$$\begin{array}{ccc} C_o(X) & \xrightarrow{T(t)} & C_o(X) \\ j \uparrow & & \uparrow j \\ C(\Gamma) & \xrightarrow[R_\tau(t)]{} & C(\Gamma) \end{array}$$

commutes. $(R_\tau(t))$ denotes the rotation semigroup of period τ (see A-I,2.5).
If X is compact, then j is a topological embedding.

<u>Proof</u>. Assume that $Ah = i\alpha h$, $\alpha > 0$, and let $\tilde{h}(x) := h(x)/|h(x)|$. Then we define j by

(3.9) $j(f) := |h|\cdot f\circ\tilde{h}$ (i.e., $(j(f))(x) = |h(x)|\cdot f(\tilde{h}(x)))$.

Obviously, j is a lattice homomorphism and because h has no zeros and $\tilde{h}$ has a dense image in Γ (Thm.3.6(c)), it follows that j is injective. For the functions $e_n \in C(\Gamma)$ given by $e_n(z) = z^n$ $(n \in \mathbb{Z})$ one has $j(e_n) = h^{[n]}$ $(n \in \mathbb{Z})$ and therefore
$T(t)\circ j(e_n) = T(t)h^{[n]} = e^{in\alpha t}\cdot h^{[n]}$ (cf. Thm.2.4) and
$j\circ R_\tau(t)(e_n) = j(e^{in\alpha t}\, e_n) = e^{in\alpha t}\cdot h^{[n]}$.
Since $\{e_n : n \in \mathbb{Z}\}$ is a total subset of $C(\Gamma)$ we have
$T(t)\circ j = j\circ R_\tau(t)$ for every $t > 0$.
If X is compact, then $\tilde{h}(X)$ is closed, hence $\tilde{h}$ is onto, moreover, $|h| \geq \varepsilon$ for some $\varepsilon > 0$ thus the definition of j implies that $\|j(f)\| > \varepsilon\|f\|$ for every $f \in C(\Gamma)$.

□

A consequence of Cor.3.8 is the following: If $\{s(A)\} \subsetneqq P\sigma(A)\cap i\mathbb{R}$, then for every $\varepsilon > 0$ there exists $g > 0$ such that $T(t)g$ and $T(s)g$ have disjoint support whenever $|s - t| = \varepsilon$. Another consequence is that there exist positive functions f_1 and f_2 such that $T(t)f_1$ and $T(t)f_2$ have disjoint support for every $t \geq 0$ (consider the images under j of two disjoint functions on $C(\Gamma)$). This observation proves the following corollary.

<u>Corollary</u> 3.9. Suppose that the hypotheses of Thm.3.6 are satisfied and that for some $t_o > 0$ we have $T(t_o)f >> 0$ whenever $f > 0$. Then $P\sigma(A)\cap i\mathbb{R} = \{0\}$.

Cor.3.9 can be applied if $T(t_o)$ is a kernel operator with strictly positive kernel. We give some examples: