

Kato's inequality characterizes the positivity of the semigroup (cf. Nagel-Uhlig (1981)). The necessity of Kato's inequality in the form given in Thm. 2.4 was first proved in [Arendt (1982), Remark 3.10] with a different proof. The proof we give here appeared in Arendt (1984). Miyajima-Okazawa (1984) use this inequality to show that a differential operator on $L^p(\mathbb{R}^n)$ which generates a positive semigroup is necessarily of order ≤ 2 and has an elliptic principal part. This result is generalized to the spaces $L^p(\Omega)$, $\Omega \subset \mathbb{R}^n$ suitable, by Miyajima (1986).

Section 3. In this section we closely follow Arendt (1984). Theorem 3.8, in a similar form but with different proof, has been obtained independently by Schep (1985).

Section 4. The characterization of domination by Kato's inequality on a Hilbert space is due to Simon (1977). Further contributions are due to Hess-Schrader-Uhlenbrock (1977) and Kishimoto-Robinson (1980). Theorem 4.3 is due to Arendt (1984b). The result on Schrödinger operators on $L^p(\mathbb{R}^n)$ stated in Example 4.7 is due to Kato (1986). The case $p = 2$ was proved in Kato (1973), where the classical Kato's inequality was established. Extensive information on Schrödinger semigroups on $L^p(\mathbb{R}^n)$ is given in Simon (1982). Other recent results on the L^p -theory of Schrödinger operators are obtained by Davies (1986), Okazawa (1984) and Voigt (1984). The existence of the modulus semigroup of semigroups with bounded, regular generator (Theorem 4.17) is due to Derndinger (1984) (in the real case). Proposition 5.15 had been proved in Schaefer-Wolff-Arendt (1978) by a completely different method.

Section 5. The characterization of generators of lattice semigroups on a Banach lattice with order continuous norm (Cor. 5.8) is due to Nagel-Uhlig (1981). An extension of this result to arbitrary Banach lattices is given by Arendt (1982) from which the proof of Prop. 5.6 is taken as well. Local closed operators having an ideal as domain (i.e., operators satisfying condition (iv) of Thm. 5.13) are investigated in detail by Nakano (1950) who calls them dilatators. Peetre (1959) characterizes differential operators by locality (see also Luxemburg (1979)). In the context of C^* -algebras local operators are investigated by Batty (1985) and Batty-Robinson (1985).