Deterministic Finite State Automata

Burak Ekici

February 28 - March 4, 2022

### Outline

A Ouick Recap

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- 1 A Quick Recap
- 2 Chomsky Hierarchy

A Ouick Recap

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strings over  $\Sigma = \{0, 1\} : 0$  0110

A Ouick Recap

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•  $\{\varepsilon, 0, 1, 00, 01, 10, 11\}$  (all strings having at most two symbols)

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- {x | x is valid program in some machine language}

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• string concatenation  $x, y \in \Sigma^* \implies xy \in \Sigma^*$  is associative:

$$(xy)z = x(yz) \quad \forall x, y, z \in \Sigma^*$$

Deterministic Finite State Automata

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Deterministic Finite State Automata

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Deterministic Finite State Automata

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•  $\#a(x)(a \in \Sigma, x \in \Sigma^*)$  denotes number of a's in x

# Definitions $(A, B \subseteq \Sigma^*)$



 $A \cup B := \{x \mid x \in A \text{ or } x \in B\}$ 

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union

A Quick Recap

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(a) asterate A\* is union of all finite powers of A

 $A^* := \bigcup_{n \ge 0} A^n = A^0 \cup A^1 \cup A^2 \cup A^3 \cup \dots = \{x_1 x_2 \cdots x_n \text{ and } x_i \in A \text{ for all } 1 \le i \le n\}$ 

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$$A^+ = AA^* := \bigcup_{n \geqslant 1} A^n$$

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 $2^A := \{Q \mid Q \subseteq A\}$ 8 power set

# Example

A Quick Recap

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• substrings of 011: 0, 1, 01, 11, 011,  $\varepsilon$ 

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A Ouick Recap 000000

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- $2^{\{1,01\}} = \{\emptyset, \{1\}, \{01\}, \{1,01\}\}$

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A Quick Recap

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A Ouick Recap

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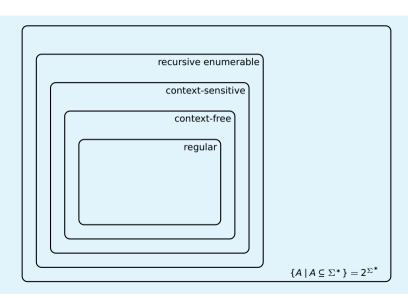
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$$A^{**} = A^*$$

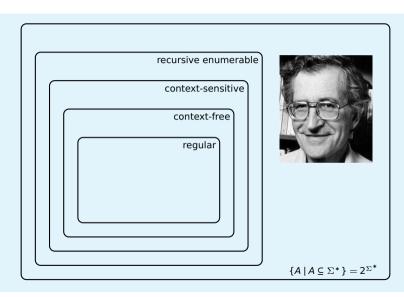
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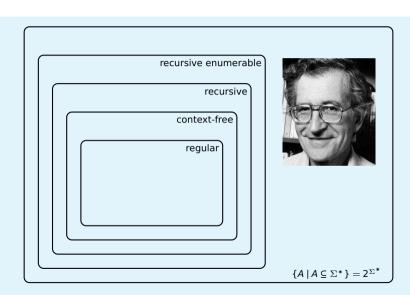
## Outline

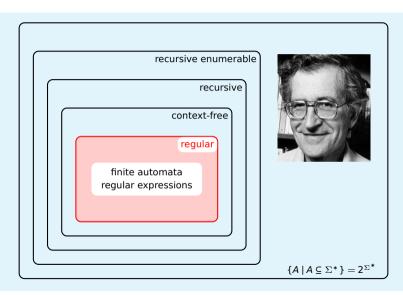
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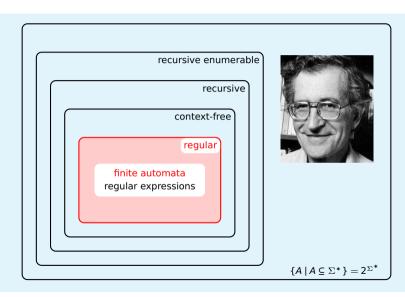




Deterministic Finite State Automata







# Outline

- 1 A Quick Recap
- 2 Chomsky Hierarchy
- 3 Deterministic Finite State Automata

# Definitions

A Ouick Recap

• deterministic finite automaton (DFA) is quintuple  $M = (Q, \Sigma, \delta, s, F)$  with

Deterministic Finite State Automata

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Deterministic Finite State Automata

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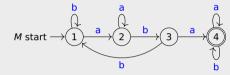
 $2 \Sigma$ : input alphabet

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Deterministic Finite State Automata

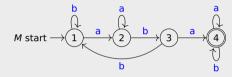
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$$M = (Q, \Sigma, \delta, s, F)$$



Deterministic Finite State Automata

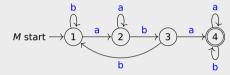
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Deterministic Finite State Automata

① 
$$Q = \{1, 2, 3, 4\}$$

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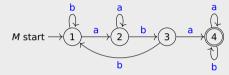


Deterministic Finite State Automata

$$\bigcirc Q = \{1, 2, 3, 4\}$$

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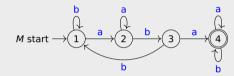
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Deterministic Finite State Automata

- $\bigcirc Q = \{1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \to Q$

$$M = (Q, \Sigma, \delta, s, F)$$



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δ	а	b
1	2	1
_	_	_

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Deterministic Finite State Automata

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**1** Q: finite set of states

 $\Sigma$ : input alphabet

**(a)**  $\delta: Q \times \Sigma \rightarrow Q:$  transition function  $4s \in O$ : start state

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Deterministic Finite State Automata

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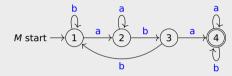
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**⑤** *F* ⊆ *Q* : final (accept) states

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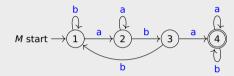


Deterministic Finite State Automata

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- 2 3
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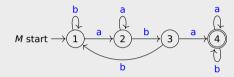


Deterministic Finite State Automata

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Deterministic Finite State Automata

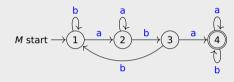
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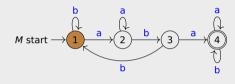
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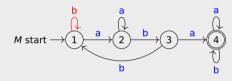


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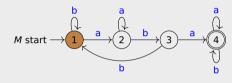
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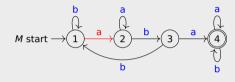


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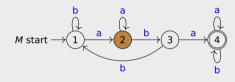
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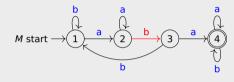
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- **4** s = 1
- **6**  $F = \{4\}$

- - 2 3

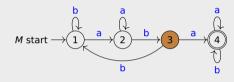
$$M = (Q, \Sigma, \delta, s, F)$$



- $\bigcirc Q = \{1, 2, 3, 4\}$
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- - 2 3

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3, 4\}$$

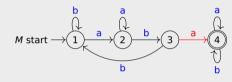
$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

$$4 s = 1$$
  
 $5 F = \{4\}$ 



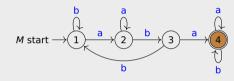
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- - 2 3

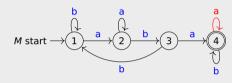
$$M = (Q, \Sigma, \delta, s, F)$$



- $\bigcirc Q = \{1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- **4** s = 1
- **6**  $F = \{4\}$

- 2 3

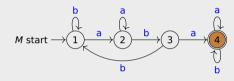
$$M = (Q, \Sigma, \delta, s, F)$$



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- 2 3

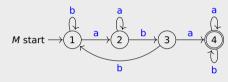
$$M = (Q, \Sigma, \delta, s, F)$$



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- $\Sigma = \{a, b\}$
- **4** s = 1
- **6**  $F = \{4\}$

- - 2 3

$$M = (Q, \Sigma, \delta, s, F)$$



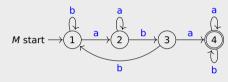
$$\bigcirc Q = \{1, 2, 3, 4\}$$

$$\delta: Q \times \Sigma \to Q$$

**4** 
$$s = 1$$

$$4s = 1$$
  
 $5F = \{4\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



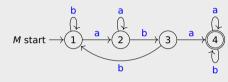
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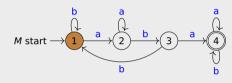
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- - 2 3

  - 4

- 3

$$M = (Q, \Sigma, \delta, s, F)$$



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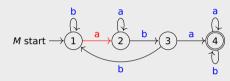
- - 2 3

  - 4

Deterministic Finite State Automata

- 3

$$M = (Q, \Sigma, \delta, s, F)$$



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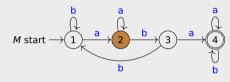
- - 2 3

  - 4

Deterministic Finite State Automata

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$$M = (Q, \Sigma, \delta, s, F)$$



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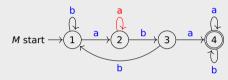
- - 2 3

  - 4

Deterministic Finite State Automata

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$$M = (Q, \Sigma, \delta, s, F)$$

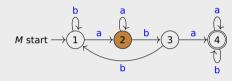


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- 4

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$$M = (Q, \Sigma, \delta, s, F)$$



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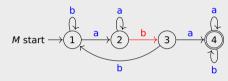
- - 2 3

  - 4

Deterministic Finite State Automata

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$$M = (Q, \Sigma, \delta, s, F)$$



- $\bigcirc Q = \{1, 2, 3, 4\}$
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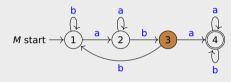
- - 2 3

  - 4

Deterministic Finite State Automata

- 3

$$M = (Q, \Sigma, \delta, s, F)$$



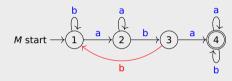
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- 2 3
- 4

Deterministic Finite State Automata

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



- $\bigcirc Q = \{1, 2, 3, 4\}$
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- **4** s = 1
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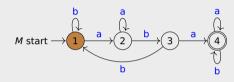
- - 2 3

  - 4

Deterministic Finite State Automata

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$$M = (Q, \Sigma, \delta, s, F)$$



- $\bigcirc Q = \{1, 2, 3, 4\}$
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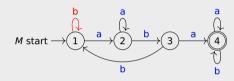
- - 2 3

  - 4

Deterministic Finite State Automata

- 3

$$M = (Q, \Sigma, \delta, s, F)$$



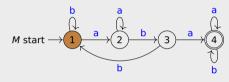
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- - 2 3

  - 4

- 3

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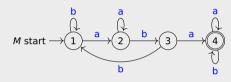
- - 2 3

  - 4

Deterministic Finite State Automata

- 3

$$M = (Q, \Sigma, \delta, s, F)$$



- $\bigcirc Q = \{1, 2, 3, 4\}$
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- **4** s = 1
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- - 2 3

  - 4

Deterministic Finite State Automata

- 3

#### **Definitions**

• deterministic finite automaton (DFA) is quintuple  $M = (Q, \Sigma, \delta, s, F)$  with

**0** 0 : finite set of states

 $\Sigma$ : input alphabet

transition function

 $4 s \in Q$ : start state ⑤  $F \subseteq Q$ : final (accept) states

•  $\hat{\delta}: Q \times \Sigma^* \to Q$  is inductively defined by

$$\widehat{\delta}(q, \varepsilon) := q$$
  $\widehat{\delta}(q, xa) := \delta(\widehat{\delta}(q, x), a)$ 

Deterministic Finite State Automata

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# Example (Unfolding of the multistep function $\widehat{\delta}$ )

Let x = abbaab over the alphabet  $\Sigma = \{a, b\}$ 

A Ouick Recap

$$\delta(\widehat{\delta}(q_0, abbaa), b)$$

first recursive call

Deterministic Finite State Automata

A Ouick Recap

$$\delta(\widehat{\delta}(q_0, abbaa), b)$$
  
 $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$ 

first recursive call second recursive call

Deterministic Finite State Automata

A Ouick Recap

$$\begin{array}{l} \delta(\widehat{\delta}(q_0,abbaa),b) \\ \delta(\delta(\widehat{\delta}(q_0,abba),a),b) \\ \delta(\delta(\delta(\widehat{\delta}(q_0,abb),a),a),b) \end{array}$$

first recursive call second recursive call third recursive call

Deterministic Finite State Automata

A Ouick Recap

 $\delta(\widehat{\delta}(q_0, abbaa), b)$  $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$  $\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$  $\delta(\delta(\delta(\delta(\delta(g_0,ab),b),a),a),b)$ 

first recursive call second recursive call third recursive call fourth recursive call

Deterministic Finite State Automata

A Ouick Recap

 $\delta(\widehat{\delta}(q_0, abbaa), b)$  $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$  $\delta(\delta(\delta(\widehat{\delta}(q_0, abb), a), a), b)$  $\delta(\delta(\delta(\delta(\delta(g_0,ab),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$ 

first recursive call second recursive call third recursive call fourth recursive call fifth recursive call

Deterministic Finite State Automata

A Ouick Recap

 $\delta(\widehat{\delta}(q_0, abbaa), b)$  $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$  $\delta(\delta(\delta(\delta(g_0,abb),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(g_0,ab),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(\delta(\widehat{q}_0, \boldsymbol{\varepsilon}), a), b), b), a), a), b)$ 

first recursive call second recursive call third recursive call fourth recursive call fifth recursive call sixth recursive call

Deterministic Finite State Automata

A Ouick Recap

 $\delta(\widehat{\delta}(q_0, abbaa), b)$  $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$  $\delta(\delta(\delta(\delta(g_0,abb),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(g_0,ab),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(\delta(\delta(g_0,\varepsilon),a),b),b),a),a),b))$  $\delta(\delta(\delta(\delta(\delta(\delta(q_0,a),b),b),a),a),b)$ 

first recursive call second recursive call third recursive call fourth recursive call fifth recursive call sixth recursive call

Deterministic Finite State Automata 

A Ouick Recap

 $\delta(\widehat{\delta}(q_0, abbaa), b)$  $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$  $\delta(\delta(\delta(\delta(g_0,abb),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(q_0,ab),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(\delta(\delta(g_0,\varepsilon),a),b),b),a),a),b))$  $\delta(\delta(\delta(\delta(\delta(\delta(q_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(a_1,b),b),a),a),b)$ 

first recursive call second recursive call third recursive call fourth recursive call fifth recursive call sixth recursive call

Deterministic Finite State Automata

assuming  $\delta(q_0, a) = q_1$ 

A Ouick Recap

 $\delta(\widehat{\delta}(q_0, abbaa), b)$  $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$  $\delta(\delta(\delta(\delta(g_0,abb),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(g_0,ab),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(\delta(\delta(g_0,\varepsilon),a),b),b),a),a),b))$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(q_1,b),b),a),a),b)$  $\delta(\delta(\delta(\delta(q_2,b),a),a),b)$ 

first recursive call second recursive call third recursive call fourth recursive call fifth recursive call sixth recursive call

Deterministic Finite State Automata

assuming  $\delta(q_0, a) = q_1$ assuming  $\delta(q_1, b) = q_2$ 

A Ouick Recap

 $\delta(\widehat{\delta}(q_0, abbaa), b)$  $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$  $\delta(\delta(\delta(\delta(g_0,abb),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(g_0,ab),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(\delta(\delta(g_0,\varepsilon),a),b),b),a),a),b))$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(q_1,b),b),a),a),b)$  $\delta(\delta(\delta(\delta(g_2,b),a),a),b)$  $\delta(\delta(\delta(a_3,a),a),b)$ 

first recursive call second recursive call third recursive call fourth recursive call fifth recursive call sixth recursive call

Deterministic Finite State Automata

assuming  $\delta(q_0, a) = q_1$ assuming  $\delta(q_1, b) = q_2$ assuming  $\delta(a_2,b)=a_3$ 

A Ouick Recap

$$\begin{split} &\delta(\widehat{\delta}(q_0,abbaa),b)\\ &\delta(\widehat{\delta}(\widehat{q}_0,abbaa),a),b)\\ &\delta(\delta(\widehat{\delta}(\widehat{q}_0,abba),a),b)\\ &\delta(\delta(\delta(\widehat{\delta}(q_0,ab),a),a),b)\\ &\delta(\delta(\delta(\delta(\widehat{\delta}(q_0,ab),b),a),a),b)\\ &\delta(\delta(\delta(\delta(\widehat{\delta}(\widehat{\delta}(q_0,a),b),b),a),a),b)\\ &\delta(\delta(\delta(\delta(\delta(\widehat{\delta}(q_0,\epsilon),a),b),b),a),a),b)\\ &\delta(\delta(\delta(\delta(\delta(\delta(q_0,\epsilon),a),b),b),a),a),b)\\ &\delta(\delta(\delta(\delta(\delta(q_1,b),b),a),a),b)\\ &\delta(\delta(\delta(\delta(q_1,b),b),a),a),b)\\ &\delta(\delta(\delta(\delta(q_3,a),a),b)\\ &\delta(\delta(\delta(q_4,a),b)\\ &\delta(\delta(q_4,a),b)\\ \end{split}$$

first recursive call second recursive call third recursive call fourth recursive call fifth recursive call sixth recursive call

Deterministic Finite State Automata

assuming  $\delta(q_0, a) = q_1$ assuming  $\delta(q_1, b) = q_2$ assuming  $\delta(a_2,b)=a_3$ assuming  $\delta(q_3, a) = q_4$ 

A Ouick Recap

 $\delta(\widehat{\delta}(q_0, abbaa), b)$  $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$  $\delta(\delta(\delta(\delta(g_0,abb),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(g_0,ab),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(\delta(\delta(g_0,\varepsilon),a),b),b),a),a),b))$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(q_1,b),b),a),a),b)$  $\delta(\delta(\delta(\delta(g_2,b),a),a),b)$  $\delta(\delta(\delta(q_3,a),a),b)$  $\delta(\delta(q_4,a),b)$  $\delta(q_5,b)$ 

first recursive call second recursive call third recursive call fourth recursive call fifth recursive call sixth recursive call

Deterministic Finite State Automata

assuming  $\delta(q_0, a) = q_1$ assuming  $\delta(a_1,b)=a_2$ assuming  $\delta(q_2, b) = q_3$ assuming  $\delta(q_3, a) = q_4$ assuming  $\delta(q_4, a) = q_5$ 

A Ouick Recap

 $\delta(\widehat{\delta}(q_0, abbaa), b)$  $\delta(\delta(\widehat{\delta}(q_0, abba), a), b)$  $\delta(\delta(\delta(\delta(g_0,abb),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(g_0,ab),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(\delta(\delta(\delta(g_0,\varepsilon),a),b),b),a),a),b))$  $\delta(\delta(\delta(\delta(\delta(\delta(g_0,a),b),b),a),a),b)$  $\delta(\delta(\delta(\delta(\delta(q_1,b),b),a),a),b)$  $\delta(\delta(\delta(\delta(q_2,b),a),a),b)$  $\delta(\delta(\delta(q_3,a),a),b)$  $\delta(\delta(q_4,a),b)$  $\delta(q_5,b)$ 96

first recursive call second recursive call third recursive call fourth recursive call fifth recursive call sixth recursive call

Deterministic Finite State Automata

assuming  $\delta(q_0, a) = q_1$ assuming  $\delta(a_1,b)=a_2$ assuming  $\delta(q_2, b) = q_3$ assuming  $\delta(q_3, a) = q_4$ assuming  $\delta(q_4, a) = q_5$ assuming  $\delta(a_5, b) = a_6$  A Ouick Recap

• deterministic finite automaton (DFA) is quintuple  $M = (Q, \Sigma, \delta, s, F)$  with

**0** 0 : finite set of states

 $\Sigma$ : input alphabet

transition function

 $4 s \in Q$ : start state  $\bigcirc$   $F \subseteq Q$ : final (accept) states

•  $\hat{\delta}: Q \times \Sigma^* \to Q$  is inductively defined by

 $\widehat{\delta}(q, \varepsilon) := q$  $\widehat{\delta}(q, xa) := \delta(\widehat{\delta}(q, x), a)$ 

Deterministic Finite State Automata

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• string  $x \in \Sigma^*$  is accepted by M if  $\widehat{\delta}(s, x) \in F$ 

A Ouick Recap

• deterministic finite automaton (DFA) is quintuple  $M = (Q, \Sigma, \delta, s, F)$  with

**0** 0 : finite set of states  $2\Sigma$ :

input alphabet transition function

 $4 s \in Q$ : start state

**⑤** *F* ⊂ *O* : final (accept) states

•  $\hat{\delta}: O \times \Sigma^* \to O$  is inductively defined by

 $\widehat{\delta}(q, \varepsilon) := q$  $\widehat{\delta}(q, xa) := \delta(\widehat{\delta}(q, x), a)$ 

Deterministic Finite State Automata

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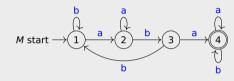
• string  $x \in \Sigma^*$  is accepted by M if  $\widehat{\delta}(s, x) \in F$ 

• string  $x \in \Sigma^*$  is rejected by M if  $\widehat{\delta}(s, x) \notin F$ 

 $\in L(M)$ 

## Example (DFAs → Regular Sets)

$$M = (Q, \Sigma, \delta, s, F)$$



- $\bigcirc Q = \{1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- **4** s = 1
- **6**  $F = \{4\}$

- - 2 3
  - 4

- $\notin L(M)$
- 2 3

A Ouick Recap

• deterministic finite automaton (DFA) is quintuple  $M = (Q, \Sigma, \delta, s, F)$  with

1 Q: finite set of states

 $\Sigma$ : input alphabet

transition function

 $4 s \in Q$ : start state **⑤** *F* ⊂ *O* : final (accept) states

•  $\hat{\delta}: O \times \Sigma^* \to O$  is inductively defined by

$$\widehat{\delta}(q, \varepsilon) := q$$
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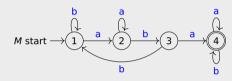
Deterministic Finite State Automata

• string  $x \in \Sigma^*$  is accepted by M if  $\widehat{\delta}(s, x) \in F$ 

• string  $x \in \Sigma^*$  is rejected by M if  $\widehat{\delta}(s, x) \notin F$ 

• language accepted by M is given by  $L(M) := \{x \mid \widehat{\delta}(s, x) \in F\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



- $\bigcirc Q = \{1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- **4** s = 1
- **6**  $F = \{4\}$

- - 2 3

  - 4

 $\in L(M)$ 

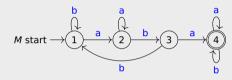
Deterministic Finite State Automata

- - $\notin L(M)$
- 2 3

$$L(M) := \{x \mid$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



- $\bigcirc Q = \{1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- $\triangle s = 1$
- **6**  $F = \{4\}$

- - 2 3

  - 4

 $\in L(M)$ 

Deterministic Finite State Automata

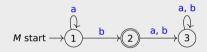
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- - $\notin L(M)$

 $L(M) := \{x \mid x \text{ contains } aba \text{ as substring}\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$

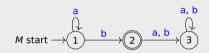
A Ouick Recap



Deterministic Finite State Automata

 $M = (Q, \Sigma, \delta, s, F)$ 

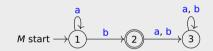
A Ouick Recap



① 
$$Q = \{1, 2, 3\}$$

 $M = (Q, \Sigma, \delta, s, F)$ 

A Ouick Recap



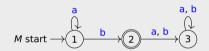
Deterministic Finite State Automata

$$\bigcirc Q = \{1, 2, 3\}$$

$$\ \ \ \Sigma = \{a,b\}$$

 $M = (Q, \Sigma, \delta, s, F)$ 

A Ouick Recap

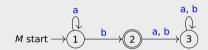


Deterministic Finite State Automata

- ①  $Q = \{1, 2, 3\}$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \to Q$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



Deterministic Finite State Automata

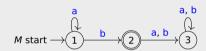
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$$\begin{array}{c|cccc}
\delta & a & b \\
1 & 1 & 2 \\
2 & 3 & 3 \\
3 & 3 & 3
\end{array}$$

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A Ouick Recap



Deterministic Finite State Automata

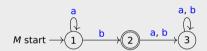
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A Ouick Recap



Deterministic Finite State Automata

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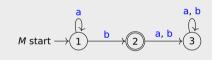
$$\Sigma = \{a, b\}$$

$$\bullet$$
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$$\mathbf{a} s = 1$$

**5** 
$$F = \{2\}$$

$$M = (Q, \Sigma, \delta, s, F)$$



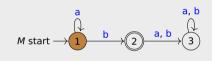
Deterministic Finite State Automata

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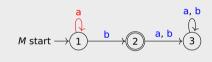
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Deterministic Finite State Automata

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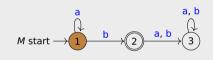
Deterministic Finite State Automata

$$\Sigma = \{a, b\}$$

$$5 = 1$$

14/31

$$M = (Q, \Sigma, \delta, s, F)$$



Deterministic Finite State Automata

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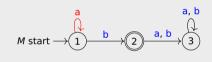
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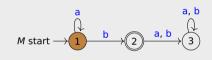
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Deterministic Finite State Automata



A Ouick Recap



$$\bigcirc Q = \{1, 2, 3\}$$

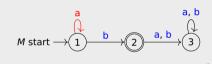
$$\Sigma = \{a, b\}$$

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$$S = 1$$
  
 $S = \{2\}$ 

Deterministic Finite State Automata

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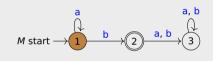
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Deterministic Finite State Automata

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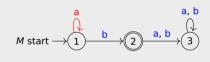
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Deterministic Finite State Automata

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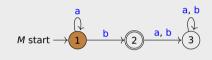
$$\delta: Q \times \Sigma \to Q$$

$$\mathbf{a} s = 1$$

$$4 s = 1$$
  
 $5 F = \{2\}$ 

Deterministic Finite State Automata

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

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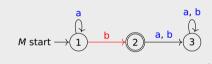
$$\mathbf{a} = \mathbf{b}$$

**5** 
$$F = \{2\}$$



Deterministic Finite State Automata

$$M = (Q, \Sigma, \delta, s, F)$$



① 
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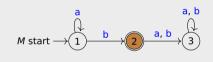
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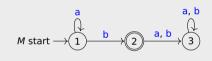
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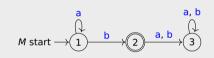
$$\mathbf{a} s = 1$$

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Deterministic Finite State Automata

 $\in L(M)$ 

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

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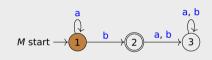
$$5 = \{2\}$$

Deterministic Finite State Automata

 $\in L(M)$ 



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$$\bigcirc Q = \{1, 2, 3\}$$

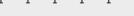
$$\Sigma = \{a, b\}$$

$$5 = 1$$

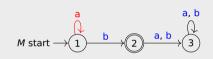
L	1	2
2	3	3
3	3	3

 $\in L(M)$ 

Deterministic Finite State Automata



$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

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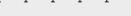
$$\mathbf{a} s = 1$$

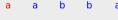
$$5 = \{2\}$$

L	1	2
2	3	3
3	3	3

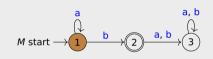
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Deterministic Finite State Automata





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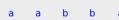


$$\Sigma = \{a, b\}$$

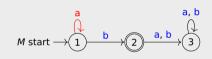
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Deterministic Finite State Automata

 $\in L(M)$ 



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$$\mathbf{a} s = 1$$

4) 
$$s = 1$$
  
5)  $F = \{2\}$ 

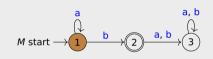
1	1	2
2	3	3
3	3	3

Deterministic Finite State Automata

 $\in L(M)$ 



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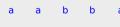
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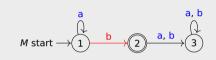
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 $\in L(M)$ 

Deterministic Finite State Automata



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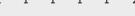


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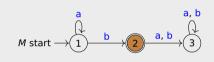
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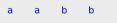
$$5 = \{2\}$$

$$\begin{array}{c|cccc} \delta & a & b \\ \hline 1 & 1 & 2 \\ \end{array}$$

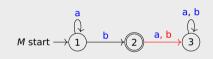
т	т.	
2	3	3
3	3	3

Deterministic Finite State Automata

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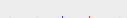
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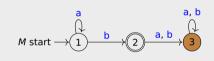
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Deterministic Finite State Automata



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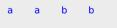


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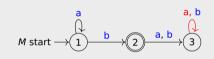


Deterministic Finite State Automata

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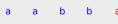
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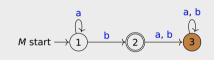
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Deterministic Finite State Automata

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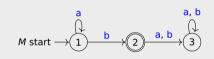
Deterministic Finite State Automata



 $\in L(M)$ 

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



$$\bigcirc Q = \{1, 2, 3\}$$

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**6** 
$$F = \{2\}$$

a a b b a 
$$\notin L(M)$$

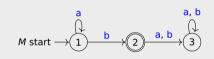
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3

Deterministic Finite State Automata

## Example (DFA → Regular Sets)

$$M = (Q, \Sigma, \delta, s, F)$$



$$\mathbb{Q} = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\mathbf{a} s = \mathbf{1}$$

$$5 = \{2\}$$

a a b b a 
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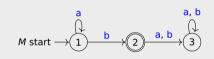
Deterministic Finite State Automata

$$L(M) := \{x \mid$$

 $\in L(M)$ 

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$$\Sigma = \{a, b\}$$

$$\mathbf{a} s = 1$$

$$5 = 1$$

a a b b a 
$$\notin L(M)$$

Deterministic Finite State Automata

$$L(M) := \{x \mid x = a^n b, n \ge 0\}$$

 $\in L(M)$ 

Deterministic Finite State Automata

A Quick Recap

The DFA M is correct with respect to predefined specs.

#### Theoren

A Ouick Recap

The DFA M is correct with respect to predefined specs. Namely, M accepts every string of the form  $a^nb$  s.t.  $n \in \mathbb{N}$ , rejecting all others.

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Deterministic Finite State Automata

Formally: 
$$\widehat{\delta}(1,x) = \begin{cases} 1 & \Longleftrightarrow x \in L(a^*) \\ 2 & \Longleftrightarrow x \in L(a^*b) \\ 3 & \Longleftrightarrow x \in L(a^*b(a+b)^+) \end{cases}$$

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#### Proof.

We argue by mathematical induction on the length of x.

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We argue by mathematical induction on the length of x.

**1** Base Case: 
$$|x| = 0 \iff x = \varepsilon \quad \widehat{\delta}(1, \varepsilon) = 1 \iff \varepsilon \in L(a^*)$$

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Deterministic Finite State Automata

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Show : *M* is correct on every *xv* for all  $v \in \Sigma = \{a, b\}$ such that |xy| = k + 1

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Deterministic Finite State Automata

0000000000000000



A Quick Recap

Proof. (cont'd)

① 
$$\widehat{\delta}(1,x) = 1$$
 and  $y = a$ 

# Proof. (cont'd)

A Ouick Recap

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = a$$
 
$$\widehat{\delta}(1,x) = 1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)}$$

$$\delta(\widehat{\delta}(1,x),a)=1 \iff xa \in L(a^*)$$

$$\delta(\widehat{\delta}(1,x),a) = 1 \quad \iff \quad xa \in L(a^*)$$

Deterministic Finite State Automata

0000000000000000

 $\widehat{\delta}(1,x)=1 \quad \text{and} \quad y=b$ 

# Proof. (cont'd)

A Ouick Recap

 $\widehat{\delta}(1,x)=1 \quad \text{and} \quad y=a$  $\widehat{\delta}(1,x) = 1 \iff x \in L(a^*) \text{ (by IH)}$ 

$$\delta(\widehat{\delta}(1,x),a) = 1 \iff xa \in L(a^*)$$

Deterministic Finite State Automata

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$$\delta(\widehat{\delta}(1,x),b) = 2 \iff xb \in L(a^*b)$$

$$\widehat{\delta}(1, x) = 1 \quad \text{and} \quad y = a$$

$$\widehat{\delta}(1, x) = 1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)}$$

$$\delta(\widehat{\delta}(1,x),a) = 1 \iff xa \in L(a^*)$$

Deterministic Finite State Automata

$$\widehat{\delta}(1, x) = 1 \quad \text{and} \quad y = b$$

$$\widehat{\delta}(1, x) = 1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)}$$

$$\widehat{\delta}(1,x)=1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)} \qquad \qquad \delta(\widehat{\delta}(1,x),b)=2 \quad \Longleftrightarrow \quad xb \in L(a^*b)$$

$$\widehat{\delta}(1,x)=2$$
 and  $y=a$ 

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = a$$

$$\widehat{\delta}(1,x) = 1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)}$$

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Deterministic Finite State Automata

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = b$$

$$\widehat{\delta}(1,x) = 1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)}$$

$$\delta(\widehat{\delta}(1,x),b) = 2 \iff xb \in L(a^*b)$$

$$\widehat{\delta}(1,x) = 2 \quad \text{and} \quad y = a$$

$$\widehat{\delta}(1,x) = 2 \quad \Longleftrightarrow \quad x \in L(a^*b) \text{ (by IH)}$$

$$\delta(\widehat{\delta}(1,x),a) = 3 \iff xa \in L(a*b(a+b)^+)$$

# Proof. (cont'd)

A Ouick Recap

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = a$$

$$\widehat{\delta}(1,x) = 1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)}$$

$$\delta(\widehat{\delta}(1,x),a) = 1 \iff xa \in L(a^*)$$

Deterministic Finite State Automata

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = b$$

$$\widehat{\delta}(1,x) = 1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)}$$

$$\delta(\widehat{\delta}(1,x),b) = 2 \iff xb \in L(a*b)$$

$$\widehat{\delta}(1,x) = 2 \quad \text{and} \quad y = a$$

$$\widehat{\delta}(1,x) = 2 \quad \Longleftrightarrow \quad x \in L(a^*b) \text{ (by IH)}$$

$$\delta(\widehat{\delta}(1,x),a) = 3 \iff xa \in L(a*b(a+b)^+)$$

$$\widehat{\delta}(1,x) = 2 \quad \text{and} \quad y = b$$

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = a$$

$$\widehat{\delta}(1,x) = 1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)}$$

$$\delta(\widehat{\delta}(1,x),a) = 1 \iff xa \in L(a^*)$$

Deterministic Finite State Automata 000000000000000000

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = b$$

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$$\delta(\widehat{\delta}(1,x),b) = 2 \iff xb \in L(a^*b)$$

§ 
$$\widehat{\delta}(1,x) = 2$$
 and  $y = a$   
 $\widehat{\delta}(1,x) = 2$   $\iff$   $x \in L(a^*b)$  (by IH)

$$\delta(\widehat{\delta}(1,x),a) = 3 \iff xa \in L(a*b(a+b)^+)$$

$$\widehat{\delta}(1, x) = 2 \quad \text{and} \quad y = b$$

$$\widehat{\delta}(1, x) = 2 \quad \Longleftrightarrow \quad x \in L(a^*b) \text{ (by IH)}$$

$$\delta(1,x) = 2$$
 and  $y = b$   
 $\widehat{\delta}(1,x) = 2 \iff x \in L(a^*b) \text{ (by IH)}$   $\delta(\widehat{\delta}(1,x),b) = 3 \iff xb \in L(a^*b(a+b)^+)$ 

## Proof. (cont'd)

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = a$$

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Deterministic Finite State Automata 000000000000000000

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = b$$

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$$\widehat{\delta}(1,x) = 2 \quad \text{and} \quad y = a$$

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$$\delta(\widehat{\delta}(1,x),b) = 3 \iff xb \in L(a*b(a+b)^+)$$

$$\widehat{\delta}(1,x) = 3 \quad \text{and} \quad y = a$$

#### Proof. (cont'd)

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = a$$

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Deterministic Finite State Automata

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$$\widehat{\delta}(1,x) = 2 \quad \text{and} \quad y = b$$

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$$\delta(1, x) = 2$$
 and  $y = b$   
 $\widehat{\delta}(1, x) = 2$   $\iff$   $x \in L(a^*b)$  (by IH)  $\delta(\widehat{\delta}(1, x), b) = 3$   $\iff$   $xb \in L(a^*b(a+b)^+)$ 

$$\widehat{\delta}(1,x) = 3 \quad \text{and} \quad y = a$$

$$\widehat{\delta}(1,x) = 3 \quad \Longleftrightarrow \quad x \in L(a^*b(a+b)^+) \text{ (by IH)} \quad \delta(\widehat{\delta}(1,x),a) = 3 \quad \Longleftrightarrow \quad xa \in L(a^*b(a+b)^+)$$

### Proof. (cont'd)

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = a$$

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 and  $y = b$   
 $\widehat{\delta}(1,x) = 2$   $\iff x \in L(a^*b)$  (by IH)  $\delta(\widehat{\delta}(1,x),b) = 3$   $\iff xb \in L(a^*b(a+b)^+)$ 

$$\widehat{\delta}(1,x) = 3 \quad \text{and} \quad y = a$$

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$$\widehat{\delta}(1,x) = 3 \quad \text{and} \quad y = b$$

Closure Properties

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = a$$

$$\widehat{\delta}(1,x) = 1 \quad \Longleftrightarrow \quad x \in L(a^*) \text{ (by IH)}$$

$$\delta(\widehat{\delta}(1,x),a) = 1 \iff xa \in L(a^*)$$

Deterministic Finite State Automata

$$\widehat{\delta}(1,x) = 1 \quad \text{and} \quad y = b$$

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§ 
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 and  $y = a$   
 $\hat{\delta}(1,x) = 2$   $\iff$   $x \in L(a^*b)$  (by IH)

$$\delta(\widehat{\delta}(1,x),a) = 3 \iff xa \in L(a^*b(a+b)^+)$$

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$$\delta(1,x)=2$$
 and  $y=b$   
 $\widehat{\delta}(1,x)=2$   $\iff$   $x \in L(a^*b)$  (by IH)  $\delta(\widehat{\delta}(1,x),b)=3$   $\iff$   $xb \in L(a^*b(a+b)^+)$ 

$$\widehat{\delta}(1,x) = 3 \quad \text{and} \quad y = a$$

$$\widehat{\delta}(1,x) = 3 \quad \Longleftrightarrow \quad x \in L(a^*b(a+b)^+) \text{ (by IH)} \quad \delta(\widehat{\delta}(1,x),a) = 3 \quad \Longleftrightarrow \quad xa \in L(a^*b(a+b)^+)$$

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$$\widehat{\delta}(1,x) = 3 \quad \Longleftrightarrow \quad x \in L(a^*b(a+b)^+) \text{ (by IH)} \quad \delta(\widehat{\delta}(1,x),b) = 3 \quad \Longleftrightarrow \quad xb \in L(a^*b(a+b)^+)$$

$$M = (Q, \Sigma, \delta, s, F)$$

$$\Sigma = \{a, b\}$$

$$M = (Q, \Sigma, \delta, s, F)$$

$$\bigcirc Q =$$

$$\Sigma = \{a, b\}$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

- **0** 0 =
- $2 \Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\triangle s =$

 $M = (Q, \Sigma, \delta, s, F)$ 

A Ouick Recap

- $\bigcirc O =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$
- $\mathbf{A} s =$
- $\bigcirc$  F =

 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

Deterministic Finite State Automata

 $M = (Q, \Sigma, \delta, s, F)$ 

A Ouick Recap

 $M \text{ start} \rightarrow \bigcirc \bigcirc \bigcirc$ 

- $\bigcirc O =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$
- $\triangle s =$
- F =

 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

Deterministic Finite State Automata

 $M = (Q, \Sigma, \delta, s, F)$ 

A Ouick Recap

 $M \text{ start} \rightarrow 1$ 

Deterministic Finite State Automata

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$$\bigcirc O =$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

$$\triangle s =$$

$$\bigcirc$$
  $F =$ 

$$M = (Q, \Sigma, \delta, s, F)$$



Deterministic Finite State Automata

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- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$
- $\triangle s =$
- F =

$$M = (Q, \Sigma, \delta, s, F)$$

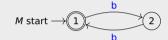


Deterministic Finite State Automata

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- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$
- $\triangle s =$
- F =

$$M = (Q, \Sigma, \delta, s, F)$$

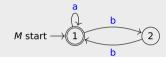


Deterministic Finite State Automata

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- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$
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- F =

$$M = (Q, \Sigma, \delta, s, F)$$



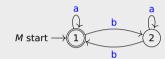
Deterministic Finite State Automata

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- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$
- $\triangle s =$
- F =

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



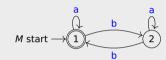
Deterministic Finite State Automata

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- $\bigcirc O =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$
- $\triangle s =$
- F =

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



Deterministic Finite State Automata

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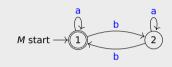
① 
$$Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

**⑤** 
$$F = \{1\}$$

$$M = (Q, \Sigma, \delta, s, F)$$



b

Deterministic Finite State Automata

b

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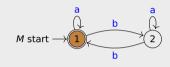
① 
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$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

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$$M = (Q, \Sigma, \delta, s, F)$$



b

Deterministic Finite State Automata

b

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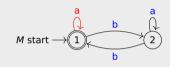
$$\bigcirc Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

**6** 
$$F = \{1\}$$

$$M = (Q, \Sigma, \delta, s, F)$$



b

Deterministic Finite State Automata

b

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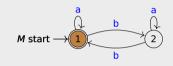
① 
$$Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

$$abrac{1}{2}$$
  $s = 1$ 

$$F = \{1\}$$

$$M = (Q, \Sigma, \delta, s, F)$$



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$$Q = \{1, 2\}$$

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b

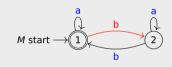
Deterministic Finite State Automata

b

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$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



b

$$\bigcirc Q = \{1, 2\}$$

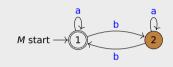
$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

$$F = \{1\}$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



Deterministic Finite State Automata

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① 
$$Q = \{1, 2\}$$

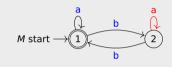
$$\Sigma = \{a, b\}$$

$$abrac{1}{2}$$
  $s = 1$ 

**6** 
$$F = \{1\}$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



b

Deterministic Finite State Automata

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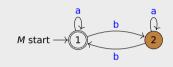
① 
$$Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

$$5 = 1$$
  
 $5 = \{1\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



Deterministic Finite State Automata

b

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① 
$$Q = \{1, 2\}$$

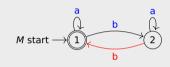
$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

**6** 
$$F = \{1\}$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



b

① 
$$Q = \{1, 2\}$$

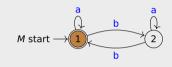
$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

**6** 
$$F = \{1\}$$



A Ouick Recap



b

Deterministic Finite State Automata

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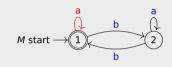
① 
$$Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

$$5 = 1$$

$$M = (Q, \Sigma, \delta, s, F)$$



b

Deterministic Finite State Automata

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$$\bigcirc Q = \{1, 2\}$$

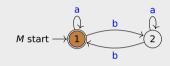
$$\Sigma = \{a, b\}$$

$$abrac{1}{2}$$
  $s = 1$ 

**6** 
$$F = \{1\}$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



b

Deterministic Finite State Automata

b

$$\bigcirc Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

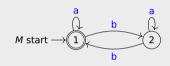
**4** 
$$s = 1$$

$$5 = 1$$
  
 $5 = \{1\}$ 

$$L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



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Deterministic Finite State Automata

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① 
$$Q = \{1, 2\}$$

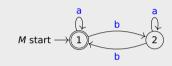
$$\Sigma = \{a, b\}$$

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⑤ 
$$F = \{1\}$$

b

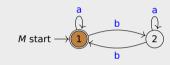
Deterministic Finite State Automata

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b

 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



а

b

① 
$$Q = \{1, 2\}$$

② 
$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \to Q$ 

**4** 
$$s = 1$$

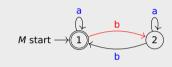
$$F = \{1\}$$



b

 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



а

b

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  $\delta: Q \times \Sigma \rightarrow Q$ 

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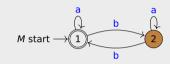
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Deterministic Finite State Automata

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 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

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$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \to Q$ 

**4** 
$$s = 1$$

⑤ 
$$F = \{1\}$$

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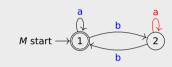
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Deterministic Finite State Automata

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 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



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$$\bigcirc Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

**4** 
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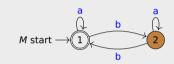
b

b

 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



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b

$$\bigcirc Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

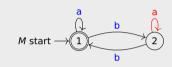
**4** 
$$s = 1$$

**6** 
$$F = \{1\}$$

b

$$L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$$

$$M = (Q, \Sigma, \delta, s, F)$$



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$$\bigcirc Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

$$F = \{1\}$$

b

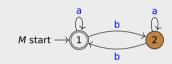
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 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

 $\in L(M)$ 

Closure Properties

$$M = (Q, \Sigma, \delta, s, F)$$



а

b

$$\bigcirc Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

**4** 
$$s = 1$$

⑤ 
$$F = \{1\}$$



2

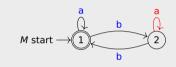
b

Deterministic Finite State Automata

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$$L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$$

$$M = (Q, \Sigma, \delta, s, F)$$



а

b

$$\bigcirc Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

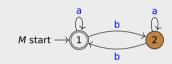
**4** 
$$s = 1$$

**6** 
$$F = \{1\}$$

b

 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



а

$$\bigcirc Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

$$F = \{1\}$$

b



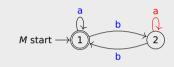
b

Deterministic Finite State Automata

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 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



а

b

2

① 
$$Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

**4** 
$$s = 1$$

$$F = \{1\}$$

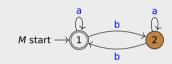
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Deterministic Finite State Automata

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$$L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$$

$$M = (Q, \Sigma, \delta, s, F)$$



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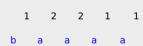
b

$$\bigcirc Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

$$F = \{1\}$$



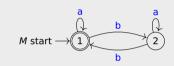
b

Deterministic Finite State Automata

00000000000000000

 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



а

① 
$$Q = \{1, 2\}$$

$$\Sigma = \{a, b\}$$

**4** 
$$s = 1$$

⑤ 
$$F = \{1\}$$



b

b

Deterministic Finite State Automata

00000000000000000

$$I(M) = [x] \times s$$

 $L(M) := \{x \mid x \text{ contains even number of } bs \text{ over } \Sigma\}$ 

$$M = (Q, \Sigma, \delta, s, F)$$

$$\Sigma = \{a, b\}$$

$$M = (Q, \Sigma, \delta, s, F)$$

$$\bigcirc Q =$$

$$\Sigma = \{a, b\}$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

- ① Q =
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \to Q$
- $\triangle s =$

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

Deterministic Finite State Automata

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \to Q$
- $\triangle s =$
- $\bigcirc$  F =

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

Deterministic Finite State Automata

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

$$M \text{ start} \longrightarrow 1$$

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \to Q$
- $\triangle s =$
- $\bigcirc$  F =

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

Deterministic Finite State Automata

$$M = (Q, \Sigma, \delta, s, F)$$

$$M \text{ start} \longrightarrow 1$$
 2

$$\bigcirc Q =$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

$$\mathbf{A} s =$$

$$\mathbf{G} F =$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

$$M \text{ start} \longrightarrow 1$$
  $\longrightarrow 2$ 

Deterministic Finite State Automata

0000000000000000

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \to Q$
- $\triangle s =$
- $\bigcirc$  F =

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

$$M \operatorname{start} \longrightarrow 1$$
  $\longrightarrow 1$   $\longrightarrow 1$ 

Deterministic Finite State Automata

0000000000000000

$$\bigcirc Q =$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

$$\mathbf{A} s =$$

$$\bigcirc F =$$

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

$$M \text{ start} \longrightarrow 1$$
  $\longrightarrow 1$   $\longrightarrow 1$ 

Deterministic Finite State Automata

0000000000000000

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \to Q$
- $\triangle s =$
- $\bigcirc$  F =

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

$$M \text{ start} \longrightarrow 1$$
  $\longrightarrow 1$   $\longrightarrow 1$ 

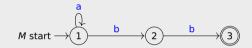
Deterministic Finite State Automata

0000000000000000

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \to Q$
- $\triangle s =$
- $\bigcirc$  F =

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap

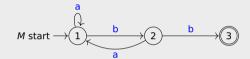


Deterministic Finite State Automata

0000000000000000

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \to Q$
- $\triangle s =$
- $\bigcirc$  F =

$$M = (Q, \Sigma, \delta, s, F)$$

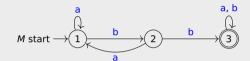


Deterministic Finite State Automata

000000000000000

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$
- $\triangle s =$
- $\bigcirc$  F =

$$M = (Q, \Sigma, \delta, s, F)$$



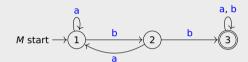
Deterministic Finite State Automata

000000000000000

- $\bigcirc Q =$
- $\Sigma = \{a, b\}$
- $\bullet$   $\delta: Q \times \Sigma \rightarrow Q$
- $\triangle s =$
- $\bigcirc$  F =

$$M = (Q, \Sigma, \delta, s, F)$$

A Ouick Recap



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

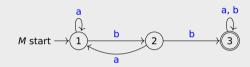
$$4 s = 1$$
  
 $5 F = {3}$ 

$$L(M)$$
:

Deterministic Finite State Automata

0000000000000000

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$F = \{3\}$$

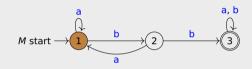
$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Deterministic Finite State Automata

0000000000000000

$$L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$$

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

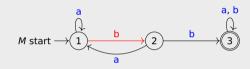
$$F = \{3\}$$

$$I(M) \cdot = \{x \mid x\}$$

Deterministic Finite State Automata

0000000000000000

$$M = (Q, \Sigma, \delta, s, F)$$



$$Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

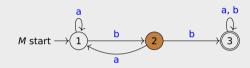
$$\mathbf{a} s = 1$$

$$5 = 1$$
  
 $5 = {3}$ 

Deterministic Finite State Automata

0000000000000000

$$M = (Q, \Sigma, \delta, s, F)$$



Deterministic Finite State Automata

0000000000000000

$$\bigcirc Q = \{1, 2, 3\}$$

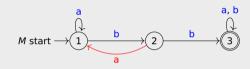
$$\Sigma = \{a, b\}$$

$$\mathbf{A} s = 1$$

$$F = \{3\}$$

$$L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$$

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

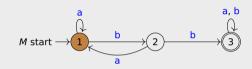
**4** 
$$s = 1$$

$$5 = 1$$

Deterministic Finite State Automata

0000000000000000

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$a s = 1$$

$$F = \{3\}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

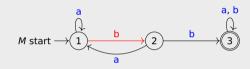
$$I(M) := \{x \mid x \in \Omega\}$$

Deterministic Finite State Automata

0000000000000000

$$L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$$

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\mathbf{a} s = 1$$

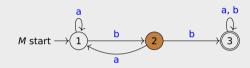
$$6F = \{3\}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Deterministic Finite State Automata

0000000000000000

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\mathbf{A} s = 1$$

$$F = \{3\}$$

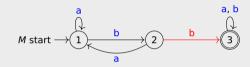
$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Deterministic Finite State Automata

0000000000000000

$$L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$$

$$M = (Q, \Sigma, \delta, s, F)$$



Deterministic Finite State Automata

0000000000000000

$$\bigcirc Q = \{1, 2, 3\}$$

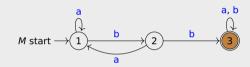
$$\Sigma = \{a, b\}$$

$$\mathbf{A} s = 1$$

$$F = \{3\}$$

$$L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$$

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\Omega = 1$$

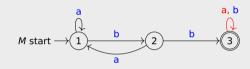
**4** 
$$s = 1$$
 **5**  $F = \{3\}$ 

$$L(M)$$
:

Deterministic Finite State Automata

0000000000000000

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\mathbf{A} s = 1$$

$$F = \{3\}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

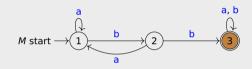
$$I(M) := \{x \mid x \text{ cont}\}$$

Deterministic Finite State Automata

0000000000000000

$$L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$$

$$M = (Q, \Sigma, \delta, s, F)$$



Deterministic Finite State Automata

0000000000000000

$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

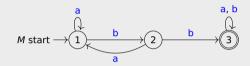
$$\delta: Q \times \Sigma \to Q$$

$$\mathbf{A} s = 1$$

$$6F = \{3\}$$

$$L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$$

$$M = (Q, \Sigma, \delta, s, F)$$



2

$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\mathbf{A} s = \mathbf{1}$$

$$GF = \{3\}$$

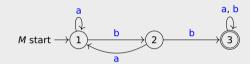
3

Deterministic Finite State Automata

0000000000000000

$$L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$$





$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

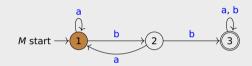
$$6F = \{3\}$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

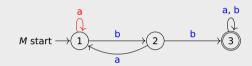
$$F = \{3\}$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

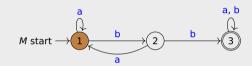
$$F = \{3\}$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

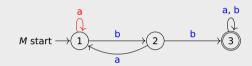
$$F = \{3\}$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

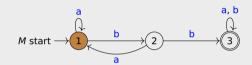
$$F = \{3\}$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 





$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

$$\mathbf{A} s = 1$$

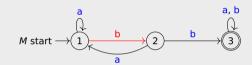
$$6F = \{3\}$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



① 
$$Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

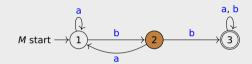
$$5 = 1$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

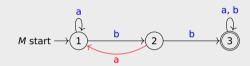
$$F = \{3\}$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 





$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

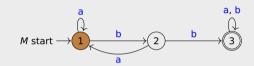
$$5 = 1$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



① 
$$Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

$$F = \{3\}$$

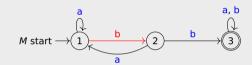
Deterministic Finite State Automata

0000000000000000



 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



① 
$$Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\mathbf{a} s = 1$$

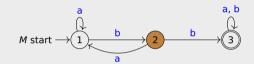
$$5 = 1$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

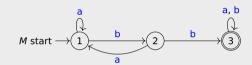
$$F = \{3\}$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

$$M = (Q, \Sigma, \delta, s, F)$$



$$\bigcirc Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b\}$$

$$\bullet$$
  $\delta: Q \times \Sigma \rightarrow Q$ 

$$F = \{3\}$$

a a b a b 
$$\notin L(M)$$

Deterministic Finite State Automata

0000000000000000

 $L(M) := \{x \mid x \text{ contains } bb \text{ as substring } \}$ 

# Outline

A Ouick Recap

- 1 A Quick Recap
- 2 Chomsky Hierarchy
- 4 Closure Properties

A Ouick Recap

regular sets are effectively closed under intersection

A Ouick Recap

regular sets are effectively closed under intersection

$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$   
 $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ 

A Ouick Recap

regular sets are effectively closed under intersection

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$ 

• 
$$A \cap B := L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, S_3, F_3)$ 

A Ouick Recap

regular sets are effectively closed under intersection

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$ 

• 
$$A \cap B := L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

A Ouick Recap

regular sets are effectively closed under intersection

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$ 

• 
$$A \cap B := L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$\bigcirc Q_3 := Q_1 \times Q_2 = \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$$

$$\bigcirc F_3 := F_1 \times F_2$$

A Ouick Recap

regular sets are effectively closed under intersection

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$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$ 

• 
$$A \cap B := L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$s_3 := (s_1, s_2)$$

regular sets are effectively closed under intersection

### Proof. (closure under intersection)

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$ 

• 
$$A \cap B := L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$\bigcirc O_3 := O_1 \times O_2 = \{(p, q) \mid p \in O_1 \text{ and } q \in O_2\}$$

$$\bigcirc F_3 := F_1 \times F_2$$

(a) 
$$S_3$$
 :=  $(S_1, S_2)$   
(b)  $S_3((p, q), a)$  :=  $(\delta_1(p, a), \delta_2(q, a))$ 

$$\forall p \in Q_1, \forall q \in Q_2, \forall a \in \Sigma$$

regular sets are effectively closed under intersection

### Proof. (closure under intersection)

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$ 

• 
$$A \cap B := L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$\bigcirc Q_3 := Q_1 \times Q_2 = \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$$

$$\bigcirc F_3 := F_1 \times F_2$$

$$s_3 := (s_1, s_2)$$

$$\textcircled{4} \ \delta_3((p,q),a) \qquad := \quad (\delta_1(p,a),\delta_2(q,a)) \qquad \forall p \in Q_1, \ \forall q \in Q_2, \ \forall a \in \Sigma$$

$$\forall p \in Q_1, \forall q \in Q_2, \forall a \in \Sigma$$

claim: 
$$\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$$

A Ouick Recap

regular sets are effectively closed under intersection

### Proof. (closure under intersection)

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ 

• 
$$A \cap B := L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$\bigcirc Q_3 := Q_1 \times Q_2 = \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$$

$$\bigcirc F_3 := F_1 \times F_2$$

$$\textcircled{4} \ \delta_3((p,q),a) := (\delta_1(p,a),\delta_2(q,a)) \qquad \forall p \in Q_1, \forall q \in Q_2, \forall a \in \Sigma$$

claim: 
$$\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$$
proof: induction on |x| next slide

# proof of the claim

A Ouick Recap

claim:  $\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$ 

claim:  $\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$ 

• base case: |x| = 0 thus  $x = \varepsilon$ 

$$\widehat{\delta_3}((p,q),\varepsilon)=(p,q)=(\widehat{\delta_1}(p,\varepsilon),\widehat{\delta_2}(q,\varepsilon))$$

 $\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x))$  $\forall x \in \Sigma^*$ claim:

• base case: |x| = 0 thus  $x = \varepsilon$ 

$$\widehat{\delta_3}((p,q),\varepsilon)=(p,q)=(\widehat{\delta_1}(p,\varepsilon),\widehat{\delta_2}(q,\varepsilon))$$

Deterministic Finite State Automata

• step case: |x| > 0 thus x = ya s.t. |y| = |x| - 1 with  $H : \widehat{\delta_3}((p,q),y) = (\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y))$ 

claim:  $\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x))$  $\forall x \in \Sigma^*$ 

• base case: |x| = 0 thus  $x = \varepsilon$ 

$$\widehat{\delta_3}((p,q),\varepsilon)=(p,q)=(\widehat{\delta_1}(p,\varepsilon),\widehat{\delta_2}(q,\varepsilon))$$

Deterministic Finite State Automata

• step case: |x| > 0 thus x = ya s.t. |y| = |x| - 1 with  $IH : \widehat{\delta_3}((p,q), y) = (\widehat{\delta_1}(p,y), \widehat{\delta_2}(q,y)))$ 

$$\widehat{\delta_3}((p,q),ya) = \delta_3(\widehat{\delta_3}((p,q),y),a)$$
 (by definition of  $\widehat{\delta_3}$ )

claim: 
$$\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$$

• base case: |x| = 0 thus  $x = \varepsilon$ 

$$\widehat{\delta_3}((p,q),\varepsilon)=(p,q)=(\widehat{\delta_1}(p,\varepsilon),\widehat{\delta_2}(q,\varepsilon))$$

Deterministic Finite State Automata

• step case: |x| > 0 thus x = ya s.t. |y| = |x| - 1 with  $H : \widehat{\delta_3}((p,q),y) = (\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y))$ 

$$\widehat{\delta_3}((p,q),ya) = \delta_3(\widehat{\delta_3}((p,q),y),a)$$
 (by definition of  $\widehat{\delta_3}$ )
$$= \delta_3((\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y)),a)$$
 (by induction hypothesis IH)

 $\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x))$  $\forall x \in \Sigma^*$ 

• base case: |x| = 0 thus  $x = \varepsilon$ 

$$\widehat{\delta_3}((p,q),\varepsilon)=(p,q)=(\widehat{\delta_1}(p,\varepsilon),\widehat{\delta_2}(q,\varepsilon))$$

Deterministic Finite State Automata

• step case: |x| > 0 thus x = ya s.t. |y| = |x| - 1 with  $H: \widehat{\delta_3}((p,q),y) = (\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y))$ 

$$\widehat{\delta_3}((p,q),ya) = \delta_3(\widehat{\delta_3}((p,q),y),a)$$
 (by definition of  $\widehat{\delta_3}$ )
$$= \delta_3((\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y)),a)$$
 (by induction hypothesis IH)
$$= (\delta_1(\widehat{\delta_1}(p,y),a),\delta_2(\widehat{\delta_2}(q,y),a))$$
 (by definition of  $\delta_3$ )

claim: 
$$\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$$

• base case: |x| = 0 thus  $x = \varepsilon$ 

$$\widehat{\delta_3}((p,q),\varepsilon)=(p,q)=(\widehat{\delta_1}(p,\varepsilon),\widehat{\delta_2}(q,\varepsilon))$$

Deterministic Finite State Automata

• step case: |x| > 0 thus x = ya s.t. |y| = |x| - 1 with  $IH : \widehat{\delta_3}((p,q),y) = (\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y)))$ 

$$\begin{array}{lll} \widehat{\delta_3}((p,q),ya) & = & \delta_3(\widehat{\delta_3}((p,q),y),a) & \text{(by definition of } \widehat{\delta_3}) \\ & = & \delta_3((\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y)),a) & \text{(by induction hypothesis IH)} \\ & = & (\delta_1(\widehat{\delta_1}(p,y),a),\delta_2(\widehat{\delta_2}(q,y),a)) & \text{(by definition of } \widehat{\delta_3}) \\ & = & (\widehat{\delta_1}(p,ya),\widehat{\delta_2}(q,ya)) & \text{(by definitions of } \widehat{\delta_1} \text{ and } \widehat{\delta_2}) \end{array}$$

claim: 
$$\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x)) \quad \forall x \in \Sigma^*$$

• base case: |x| = 0 thus  $x = \varepsilon$ 

$$\widehat{\delta_3}((p,q),\varepsilon)=(p,q)=(\widehat{\delta_1}(p,\varepsilon),\widehat{\delta_2}(q,\varepsilon))$$

Deterministic Finite State Automata

• step case: |x| > 0 thus x = ya s.t. |y| = |x| - 1 with  $IH : \widehat{\delta_3}((p,q),y) = (\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y)))$ 

$$\begin{array}{lll} \widehat{\delta_3}((p,q),ya) & = & \delta_3(\widehat{\delta_3}((p,q),y),a) & \text{(by definition of } \widehat{\delta_3}) \\ & = & \delta_3((\widehat{\delta_1}(p,y),\widehat{\delta_2}(q,y)),a) & \text{(by induction hypothesis IH)} \\ & = & (\delta_1(\widehat{\delta_1}(p,y),a),\delta_2(\widehat{\delta_2}(q,y),a)) & \text{(by definition of } \delta_3) \\ & = & (\widehat{\delta_1}(p,ya),\widehat{\delta_2}(q,ya)) & \text{(by definitions of } \widehat{\delta_1} \text{ and } \widehat{\delta_2}) \\ & = & (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x)) & \text{(by definitions of } \widehat{\delta_1} \text{ and } \widehat{\delta_2}) \end{array}$$

Proof. (closure under intersection (cont'd))

statement:  $L(M_3) = L(M_1) \cap L(M_2)$ 

A Ouick Recap

# Proof. (closure under intersection (cont'd))

statement:  $L(M_3) = L(M_1) \cap L(M_2)$ 

A Ouick Recap

$$\forall x \in \Sigma^*, x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3$$

(by definition of acceptance)

statement:  $L(M_3) = L(M_1) \cap L(M_2)$ 

A Ouick Recap

$$\forall x \in \Sigma^*, x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3$$

$$\iff$$
  $\widehat{\delta_3}((s_1,s_2),x) \in F_1 \times F_2$ 

(by definition of acceptance) (by definition of  $s_3$  and  $F_3$ )

statement:  $L(M_3) = L(M_1) \cap L(M_2)$ 

A Ouick Recap

$$\forall x \in \Sigma^*, x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3$$

$$\iff$$
  $\widehat{\delta_3}((s_1, s_2), x) \in F_1 \times F_2$ 

$$\iff$$
  $(\widehat{\delta_1}(s_1,x),\widehat{\delta_2}(s_2,x)) \in F_1 \times F_2$ 

(by definition of acceptance)

Deterministic Finite State Automata

(by definition of  $s_3$  and  $F_3$ ) (by claim proven in slide 21) statement:  $L(M_3) = L(M_1) \cap L(M_2)$ 

A Ouick Recap

$$\forall x \in \Sigma^*, x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3$$

$$\iff \widehat{\delta_3}((s_1,s_2),x) \in F_1 \times F_2$$

$$\iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in F_1 \times F_2$$

$$\iff$$
  $\widehat{\delta_1}(s_1, x) \in F_1 \text{ and } \widehat{\delta_2}(s_2, x) \in F_2$ 

(by definition of acceptance)

Deterministic Finite State Automata

(by definition of  $s_3$  and  $F_3$ )

(by claim proven in slide 21) (by definition of product) statement:  $L(M_3) = L(M_1) \cap L(M_2)$ 

A Ouick Recap

$$\forall x \in \Sigma^*, x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3$$

$$\iff \widehat{\delta_3}((s_1, s_2), x) \in F_1 \times F_2$$

$$\iff (\widehat{\delta_1}(s_1,x),\widehat{\delta_2}(s_2,x)) \in F_1 \times F_2$$

$$\iff$$
  $\widehat{\delta_1}(s_1, x) \in F_1 \text{ and } \widehat{\delta_2}(s_2, x) \in F_2$ 

$$\iff$$
  $x \in L(M_1)$  and  $x \in L(M_2)$ 

(by definition of acceptance)

Deterministic Finite State Automata

(by definition of  $s_3$  and  $F_3$ )

(by claim proven in slide 21)

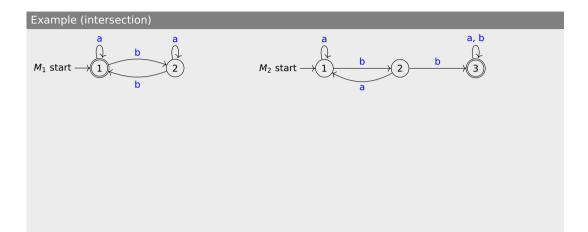
(by definition of product)

(by definition of acceptance)

statement: 
$$L(M_3) = L(M_1) \cap L(M_2)$$

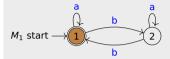
$$\begin{array}{ll} \forall x \in \Sigma^*, \, x \in L(M_3) & \iff & \widehat{\delta_3}(s_3, x) \in F_3 \\ & \iff & \widehat{\delta_3}((s_1, s_2), x) \in F_1 \times F_2 \\ & \iff & (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in F_1 \times F_2 \\ & \iff & \widehat{\delta_1}(s_1, x) \in F_1 \text{ and } \widehat{\delta_2}(s_2, x) \in F_2 \\ & \iff & x \in L(M_1) \text{ and } x \in L(M_2) \\ & \iff & x \in L(M_1) \cap L(M_2) \end{array}$$

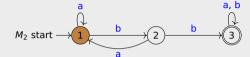
(by definition of acceptance) (by definition of  $s_3$  and  $F_3$ ) (by claim proven in slide 21) (by definition of product) (by definition of acceptance) (by definition of intersection)



 $L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring}\}$ 

# Example (intersection)

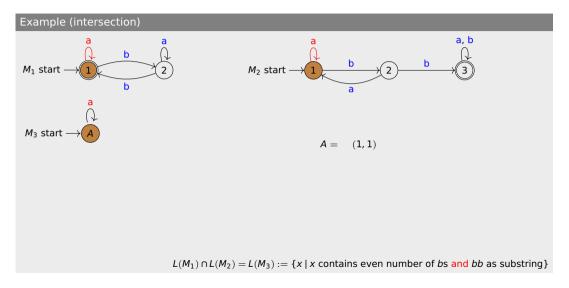


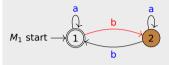


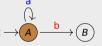
 $M_3$  start  $\longrightarrow$  A

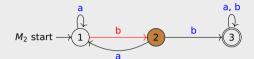
$$A = (1, 1)$$

 $L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring}\}$ 





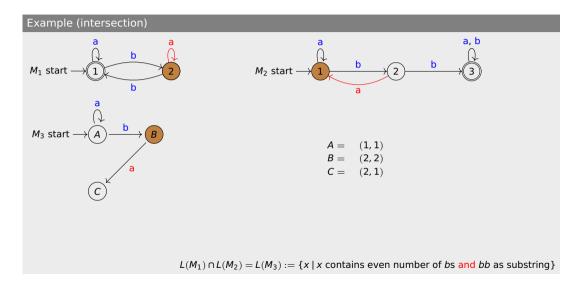


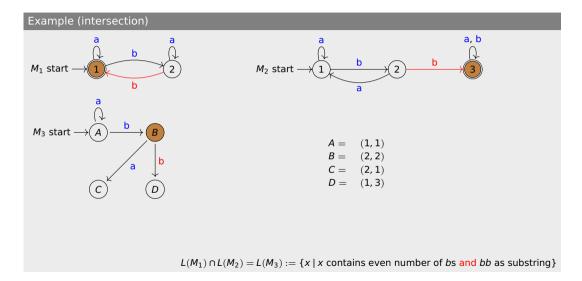


Deterministic Finite State Automata

$$A = (1, 1)$$
  
 $B = (2, 2)$ 

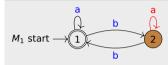
 $L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring}\}$ 

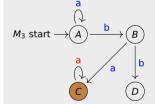


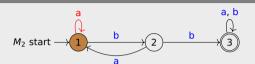


## Example (intersection)

A Ouick Recap







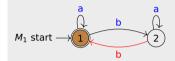
$$A = (1, 1)$$
  
 $B = (2, 2)$ 

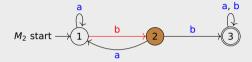
$$B = (2, 2)$$

$$C = (2, 1)$$

$$D = (1, 3)$$

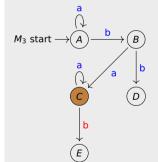
 $L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring}\}$ 





(1, 1)

Deterministic Finite State Automata

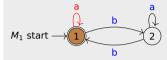


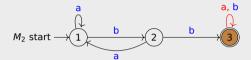
$$B = (2, 2)$$
  
 $C = (2, 1)$ 

$$D = (1,3)$$

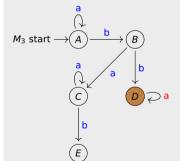
$$D = (1,3)$$
$$E = (1,2)$$

 $L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring} \}$ 





Deterministic Finite State Automata

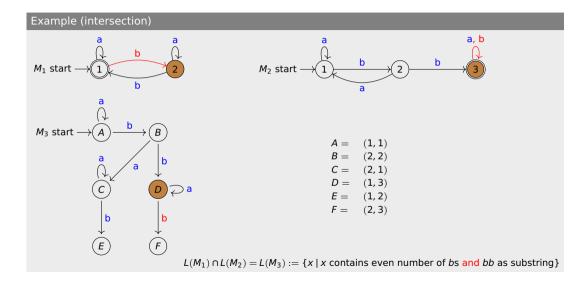


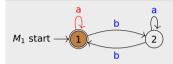
$$A = (1, 1)$$
  
 $B = (2, 2)$   
 $C = (2, 1)$ 

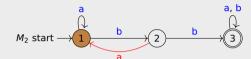
$$D = (1,3)$$

$$E = (1, 2)$$

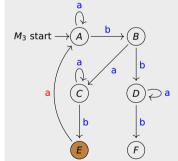
 $L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring} \}$ 







Deterministic Finite State Automata



$$A = (1, 1)$$

$$B = (2, 2)$$

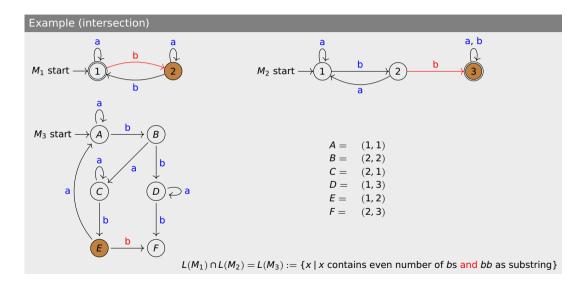
$$C = (2, 1)$$

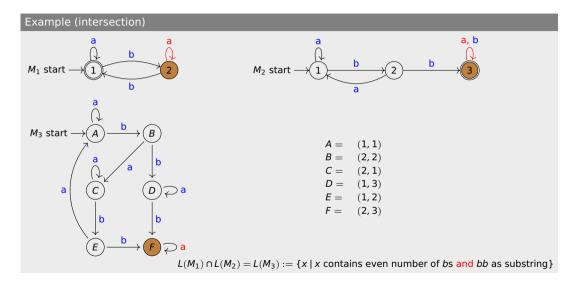
$$D = (1, 3)$$

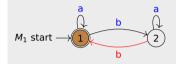
$$E = (1, 2)$$

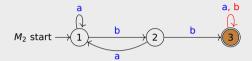
$$F = (2, 3)$$

 $L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring} \}$ 

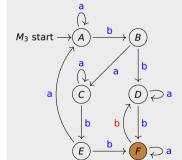








Deterministic Finite State Automata



$$A = (1, 1)$$

$$B = (2, 2)$$

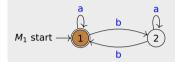
$$C = (2, 1)$$

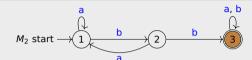
$$D = (1, 3)$$

$$E = (1, 2)$$

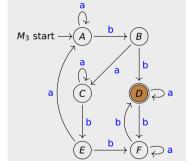
$$F = (2, 3)$$

 $L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring} \}$ 





Deterministic Finite State Automata



$$A = (1, 1)$$

$$B = (2, 2)$$

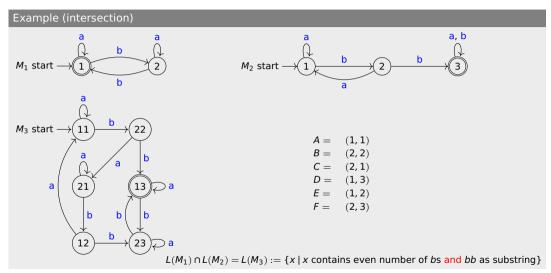
$$C = (2, 1)$$

$$D = (1, 3)$$

$$E = (1, 2)$$

$$F = (2, 3)$$

 $L(M_1) \cap L(M_2) = L(M_3) := \{x \mid x \text{ contains even number of } bs \text{ and } bb \text{ as substring} \}$ 



A Quick Recap

regular sets are effectively closed under complement

A Ouick Recap

regular sets are effectively closed under complement

### Proof. (closure under complement)

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ 

A Ouick Recap

regular sets are effectively closed under complement

- $A = L(M_1)$  for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- $\sim A := \Sigma^* A$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$

A Ouick Recap

regular sets are effectively closed under complement

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ 

• 
$$\sim A := \Sigma^* - A$$
 for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ 

A Ouick Recap

regular sets are effectively closed under complement

- $A = L(M_1)$  for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
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A Ouick Recap

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A Ouick Recap

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- $A = L(M_1)$  for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- $\sim A := \Sigma^* A$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ 

  - $\bigcirc s_2 := s_1$
  - $\emptyset \delta_2(p,a) := \delta_1(p,a) \quad \forall p \in Q_1, \forall a \in \Sigma$

A Ouick Recap

regular sets are effectively closed under complement

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ 

• 
$$\sim A := \Sigma^* - A$$
 for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ 

$$Q F_2 := Q_1 - F_1$$

$$s_2 := s_1$$

• obvious claim: 
$$\widehat{\delta_2}(p, x) = \widehat{\delta_1}(p, x) \quad \forall x \in \Sigma^*$$

# Proof. (closure under complement (cont'd))

statement:  $L(M_2) = \Sigma^* - L(M_1)$ 

A Ouick Recap

statement:  $L(M_2) = \Sigma^* - L(M_1)$ 

A Ouick Recap

$$\forall x \in \Sigma^*, x \in L(M_2) \iff \widehat{\delta_2}(s_2, x) \in F_2$$

(by definition of acceptance)

statement: 
$$L(M_2) = \Sigma^* - L(M_1)$$

$$\forall x \in \Sigma^*, \, x \in L(M_2) \quad \iff \quad \widehat{\delta_2}(s_2, x) \in F_2 \\ \iff \quad \widehat{\delta_1}(s_2, x) \in F_2$$

(by definition of acceptance) (by the obvious claim in slide 24)

statement:  $L(M_2) = \Sigma^* - L(M_1)$ 

A Ouick Recap

$$\begin{array}{ll} \forall x \in \Sigma^*, \, x \in L(M_2) & \Longleftrightarrow & \widehat{\delta_2}(s_2, x) \in F_2 \\ & \Longleftrightarrow & \widehat{\delta_1}(s_2, x) \in F_2 \\ & \Longleftrightarrow & \widehat{\delta_1}(s_1, x) \in Q_1 - F_1 \end{array}$$

(by definition of acceptance) (by the obvious claim in slide 24) (by definitions of  $s_2$  and  $F_2$ )

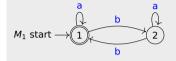
statement: 
$$L(M_2) = \Sigma^* - L(M_1)$$

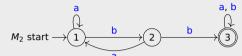
$$\forall x \in \Sigma^*, x \in L(M_2) \iff \widehat{\delta_2}(s_2, x) \in F_2 \qquad \text{(by definition of acceptance)} \\ \iff \widehat{\delta_1}(s_2, x) \in F_2 \qquad \text{(by the obvious claim in slide 24)} \\ \iff \widehat{\delta_1}(s_1, x) \in Q_1 - F_1 \qquad \text{(by definition of } s_2 \text{ and } F_2) \\ \iff \widehat{\delta_1}(s_1, x) \in Q_1 \text{ and } \widehat{\delta_1}(s_1, x) \notin F_1 \qquad \text{(by definition of set difference)}$$

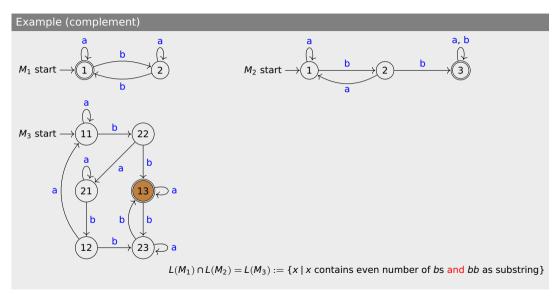
statement: 
$$L(M_2) = \Sigma^* - L(M_1)$$

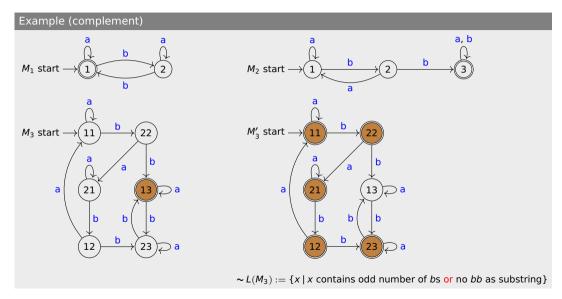
$$\begin{array}{ll} \forall x \in \Sigma^*, \, x \in L(M_2) & \iff & \widehat{\delta_2}(s_2, x) \in F_2 \\ & \iff & \widehat{\delta_1}(s_2, x) \in F_2 \\ & \iff & \widehat{\delta_1}(s_1, x) \in Q_1 - F_1 \\ & \iff & \widehat{\delta_1}(s_1, x) \in Q_1 \text{ and } \widehat{\delta_1}(s_1, x) \notin F_1 \\ & \iff & x \notin L(M_1) \end{array}$$

(by definition of acceptance) (by the obvious claim in slide 24) (by definitions of  $s_2$  and  $F_2$ ) (by definition of set difference) (by definition of acceptance)









### Theoren

A Quick Recap

regular sets are effectively closed under union

### Theoren

A Ouick Recap

regular sets are effectively closed under union

# Proof. (closure under union)

 $A \cup B = \sim ((\sim A) \cap (\sim B))$ 

A Quick Recap

regular sets are effectively closed under union

A Ouick Recap

regular sets are effectively closed under union

$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ 

$$B = L(M_2)$$
 for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ 

A Ouick Recap

regular sets are effectively closed under union

$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ 

$$B = L(M_2)$$
 for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ 

• 
$$A \cup B = L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

A Ouick Recap

regular sets are effectively closed under union

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$ 

• 
$$A \cup B = L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$\bigcirc Q_3 = Q_1 \times Q_2 := \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$$

A Ouick Recap

regular sets are effectively closed under union

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ 

• 
$$A \cup B = L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$\bigcirc Q_3 = Q_1 \times Q_2 := \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$$

A Ouick Recap

regular sets are effectively closed under union

- $A = L(M_1)$  for DFA  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$
- $A \cup B = L(M_3)$  for DFA  $M_3 = (Q_3, \Sigma, \delta_3, S_3, F_3)$ 
  - $\bigcirc Q_3 = Q_1 \times Q_2 := \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$

regular sets are effectively closed under union

### Proof. (closure under union – explicit construction)

```
• A = L(M_1) for DFA M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)
• B = L(M_2) for DFA M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)
```

• 
$$A \cup B = L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$\bigcirc O_3 = O_1 \times O_2 := \{(p,q) \mid p \in O_1 \text{ and } q \in O_2\}$$

§ 
$$s_3 := (s_1, s_2)$$

$$\forall p \in Q_1, \forall q \in Q_2, \forall a \in \Sigma$$

regular sets are effectively closed under union

### Proof. (closure under union – explicit construction)

• 
$$A = L(M_1)$$
 for DFA  $M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)$   
•  $B = L(M_2)$  for DFA  $M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$ 

• 
$$A \cup B = L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$\bigcirc O_3 = O_1 \times O_2 := \{(p,q) \mid p \in O_1 \text{ and } q \in O_2\}$$

claim: 
$$\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x))$$
  $\forall x \in \Sigma^*$ 

regular sets are effectively closed under union

### Proof. (closure under union – explicit construction)

```
 A = L(M_1) \quad \text{for DFA} \quad M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1) 
 B = L(M_2) \quad \text{for DFA} \quad M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)
```

• 
$$A \cup B = L(M_3)$$
 for DFA  $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$ 

$$\bigcirc Q_3 = Q_1 \times Q_2 := \{(p,q) \mid p \in Q_1 \text{ and } q \in Q_2\}$$

$$s_3 := (s_1, s_2)$$

claim: 
$$\widehat{\delta_3}((p,q),x) = (\widehat{\delta_1}(p,x),\widehat{\delta_2}(q,x))$$
  $\forall x \in \Sigma^*$ 

proof: induction on |x| – skipped (follows exact same steps with that is given at slide #21)

Proof. (closure under union – explicit construction (cont'd))

statement:  $L(M_3) = L(M_1) \cup L(M_2)$ 

A Ouick Recap

statement: 
$$L(M_3) = L(M_1) \cup L(M_2)$$

A Ouick Recap

$$\forall x \in \Sigma^*, \, x \in L(M_3) \quad \Longleftrightarrow \quad \widehat{\delta_3}(s_3, x) \in F_3$$

$$\begin{array}{lll} \text{statement:} & L(M_3) = L(M_1) \cup L(M_2) \\ \forall x \in \Sigma^*, \, x \in L(M_3) & \Longleftrightarrow & \widehat{\delta_3}(s_3, x) \in F_3 \\ & \Longleftrightarrow & \widehat{\delta_3}((s_1, s_2), x) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \end{array}$$

A Ouick Recap

$$\begin{array}{lll} \text{statement:} & L(M_3) = L(M_1) \cup L(M_2) \\ \forall x \in \Sigma^*, \, x \in L(M_3) & \iff & \widehat{\delta_3}(s_3, x) \in F_3 \\ & \iff & \widehat{\delta_3}((s_1, s_2), x) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\ & \iff & (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2) \\ \end{array}$$

```
statement: L(M_3) = L(M_1) \cup L(M_2)
\forall x \in \Sigma^*, x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3
                                         \iff \widehat{\delta_3}((s_1, s_2), x) \in (F_1 \times Q_2) \cup (Q_1 \times F_2)
                                          \iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2)
                                          \iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) or (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (Q_1 \times F_2)
```

```
statement: L(M_3) = L(M_1) \cup L(M_2)
\forall x \in \Sigma^*, x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3
                                            \iff \widehat{\delta_3}((S_1, S_2), X) \in (F_1 \times Q_2) \cup (Q_1 \times F_2)
                                             \iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2)
                                            \iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) or (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (Q_1 \times F_2)
                                            \iff \left(\widehat{\delta_1}(s_1, x) \in F_1 \text{ and } \widehat{\delta_2}(s_2, x) \in Q_2\right) or \left(\widehat{\delta_1}(s_1, x) \in Q_1 \text{ and } \widehat{\delta_2}(s_2, x) \in F_2\right)
```

```
statement: L(M_3) = L(M_1) \cup L(M_2)
\forall x \in \Sigma^*, x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3
                                           \iff \widehat{\delta_3}((S_1, S_2), X) \in (F_1 \times O_2) \cup (O_1 \times F_2)
                                           \iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2)
                                           \iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) or (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (Q_1 \times F_2)
                                           \iff \left(\widehat{\delta_1}(s_1, x) \in F_1 \text{ and } \widehat{\delta_2}(s_2, x) \in Q_2\right) or \left(\widehat{\delta_1}(s_1, x) \in Q_1 \text{ and } \widehat{\delta_2}(s_2, x) \in F_2\right)
                                           \iff x \in L(M_1) or x \in L(M_2)
```

```
statement: L(M_3) = L(M_1) \cup L(M_2)
\forall x \in \Sigma^*, x \in L(M_3) \iff \widehat{\delta_3}(s_3, x) \in F_3
                                          \iff \widehat{\delta_3}((S_1, S_2), X) \in (F_1 \times Q_2) \cup (Q_1 \times F_2)
                                          \iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) \cup (Q_1 \times F_2)
                                          \iff (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (F_1 \times Q_2) or (\widehat{\delta_1}(s_1, x), \widehat{\delta_2}(s_2, x)) \in (Q_1 \times F_2)
                                          \iff \left(\widehat{\delta_1}(s_1, x) \in F_1 \text{ and } \widehat{\delta_2}(s_2, x) \in Q_2\right) or \left(\widehat{\delta_1}(s_1, x) \in Q_1 \text{ and } \widehat{\delta_2}(s_2, x) \in F_2\right)
                                          \iff x \in L(M_1) or x \in L(M_2)
                                          \iff x \in L(M_1) \cup L(M_2)
```

