show that $v = 0$. Considering $A - \mu$ and $B - \mu$ for some $\mu > s(A)$ instead of A and B we may assume that $s(A) < 0$. Then there exists a strictly positive set $M' \subset E'$ such that

$$(4.5) \qquad \phi \in D(A') \text{ and } A'\phi \leq 0 \text{ for all } \phi \in M'$$

(see the proof of Proposition 3.5).
Let $\phi \in M'$ and p be the seminorm given by $p(f) = \langle |f|,\phi\rangle$. We show that B is p-dissipative (see end of A-II,Sec.2).
Let $f \in D(B)$, $\psi = (\text{sign } \bar{f})'\phi$. Then it is easy to see that $\psi \in dp(f)$. Moreover, by (4.4) and (4.5) one obtains that $\mathrm{Re}\langle Bf,\psi\rangle = \mathrm{Re}\langle(\text{sign } \bar{f})Bf,\phi\rangle \leq \langle |f|,A'\phi\rangle \leq 0$. Thus B is p-dissipative. By the proof of A-II,Prop.2.9 one sees that $p(v) = 0$; i.e., $\langle |v|,\phi\rangle = 0$. Since $\phi \in M'$ was arbitrary we conclude that $v = 0$.

2. Let $\lambda > \lambda_o := \max\{s(A),0\}$. We show that for $f \in D(B)$,

$$(4.6) \qquad g = (\lambda - B)f \text{ implies } |f| \leq R(\lambda,A)|g|.$$

Let $\psi \in E'_+$. We have to show that $\langle |f|,\psi\rangle \leq \langle R(\lambda,A)|g|,\psi\rangle$.
Let $\phi = R(\lambda,A)'\psi \in D(A')_+$. Then by (4.4)

$$\begin{aligned}\langle |f|,\psi\rangle &= \langle |f|, (\lambda - A')\phi\rangle = \mathrm{Re}\langle(\text{sign } \bar{f})(\lambda f),\phi\rangle - \langle |f|,A'\phi\rangle \\ &\leq \mathrm{Re}\langle(\text{sign } \bar{f})(\lambda - B)f,\phi\rangle = \mathrm{Re}\langle(\text{sign } \bar{f})g,\phi\rangle \\ &\leq \langle |g|,\phi\rangle = \langle R(\lambda,A)|g|,\psi\rangle .\end{aligned}$$

It follows from (4.6) that for $\lambda > \lambda_o$ and $f \in D(\bar{B})$

$$(4.7) \qquad g = (\lambda - \bar{B})f \text{ implies } |f| \leq R(\lambda,A)|g|.$$

In particular, $(\lambda - \bar{B})$ is injective for $\lambda > \lambda_o$. Moreover,

$$(4.8) \qquad |R(\lambda,\bar{B})g| \leq R(\lambda,A)|g| \text{ for all } g \in E \text{ whenever } \lambda_o < \lambda \in \rho(\bar{B}) .$$

Assume now that $\mu > \lambda_o$ such that $(\mu - B)D(B)$ is dense in E . Then $(\mu - \bar{B})D(\bar{B}) = E$. (Indeed, let $h \in E$. There exists $f_n \in D(B)$ such that $g_n := (\mu - B)f_n \to h$ $(n \to \infty)$. By (4.6) it follows that $|f_n - f_m| \leq R(\lambda,A)|g_n - g_m|$. Thus (f_n) is a Cauchy sequence. Let $f = \lim_{n\to\infty} f_n$. Then $f \in D(\bar{B})$ and $(\mu - \bar{B})f = h$.) Thus $\mu \in \rho(\bar{B})$.
It follows from the hypothesis that there esists $\lambda_1 \in \rho(\bar{B})$ such that $\lambda_o < \lambda_1$. Since $R(\lambda,A) \leq R(\lambda_1,A)$ (by B-II,Lemma 1.9), it follows from (4.8) that $\|R(\lambda,\bar{B})\| \leq \|R(\lambda,A)\| \leq \|R(\lambda_1,A)\| := c$; hence $\mathrm{dist}(\lambda,\sigma(\bar{B})) = r(R(\lambda,\bar{B}))^{-1} \geq \|R(\lambda,\bar{B})\|^{-1} \geq 1/c$ for all $\lambda \in \rho(\bar{B}) \cap [\lambda_1,\infty]$. This implies that $[\lambda_1,\infty) \subset \rho(\bar{B})$. Moreover, it follows from (4.8) that

$$(4.9) \qquad |R(\lambda,\bar{B})^n f| \leq R(\lambda,A)^n|f| \qquad (f \in E,\ n \in \mathbb{N} ,\ \lambda_1 < \lambda) .$$