CMPE 322/327 - Theory of Computation

Week 7: An Overview for the Midterm

Burak Ekici

April 4-8, 2022

Outline

1. Midterm
2. Finite Automata
3. Regular Expressions

Logistics & Questions

- on April the 10th in between 13h00 – 15h30 (UTC+3 İstanbul Time)

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  - $\text{NFA}_\varepsilon \xrightarrow{\varepsilon\text{–elimination}} \text{NFA} \xrightarrow{\text{subset construction}} \text{DFA} \xrightarrow{\text{minimization}} \text{minimal DFA}$ (with all intermediate steps clearly stated)

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  - $\mathrm{NFA}_\varepsilon \xrightarrow{\varepsilon-\text{elimination}} \mathrm{NFA} \xrightarrow{\text{subset construction}} \mathrm{DFA} \xrightarrow{\text{minimization}}$ minimal DFA (with all intermediate steps clearly stated)
  - $\mathrm{NFA}_\varepsilon \rightarrow$ regular expressions (with all recursive calls to the algorithm (definition) clearly traced and computed)

Topics Covered Thus Far

deterministic finite state machines (DFA), nondeterminism in finite state machines (NFA), closure properties of regular sets, ε-transitions (NFA_ε), homomorphisms, minimization, Myhill-Nerode relations, regular expressions, derivatives, Kleene algebra

Topics Appear in the Midterm

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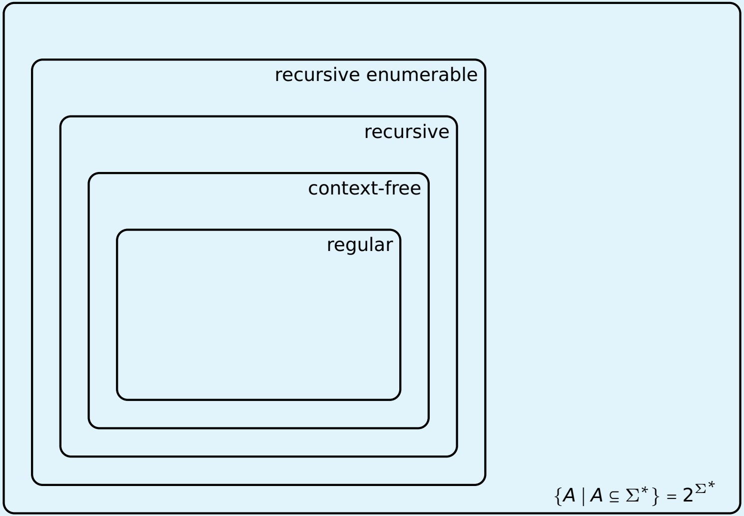
Outline

1 Midterm

2 Finite Automata

3 Regular Expressions


recursive enumerable
recursive
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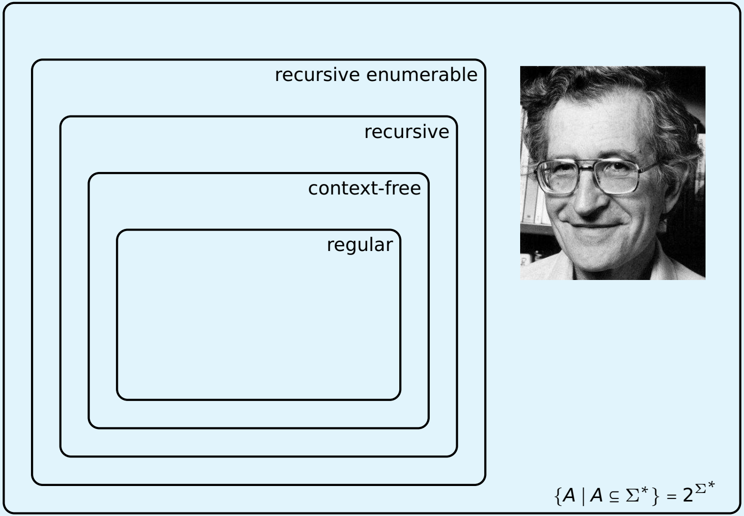
regular
$\{A \mid A \subseteq \Sigma^*\} = 2^{\Sigma^*}$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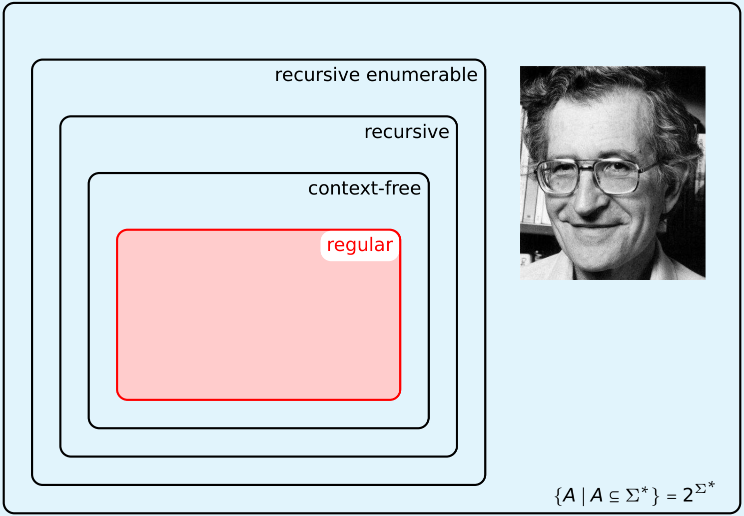
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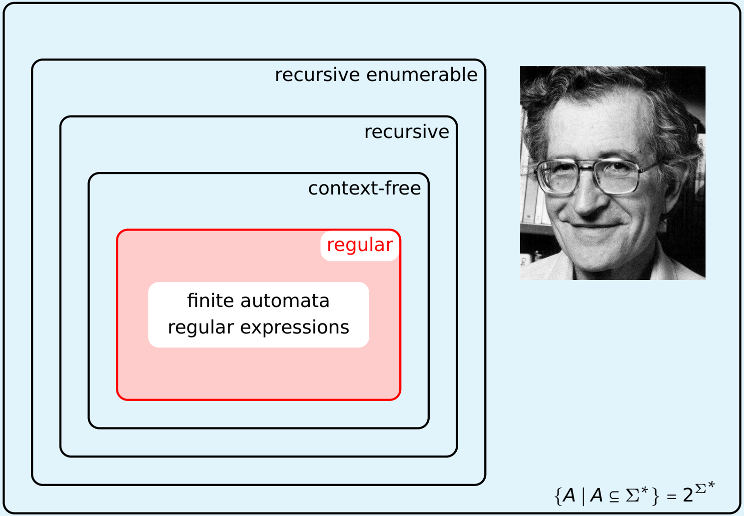
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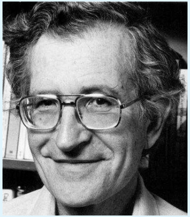

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Topics Appear in the Midterm (Finite Automata)

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Definition

set $A \subseteq \Sigma^*$ is regular if $A = L(M)$ for some DFA M

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Theorem

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  1. $\Delta_2(q, a) \quad := \quad \widehat{\Delta}_1(\{q\}, a) \qquad\qquad \forall q \in Q \ \forall a \in \Sigma$

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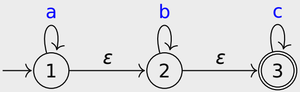
  ➋ $F_2 \quad := \quad \{q \mid C_\varepsilon(\{q\}) \cap F_1 \neq \varnothing\}$

Example (ε-elimination)



Example (ε-elimination)

NFA$_\varepsilon$ $N_1 = (\{1, 2, 3\}, \{a, b, c\}, \varepsilon, \Delta_1, \{1\}, \{3\})$ with

Δ_1	a	b	c	ε
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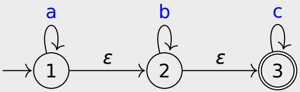

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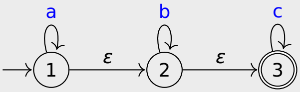

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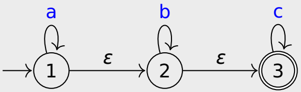

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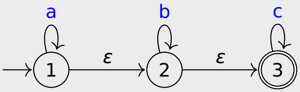

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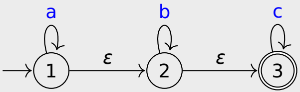

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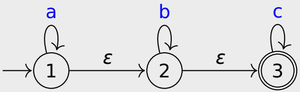

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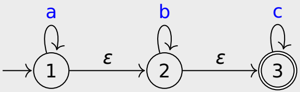

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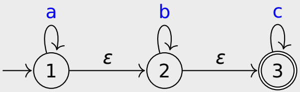

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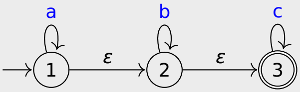

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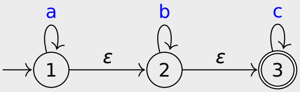

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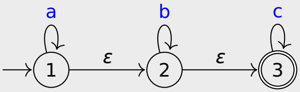

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Example (ε-elimination)

NFA$_\varepsilon$ $N_1 = (\{1,2,3\}, \{a,b,c\}, \varepsilon, \Delta_1, \{1\}, \{3\})$ with

-

Δ_1	a	b	c	ε
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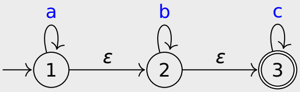

NFA $N_2 = (\{1,2,3\}, \{a,b,c\}, \Delta_2, \{1\}, F_2)$ with

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Δ	a	b	c
1	$\{1,2,3\}$	$\{2{,}3\}$	

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&= \quad C_\varepsilon(\Delta_1(1,b)) \cup C_\varepsilon(\Delta_1(2,b)) \cup C_\varepsilon(\Delta_1(3,b)) \\
&= \quad C_\varepsilon(\varnothing) \cup C_\varepsilon(\{2\}) \cup C_\varepsilon(\varnothing) \\
&= \quad \{2,3\}
\end{aligned}$$

Example (ε-elimination cont'd)

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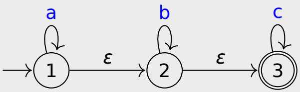

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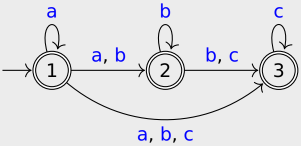

Effective Computation (subset construction)

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Effective Computation (subset construction)

- NFA $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$
- $L(N) = L(M)$ for some DFA $M = (Q_M, \Sigma, \delta_M, s_M, F_M)$ with

Effective Computation (subset construction)

- NFA $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$
- $L(N) = L(M)$ for some DFA $M = (Q_M, \Sigma, \delta_M, s_M, F_M)$ with
  1. $Q_M := 2^{Q_N}$

Effective Computation (subset construction)

- NFA $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$
- $L(N) = L(M)$ for some DFA $M = (Q_M, \Sigma, \delta_M, s_M, F_M)$ with
  1. $Q_M := 2^{Q_N}$
  2. $\delta_M(A, a) := \widehat{\Delta}(A, a) \quad \forall A \subseteq Q_N \quad \forall a \in \Sigma$

Effective Computation (subset construction)

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  3. $s_M := S_N$

Effective Computation (subset construction)

- NFA $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$
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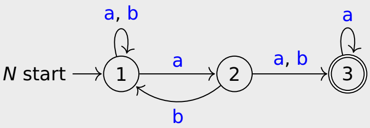
  2. $\delta_M(A, a) := \widehat{\Delta}(A, a) \quad \forall A \subseteq Q_N \ \ \forall a \in \Sigma$
  3. $s_M := S_N$
  4. $F_M := \{A \subseteq Q_N \mid A \cap F_N \neq \varnothing\}$

Example (subset construction)



$A = \varnothing$ $\quad E = \{1, 2\}$

$B = \{1\}$ $\quad F = \{1, 3\}$

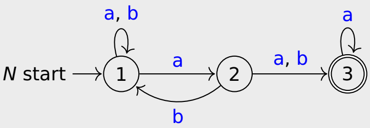
$C = \{2\}$ $\quad G = \{2, 3\}$

$D = \{3\}$ $\quad H = \{1, 2, 3\}$

Δ	a	b

Δ	a	b

Example (subset construction)



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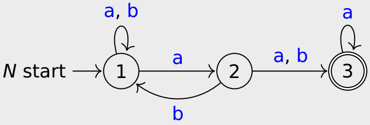
$C = \{2\}$ $\quad$ $G = \{2, 3\}$

$D = \{3\}$ $\quad$ $H = \{1, 2, 3\}$

Δ	a	b
A	A	A

Δ	a	b

Example (subset construction)



$A = \emptyset$ $\quad E = \{1, 2\}$

$B = \{1\}$ $\quad F = \{1, 3\}$

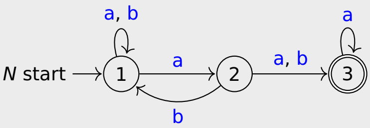
$C = \{2\}$ $\quad G = \{2, 3\}$

$D = \{3\}$ $\quad H = \{1, 2, 3\}$

Δ	a	b
A	A	A
B	E	B

Δ	a	b

Example (subset construction)

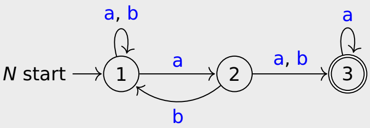


$A = \varnothing$
$B = \{1\}$
$C = \{2\}$
$D = \{3\}$
$E = \{1, 2\}$
$F = \{1, 3\}$
$G = \{2, 3\}$
$H = \{1, 2, 3\}$

Δ	a	b
A	A	A
B	E	B
C	D	F

Δ	a	b

Example (subset construction)



$A = \varnothing$ $\quad E = \{1, 2\}$

$B = \{1\}$ $\quad F = \{1, 3\}$

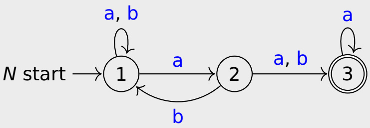
$C = \{2\}$ $\quad G = \{2, 3\}$

$D = \{3\}$ $\quad H = \{1, 2, 3\}$

Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b

Example (subset construction)



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$B = \{1\}$ $\quad F = \{1, 3\}$

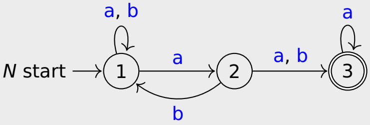
$C = \{2\}$ $\quad G = \{2, 3\}$

$D = \{3\}$ $\quad H = \{1, 2, 3\}$

Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

Example (subset construction)



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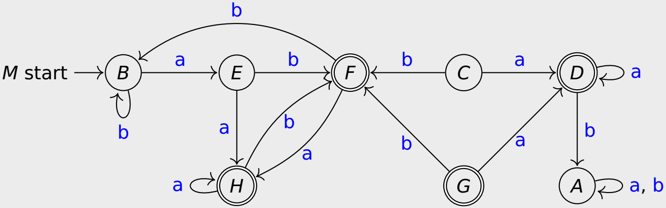
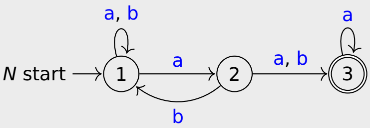
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F



Example (subset construction)



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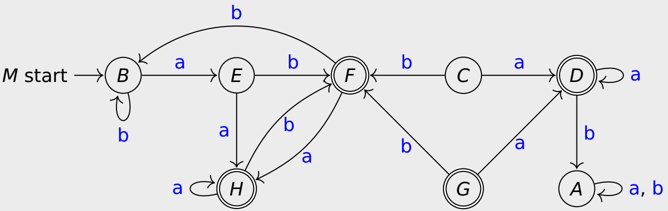
$D = \{3\}$ $\quad$ $H = \{1, 2, 3\}$

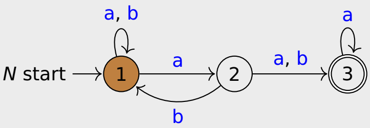
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



$A = \varnothing$ $E = \{1, 2\}$
$B = \{1\}$ $F = \{1, 3\}$
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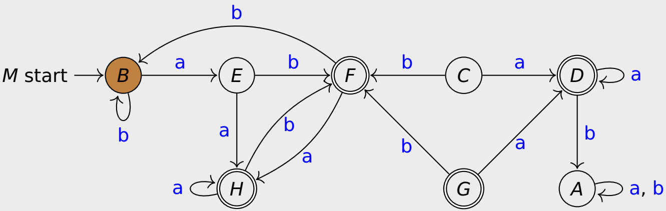
$D = \{3\}$ $H = \{1, 2, 3\}$

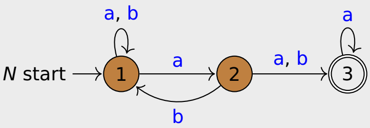
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



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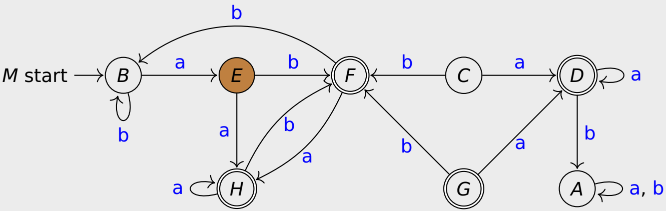
$D = \{3\}$ $\quad$ $H = \{1, 2, 3\}$

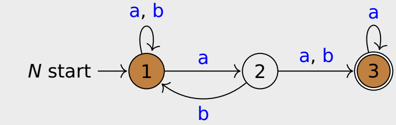
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)

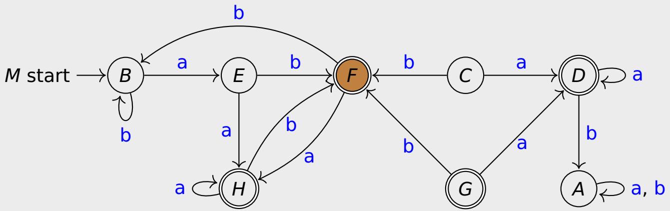


$A = \varnothing$ $E = \{1, 2\}$
$B = \{1\}$ $F = \{1, 3\}$
$C = \{2\}$ $G = \{2, 3\}$
$D = \{3\}$ $H = \{1, 2, 3\}$

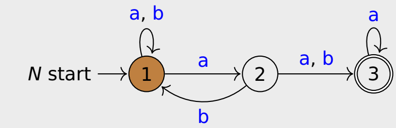
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



$A = \varnothing$ $\quad$ $E = \{1, 2\}$

$B = \{1\}$ $\quad$ $F = \{1, 3\}$

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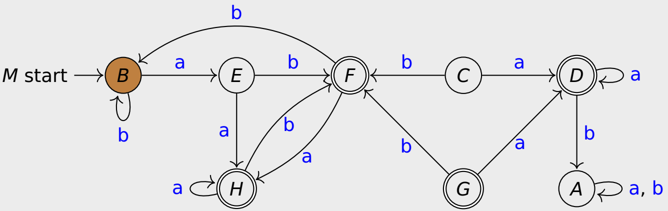
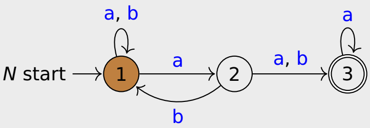
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



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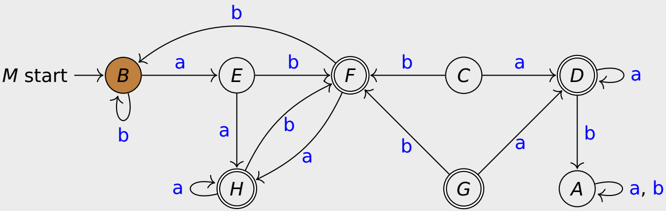
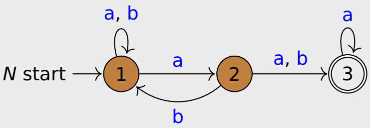
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



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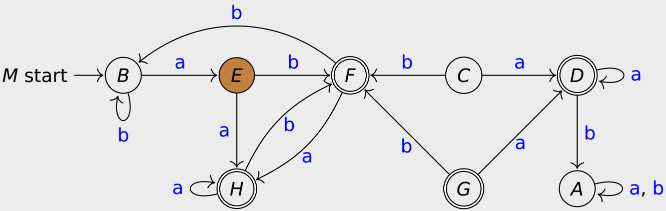
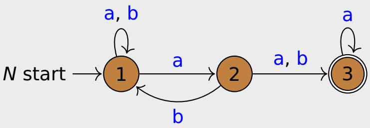
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



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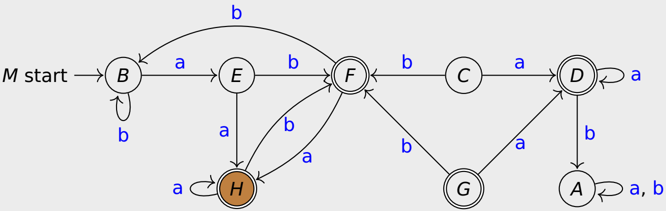
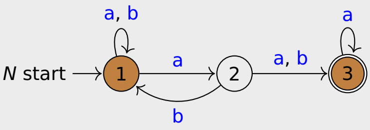
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



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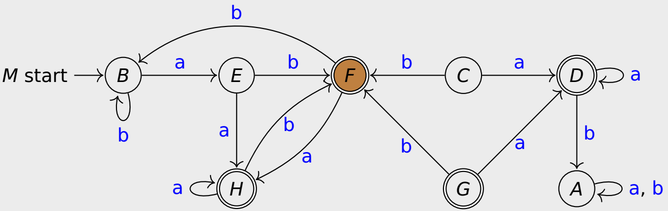
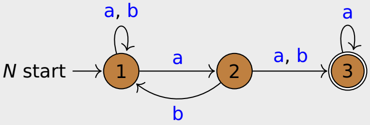
H	H	F

abbbaababbabbbaababba



Example (subset construction)



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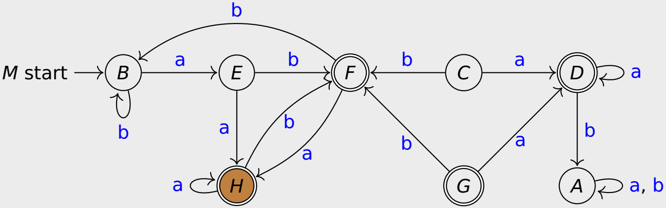
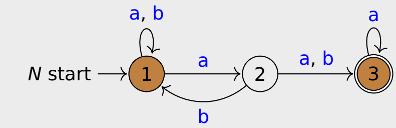
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaabab babbbaababba



Example (subset construction)



$A = \varnothing$ $\quad B = \{1\}$ $\quad C = \{2\}$ $\quad D = \{3\}$

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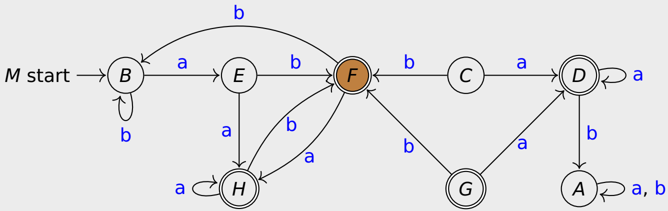
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



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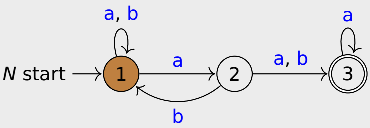

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Δ	a	b
A	A	A
B	E	B
C	D	F
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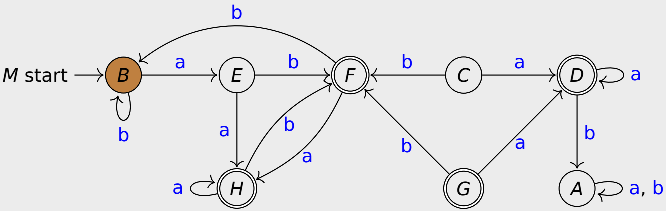
Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



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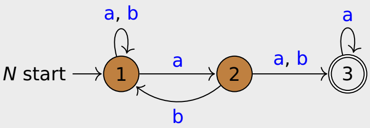

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Δ	a	b
A	A	A
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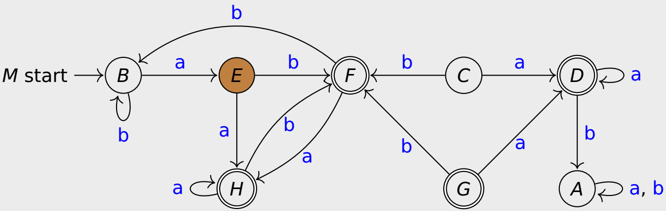
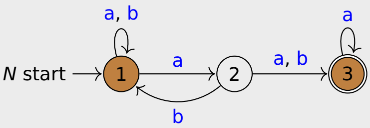
Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



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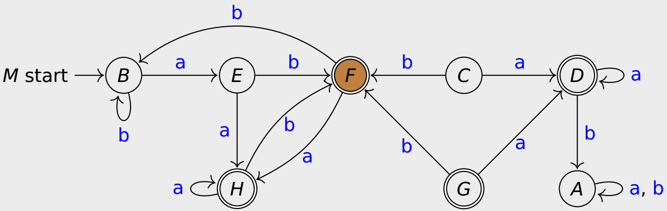
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbbabbbaababba



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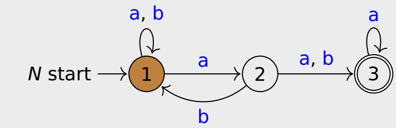

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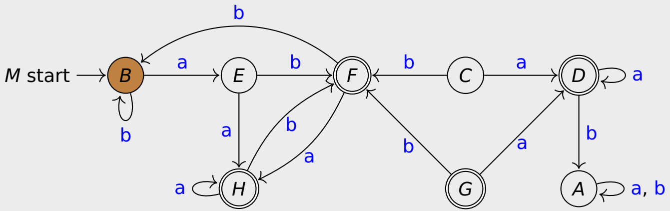
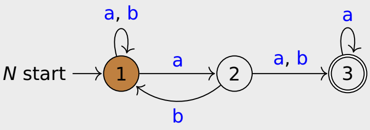
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



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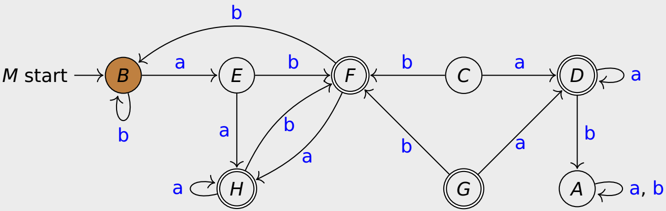
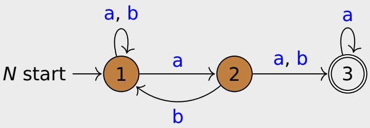
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



$A = \varnothing$ $\quad$ $E = \{1, 2\}$

$B = \{1\}$ $\quad$ $F = \{1, 3\}$

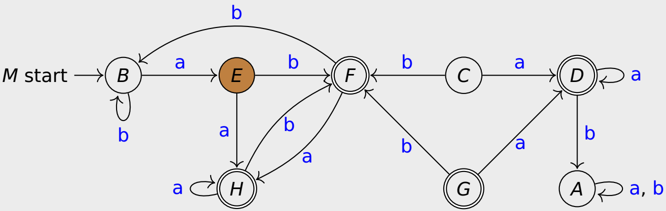
$C = \{2\}$ $\quad$ $G = \{2, 3\}$

$D = \{3\}$ $\quad$ $H = \{1, 2, 3\}$

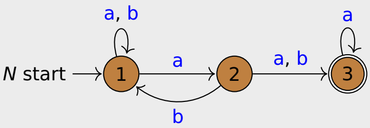
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



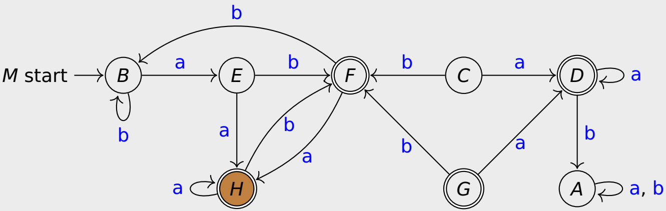
$A = \varnothing$ $B = \{1\}$ $C = \{2\}$ $D = \{3\}$

$E = \{1,2\}$ $F = \{1,3\}$ $G = \{2,3\}$ $H = \{1,2,3\}$

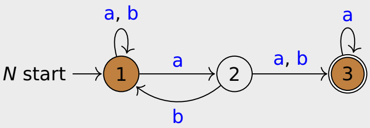
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



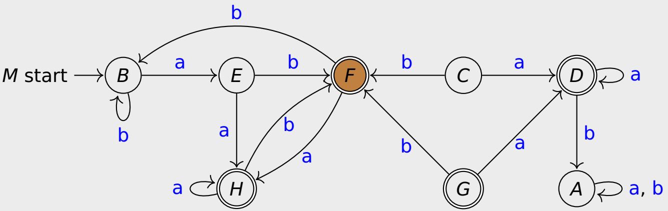
$A = \varnothing$ $\quad B = \{1\}$ $\quad C = \{2\}$ $\quad D = \{3\}$

$E = \{1, 2\}$ $\quad F = \{1, 3\}$ $\quad G = \{2, 3\}$ $\quad H = \{1, 2, 3\}$

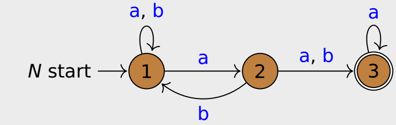
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



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Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

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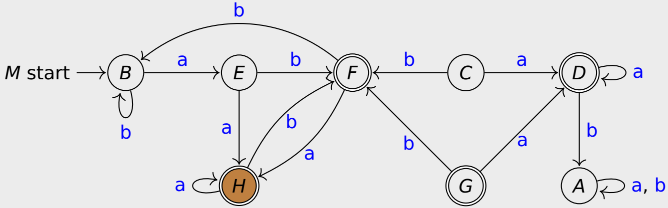
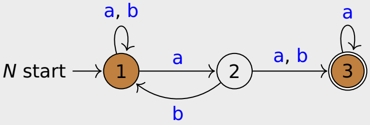


Example (subset construction)



$A = \varnothing$ $\quad$ $E = \{1, 2\}$

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Δ	a	b
A	A	A
B	E	B
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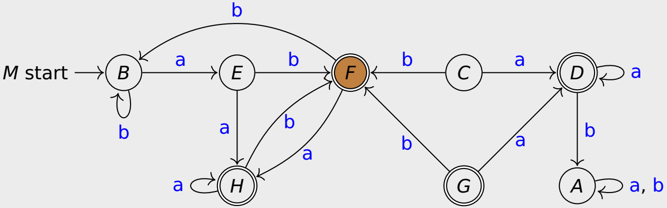
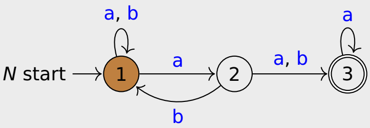
Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababbba



Example (subset construction)



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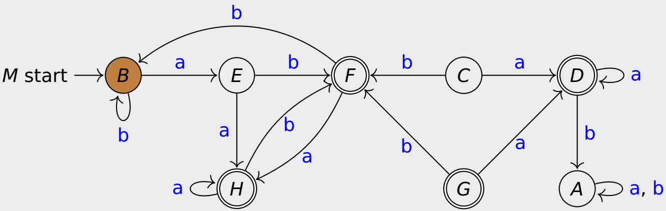
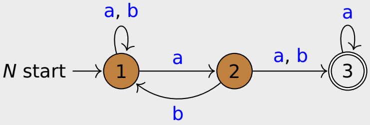
Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

abbbaababbabbbaababba



Example (subset construction)



$A = \varnothing$ $E = \{1, 2\}$
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Δ	a	b
A	A	A
B	E	B
C	D	F
D	D	A

Δ	a	b
E	H	F
F	H	B
G	D	F
H	H	F

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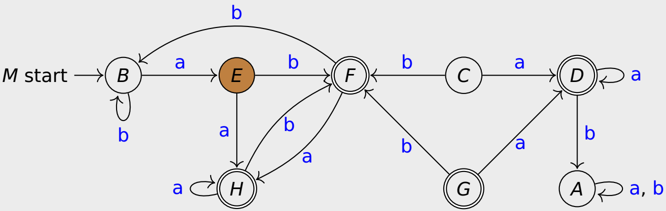
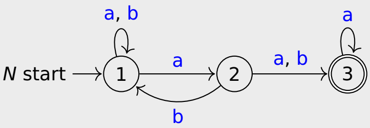


Example (subset construction)



$B = \{1\}$

$E = \{1, 2\}$
$F = \{1, 3\}$

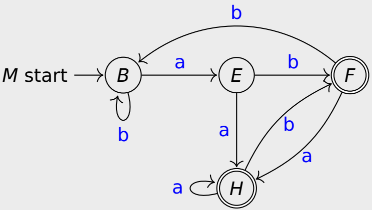
$H = \{1, 2, 3\}$

Δ	a	b
B	E	B

Δ	a	b
E	H	F
F	H	B
H	H	F

abbbaababbabbbaababba

remove inaccessible states



Minimization Algorithm

DFA $M = (Q, \Sigma, \delta, s, F)$

Minimization Algorithm

DFA $M = (Q, \Sigma, \delta, s, F)$

1. remove inaccessible states

Minimization Algorithm

DFA $M = (Q, \Sigma, \delta, s, F)$

1. remove inaccessible states
2. for every two different states, determine whether they are distinguishable (marking)

Minimization Algorithm

DFA $M = (Q, \Sigma, \delta, s, F)$

1. remove inaccessible states
2. for every two different states, determine whether they are distinguishable (marking)
3. collapse indistinguishable states

Minimization Algorithm

DFA $M = (Q, \Sigma, \delta, s, F)$

1. remove inaccessible states
2. for every two different states, determine whether they are distinguishable (marking)
3. collapse indistinguishable states

Marking Algorithm

given DFA $M = (Q, \Sigma, \delta, s, F)$ without inaccessible states

1. tabulate all unordered pairs $\{p, q\}$ with $p, q \in Q$, initially unmarked

Minimization Algorithm

DFA $M = (Q, \Sigma, \delta, s, F)$

1. remove inaccessible states
2. for every two different states, determine whether they are distinguishable (marking)
3. collapse indistinguishable states

Marking Algorithm

given DFA $M = (Q, \Sigma, \delta, s, F)$ without inaccessible states

1. tabulate all unordered pairs $\{p, q\}$ with $p, q \in Q$, initially unmarked
2. mark $\{p, q\}$ if $p \in F$ and $q \notin F$ or vice versa

Minimization Algorithm

DFA $M = (Q, \Sigma, \delta, s, F)$

1. remove inaccessible states
2. for every two different states, determine whether they are distinguishable (marking)
3. collapse indistinguishable states

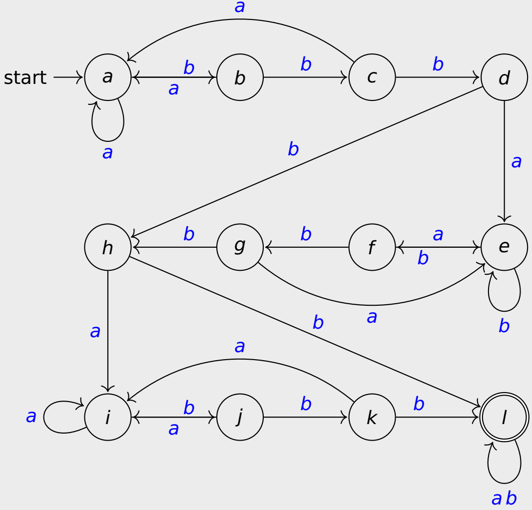
Marking Algorithm

given DFA $M = (Q, \Sigma, \delta, s, F)$ without inaccessible states

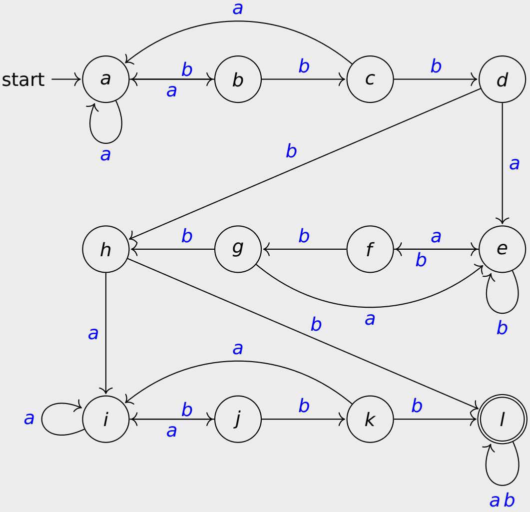
1. tabulate all unordered pairs $\{p, q\}$ with $p, q \in Q$, initially unmarked
2. mark $\{p, q\}$ if $p \in F$ and $q \notin F$ or vice versa
3. repeat until no change:

$$\text{mark } \{p, q\} \text{ if } \{\delta(p, a), \delta(q, a)\} \text{ is marked for some } a \in \Sigma$$

Example



Example



a
b
c
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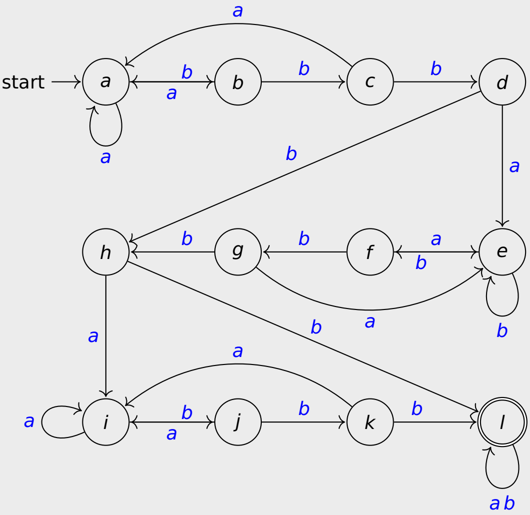
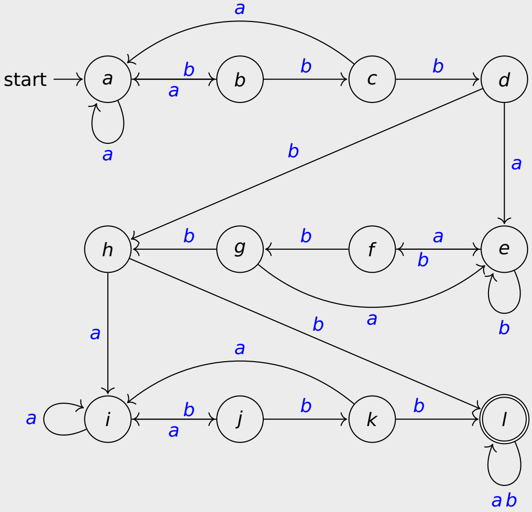
e
f
g
h
i
j
k
✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ l

Example



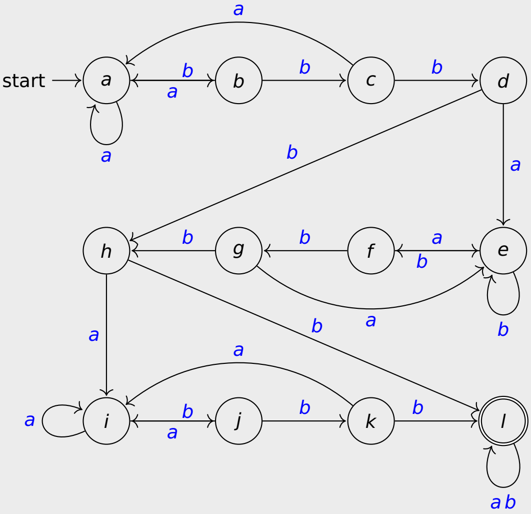
	a	b	c	d	e	f	g	h	i	j	k
a											
b											
c											
d											
e											
f											
g											
h	✓	✓	✓	✓	✓	✓	✓				
i								✓			
j								✓			
k	✓	✓	✓	✓	✓	✓	✓		✓	✓	
l	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Example



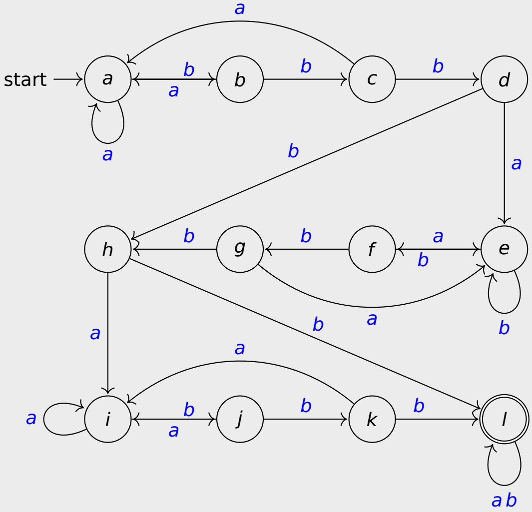
	a	b	c	d	e	f	g	h	i	j	k
b											
c											
d	✓	✓	✓								
e				✓							
f				✓							
g	✓	✓	✓		✓	✓					
h	✓	✓	✓	✓	✓	✓	✓				
i				✓			✓	✓			
j	✓	✓	✓		✓	✓		✓	✓		
k	✓	✓	✓	✓	✓	✓	✓		✓	✓	
l	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Example



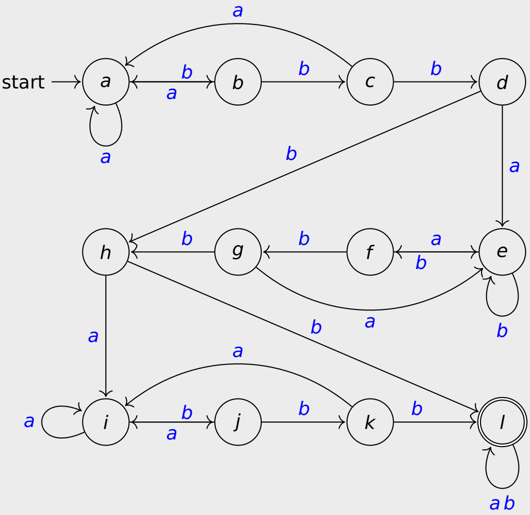
	a	b	c	d	e	f	g	h	i	j	k
a											
b											
c	✓	✓									
d	✓	✓	✓								
e			✓	✓							
f	✓	✓		✓	✓						
g	✓	✓	✓		✓	✓					
h	✓	✓	✓	✓	✓	✓	✓				
i	✓	✓		✓	✓		✓	✓			
j	✓	✓	✓		✓	✓		✓	✓		
k	✓	✓	✓	✓	✓	✓	✓		✓	✓	
l	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Example



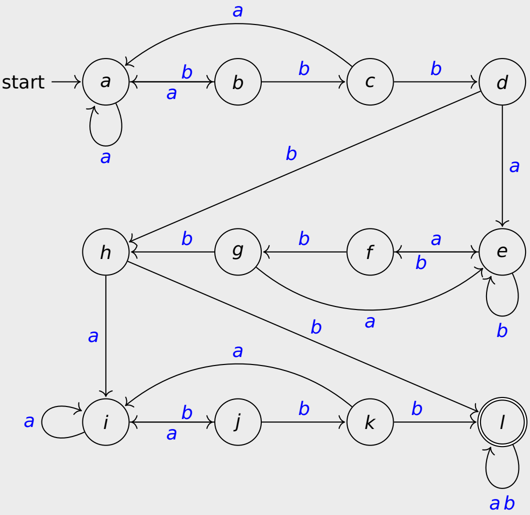
a											
✓	b										
✓	✓	c									
✓	✓	✓	d								
✓		✓	✓	e							
✓	✓		✓	✓	f						
✓	✓	✓		✓	✓	g					
✓	✓	✓	✓	✓	✓	✓	h				
✓	✓	✓	✓	✓	✓	✓	✓	i			
✓	✓	✓	✓	✓	✓	✓	✓	✓	j		
✓	✓	✓	✓	✓	✓	✓		✓	✓	k	
✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	l

Example



a											
✓	*b*										
✓	✓	*c*									
✓	✓	✓	*d*								
✓	✓	✓	✓	*e*							
✓	✓	✓	✓	✓	*f*						
✓	✓	✓		✓	✓	*g*					
✓	✓	✓	✓	✓	✓	✓	*h*				
✓	✓	✓	✓	✓	✓	✓	✓	*i*			
✓	✓	✓	✓	✓	✓	✓	✓	✓	*j*		
✓	✓	✓	✓	✓	✓	✓		✓	✓	*k*	
✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	*l*

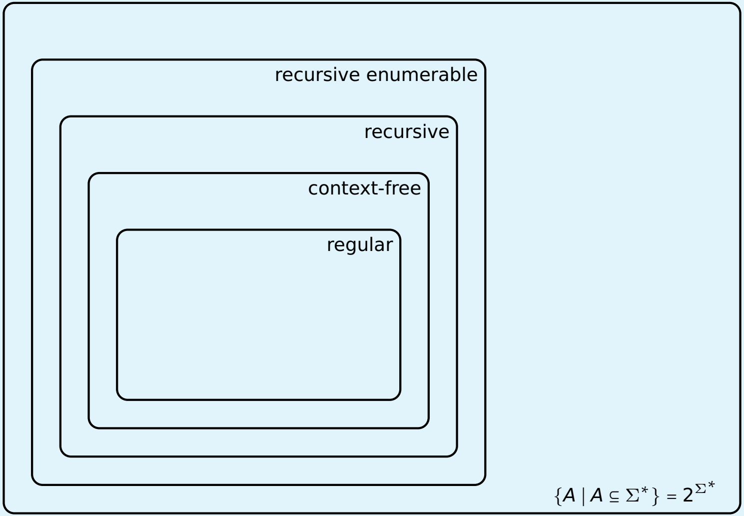
Example



a											
✓	*b*										
✓	✓	*c*									
✓	✓	✓	*d*								
✓	✓	✓	✓	*e*							
✓	✓	✓	✓	✓	*f*						
✓	✓	✓		✓	✓	*g*					
✓	✓	✓	✓	✓	✓	✓	*h*				
✓	✓	✓	✓	✓	✓	✓	✓	*i*			
✓	✓	✓	✓	✓	✓	✓	✓	✓	*j*		
✓	✓	✓	✓	✓	✓	✓		✓	✓	*k*	
✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	*l*

states *d*, *g* and *h*, *k* can be merged

Outline

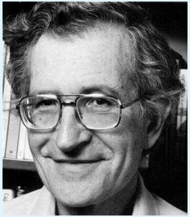

recursive enumerable
recursive
context-free
regular
$\{A \mid A \subseteq \Sigma^*\} = 2^{\Sigma^*}$

recursive enumerable

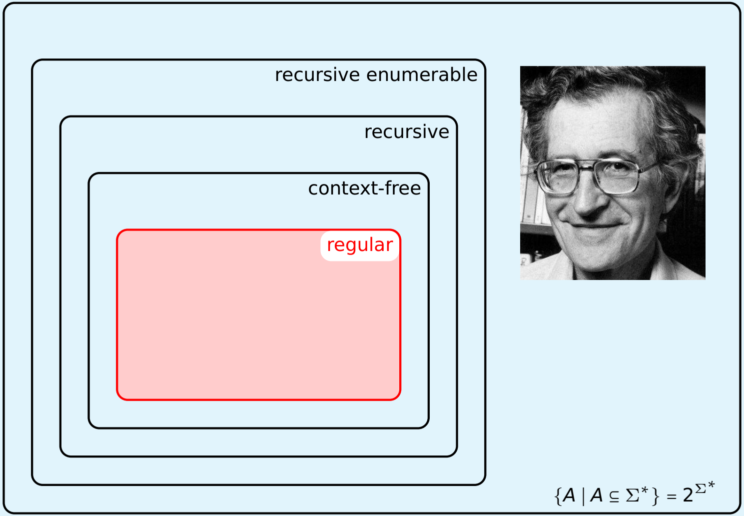
recursive

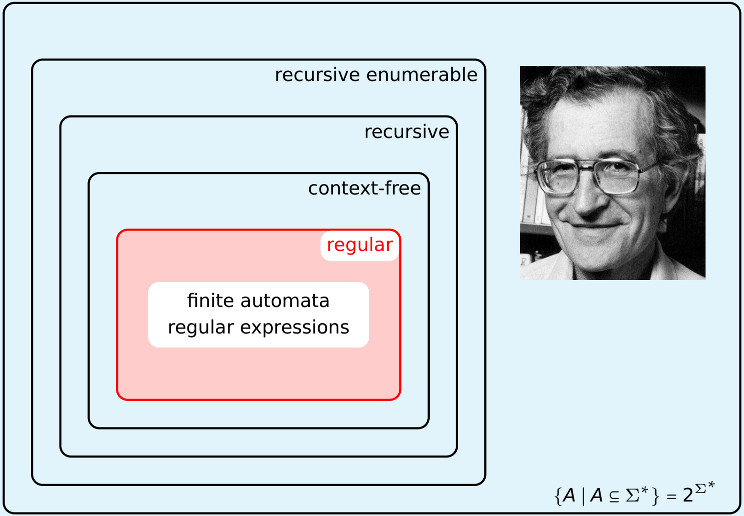
context-free

regular



$\{A \mid A \subseteq \Sigma^*\} = 2^{\Sigma^*}$


recursive enumerable
recursive
context-free
regular
$\{A \mid A \subseteq \Sigma^*\} = 2^{\Sigma^*}$


recursive enumerable
recursive
context-free
regular
finite automata
regular expressions
$\{A \mid A \subseteq \Sigma^*\} = 2^{\Sigma^*}$

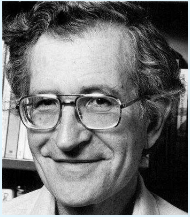
recursive enumerable

recursive

context-free

regular

finite automata
regular expressions



$\{A \mid A \subseteq \Sigma^*\} = 2^{\Sigma^*}$

Topics Appear in the Midterm (Regular Expressions)

DFAs to regular expressions

Definition

regular expressions are restricted patterns which use only

$$\boldsymbol{a} \in \Sigma \quad \boldsymbol{\varepsilon} \quad \varnothing \quad \alpha + \beta \quad \alpha^* \quad \alpha\beta$$

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$$\boldsymbol{a} \in \Sigma \quad \boldsymbol{\varepsilon} \quad \varnothing \quad \alpha + \beta \quad \alpha^* \quad \alpha\beta$$

Theorem

finite automata and regular expressions are equivalent:

Definition

regular expressions are restricted patterns which use only

$$\boldsymbol{a} \in \Sigma \quad \boldsymbol{\varepsilon} \quad \varnothing \quad \alpha + \beta \quad \alpha^* \quad \alpha\beta$$

Theorem

finite automata and regular expressions are equivalent:

for all $A \subseteq \Sigma^*$ ➀ A is regular

$\iff$ ➁ $A = L(\alpha)$ for some regular expression α

Proof. (1 $\Longrightarrow$ 2 – An idea)

given NFA$_\varepsilon$ $N_\varepsilon = (Q, \Sigma, \varepsilon, \Delta, S, F)$

$\forall Y \subseteq Q \quad \forall u, v \in Q$ construct regular expression α_{uv}^{Y} such that

$x \in L(\alpha_{uv}^{Y}) \iff$ $\boxed{\text{$\exists$ a path from u to v labeled x ($v \in \widehat{\Delta}(\{u\}, x)$) such that all intermediate states belong to Y}}$

Proof. (① $\Longrightarrow$ ② – An idea)

given NFA$_\varepsilon$ $N_\varepsilon = (Q, \Sigma, \varepsilon, \Delta, S, F)$

$\forall Y \subseteq Q \quad \forall u, v \in Q$ construct regular expression α_{uv}^{Y} such that

$x \in L(\alpha_{uv}^{Y}) \iff$ ∃ a path from u to v labeled x ($v \in \widehat{\Delta}(\{u\}, x)$) such that all intermediate states belong to Y

Definitions

- $\alpha_{uv}^{\varnothing} := \begin{cases} \boldsymbol{a_1} + \ldots + \boldsymbol{a_k} & \text{if } u \neq v \text{ and } k > 0 \\ \boldsymbol{\varnothing} & \text{if } u \neq v \text{ and } k = 0 \\ \boldsymbol{a_1} + \ldots + \boldsymbol{a_k} + \boldsymbol{\varepsilon} & \text{if } u = v \text{ and } k > 0 \\ \boldsymbol{\varepsilon} & \text{if } u = v \text{ and } k = 0 \end{cases}$

$\{a_1, \ldots, a_k\} := \{a \in \Sigma \cup \{\varepsilon\} \mid v \in \Delta(u, a)\}$

Proof. (1 $\Longrightarrow$ 2 – An idea)

given NFA$_\varepsilon$ $N_\varepsilon = (Q, \Sigma, \varepsilon, \Delta, S, F)$

$\forall Y \subseteq Q \quad \forall u, v \in Q$ construct regular expression α_{uv}^Y such that

$x \in L(\alpha_{uv}^Y) \iff$ $\exists$ a path from u to v labeled x ($v \in \widehat{\Delta}(\{u\}, x)$) such that all intermediate states belong to Y

Definitions

- $$\alpha_{uv}^{\varnothing} := \begin{cases} \boldsymbol{a_1} + \ldots + \boldsymbol{a_k} & \text{if } u \neq v \text{ and } k > 0 \\ \varnothing & \text{if } u \neq v \text{ and } k = 0 \\ \boldsymbol{a_1} + \ldots + \boldsymbol{a_k} + \boldsymbol{\varepsilon} & \text{if } u = v \text{ and } k > 0 \\ \boldsymbol{\varepsilon} & \text{if } u = v \text{ and } k = 0 \end{cases}$$ $\{a_1, \ldots, a_k\} := \{a \in \Sigma \cup \{\varepsilon\} \mid v \in \Delta(u, a)\}$
- $\alpha_{uv}^Y := \alpha_{uv}^{Y-\{q\}} + \alpha_{uq}^{Y-\{q\}} (\alpha_{qq}^{Y-\{q\}})^* \alpha_{qv}^{Y-\{q\}}$ for some fixed $q \in Y$

Proof. (1 $\Longrightarrow$ 2 – An idea)

given NFA$_\varepsilon$ $N_\varepsilon = (Q, \Sigma, \varepsilon, \Delta, S, F)$

$\forall Y \subseteq Q \quad \forall u, v \in Q$ construct regular expression α_{uv}^{Y} such that

$x \in L(\alpha_{uv}^{Y}) \iff$ $\boxed{\exists \text{ a path from } u \text{ to } v \text{ labeled } x\ (v \in \widehat{\Delta}(\{u\}, x)) \text{ such that all intermediate states belong to } Y}$

Theorem

$$L(N_\varepsilon) = L\left(\sum_{s \in S, t \in F} \alpha_{st}^{Q} \right)$$

Thanks! & Questions?