Algorithm Sketch

Ugur Yildirim

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Initialization Step

1) Check that:
   - D is connected
   - $g(x)$ is continuous and differentiable everywhere in D
   - $h(x) = ln(g(x))$ is concave everywhere in D

2) Initialize `T_k` (vector with k elements):
   - $T_k = \{x_i; i = 1, ..., k\}$, where $x_1 \leq ... \leq x_k$ are the k abscissae in D where we will evaluate $h(x)$ and $h'(x)$
   - If D unbounded on the left, chose x_1 s.t. $h'(x_1) > 0$
   - If D unbounded on the right, chose x_k s.t. $h'(x_k) < 0$

3) Evaluate $h(x)$ and $h'(x)$ on T_k and store these as two length k vectors, say `h_x` and `h_prime_x`

4) Calculate `z` (vector with $k+1$ elements):
   - z_0 = lower bound of D (or $-\infty$ if D is not bounded below)
   - For $j = 1, ..., k-1$, $z_j = \frac{h(x_{j+1}) - h(x_j) - x_{j+1}h'(x_{j+1}) + x_j h'(x_j)}{h'(x_j) - h'(x_{j+1})}$ (these are the points at which the tangents to $h(x)$ at x_j and x_{j+1} intersect)
   - z_k = upper bound of D (or $+\infty$ if D is not bounded above)

5) Find `u_k` (this is the piecewise linear upper hull formed by tangents to $h(x)$ at T_k):
   - $u_k(x) = h(x_j) + (x - x_j)h'(x_j)$, where $x \in [z_{j-1}, z_j]$ and $j = 1, ..., k$
   - NB: $exp(u_k(x))$ is the rejection envelope on T_k

6) Find `s_k`:
   - $s_k(x) = \frac{exp(u_k(x))}{\int_D exp(u_k(x'))dx'}$

7) Find `l_k` (this is the piecewise linear lower hull formed by connecting adjacent points on $h(x)$ where T_k is evaluated)
   - $l_k(x) = \frac{(x_{j+1} - x)h(x_j) + (x - x_j)h(x_{j+1})}{x_{j+1} - x_j}$, where $x \in [x_j, x_{j+1}]$ and $j = 1, ..., k-1$
   - For $x < x_1$ or $x > x_k$, define $l_k(x) = -\infty$
   - NB: $exp(l_k(x))$ is the squezzing function on T_k

Sampling Step

1) Sample a value x^* from $s_k(x)$

2) Sample a value w independently from $Unif(0, 1)$

3) Perform the test:
   - If $w \leq exp\{l_k(x^*) - u_k(x^*)\}$:
     - Accept x^*
   - Else:

- Evaluate $h(x^*)$ and $h'(x^*)$
- If $w \leq h(x^*) - u_k(x^*)\}$:
  - Accept x^*
- Else:
  - Reject x^*

Updating Step

1) Follow this recipe:

- If $h(x^*)$ and $h'(x^*)$ were evaluated in Sampling Step:
  - Include x^* in T_k to form T_{k+1}
  - Relabel the x_i in T_k in ascending order
  - Construct new functions $u_{k+1}(x)$, $s_{k+1}(x)$, and $l_{k+1}(x)$
  - Increment k
  - Return to Sampling Step if n points have not been sampled yet
- Else:
  - No update necessary, repeat Sampling Step if n points have not been sampled yet