

## Assignment IV (20 pts)

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Assigned : May the 20<sup>th</sup>, 19h00Due : May the 27<sup>th</sup>, 19h00**Q1. (8 pts)** Design Turing Machine (TM)
 $M = (\{\dots, \text{halt}, \text{halt-reject}\}, \{a\}, \{a, b, 0, 1, \vdash, \_, \#, >, +, x\}, \vdash, \_, \delta, s, \text{halt}, \text{halt-reject})$ 
that takes  $\vdash a^m ? a^n \#$ , behaves as

$$\begin{cases} \text{comparator} & \text{if ? is '>'} \\ \text{multiplier} & \text{if ? is 'x'} \\ \text{adder} & \text{if ? is '+'} \end{cases}$$

and halts with

$$\begin{cases} \begin{cases} \dots \# 1 & \text{if } m > n \\ \dots \# 0 & \text{otherwise} \end{cases} & \text{if input string is } \vdash a^m > a^n \# \\ \dots \# a^{m \times n} & \text{if input string is } \vdash a^m x a^n \# \\ \dots \# a^{m+n} & \text{if input string is } \vdash a^m + a^n \# \end{cases}$$

written on its tape.

Below are a few examples to the input-output harmony of the intended TM:

Input	Output
$\vdash aaaa > aaaa \# \_ \omega$	$\dots \# 0 \_ \omega$
$\vdash aaaaa > aaaa \# \_ \omega$	$\dots \# 1 \_ \omega$
$\vdash aaxaaaa \# \_ \omega$	$\dots \# aaaaaaaa \_ \omega$
$\vdash aaaaa + aa \# \_ \omega$	$\dots \# aaaaaaa \_ \omega$
$\vdash > aaa \# \_ \omega$	$\dots \# 0 \_ \omega$
$\vdash aaa > \# \_ \omega$	$\dots \# 1 \_ \omega$
$\vdash aaxx \# \_ \omega$	$\dots \# \_ \omega$
$\vdash xaaaaa \# \_ \omega$	$\dots \# \_ \omega$
$\vdash aaa + \# \_ \omega$	$\dots \# aaa \_ \omega$
$\vdash +aaaaa \# \_ \omega$	$\dots \# aaaaa \_ \omega$
$\vdash aaat a + aaaaaaa \_ \omega$	reject
$\vdash aaxaaa uaaa \_ \omega$	reject
$\vdots$	$\vdots$

**Important.** Implement the machine  $M$  in [Morphett's TM simulator](#), and explain your implementation in a few comment-out lines. Note that TMs designated **elsewise** will be graded **zero**.

**A1. Turing Machine**

$M = (\{\text{prep, backa, backm, backc, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, halt, halt-reject}\}, \{a\}, \{a, b, 0, 1, \vdash, \_, \#\}, \vdash, \_, \delta, \text{prep, halt, halt-reject})$

with transition function  $\delta$  available [here](#) performs as intended.

**Q2. (8 pts) Design a total Turing Machine (TM)**

$M = (\{\dots, \text{halt-accept, halt-reject}\}, \{a\}, \{a, b, x, 1, \vdash, \_, \#\}, \vdash, \_, \delta, s, \text{halt-accept, halt-reject})$

that accepts the input  $\vdash a^n$  if  $n = \sum_{i=0}^{m \in \mathbb{N}} i$ , and rejects otherwise.

Below are a few examples to the input-output harmony of the intended TM:

Input	Output
$\vdash \_^\omega$	accept
$\vdash a \_^\omega$	accept
$\vdash aaa \_^\omega$	accept
$\vdash aaaaaa \_^\omega$	accept
$\vdash aaaaaaaaaa \_^\omega$	accept
$\vdash aaaaaaaaaaaaaa \_^\omega$	accept
$\vdash aa \_^\omega$	reject
$\vdash aaaa \_^\omega$	reject
$\vdash aaaaa \_^\omega$	reject
$\vdash aaac \_^\omega$	reject
$\vdots$	$\vdots$

**Important.** Implement the machine  $M$  in [Morphett's TM simulator](#), and explain your implementation in a few comment-out lines. Note that TMs designated [elsewise](#) will be graded [zero](#).

**A2. Turing Machine**

$M = (\{\text{prep, prep1, back, 1, 2, 3, 4, halt-accept, halt-reject}\}, \{a\}, \{b, x, 1, \vdash, \_, \#\}, \vdash, \_, \delta, \text{prep, halt-accept, halt-reject})$

with transition function  $\delta$  available [here](#) performs as intended.

**Q3. (4 pts)** Considering following context free grammar  $G = (\{S, A, B, K, U, T, V, W, Y, Z\}, \{a, b\}, P, S)$  with below production rules

$S \rightarrow AV \mid AB \mid SB \mid WY \mid ZV \mid BV \mid ZB \mid BB \mid UU \mid a \mid b$   
 $U \rightarrow b \quad V \rightarrow SB \quad W \rightarrow SU \quad Y \rightarrow US \quad Z \rightarrow BA \quad T \rightarrow UA \quad K \rightarrow SA$   
 $A \rightarrow TK \mid TA \mid US \mid a \mid b$

decide employing the [Cocke Kasami Younger \(CKY\)](#) algorithm whether the string " $x = aabab$ " belongs to the language  $L(G)$ .

**Important.** Recall that CKY algorithm functions on grammars in Chomsky Normal Form (CNF). Therefore make sure before employing the algorithm that  $G$  is already in CNF; transform  $G$  into an equivalent grammar in CNF, otherwise.

### A3.

$x = aabab$

$$\begin{aligned} T_{01} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \xrightarrow{*}_G a\} \\ &= \{S, A\} = T_{12} = T_{34} \end{aligned}$$

$x = aabab$

$$\begin{aligned} T_{23} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \xrightarrow{*}_G b\} \\ &= \{S, A, U\} = T_{45} \end{aligned}$$

0					
{S, A}	1				
	{S, A}	2			
		{S, A, U}	3		
			{S, A}	4	
				{S, A, U}	5

$x = aabab$

$$\begin{aligned} T_{02} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{12}\} \\ &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{S, A\} \text{ and } Z \in \{S, A\}\} \\ &= \{K\} \end{aligned}$$

$x = aabab$

$$\begin{aligned} T_{13} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{12} \text{ and } Z \in T_{23}\} \\ &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{S, A\} \text{ and } Z \in \{S, A, U\}\} \\ &= \{K, W\} = T_{35} \end{aligned}$$

$x = aabab$

$$\begin{aligned} T_{24} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{23} \text{ and } Z \in T_{34}\} \\ &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{S, A, U\} \text{ and } Z \in \{S, A\}\} \\ &= \{K, A, T\} \end{aligned}$$

0					
{S, A}	1				
{K}	{S, A}	2			
	{K, W}	{S, A, U}	3		
		{K, A, Y, T}	{S, A}	4	
			{K, W}	{S, A, U}	5

$x = aabab$

$$\begin{aligned}
 T_{03} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{13}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{02} \text{ and } Z \in T_{23}\} \\
 &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{S, A\} \text{ and } Z \in \{K, W\}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{K\} \text{ and } Z \in \{S, A, U\}\} \\
 &= \emptyset
 \end{aligned}$$

$x = aabab$

$$\begin{aligned}
 T_{14} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{12} \text{ and } Z \in T_{24}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{13} \text{ and } Z \in T_{34}\} \\
 &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{S, A\} \text{ and } Z \in \{K, A, T\}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{K, W\} \text{ and } Z \in \{S, A\}\} \\
 &= \{K\}
 \end{aligned}$$

$x = aabab$

$$\begin{aligned}
 T_{25} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{23} \text{ and } Z \in T_{35}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{24} \text{ and } Z \in T_{45}\} \\
 &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{S, A, U\} \text{ and } Z \in \{K, W\}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{K, A, Y, T\} \text{ and } Z \in \{S, A, U\}\} \\
 &= \{A\}
 \end{aligned}$$

0					
{S, A}	1				
{K}	{S, A}	2			
$\emptyset$	{K, W}	{S, A, U}	3		
	{K}	{K, A, Y, T}	{S, A}	4	
		{A}	{K, W}	{S, A, U}	5

$x = aabab$

$$\begin{aligned}
 T_{04} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{14}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{02} \text{ and } Z \in T_{24}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{03} \text{ and } Z \in T_{34}\} \\
 &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{S, A\} \text{ and } Z \in \{K\}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{K\} \text{ and } Z \in \{K, A, Y, T\}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \emptyset \text{ and } Z \in \{S, A\}\} \\
 &= \emptyset
 \end{aligned}$$

$x = aabab$

$$\begin{aligned}
 T_{15} &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{12} \text{ and } Z \in T_{25}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{13} \text{ and } Z \in T_{35}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{14} \text{ and } Z \in T_{45}\} \\
 &= \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{S, A\} \text{ and } Z \in \{A\}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{K, W\} \text{ and } Z \in \{K, W\}\} \cup \\
 &\quad \{X \in \{S, A, B, K, U, T, V, W, Y, Z\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{K\} \text{ and } Z \in \{S, A, U\}\} \\
 &= \{K\}
 \end{aligned}$$

0					
{S, A}	1				
{K}	{S, A}	2			
∅	{K, W}	{S, A, U}	3		
∅	{K}	{K, A, Y, T}	{S, A}	4	
	{K}	{A}	{K, W}	{S, A, U}	5

$x = aabab$

$$\begin{aligned}
 T_{05} &= \{X \in \{S, A, B, C, D, U, T, V, W\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{15}\} \cup \\
 &\quad \{X \in \{S, A, B, C, D, U, T, V, W\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{02} \text{ and } Z \in T_{25}\} \cup \\
 &\quad \{X \in \{S, A, B, C, D, U, T, V, W\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{03} \text{ and } Z \in T_{35}\} \cup \\
 &\quad \{X \in \{S, A, B, C, D, U, T, V, W\} \mid X \rightarrow YZ \in P \text{ with } Y \in T_{04} \text{ and } Z \in T_{45}\} \\
 &= \{X \in \{S, A, B, C, D, U, T, V, W\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{S, A\} \text{ and } Z \in \{K\}\} \cup \\
 &\quad \{X \in \{S, A, B, C, D, U, T, V, W\} \mid X \rightarrow YZ \in P \text{ with } Y \in \{K\} \text{ and } Z \in \{A\}\} \cup \\
 &\quad \{X \in \{S, A, B, C, D, U, T, V, W\} \mid X \rightarrow YZ \in P \text{ with } Y \in \emptyset \text{ and } Z \in \{K, W\}\} \cup \\
 &\quad \{X \in \{S, A, B, C, D, U, T, V, W\} \mid X \rightarrow YZ \in P \text{ with } Y \in \emptyset \text{ and } Z \in \{S, A, U\}\} \\
 &= \emptyset
 \end{aligned}$$

0					
{S, A}	1				
{K}	{S, A}	2			
∅	{K, W}	{S, A, U}	3		
∅	{K}	{K, A, Y, T}	{S, A}	4	
∅	{K}	{A}	{K, W}	{S, A, U}	5

$$S \notin T_{05} \implies x \notin L(G)$$

**Important Notice:**

- Collaboration is strictly and positively prohibited; lowers your score to 0 if detected.
- Any submission after 19h00 on May the 27<sup>th</sup> will NOT be accepted. Please beware and respect the deadline!
- Submission policy:
  1. considering **Q1** and **Q2**, first implement TMs in Morphett's TM simulator, then copy-and-paste your code in separate text files respectively named **A1.txt** and **A2.txt**,
  2. as for **Q3**, write your answer down on a piece of paper, scan it into a PDF file named **A3.pdf**,
  3. and then submit all files **A1.txt**, **A2.txt** and **A3.pdf** in raw form. Please do not compress files!
- Make sure that your handwriting in **A3.pdf** is decent and readable.