Quiz II (10 pts)

Burak Ekici

Assigned : April the 30th, 20h15
Duration : 60 minutes

Q1. (8 pts) Let $\alpha = a(bca)^*bc$ and $\beta = ab(cab)^*c$ be a pair of regular expressions defined over the alphabet $\Sigma = \{a, b, c\}$. Decide whether $\alpha \equiv \beta$ employing derivatives and bisimulation. Justify your reasoning.

A1. We start with partially deriving the expression α with respect to the letters a, b and c until no new expression is generated:

$$\begin{aligned} \alpha_a &= a_a(bca)^*bc \\ &= \varepsilon(bca)^*bc \\ &= (bca)^*bc =: \alpha_1 \end{aligned} \qquad \alpha_b = \emptyset \qquad \alpha_c = \emptyset$$

$$(\alpha_1)_a = \emptyset \qquad \begin{aligned} (\alpha_1)_b &= ((bca)^*)_b bc + (bc)_b \\ &= (bca)_b(bca)^*bc + \varepsilon c \\ &= \varepsilon ca(bca)^*bc + c \\ &= ca(bca)^*bc + c =: \alpha_2 \end{aligned} \qquad (\alpha_1)_c = \emptyset$$

$$(\alpha_2)_a = \emptyset \qquad (\alpha_2)_b = \emptyset \qquad \begin{aligned} (\alpha_2)_c &= (ca)_c(bca)^*bc + c_c \\ &= \varepsilon a(bca)^*bc + \varepsilon \\ &= a(bca)^*bc + \varepsilon =: \alpha_3 \end{aligned}$$

$$\begin{aligned} (\alpha_3)_a &= a_a(bca)^*bc + \varepsilon_a \\ &= \varepsilon(bca)^*bc + \emptyset \\ &= (bca)^*bc = \alpha_1 \end{aligned} \qquad (\alpha_3)_b = \emptyset \qquad (\alpha_3)_c = \emptyset$$

$$(\emptyset)_a = \emptyset \qquad (\emptyset)_b = \emptyset \qquad (\emptyset)_c = \emptyset$$

We apply the same procedure above for the expression β:

$$\begin{aligned} \beta_a &= a_ab(cab)^*c \\ &= \varepsilon b(cab)^*c \\ &= b(cab)^*c =: \beta_1 \end{aligned} \qquad \beta_b = \emptyset \qquad \beta_c = \emptyset$$

$$(\beta_1)_a = \emptyset \qquad \begin{aligned} (\beta_1)_b &= b_b(cab)^*c \\ &= \varepsilon(cab)^*c \\ &= (cab)^*c =: \beta_2 \end{aligned} \qquad (\beta_1)_c = \emptyset$$

$$
\begin{array}{rcl}
(\beta_2)_a & = & \emptyset
\end{array}
\qquad
\begin{array}{rcl}
(\beta_2)_b & = & \emptyset
\end{array}
\qquad
\begin{array}{rcl}
(\beta_2)_c & = & ((cab)^*)_c c + c_c \\
& = & (cab)_c (cab)^* c + \varepsilon \\
& = & c_c ab(cab)^* c + \varepsilon \\
& = & \varepsilon ab(cab)^* c + \varepsilon \\
& = & ab(cab)^* c + \varepsilon =: \beta_3
\end{array}
$$

$$
\begin{array}{rcl}
(\beta_3)_a & = & a_a b(cab)^* c + \varepsilon_a \\
& = & \varepsilon b(cab)^* c + \emptyset \\
& = & b(cab)^* c = \beta_1
\end{array}
\qquad
\begin{array}{rcl}
(\beta_3)_b & = & \emptyset
\end{array}
\qquad
\begin{array}{rcl}
(\beta_3)_c & = & \emptyset
\end{array}
$$

$$
(\emptyset)_a = \emptyset \qquad (\emptyset)_b = \emptyset \qquad (\emptyset)_c = \emptyset
$$

We have the following derivative tables:

	a	b	c	
α	α_1	$\emptyset$	$\emptyset$	$\uparrow$
α_1	$\emptyset$	α_2	$\emptyset$	$\uparrow$
α_2	$\emptyset$	$\emptyset$	α_3	$\uparrow$
α_3	α_1	$\emptyset$	$\emptyset$	$\downarrow$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\uparrow$

	a	b	c	
β	β_1	$\emptyset$	$\emptyset$	$\uparrow$
β_1	$\emptyset$	β_2	$\emptyset$	$\uparrow$
β_2	$\emptyset$	$\emptyset$	β_3	$\uparrow$
β_3	β_1	$\emptyset$	$\emptyset$	$\downarrow$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\uparrow$

Therefore, the fact that $\alpha \equiv \beta$ follows by the bisimulation $\sim$ that satisfies

$$
\begin{array}{rcl}
L(\alpha) & \sim & L(\beta) \\
L(\alpha_1) & \sim & L(\beta_1) \\
L(\alpha_2) & \sim & L(\beta_2) \\
L(\alpha_3) & \sim & L(\beta_3) \\
L(\emptyset) & \sim & L(\emptyset).
\end{array}
$$

Q2. (2 pts) Simplify the regular expression

$$
\alpha := \varepsilon + (a+b+c)((a^*b)^*a^*c)^*(a^*b)^*(\varepsilon + a + aaa^*)
$$

defined over the alphabet $\Sigma = \{a, b, c\}$ as much as possible benefiting Kleene Algebra axioms and rules (A.1) – (A.17). Clearly show simplification steps.

A2.

$$
\begin{array}{rcl}
\alpha & := & \varepsilon + (a+b+c)((a^*b)^*a^*c)^*(a^*b)^*(\varepsilon + a + aaa^*) \\
& = & \varepsilon + (a+b+c)((a^*b)^*a^*c)^*(a^*b)^*(\varepsilon + a(\varepsilon + aa^*)) \\
& = & \varepsilon + (a+b+c)((a^*b)^*a^*c)^*(a^*b)^*(\varepsilon + aa^*) \\
& = & \varepsilon + (a+b+c)((a^*b)^*a^*c)^*(a^*b)^*a^* \\
& = & \varepsilon + (a+b+c)((a^*b)^*a^*c)^*(a+b)^* \\
& = & \varepsilon + (a+b+c)((a+b)^*c)^*(a+b)^* \\
& = & \varepsilon + (a+b+c)(a+b+c)^* \\
& = & (a+b+c)^*
\end{array}
$$

Important Notice:

- Collaboration is strictly and positively prohibited; lowers your score to 0 if detected.
- Any submission after 60 minutes will NOT be accepted. Please beware and respect the deadline!
- All handwritten answers should somehow be scanned into a single pdf file, and only then submitted. Make sure that your handwriting is decent and readable.