Istanbul Technical University
Faculty of Computer and Informatics
Computer Engineering Department

# BLG 354E
# Signals & Systems for Computer Engineering
# Homework I

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# Contents

# 1 Question I

**Figure 1:** Diagram of taking photo and projecting it.

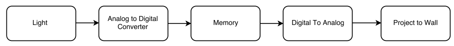



Diagram.pdf

# 2 Question II

The heartbeat records is a signal transformed from analog to digital in this example. Heart is periodically beating and it creates oscillation and it is a signal actually.

Voice record is same like heart beat but it does not have to periodic. Sound is oscillation of particles in the environment. It is signal too.

Images are kind of representation of light in the digital environment by help of some techniques. It is also signal.

# 3 Question III

Roots are

$$z_1 = -\sqrt[8]{-1}$$

$$z_2 = \sqrt[8]{-1}$$

$$z_3 = -(-1)^{\frac{5}{8}}$$

$$z_4 = (-1)^{\frac{5}{8}}$$

# 4 Question IV

$$\text{Taylor Expansion of } e^x$$
$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$
$$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} \;\; ...$$
$$= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} \;\; ...$$
$$\text{Group them..}$$
$$= (1 - \frac{x^2}{2!} + \frac{x^4}{4!} -) + i * (x - \frac{x^3}{3!} + \frac{x^5}{5!} -)$$
$$= \cos x + i \sin x$$

# 5 Question V

## 5.1 Part A - Odd Function

Let's $f(x) = y$ a function with one parameter. If $f(x) = -f(-x)$ we call them odd functions. For example sin is a odd function. $\sin \pi = -\sin -\pi$

## 5.2 Part B - Even Function

Let's $f(x) = y$ a function with one parameter. If $f(x) = f(-x)$ we call them odd functions. For example cos is a odd function. $\cos \pi = \cos -\pi$

## 5.3 Part C - Matches

$\sin \theta = \cos(\theta - \pi/2)$, $\cos(\theta + 2\pi k) = \cos \theta$ if k is integer, $\cos -\theta = \cos \theta$, $\sin -\theta = -\sin \theta$, $\sin(\pi k) = 0$ if k is integer $\cos(2\pi k) = 1$ if k is integer, $\cos(2\pi(k + 1/2)) = -1$ if k is integer.

## 5.4 Part D - Derivations

## 5.5 i

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$
$$\cos^2 \theta + \sin^2 \theta = \frac{e^{2i\theta} + 2 + e^{-2i\theta}}{4} - \frac{e^{2i\theta} - 2 + e^{-2i\theta}}{4} = \frac{2 - (-2)}{4} = 1$$

### 5.6 ii

$$\cos^2\theta - \sin^2\theta = \frac{e^{2i\theta} + 2 + e^{-2i\theta}}{4} + \frac{e^{2i\theta} - 2 + e^{-2i\theta}}{4}$$
$$= \frac{2(e^{2i\theta} + e^{-2i\theta})}{4} = \cos 2\theta$$

### 5.7 iii

$$2\sin\theta\cos\theta = 2 * \frac{e^{i\theta} + e^{-i\theta}}{2} * \frac{e^{i\theta} - e^{-i\theta}}{2i} = \frac{2(e^{2i\theta} - e^{-2i\theta})}{4i} = \sin 2\theta$$

### 5.8 iv

From Euler expansion arithmetic operations.

### 5.9 v

From Euler expansion arithmetic operations.

## 6 Question VI

$$\textstyle\sum_{k=1}^{N} A_k \cos(\omega t + \theta) = \sum_{k=1}^{N} \mathrm{Re}\{A_k e^{j\omega_0 t + \phi_k}\}$$
$$= \mathrm{Re}\{\textstyle\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\}$$
$$= \mathrm{Re}\{(\textstyle\sum_{k=1}^{N} A_k e^{j\phi_k}) e^{j\omega_0 t}\}$$
$$= \mathrm{Re}\{(Ae^{j\phi}) e^{j\omega_0 t}\}$$
$$= \mathrm{Re}\{(Ae^{j(\omega_0 t + \phi)}\}$$
$$A = cos(\omega_0 t + \phi)$$

## 7 Question VII

### 7.1 Part I

$$z_1(t) = \cos(\omega t - \pi/3), z_2(t) = 3\sin(\omega t - \frac{5}{4}\pi) = 3\cos(\omega t - \frac{7}{4}\pi), z_3(t) = 2\cos(\omega t 4.7124)$$
$$x(t) = z_1(t) + z_2(t) + z_3(t)$$

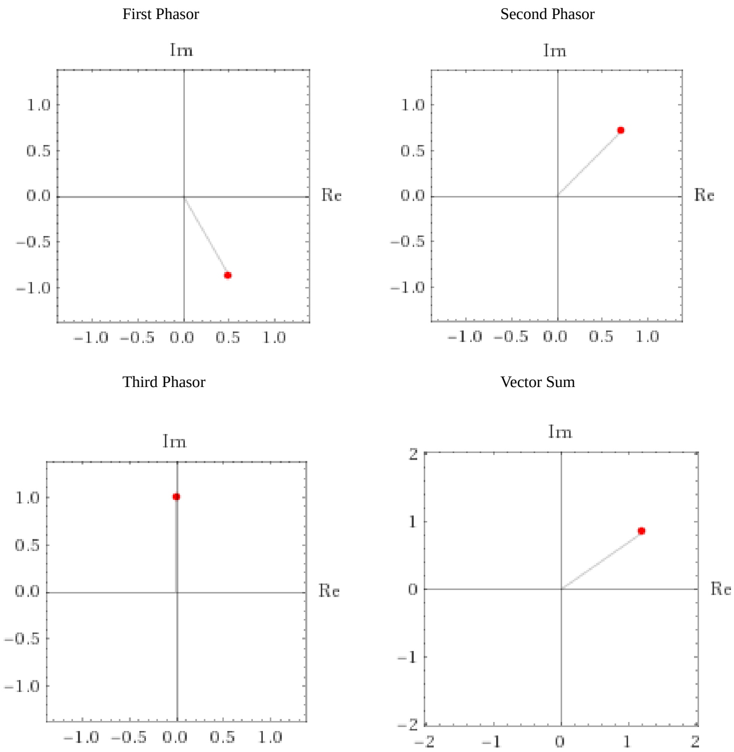
$e^{-\pi j/3} + e^{-7\pi j/4} + e^{-3\pi j/2j}$ then we can sum phasors like vector sum.

Result is $r = 1.47, \theta = 34.86\ degree, 0.60\ rad$, I used calculator for it.

$$x(t) = 6\cos(\omega t - 0.19\pi)$$

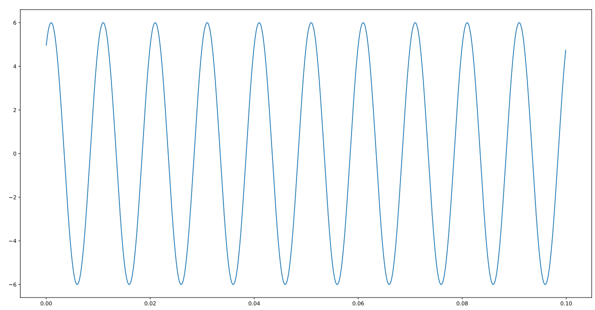
## 7.2 Part II

**Figure 2:** Polar Coordinates of Phasors, created by wolframalpha




## 7.3 Part III

**Figure 3:** 500 hz frequency cosine signal with $0.19\pi$ phase (x axis time)



# 8 Question VIII

## 8.1 Part I

## 8.2 Part II

The signal is periodic because we can find $f_0 = gcd(f_k)$ and $f_0$ is an integer. $f_0 = 5$ for this case and period is $t_0 = \frac{1}{f_0} = 0.2$

## 8.3 Part III

The fundamental frequency $f_0 = 5$

# 9 Question IX

Multiplication of sinusoids..

## 9.1 Part I

## 9.2 Part II

## 9.3 Part III

$-7\cos(9990\pi t + \pi/6) + -7\cos(10010\pi t + \pi/6)$

# 10 Question X

We can take integral of products of functions. If two functions satisfy this $\int f(x)g(x)dx = 0$ we called them as orthogonal functions. From Euler expansion arithmetic operations. We can user Euler expansion for make the integral easy may be I am not even sure.

# 11 Question XI

It occurs when we are calculating sum of Fourier series. It makes jumps that can be seen at figure below.

Photo taken from Wikipedia article about Gibbs phenomenon.