# Assignment III (20 pts)

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Assigned : May the 12th, 19h00
Due : May the 19th, 19h00

**Q1. (4 pts)** Transform the grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$ with below production rules $P$

$$S \rightarrow ASB \mid B \mid a \mid \varepsilon$$
$$A \rightarrow bASA \mid a \mid \varepsilon$$
$$B \rightarrow SbbS \mid BASB \mid b$$

into an equivalent grammar $G'$ in Chomsky Normal Form. Clearly show intermediate steps.

**A1.** We start with eliminating the $\varepsilon$-production $S \rightarrow \varepsilon$:

$S \rightarrow ASB \mid AB \mid B \mid a$

$A \rightarrow bASA \mid bAA \mid a \mid \varepsilon$

$B \rightarrow SbbS \mid BASB \mid bbS \mid Sbb \mid BAB \mid bb \mid b$

We then eliminate the $\varepsilon$-production $A \rightarrow \varepsilon$:

$S \rightarrow ASB \mid AB \mid SB \mid B \mid a$

$A \rightarrow bASA \mid bSA \mid bAS \mid bAA \mid bA \mid bS \mid a \mid b$

$B \rightarrow SbbS \mid BASB \mid bbS \mid Sbb \mid BAB \mid BSB \mid BB \mid bb \mid b$

Next we eliminate the unit-production $S \rightarrow B$:

$S \rightarrow ASB \mid AB \mid SB \mid SbbS \mid BASB \mid bbS \mid Sbb \mid BAB \mid BSB \mid BB \mid bb \mid b \mid a$

$A \rightarrow bASA \mid bSA \mid bAS \mid bAA \mid bA \mid bS \mid a \mid b$

$B \rightarrow SbbS \mid BASB \mid bbS \mid Sbb \mid BAB \mid BSB \mid BB \mid bb \mid b$

We now introduce new nonterminals:

$U \rightarrow b$

$S \rightarrow ASB \mid AB \mid SB \mid SUUS \mid BASB \mid UUS \mid SUU \mid BAB \mid BSB \mid BB \mid UU \mid b \mid a$

$A \rightarrow UASA \mid USA \mid UAS \mid UAA \mid UA \mid US \mid a \mid b$

$B \rightarrow SUUS \mid BASB \mid UUS \mid SUU \mid BAB \mid BSB \mid BB \mid UU \mid b$

Splitting the right-hand sides long right hand sizes produces an equivalent grammar $G'$ in Chomsky normal form:

$U \rightarrow b \quad V \rightarrow SB \quad W \rightarrow SU \quad Y \rightarrow US \quad Z \rightarrow BA \quad T \rightarrow UA \quad K \rightarrow SA$

$S \rightarrow AV \mid AB \mid SB \mid WY \mid ZV \mid UY \mid WU \mid BV \mid ZB \mid BB \mid UU \mid a \mid b$

$A \rightarrow TK \mid YA \mid UK \mid TS \mid TA \mid UA \mid US \mid a \mid b$

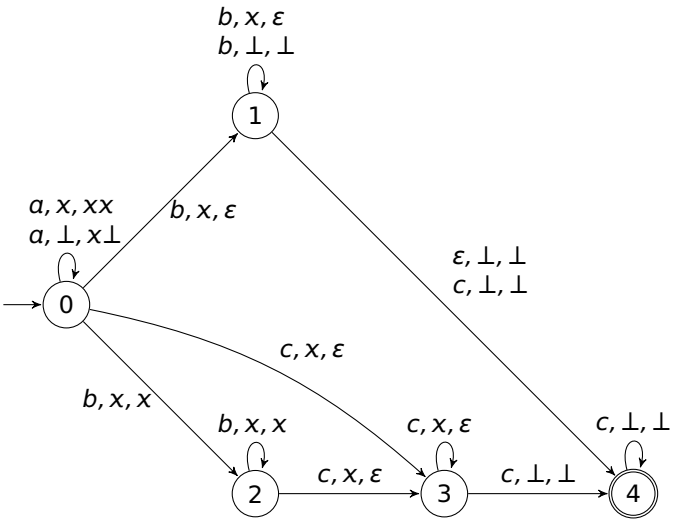
$B \to WY \mid ZV \mid UY \mid WU \mid ZB \mid BV \mid BB \mid UU \mid b.$

**Q2. (8 pts)** Design a non-deterministic push down automaton (NPDA) $N = (Q, \{a, b, c\}, \{\bot, x\}, \delta, s, \bot, F)$ that recognizes the language

$$A := \{a^i b^j c^k \mid k > i > 0 \text{ or } j \geqslant i > 0\}.$$

Justify your design in a few lines.

**A2.** The NPDA $N = (\{0, 1, 2, 3, 4\}, \{a, b\}, \{\bot, x\}, \delta, 0, \bot, \{4\})$ with set of transitions $\delta$ depicted in below state diagram



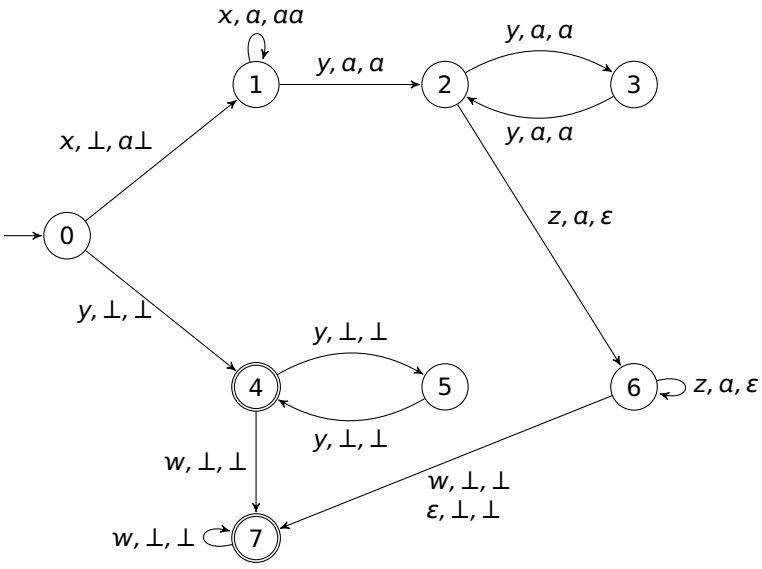

accepts the set $A$ by final state. In state 0, we first push all $a$'s on the stack. Then we non-deterministically guess if $k > i > 0$ or $j \geqslant i > 0$ holds. The former is checked in states 2, 3 and 4 while the latter is tested in states 1 and 4.

**Q3. (8 pts)** Design a non-deterministic push down automaton (NPDA) $N = (Q, \{x, y, z, w\}, \{\bot, a\}, \delta, s, \bot, F)$ that recognizes the language

$$A := \{x^i y^j z^k w^m \mid i = k \geqslant 0, j \text{ is odd and } m \geqslant 0\}.$$

Justify your design in a few lines.

**A3.** The NPDA $N = (\{0, 1, 2, 3, 4, 5, 6, 7\}, \{x, y, z, w\}, \{\bot, a\}, \delta, 0, \bot, \{4, 7\})$ with transitions depicted in below state diagram




accepts the set $A$ by final state.

If the input string starts with $x$s, the machine pushes an $a$ onto the stack for every consumption of $x$ in states 0 and 1. It is then guaranteed that the string contains odd number of $y$s following $x$s and some $z$s in what follows if the machine happens to advance into the state 6. There, it pops an $a$ off the stack upon reading a $z$. The number of $z$s and $x$s should be equal if the stack contains no $a$s, only the bottom ($\perp$) symbol immediately after all $z$s are consumed. The machine then evolves into the state 7 and accepts if the number of $w$s at the end of the string is strictly greater than or equal to zero.

Otherwise, the string must start with even number of $y$s followed by zero or more $w$s to be accepted. This situation is checked with the states 0, 4, 5, and 7. Notice also that in this case, the machine does not make use of the stack properly as all transitions are performed alongside popping the bottom of the stack off and pushing it back onto the stack.

**Important Notice:**

- Collaboration is strictly and positively prohibited; lowers your score to 0 if detected.
- Any submission after 19h00 on May the 19$^{th}$ will NOT be accepted. Please beware and respect the deadline!
- All handwritten answers should somehow be scanned into a single pdf file, and only then submitted. Make sure that your handwriting is decent and readable.