CMPE 322/327 - Theory of Computation

Week 9: Chomsky Normal Form & Pumping Lemma – CKY Algorithm

Burak Ekici

April 18-22, 2022

Outline

New Algorithm for Testing Equivalence of NFAs

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- bisimulation up to congruence

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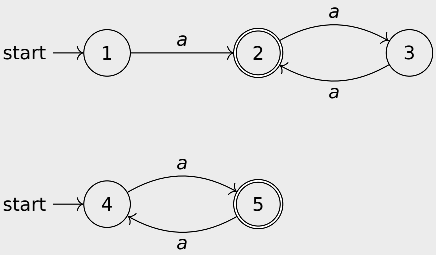
Example



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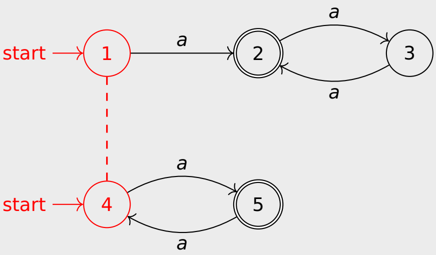

$1 \sim 4$?

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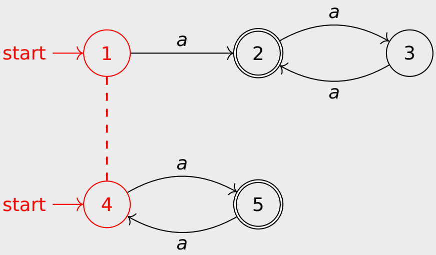

$1 \sim 4$? $1\uparrow$ $4\uparrow$

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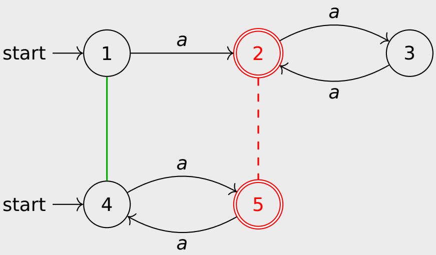

$1 \sim 4$ $\quad 1\uparrow \quad 4\uparrow$

$2 \sim 5$?

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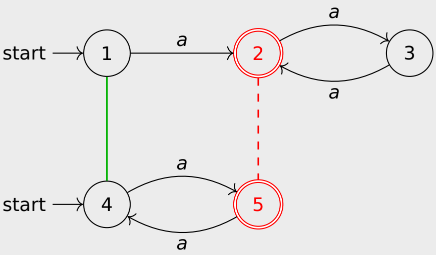

$1 \sim 4$ $\quad 1\uparrow \quad 4\uparrow$

$2 \sim 5$? $\quad 2\downarrow \quad 5\downarrow$

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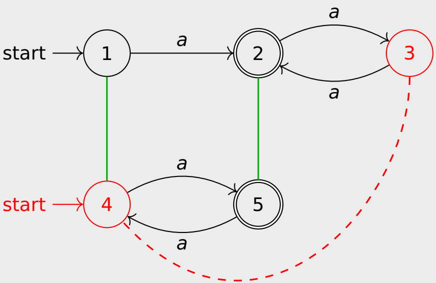

$1 \sim 4$ $\quad 1\!\uparrow \quad 4\!\uparrow$

$2 \sim 5$ $\quad 2\!\downarrow \quad 5\!\downarrow$

$3 \sim 4$?

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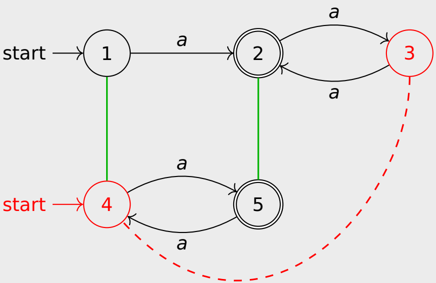

$1 \sim 4$	$1\uparrow$	$4\uparrow$
$2 \sim 5$	$2\downarrow$	$5\downarrow$
$3 \sim 4$?	$3\uparrow$	$4\uparrow$

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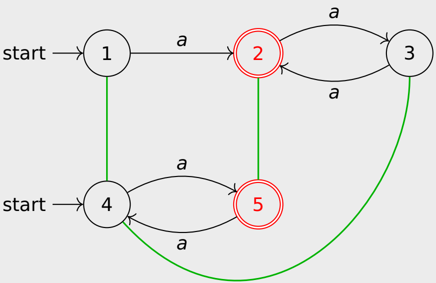

$1 \sim 4$ $\quad 1\uparrow \quad 4\uparrow$
$2 \sim 5$ $\quad 2\downarrow \quad 5\downarrow$
$3 \sim 4$ $\quad 3\uparrow \quad 4\uparrow$
$2 \sim 5$

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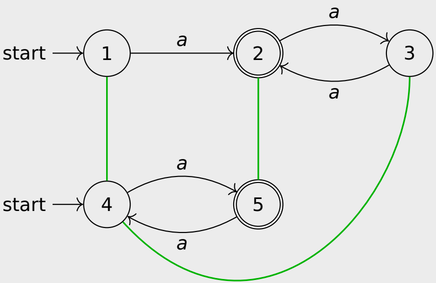

$1 \sim 4$ $\quad 1\uparrow \quad 4\uparrow$

$2 \sim 5$ $\quad 2\downarrow \quad 5\downarrow$

$3 \sim 4$ $\quad 3\uparrow \quad 4\uparrow$

$2 \sim 5$ $\quad \sim$ is bisimulation

Remark

bisimulation relates states with same observable behaviour

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Definition

- bisimulation is binary relation R on states Q of DFA $M = (Q, \Sigma, \delta, s, F)$ such that for all $p\,R\,q$
  1. $p \in F \iff q \in F$

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Example

- $=$ (identity relation)

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Example

- $=$ (identity relation)
- $\approx$ (indistinguishability relation of lecture 5)

Remark

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Remarks

- bisimilarity is equivalence relation

Remark

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  1. $p \in F \quad \Longleftrightarrow \quad q \in F$
  2. $\delta(p, a)\ R\ \delta(q, a) \quad$ for all $a \in \Sigma$
- states p and q are bisimilar ($p \sim q$) if $p\ R\ q$ for some bisimulation R

Remarks

- bisimilarity is equivalence relation
- $L(M, p) := \{x \in \Sigma^* \mid \widehat{\delta}(p, x) \in F\}$

Remark

bisimulation relates states with same observable behaviour

Definition

- **bisimulation** is binary relation R on states Q of DFA $M = (Q, \Sigma, \delta, s, F)$ such that for all $p\,R\,q$
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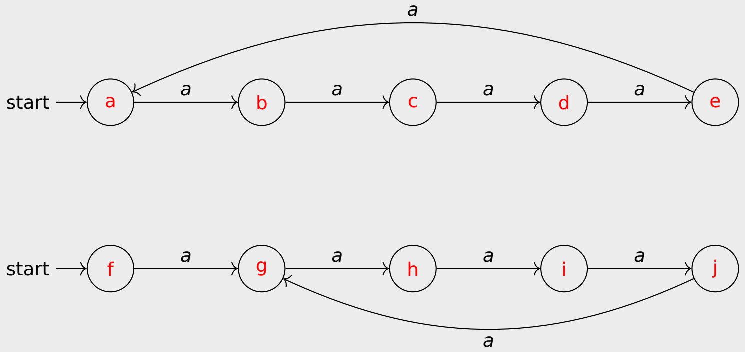
Remarks

- bisimilarity is equivalence relation
- $L(M, p) := \{x \in \Sigma^* \mid \widehat{\delta}(p, x) \in F\}$
- $p \sim q \quad \Longleftrightarrow \quad L(M, p) \sim L(M, q)$

Example



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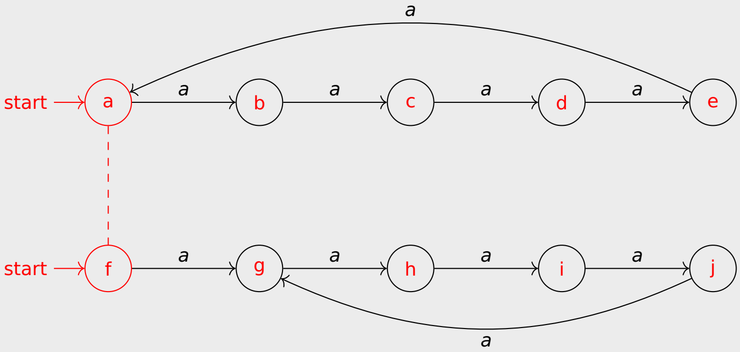

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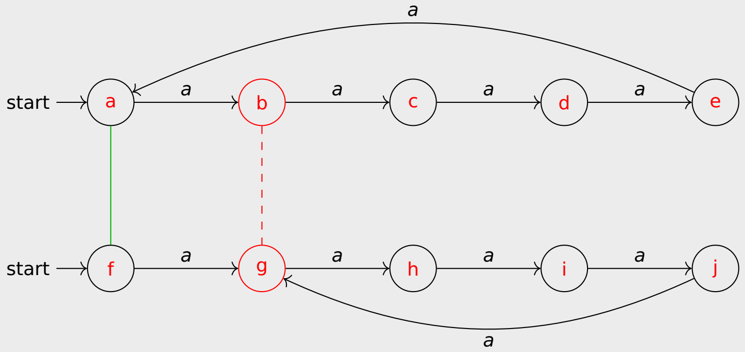
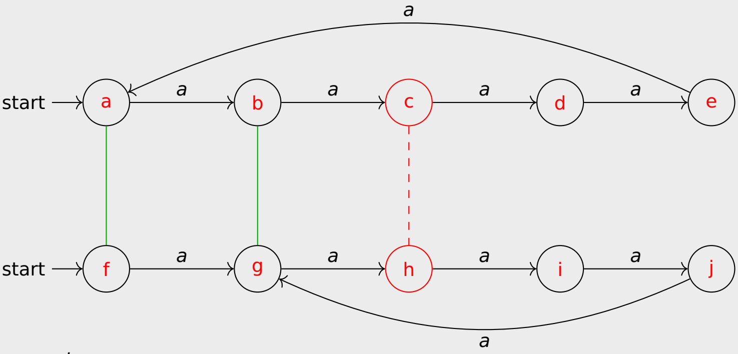


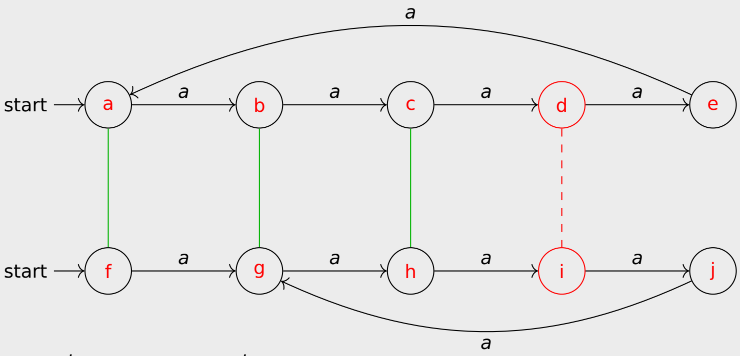
$a \sim f$

Example



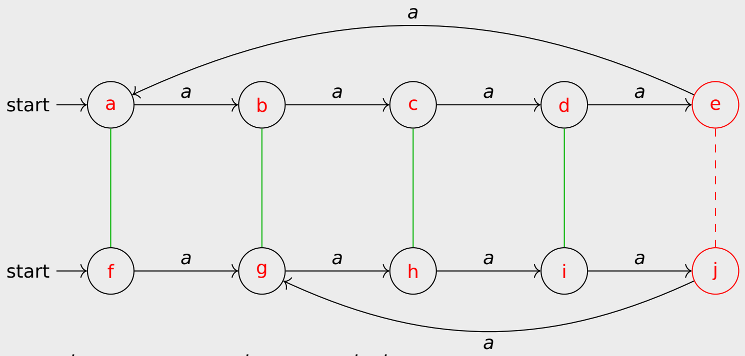
$a \sim f$ $b \sim g$

Example



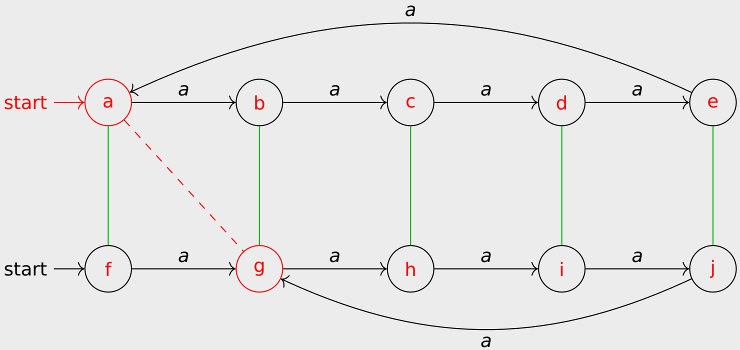
$a \sim f$ $b \sim g$ $c \sim h$

Example



$a \sim f$ $\quad b \sim g$ $\quad c \sim h$ $\quad d \sim i$

Example



$a \sim f$ $b \sim g$ $c \sim h$ $d \sim i$ $e \sim j$

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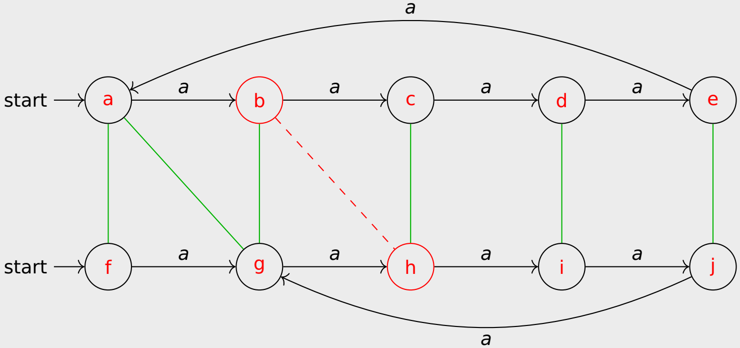
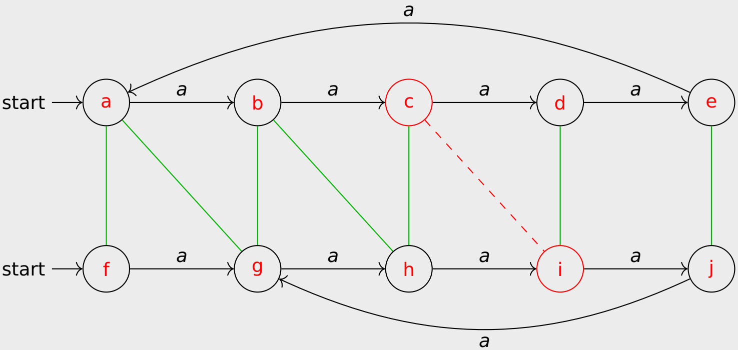


$a \sim f$ $\quad$ $b \sim g$ $\quad$ $c \sim h$ $\quad$ $d \sim i$ $\quad$ $e \sim j$ $\quad$ $a \sim g$

Example



$a \sim f$ $b \sim g$ $c \sim h$ $d \sim i$ $e \sim j$ $a \sim g$ $b \sim h$

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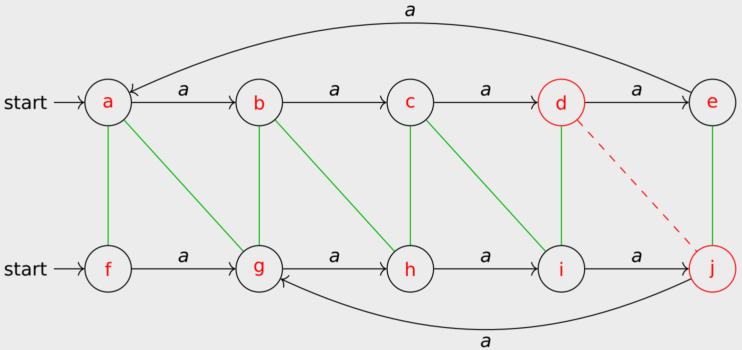

$a \sim f$ $b \sim g$ $c \sim h$ $d \sim i$ $e \sim j$ $a \sim g$ $b \sim h$

$c \sim i$

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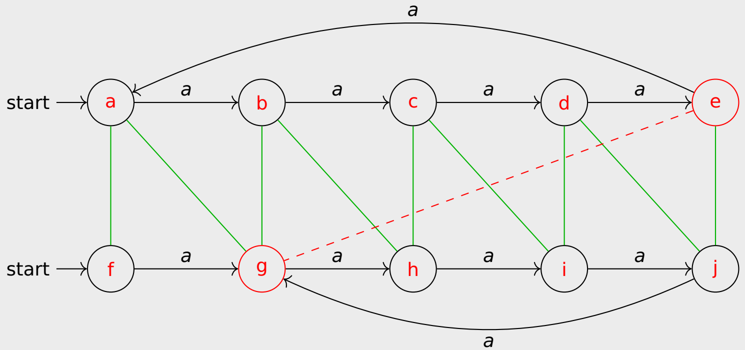

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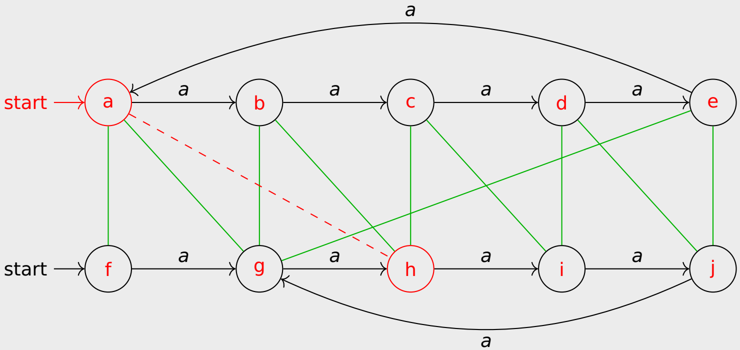

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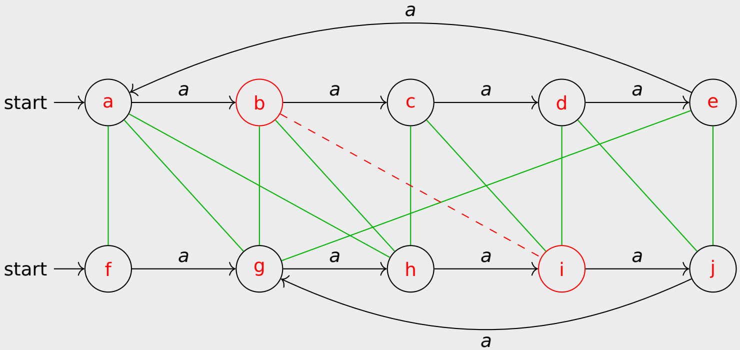

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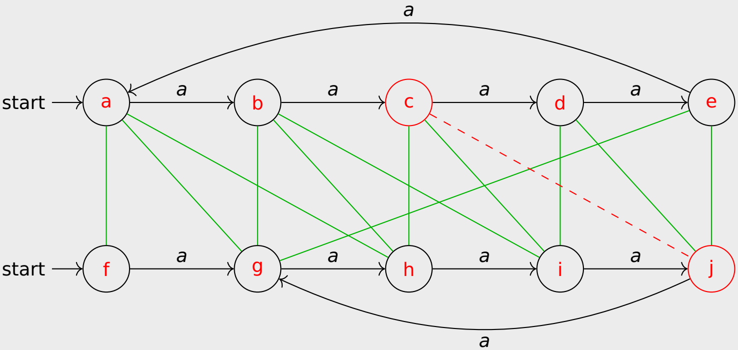

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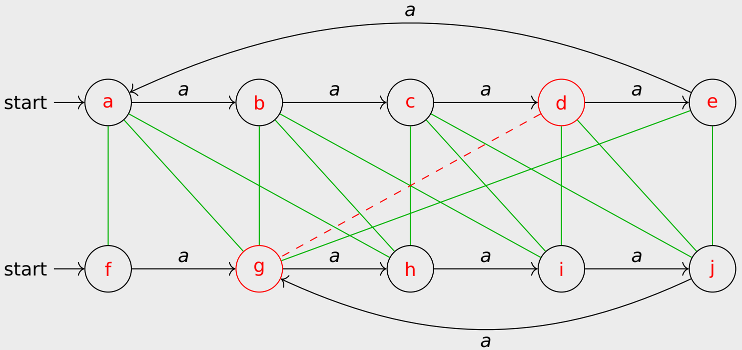

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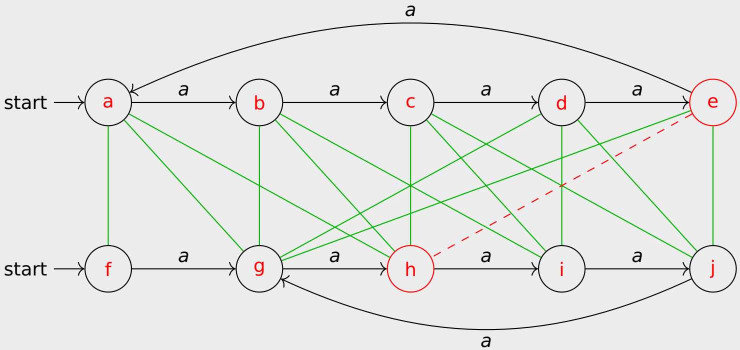

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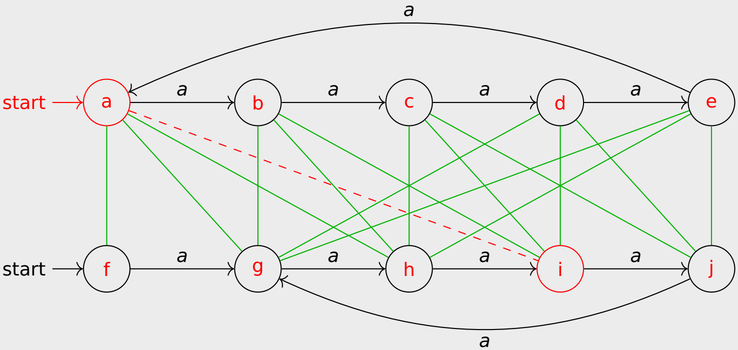
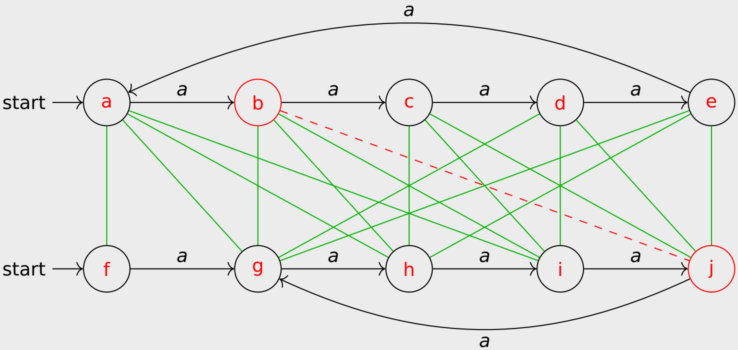


$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$						

Example



$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$	$a \sim i$					

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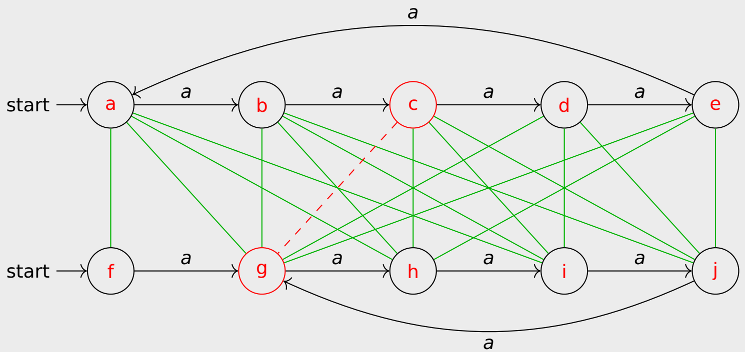

$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$	$a \sim i$	$b \sim j$				

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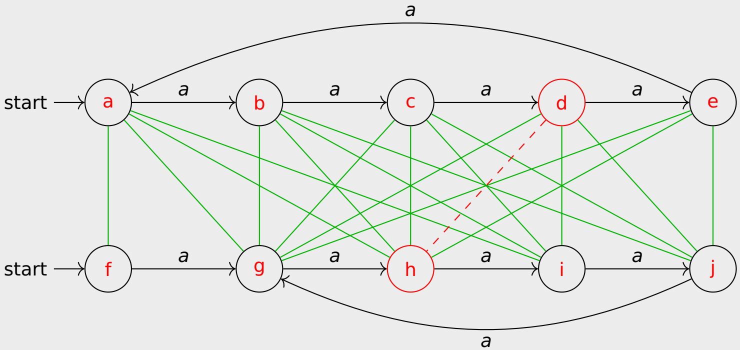

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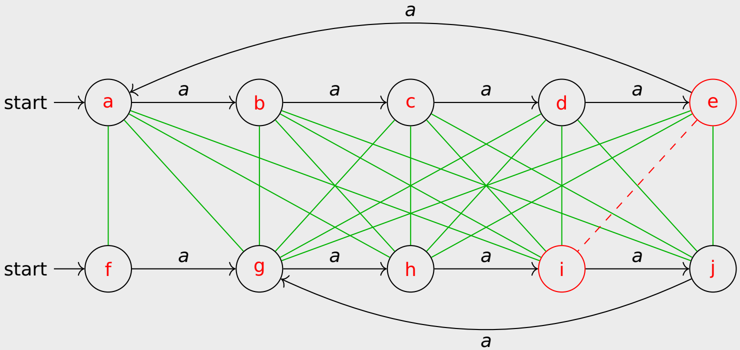

$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
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$e \sim h$	$a \sim i$	$b \sim j$	$c \sim g$	$d \sim h$		

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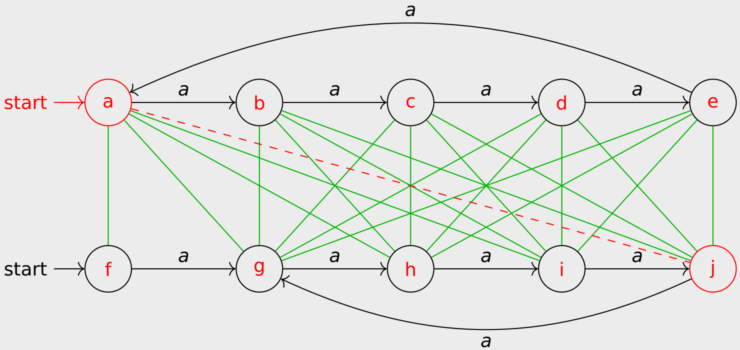

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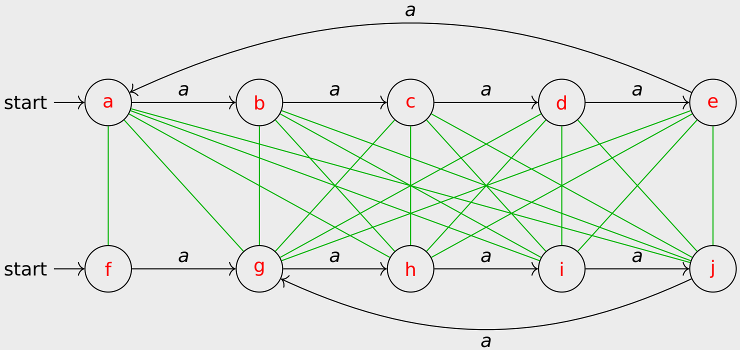

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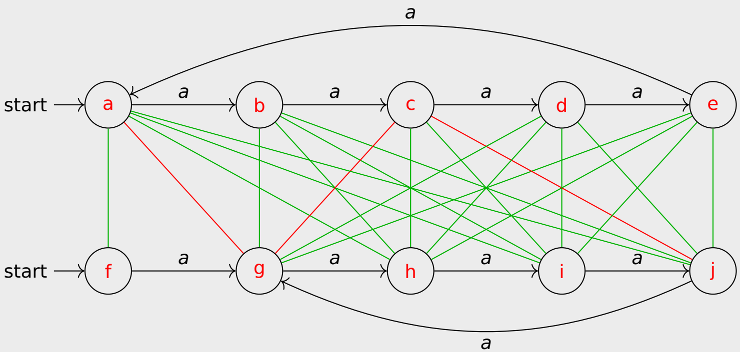
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$	$a \sim i$	$b \sim j$	$c \sim g$	$d \sim h$	$e \sim i$	$a \sim j$

bisimulation

21 comparisons

Example



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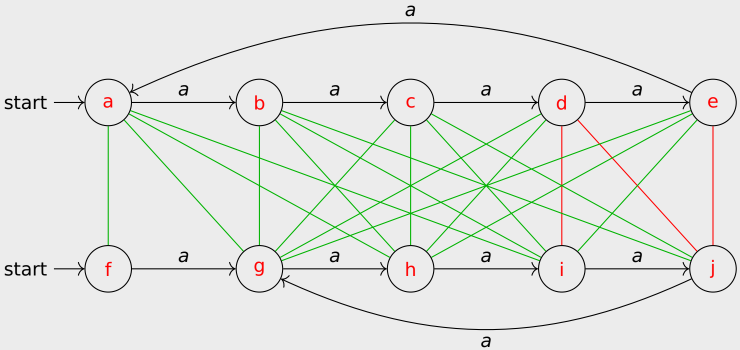
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$	$a \sim i$	$b \sim j$	$c \sim g$	$d \sim h$	$e \sim i$	$a \sim j$

bisimulation

$a \sim j$ follows from $a \sim g, c \sim g, c \sim j$

21 comparisons

Example



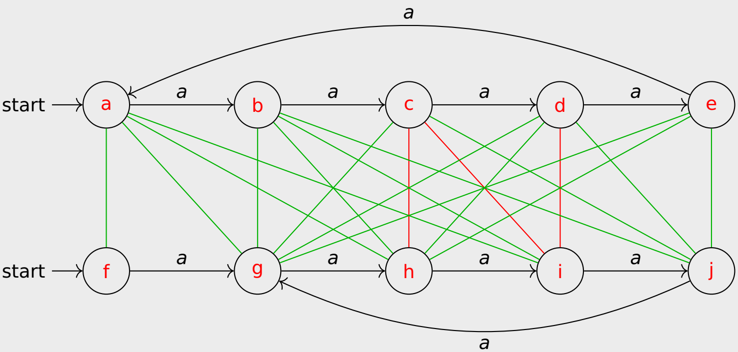
$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$	$a \sim i$	$b \sim j$	$c \sim g$	$d \sim h$	$e \sim i$	

bisimulation

$e \sim i$ follows from $d \sim i, d \sim j, e \sim j$

20 comparisons

Example



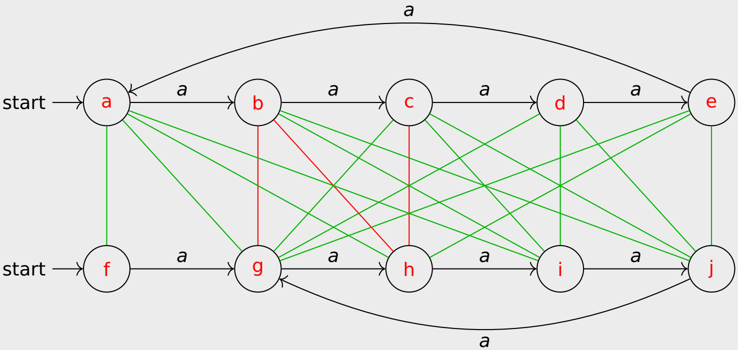
$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$	$a \sim i$	$b \sim j$	$c \sim g$	$d \sim h$		

bisimulation

$d \sim h$ follows from $c \sim h, c \sim i, d \sim i$

19 comparisons

Example



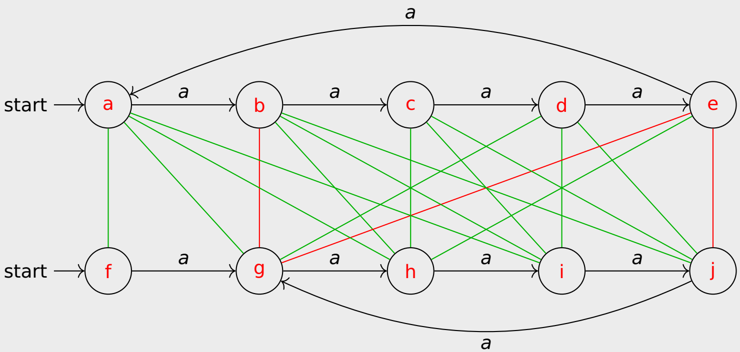
$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$	$a \sim i$	$b \sim j$	$c \sim g$			

bisimulation

$c \sim g$ follows from $b \sim g, b \sim h, c \sim h$

18 comparisons

Example



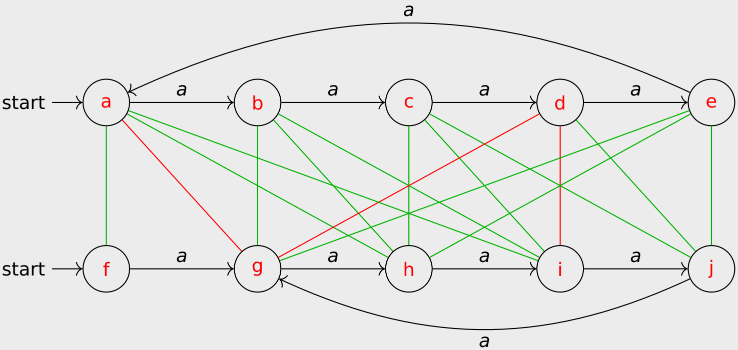
$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$	$a \sim i$	$b \sim j$				

bisimulation

17 comparisons

$b \sim j$ follows from $b \sim g, e \sim g, e \sim j$

Example



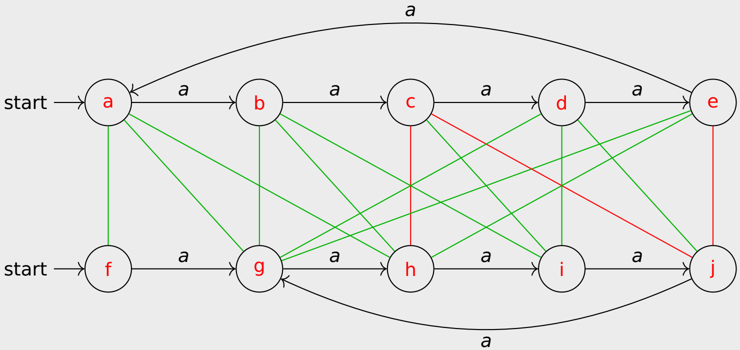
$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$
$e \sim h$	$a \sim i$					

bisimulation

$a \sim i$ follows from $a \sim g, d \sim g, d \sim i$

16 comparisons

Example



$a \sim f$ $b \sim g$ $c \sim h$ $d \sim i$ $e \sim j$ $a \sim g$ $b \sim h$

$c \sim i$ $d \sim j$ $e \sim g$ $a \sim h$ $b \sim i$ $c \sim j$ $d \sim g$

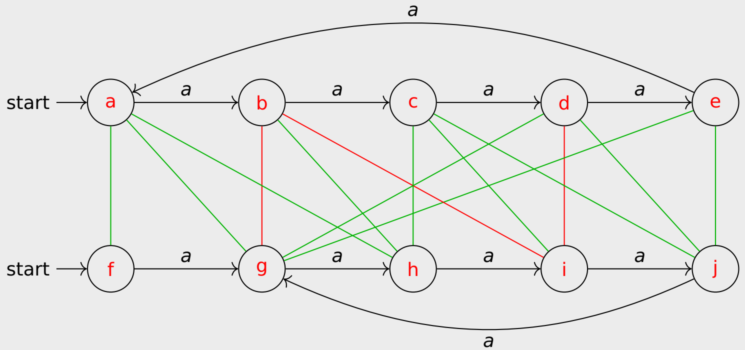
$e \sim h$

bisimulation

$e \sim h$ follows from $c \sim h, c \sim j, e \sim j$

15 comparisons

Example



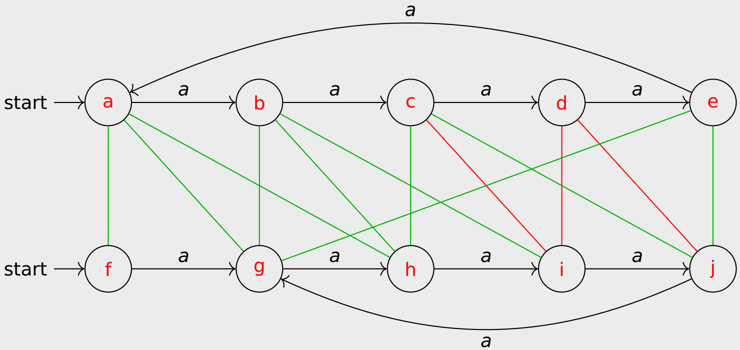
$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$	$c \sim j$	$d \sim g$

bisimulation

$d \sim g$ follows from $b \sim g$, $b \sim i$, $d \sim i$

14 comparisons

Example



$a \sim f$ $b \sim g$ $c \sim h$ $d \sim i$ $e \sim j$ $a \sim g$ $b \sim h$

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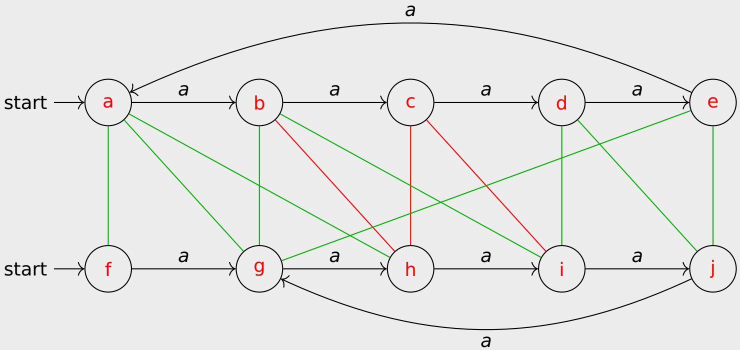
bisimulation

$c \sim j$ follows from $c \sim i, d \sim i, d \sim j$

13 comparisons

Example



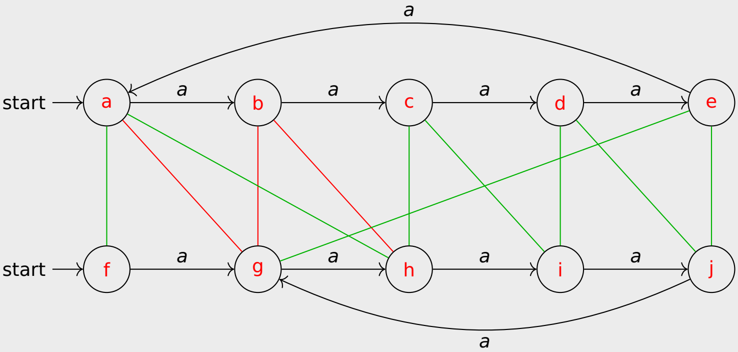
$a \sim f$	$b \sim g$	$c \sim h$	$d \sim i$	$e \sim j$	$a \sim g$	$b \sim h$
$c \sim i$	$d \sim j$	$e \sim g$	$a \sim h$	$b \sim i$		

bisimulation

$b \sim i$ follows from $b \sim h, c \sim h, c \sim i$

12 comparisons

Example



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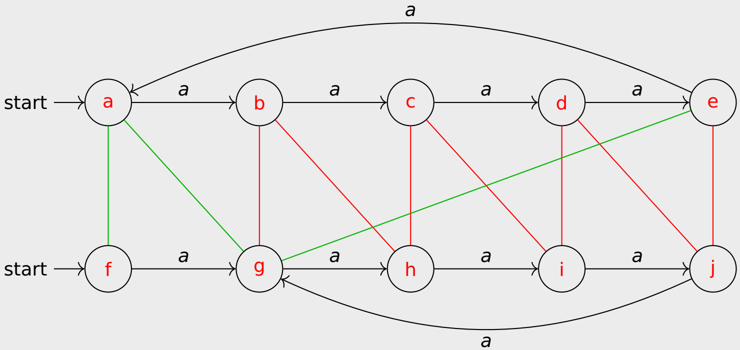
$c \sim i$ $d \sim j$ $e \sim g$ $a \sim h$

bisimulation

$a \sim h$ follows from $a \sim g, b \sim g, b \sim h$

11 comparisons

Example



$a \sim f$ $b \sim g$ $c \sim h$ $d \sim i$ $e \sim j$ $a \sim g$ $b \sim h$

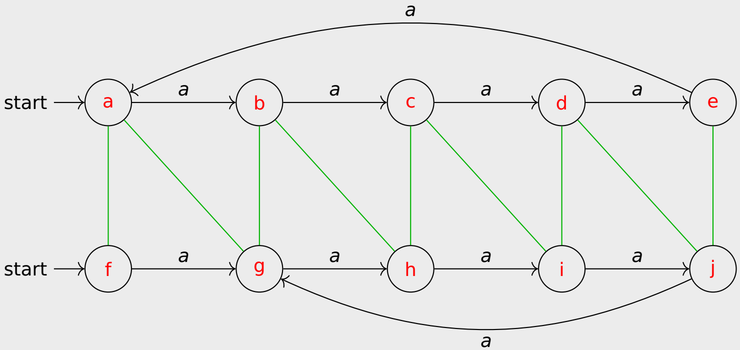
$c \sim i$ $d \sim j$ $e \sim g$

bisimulation

$e \sim g$ follows from $b \sim g, b \sim h, c \sim h, c \sim i, d \sim i, d \sim j, e \sim j$

10 comparisons

Example



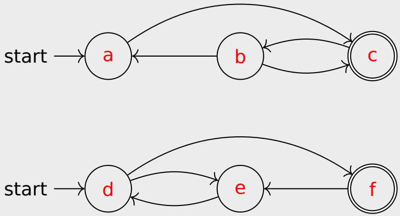
$a \sim f$ $b \sim g$ $c \sim h$ $d \sim i$ $e \sim j$ $a \sim g$ $b \sim h$
$c \sim i$ $d \sim j$

bisimulation up to equivalence

9 instead of 21 comparisons

for NFAs on-the-fly determinization is incorporated

Example



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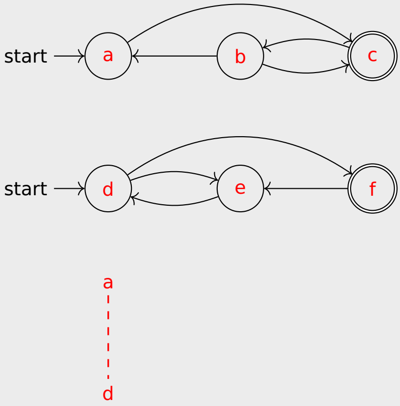

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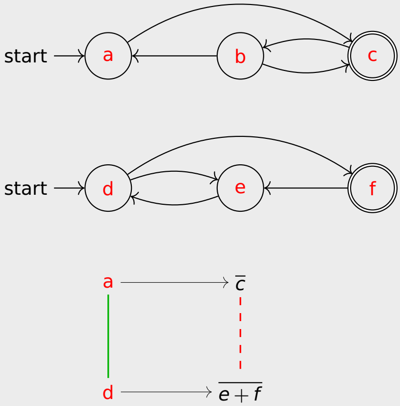

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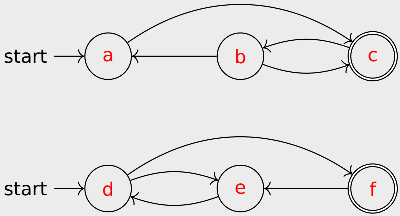

$$\begin{tikzcd}
a \arrow[r] \arrow[d, dash] & \overline{c} \arrow[r] \arrow[d, dash] & b \arrow[d, dash, dashed] \\
d \arrow[r] & \overline{e+f} \arrow[r] & d+e
\end{tikzcd}$$

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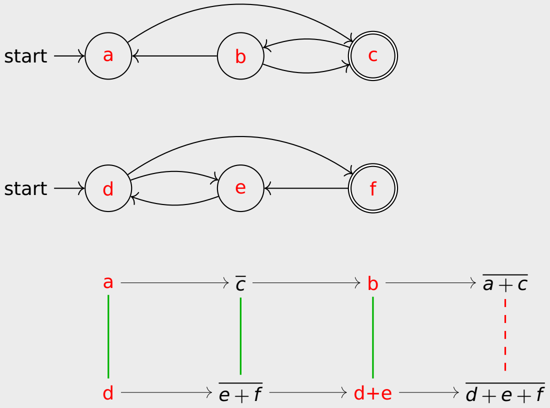

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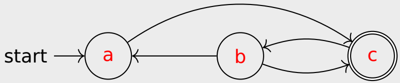

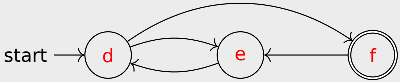

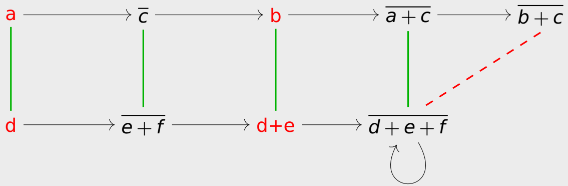

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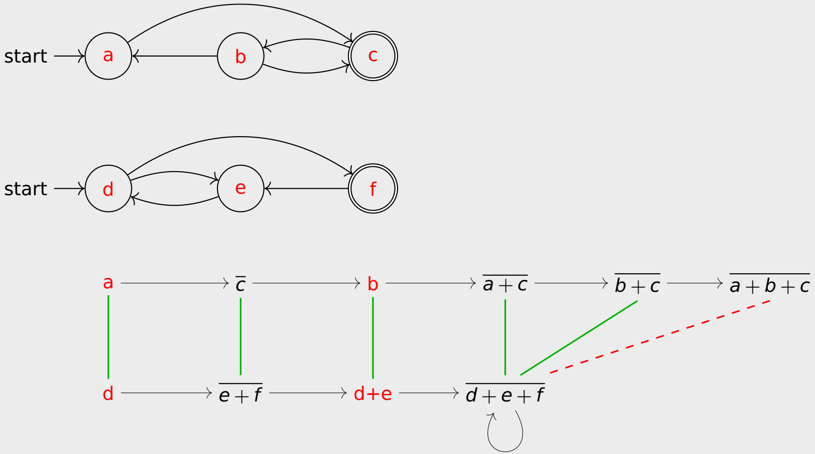

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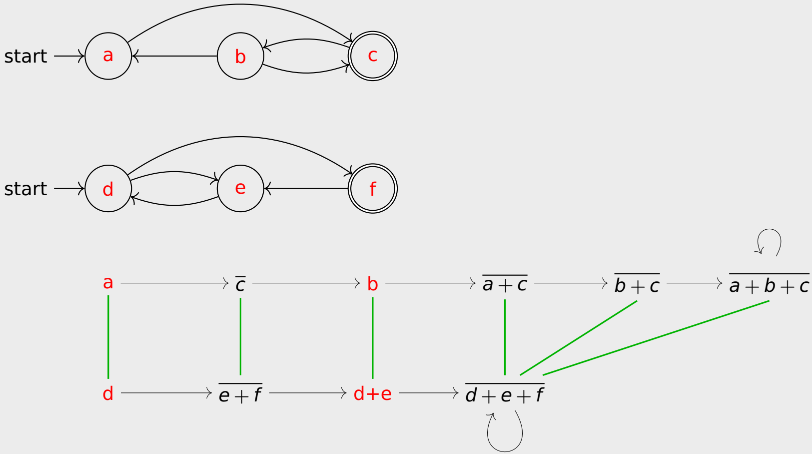

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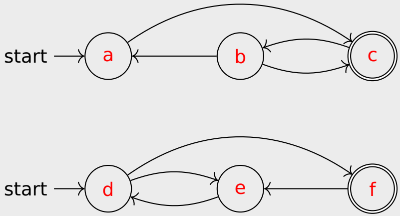

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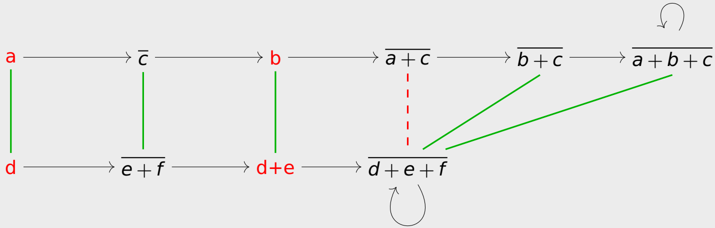
$\implies \ a + c \sim d + e + f$



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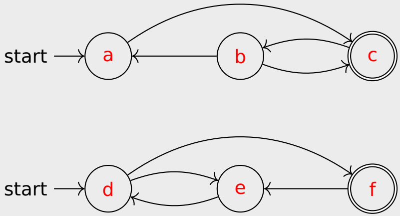

$$a \sim d \ \wedge \ c \sim e + f$$
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$$\begin{tikzcd}
a \arrow[r] \arrow[d, no head] & \overline{c} \arrow[r] \arrow[d, no head] & b \arrow[d, no head] \\
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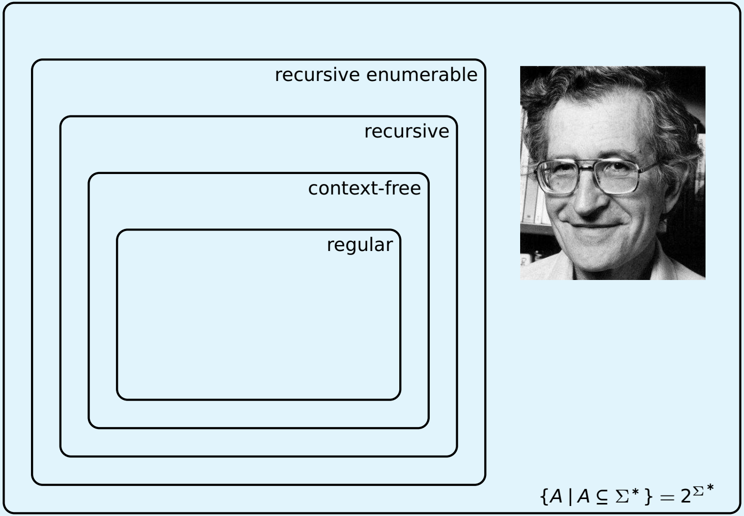

recursive enumerable
recursive
context-free
regular
$\{A \mid A \subseteq \Sigma^*\} = 2^{\Sigma^*}$

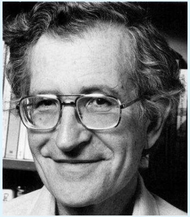
recursive enumerable

recursive

context-free

push-down automata

context-free grammars



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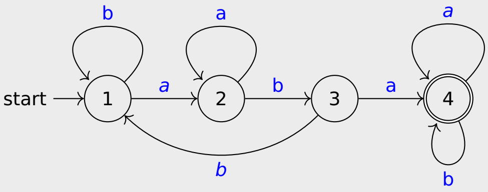
- set $B \subseteq \Sigma^*$ is context-free if $B = L(G)$ for some CFG G

Example

DFA M



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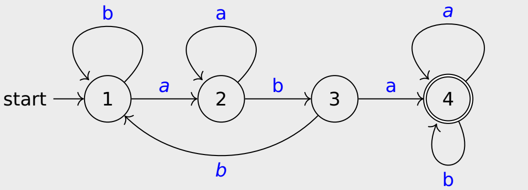

CFG G_M

$$1 \to a\,2 \mid b\,1$$
$$2 \to a\,2 \mid b\,3$$
$$3 \to a\,4 \mid b\,1$$
$$4 \to a\,4 \mid b\,4 \mid \varepsilon$$

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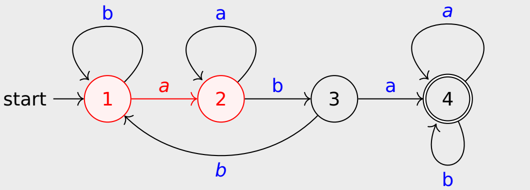

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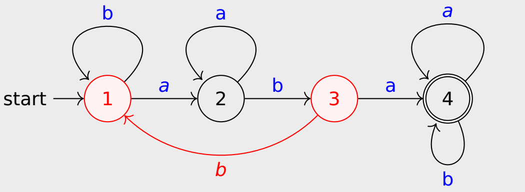

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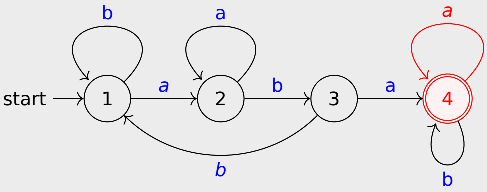

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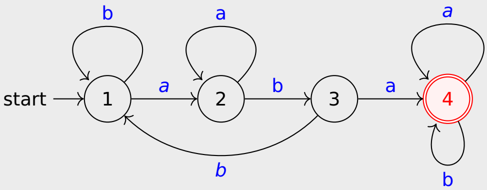

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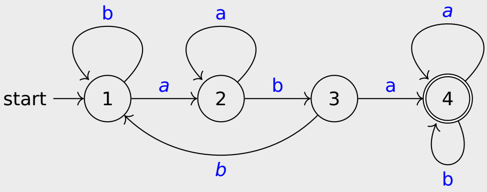

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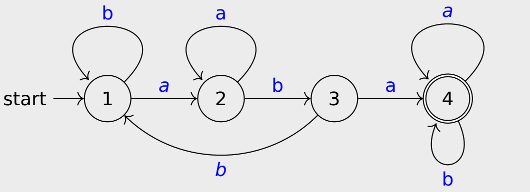
DFA 1

CFG 1

Example

DFA M



CFG G_M

$1 \rightarrow a\,2 \mid b\,1$

$2 \rightarrow a\,2 \mid b\,3$

$3 \rightarrow a\,4 \mid b\,1$

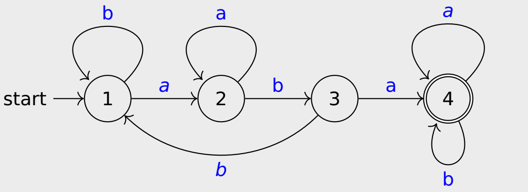
$4 \rightarrow a\,4 \mid b\,4 \mid \varepsilon$

DFA $1 \xrightarrow{b} 1$

CFG $1 \rightarrow b\,1$

Example

DFA M



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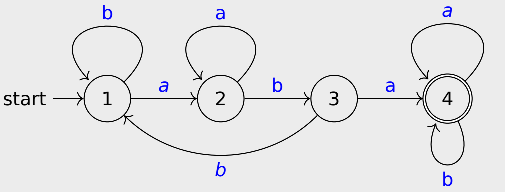
DFA $1 \xrightarrow{b} 1 \xrightarrow{a} 2$

CFG $1 \rightarrow b\,1 \rightarrow b\,a\,2$

Example

DFA M



CFG G_M

$$1 \to a\,2 \mid b\,1 \qquad 3 \to a\,4 \mid b\,1$$
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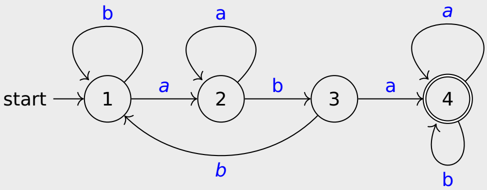
DFA $\quad 1\ ^{b}\ 1\ ^{a}\ 2\ ^{b}\ 3$

CFG $\quad 1 \to b\,1 \to b\,a\,2 \to b\,a\,b\,3$

Example

DFA M



CFG G_M

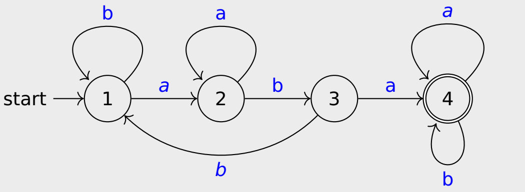
$$1 \to a\,2 \mid b\,1 \qquad 3 \to a\,4 \mid b\,1$$
$$2 \to a\,2 \mid b\,3 \qquad 4 \to a\,4 \mid b\,4 \mid \varepsilon$$

DFA $\quad 1 \xrightarrow{b} 1 \xrightarrow{a} 2 \xrightarrow{b} 3 \xrightarrow{a} 4$

CFG $\quad 1 \to b\,1 \to b\,a\,2 \to b\,a\,b\,3 \to b\,a\,b\,a\,4$

Example

DFA M



CFG G_M

$$1 \to a\,2 \mid b\,1 \qquad 3 \to a\,4 \mid b\,1$$

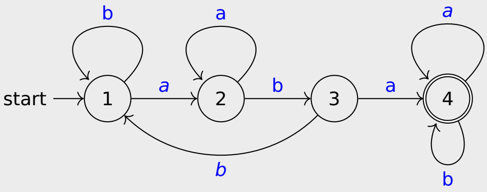
$$2 \to a\,2 \mid b\,3 \qquad 4 \to a\,4 \mid b\,4 \mid \varepsilon$$

DFA $\quad 1 \xrightarrow{b} 1 \xrightarrow{a} 2 \xrightarrow{b} 3 \xrightarrow{a} 4 \xrightarrow{a} 4$

CFG $\quad 1 \to b\,1 \to b\,a\,2 \to b\,a\,b\,3 \to b\,a\,b\,a\,4 \to b\,a\,b\,a\,a\,4$

Example

DFA M



CFG G_M

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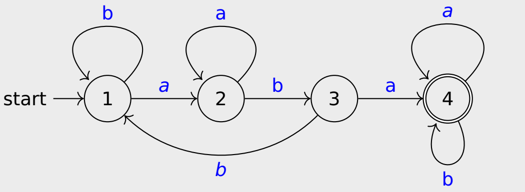
CFG $1 \to b\,1 \to b\,a\,2 \to b\,a\,b\,3 \to b\,a\,b\,a\,4 \to b\,a\,b\,a\,a\,4 \to b\,a\,b\,a\,a$

Example

DFA M



CFG G_M

$$1 \to a\,2 \mid b\,1 \qquad 3 \to a\,4 \mid b\,1$$
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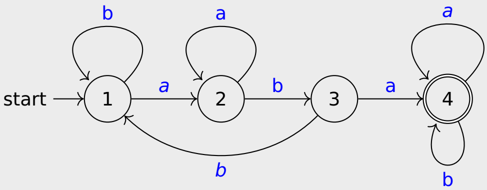
DFA $\quad 1 \overset{b}{\ } 1 \overset{a}{\ } 2 \overset{b}{\ } 3 \overset{a}{\ } 4 \overset{a}{\ } 4 \in L(M)$

CFG $\quad 1 \to b\,1 \to b\,a\,2 \to b\,a\,b\,3 \to b\,a\,b\,a\,4 \to b\,a\,b\,a\,a\,4 \to b\,a\,b\,a\,a \in L(G_M)$

Example

DFA M



strongly right-linear CFG G_M

$$1 \to a\,2 \mid b\,1 \qquad 3 \to a\,4 \mid b\,1$$
$$2 \to a\,2 \mid b\,3 \qquad 4 \to a\,4 \mid b\,4 \mid \varepsilon$$

DFA $\quad 1 \xrightarrow{b} 1 \xrightarrow{a} 2 \xrightarrow{b} 3 \xrightarrow{a} 4 \xrightarrow{a} 4 \in L(M)$

CFG $\quad 1 \to b\,1 \to b\,a\,2 \to b\,a\,b\,3 \to b\,a\,b\,a\,4 \to b\,a\,b\,a\,a\,4 \to b\,a\,b\,a\,a \in L(G_M)$

Definition

CFG $G = (N, \Sigma, P, S)$ is strongly right-linear if

$$\alpha = aB \in \Sigma N \quad \text{or} \quad \alpha = \varepsilon$$

for all $A \to \alpha$ in P

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Lemma

L is regular $\iff$ L is generated by strongly right-linear CFG

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Lemma

L is regular $\iff$ L is generated by strongly right-linear CFG

Proof. ($\Longrightarrow$)

- DFA $M = (Q, \Sigma, \delta, s, F)$
- $L(M) = L(G_M)$ for strongly right-linear CFG $G_M = \{Q, \Sigma, P, s\}$ with

$$P = \{p \to aq \mid \delta(p, a) = q\} \cup \{q \to \varepsilon \mid q \in F\}$$

Definition

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Lemma

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Proof. ($\Longleftarrow$)

- strongly right-linear CFG $G = (N, \Sigma, P, S)$
- $L(G) = L(M_G)$ for NFA $M_G = (N, \Delta, \{S\}, F)$ with

$$\Delta(A, a) = \{B \mid A \to aB \in P\} \quad \text{and} \quad F = \{A \mid A \to \varepsilon \in P\}$$

Definition

CFG $G = (N, \Sigma, P, S)$ is strongly right-linear if

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Lemma

L is regular $\iff$ L is generated by strongly right-linear CFG

Corollary

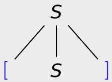
every regular set is context-free

Example

CFG G: $S \to [S] \mid S\,S \mid \varepsilon$

1. $S \xrightarrow[G]{1} [S]$

parse tree



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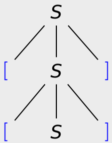
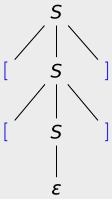


Example

CFG G: $S \to [S] \mid SS \mid \varepsilon$

1. $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[]]$

parse tree



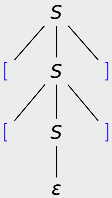
Example

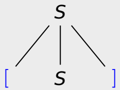
CFG G: $S \to [S] \mid SS \mid \varepsilon$

1. $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[]]$

2. $S \xrightarrow[G]{1} [S]$

parse trees





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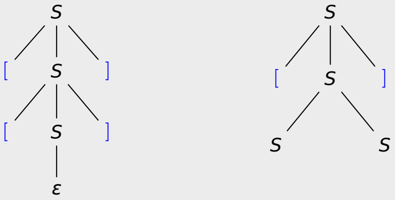

Example

CFG G: $S \rightarrow [S] \mid SS \mid \varepsilon$

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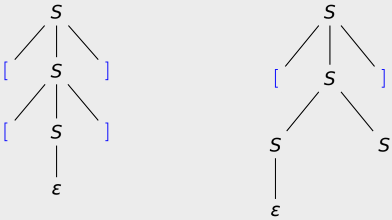

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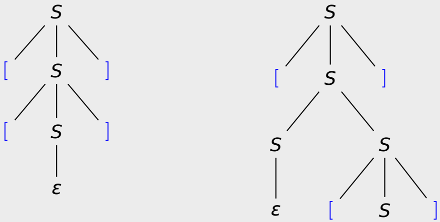
parse trees



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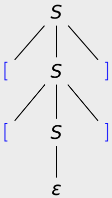
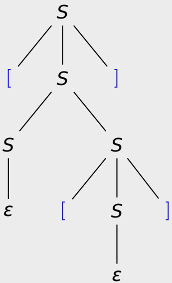


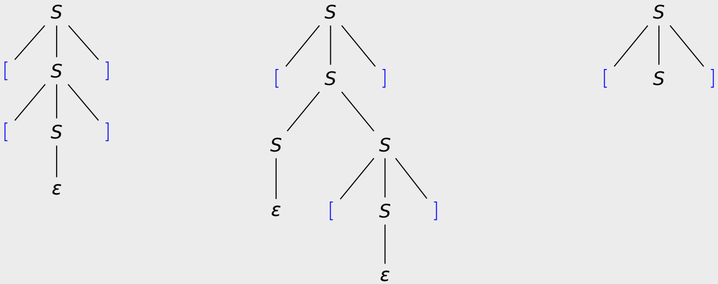


Example

CFG G: $S \to [S] \mid S\,S \mid \varepsilon$

1. $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[]]$
2. $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [S\,S] \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[]]$
3. $S \xrightarrow[G]{1} [S]$

parse trees



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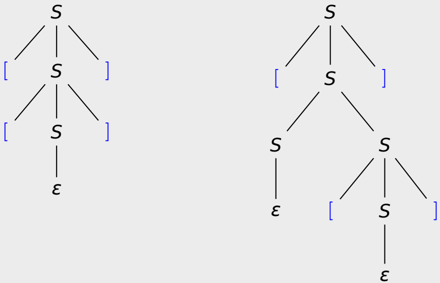

Example

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① $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[]]$

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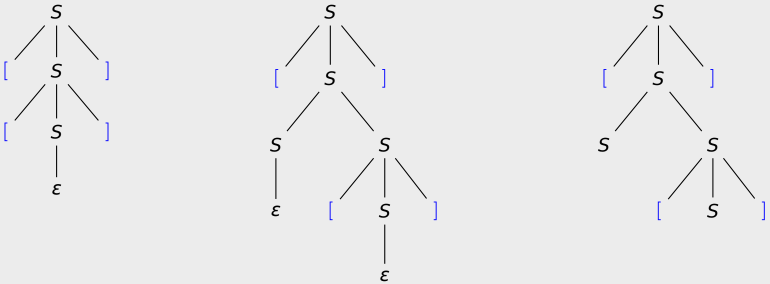
parse trees



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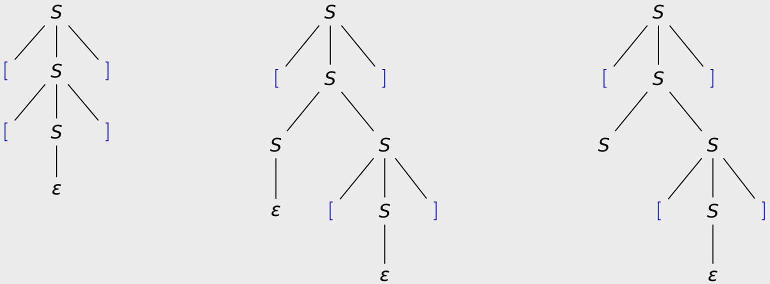

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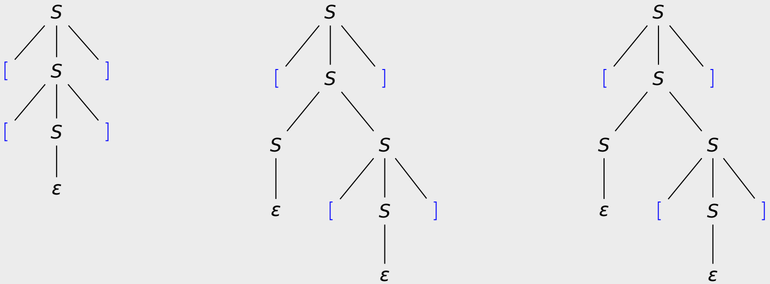

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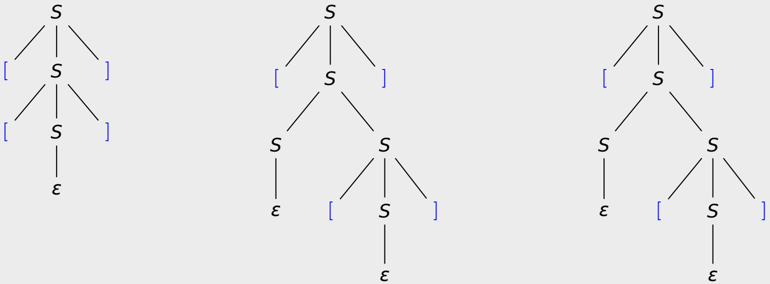

Definition

- CFG is ambiguous if some string has different parse trees

Example

- CFG G : $S \to S \times S \mid S + S \mid \mathit{int}$

- G is ambiguous

  with G $7 + 5 \times 2$ could be parsed as $7 + (5 \times 2)$ and $(7 + 5) \times 2$

Example

- CFG G : $S \rightarrow S \times S \mid S + S \mid int$
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  $U \rightarrow int \mid (S)$
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- G is ambiguous G' is unambiguous $L(G) = L(G')$

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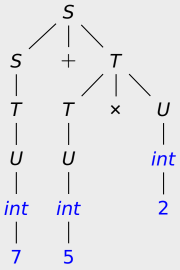
  with G' $7 + 5 \times 2$ could only be parsed as $7 + (5 \times 2)$



Example

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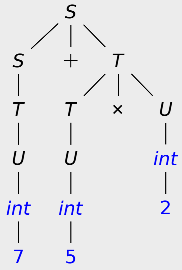
  with G $\quad 7 + 5 \times 2$ could be parsed as $\quad 7 + (5 \times 2)$ and $(7 + 5) \times 2$
  with G' $\quad 7 + 5 \times 2$ could only be parsed as $\quad 7 + (5 \times 2)$
- (if applicable) one way to remove ambiguity is to benefit from precedence of operators



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let $A = L(G)$ such that G:

$$S \to T \mid W$$

$$\begin{array}{ll} T \to UV & W \to XY \\ U \to aUb \mid \varepsilon & X \to aX \mid \varepsilon \\ V \to cV \mid \varepsilon & Y \to bYc \mid \varepsilon \end{array}$$

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Lemma

there is no CFG G' such that $L(G')$ is unambiguous with $L(G') = A$

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Lemma

there is no CFG G' such that $L(G')$ is unambiguous with $L(G') = A$

Remark

1. given an ambiguous CFG G, the language $L(G)$ may or may not be ambiguous

Remark

there exist context-free sets without unambiguous grammars (inherently ambiguous)

Example

$A = \{a^i b^j c^k \mid i = j \text{ or } j = k\}$ is context-free and inherently ambiguous

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let $A = L(G)$ such that G:

$$S \to T \mid W$$

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the union we used has a non-empty intersection, where letters a, b and c all are in equal number

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there is no CFG G' such that $L(G')$ is unambiguous with $L(G') = A$

Remark

1. given an ambiguous CFG G, the language $L(G)$ may or may not be ambiguous
2. there is no algorithm to convert ambiguous CFG to unambiguous CFG

Remark

there exist context-free sets without unambiguous grammars (inherently ambiguous)

Example

$A = \{a^i b^j c^k \mid i = j \text{ or } j = k\}$ is context-free and inherently ambiguous

$A = \{a^i b^i c^k\} \cup \{a^i b^j c^j\}$

let $A = L(G)$ such that G:

$$S \to T \mid W$$

$$\begin{array}{ll} T \to UV & W \to XY \\ U \to aUb \mid \varepsilon & X \to aX \mid \varepsilon \\ V \to cV \mid \varepsilon & Y \to bYc \mid \varepsilon \end{array}$$

the union we used has a non-empty intersection, where letters a, b and c all are in equal number

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there is no CFG G' such that $L(G')$ is unambiguous with $L(G') = A$

Remark

1. given an ambiguous CFG G, the language $L(G)$ may or may not be ambiguous
2. there is no algorithm to convert ambiguous CFG to unambiguous CFG
3. unambiguous context free languages can be parsed by deterministic push down automata

Outline

Definitions

CFG $G = (N, \Sigma, P, S)$ is in

- Chomsky normal form if for all $A \to \alpha$ in P $\quad \alpha = B\,C \in N^2 \quad$ or $\quad \alpha = a \in \Sigma$

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every context-free set not containing ϵ is generated by CFG in

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2. Chomsky normal form

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membership $x \in L(G)$ is easily decidable for CFG G in Greibach normal form

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Theorem

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1. Greibach normal form
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Application

membership $x \in L(G)$ is easily decidable for CFG G in Chomsky normal form:

generate all derivations of length $2|x| - 1$ *and test whether one of them ends in* x

Definitions

- ε-production: $A \rightarrow \varepsilon$

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- unit production: $A \rightarrow B$

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- ε-production: $A \to \varepsilon$
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every context-free set is generated by CFG without unit productions

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every context-free set is generated by CFG without unit productions

Proof.

- $\widehat{P}$ is smallest set containing P such that

$$A \to B \in \widehat{P} \quad \wedge \quad B \to \alpha \in \widehat{P} \quad \Longrightarrow \quad A \to \alpha \in \widehat{P}$$

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- remove all unit productions from $\widehat{P}$

Definitions

- ε-production: $A \to \varepsilon$
- unit production: $A \to B$

Lemma

every context-free set not containing ε is generated by CFG without ε-productions

Proof.

- $\widehat{P}$ is smallest set containing P such that

$$A \to \alpha B \beta \in \widehat{P} \quad \wedge \quad B \to \varepsilon \in \widehat{P} \quad \Longrightarrow \quad A \to \alpha \beta \in \widehat{P}$$

- remove all ε-productions from $\widehat{P}$

Definitions

- ϵ-production: $A \to \epsilon$
- unit production: $A \to B$

Lemma

every context-free set not containing ϵ is generated by CFG without ϵ and unit productions

Proof.

- $\widehat{P}$ is smallest set containing P such that

$$
\begin{aligned}
A \to B \in \widehat{P} \quad &\wedge \quad B \to \alpha \in \widehat{P} \quad \Longrightarrow \quad A \to \alpha \in \widehat{P} \\
A \to \alpha B \beta \in \widehat{P} \quad &\wedge \quad B \to \epsilon \in \widehat{P} \quad \Longrightarrow \quad A \to \alpha \beta \in \widehat{P}
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Proof.

- remove ε and unit productions

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Proof.

- remove ε and unit productions
- introduce $A_a \to a$ for every $a \in \Sigma$

Example

$S \to S\,S \qquad S \to [\,S\,] \qquad S \to [\,]$

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Example

$$S \to S\,S \qquad S \to A\,S\,B \qquad S \to A\,B$$

$$A \to [\qquad B \to]$$

Theorem

every context-free set not containing ε is generated by CFG in Chomsky normal form

Proof.

- remove ε and unit productions
- introduce $A_a \to a$ for every $a \in \Sigma$
- split long right-hand sides of productions

Example

$$S \to SS \qquad S \to ASB \qquad S \to AB$$
$$A \to [\qquad B \to]$$

Theorem

every context-free set not containing ϵ is generated by CFG in Chomsky normal form

Proof.

- remove ϵ and unit productions
- introduce $A_a \to a$ for every $a \in \Sigma$
- split long right-hand sides of productions

Example

$$S \to S\,S \qquad S \to A\,C \qquad S \to A\,B$$
$$A \to [\qquad B \to] \qquad C \to S\,B$$

Example

CFG G: $S \to X\mathrm{b}S \mid XY\mathrm{b} \mid YXZ \quad X \to Z \mid \varepsilon \quad Y \to \mathrm{b}XY \mid \varepsilon \quad Z \to \mathrm{a}$
remove ε and unit productions

$$S \to X\mathrm{b}S \mid XY\mathrm{b} \mid YXZ \qquad X \to Z \mid \varepsilon \qquad Y \to \mathrm{b}XY \mid \varepsilon \qquad Z \to \mathrm{a}$$

Example

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$$S \to \mathrm{b}S$$

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$$S \to \mathrm{b}S \mid Y\mathrm{b}$$

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$$S \to \mathrm{b}S \mid Y\mathrm{b} \mid YZ \qquad Y \to \mathrm{b}Y$$

Example

CFG G: $S \to X\mathrm{b}S \mid XY\mathrm{b} \mid YXZ \qquad X \to Z \mid \varepsilon \qquad Y \to \mathrm{b}XY \mid \varepsilon \qquad Z \to \mathrm{a}$
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introduce new non-terminals

$$B \to \text{b}$$

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introduce new non-terminals

$$B \to \mathrm{b} \qquad S \to XBS \mid XYB \mid YXZ \mid BS \mid YB \mid YZ \mid XB \mid XZ \mid \mathrm{b} \mid \mathrm{a}$$

$$X \to \mathrm{a} \qquad Y \to BXY \mid BY \mid BX \mid \mathrm{b} \qquad Z \to \mathrm{a}$$

Example

CFG G: $S \to X\mathrm{b}S \mid XY\mathrm{b} \mid YXZ \qquad X \to Z \mid \varepsilon \qquad Y \to \mathrm{b}XY \mid \varepsilon \qquad Z \to \mathrm{a}$

remove ε and unit productions

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split long right-hand sides

$$B \to \mathrm{b} \qquad S \to TS \mid UB \mid VZ \mid BS \mid YB \mid YZ \mid XB \mid XZ \mid \mathrm{b} \mid \mathrm{a}$$

$$X \to \mathrm{a} \qquad Y \to BU \mid BY \mid BX \mid \mathrm{b} \qquad Z \to \mathrm{a}$$

$$T \to XB \qquad U \to XY \qquad V \to YX$$

Outline

Pumping Lemma

$$
A \text{ is context-free} \implies \begin{cases} \exists\, k & \\ \forall\, z \in A & \text{with } |z| \geqslant k \\ \exists\, u, v, w, x, y & \text{with } \begin{cases} z = uvwxy \\ |vwx| \leqslant k \\ vx \neq \varepsilon \end{cases} \\ \forall\, i \geqslant 0 & uv^i w x^i y \in A \end{cases}
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Pumping Lemma

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Proof. (Idea)

take $k = 2^{n+1}$ where n is number of nonterminals of any CFG in Chomsky normal form that accepts $A - \{\varepsilon\}$

Example

$L = \{a^i b^i \mid i > 0\}$ Chomsky normal form G: $S \to AC \mid AB$ $C \to SB$ $A \to a$ $B \to b$

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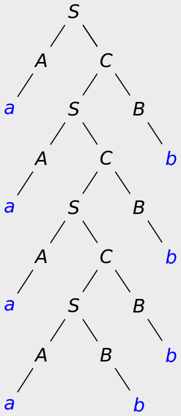

parse tree for *aaaabbbb*

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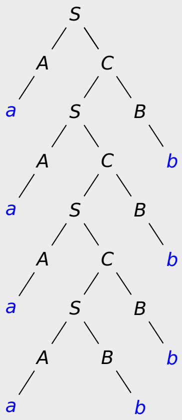

parse tree for *aaaabbbb*

long string $\implies$ long path in parse tree

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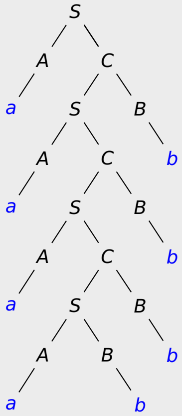

parse tree for *aaaabbbb*

long string $\Longrightarrow$ long path in parse tree

(at depth m at most 2^m symbols)

Example

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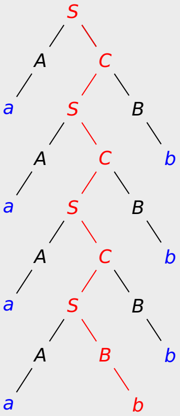


parse tree for *aaaabbbb*

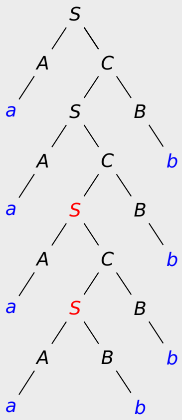
long string $\Longrightarrow$ long path in parse tree

(at depth m at most 2^m symbols)

consider longest path

Example

$L = \{\mathrm{a}^i\mathrm{b}^i \mid i > 0\}$ Chomsky normal form G: $S \to AC \mid AB$ $\quad C \to SB$ $\quad A \to \mathrm{a}$ $\quad B \to \mathrm{b}$



parse tree for *aaaabbbb*

long string $\implies$ long path in parse tree

(at depth m at most 2^m symbols)

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look for repetition of nonterminals near bottom

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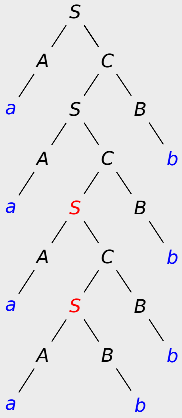

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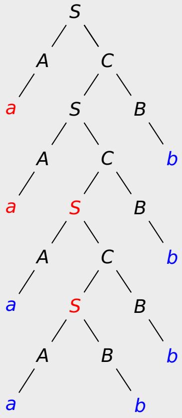

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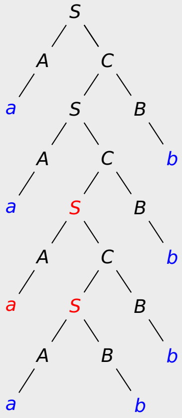

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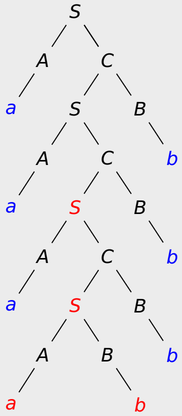

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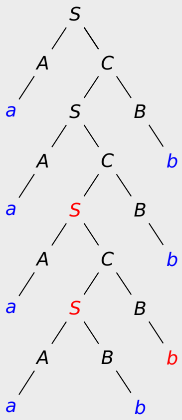

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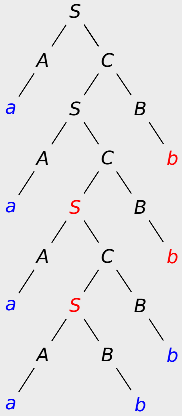

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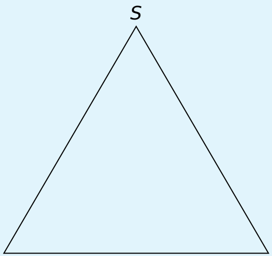
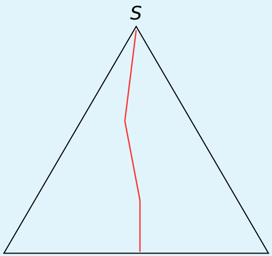
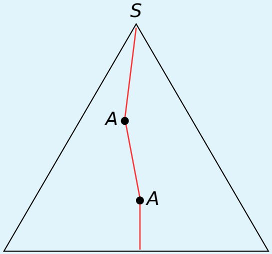
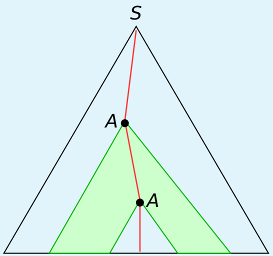
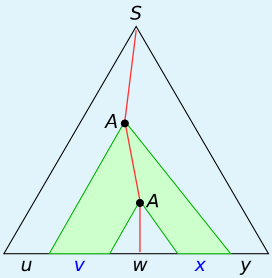

S


S


S
A
A


S
A
A


S
A
A
u
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w
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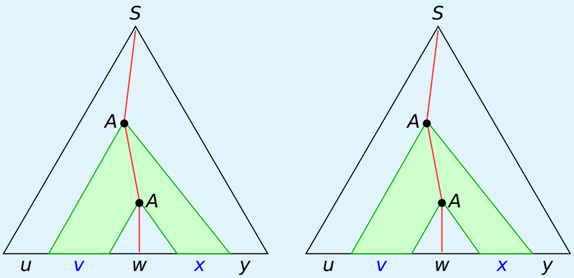
y



$S \xrightarrow[G]{*} uAy \qquad A \xrightarrow[G]{*} vAx \qquad A \xrightarrow[G]{*} w$

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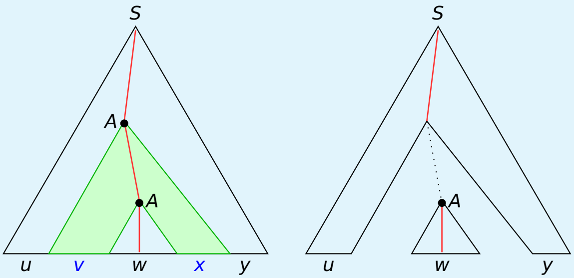

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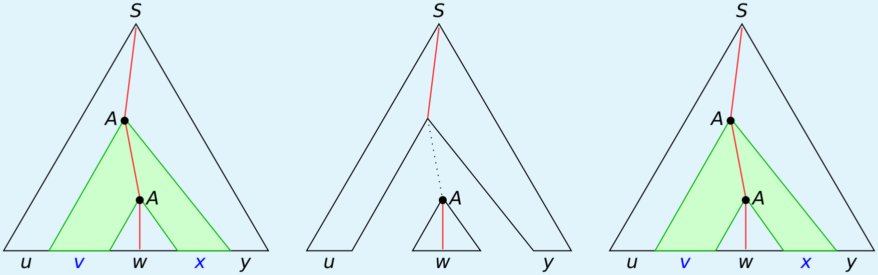

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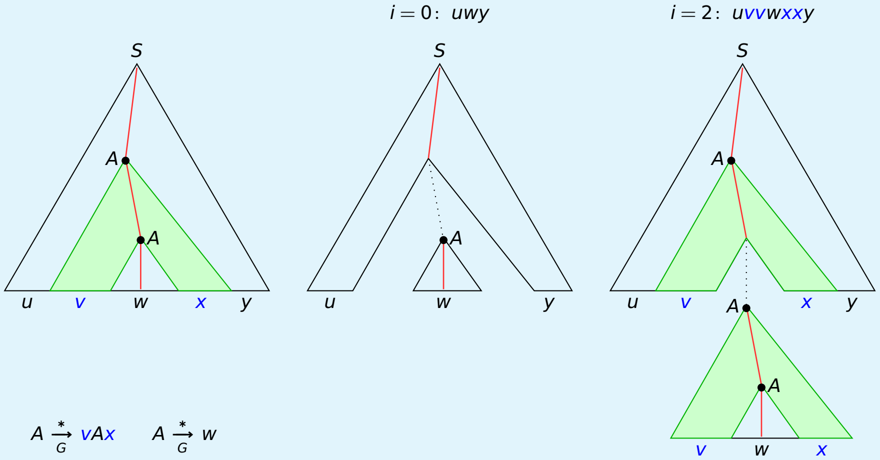

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Pumping Lemma (Contrapositive)

$$\left.\begin{array}{ll} \forall\, k & \\ \exists\, z \in A & \text{with } |z| \geqslant k \\ \forall\, u, v, w, x, y & \text{with } \begin{cases} z = uvwxy \\ |vwx| \leqslant k \\ vx \neq \varepsilon \end{cases} \\ \exists\, i \geqslant 0 & \text{with } uv^iwx^iy \notin A \end{array}\right\} \implies A \text{ is not context-free}$$

Example

$A = \{a^i b^i c^i \mid i \geqslant 0\}$ is not context-free

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- choose $\boxed{z = a^k b^k c^k}$ check: $z \in A$ $|z| = 3k \geqslant k$

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2. vwx has no c's: uv^iwx^iy has more c's than b's or a's

Example

$B = \{\mathrm{a}^i\mathrm{b}^j\mathrm{c}^k \mid i < j < k\}$ is not context-free

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  $|uv^iwx^iy| = p + p\,|vx| = p\,(1 + |vx|)$ is not prime

Outline

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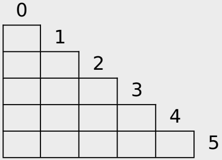
- $x \in L(G) \quad \Longleftrightarrow \quad S \in T_{0|x|}$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$



$$x = baaba$$

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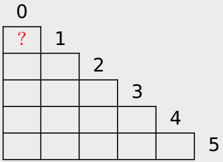

$$x = baaba$$

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$$T_{01} = \{X \in N \mid X \xrightarrow[G]{*} \mathrm{b}\}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \mathrm{a} \quad B \to CC \mid \mathrm{b} \quad C \to AB \mid \mathrm{a}$

0					
?	1				
		2			
			3		
				4	
					5

$$x = baaba$$

$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$\begin{aligned} T_{01} &= \{X \in N \mid X \xrightarrow[G]{*} \mathrm{b}\} \\ &= \{X \in N \mid X \to \mathrm{b}\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \mathrm{a} \quad B \to CC \mid \mathrm{b} \quad C \to AB \mid \mathrm{a}$

	0	1	2	3	4
1	$\{B\}$				
2					
3					
4				$\{B\}$	
5					

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$$\begin{aligned} T_{34} = T_{01} &= \{X \in N \mid X \xrightarrow[G]{*} \mathrm{b}\} \\ &= \{X \in N \mid X \to \mathrm{b}\} \\ &= \{B\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \mathrm{a} \quad B \to CC \mid \mathrm{b} \quad C \to AB \mid \mathrm{a}$

0					
$\{B\}$	1				
	?	2			
		?	3		
			$\{B\}$	4	
				?	5

$$x = baaba$$
$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$\begin{aligned} T_{12} = T_{23} = T_{45} &= \{X \in N \mid X \xrightarrow[G]{*} \mathrm{a}\} \\ &= \{X \in N \mid X \to \mathrm{a}\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \mathsf{a} \quad B \to CC \mid \mathsf{b} \quad C \to AB \mid \mathsf{a}$

0	1	2	3	4	
$\{B\}$					
	$\{A, C\}$				
		$\{A, C\}$			
			$\{B\}$		
				$\{A, C\}$	5

$$
\begin{aligned}
x &= \mathsf{baaba} \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{12} = T_{23} = T_{45} &= \{X \in N \mid X \xrightarrow[G]{*} \mathsf{a}\} \\
&= \{X \in N \mid X \to \mathsf{a}\} \\
&= \{A, C\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid a \quad B \to CC \mid b \quad C \to AB \mid a$

0	1	2	3	4	
$\{B\}$	1				
?	$\{A, C\}$	2			
		$\{A, C\}$	3		
			$\{B\}$	4	
				$\{A, C\}$	5

$$x = baaba$$
$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$T_{02} = \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{12}\}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$

0	1	2	3	4	5
$\{B\}$					
?	$\{A, C\}$				
		$\{A, C\}$			
			$\{B\}$		
				$\{A, C\}$	

$$x = baaba$$

$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$\begin{aligned} T_{02} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{12}\} \\ &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{A, C\}\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC$ $\quad A \to BA \mid \text{a}$ $\quad B \to CC \mid \text{b}$ $\quad C \to AB \mid \text{a}$

0	1	2	3	4	5
$\{B\}$					
?	$\{A, C\}$				
		$\{A, C\}$			
			$\{B\}$		
				$\{A, C\}$	

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Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
		$\{A, C\}$	3		
			$\{B\}$	4	
				$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{02} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{12}\} \\
&= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{A, C\}\} \\
&= \{X \in N \mid X \to BA \in P \text{ or } X \to BC \in P\} \\
&= \{S, A\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \mathrm{a} \quad B \to CC \mid \mathrm{b} \quad C \to AB \mid \mathrm{a}$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
	?	$\{A, C\}$	3		
			$\{B\}$	4	
				$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{13} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{12} \text{ and } Z \in T_{23}\} \\
&= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A, C\} \text{ and } Z \in \{A, C\}\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
	$\{B\}$	$\{A, C\}$	3		
			$\{B\}$	4	
				$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{13} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{12} \text{ and } Z \in T_{23}\} \\
&= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A, C\} \text{ and } Z \in \{A, C\}\} \\
&= \{B\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \textcolor{blue}{a} \quad B \to CC \mid \textcolor{blue}{b} \quad C \to AB \mid \textcolor{blue}{a}$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
	$\{B\}$	$\{A, C\}$	3		
		?	$\{B\}$	4	
				$\{A, C\}$	5

$$\begin{aligned} x &= baaba \\ T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\} \end{aligned}$$

$$\begin{aligned} T_{24} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{23} \text{ and } Z \in T_{34}\} \\ &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A, C\} \text{ and } Z \in \{B\}\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \mathrm{a} \quad B \to CC \mid \mathrm{b} \quad C \to AB \mid \mathrm{a}$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
	$\{B\}$	$\{A, C\}$	3		
		$\{S, C\}$	$\{B\}$	4	
				$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{24} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{23} \text{ and } Z \in T_{34}\} \\
&= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A, C\} \text{ and } Z \in \{B\}\} \\
&= \{S, C\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \rightarrow AB \mid BC \quad A \rightarrow BA \mid \mathrm{a} \quad B \rightarrow CC \mid \mathrm{b} \quad C \rightarrow AB \mid \mathrm{a}$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
	$\{B\}$	$\{A, C\}$	3		
		$\{S, C\}$	$\{B\}$	4	
			?	$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{35} &= \{X \in N \mid X \rightarrow YZ \in P \text{ with } Y \in T_{34} \text{ and } Z \in T_{45}\} \\
&= \{X \in N \mid X \rightarrow YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{A, C\}\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \mathrm{a} \quad B \to CC \mid \mathrm{b} \quad C \to AB \mid \mathrm{a}$

	0	1	2	3	4
1	$\{B\}$				
2	$\{S, A\}$	$\{A, C\}$			
3		$\{B\}$	$\{A, C\}$		
4			$\{S, C\}$	$\{B\}$	
5				$\{S, A\}$	$\{A, C\}$

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{35} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{34} \text{ and } Z \in T_{45}\} \\
&= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{A, C\}\} \\
&= \{S, A\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$

0	1	2	3	4	
$\{B\}$					
$\{S,A\}$	$\{A,C\}$				
?	$\{B\}$	$\{A,C\}$			
		$\{S,C\}$	$\{B\}$		
			$\{S,A\}$	$\{A,C\}$	5

$$x = baaba$$

$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$\begin{aligned} T_{03} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{13}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{02} \text{ and } Z \in T_{23}\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC$ $\quad A \to BA \mid a$ $\quad B \to CC \mid b$ $\quad C \to AB \mid a$

0	1	2	3	4	
$\{B\}$	1				
$\{S,A\}$	$\{A,C\}$	2			
?	$\{B\}$	$\{A,C\}$	3		
		$\{S,C\}$	$\{B\}$	4	
			$\{S,A\}$	$\{A,C\}$	5

$$x = baaba$$

$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$\begin{aligned} T_{03} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{13}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{02} \text{ and } Z \in T_{23}\} \\ &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{B\}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{S,A\} \text{ and } Z \in \{A,C\}\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid a \quad B \to CC \mid b \quad C \to AB \mid a$

0	1	2	3	4	
$\{B\}$					
$\{S,A\}$	$\{A,C\}$				
$\varnothing$	$\{B\}$	$\{A,C\}$			
		$\{S,C\}$	$\{B\}$		
			$\{S,A\}$	$\{A,C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{03} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{13}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{02} \text{ and } Z \in T_{23}\} \\
&= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{B\}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{S,A\} \text{ and } Z \in \{A,C\}\} \\
&= \varnothing
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid a \quad B \to CC \mid b \quad C \to AB \mid a$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
$\varnothing$	$\{B\}$	$\{A, C\}$	3		
	?	$\{S, C\}$	$\{B\}$	4	
			$\{S, A\}$	$\{A, C\}$	5

$$x = baaba$$
$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$T_{14} = \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{12} \text{ and } Z \in T_{24}\}$$
$$\cup \ \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{13} \text{ and } Z \in T_{34}\}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
$\varnothing$	$\{B\}$	$\{A, C\}$	3		
	?	$\{S, C\}$	$\{B\}$	4	
			$\{S, A\}$	$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{14} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{12} \text{ and } Z \in T_{24}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{13} \text{ and } Z \in T_{34}\} \\
&= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A, C\} \text{ and } Z \in \{S, C\}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{B\}\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid a \quad B \to CC \mid b \quad C \to AB \mid a$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
$\varnothing$	$\{B\}$	$\{A, C\}$	3		
	$\{B\}$	$\{S, C\}$	$\{B\}$	4	
			$\{S, A\}$	$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{14} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{12} \text{ and } Z \in T_{24}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{13} \text{ and } Z \in T_{34}\} \\
&= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A, C\} \text{ and } Z \in \{S, C\}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{B\}\} \\
&= \{B\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC$ $\quad A \to BA \mid \textcolor{blue}{\mathrm{a}}$ $\quad B \to CC \mid \textcolor{blue}{\mathrm{b}}$ $\quad C \to AB \mid \textcolor{blue}{\mathrm{a}}$

0					
$\{B\}$	1				
$\{S,A\}$	$\{A,C\}$	2			
$\varnothing$	$\{B\}$	$\{A,C\}$	3		
	$\{B\}$	$\{S,C\}$	$\{B\}$	4	
		?	$\{S,A\}$	$\{A,C\}$	5

$$x = \textcolor{blue}{ba}\textcolor{red}{aba}$$

$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$\begin{aligned} T_{25} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{23} \text{ and } Z \in T_{35}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{24} \text{ and } Z \in T_{45}\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC$ $\quad A \to BA \mid \text{a}$ $\quad B \to CC \mid \text{b}$ $\quad C \to AB \mid \text{a}$

0	1	2	3	4	
$\{B\}$					
$\{S, A\}$	$\{A, C\}$				
$\varnothing$	$\{B\}$	$\{A, C\}$			
	$\{B\}$	$\{S, C\}$	$\{B\}$		
		?	$\{S, A\}$	$\{A, C\}$	5

$$x = baaba$$

$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$\begin{aligned} T_{25} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{23} \text{ and } Z \in T_{35}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{24} \text{ and } Z \in T_{45}\} \\ &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A, C\} \text{ and } Z \in \{S, A\}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{S, C\} \text{ and } Z \in \{A, C\}\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid a \quad B \to CC \mid b \quad C \to AB \mid a$

0	1	2	3	4	
$\{B\}$					
$\{S, A\}$	$\{A, C\}$				
$\varnothing$	$\{B\}$	$\{A, C\}$			
	$\{B\}$	$\{S, C\}$	$\{B\}$		
		$\{B\}$	$\{S, A\}$	$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{25} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{23} \text{ and } Z \in T_{35}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{24} \text{ and } Z \in T_{45}\} \\
&= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A, C\} \text{ and } Z \in \{S, A\}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{S, C\} \text{ and } Z \in \{A, C\}\} \\
&= \{B\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
$\varnothing$	$\{B\}$	$\{A, C\}$	3		
?	$\{B\}$	$\{S, C\}$	$\{B\}$	4	
		$\{B\}$	$\{S, A\}$	$\{A, C\}$	5

$$x = baaba$$

$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$\begin{aligned} T_{04} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{14}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{02} \text{ and } Z \in T_{24}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{03} \text{ and } Z \in T_{34}\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$

0					
$\{B\}$	1				
$\{S, A\}$	$\{A, C\}$	2			
$\varnothing$	$\{B\}$	$\{A, C\}$	3		
?	$\{B\}$	$\{S, C\}$	$\{B\}$	4	
		$\{B\}$	$\{S, A\}$	$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{04} = \; & \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{B\}\} \\
\cup \; & \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{S, A\} \text{ and } Z \in \{S, C\}\} \\
\cup \; & \{X \in N \mid X \to YZ \in P \text{ with } Y \in \varnothing \text{ and } Z \in \{B\}\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid a \quad B \to CC \mid b \quad C \to AB \mid a$

0	1	2	3	4	
$\{B\}$					
$\{S, A\}$	$\{A, C\}$				
$\varnothing$	$\{B\}$	$\{A, C\}$			
$\varnothing$	$\{B\}$	$\{S, C\}$	$\{B\}$		
		$\{B\}$	$\{S, A\}$	$\{A, C\}$	5

$$\begin{aligned} x &= baaba \\ T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\} \end{aligned}$$

$$\begin{aligned} T_{04} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{B\}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{S, A\} \text{ and } Z \in \{S, C\}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \varnothing \text{ and } Z \in \{B\}\} \\ &= \varnothing \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \mathrm{a} \quad B \to CC \mid \mathrm{b} \quad C \to AB \mid \mathrm{a}$

0					
$\{B\}$	1				
$\{S,A\}$	$\{A,C\}$	2			
$\varnothing$	$\{B\}$	$\{A,C\}$	3		
$\varnothing$	$\{B\}$	$\{S,C\}$	$\{B\}$	4	
	?	$\{B\}$	$\{S,A\}$	$\{A,C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{15} = &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{12} \text{ and } Z \in T_{25}\} \\
\cup &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{13} \text{ and } Z \in T_{35}\} \\
\cup &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{14} \text{ and } Z \in T_{45}\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid a \quad B \to CC \mid b \quad C \to AB \mid a$

$$
\begin{array}{|c|c|c|c|c|l}
\multicolumn{1}{c}{0} \\
\cline{1-1}
\{B\} & \multicolumn{1}{c}{1} \\
\cline{1-2}
\{S,A\} & \{A,C\} & \multicolumn{1}{c}{2} \\
\cline{1-3}
\varnothing & \{B\} & \{A,C\} & \multicolumn{1}{c}{3} \\
\cline{1-4}
\varnothing & \{B\} & \{S,C\} & \{B\} & \multicolumn{1}{c}{4} \\
\cline{1-5}
 & ? & \{B\} & \{S,A\} & \{A,C\} & 5 \\
\cline{1-5}
\end{array}
$$

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{15} = &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A,C\} \text{ and } Z \in \{B\}\} \\
\cup &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{S,A\}\} \\
\cup &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{A,C\}\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid a \quad B \to CC \mid b \quad C \to AB \mid a$

0	1	2	3	4	
$\{B\}$					
$\{S, A\}$	$\{A, C\}$				
$\varnothing$	$\{B\}$	$\{A, C\}$			
$\varnothing$	$\{B\}$	$\{S, C\}$	$\{B\}$		
	$\{S, A, C\}$	$\{B\}$	$\{S, A\}$	$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{15} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{A, C\} \text{ and } Z \in \{B\}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{S, A\}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{A, C\}\} \\
&= \{S, A, C\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$

0	1	2	3	4	
$\{B\}$					
$\{S, A\}$	$\{A, C\}$				
$\varnothing$	$\{B\}$	$\{A, C\}$			
$\varnothing$	$\{B\}$	$\{S, C\}$	$\{B\}$		
?	$\{S, A, C\}$	$\{B\}$	$\{S, A\}$	$\{A, C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{05} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{01} \text{ and } Z \in T_{15}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{02} \text{ and } Z \in T_{25}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{03} \text{ and } Z \in T_{35}\} \\
&\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in T_{04} \text{ and } Z \in T_{45}\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \mathrm{a} \quad B \to CC \mid \mathrm{b} \quad C \to AB \mid \mathrm{a}$

0					
$\{B\}$	1				
$\{S,A\}$	$\{A,C\}$	2			
$\varnothing$	$\{B\}$	$\{A,C\}$	3		
$\varnothing$	$\{B\}$	$\{S,C\}$	$\{B\}$	4	
?	$\{S,A,C\}$	$\{B\}$	$\{S,A\}$	$\{A,C\}$	5

$$
\begin{aligned}
x &= baaba \\
T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}
\end{aligned}
$$

$$
\begin{aligned}
T_{05} = &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{S,A,C\}\} \\
\cup &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{S,A\} \text{ and } Z \in \{B\}\} \\
\cup &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in \varnothing \text{ and } Z \in \{S,A\}\} \\
\cup &\ \{X \in N \mid X \to YZ \in P \text{ with } Y \in \varnothing \text{ and } Z \in \{A,C\}\}
\end{aligned}
$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid a \quad B \to CC \mid b \quad C \to AB \mid a$

0	1	2	3	4	
$\{B\}$					
$\{S,A\}$	$\{A,C\}$				
$\varnothing$	$\{B\}$	$\{A,C\}$			
$\varnothing$	$\{B\}$	$\{S,C\}$	$\{B\}$		
$\{S,A,C\}$	$\{S,A,C\}$	$\{B\}$	$\{S,A\}$	$\{A,C\}$	5

$$\begin{aligned} x &= baaba \\ T_{ij} &= \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\} \end{aligned}$$

$$\begin{aligned} T_{05} &= \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{B\} \text{ and } Z \in \{S,A,C\}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \{S,A\} \text{ and } Z \in \{B\}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \varnothing \text{ and } Z \in \{S,A\}\} \\ &\cup \{X \in N \mid X \to YZ \in P \text{ with } Y \in \varnothing \text{ and } Z \in \{A,C\}\} \\ &= \{S,A,C\} \end{aligned}$$

Example

CFG G in Chomsky normal form: $S \to AB \mid BC \quad A \to BA \mid \text{a} \quad B \to CC \mid \text{b} \quad C \to AB \mid \text{a}$

0					
$\{B\}$	1				
$\{S,A\}$	$\{A,C\}$	2			
$\varnothing$	$\{B\}$	$\{A,C\}$	3		
$\varnothing$	$\{B\}$	$\{S,C\}$	$\{B\}$	4	
$\{S,A,C\}$	$\{S,A,C\}$	$\{B\}$	$\{S,A\}$	$\{A,C\}$	5

$$x = baaba$$
$$T_{ij} = \{X \in N \mid X \xrightarrow[G]{*} x_{ij}\}$$

$$S \in T_{05} \implies x \in L(G)$$

Thanks! & Questions?