ISTANBUL TECHNICAL UNIVERSITY
Computer Engineering Department

BLG354E Signals and Systems for CE

Midterm Exam: Spring 2018 Term

Date: 09 April 2018

Time: 17:30 - 19:20

Rooms: EEB 4104, EEB 5203 and EEB 5204

Student Name:

Student ID#:

- **Closed Book and Closed Notes; 4 problems (105 points total); 110 minutes.**
- **YOUR FINAL RESULTS to Problems should be in the BOXES provided. OTHERWISE, your solution will NOT be graded.**
- **Be sure to justify your answers and show all your work, just writing the result does NOT get you any credit.**
- **Please make sure your full name is on all sheets. DO IT NOW!**
- **Make sure you return all the sheets of the exam before you leave the exam.**
- **Cheating will be penalized according to the university disciplinary policy.**
- **Good luck!**

Please leave the rest of this page blank for use by the graders:

Problem	No. of Points	Score	Grader
1	30		
2	25		
3	25		
4	25		
Total	105		

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Problem 1: Each part is 3 points.

1.1. For which of two signals with given Fourier Series (FS) coefficients, the FS approximation of the signal converges faster to the original signal as we add more FS coefficients? (Explain your reasoning briefly):

i) $a_k = \begin{cases} \frac{1}{j\pi k}, k = \mp1, \mp3, \dots \\ 0, k = \mp2, \mp4, \dots \\ \frac{1}{2}, k = 0 \end{cases}$ ii) $a_k = \begin{cases} \frac{1}{j\pi^2 k^2}, k = \mp1, \mp3, \dots \\ 0, k = \mp2, \mp4, \dots \\ \frac{1}{2}, k = 0 \end{cases}$

1.2. Rate of oscillation of discrete time signal increases as ω increases up to ……..

1.3. The reason of Gibbs phenomenon is ………………………………

1.4. What are the frequency values f_1 and f_2 of the following beat signal: $\sin(2\pi 210t) + sin(2\pi 200t) = 2\cos(2\pi f_1 t) sin(2\pi f_2 t)$?

1.5. Calculate Fourier Transform of $\delta(t)$ using Fourier Transform formula: $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$.

1.6. $y(t) = \frac{dx}{dt} + x(t)$, determine system properties: Linear or nonlinear; Time invariant or time varying; Causal or non-causal)

1.7. Express $u[n+2] + u[n+1] - u[n] - u[n-3]$ in terms of impulse sequences:

1.8. Calculate Discrete Fourier Transform coefficients of $x[0] = 8, x[1] = 4, x[2] = 8, x[3] = 4$ (Recall $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk}$).

1.9. Add two sinusoids: $x_1(t) = 2\sin(5\pi t - \frac{\pi}{2})$ and $x_2(t) = 4\cos(5\pi t + \frac{\pi}{6})$ to obtain another sinusoid. Use phasor addition.

1.10. Fourier Transform of $x(t) = \sin(\omega_0 t)$ is equal to ………..

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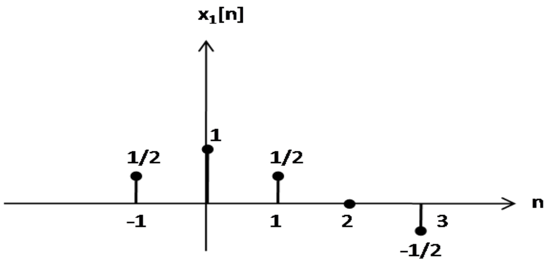
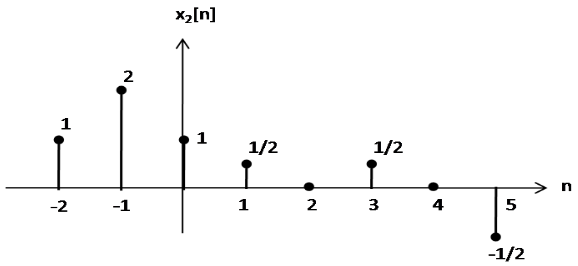
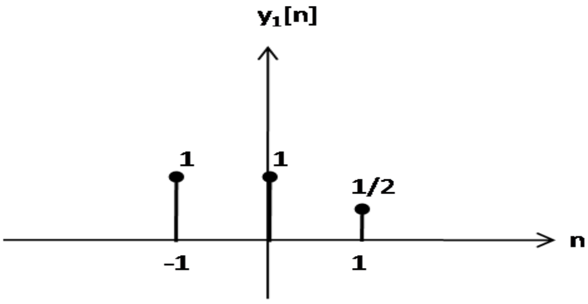
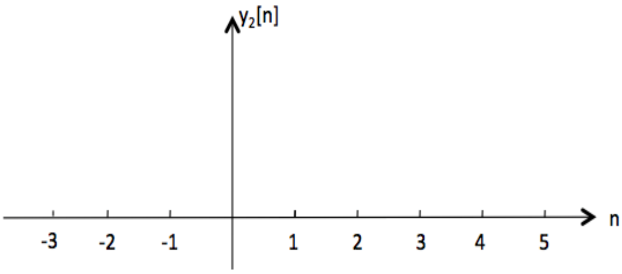
Problem 2: Problem parts (a) and (b) are not related.

2. (a) Consider a linear time-invariant system whose response to $x_1[n]$ is the signal $y_1[n]$. Sketch carefully the response of the system $y_2[n]$ to the input $x_2[n]$ in the given axis below. Justify your reasoning clearly.



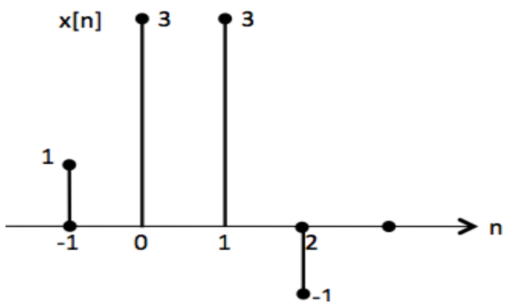


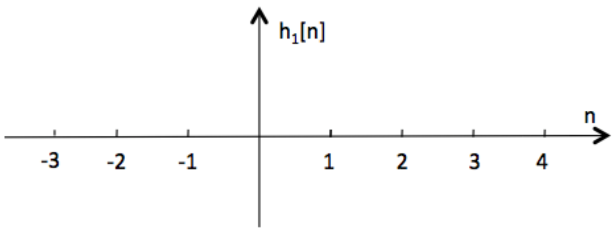


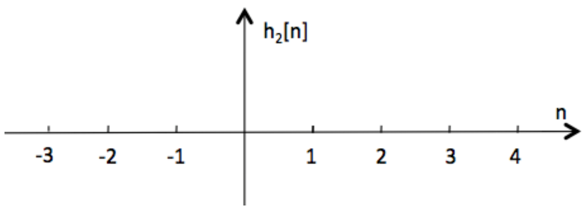


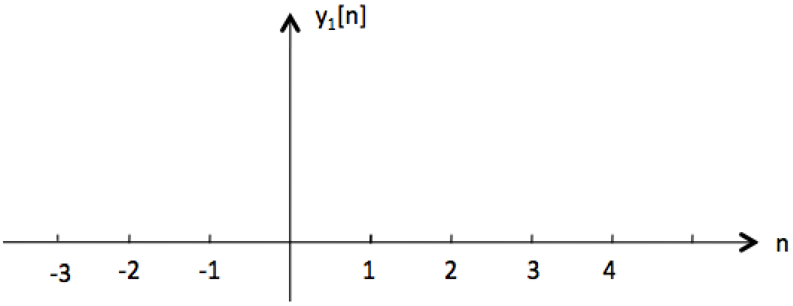
(b) A discrete time signal $x[n]$, whose plot is given below, is provided as an input to two systems that you will define:

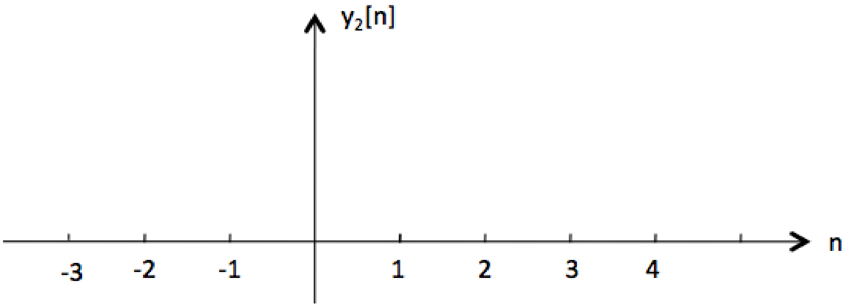
S1: a 2-pt filter that detects abrupt changes in the signal; S2: a 3-pt filter that smooths the signal. Plot the impulse responses $h_1[n]$ and $h_2[n]$ corresponding to S1 and S2, respectively, in the given axes below. Also, plot the output of the system S1, i.e. $y_1[n]$, and output of S2, $y_2[n]$, in response to input x[n] in the given axes below. Justify your solution.











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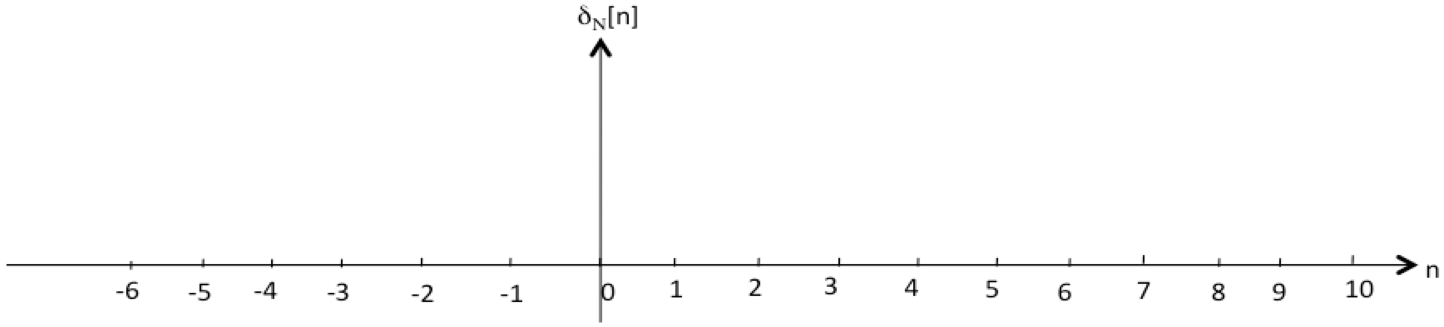
Problem 3. Consider the periodic impulse train: $\delta_N[n] = \sum_{m=-\infty}^{\infty} \delta[n - mN]$.

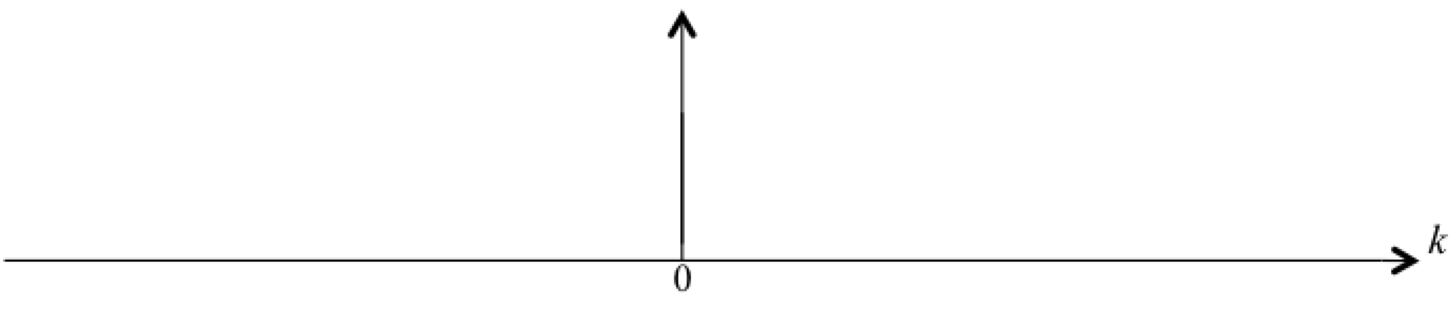
3.1. Plot $d_N[n]$ for N=4.



3.2. Calculate the Discrete Time Fourier Series coefficients c_k of $d_N[n]$, for all k integers: $c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$

3.3. Using the coefficients calculated in 3.2, write down the Fourier series representation of $d_N[n]$.

3.4. Plot the frequency spectrum of $d_N[n]$. State which frequency k=1 corresponds to?



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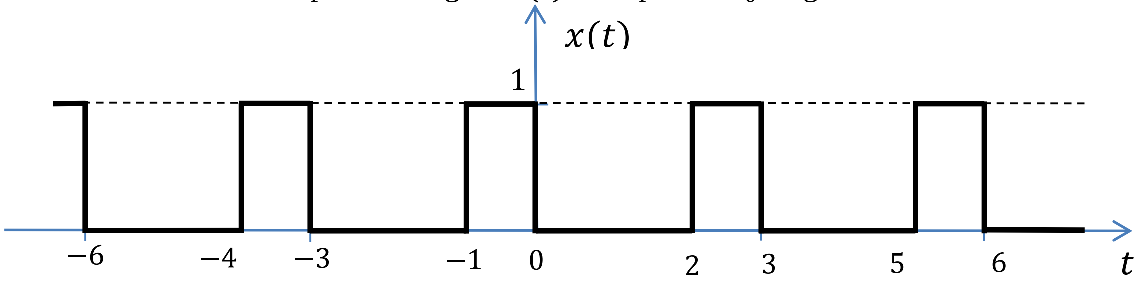
Problem 4: A continuous time periodic signal $x(t)$ with period T_0 is given below:



a) Derive a general formula for the Fourier series coefficients: a_0 and a_k (k: integers, $k \neq 0$) for $x(t)$. $a_k = \frac{1}{T_o}\int_{To} x(t)e^{-j2\pi f_0 kt}dt$

b) Compute the Fourier series coefficients a_k for $-3 \leq k \leq 3$ in <u>polar form</u> and plot the spectrum for those harmonics.

(Use $\theta \triangleq \text{atan}\left(\frac{\sqrt{3}}{3}\right)$ if needed.)