in M_1^+ such that

$$\sigma(M'',M')\text{-}\lim_\gamma z_\gamma = s(\psi_1^{(n)}) .$$

Then $\lim_\gamma \psi_1^{(n)}(z_\gamma) = 1$ whereas $\lim_\gamma \psi_2^{(n)}(z_\gamma) = 0$. Let z be a $\sigma(M,M_*)$-accumulation point of (z_γ) in M_+ . Since every $\psi_i^{(n)}$ is normal, $\psi_1^{(n)}(z) = 1$ whereas $\psi_2^{(n)}(z) = 0$. The first condition implies $z \neq 0$ whereas the second shows that $\psi_2^{(n)}$ cannot be faithful. Since this is a contradiction, it follows $\dim \mathrm{Fix}(R') = 1$ which proves (d).

□

The next corollary is an easy application of Theorem 4.4 and of D-III, Proposition 2.3.

Corollary 4.5. Let $\mathcal{T}$ be an identity preserving semigroup of Schwarz type on the predual of a W*-algebra M . Then the following assertions are equivalent:

(a) $\mathcal{T}$ is uniformly ergodic with finite dimensional fixed space.

(b) The adjoint weak*-semigroup is strongly ergodic with finite dimensional fixed space.

(c) Every $\mathcal{T}''$-invariant state is normal.

Proof. If (a) is fulfilled then the semigroup $\mathcal{T}$ is strongly ergodic on M_* . Since

$$\dim \mathrm{Fix}(\mathcal{T}) = \dim \mathrm{Fix}(\mathcal{T}') < \infty$$

there exist normal states $\phi_1,\ldots,\phi_n$ in $\mathrm{Fix}(\mathcal{T})$ and $x_1,\ldots,x_k$ in $\mathrm{Fix}(\mathcal{T}')$ such that $\phi_n(x_m) = \delta_{n,m}$ $(1 \leq n,m \leq k)$ and

$$P = \sum_{i=1}^{k} \phi_i \otimes x_i$$

is the associated ergodic projection. If $(C(s))_{s>0}$ is the family of Césaro means of $\mathcal{T}$, then