Student Information

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Answer 1

Say, we have n vertices that have the degree at least 4.
If we add all degrees of the vertices then this will be at least $4n$. Let v be vertex and V be set of the vertices in the graph. We can easily say,

$$4n \leq \sum_{v \in V} deg(v)$$

By Handshaking Theorem, we know that

$$2(Number\ of\ Edges) = \sum_{v \in V} deg(v)$$

So we can pass it into the previous inequality,

$$4n \leq 46$$

$$n \leq 11.5$$

This means number of vertices can be 11 at most.
Just a quick sanity check since these a small numbers, we can easily build this graph with 10 vertices having double loops and one vertex having one triple loop. Then we would have 11 vertices.

Answer 2

Assuming that G is simple graph with n vertices.
Dirac's Theorem states that if each of the vertices has degree $d \geq \frac{n}{2}$ where $n \geq 3$, then the graph G contains Hamiltonian Circuit. Since this is still a circuit it must start and finish at the same vertex. Let's call this vertex v_1, the vertex after v_1 v_2 and the vertex just before the circuit is completed v_k (i.e. v_2 is the second vertex traveled in the circuit, v_k is the vertex just before completing the circuit at v_1 again). Since v_1 is the first and the last vertex in the circuit, if we remove it and all the edges incident to it then we will not have a Hamiltonian Circuit but we will have Hamiltonian Path. It will start at v_2 and it will end at v_k. So this way, we always have a Hamiltonian Path if we remove the starting and the ending vertex (i.e. v_1). We would have one less vertex so this is possible if each of the vertices of the simple graph G, has degree $d \geq \frac{n-1}{2}$ and $n \geq 3$.

Answer 3

A bipartite graph G is a graph whose set of vertices V can be partitioned into two nonempty subsets V_1 and V_2 such that every edge in G connects V_1 and V_2. Therefore, the first neighbors of vertices in V_1 are contained in V_2 and vice versa. Let's say 1, 2, ..., m belong to the subset V_1 and vertices $p+1,\ p+2,\ ...,\ p+n$ belong to V_2 then the corresponding adjacency matrix is going to be,

$$A = \begin{bmatrix} 0 & B \\ B^T & 0 \end{bmatrix}$$

where B is a sub-matrix with dimensions m x n, B^T is its transpose and 0 are the zero matrices of the size determined by the sub-matrices. Then we can say,

$$A^2 = \begin{bmatrix} BB^T & 0 \\ 0 & B^TB \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & BB^TB \\ B^TBB^T & 0 \end{bmatrix}$$

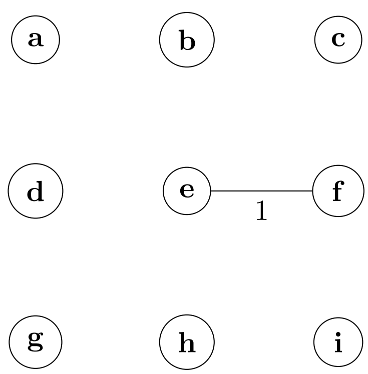
Considering this pattern we can easily determine that A^k always has 0 on its diagonal when k is odd. Since 37 is an odd number the final matrix will have the form,

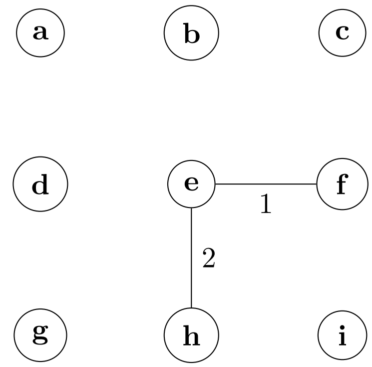
$$A^{37} = \begin{bmatrix} 0 & BB^TBB^T\ ...\ B^TB \\ B^TBB^TB\ ...\ BB^T & 0 \end{bmatrix}$$

So as we can see the diagonal is all 0.

Answer 4

a.

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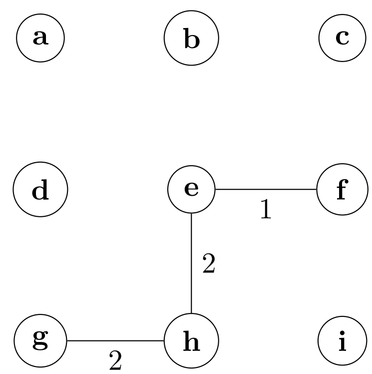
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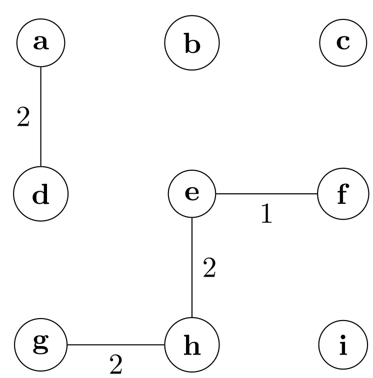
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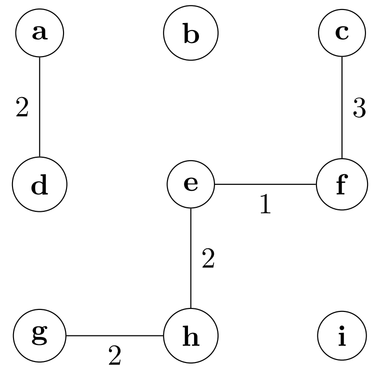
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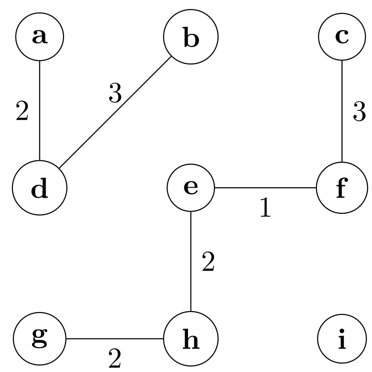
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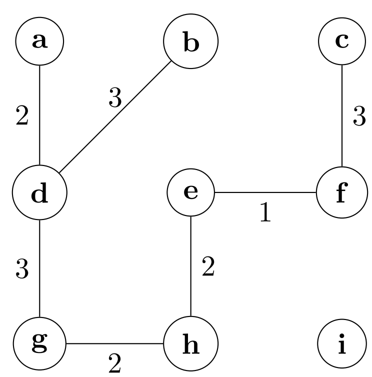
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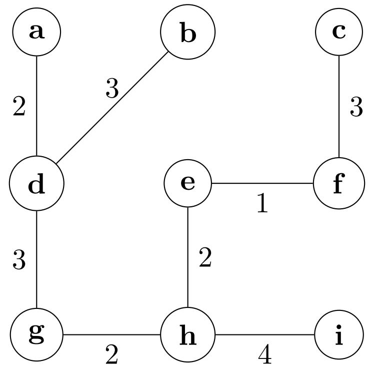

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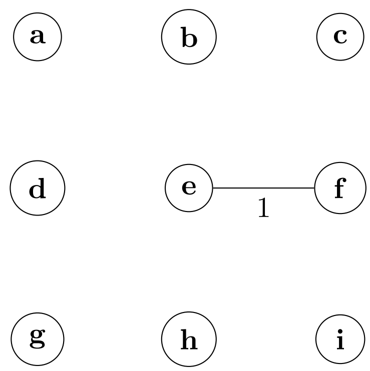

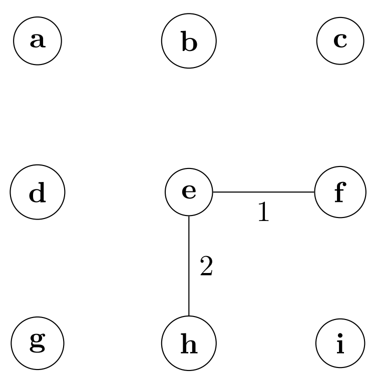

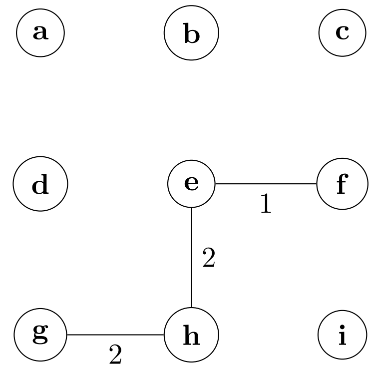
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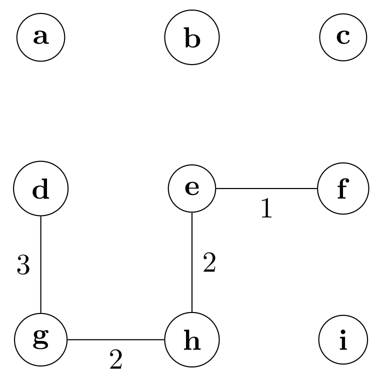
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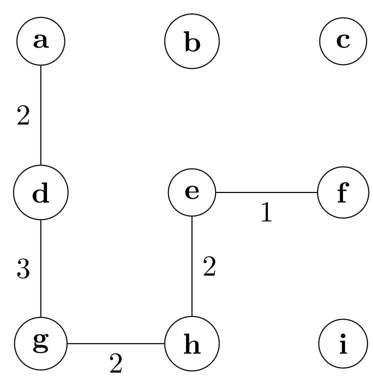
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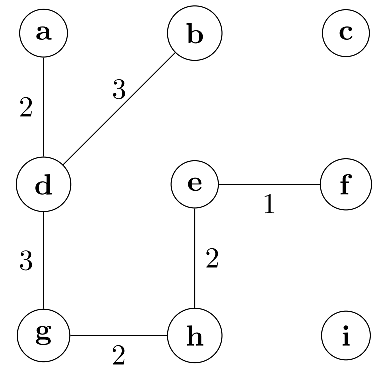
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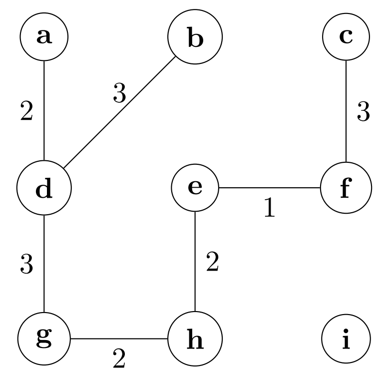
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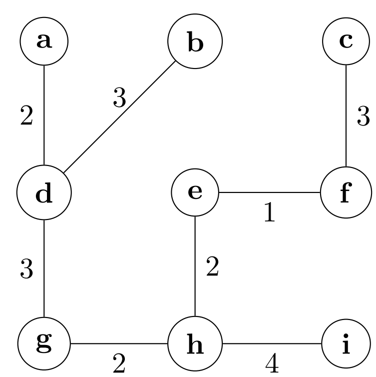
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