BLG435E
Artificial Intelligence

Lecture 3: Problem-Solving

Outline

- Problem-solving agents
- Problem formulation
- Uninformed search strategies
- Informed search strategies
  - Best-first search
  - A* search
  - Heuristics

Problem Solving

- Goal formulation
  - limiting objectives
  - Uses current situation and agent's performance measure
  - A set of world states
- Problem formulation
  - What actions and states to consider
- Search procedure
  - Problem → solution in the form of action sequence

Problem-solving Agents

```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
  persistent: seq, an action sequence, initially empty
              state, some description of the current world state
              goal, a goal, initially null
              problem, a problem formulation

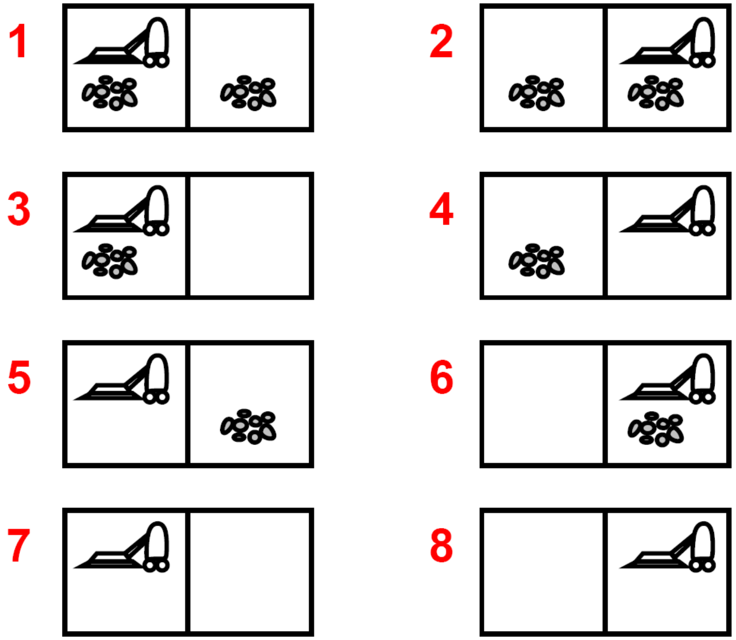
  state ← UPDATE-STATE(state, percept)
  if seq is empty then
      goal ← FORMULATE-GOAL(state)
      problem ← FORMULATE-PROBLEM(state, goal)
      seq ← SEARCH(problem)
      if seq = failure then return a null action
  action ← FIRST(seq)
  seq ← REST(seq)
  return action
```

Well-defined Problems

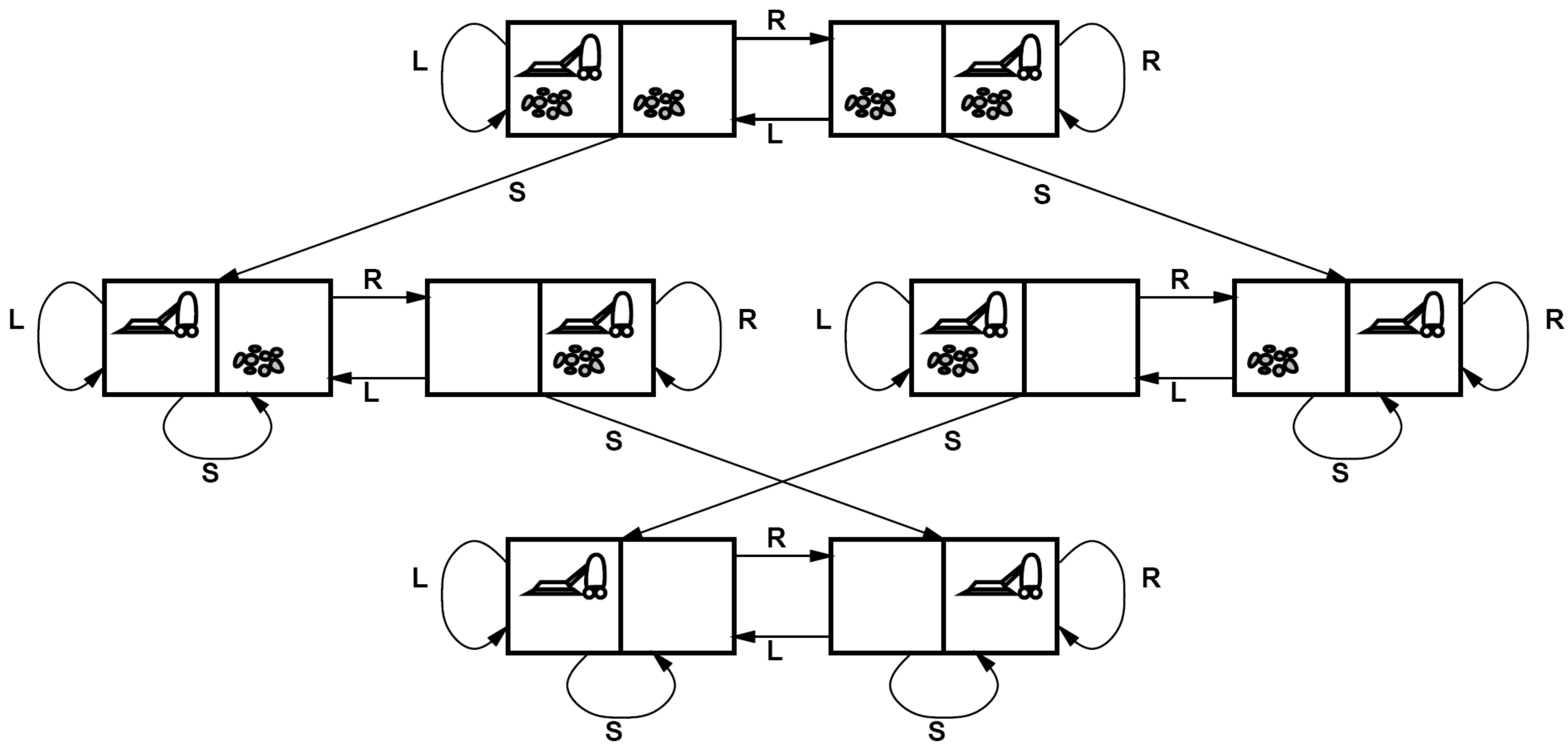
- Initial state
- Actions
- Transition model
- State space
  - Path in the state space
  - Size of the search space
- Goal test
  - Explicit vs. implicit
- Path cost
  - Step cost
- The solution to the problem
  - Optimal solution
- Abstraction

Vacuum-world Problem

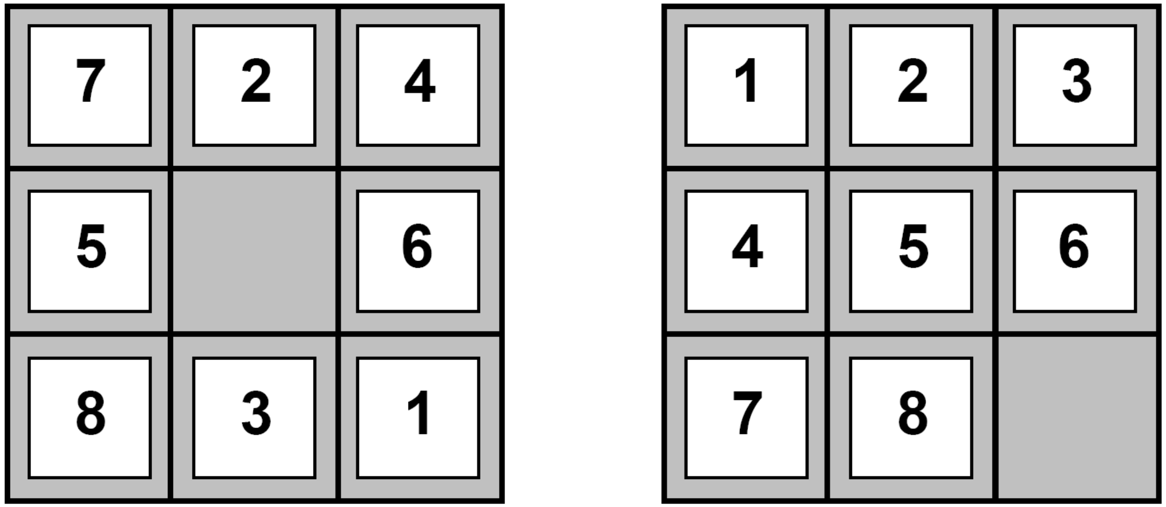
- Single state, start in #5
- Solution?



Vacuum-world State Space Graph



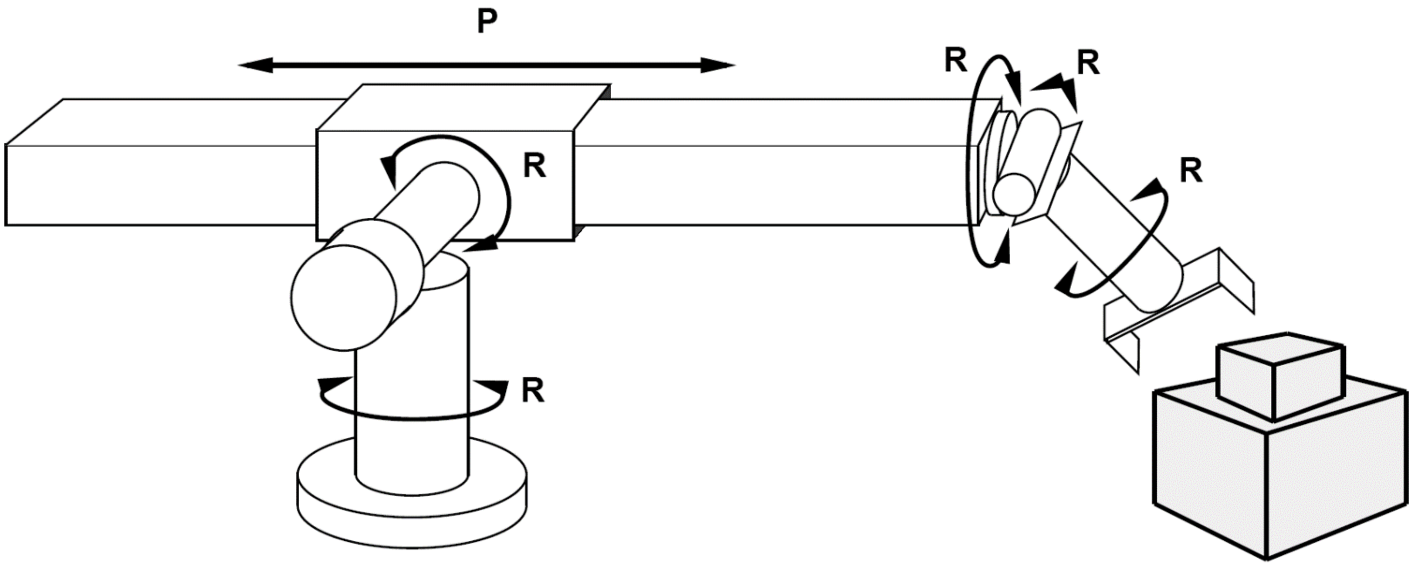
8-Puzzle



Start State

Goal State

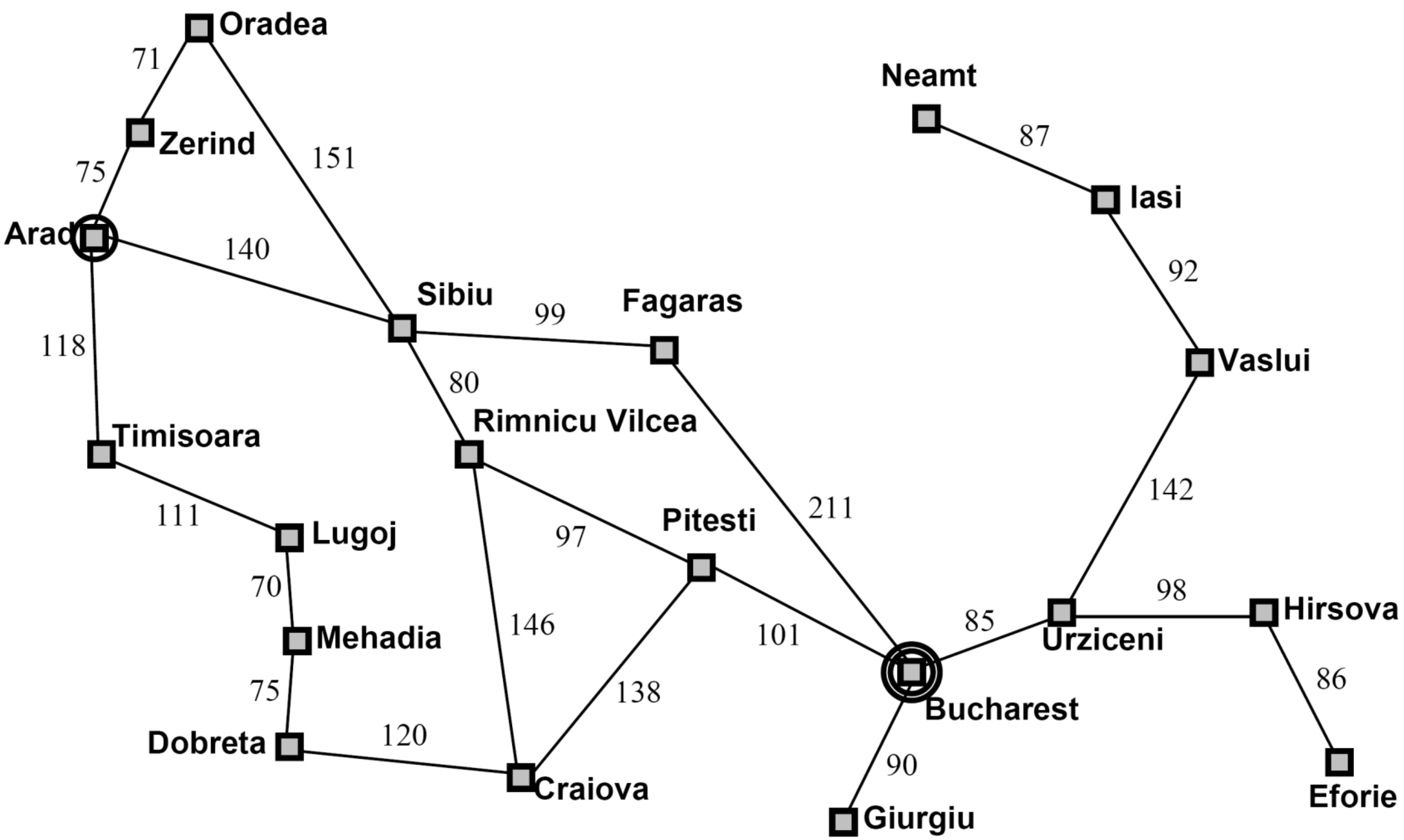
Robotic Assembly



Romania Problem

- On holiday in Romania, currently in Arad. Flight leaves tomorrow from Bucharest
- Formulate goal:
  - Be in Bucharest
- Formulate problem:
  - States: various cities
  - Actions: drive between cities
- Find solution: sequence of actions

Romania



Oradea
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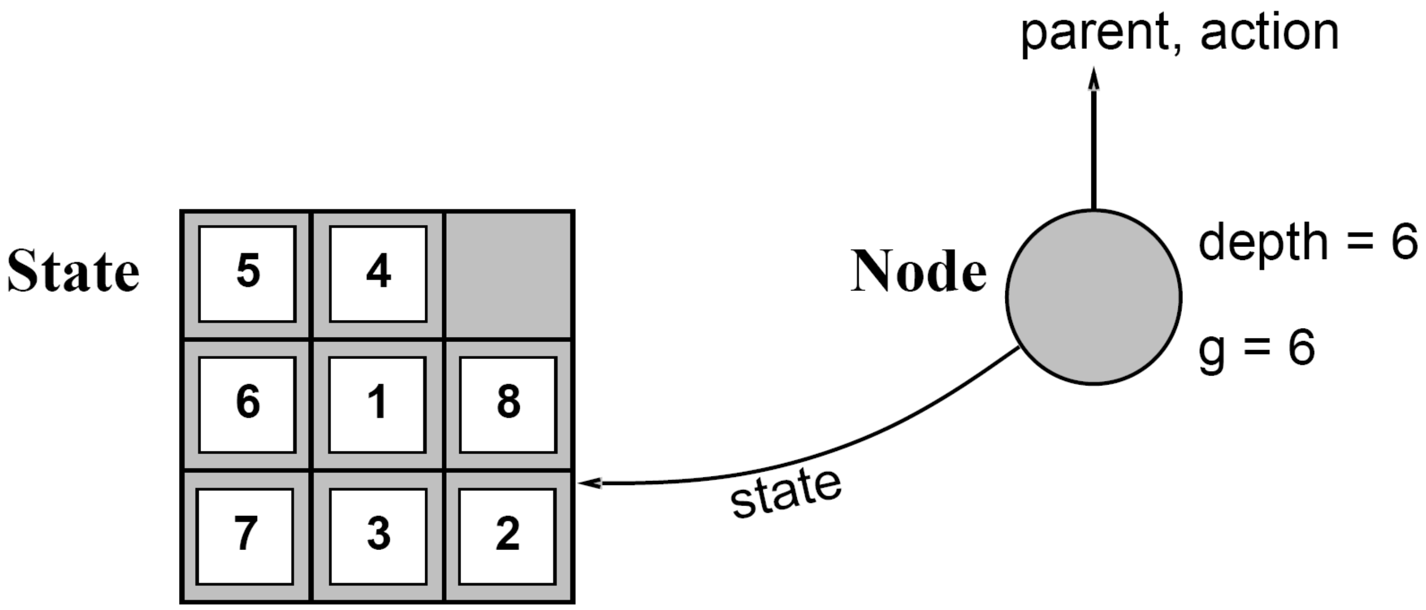
Implementation: states vs. nodes



Node data structure

- **State**: the state to which the node corresponds
- **Parent-node**: the node in the search tree that generated this node
- **Action**: action that was applied
- **Path-cost**: the cost from the initial state to the node - $g(n)$
- **Depth**: the number of steps along the path from the initial state

Node data structure

- Nodes = states ?
- Frontier: the collection of nodes that have been generated but not yet expanded
- Implementation of frontier? Set?

Tree and Graph -Search Algorithms

- Off-line, simulated exploration of the state space by expanding states

```
function TREE-SEARCH(problem) returns a solution, or failure
  initialize the frontier using the initial state of problem
  loop do
     if the frontier is empty then return failure
     choose a leaf node and remove it from the frontier
     if the node contains a goal state then return the corresponding solution
     expand the chosen node, adding the resulting nodes to the frontier
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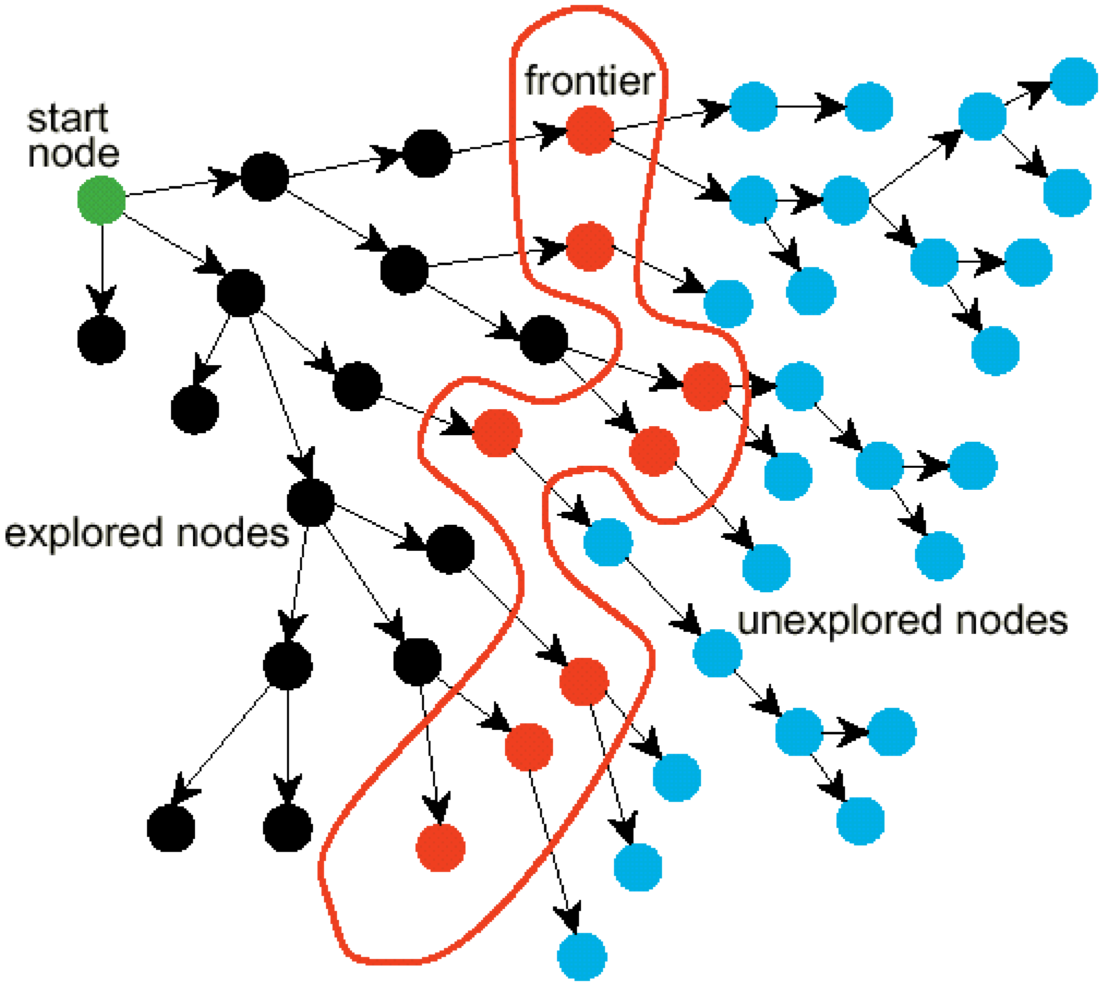
```

- Choosing, testing, expanding

Nodes in The Search Tree

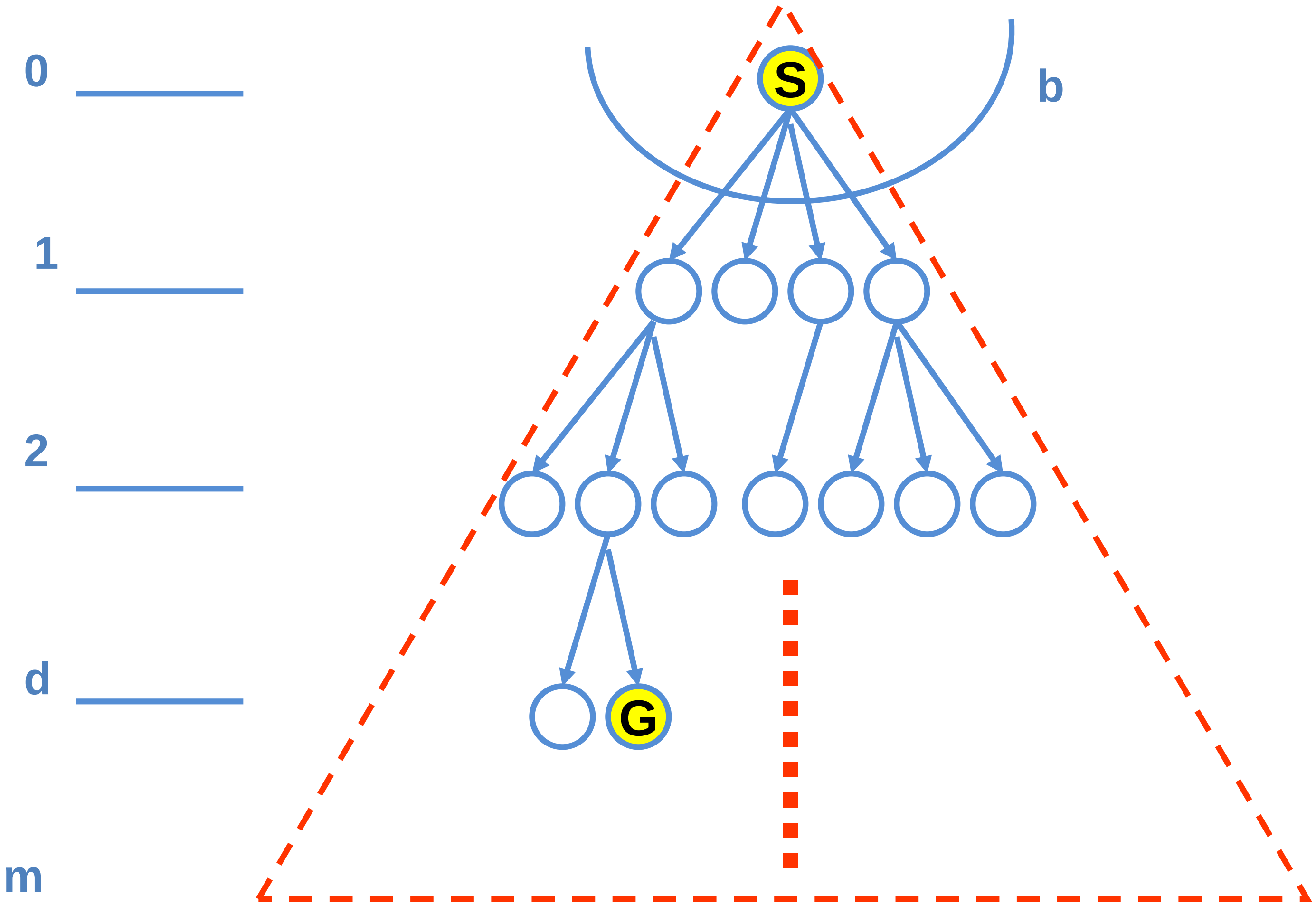


Measuring Problem-Solving Performance

- Strategy = order of node expansion

- Completeness
- Optimality
- Time complexity
- Space complexity
  - b: maximum branching factor
  - d: depth of the least-cost solution
  - m: maximum depth of the search tree

Measuring Problem-Solving Performance



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Uninformed Search Strategies

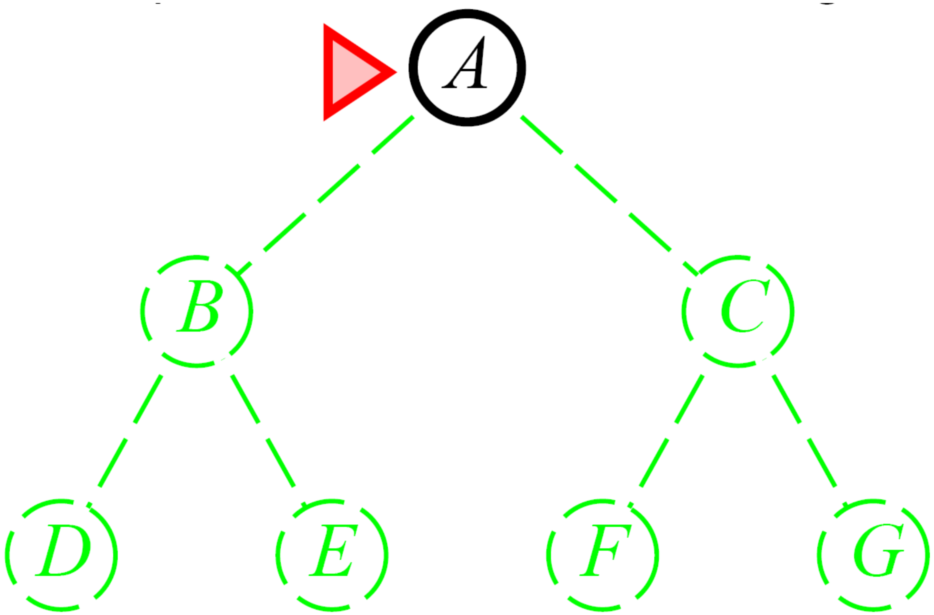
Uninformed Search Strategies

- Use information only available in the problem definition

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative Deepening search

Breadth-first Search (BFS)

- Expand shallowest unexpanded node
- Frontier is a FIFO queue



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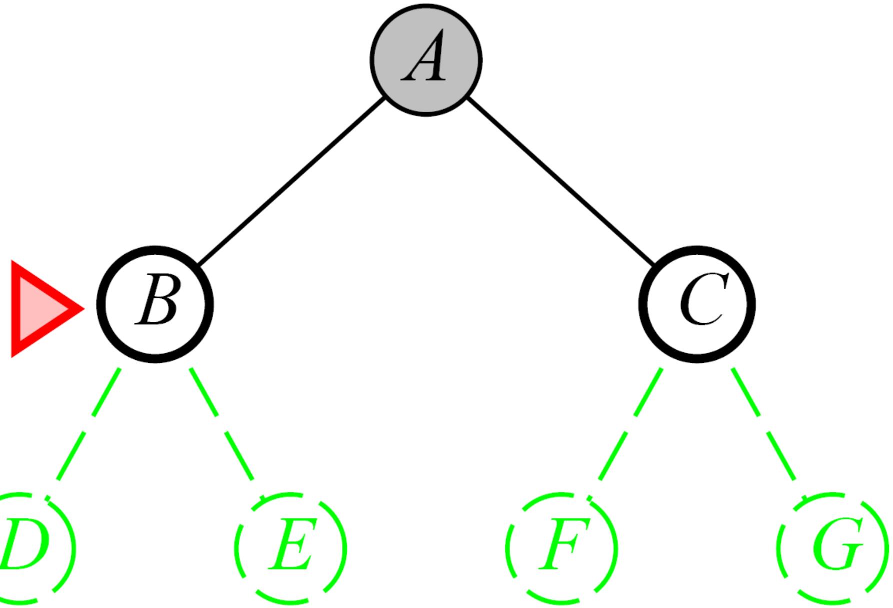

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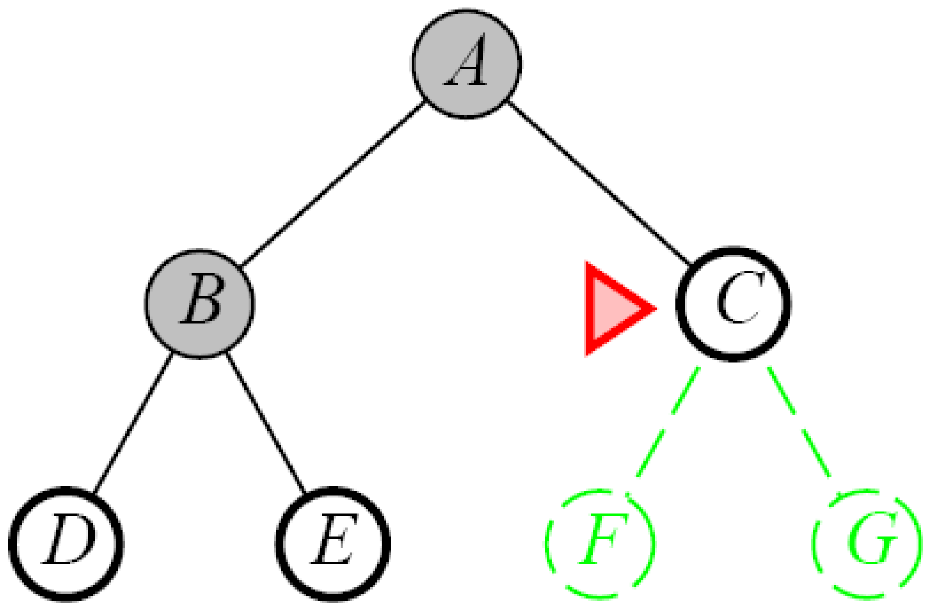

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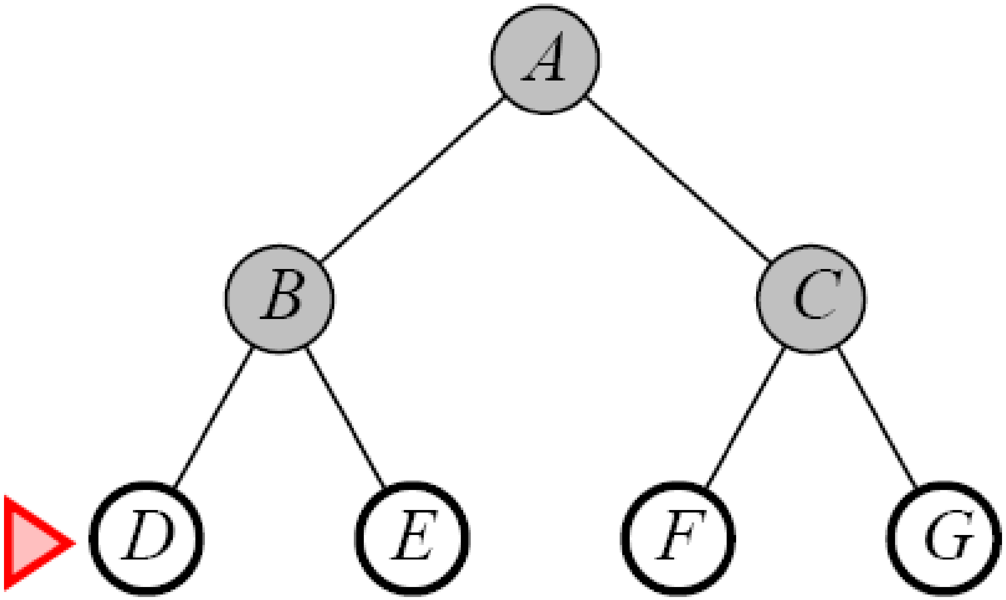


Properties of BFS

- Complete?
- Optimal?
- Time complexity?
- Space complexity?

Properties of BFS

- Complete? Yes, if b is limited
- Optimal? Optimal, if path cost is nondecreasing
- Time complexity? $O(b^d)$
- Space complexity? $O(b^d)$ keeps every node in memory

BFS – Time/Space Requirements

- b = 10, 1.000.000 can be generated per sec

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

- A node requires 1000 bytes of storage

Uniform-cost Search (UCS)

- Expand least-cost (g(n)) unexpanded node
- Frontier = queue ordered by path cost
- What if all costs are the same?

- Complete?
- Optimal?
- Time complexity?
- Space complexity?

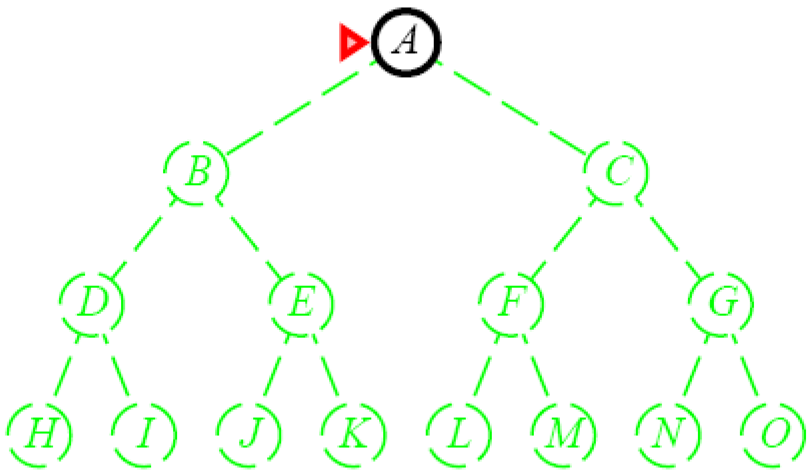
Uniform-cost Search (UCS)

- Expand least-cost unexpanded node
- Frontier = queue ordered by path cost
- What if all costs are the same?

- Complete? Yes, if step cost > ε
- Optimal? Yes,
- Time complexity? $O(b^{1+\lfloor C^*/\varepsilon \rfloor})$
- Space complexity? $O(b^{1+\lfloor C^*/\varepsilon \rfloor})$

Depth-first Search (DFS)

- Expand deepest unexpanded node
- Frontier = LIFO queue



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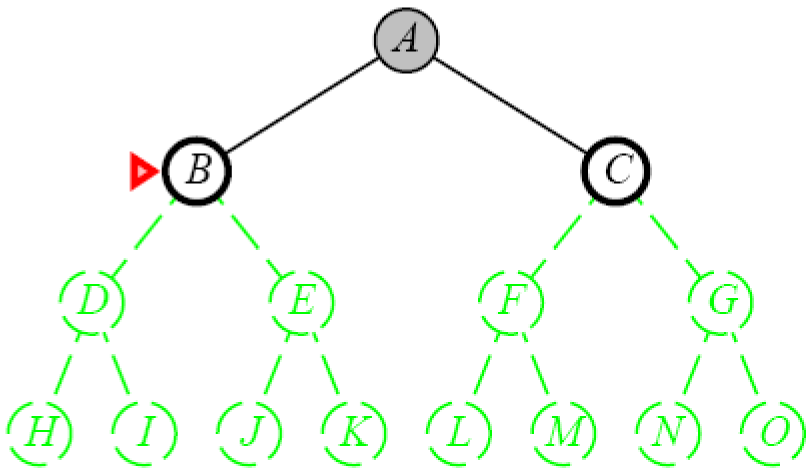

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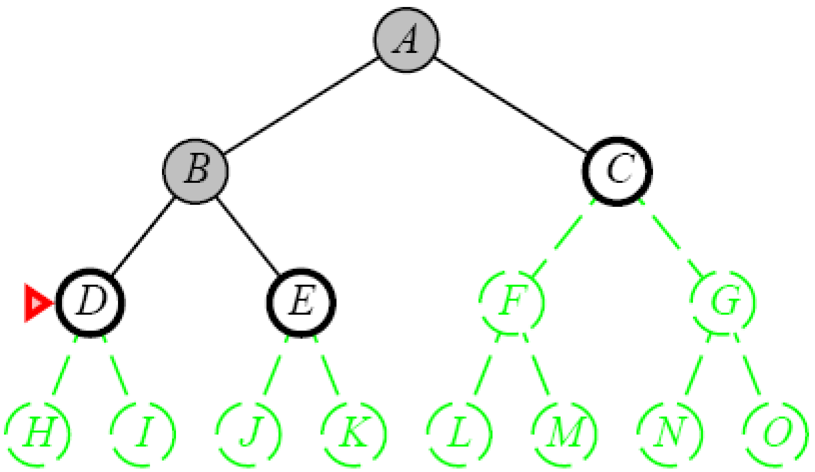

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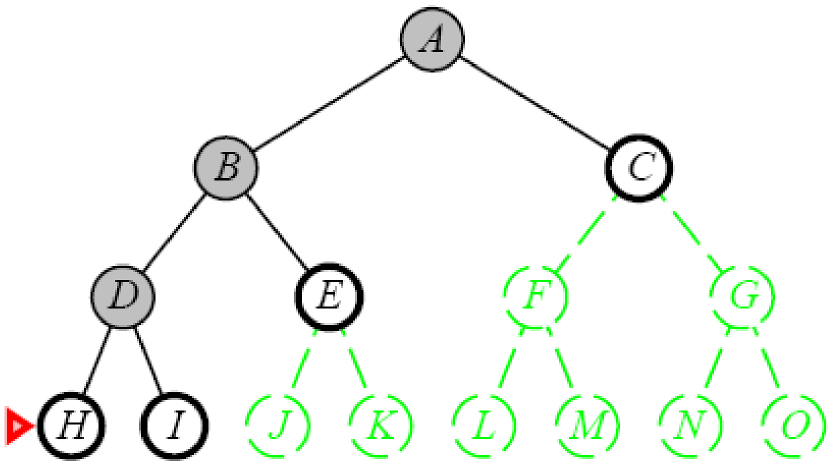

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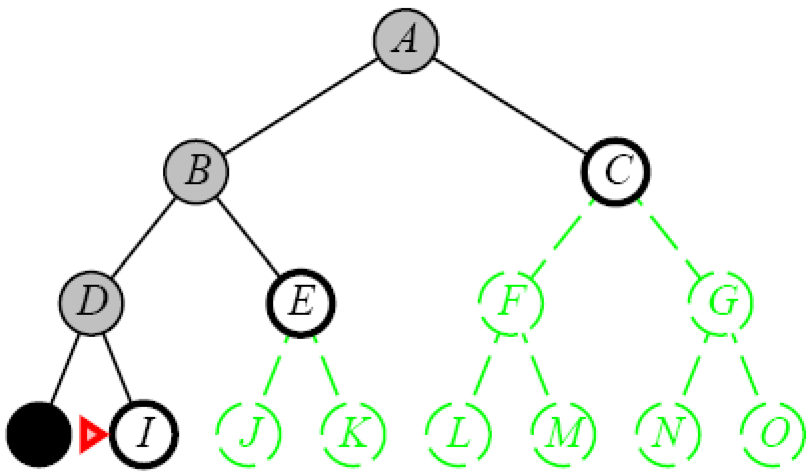

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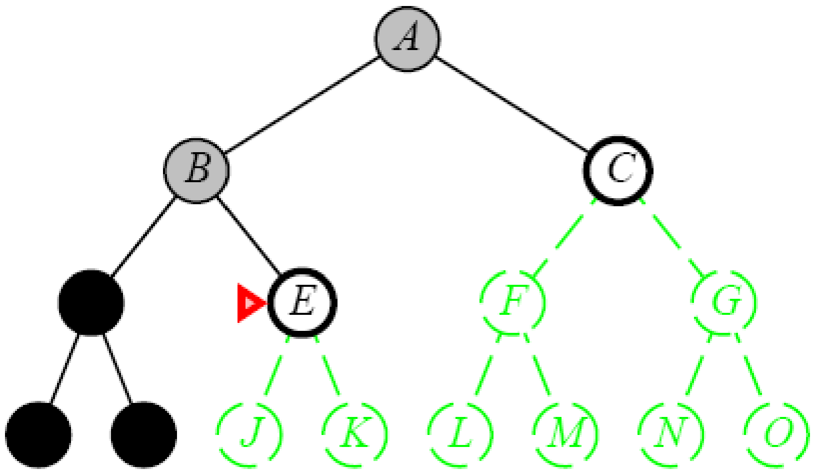

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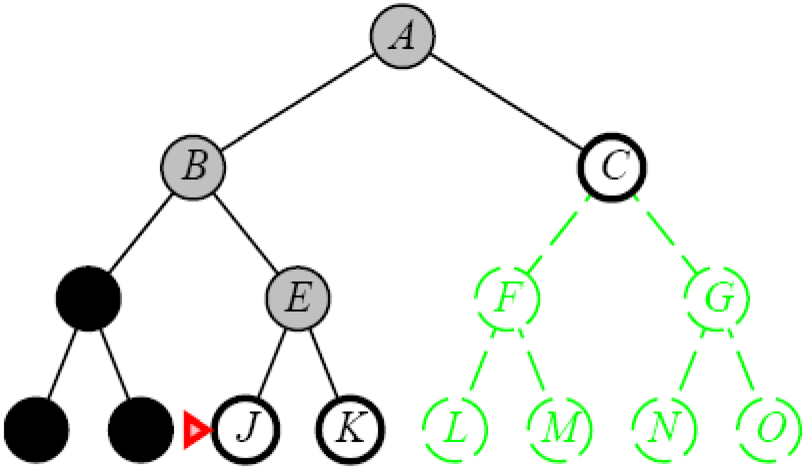

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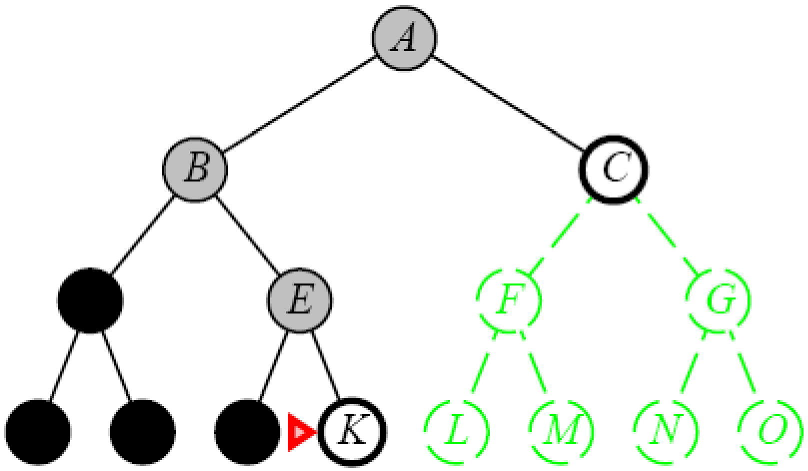

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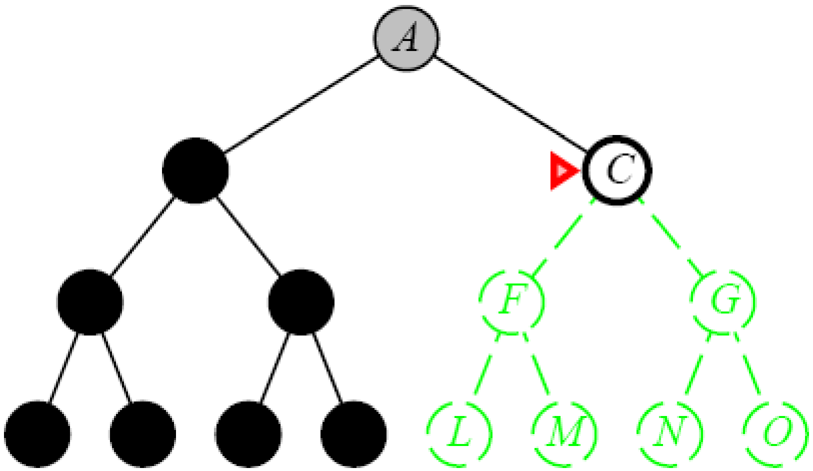

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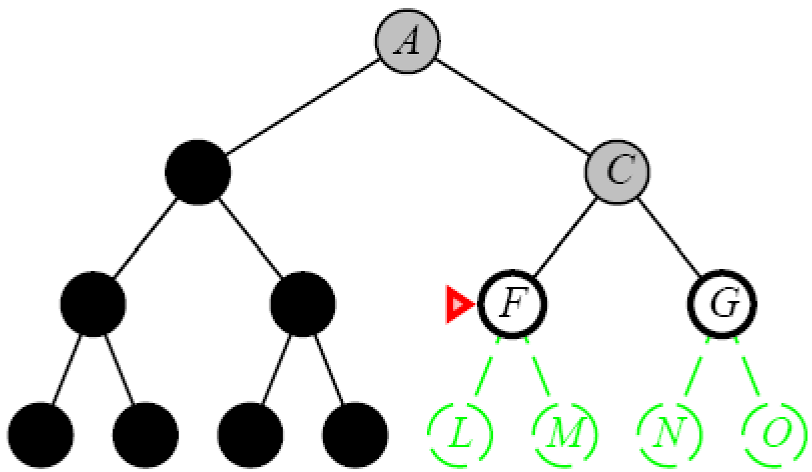

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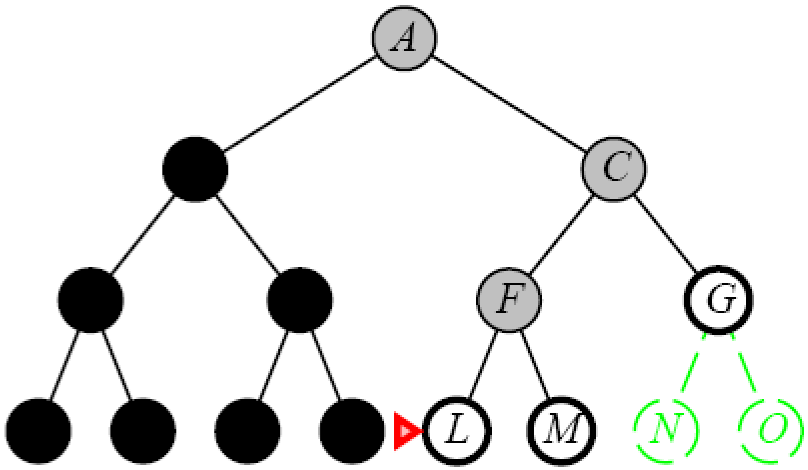

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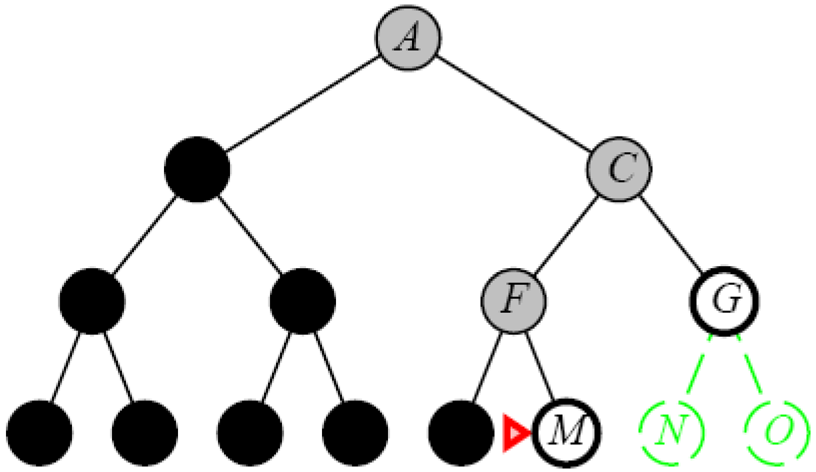


Properties of DFS

- Complete?
- Optimal?
- Time complexity?
- Space complexity?

Properties of DFS

- Complete? No
- Optimal? No
- Time complexity? $O(b^m)$
- Space complexity? $O(bm)$

Depth-limited Search

- Depth-first search with depth limit: *l*

```
function DEPTH-LIMITED-SEARCH(problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST(problem, STATE[node]) then return node
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Iterative Deepening Search (IDS)

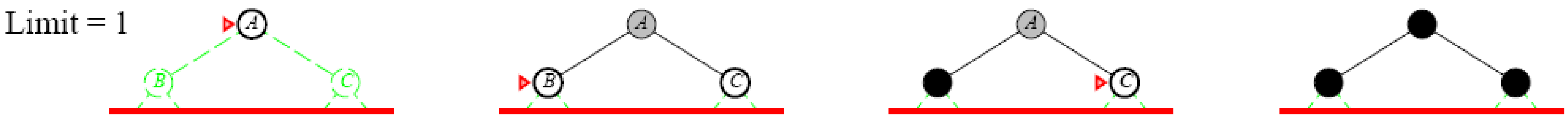
```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

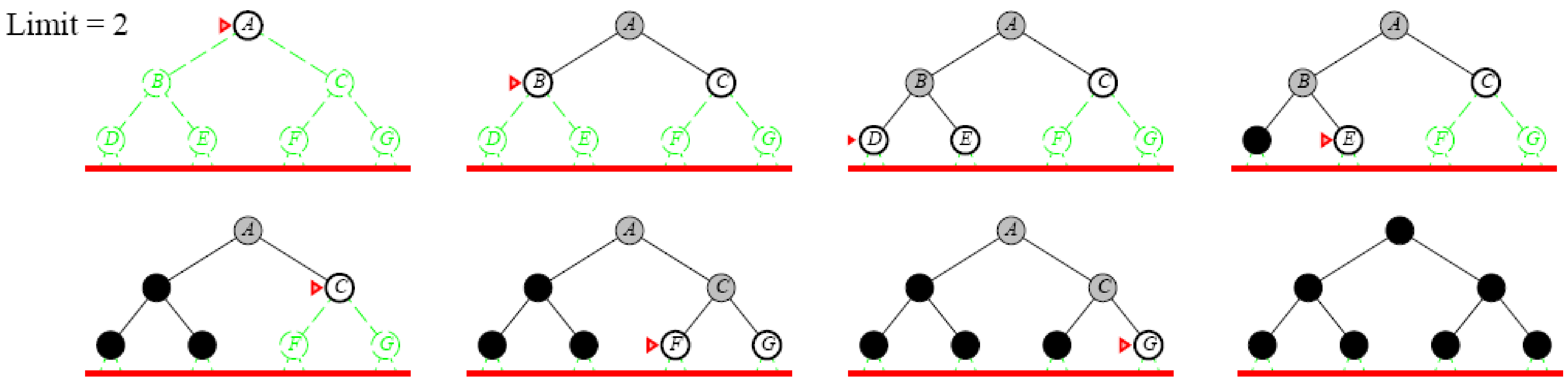
Iterative Deepening Search (IDS)

Limit = 0

Iterative Deepening Search (IDS)



Iterative Deepening Search (IDS)



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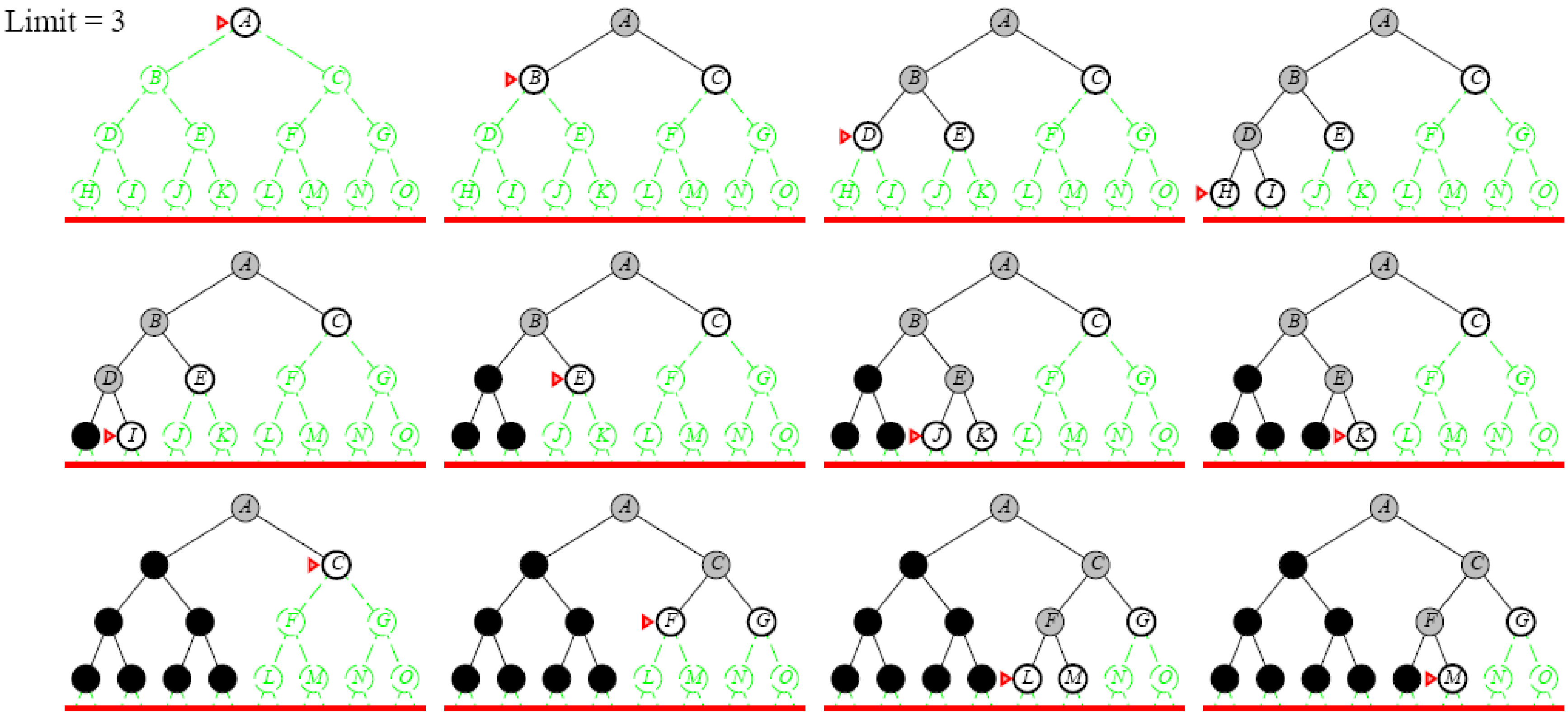


Properties of IDS

- Complete?
- Optimal?
- Time complexity?
- Space complexity?

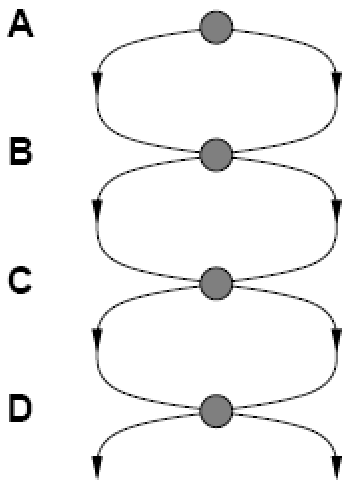
Properties of IDS

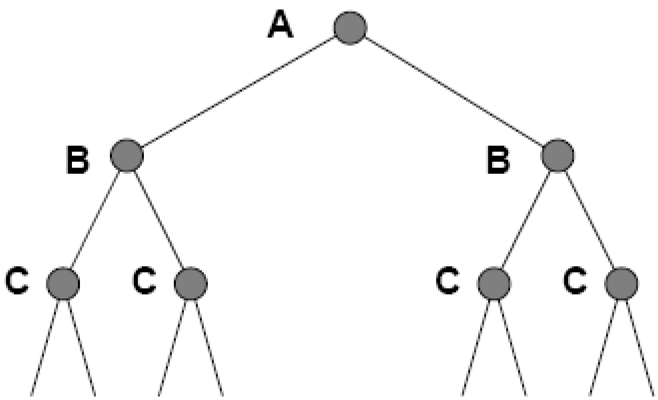
- Complete? Yes, if b is limited
- Optimal? Yes, if path cost is nondecreasing
- Time complexity? $O(b^d)$
- Space complexity? $O(bd)$

Summary of Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^d	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^d	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

Avoiding Repeating States





General Graph-search Algorithm

function GRAPH-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  initialize the explored set to be empty
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    add the node to the explored set
    expand the chosen node, adding the resulting nodes to the frontier
      only if not in the frontier or explored set

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Informed Search Strategies

Tree-Search Algorithms-Revisited

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function TREE-SEARCH(problem) returns a solution, or failure
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```

Best-first Search

- Node is selected based on $f(n)$
  - Estimate of desirability
  - The node with lowest evaluation is selected

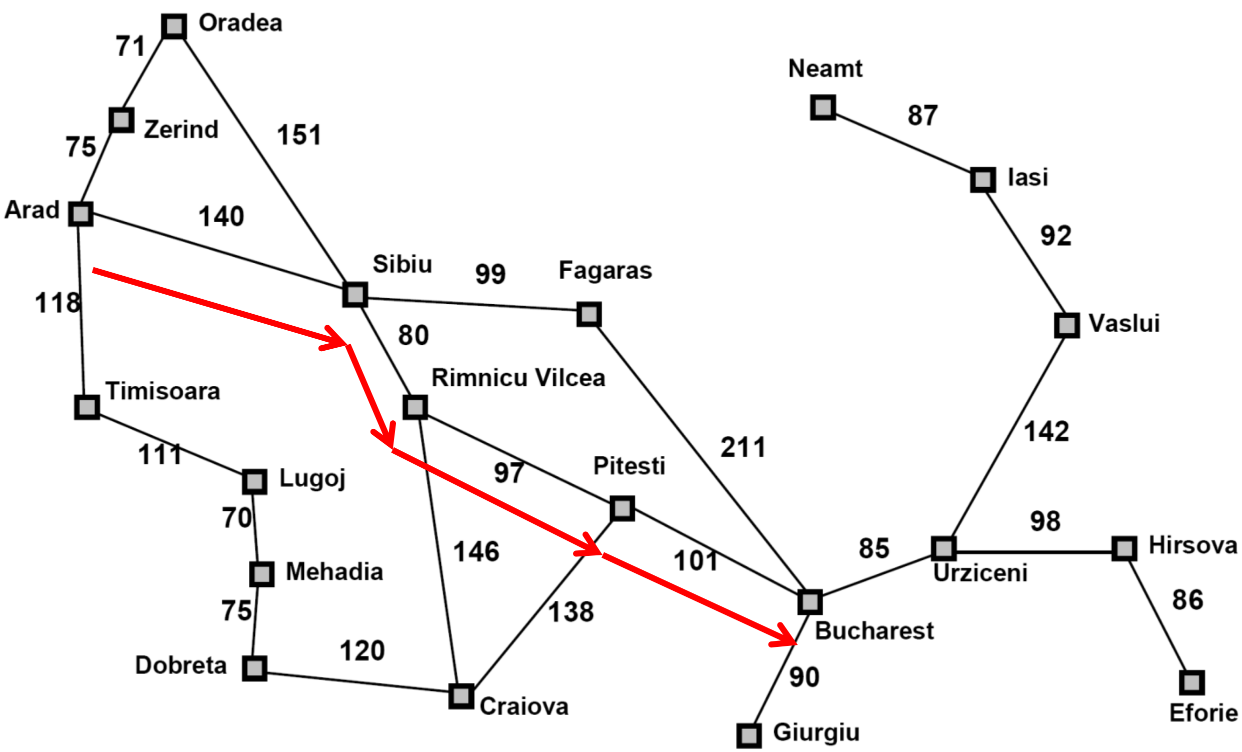
- Frontier is a queue sorted in decreasing order of desirability

- A key component: heuristic function
  - $h(n)$ = estimated cost of the cheapest path from node n to a goal node

Greedy Best-First Search

- Expands the node that is closest to the goal
- $f(n) = h(n)$
- $h_{SLD}(n) =$ Straight-line distance heuristic

Romania Problem Step Costs



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
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Romania Problem–Greedy Search

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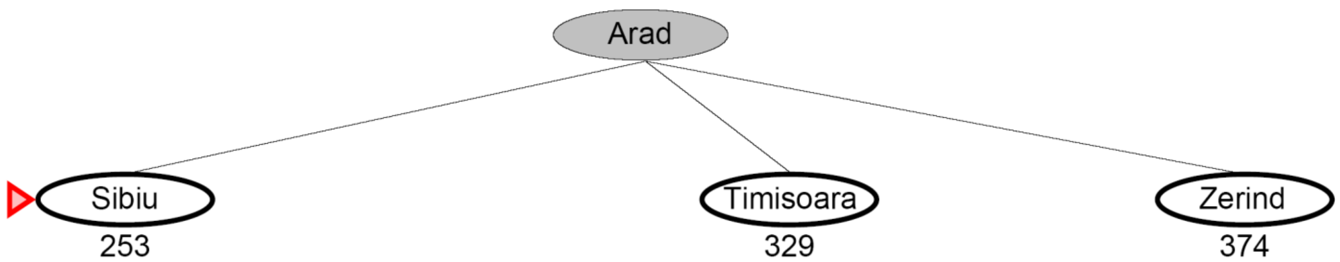

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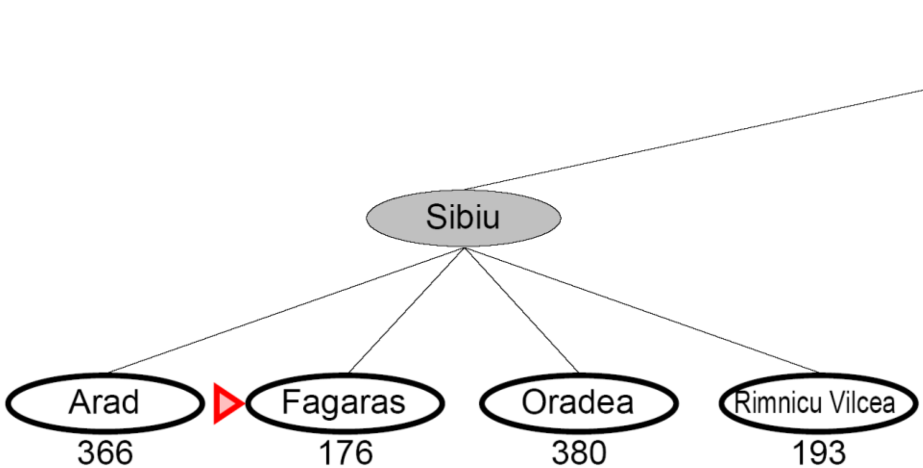

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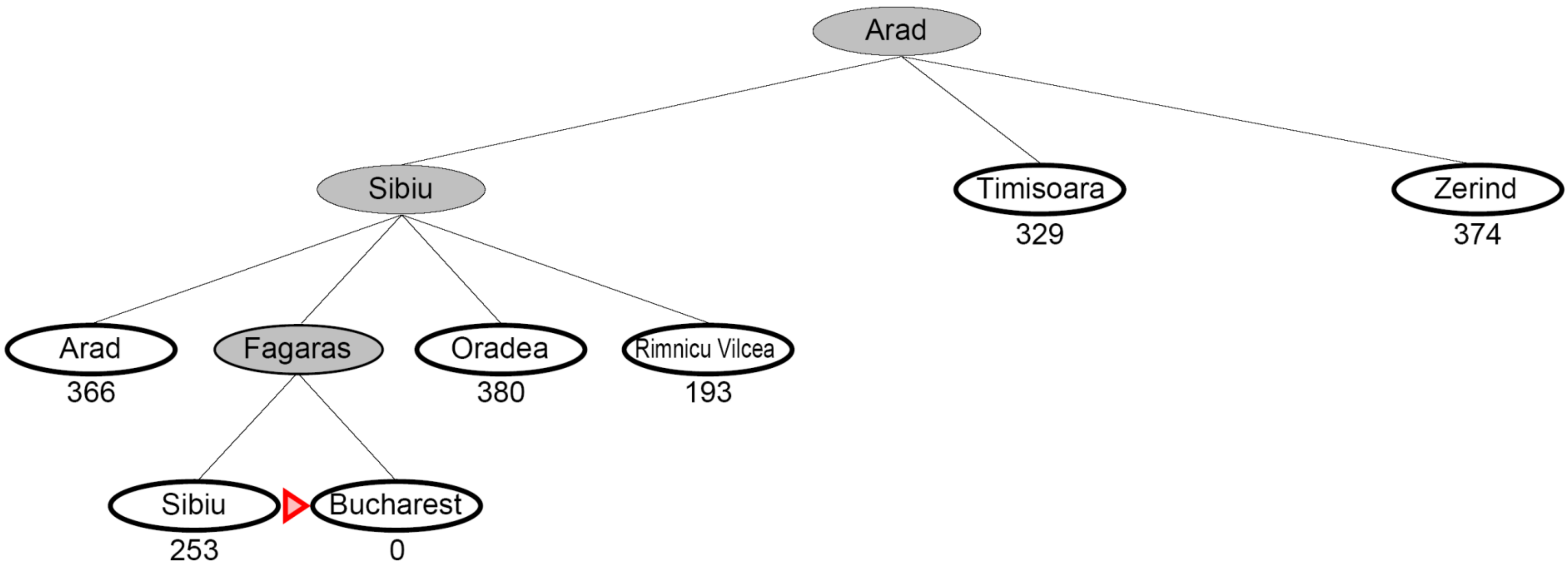
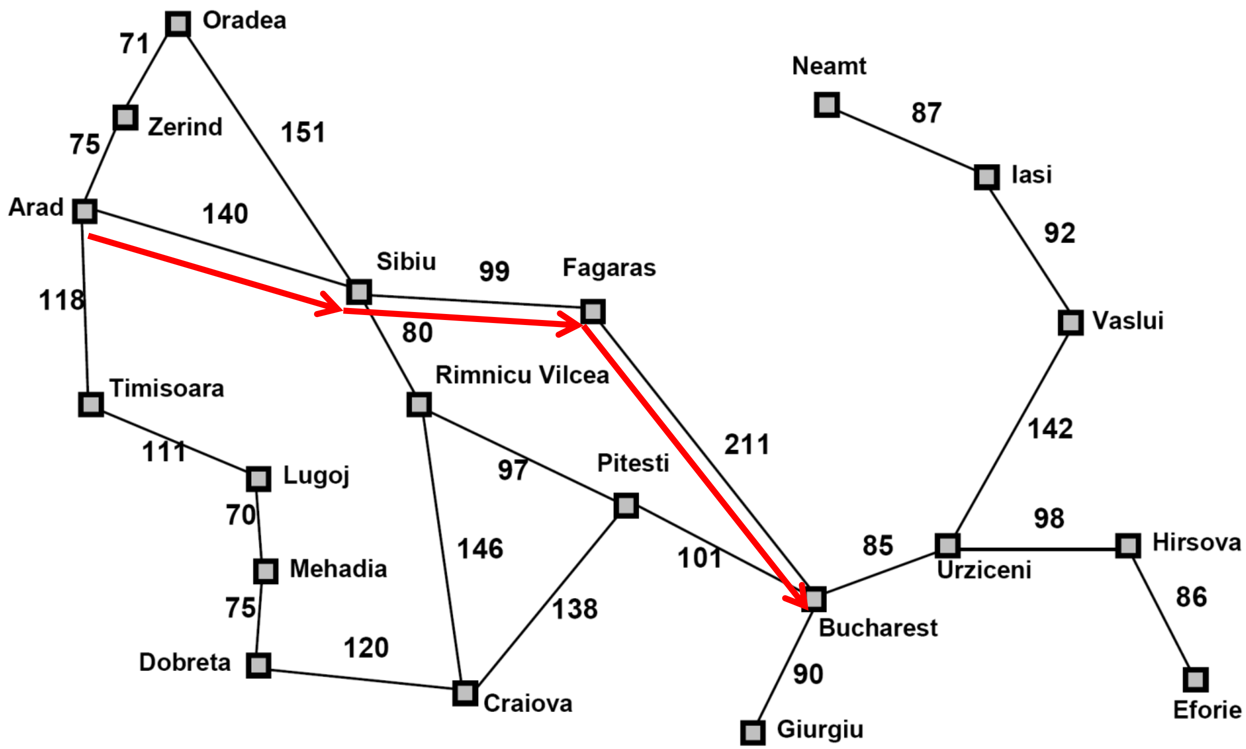


Romania Problem – Greedy Solution



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Properties of Greedy Search

- Complete? No, can stuck in loops
- Time? $O(b^m)$, can be improved with a good heuristic
- Space? $O(b^m)$
- Optimality? No

A* Search

- The most well-known form of best-first search
- Avoiding paths that are already expensive

- $f(n) = g(n) + h(n)$
  - $g(n)$ = the path cost from the start node to node n

- $f(n)$ becomes the estimated cost of the cheapest solution through n

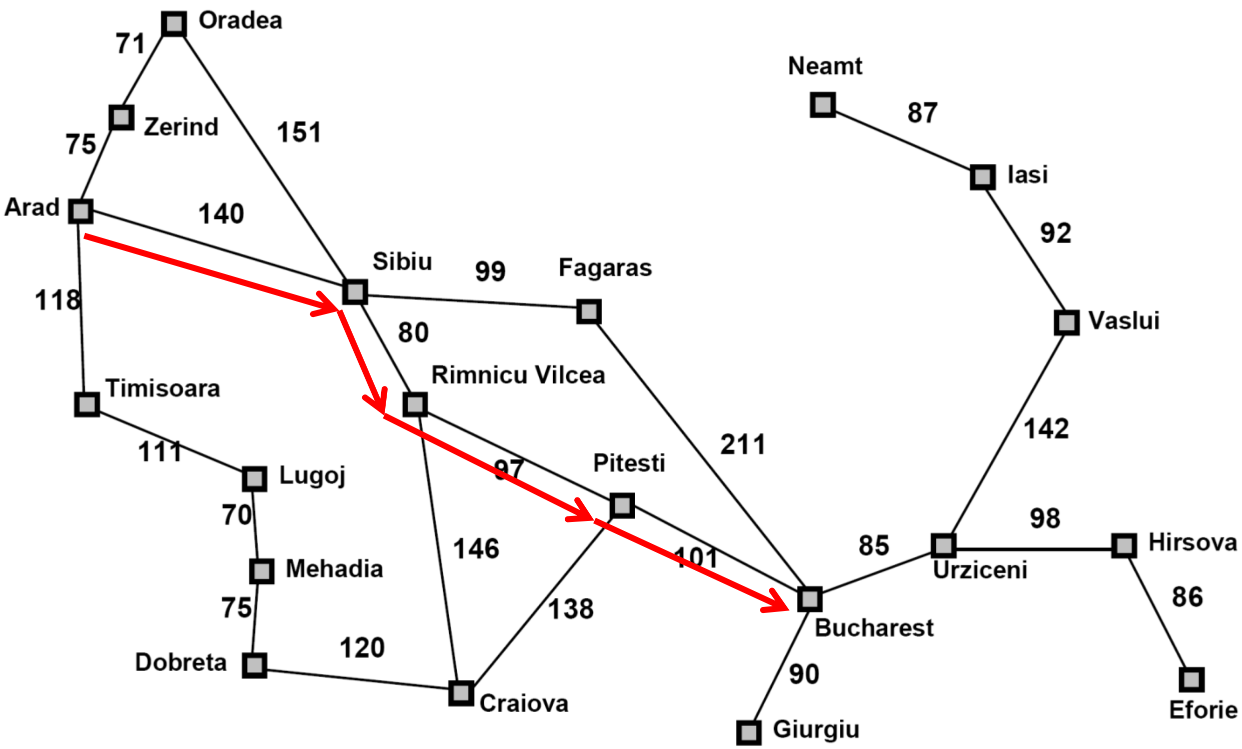
Admissible Heuristic

- Admissible heuristic:
  - $h(n) \leq h(n)^*$ (true cost from n)
  - $h(n) \geq 0$ and $h(G) = 0$

- A* (with tree-search) is optimal if $h(n)$ is admissible

- $h_{SLD}(n)$ admissible?

Romania Problem – A* Search



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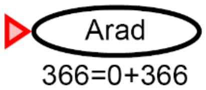
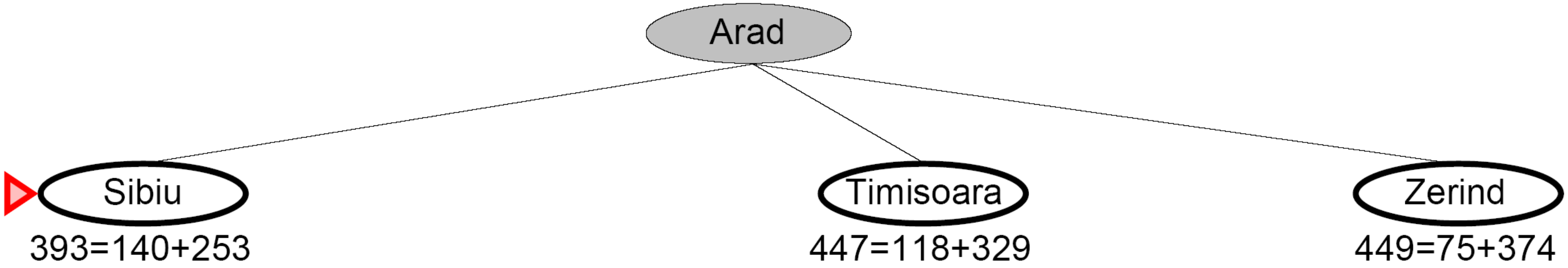


- $g(n)$ values-computed from the step costs
- $h_{SLD}(n)$ values-known initially

Romania Problem – A* Search



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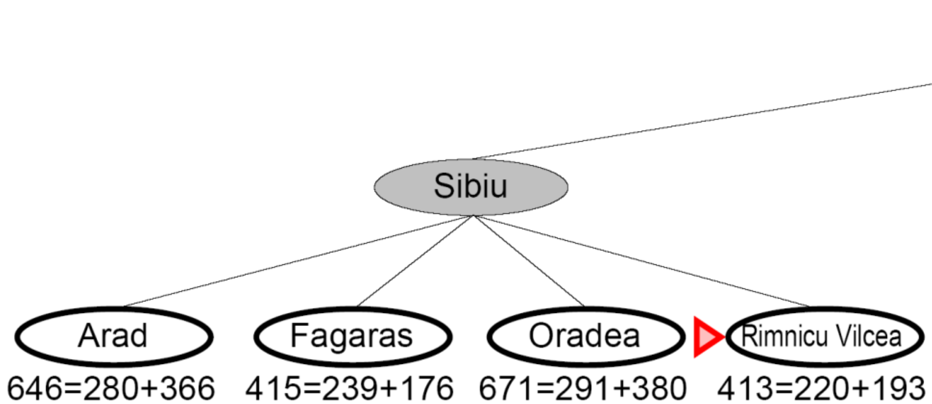
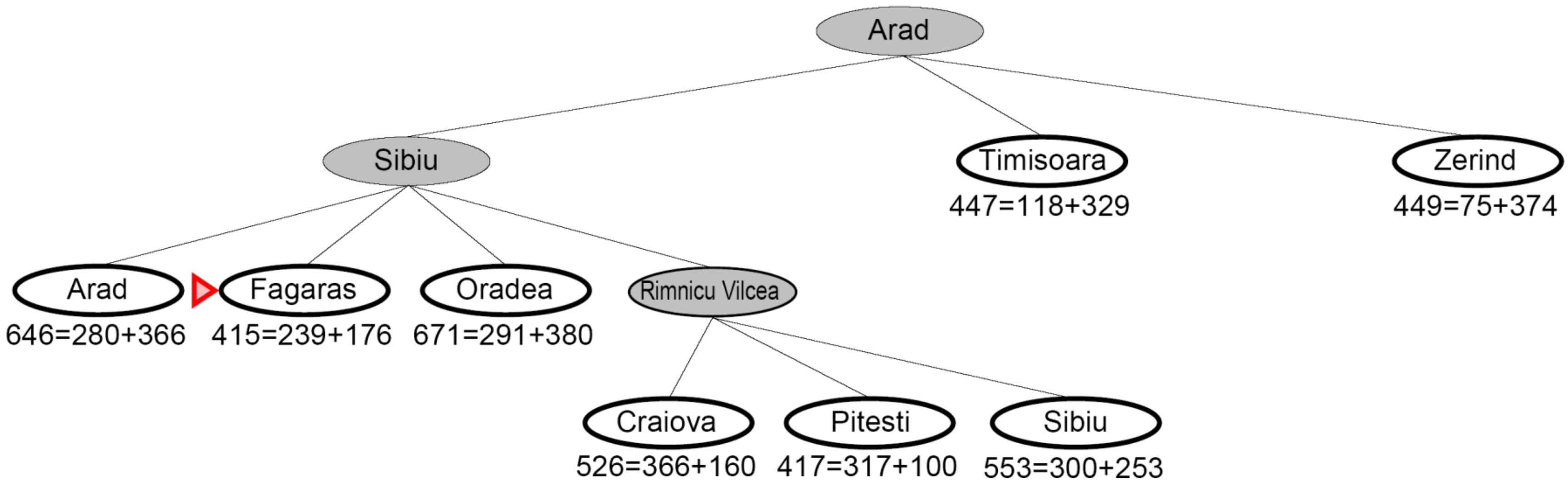
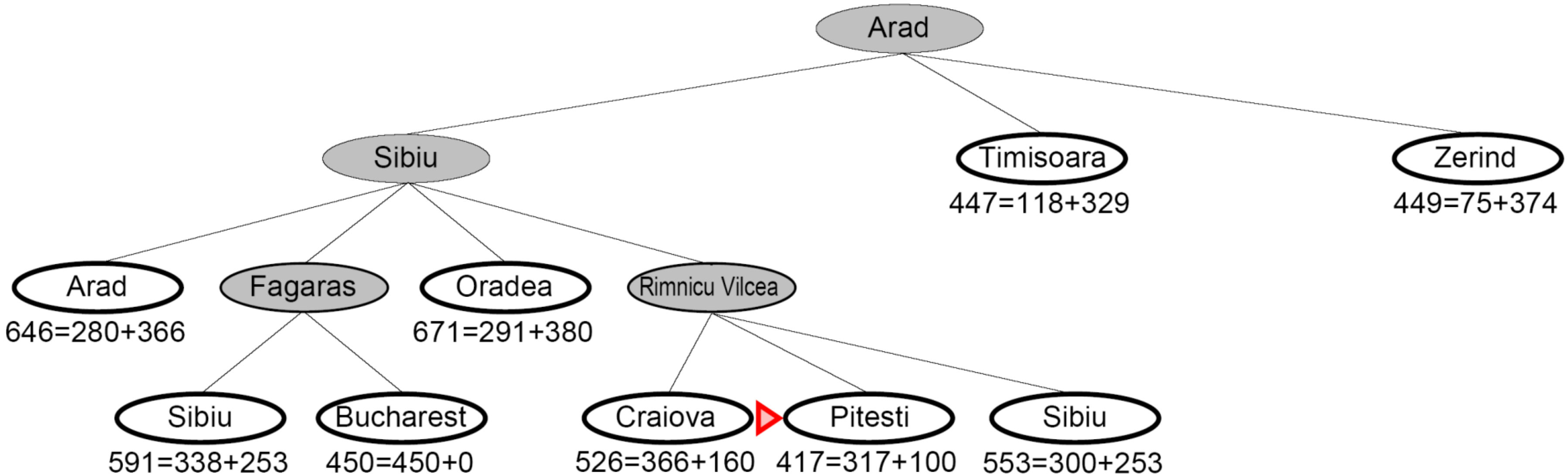


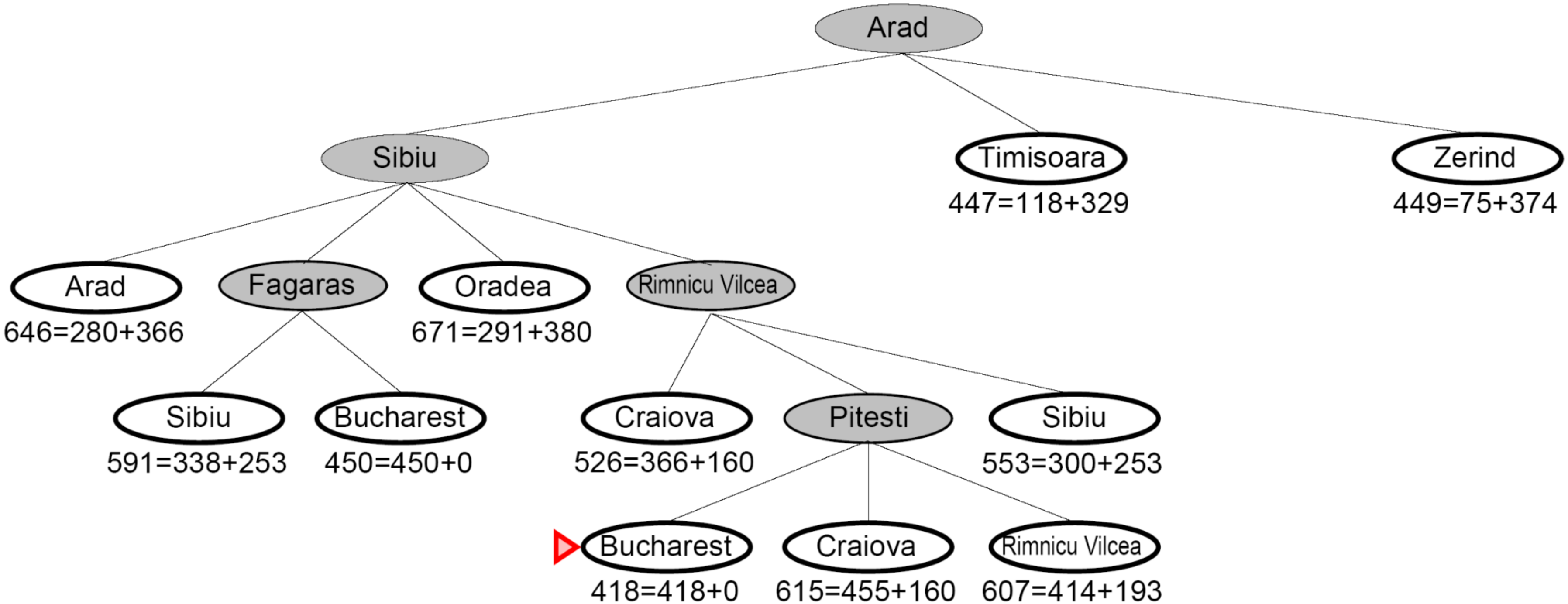
Romania Problem – A* Search



Romania Problem – A* Search

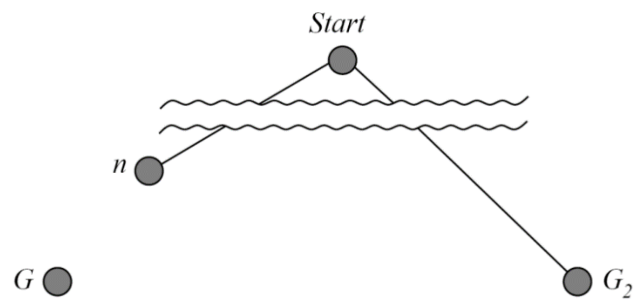


Romania Problem – A* Search



Optimality of A* (standard proof)

- Suppose a suboptimal goal G_2
- Let n be an unexpanded node on a shortest path to an optimal goal G_1



$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\ &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

- Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

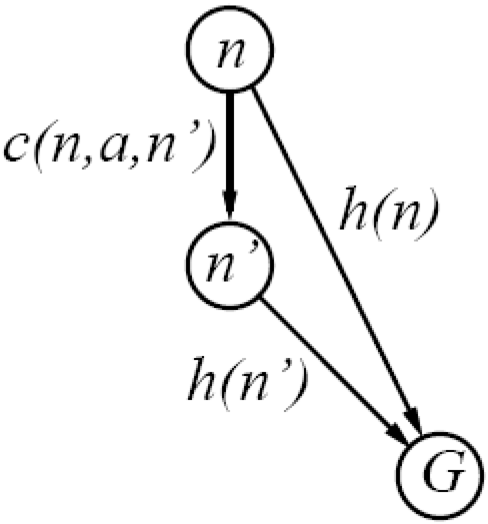
Consistency

A heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$



I.e., $f(n)$ is nondecreasing along any path.

When do we need a consistent heuristic?

- We need an admissable and consistent heuristic for the graph search version of A*

- Why?

Properties of A*

- Complete? Yes, unless there are infinitely many nodes with f < C*
- Time? Exponential in
  - [relative error in h * length of solution]
- Space? Keeps all nodes in memory
- Optimality? Yes

To reduce the space complexity

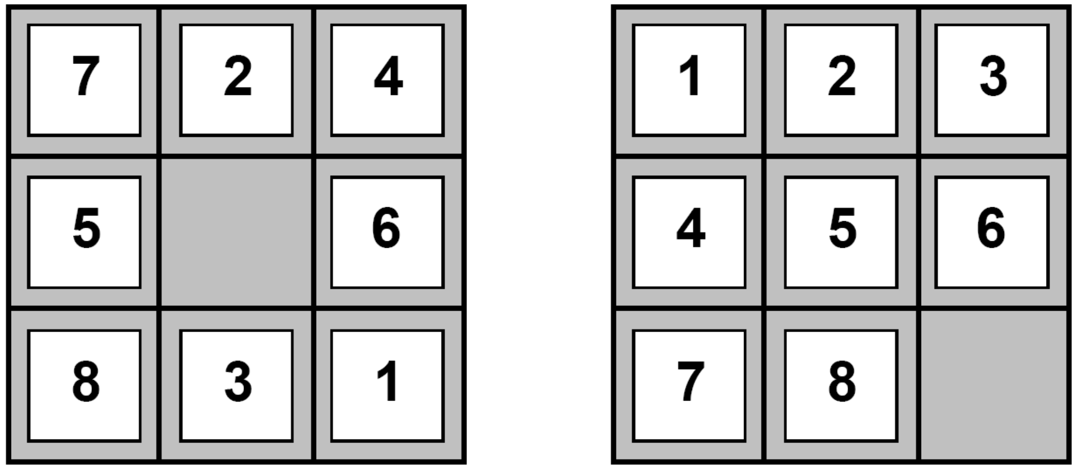
- Iterative deepening A* (IDA*)
  - Cut-off used is the f-cost

- Recursive best-first search (RBFS)
  - f_limit of best alternative path

- Memory-Bounded A* (MA*)
- Simplified MA* (SMA*)
  - drops the worst leaf node when memory is full

Admissible Heuristics

- 8-puzzle



Start State   Goal State

- $h_1(n)$ = the number of misplaced tiles
- $h_2(n)$ = the total Manhattan distance

Dominance

- if $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 dominates h_1, and is better for search

- Given any admissible heuristics h_a, h_b

  $$h(n) = \max(h_a(n) , h_b(n))$$

is also admissible and dominates h_a, h_b