Assignment IV (20 pts)

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Assigned : May the 20th, 19h00
Due : May the 27th, 19h00

Q1. (8 pts) Design Turing Machine (TM)

$$M = (\{\cdots, \text{halt}, \text{halt-reject}\}, \{a\}, \{a, b, 0, 1, \vdash, \_, \#, >, +, x\}, \vdash, \_, \delta, s, \text{halt}, \text{halt-reject})$$

that takes $\vdash a^m ? a^n \#$, behaves as

$$\begin{cases} \textit{comparator} & \text{if ? is '>'} \\ \textit{multiplier} & \text{if ? is 'x'} \\ \textit{adder} & \text{if ? is '+'} \end{cases}$$

and halts with

$$\begin{cases} \begin{cases} \cdots \#1 \text{ if m > n} \\ \cdots \#0 \text{ otherwise} \end{cases} & \text{if input string is } \vdash a^m > a^n \# \\ \cdots \#a^{m \times n} & \text{if input string is } \vdash a^m x a^n \# \\ \cdots \#a^{m+n} & \text{if input string is } \vdash a^m + a^n \# \end{cases}$$

written on its tape.

Below are a few examples to the input-output harmony of the intended TM:

Input	Output
$\vdash aaaa > aaaa\#\_^\omega$	$\cdots \#0\_^\omega$
$\vdash aaaaa > aaaa\#\_^\omega$	$\cdots \#1\_^\omega$
$\vdash aaxaaaa\#\_^\omega$	$\cdots \#aaaaaaaa\_^\omega$
$\vdash aaaaa + aa\#\_^\omega$	$\cdots \#aaaaaaa\_^\omega$
$\vdash > aaa\#\_^\omega$	$\cdots \#0\_^\omega$
$\vdash aaa > \#\_^\omega$	$\cdots \#1\_^\omega$
$\vdash aaax\#\_^\omega$	$\cdots \#\_^\omega$
$\vdash xaaaaa\#\_^\omega$	$\cdots \#\_^\omega$
$\vdash aaa + \#\_^\omega$	$\cdots \#aaa\_^\omega$
$\vdash +aaaaa\#\_^\omega$	$\cdots \#aaaaa\_^\omega$
$\vdash aaata + aaaaaaa\_^\omega$	reject
$\vdash aaxaaauaaa\_^\omega$	reject
$\vdots$	$\vdots$

Important. Implement the machine M in Morphett's TM simulator, and explain your implementation in a few comment-out lines. Note that TMs designated elsewise will be graded zero.

Q2. (8 pts) Design a total Turing Machine (TM)

$$M = (\{\cdots, \text{halt-accept}, \text{halt-reject}\}, \{a\}, \{a, b, x, 1, \vdash, \_, \#\}, \vdash, \_, \delta, s, \text{halt-accept}, \text{halt-reject})$$

that accepts the input $\vdash a^n$ if $n = \sum_{i=0}^{m \in \mathbb{N}} i$, and rejects otherwise.

Below are a few examples to the input-output harmony of the intended TM:

Input	Output
$\vdash \_^\omega$	accept
$\vdash a\_^\omega$	accept
$\vdash aaa\_^\omega$	accept
$\vdash aaaaaa\_^\omega$	accept
$\vdash aaaaaaaaaa\_^\omega$	accept
$\vdash aaaaaaaaaaaaaaa\_^\omega$	accept
$\vdash aa\_^\omega$	reject
$\vdash aaaa\_^\omega$	reject
$\vdash aaaaa\_^\omega$	reject
$\vdash aaac\_^\omega$	reject
$\vdots$	$\vdots$

Important. Implement the machine M in Morphett's TM simulator, and explain your implementation in a few comment-out lines. Note that TMs designated elsewise will be graded zero.

Q3. (4 pts) Considering following context free grammar $G = (\{S, A, B, K, U, T, V, W, Y, Z\}, \{a, b\}, P, S)$ with below production rules

$$S \to AV \mid AB \mid SB \mid WY \mid ZV \mid BV \mid ZB \mid BB \mid UU \mid a \mid b$$
$$U \to b \quad V \to SB \quad W \to SU \quad Y \to US \quad Z \to BA \quad T \to UA \quad K \to SA$$
$$A \to TK \mid TA \mid US \mid a \mid b$$

decide employing the Cocke Kasami Younger (CKY) algorithm whether the string "$x = aabab$" belongs to the language $L(G)$.

Important. Recall that CKY algorithm functions on grammars in Chomsky Normal Form (CNF). Therefore make sure before employing the algorithm that G is already in CNF; transform G into an equivalent grammar in CNF, otherwise.

Important Notice:

- Collaboration is strictly and positively prohibited; lowers your score to 0 if detected.
- Any submission after 19h00 on May the 27th will NOT be accepted. Please beware and respect the deadline!
- Submission policy:
  1. considering **Q1** and **Q2**, first implement TMs in Morphett's TM simulator, then copy-and-paste your code in separate text files respectively named **A1.txt** and **A2.txt**,
  2. as for **Q3**, write your answer down on a piece of paper, scan it into a PDF file named **A3.pdf**,
  3. and then submit all files **A1.txt**, **A2.txt** and **A3.pdf** in raw form. Please do not compress files!
- Make sure that your handwriting in **A3.pdf** is decent and readable.