In [Bratteli-Robinson (1979)] it is shown that $\mathcal{T}$ is a semigroup of Schwarz type if and only if $\mu R(\mu,A)$ is a Schwarz map for every $\mu\in\mathbb{R}_+$. Here we relate the domination of two semigroups to an inequality for the corresponding resolvent operator . This inequality will be needed later.

Theorem 2.1. Let $\mathcal{T} = (T(t))_{t\geq 0}$ be a semigroup of Schwarz type and $\mathcal{T} = (S(t))_{t\geq 0}$ a semigroup on a C*-algebra M with generators A and B , respectively. If

$$(*) \qquad (S(t)x)(S(t)x)^* \leq T(t)xx^*$$

for all $x\in M$ and $t\in\mathbb{R}_+$, then

$$(\mu R(\mu,B)x)(\mu R(\mu,B)x)^* \leq \mu R(\mu,A)xx^*$$

for all $x\in M$ and $\mu\in\mathbb{R}_+$. The same result holds if $\mathcal{T}$ is a weak*-semigroup of Schwarz type and $\mathcal{S}$ is a weak*-semigroup on a W*-algebra M such that $(*)$ is fulfilled.

Proof. From the assumption $(*)$ it follows that

$$\begin{aligned} 0 &\leq (S(r)x - S(t)x)(S(r)x - S(t)x)^* = \\ &= (S(r)x)(S(r)x)^* - (S(r)x)(S(t)x)^* - \\ &\qquad - (S(t)x)(S(r)x)^* + (S(t)x)(S(t)x)^* \leq \\ &\leq T(r)xx^* + T(t)xx^* - (S(r)x)(S(t)x)^* - \\ &\qquad - (S(t)x)(S(r)x)^* \end{aligned}$$

for every $r,t\in\mathbb{R}_+$. Hence

$$(S(r)x)(S(t)x)^* + (S(t)x)(S(r)x)^* \leq T(r)xx^* + T(t)xx^*.$$

Obviously, $\|S(t)\| \leq 1$ for all $t\in\mathbb{R}_+$. Then for all $\mu\in\mathbb{R}_+$ and $x\in M$:

$$(R(\mu,B)x)(R(\mu,B)x)^* = (\int_0^\infty e^{-\mu r}S(r)x\,dr)(\int_0^\infty e^{-\mu t}S(t)x\,dt)^* =$$