CMPE 322/327 - Theory of Computation

Week 2: Deterministic Finite Automata & Closure Properties

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Outline

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- $\{x \mid x$ is valid program in some machine language$\}$

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8. power set $2^A := \{Q \mid Q \subseteq A\}$

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- $2^{\{1,01\}} = \{\emptyset, \{1\}, \{01\}, \{1, 01\}\}$

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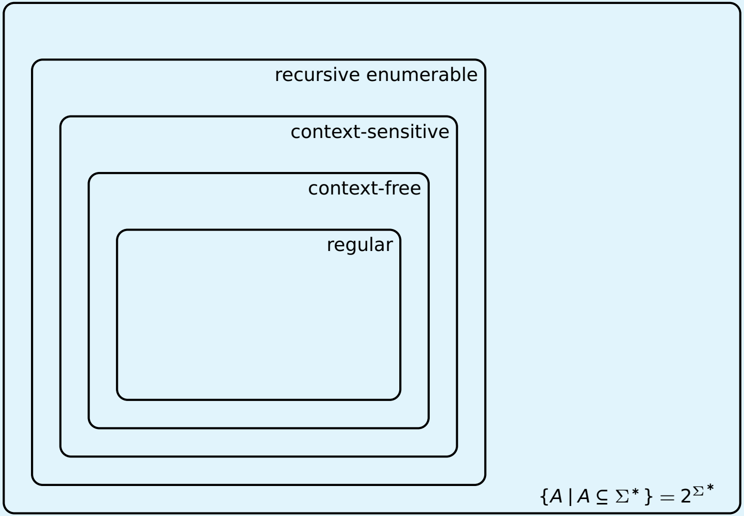
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context-sensitive
context-free
regular
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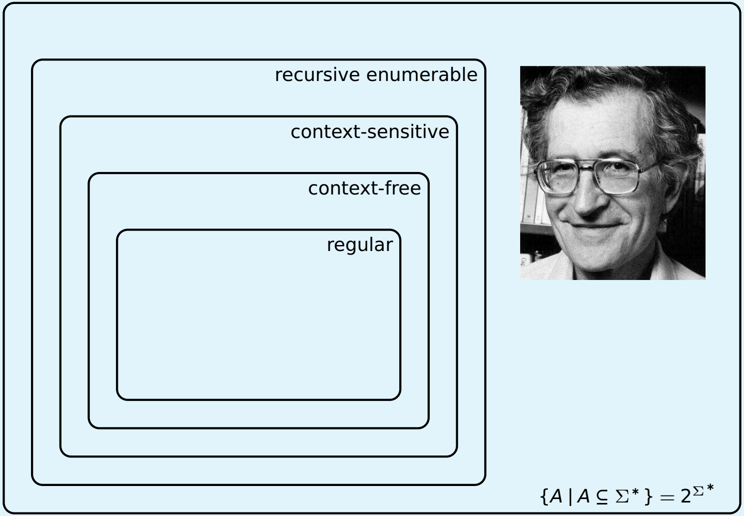
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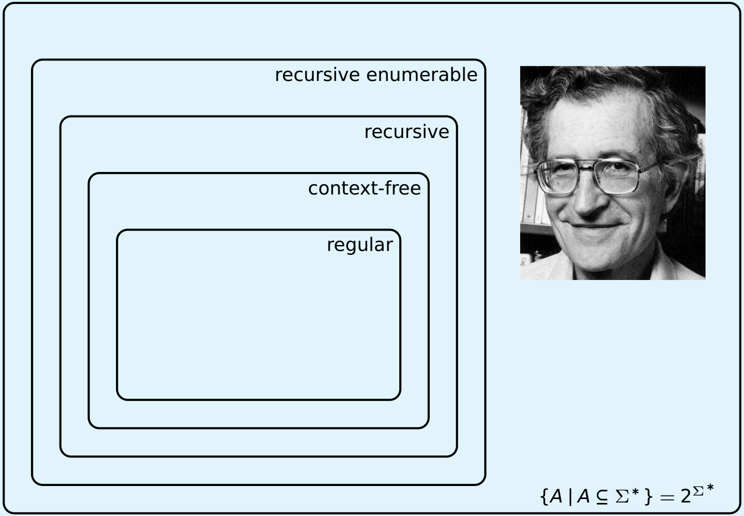
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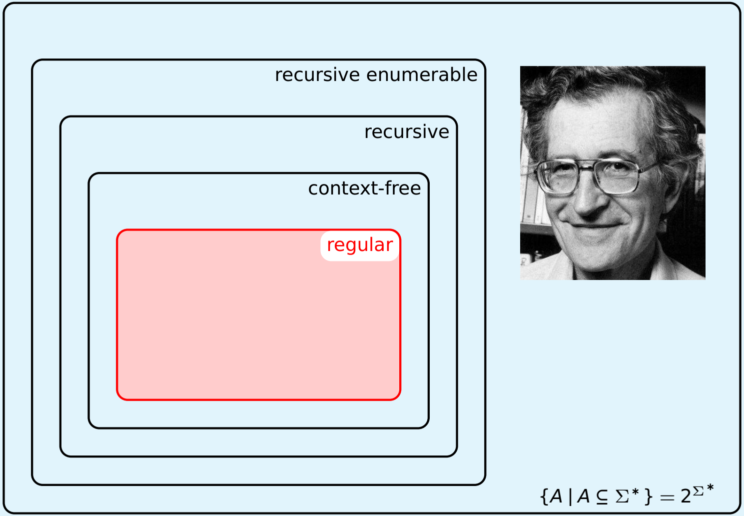
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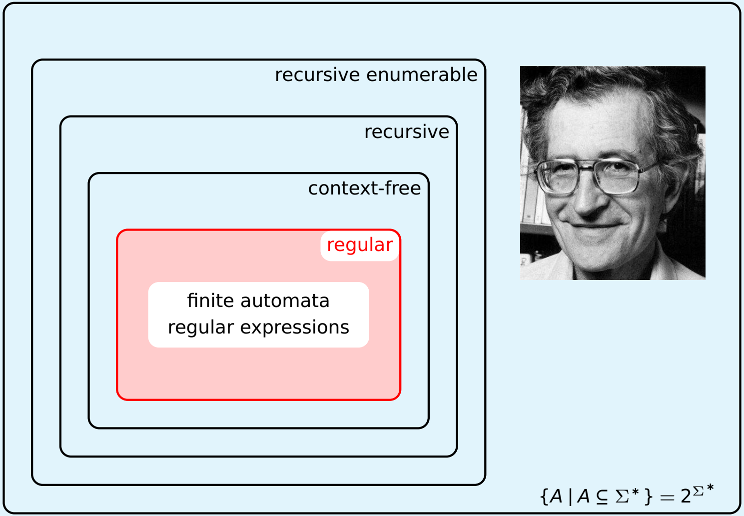
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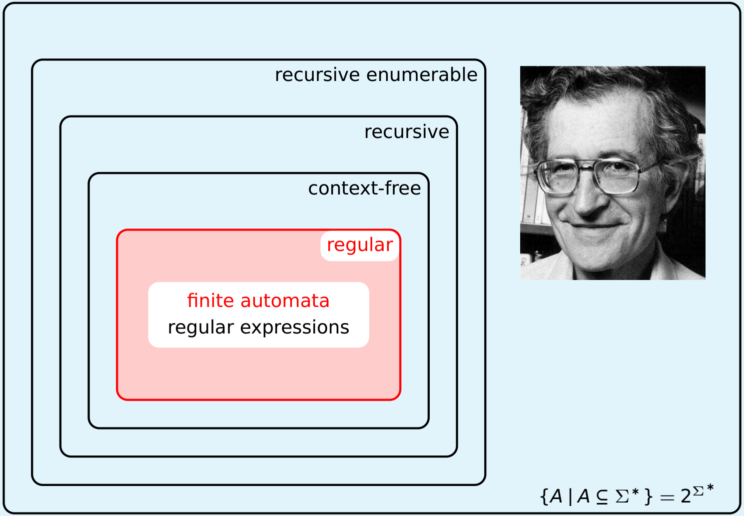
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Example (DFAs → Regular Sets)

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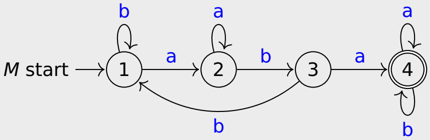

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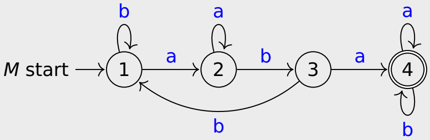

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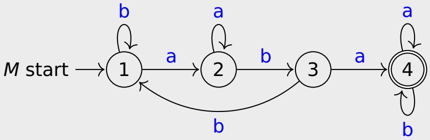

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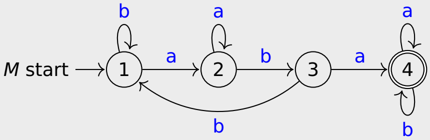

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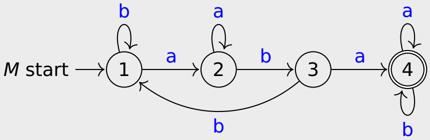

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  5. $F \subseteq Q$: final (accept) states

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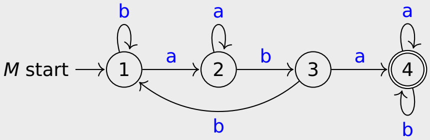

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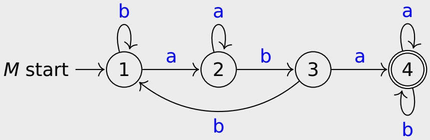

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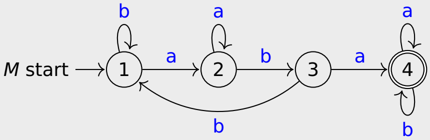

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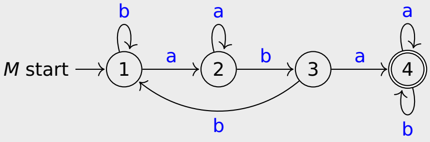

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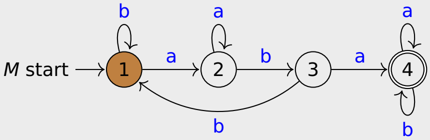

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4	4	4

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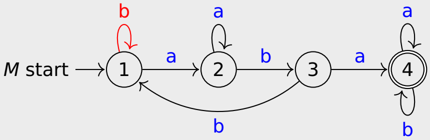
$M = (Q, \Sigma, \delta, s, F)$



b a b a a

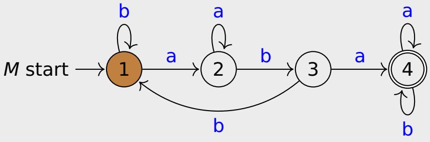
1 1 2 3 4 4

1. $Q = \{1, 2, 3, 4\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{4\}$

δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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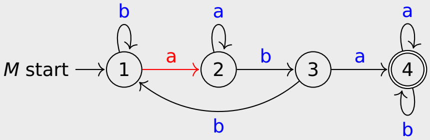
δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

	b		a		b		a		a	
1		1		2		3		4		4

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



	b	a	b	a	a
1	1	2	3	4	4

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δ	a	b
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2	2	3
3	4	1
4	4	4

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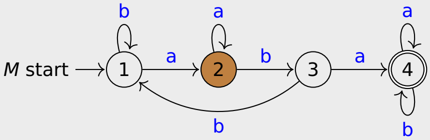

b	a	b	a	a	
1	1	2	3	4	4

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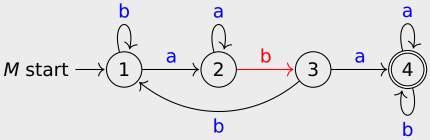

b a b a a

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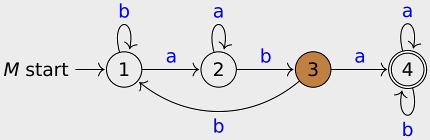

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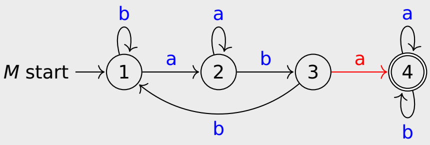
3	4	1
4	4	4

b a b a a

1 1 2 3 4 4

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



b	a	b	a	a	
1	1	2	3	4	4

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δ	a	b
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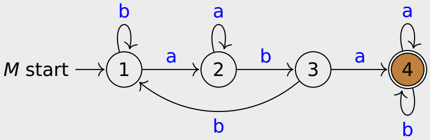
Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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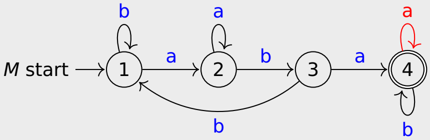
δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

b a b a a

1 1 2 3 4 4

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



b	a	b	a	a	
1	1	2	3	4	4

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δ	a	b
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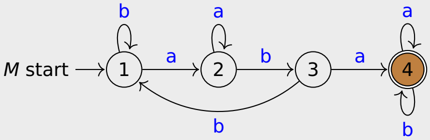
Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



b a b a a

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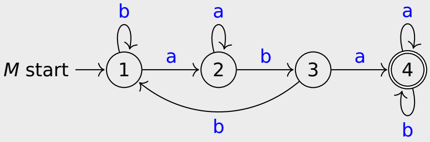
Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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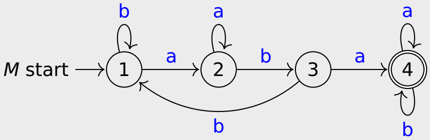
δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

	b	a	b	a	a	
1	1	2	3	4	4	

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



b a b a a

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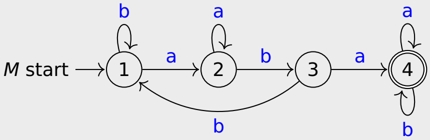

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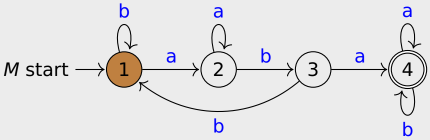
δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

	b		a		b		a		a	
1		1		2		3		4		4

	a		a		b		b		b	
1		2		2		3		1		1

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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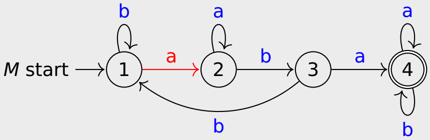
δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

	b		a		b		a		a	
1		1		2		3		4		4

	a		a		b		b		b	
1		2		2		3		1		1

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

b a b a a

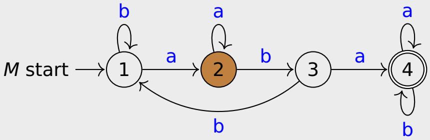
1 1 2 3 4 4

a a b b b

1 2 2 3 1 1

Example (DFAs $\to$ Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

b a b a a

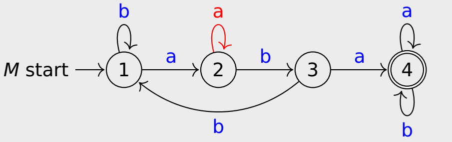
1 1 2 3 4 4

a a b b b

1 2 2 3 1 1

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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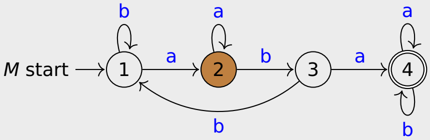
δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

	b		a		b		a		a	
1		1		2		3		4		4

	a		a		b		b		b	
1		2		2		3		1		1

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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2	2	3
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b a b a a

1 1 2 3 4 4

a a b b b

1 2 2 3 1 1

Example (DFAs → Regular Sets)

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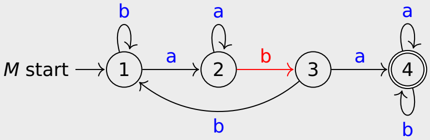

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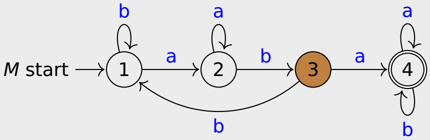
δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

	b		a		b		a		a	
1		1		2		3		4		4

	a		a		b		b		b	
1		2		2		3		1		1

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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1	2	1
2	2	3
3	4	1
4	4	4

	b		a		b		a		a	
1		1		2		3		4		4

	a		a		b		b		b	
1		2		2		3		1		1

Example (DFAs → Regular Sets)

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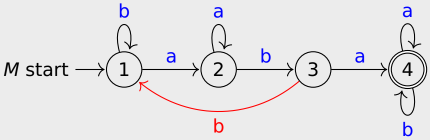

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δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

b a b a a

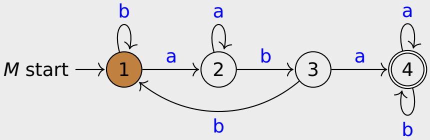
1 1 2 3 4 4

a a b b b

1 2 2 3 1 1

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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4. $s = 1$
5. $F = \{4\}$

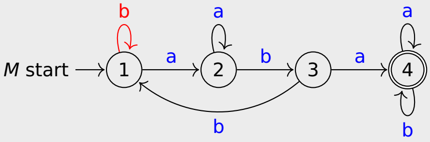
δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

	b	a	b	a	a
1	1	2	3	4	4

	a	a	b	b	b
1	2	2	3	1	1

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



$$1 \xrightarrow{b} 1 \xrightarrow{a} 2 \xrightarrow{b} 3 \xrightarrow{a} 4 \xrightarrow{a} 4$$

$$1 \xrightarrow{a} 2 \xrightarrow{a} 2 \xrightarrow{b} 3 \xrightarrow{b} 1 \xrightarrow{b} 1$$

1. $Q = \{1, 2, 3, 4\}$
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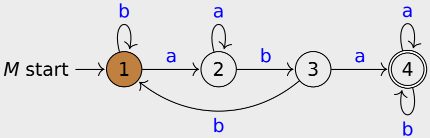

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1	2	1
2	2	3
3	4	1
4	4	4

b a b a a

1 1 2 3 4 4

a a b b b

1 2 2 3 1 1

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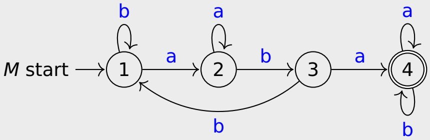
$M = (Q, \Sigma, \delta, s, F)$



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δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

b a b a a

1 1 2 3 4 4

a a b b b

1 2 2 3 1 1

Definitions

- deterministic finite automaton (DFA) is quintuple $M = (Q, \Sigma, \delta, s, F)$ with
  1. Q : finite set of states
  2. Σ : input alphabet
  3. $\delta : Q \times \Sigma \to Q$: transition function
  4. $s \in Q$: start state
  5. $F \subseteq Q$: final (accept) states
- $\widehat{\delta} : Q \times \Sigma^* \to Q$ is inductively defined by

$$\widehat{\delta}(q, \epsilon) := q \qquad \widehat{\delta}(q, xa) := \delta(\widehat{\delta}(q, x), a)$$

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$ first recursive call

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call
$\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, a), b), b), a), a), b)$	fifth recursive call

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call
$\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, a), b), b), a), a), b)$	fifth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, \varepsilon), a), b), b), a), a), b)$	sixth recursive call

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call
$\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, a), b), b), a), a), b)$	fifth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, \varepsilon), a), b), b), a), a), b)$	sixth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(q_0, a), b), b), a), a), b)$	

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call
$\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, a), b), b), a), a), b)$	fifth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, \varepsilon), a), b), b), a), a), b)$	sixth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(q_0, a), b), b), a), a), b)$	
$\delta(\delta(\delta(\delta(\delta(q_1, b), b), a), a), b)$	assuming $\delta(q_0, a) = q_1$

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call
$\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, a), b), b), a), a), b)$	fifth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, \varepsilon), a), b), b), a), a), b)$	sixth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(q_0, a), b), b), a), a), b)$	
$\delta(\delta(\delta(\delta(\delta(q_1, b), b), a), a), b)$	assuming $\delta(q_0, a) = q_1$
$\delta(\delta(\delta(\delta(q_2, b), a), a), b)$	assuming $\delta(q_1, b) = q_2$

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call
$\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, a), b), b), a), a), b)$	fifth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, \varepsilon), a), b), b), a), a), b)$	sixth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(q_0, a), b), b), a), a), b)$	
$\delta(\delta(\delta(\delta(\delta(q_1, b), b), a), a), b)$	assuming $\delta(q_0, a) = q_1$
$\delta(\delta(\delta(\delta(q_2, b), a), a), b)$	assuming $\delta(q_1, b) = q_2$
$\delta(\delta(\delta(q_3, a), a), b)$	assuming $\delta(q_2, b) = q_3$

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call
$\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, a), b), b), a), a), b)$	fifth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, \varepsilon), a), b), b), a), a), b)$	sixth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(q_0, a), b), b), a), a), b)$	
$\delta(\delta(\delta(\delta(\delta(q_1, b), b), a), a), b)$	assuming $\delta(q_0, a) = q_1$
$\delta(\delta(\delta(\delta(q_2, b), a), a), b)$	assuming $\delta(q_1, b) = q_2$
$\delta(\delta(\delta(q_3, a), a), b)$	assuming $\delta(q_2, b) = q_3$
$\delta(\delta(q_4, a), b)$	assuming $\delta(q_3, a) = q_4$

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call
$\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, a), b), b), a), a), b)$	fifth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, \varepsilon), a), b), b), a), a), b)$	sixth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(q_0, a), b), b), a), a), b)$	
$\delta(\delta(\delta(\delta(\delta(q_1, b), b), a), a), b)$	assuming $\delta(q_0, a) = q_1$
$\delta(\delta(\delta(\delta(q_2, b), a), a), b)$	assuming $\delta(q_1, b) = q_2$
$\delta(\delta(\delta(q_3, a), a), b)$	assuming $\delta(q_2, b) = q_3$
$\delta(\delta(q_4, a), b)$	assuming $\delta(q_3, a) = q_4$
$\delta(q_5, b)$	assuming $\delta(q_4, a) = q_5$

Example (Unfolding of the multistep function $\widehat{\delta}$)

Let $x = abbaab$ over the alphabet $\Sigma = \{a, b\}$

$\delta(\widehat{\delta}(q_0, abbaa), b)$	first recursive call
$\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$	second recursive call
$\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$	third recursive call
$\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, ab), b), a), a), b)$	fourth recursive call
$\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, a), b), b), a), a), b)$	fifth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0, \varepsilon), a), b), b), a), a), b)$	sixth recursive call
$\delta(\delta(\delta(\delta(\delta(\delta(q_0, a), b), b), a), a), b)$	
$\delta(\delta(\delta(\delta(\delta(q_1, b), b), a), a), b)$	assuming $\delta(q_0, a) = q_1$
$\delta(\delta(\delta(\delta(q_2, b), a), a), b)$	assuming $\delta(q_1, b) = q_2$
$\delta(\delta(\delta(q_3, a), a), b)$	assuming $\delta(q_2, b) = q_3$
$\delta(\delta(q_4, a), b)$	assuming $\delta(q_3, a) = q_4$
$\delta(q_5, b)$	assuming $\delta(q_4, a) = q_5$
q_6	assuming $\delta(q_5, b) = q_6$

Definitions

- deterministic finite automaton (DFA) is quintuple $M = (Q, \Sigma, \delta, s, F)$ with
  1. Q : finite set of states
  2. Σ : input alphabet
  3. $\delta : Q \times \Sigma \to Q$: transition function
  4. $s \in Q$: start state
  5. $F \subseteq Q$: final (accept) states
- $\widehat{\delta} : Q \times \Sigma^* \to Q$ is inductively defined by

$$\widehat{\delta}(q, \epsilon) := q \qquad \widehat{\delta}(q, xa) := \delta(\widehat{\delta}(q, x), a)$$

- string $x \in \Sigma^*$ is accepted by M if $\widehat{\delta}(s, x) \in F$

Definitions

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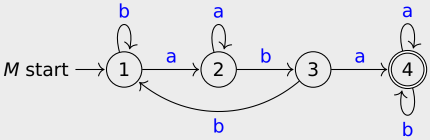
- string $x \in \Sigma^*$ is rejected by M if $\widehat{\delta}(s, x) \notin F$

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



1. $Q = \{1, 2, 3, 4\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{4\}$

δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

b a b a a $\in L(M)$

1 1 2 3 4 4

a a b b b $\notin L(M)$

1 2 2 3 1 1

Definitions

- deterministic finite automaton (DFA) is quintuple $M = (Q, \Sigma, \delta, s, F)$ with
  1. Q : finite set of states
  2. Σ : input alphabet
  3. $\delta : Q \times \Sigma \to Q$: transition function
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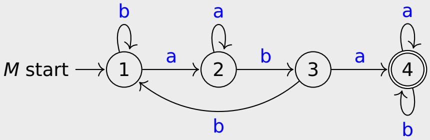
- $\widehat{\delta} : Q \times \Sigma^* \to Q$ is inductively defined by

$$\widehat{\delta}(q, \varepsilon) := q \qquad \widehat{\delta}(q, xa) := \delta(\widehat{\delta}(q, x), a)$$

- string $x \in \Sigma^*$ is accepted by M if $\widehat{\delta}(s, x) \in F$
- string $x \in \Sigma^*$ is rejected by M if $\widehat{\delta}(s, x) \notin F$
- language accepted by M is given by $L(M) := \{x \mid \widehat{\delta}(s, x) \in F\}$

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



1. $Q = \{1, 2, 3, 4\}$
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3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{4\}$

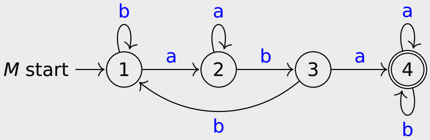
δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

	b	a	b	a	a	$\in L(M)$
1	1	2	3	4	4	
	a	a	b	b	b	$\notin L(M)$
1	2	2	3	1	1	

$L(M) := \{x \mid \qquad \}$

Example (DFAs → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{4\}$

δ	a	b
1	2	1
2	2	3
3	4	1
4	4	4

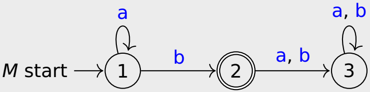
	b	a	b	a	a	$\in L(M)$
1	1	2	3	4	4	

	a	a	b	b	b	$\notin L(M)$
1	2	2	3	1	1	

$L(M) := \{x \mid x \text{ contains } aba \text{ as substring}\}$

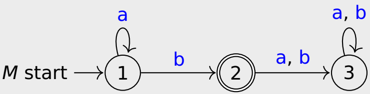
Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



Example (DFA → Regular Sets)

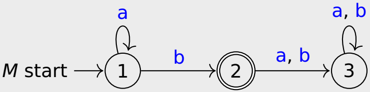
$M = (Q, \Sigma, \delta, s, F)$



1. $Q = \{1, 2, 3\}$

Example (DFA → Regular Sets)

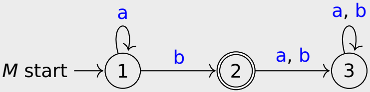
$M = (Q, \Sigma, \delta, s, F)$



1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$

Example (DFA → Regular Sets)

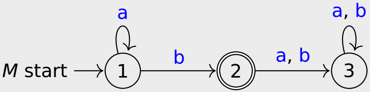
$M = (Q, \Sigma, \delta, s, F)$



1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$

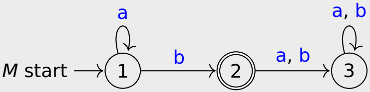


1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \rightarrow Q$

δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



1. $Q = \{1, 2, 3\}$
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δ	a	b
1	1	2
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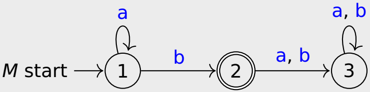
Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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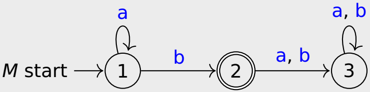
Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



a a a a b

1 1 1 1 1 2

1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
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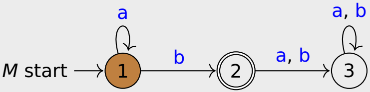
δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



a a a a b

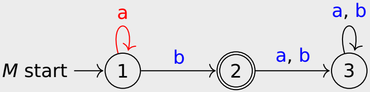
1 1 1 1 1 2

1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{2\}$

δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



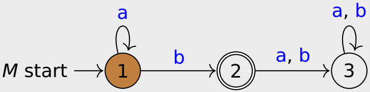
a	a	a	a	b	
1	1	1	1	1	2

1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{2\}$

δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



a a a a b

1 1 1 1 1 2

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3. $\delta : Q \times \Sigma \to Q$
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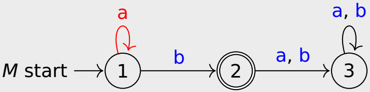
5. $F = \{2\}$

δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



	a	a	a	a	b
1	1	1	1	1	2

1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
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5. $F = \{2\}$

δ	a	b
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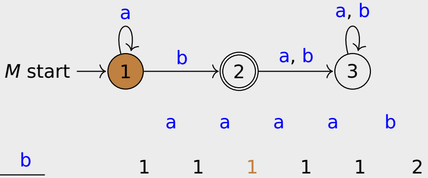
$M = (Q, \Sigma, \delta, s, F)$



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2	3	3
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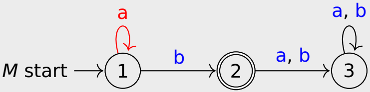
Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



a a a a b

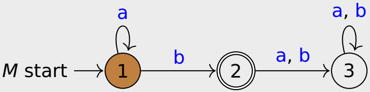
1 1 1 1 1 2

1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{2\}$

δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



a a a a b

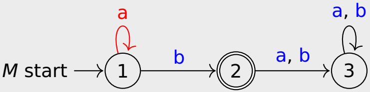
1 1 1 1 1 2

1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{2\}$

δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



a a a a b

1 1 1 1 1 2

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δ	a	b
1	1	2
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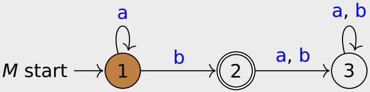
$M = (Q, \Sigma, \delta, s, F)$



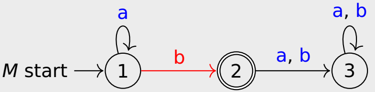
a	a	a	a	b	
1	1	1	1	1	2

1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{2\}$

δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



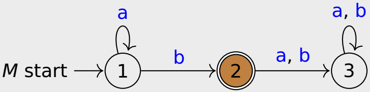
a	a	a	a	b	
1	1	1	1	1	2

1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{2\}$

δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



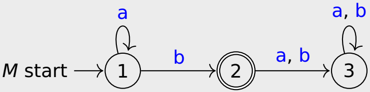
1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{2\}$

δ	a	b
1	1	2
2	3	3
3	3	3

	a	a	a	a	b
1	1	1	1	1	2

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



a a a a b $\in L(M)$

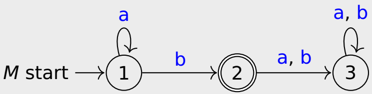
1 1 1 1 1 2

1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{2\}$

δ	a	b
1	1	2
2	3	3
3	3	3

Example (DFA → Regular Sets)

$M = (Q, \Sigma, \delta, s, F)$



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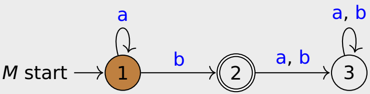

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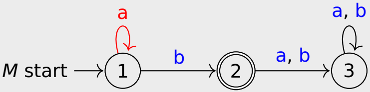

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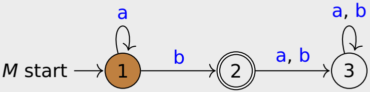

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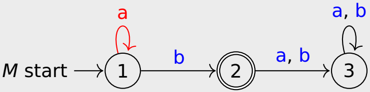

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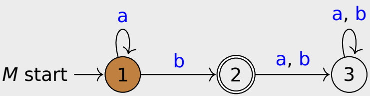

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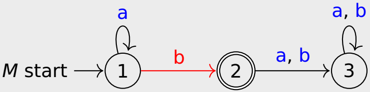

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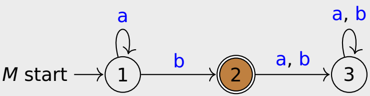

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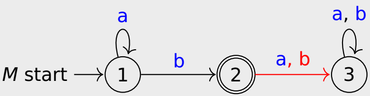

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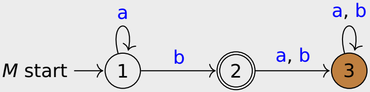

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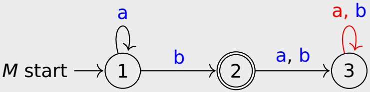

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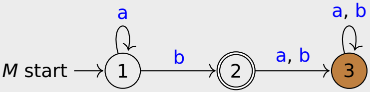

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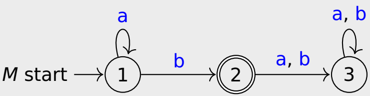

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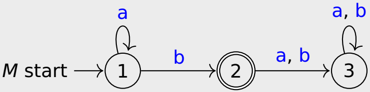

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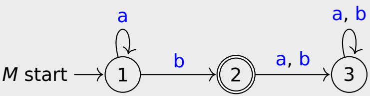

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Proof. (cont'd)

1. $\widehat{\delta}(1, x) = 1 \quad \text{and} \quad y = a$

Proof. (cont'd)

1. $\widehat{\delta}(1,x)=1$ and $y=a$

   $\widehat{\delta}(1,x)=1 \iff x\in L(a^*)$ (by IH) $\delta(\widehat{\delta}(1,x),a)=1 \iff xa\in L(a^*)$

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Proof. (cont'd)

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   $\widehat{\delta}(1,x)=1 \iff x\in L(a^*)$ (by IH) $\quad \delta(\widehat{\delta}(1,x),a)=1 \iff xa\in L(a^*)$

2. $\widehat{\delta}(1,x)=1$ and $y=b$
   $\widehat{\delta}(1,x)=1 \iff x\in L(a^*)$ (by IH) $\quad \delta(\widehat{\delta}(1,x),b)=2 \iff xb\in L(a^*b)$

3. $\widehat{\delta}(1,x)=2$ and $y=a$
   $\widehat{\delta}(1,x)=2 \iff x\in L(a^*b)$ (by IH) $\quad \delta(\widehat{\delta}(1,x),a)=3 \iff xa\in L\big(a^*b(a+b)^+\big)$

4. $\widehat{\delta}(1,x)=2$ and $y=b$
   $\widehat{\delta}(1,x)=2 \iff x\in L(a^*b)$ (by IH) $\quad \delta(\widehat{\delta}(1,x),b)=3 \iff xb\in L\big(a^*b(a+b)^+\big)$

5. $\widehat{\delta}(1,x)=3$ and $y=a$
   $\widehat{\delta}(1,x)=3 \iff x\in L\big(a^*b(a+b)^+\big)$ (by IH) $\quad \delta(\widehat{\delta}(1,x),a)=3 \iff xa\in L\big(a^*b(a+b)^+\big)$

6. $\widehat{\delta}(1,x)=3$ and $y=b$
   $\widehat{\delta}(1,x)=3 \iff x\in L\big(a^*b(a+b)^+\big)$ (by IH) $\quad \delta(\widehat{\delta}(1,x),b)=3 \iff xb\in L\big(a^*b(a+b)^+\big)$

□

Example (Regular Sets → DFA)

$M = (Q, \Sigma, \delta, s, F)$

2. $\Sigma = \{a, b\}$

$L(M) := \{x \mid x \text{ contains even number of } b\text{s over } \Sigma\}$

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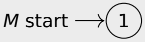

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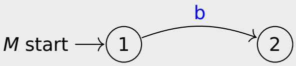

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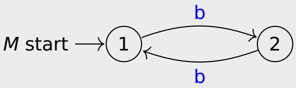

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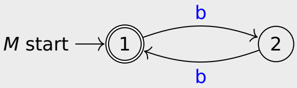

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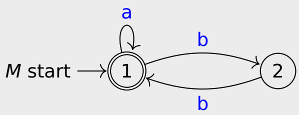

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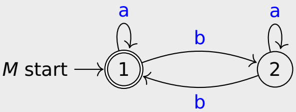

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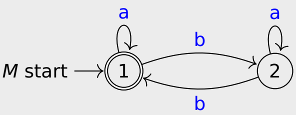

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δ	a	b
1	1	2
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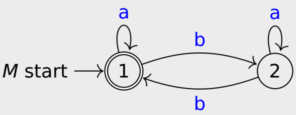

a	b	a	b	a	
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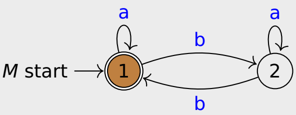

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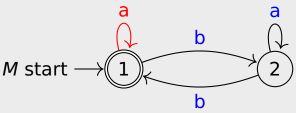

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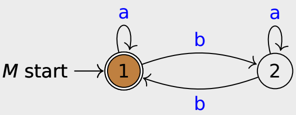

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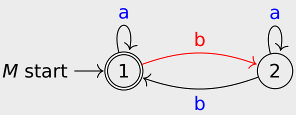

a b a b a

1 1 2 2 1 1

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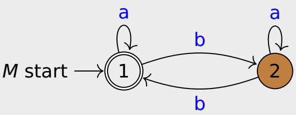

a	b	a	b	a	
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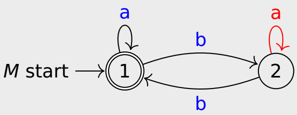

a	b	a	b	a	
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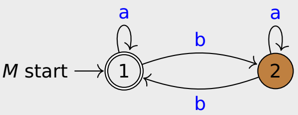

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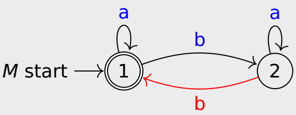

a	b	a	b	a	
1	1	2	2	1	1

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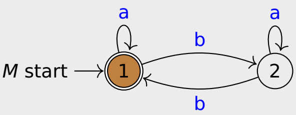

	a	b	a	b	a	
1	1	2	2	1	1	

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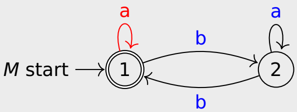

a b a b a

1 1 2 2 1 1

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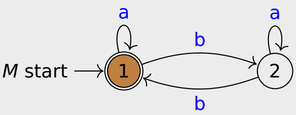

	a	b	a	b	a	
1	1	2	2	1	1	

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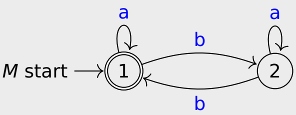

a	b	a	b	a	$\in L(M)$

1 1 2 2 1 1

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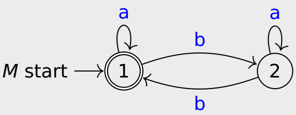

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2	2	1

	a	b	a	b	a	$\in L(M)$
1	1	2	2	1	1	

	b	a	a	a	a
1	2	2	2	2	2

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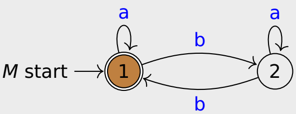

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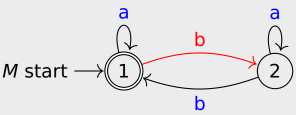

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1	1	2
2	2	1

	a	b	a	b	a	$\in L(M)$
1	2	2	1	1		

	b	a	a	a	a
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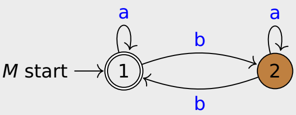

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δ	a	b
1	1	2
2	2	1

a b a b a $\in L(M)$

1 2 2 1 1

b a a a a

1 2 2 2 2 2

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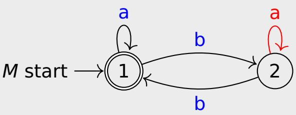

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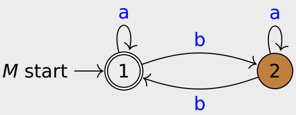

$a \quad b \quad a \quad b \quad a \quad \in L(M)$

$1 \quad 2 \quad 2 \quad 1 \quad 1$

$b \quad a \quad a \quad a \quad a$

$1 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2$

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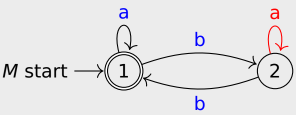

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Example (Regular Sets → DFA)

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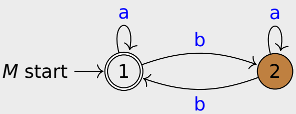

1. $Q = \{1, 2\}$
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3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{1\}$

δ	a	b
1	1	2
2	2	1

a b a b a $\in L(M)$

1 2 2 1 1

b a a a a

1 2 2 2 2 2

$L(M) := \{x \mid x \text{ contains even number of } b\text{s over } \Sigma\}$

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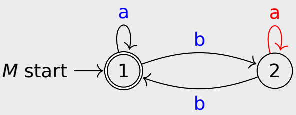

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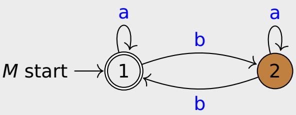

a b a b a $\in L(M)$

1 2 2 1 1

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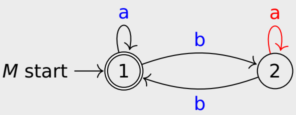

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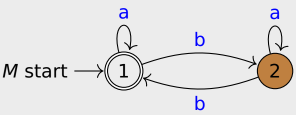

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a	b	a	b	a	$\in L(M)$
1	2	2	1	1	

	b	a	a	a	a
1	2	2	2	2	2

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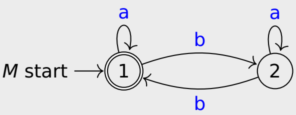
Example (Regular Sets → DFA)

$M = (Q, \Sigma, \delta, s, F)$



a b a b a $\in L(M)$

1 2 2 1 1

b a a a a $\notin L(M)$

1 2 2 2 2 2

1. $Q = \{1, 2\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
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Example (Regular Sets → DFA)

$M = (Q, \Sigma, \delta, s, F)$

2. $\Sigma = \{a, b\}$

$L(M) := \{x \mid x \text{ contains } bb \text{ as substring }\}$

Example (Regular Sets → DFA)

$M = (Q, \Sigma, \delta, s, F)$

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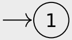
Example (Regular Sets → DFA)

$M = (Q, \Sigma, \delta, s, F)$

M start



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Example (Regular Sets → DFA)

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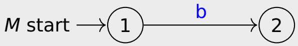

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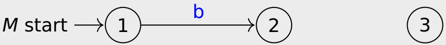

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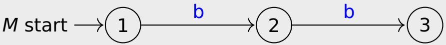

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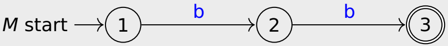

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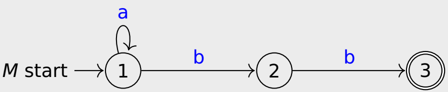

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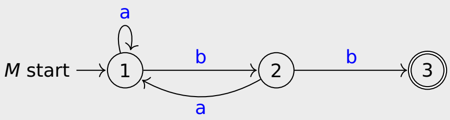

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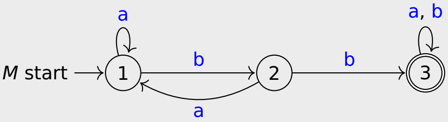

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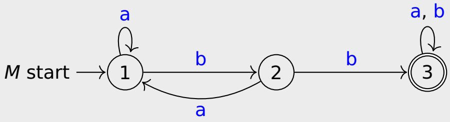

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4. $s = 1$
5. $F = \{3\}$

δ	a	b
1	1	2
2	1	3
3	3	3

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Example (Regular Sets → DFA)

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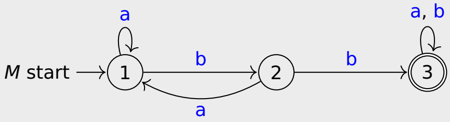

b	a	b	b	a	
1	2	1	2	3	3

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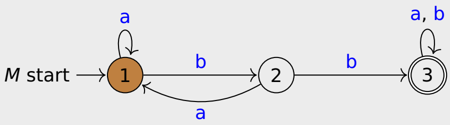

b a b b a

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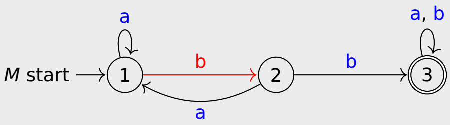

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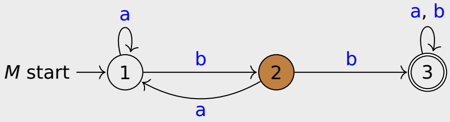

	b	a	b	b	a
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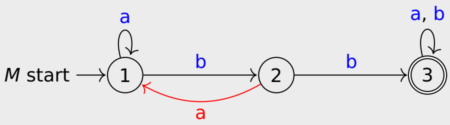

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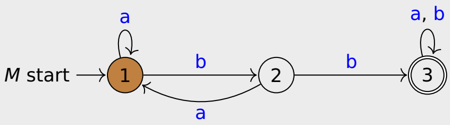

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	b		a		b		b		a	
1		2		1		2		3		3

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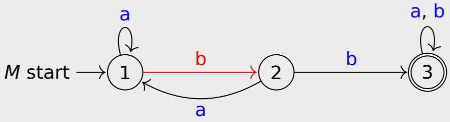

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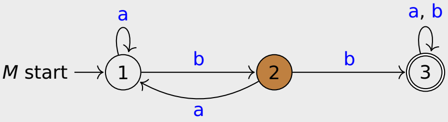

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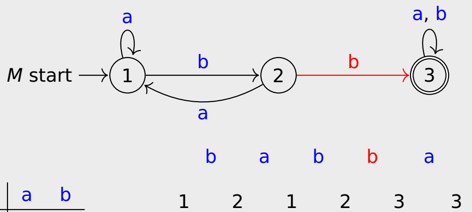

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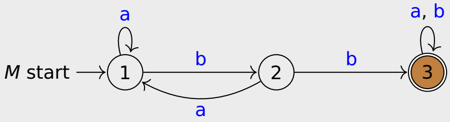

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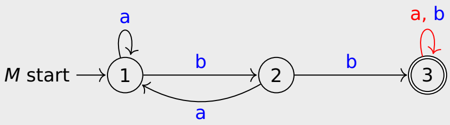

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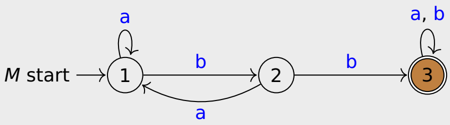

b a b b a

1 2 1 2 3 3

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Example (Regular Sets → DFA)

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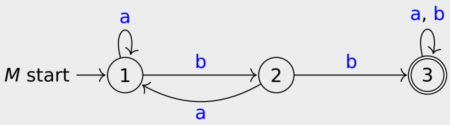

b a b b a $\in L(M)$

1 2 1 2 3 3

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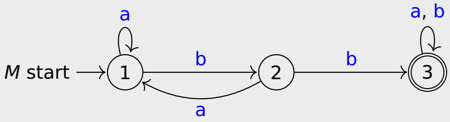

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3	3	3

	b	a	b	b	a	$\in L(M)$
1	2	1	2	3	3	
	a	a	b	a	b	
1	1	1	2	1	2	

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Example (Regular Sets → DFA)

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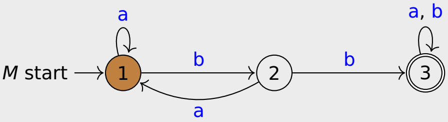

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1	1	2
2	1	3
3	3	3

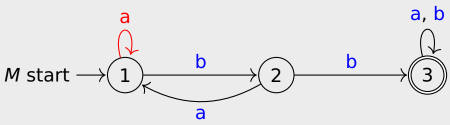
	b		a		b		b		a		$\in L(M)$
1		2		1		2		3		3	

	a		a		b		a		b	
1		1		1		2		1		2

$L(M) := \{x \mid x \text{ contains } bb \text{ as substring }\}$

Example (Regular Sets → DFA)

$M = (Q, \Sigma, \delta, s, F)$



1. $Q = \{1, 2, 3\}$
2. $\Sigma = \{a, b\}$
3. $\delta : Q \times \Sigma \to Q$
4. $s = 1$
5. $F = \{3\}$

δ	a	b
1	1	2
2	1	3
3	3	3

	b		a		b		b		a		$\in L(M)$
1		2		1		2		3		3	

	a		a		b		a		b	
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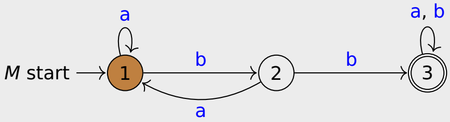

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3	3	3

	b	a	b	b	a	$\in L(M)$
1	2	1	2	3	3	

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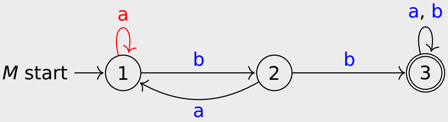

	b	a	b	b	a	$\in L(M)$
1	2	1	2	3	3	

	a	a	b	a	b
1	1	1	2	1	2

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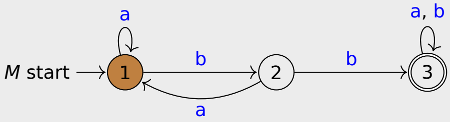

b a b b a $\in L(M)$

1 2 1 2 3 3

a a b a b

1 1 1 2 1 2

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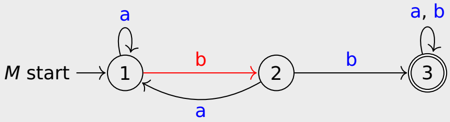

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1		1		1		2		1		2

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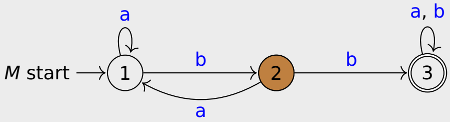

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1 1 1 2 1 2

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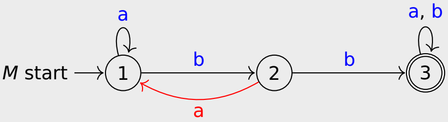

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1	2	1	2	3	3	
	a	a	b	a	b	
1	1	1	2	1	2	

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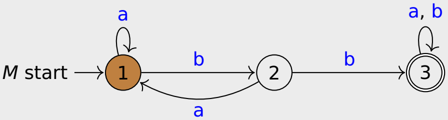

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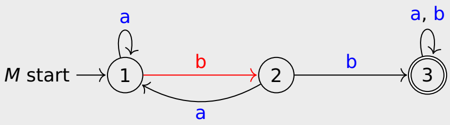

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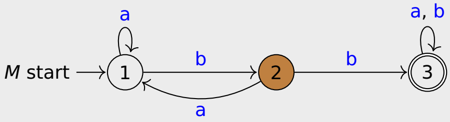

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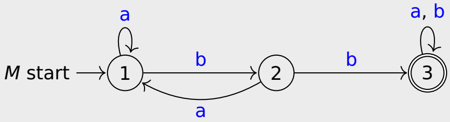

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δ	a	b
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1	2	1	2	3	3	
	a	a	b	a	b	$\notin L(M)$
1	1	1	2	1	2	

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Outline

Theorem

regular sets are effectively closed under intersection

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Proof. (closure under intersection)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
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  1. $Q_3 := Q_1 \times Q_2 \quad = \quad \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$

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2. $F_3 \quad := \quad F_1 \times F_2$
3. $s_3 \quad := \quad (s_1, s_2)$
4. $\delta_3((p,q),a) \quad := \quad (\delta_1(p,a), \delta_2(q,a)) \qquad \forall p \in Q_1,\ \forall q \in Q_2,\ \forall a \in \Sigma$

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- claim: $\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x), \widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$

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Proof. (closure under intersection)

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- $A \cap B := L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$

  1. $Q_3 := Q_1 \times Q_2 = \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
  2. $F_3 := F_1 \times F_2$
  3. $s_3 := (s_1, s_2)$
  4. $\delta_3((p,q),a) := (\delta_1(p,a), \delta_2(q,a)) \quad \forall p \in Q_1, \forall q \in Q_2, \forall a \in \Sigma$

- claim: $\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x), \widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$
  proof: induction on $|x|$ next slide

proof of the claim

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- base case: $|x| = 0$ thus $x = \varepsilon$

$$\widehat{\delta_3}((p,q),\varepsilon) = (p,q) = (\widehat{\delta_1}(p,\varepsilon), \widehat{\delta_2}(q,\varepsilon))$$

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- step case: $|x| > 0$ thus $x = ya$ s.t. $|y| = |x| - 1$ with IH : $\widehat{\delta_3}((p,q),y) = (\widehat{\delta_1}(p,y), \widehat{\delta_2}(q,y)))$

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- step case: $|x| > 0$ thus $x = ya$ s.t. $|y| = |x| - 1$ with $\text{IH} : \widehat{\delta_3}((p,q),y) = (\widehat{\delta_1}(p,y), \widehat{\delta_2}(q,y)))$

$$\widehat{\delta_3}((p,q),ya) \quad = \quad \delta_3(\widehat{\delta_3}((p,q),y),a) \qquad \text{(by definition of } \widehat{\delta_3}\text{)}$$

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$$\begin{aligned} \widehat{\delta_3}((p,q),ya) &= \delta_3(\widehat{\delta_3}((p,q),y),a) && \text{(by definition of } \widehat{\delta_3}\text{)} \\ &= \delta_3((\widehat{\delta_1}(p,y), \widehat{\delta_2}(q,y)),a) && \text{(by induction hypothesis IH)} \end{aligned}$$

proof of the claim

claim: $\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x), \widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$

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$$\begin{aligned}
\widehat{\delta_3}((p,q),ya) &= \delta_3(\widehat{\delta_3}((p,q),y),a) && \text{(by definition of } \widehat{\delta_3}\text{)} \\
&= \delta_3((\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y)),a) && \text{(by induction hypothesis IH)} \\
&= (\delta_1(\widehat{\delta_1}(p,y),a),\delta_2(\widehat{\delta_2}(q,y),a)) && \text{(by definition of } \delta_3\text{)}
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proof of the claim

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&= \delta_3((\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y)),a) && \text{(by induction hypothesis IH)}\\
&= (\delta_1(\widehat{\delta_1}(p,y),a),\delta_2(\widehat{\delta_2}(q,y),a)) && \text{(by definition of } \delta_3\text{)}\\
&= (\widehat{\delta_1}(p,ya),\widehat{\delta_2}(q,ya)) && \text{(by definitions of } \widehat{\delta_1} \text{ and } \widehat{\delta_2}\text{)}
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&= \delta_3((\widehat{\delta_1}(p,y), \widehat{\delta_2}(q,y)),a) && \text{(by induction hypothesis IH)} \\
&= (\delta_1(\widehat{\delta_1}(p,y),a), \delta_2(\widehat{\delta_2}(q,y),a)) && \text{(by definition of } \delta_3\text{)} \\
&= (\widehat{\delta_1}(p,ya), \widehat{\delta_2}(q,ya)) && \text{(by definitions of } \widehat{\delta_1} \text{ and } \widehat{\delta_2}\text{)} \\
&= (\widehat{\delta_1}(p,x), \widehat{\delta_2}(q,x))
\end{aligned}$$

$\square$

Proof. (closure under intersection (cont'd))

statement: $L(M_3) = L(M_1) \cap L(M_2)$

Proof. (closure under intersection (cont'd))

statement: $L(M_3) = L(M_1) \cap L(M_2)$

$$\forall x \in \Sigma^*,\ x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3 \qquad \text{(by definition of acceptance)}$$

Proof. (closure under intersection (cont'd))

statement: $L(M_3) = L(M_1) \cap L(M_2)$

$$\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_3) &\iff \widehat{\delta_3}(s_3, x) \in F_3 && \text{(by definition of acceptance)}\\
&\iff \widehat{\delta_3}((s_1, s_2), x) \in F_1 \times F_2 && \text{(by definition of } s_3 \text{ and } F_3\text{)}
\end{aligned}$$

Proof. (closure under intersection (cont'd))

statement: $L(M_3) = L(M_1) \cap L(M_2)$

$$
\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_3) &\iff \widehat{\delta_3}(s_3, x) \in F_3 && \text{(by definition of acceptance)} \\
&\iff \widehat{\delta_3}((s_1, s_2), x) \in F_1 \times F_2 && \text{(by definition of } s_3 \text{ and } F_3\text{)} \\
&\iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in F_1 \times F_2 && \text{(by claim proven in slide 21)}
\end{aligned}
$$

Proof. (closure under intersection (cont'd))

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$$
\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_3) &\iff \widehat{\delta_3}(s_3, x) \in F_3 && \text{(by definition of acceptance)}\\
&\iff \widehat{\delta_3}((s_1, s_2), x) \in F_1 \times F_2 && \text{(by definition of } s_3 \text{ and } F_3\text{)}\\
&\iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in F_1 \times F_2 && \text{(by claim proven in slide 21)}\\
&\iff \widehat{\delta_1}(s_1, x) \in F_1 \text{ and } \widehat{\delta_2}(s_2, x) \in F_2 && \text{(by definition of product)}
\end{aligned}
$$

Proof. (closure under intersection (cont'd))

statement: $L(M_3) = L(M_1) \cap L(M_2)$

$$
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\forall x \in \Sigma^*,\ x \in L(M_3) &\iff \widehat{\delta_3}(s_3, x) \in F_3 && \text{(by definition of acceptance)}\\
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&\iff \widehat{\delta_1}(s_1, x) \in F_1 \text{ and } \widehat{\delta_2}(s_2, x) \in F_2 && \text{(by definition of product)}\\
&\iff x \in L(M_1) \text{ and } x \in L(M_2) && \text{(by definition of acceptance)}
\end{aligned}
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Proof. (closure under intersection (cont'd))

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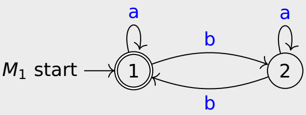
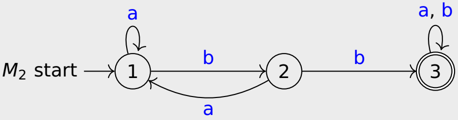
&\iff x \in L(M_1) \text{ and } x \in L(M_2) && \text{(by definition of acceptance)}\\
&\iff x \in L(M_1) \cap L(M_2) && \text{(by definition of intersection)}
\end{aligned}
$$

$\square$

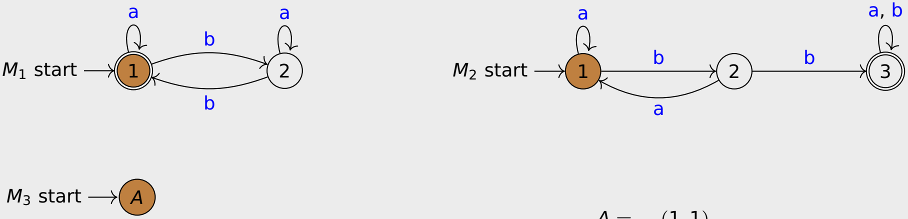
Example (intersection)





$L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x$ contains even number of bs and bb as substring$\}$

Example (intersection)



$A = \quad (1,1)$

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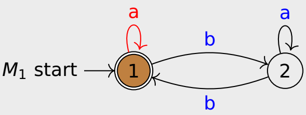

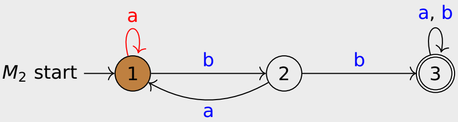

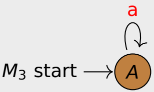

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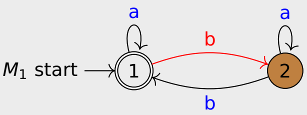

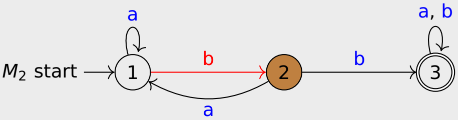

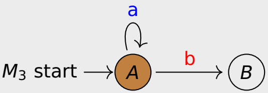

$$\begin{aligned} A &= \quad (1,1) \\ B &= \quad (2,2) \end{aligned}$$

$L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x$ contains even number of *b*s and *bb* as substring$\}$

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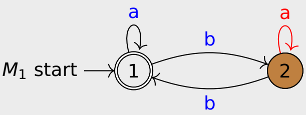
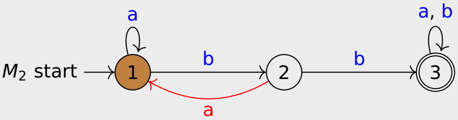
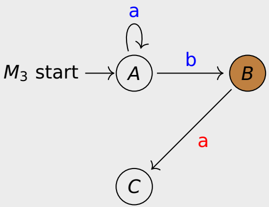


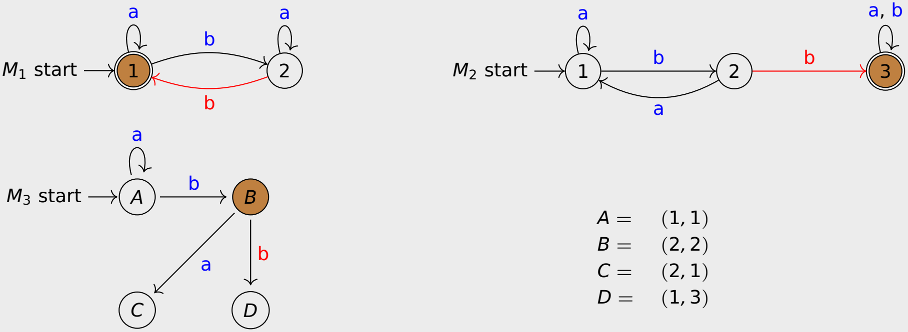




$$\begin{aligned} A &= (1,1) \\ B &= (2,2) \\ C &= (2,1) \end{aligned}$$

$L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring}\}$

Example (intersection)



$$
\begin{aligned}
A &= (1,1) \\
B &= (2,2) \\
C &= (2,1) \\
D &= (1,3)
\end{aligned}
$$

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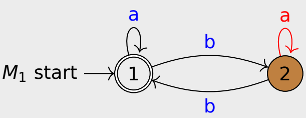

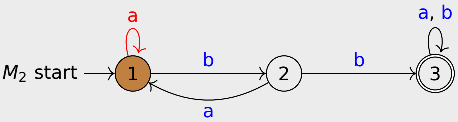

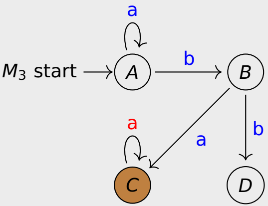

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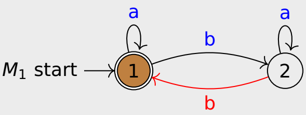

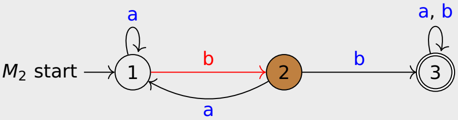

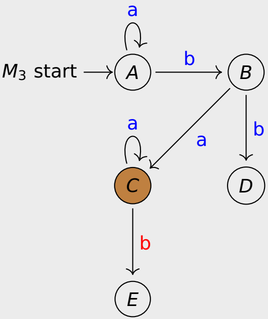

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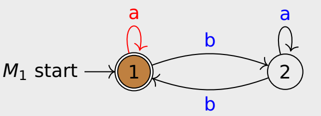

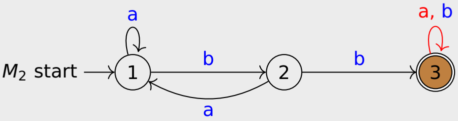

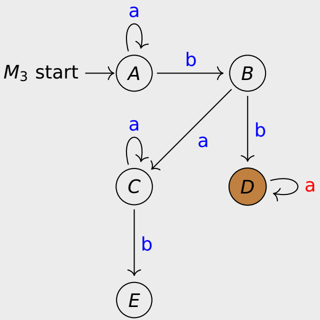

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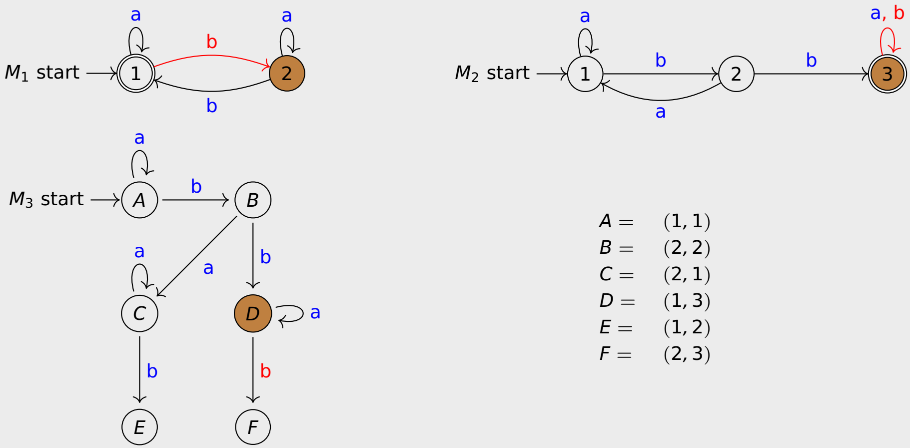

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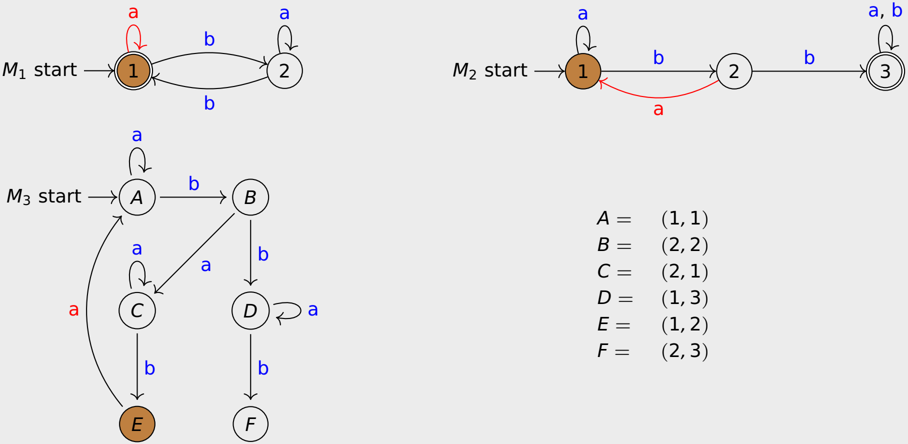

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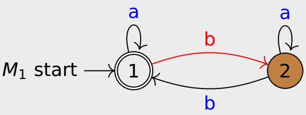

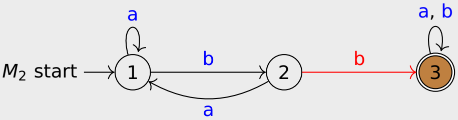

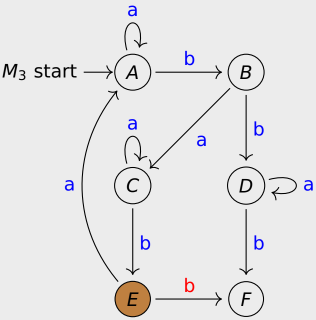

$$\begin{aligned} A &= \quad (1,1) \\ B &= \quad (2,2) \\ C &= \quad (2,1) \\ D &= \quad (1,3) \\ E &= \quad (1,2) \\ F &= \quad (2,3) \end{aligned}$$

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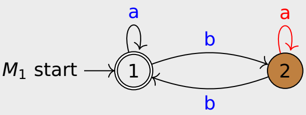

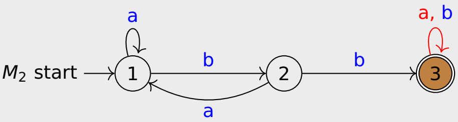

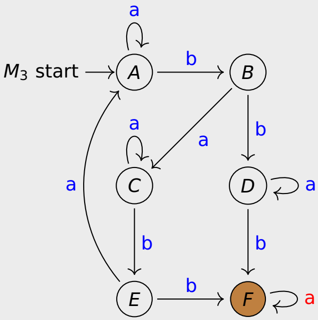

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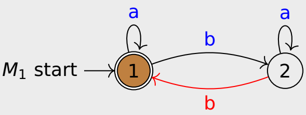

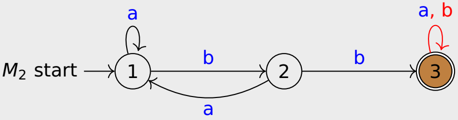

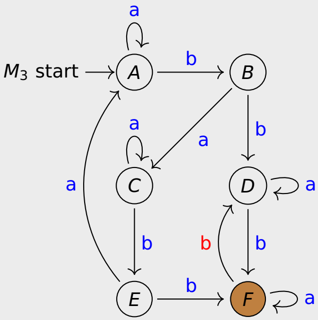

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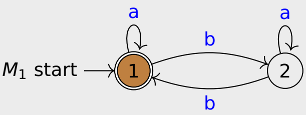

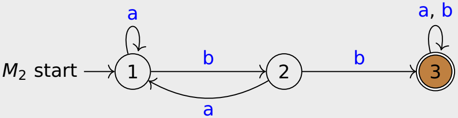

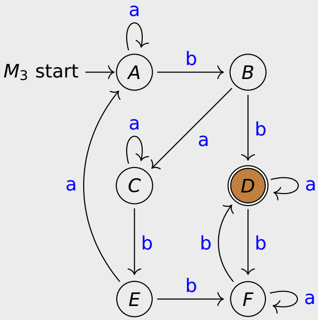

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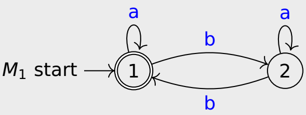

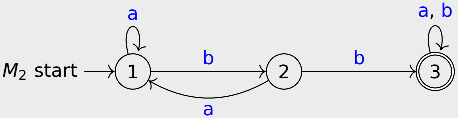

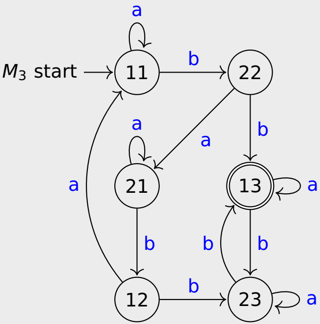

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$L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x$ contains even number of *bs* and *bb* as substring$\}$

Theorem

regular sets are effectively closed under complement

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Proof. (closure under complement)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$

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Proof. (closure under complement)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- $\sim A := \Sigma^* - A$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$

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    1. $Q_2 := Q_1$

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1. $Q_2 := Q_1$
2. $F_2 := Q_1 - F_1$

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1. $Q_2 := Q_1$
2. $F_2 := Q_1 - F_1$
3. $s_2 := s_1$

Theorem

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Proof. (closure under complement)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- $\sim A := \Sigma^* - A$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$

1. $Q_2 := Q_1$
2. $F_2 := Q_1 - F_1$
3. $s_2 := s_1$
4. $\delta_2(p, a) := \delta_1(p, a) \quad \forall p \in Q_1, \forall a \in \Sigma$

Theorem

regular sets are effectively closed under complement

Proof. (closure under complement)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- $\sim A := \Sigma^* - A$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$

  1. $Q_2 := Q_1$
  2. $F_2 := Q_1 - F_1$
  3. $s_2 := s_1$
  4. $\delta_2(p, a) := \delta_1(p, a) \quad \forall p \in Q_1, \forall a \in \Sigma$

- obvious claim: $\widehat{\delta_2}(p, x) = \widehat{\delta_1}(p, x) \quad \forall x \in \Sigma^*$

Proof. (closure under complement (cont'd))

statement: $L(M_2) = \Sigma^* - L(M_1)$

Proof. (closure under complement (cont'd))

statement: $L(M_2) = \Sigma^* - L(M_1)$

$$\forall x \in \Sigma^*,\ x \in L(M_2) \iff \widehat{\delta_2}(s_2, x) \in F_2 \qquad \text{(by definition of acceptance)}$$

Proof. (closure under complement (cont'd))

statement: $L(M_2) = \Sigma^* - L(M_1)$

$$\begin{aligned} \forall x \in \Sigma^*,\ x \in L(M_2) &\iff \widehat{\delta_2}(s_2, x) \in F_2 && \text{(by definition of acceptance)} \\ &\iff \widehat{\delta_1}(s_2, x) \in F_2 && \text{(by the obvious claim in slide 24)} \end{aligned}$$

Proof. (closure under complement (cont'd))

statement: $L(M_2) = \Sigma^* - L(M_1)$

$$
\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_2) &\iff \widehat{\delta_2}(s_2, x) \in F_2 && \text{(by definition of acceptance)} \\
&\iff \widehat{\delta_1}(s_2, x) \in F_2 && \text{(by the obvious claim in slide 24)} \\
&\iff \widehat{\delta_1}(s_1, x) \in Q_1 - F_1 && \text{(by definitions of } s_2 \text{ and } F_2\text{)}
\end{aligned}
$$

Proof. (closure under complement (cont'd))

statement: $L(M_2) = \Sigma^* - L(M_1)$

$$
\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_2) &\iff \widehat{\delta_2}(s_2, x) \in F_2 && \text{(by definition of acceptance)} \\
&\iff \widehat{\delta_1}(s_2, x) \in F_2 && \text{(by the obvious claim in slide 24)} \\
&\iff \widehat{\delta_1}(s_1, x) \in Q_1 - F_1 && \text{(by definitions of } s_2 \text{ and } F_2\text{)} \\
&\iff \widehat{\delta_1}(s_1, x) \in Q_1 \text{ and } \widehat{\delta_1}(s_1, x) \notin F_1 && \text{(by definition of set difference)}
\end{aligned}
$$

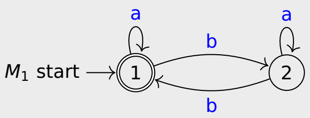
Proof. (closure under complement (cont'd))

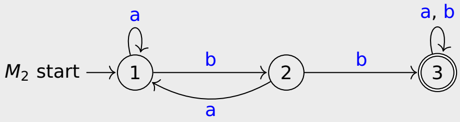
statement: $L(M_2) = \Sigma^* - L(M_1)$

$$
\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_2) &\iff \widehat{\delta_2}(s_2, x) \in F_2 && \text{(by definition of acceptance)} \\
&\iff \widehat{\delta_1}(s_2, x) \in F_2 && \text{(by the obvious claim in slide 24)} \\
&\iff \widehat{\delta_1}(s_1, x) \in Q_1 - F_1 && \text{(by definitions of } s_2 \text{ and } F_2\text{)} \\
&\iff \widehat{\delta_1}(s_1, x) \in Q_1 \text{ and } \widehat{\delta_1}(s_1, x) \notin F_1 && \text{(by definition of set difference)} \\
&\iff x \notin L(M_1) && \text{(by definition of acceptance)}
\end{aligned}
$$

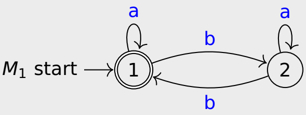
□

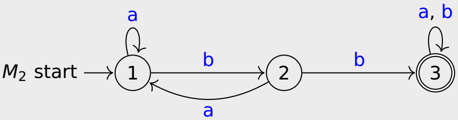
Example (complement)

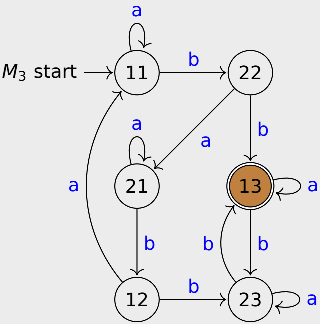




Example (complement)

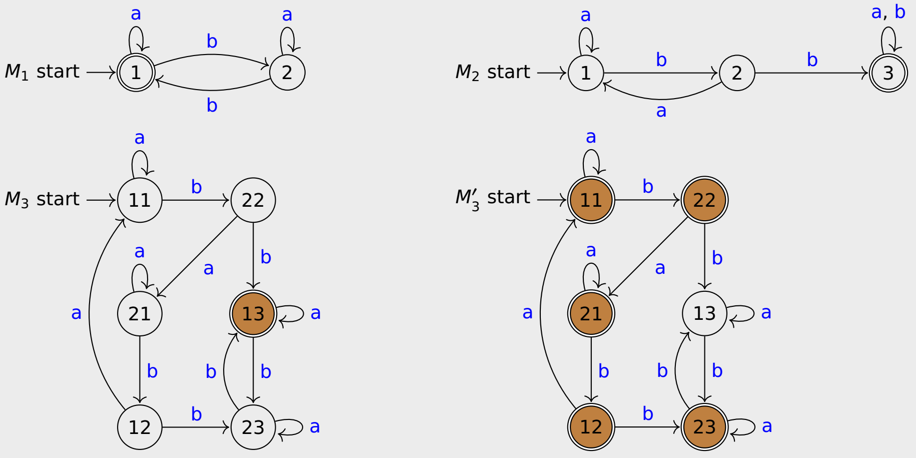






$L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x$ contains even number of bs and bb as substring$\}$

Example (complement)



$\sim L(M_3) := \{x \mid x$ contains odd number of bs or no bb as substring$\}$

Theorem

regular sets are effectively closed under union

Theorem

regular sets are effectively closed under union

Proof. (closure under union)

$A \cup B = \sim((\sim A) \cap (\sim B))$

Theorem

regular sets are effectively closed under union

Theorem

regular sets are effectively closed under union

Proof. (closure under union – explicit construction)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
  $B = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$

Theorem

regular sets are effectively closed under union

Proof. (closure under union – explicit construction)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
  $B = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$
- $A \cup B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$

Theorem

regular sets are effectively closed under union

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- $A \cup B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$

  1. $Q_3 = Q_1 \times Q_2 \quad := \quad \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$

Theorem

regular sets are effectively closed under union

Proof. (closure under union – explicit construction)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
  $B = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$
- $A \cup B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$

  1. $Q_3 = Q_1 \times Q_2 \quad := \quad \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
  2. $F_3 \quad := \quad (F_1 \times Q_2) \cup (Q_1 \times F_2)$

Theorem

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  1. $Q_3 = Q_1 \times Q_2 \quad := \quad \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
  2. $F_3 \quad := \quad (F_1 \times Q_2) \cup (Q_1 \times F_2)$
  3. $s_3 \quad := \quad (s_1, s_2)$

Theorem

regular sets are effectively closed under union

Proof. (closure under union – explicit construction)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
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- $A \cup B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$

1. $Q_3 = Q_1 \times Q_2 \quad := \quad \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
2. $F_3 \quad := \quad (F_1 \times Q_2) \cup (Q_1 \times F_2)$
3. $s_3 \quad := \quad (s_1, s_2)$
4. $\delta_3((p, q), a) \quad := \quad (\delta_1(p, a), \delta_2(q, a)) \qquad \forall p \in Q_1, \forall q \in Q_2, \forall a \in \Sigma$

Theorem

regular sets are effectively closed under union

Proof. (closure under union – explicit construction)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
  $B = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$
- $A \cup B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$

  ① $Q_3 = Q_1 \times Q_2 := \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
  ② $F_3 := (F_1 \times Q_2) \cup (Q_1 \times F_2)$
  ③ $s_3 := (s_1, s_2)$
  ④ $\delta_3((p,q),a) := (\delta_1(p,a), \delta_2(q,a)) \quad \forall p \in Q_1,\ \forall q \in Q_2,\ \forall a \in \Sigma$

- claim: $\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x), \widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$

Theorem

regular sets are effectively closed under union

Proof. (closure under union – explicit construction)

- $A = L(M_1)$ for DFA $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
  $B = L(M_2)$ for DFA $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$
- $A \cup B = L(M_3)$ for DFA $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$

  1. $Q_3 = Q_1 \times Q_2 \quad := \quad \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
  2. $F_3 \quad := \quad (F_1 \times Q_2) \cup (Q_1 \times F_2)$
  3. $s_3 \quad := \quad (s_1, s_2)$
  4. $\delta_3((p, q), a) \quad := \quad (\delta_1(p, a), \delta_2(q, a)) \qquad \forall p \in Q_1, \forall q \in Q_2, \forall a \in \Sigma$

- claim: $\widehat{\delta_3}((p, q), x) = (\widehat{\delta_1}(p, x), \widehat{\delta_2}(q, x)) \qquad \forall x \in \Sigma^*$
  proof: induction on $|x|$ – skipped (follows exact same steps with that is given at slide #21)

Proof. (closure under union – explicit construction (cont'd))

statement: $L(M_3) = L(M_1) \cup L(M_2)$

Proof. (closure under union – explicit construction (cont'd))

statement: $L(M_3) = L(M_1) \cup L(M_2)$

$\forall x \in \Sigma^*, x \in L(M_3) \quad \iff \quad \widehat{\delta_3}(s_3, x) \in F_3$

Proof. (closure under union – explicit construction (cont'd))

statement: $L(M_3) = L(M_1) \cup L(M_2)$

$$\begin{aligned} \forall x \in \Sigma^*,\ x \in L(M_3) \quad &\iff \quad \widehat{\delta_3}(s_3, x) \in F_3 \\ &\iff \quad \widehat{\delta_3}((s_1, s_2), x) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \end{aligned}$$

Proof. (closure under union – explicit construction (cont'd))

statement: $L(M_3) = L(M_1) \cup L(M_2)$

$$
\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_3) \quad &\Longleftrightarrow \quad \widehat{\delta_3}(s_3, x) \in F_3 \\
&\Longleftrightarrow \quad \widehat{\delta_3}((s_1, s_2), x) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\
&\Longleftrightarrow \quad (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2)
\end{aligned}
$$

Proof. (closure under union – explicit construction (cont'd))

statement: $L(M_3) = L(M_1) \cup L(M_2)$

$$\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_3) \quad &\Longleftrightarrow \quad \widehat{\delta_3}(s_3, x) \in F_3 \\
&\Longleftrightarrow \quad \widehat{\delta_3}((s_1, s_2), x) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\
&\Longleftrightarrow \quad (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\
&\Longleftrightarrow \quad (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \ \text{ or } \ (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (Q_1 \times F_2)
\end{aligned}$$

Proof. (closure under union – explicit construction (cont'd))

statement: $L(M_3) = L(M_1) \cup L(M_2)$

$$\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_3) \quad &\Longleftrightarrow \quad \widehat{\delta_3}(s_3, x) \in F_3 \\
&\Longleftrightarrow \quad \widehat{\delta_3}((s_1, s_2), x) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\
&\Longleftrightarrow \quad (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\
&\Longleftrightarrow \quad (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \ \text{ or } \ (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (Q_1 \times F_2) \\
&\Longleftrightarrow \quad \left(\widehat{\delta_1}(s_1, x) \in F_1 \ \text{ and } \ \widehat{\delta_2}(s_2, x) \in Q_2\right) \ \text{ or } \ \left(\widehat{\delta_1}(s_1, x) \in Q_1 \ \text{ and } \ \widehat{\delta_2}(s_2, x) \in F_2\right)
\end{aligned}$$

Proof. (closure under union – explicit construction (cont'd))

statement: $L(M_3) = L(M_1) \cup L(M_2)$

$$\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_3) \quad &\Longleftrightarrow \quad \widehat{\delta_3}(s_3, x) \in F_3 \\
&\Longleftrightarrow \quad \widehat{\delta_3}((s_1, s_2), x) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\
&\Longleftrightarrow \quad (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\
&\Longleftrightarrow \quad (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \ \text{ or } \ (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (Q_1 \times F_2) \\
&\Longleftrightarrow \quad \Big(\widehat{\delta_1}(s_1, x) \in F_1 \ \text{ and } \ \widehat{\delta_2}(s_2, x) \in Q_2\Big) \ \text{ or } \ \Big(\widehat{\delta_1}(s_1, x) \in Q_1 \ \text{ and } \ \widehat{\delta_2}(s_2, x) \in F_2\Big) \\
&\Longleftrightarrow \quad x \in L(M_1) \text{ or } x \in L(M_2)
\end{aligned}$$

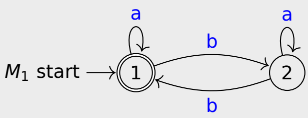
Proof. (closure under union – explicit construction (cont'd))

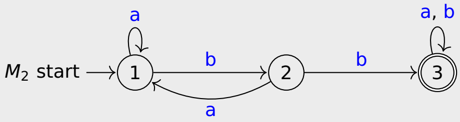
statement: $L(M_3) = L(M_1) \cup L(M_2)$

$$
\begin{aligned}
\forall x \in \Sigma^*,\ x \in L(M_3) \quad &\iff \quad \widehat{\delta_3}(s_3, x) \in F_3 \\
&\iff \quad \widehat{\delta_3}((s_1, s_2), x) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\
&\iff \quad (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\
&\iff \quad (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \ \text{ or } \ (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (Q_1 \times F_2) \\
&\iff \quad \left(\widehat{\delta_1}(s_1, x) \in F_1 \ \text{ and } \ \widehat{\delta_2}(s_2, x) \in Q_2\right) \ \text{ or } \ \left(\widehat{\delta_1}(s_1, x) \in Q_1 \ \text{ and } \ \widehat{\delta_2}(s_2, x) \in F_2\right) \\
&\iff \quad x \in L(M_1) \text{ or } x \in L(M_2) \\
&\iff \quad x \in L(M_1) \cup L(M_2)
\end{aligned}
$$

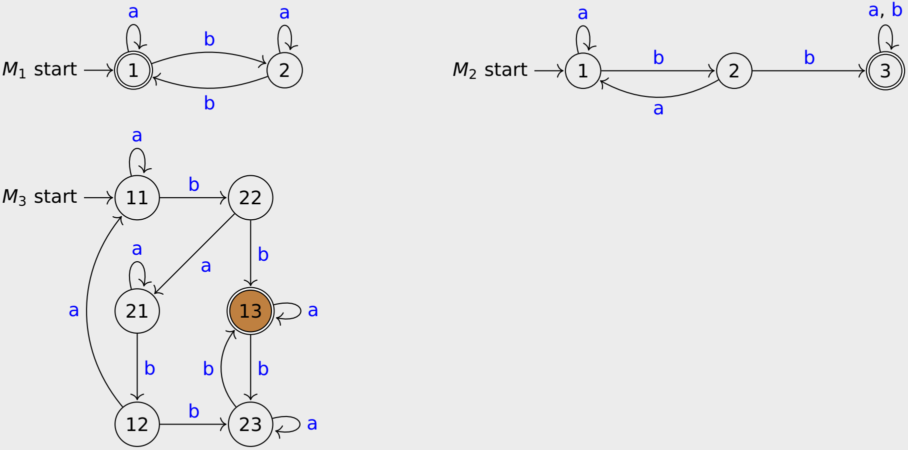
□

Example (union)



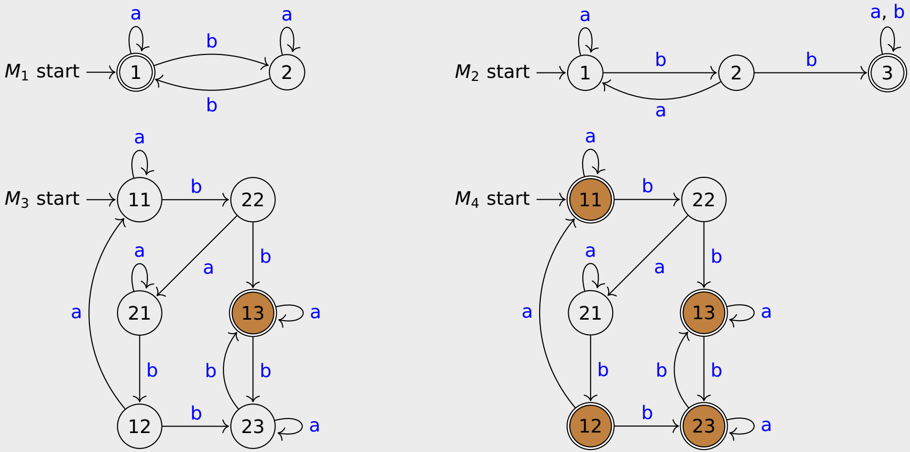


Example (union)



$L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring}\}$

Example (union)



$L(M_1) \cup L(M_2) = L(M_4) := \{x \mid x$ contains even number of bs or bb as substring$\}$

Thanks! & Questions?