Quiz III (10 pts)

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Assigned : May the 19th, 20h00
Duration : 60 minutes

Q1. (7 pts) Design a Turing Machine (TM)

$$M = (\{\ldots, \text{halt}, \text{halt-reject}\}, \{a, b\}, \{a, b, \vdash, \_, X\}, \vdash, \_, \delta, s, t, r)$$

which on input $x\#y$ (such that $x, y \in \{a, b\}^\star$) halts with

$$\begin{cases} 1 & \text{if } 2 * \#a(x) > \#a(y) \\ 0 & \text{if } 2 * \#a(x) = \#a(y) \\ 2 & \text{if } 2 * \#a(x) < \#a(y) \end{cases}$$

written on its tape.

Note that $\#a(y)$ denotes the number of as in the string y. Similarly, $2 * \#a(x)$ connotes the double amount of as in the string x. Below are a few examples to the input-output harmony of the intended TM:

Input	Output
$\vdash abbbaa\#aababababaa\#\_^\omega$	$\vdash \cdots\#\cdots\#0$
$\vdash abbababaa\#aababababaa\#\_^\omega$	$\vdash \cdots\#\cdots\#1$
$\vdash abbbaa\#aababaabaa\#\_^\omega$	$\vdash \cdots\#\cdots\#2$
$\vdots$	$\vdots$

Important. Implement the machine M in Morphett's TM simulator, and explain your implementation in a few comment-out lines. Note that TMs designated elsewise will be graded zero.

A1. Turing Machine

$$M = (\{1, 2, 3, 4, 5, 6, 7, \text{halt}, \text{halt-reject}\}, \{a, b\}, \{a, b, \vdash, \_, X\}, \vdash, \_, \delta, 1, \text{halt}, \text{halt-reject})$$

with the below stated transition function δ

```
1 ⊢ ⊢ r 1
1 b b r 1
1 a ⊢ r 2
1 # # r 7
1 * * * halt-reject

2 a a r 2
2 b b r 2
2 # # r 3
2 * * * halt-reject

3 a X r 4
3 b b r 3
3 X X r 3
3 # # r 6
3 * * * halt-reject

4 a X r 5
4 b b r 4
4 # # r 6
4 * * * halt-reject

5 a a l 5
5 b b l 5
5 X X l 5
5 # # l 5
5 ⊢ ⊢ r 1
5 * * * halt-reject

6 _ 1 l halt
6 * * * halt-reject

7 X X r 7
7 b b r 7
7 # # r 7
7 _ 0 l halt
7 a a r 8
7 * * * halt-reject

8 a a r 8
8 b b r 8
8 # # r 8
8 _ 2 l halt
8 * * * halt-reject
```

computes the intended function. Click here to load the code in your browser.

Q2. (3 pts) Prove employing contra-positive of the Pumping Lemma if the set

$$A := \{x^k y^m z^n \mid k \geqslant n \text{ and } m \text{ is even}\}$$

cannot be context free. Otherwise, construct a context-free grammar (CFG) that generates the set A.

A2. The set $A := \{x^k y^m z^n \mid k \geqslant n \text{ and } m \text{ is even}\}$ is context-free as the context-free grammar

$$G = \big(\{S, A\}, \{x, y, z\}, P, S\big)$$

with below set of production rules P generates A:

$S \to xSz \mid xS \mid A$
$A \to yAy \mid \varepsilon$.

Important Notice:

- Collaboration is strictly and positively prohibited; lowers your score to 0 if detected.
- Any submission after 60 minutes will NOT be accepted. Please be aware and respect the deadline!
- Submission policy:
  1. considering **Q1**, first implement a TM in Morphett's TM simulator, then copy-and-paste your code in a text file named **A1.txt**,
  2. as for **Q2**, write your answer down on a piece of paper, scan it into a PDF file named **A2.pdf**,
  3. and then submit both files **A1.txt** and **A2.pdf**.
- Make sure that your handwriting in **A2.pdf** is decent and readable.