Quiz I (10 pts)

Burak Ekici

Assigned : March the 17th, 20h00
Duration : 60 minutes

Q1 (7 pts). Design a deterministic finite automaton (DFA) $M = (Q, \Sigma = \{a, b\}, \delta, s, F)$ that recognizes the language $\mathcal{L} := \{x \in \Sigma^* \mid \#a(x) \geqslant 3 \ \wedge \ \#b(x) \leqslant 2\}$. Recall that $\#a(x)$ and $\#b(x)$ respectively denote the number of 'a's and number of 'b's contained in a given string x.

Below are a few examples to the input-output harmony of the intended DFA:

string	reaction of M
aaa	accepted
baaab	accepted
babaa	accepted
baaaba	accepted
bb	rejected
bab	rejected
baab	rejected
baabaab	rejected
⋮	⋮

Hint: Individually design automata M_1 and M_2 recognizing languages $\{x \in \Sigma^* \mid \#a(x) \geqslant 3\}$ and $\{x \in \Sigma^* \mid \#b(x) \leqslant 2\}$ respectively, and only then compute the product automaton $M = M_1 \times M_2$ accepting the set of strings $\mathcal{L}$.

Q2 (3 pts). We recursively define a function $|\cdot| : \text{String} \to \mathbb{N}$ that computes the length of a given string defined over some alphabet Σ as follows:

$$|\varepsilon| := 0 \qquad |xa| := |x| + 1 \quad \text{for all } a \in \Sigma.$$

Prove employing the mathematical induction principle that below equation

$$|xy| = |x| + |y|$$

holds for all strings $x, y \in \Sigma^*$.

Hint: Argue by induction over the length of the string y.

Important Notice:

- Collaboration is strictly and positively prohibited; lowers your score to 0 if detected.
- Any submission after 60 minutes will NOT be accepted. Please beware and respect the deadline!
- All handwritten answers should somehow be scanned into a single pdf file, and only then submitted. Make sure that your handwriting is decent and readable.