Assignment II (20 pts)

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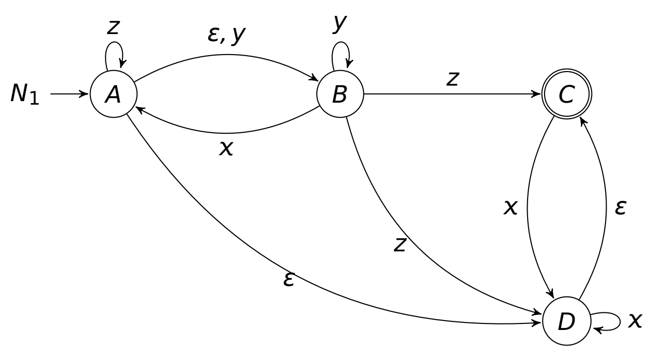
Assigned	:	March the 31st, 23h55
Due	:	April the 8th, 23h55

Q1. (15 pts) Given an NFA$_\varepsilon$ $N_1 = (\{A, B, C, D\}, \{x, y, z\}, \varepsilon, \Delta_1, \{A\}, \{C\})$ with the below state diagram



a) **(5 pts)** employ ε-elimination over N_1 to obtain an equivalent NFA $N_2 = (\{A, B, C, D\}, \{x, y, z\}, \Delta_2, \{A\}, F_2)$ with no ε-transitions. Clearly show intermediate steps.

b) **(5 pts)** apply subset construction algorithm to the NFA N_2 so as to get an equivalent DFA $D = (Q, \{x, y, z\}, \delta, s, F)$. Clearly show intermediate steps.

c) **(5 pts)** minimize the DFA D benefiting the marking algorithm. Justify your reasoning.

A1.

a) To start with, we compute ε-closure of below singleton sets:

$$C_\varepsilon(\{A\}) = \{A, B, C, D\} \quad C_\varepsilon(\{B\}) = \{B\} \quad C_\varepsilon(\{C\}) = \{C\} \quad C_\varepsilon(\{D\}) = \{C, D\}$$

We then apply ε-elimination to compute the transition function Δ_2 for the NFA N_2:

$$\begin{aligned}
\Delta_2(A, x) &= \widehat{\Delta}_1(\{A\}, x) \\
&= \bigcup\{C_\varepsilon(\Delta_1(q, x)) \mid q \in \widehat{\Delta}_1(\{A\}, \varepsilon)\} \\
&= C_\varepsilon(\Delta_1(A, x)) \cup C_\varepsilon(\Delta_1(B, x)) \cup \\
&\quad\; C_\varepsilon(\Delta_1(C, x)) \cup C_\varepsilon(\Delta_1(D, x)) \\
&= C_\varepsilon(\emptyset) \cup C_\varepsilon(\{A\}) \cup C_\varepsilon(\{D\}) \cup C_\varepsilon(\{D\}) \\
&= \emptyset \cup \{A, B, C, D\} \cup \{C, D\} \cup \{C, D\} \\
&= \{A, B, C, D\}
\end{aligned}$$

$$\begin{aligned}
\Delta_2(A, y) &= \widehat{\Delta}_1(\{A\}, y) \\
&= \bigcup\{C_\varepsilon(\Delta_1(q, y)) \mid q \in \widehat{\Delta}_1(\{A\}, \varepsilon)\} \\
&= C_\varepsilon(\Delta_1(A, y)) \cup C_\varepsilon(\Delta_1(B, y)) \cup \\
&\quad\; C_\varepsilon(\Delta_1(C, y)) \cup C_\varepsilon(\Delta_1(D, y)) \\
&= C_\varepsilon(\{B\}) \cup C_\varepsilon(\{B\}) \cup C_\varepsilon(\emptyset) \cup C_\varepsilon(\emptyset) \\
&= \{B\} \cup \{B\} \cup \emptyset \cup \emptyset \\
&= \{B\}
\end{aligned}$$

$$\begin{aligned}\Delta_2(A,z) &= \widehat{\Delta}_1(\{A\},z)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,z)) \mid q \in \widehat{\Delta}_1(\{A\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(A,z)) \cup C_\varepsilon(\Delta_1(B,z)) \cup\\ &\quad\ C_\varepsilon(\Delta_1(C,z)) \cup C_\varepsilon(\Delta_1(D,z))\\ &= C_\varepsilon(\{A\}) \cup C_\varepsilon(\{C,D\}) \cup C_\varepsilon(\varnothing) \cup C_\varepsilon(\varnothing)\\ &= \{A,B,C,D\} \cup \{C,D\} \cup \varnothing \cup \varnothing\\ &= \{A,B,C,D\}\end{aligned}$$

$$\begin{aligned}\Delta_2(B,x) &= \widehat{\Delta}_1(\{B\},x)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,x)) \mid q \in \widehat{\Delta}_1(\{B\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(B,x))\\ &= C_\varepsilon(\{A\})\\ &= \{A,B,C,D\}\end{aligned}$$

$$\begin{aligned}\Delta_2(B,y) &= \widehat{\Delta}_1(\{B\},y)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,y)) \mid q \in \widehat{\Delta}_1(\{B\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(B,y))\\ &= C_\varepsilon(\{B\})\\ &= \{B\}\end{aligned}$$

$$\begin{aligned}\Delta_2(B,z) &= \widehat{\Delta}_1(\{B\},z)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,z)) \mid q \in \widehat{\Delta}_1(\{B\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(B,z))\\ &= C_\varepsilon(\{C,D\})\\ &= \{C,D\}\end{aligned}$$

$$\begin{aligned}\Delta_2(C,x) &= \widehat{\Delta}_1(\{C\},x)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,x)) \mid q \in \widehat{\Delta}_1(\{C\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(C,x))\\ &= C_\varepsilon(\{D\})\\ &= \{C,D\}\end{aligned}$$

$$\begin{aligned}\Delta_2(C,y) &= \widehat{\Delta}_1(\{C\},y)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,y)) \mid q \in \widehat{\Delta}_1(\{C\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(C,y))\\ &= C_\varepsilon(\varnothing)\\ &= \varnothing\end{aligned}$$

$$\begin{aligned}\Delta_2(C,z) &= \widehat{\Delta}_1(\{C\},z)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,z)) \mid q \in \widehat{\Delta}_1(\{C\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(C,z))\\ &= C_\varepsilon(\varnothing)\\ &= \varnothing\end{aligned}$$

$$\begin{aligned}\Delta_2(D,x) &= \widehat{\Delta}_1(\{D\},x)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,x)) \mid q \in \widehat{\Delta}_1(\{D\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(C,x)) \cup C_\varepsilon(\Delta_1(D,x))\\ &= C_\varepsilon(\{D\}) \cup C_\varepsilon(\{D\})\\ &= \{C,D\} \cup \{C,D\}\\ &= \{C,D\}\end{aligned}$$

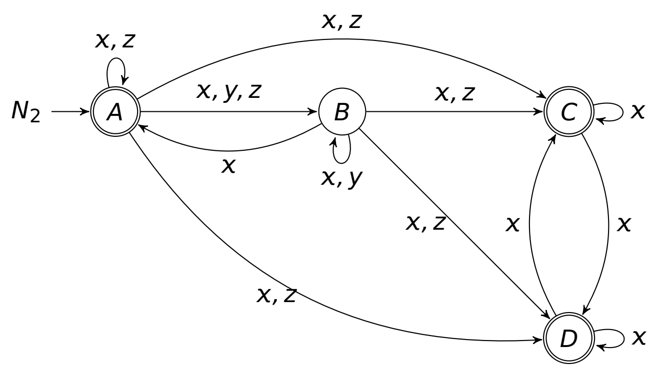
$$\begin{aligned}\Delta_2(D,y) &= \widehat{\Delta}_1(\{D\},y)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,y)) \mid q \in \widehat{\Delta}_1(\{D\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(C,y)) \cup C_\varepsilon(\Delta_1(D,y))\\ &= C_\varepsilon(\varnothing) \cup C_\varepsilon(\varnothing)\\ &= \varnothing\end{aligned}$$

$$\begin{aligned}\Delta_2(D,z) &= \widehat{\Delta}_1(\{D\},z)\\ &= \bigcup\{C_\varepsilon(\Delta_1(q,z)) \mid q \in \widehat{\Delta}_1(\{D\},\varepsilon)\}\\ &= C_\varepsilon(\Delta_1(C,z)) \cup C_\varepsilon(\Delta_1(D,z))\\ &= C_\varepsilon(\varnothing) \cup C_\varepsilon(\varnothing)\\ &= \varnothing\end{aligned}$$

The set of final states for N_2 is computed as follows:

$$F_2 \quad := \quad \{q \mid C_\varepsilon(\{q\}) \cap F_1 \neq \varnothing\} \quad = \quad \{A,C,D\}.$$

Therefore, the state diagram for N_2 looks like:



b) Let us now apply subset construction over the NFA N_2 to obtain an equivalent DFA $D = (Q, \{x, y, z\}, \delta, s, F)$:

$$
\begin{aligned}
\delta(\{A\}, x) &= \widehat{\Delta}_2(\{A\}, x) \\
&= \{A, B, C, D\} \\
\delta(\{A\}, z) &= \widehat{\Delta}_2(\{A\}, z) \\
&= \{A, B, C, D\} \\
\delta(\{B\}, y) &= \widehat{\Delta}_2(\{B\}, y) \\
&= \{B\} \\
\delta(\{C, D\}, x) &= \widehat{\Delta}_2(\{C, D\}, x) \\
&= \widehat{\Delta}_2(\{C\}, x) \cup \widehat{\Delta}_2(\{D\}, x) \\
&= \{C, D\} \cup \{C, D\} \\
& \{C, D\} \\
\delta(\{C, D\}, z) &= \widehat{\Delta}_2(\{C, D\}, z) \\
&= \widehat{\Delta}_2(\{C\}, z) \cup \widehat{\Delta}_2(\{D\}, z) \\
&= \varnothing \cup \varnothing \\
&= \varnothing
\end{aligned}
\qquad
\begin{aligned}
\delta(\{A\}, y) &= \widehat{\Delta}_2(\{A\}, y) \\
&= \{B\} \\
\delta(\{B\}, x) &= \widehat{\Delta}_2(\{B\}, x) \\
&= \{A, B, C, D\} \\
\delta(\{B\}, z) &= \widehat{\Delta}_2(\{B\}, z) \\
&= \{C, D\} \\
\delta(\{C, D\}, y) &= \widehat{\Delta}_2(\{C, D\}, y) \\
&= \widehat{\Delta}_2(\{C\}, y) \cup \widehat{\Delta}_2(\{D\}, y) \\
&= \varnothing \cup \varnothing \\
&= \varnothing \\
\delta(\{A, B, C, D\}, x) &= \widehat{\Delta}_2(\{A, B, C, D\}, x) \\
&= \widehat{\Delta}_2(\{A\}, x) \cup \widehat{\Delta}_2(\{B\}, x) \cup \\
& \widehat{\Delta}_2(\{C\}, x) \cup \widehat{\Delta}_2(\{D\}, x) \\
&= \{A, B, C, D\} \cup \{A, B, C, D\} \cup \\
& \{C, D\} \cup \{C, D\} \\
&= \{A, B, C, D\}
\end{aligned}
$$

$$
\begin{aligned}
\delta(\{A, B, C, D\}, y) &= \widehat{\Delta}_2(\{A, B, C, D\}, y) \\
&= \widehat{\Delta}_2(\{A\}, y) \cup \widehat{\Delta}_2(\{B\}, y) \cup \\
& \widehat{\Delta}_2(\{C\}, y) \cup \widehat{\Delta}_2(\{D\}, y) \\
&= \{B\} \cup \{B\} \cup \varnothing \cup \varnothing \\
&= \{B\} \\
\delta(\varnothing, x) &= \widehat{\Delta}_2(\varnothing, x) \\
&= \varnothing \\
\delta(\varnothing, z) &= \widehat{\Delta}_2(\varnothing, z) \\
&= \varnothing
\end{aligned}
\qquad
\begin{aligned}
\delta(\{A, B, C, D\}, z) &= \widehat{\Delta}_2(\{A, B, C, D\}, z) \\
&= \widehat{\Delta}_2(\{A\}, z) \cup \widehat{\Delta}_2(\{B\}, z) \cup \\
& \widehat{\Delta}_2(\{C\}, z) \cup \widehat{\Delta}_2(\{D\}, z) \\
&= \{A, B, C, D\} \cup \{C, D\} \cup \varnothing \cup \varnothing \\
&= \{A, B, C, D\} \\
\delta(\varnothing, y) &= \widehat{\Delta}_2(\varnothing, y) \\
&= \varnothing
\end{aligned}
$$

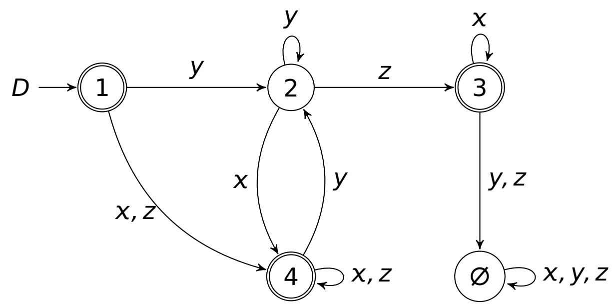
The set of final states F for the DFA D is given as follows:

$$F \quad := \quad \{A \subseteq Q_{N_2} \mid A \cap F_{N_2} \neq \varnothing\} \quad = \quad \{\{A\}, \{C, D\}, \{A, B, C, D\}\}.$$

Obviously,

$$s \quad := \quad S_{N_2} \quad = \quad \{A\}.$$

Given all these, we depict below the state diagram for the DFA D:



where

$$1 \quad = \quad \{A\} \qquad\qquad 2 \quad = \quad \{B\} \qquad\qquad 3 \quad = \quad \{C, D\} \qquad\qquad 4 \quad = \quad \{A, B, C, D\}.$$

c) We now check whether D is the minimal DFA with the above configuration. Observe that D has no inaccessible states. We can then employ the marking algorithm to perform the (in)distinguishability test for each pair of states.

As final and non-final states are distinguishable, we mark them in the below tabular right from the starch:

1				
$\checkmark$	2			
	$\checkmark$	3		
	$\checkmark$		4	
$\checkmark$		$\checkmark$	$\checkmark$	$\varnothing$

We then compare pairs of states in the below given order, and resume accordingly:

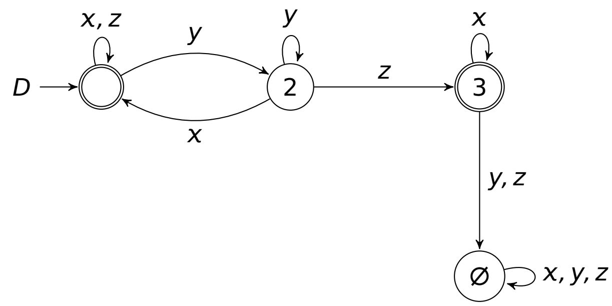
$$\{4, 3\} \quad \xrightarrow{z} \quad \{4, \varnothing\} \quad \text{mark } (4, 3) \quad \text{as } (4, \varnothing) \text{ is already marked}$$
$$\{1, 3\} \quad \xrightarrow{z} \quad \{4, \varnothing\} \quad \text{mark } (1, 3) \quad \text{as } (4, \varnothing) \text{ is already marked}$$
$$\{2, \varnothing\} \quad \xrightarrow{x} \quad \{4, \varnothing\} \quad \text{mark } (2, \varnothing) \quad \text{as } (4, \varnothing) \text{ is already marked}$$

1				
$\checkmark$	2			
$\checkmark$	$\checkmark$	3		
	$\checkmark$	$\checkmark$	4	
$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\varnothing$

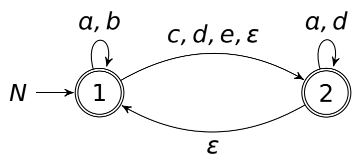
We cannot mark the pair $(1, 4)$ as the states 1 and 4 are indistinguishable:

$$\{1, 4\} \quad \xrightarrow{x} \quad \{4, 4\}$$
$$\{1, 4\} \quad \xrightarrow{y} \quad \{2, 2\}$$
$$\{1, 4\} \quad \xrightarrow{z} \quad \{4, 4\}$$

Therefore, we collapse states 1 and 4 to obtain the minimal DFA for D:



Q2. (5 pts) Given a NFA$_\varepsilon$ $N = (\{1,2\}, \{a,b,c,d,e\}, \varepsilon, \Delta, \{1\}, \{1,2\})$ with below depicted state diagram



compute the regular expression α such that $\mathcal{L}(\alpha) = \mathcal{L}(N)$ employing the algorithm (definition) given in w4.pdf, slide #18.

A2.

By specializing the theorem given in w4.pdf on slide #18, we obtain that $\mathcal{L}(N) = \alpha_{12}^{\{1,2\}} + \alpha_{11}^{\{1,2\}}$.

- The unfolding of the algorithm in computing the expression $\alpha_{12}^{\{1,2\}}$ is itemized as follows.

  1. 1st recursive call:

  $$\alpha_{12}^{\{1,2\}} \quad = \quad \alpha_{12}^{\{1\}} + \alpha_{12}^{\{1\}}(\alpha_{22}^{\{1\}})^* \alpha_{22}^{\{1\}} \qquad u = 1, \mathbf{q = 2}, v = 2$$

  2. 2nd recursive call:

  $$\begin{aligned} \alpha_{12}^{\{1\}} &= \alpha_{12}^{\varnothing} + \alpha_{11}^{\varnothing}(\alpha_{11}^{\varnothing})^* \alpha_{12}^{\varnothing} \qquad u = 1, \mathbf{q = 1}, v = 2 \\ \alpha_{22}^{\{1\}} &= \alpha_{22}^{\varnothing} + \alpha_{21}^{\varnothing}(\alpha_{11}^{\varnothing})^* \alpha_{12}^{\varnothing} \qquad u = 2, \mathbf{q = 1}, v = 2 \end{aligned}$$

  3. In the 3rd recursive call, the algorithm reaches the base case:

  $$\begin{aligned} \alpha_{12}^{\varnothing} &= \mathbf{c} + \mathbf{d} + \mathbf{e} + \boldsymbol{\varepsilon} \\ \alpha_{11}^{\varnothing} &= \mathbf{a} + \mathbf{b} + \boldsymbol{\varepsilon} \\ \alpha_{22}^{\varnothing} &= \mathbf{a} + \mathbf{d} + \boldsymbol{\varepsilon} \\ \alpha_{21}^{\varnothing} &= \boldsymbol{\varepsilon} \end{aligned}$$

  4. At this stage, it folds back:

  $$\begin{aligned} \alpha_{12}^{\{1\}} &= \mathbf{(c + d + e + \boldsymbol{\varepsilon}) + \big[(a + b + \boldsymbol{\varepsilon})(a + b + \boldsymbol{\varepsilon})^*(c + d + e + \boldsymbol{\varepsilon})\big]} \\ \alpha_{22}^{\{1\}} &= \mathbf{(a + d + \boldsymbol{\varepsilon}) + \big[(\boldsymbol{\varepsilon})(a + b + \boldsymbol{\varepsilon})^*(c + d + e + \boldsymbol{\varepsilon})\big]} \end{aligned}$$

  Therefore,

  $$\begin{aligned} \alpha_{12}^{\{1,2\}} = \; & \big(((\mathbf{c} + \mathbf{d} + \mathbf{e} + \boldsymbol{\varepsilon}) + [(\mathbf{a} + \mathbf{b} + \boldsymbol{\varepsilon})(\mathbf{a} + \mathbf{b} + \boldsymbol{\varepsilon})^*(\mathbf{c} + \mathbf{d} + \mathbf{e} + \boldsymbol{\varepsilon})]) + \\ & ((\mathbf{c} + \mathbf{d} + \mathbf{e} + \boldsymbol{\varepsilon}) + [(\mathbf{a} + \mathbf{b} + \boldsymbol{\varepsilon})(\mathbf{a} + \mathbf{b} + \boldsymbol{\varepsilon})^*(\mathbf{c} + \mathbf{d} + \mathbf{e} + \boldsymbol{\varepsilon})])((\mathbf{a} + \mathbf{d} + \boldsymbol{\varepsilon}) + [(\boldsymbol{\varepsilon})(\mathbf{a} + \mathbf{b} + \boldsymbol{\varepsilon})^*(\mathbf{c} + \mathbf{d} + \mathbf{e} + \boldsymbol{\varepsilon})])^* \\ & ((\mathbf{a} + \mathbf{d} + \boldsymbol{\varepsilon}) + [(\boldsymbol{\varepsilon})(\mathbf{a} + \mathbf{b} + \boldsymbol{\varepsilon})^*(\mathbf{c} + \mathbf{d} + \mathbf{e} + \boldsymbol{\varepsilon})])\big) \end{aligned}$$

- The unfolding of the algorithm in computing the expression $\alpha_{11}^{\{1,2\}}$ is summarized in the following.

  1. 1st recursive call:

  $$\alpha_{11}^{\{1,2\}} \quad = \quad \alpha_{11}^{\{2\}} + \alpha_{11}^{\{2\}}(\alpha_{11}^{\{2\}})^* \alpha_{11}^{\{2\}} \qquad u = 1, \mathbf{q = 1}, v = 1$$

  2. 2nd recursive call:

$$\alpha_{11}^{\{2\}} = \alpha_{11}^{\emptyset} + \alpha_{12}^{\emptyset}(\alpha_{22}^{\emptyset})^{*}\alpha_{21}^{\emptyset} \quad u = 1, \mathbf{q = 2}, v = 1$$

3. In the 3rd recursive call, the algorithm reaches the base case:

$$\begin{aligned}
\alpha_{12}^{\emptyset} &= \mathbf{c} + \mathbf{d} + \mathbf{e} + \boldsymbol{\varepsilon} \\
\alpha_{11}^{\emptyset} &= \mathbf{a} + \mathbf{b} + \boldsymbol{\varepsilon} \\
\alpha_{22}^{\emptyset} &= \mathbf{a} + \mathbf{d} + \boldsymbol{\varepsilon} \\
\alpha_{21}^{\emptyset} &= \boldsymbol{\varepsilon}
\end{aligned}$$

4. At this stage, it folds back:

$$\alpha_{11}^{\{2\}} = (\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon}) + \left[(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})^{*}(\boldsymbol{\varepsilon})\right]$$

Therefore,

$$\begin{aligned}
\alpha_{11}^{\{1,2\}} = \; & \Big(((\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})+\left[(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})^{*}(\boldsymbol{\varepsilon})\right])+ \\
& ((\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})+\left[(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})^{*}(\boldsymbol{\varepsilon})\right])((\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})+\left[(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})^{*}(\boldsymbol{\varepsilon})\right])^{*} \\
& ((\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})+\left[(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})^{*}(\boldsymbol{\varepsilon})\right])\Big)
\end{aligned}$$

- Finally,

$$\begin{aligned}
\alpha_{12}^{\{1,2\}} + \alpha_{11}^{\{1,2\}} = \; & \Big(((\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})+\left[(\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})^{*}(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})\right])+ \\
& ((\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})+\left[(\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})^{*}(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})\right])((\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})+\left[(\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})^{*}(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})\right])^{*} \\
& ((\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})+\left[(\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})^{*}(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})\right])\Big)+ \\
& \Big(((\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})+\left[(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})^{*}(\boldsymbol{\varepsilon})\right])+ \\
& ((\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})+\left[(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})^{*}(\boldsymbol{\varepsilon})\right])((\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})+\left[(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})^{*}(\boldsymbol{\varepsilon})\right])^{*} \\
& ((\mathbf{a}+\mathbf{b}+\boldsymbol{\varepsilon})+\left[(\mathbf{c}+\mathbf{d}+\mathbf{e}+\boldsymbol{\varepsilon})(\mathbf{a}+\mathbf{d}+\boldsymbol{\varepsilon})^{*}(\boldsymbol{\varepsilon})\right])\Big)
\end{aligned}$$

Important Notice:

- Collaboration is strictly and positively prohibited; lowers your score to 0 if detected.
- Any submission after 23h55 on April the 8th will NOT be accepted. Please beware and respect the deadline!
- All handwritten answers should somehow be scanned into a single pdf file, and only then submitted. Make sure that your handwriting is decent and readable.