Student Information

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Answer 1

Table 1: Question 1.1

p	q	$\neg q$	$p \to q$	$\neg q \land (p \to q)$	$\neg p$	$(\neg q \land (p \to q)) \to \neg p$
T	F	T	F	F	F	T
F	T	F	T	F	T	T
T	T	F	T	F	F	T
F	F	T	T	T	T	T

Table 2: Question 1.2

p	q	r	$p \lor q$	$\neg p$	$\neg p \lor r$	$(p \lor q) \land (\neg p \lor r)$	$q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$
T	T	T	T	F	T	T	T	T
F	T	T	T	T	T	T	T	T
T	F	T	T	F	T	T	T	T
F	F	T	F	T	T	F	T	T
T	T	F	T	F	F	F	T	T
F	T	F	T	T	T	T	T	T
T	F	F	T	F	F	F	F	T
F	F	F	F	T	T	F	F	T

Answer 2

$$
\begin{aligned}
(p \to q) \lor (p \to r) &\equiv (\neg p \lor q) \lor (p \to r) && \textit{Using Table 7} \\
&\equiv (\neg p \lor q) \lor (\neg p \lor r) && \textit{Using Table 7} \\
&\equiv (q \lor r) \lor (\neg p \lor \neg p) && \textit{Commutative Laws} \\
&\equiv (q \lor r) \lor \neg p && \textit{Idempotent Laws} \\
&\equiv \neg(q \lor r) \to \neg p && \textit{Using Table 7} \\
&\equiv (\neg q \land \neg r) \to \neg p && \textit{De Morgan's Laws}
\end{aligned} \tag{1}
$$

Answer 3

Question 3.1

a) Every cat is friend with at least one dog.
b) There exists at least one cat that is friend with all dogs.

Question 3.2

$$a) \quad \forall x \forall y \{[Eats(x,y) \rightarrow Meal(y)] \rightarrow Customer(x)\}$$

$$b) \quad \neg \forall x \{Chef(x) \rightarrow \forall y [Meal(y) \rightarrow Cooks(x,y)]\}$$

$$c) \quad \exists x \{Customer(x) \wedge \exists y [Chef(y) \wedge \forall z ((Cooks(y,z) \rightarrow Meal(z)) \rightarrow Eats(x,z))]\}$$

$$d) \quad \forall x \{Chef(x) \rightarrow \exists y [Knows(x,y) \wedge Chef(y) \wedge \forall z ((Cooks(y,z) \rightarrow Meal(z)) \rightarrow \neg Cooks(x,z))]\}$$

Answer 4

Table 3: Question 4

p	q	$\neg p$	$p \rightarrow q$	$\neg q$
T	T	F	T	F
T	F	F	F	T
F	**T**	**T**	**T**	**F**
F	**F**	**T**	**T**	**T**

Considering the last two lines of the Table 3, we can see that when $\neg p$ is true, as given in the premise, $\neg q$ can either be true or false. In conclusion, false implies anything. That's why given entailment cannot be a deduction rule in a sound deductive system.

Answer 5

Table 4: Question 5

1	$p \to q$	*premise*
2	$q \to r$	*premise*
3	$r \to p$	*premise*
4	q	*assumption*
5	r	$\to$e $2, 4$
6	p	$\to$e $3, 5$
7	$q \to p$	$\to$i $4 - 6$
8	$p \iff q$	$\iff$ i $1, 7$
9	p	*assumption*
10	q	$\to$e $1, 9$
11	r	$\to$e $2, 10$
12	$p \to r$	$\to$i $9 - 11$
13	$p \iff r$	$\iff$ i $3, 12$
14	$(p \iff q) \wedge (p \iff r)$	$\wedge i$ $8, 13$

Answer 6

Table 5: Question 6

1	$\forall x(Q(x) \to R(x))$	*premise*
2	$\exists x(P(x) \to Q(x))$	*premise*
3	$\forall x P(x)$	*premise*
4	$P(c) \to Q(c)$	*assumption*
5	$Q(c) \to R(c)$	$\forall$e 1
6	$P(c)$	$\forall$e 3
7	$Q(c)$	$\to$e $4, 6$
8	$R(c)$	$\to$e $5, 7$
9	$P(c) \wedge R(c)$	$\wedge$i $6, 8$
10	$\exists x(P(x) \wedge R(x))$	$\exists$i 9
11	$\exists x(P(x) \wedge R(x))$	$\exists$e $2, 4 - 10$