Types of Differential Equations

Ordinary Differential Equations

A differential equations that depend on only one independent variables.

- Order of Differential Equation (ODE)
  The highest order of derivative involved in the equation
- First Order ODE
$$f(y)\frac{dy}{dx} = g(x)$$
- First Order Linear ODE
$$y'(x) + p(x)y(x) = q(x)$$
- Second Order ODE
$$y"(x) + y'(x) + y(x) = x$$

Partial Differential Equations

A differential equations that depend on two or more independent variables

Solving Differential Equations

1st Order Linear ODE

Use the integrating factor method to solve for the general form solution such that the integrating factor is

$$e^{\int p(x)dx}$$

then distribute the integrating factor

2nd Order ODE General Solution of Superposition

$$y(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

Case 1: $r_1, r_2 \in \mathbb{R}$

- 2 real roots then same as above
- 1 real roots
$$y(x) = c_1 e^{rx} + c_2 x e^{rx}$$

Case 2: If $r_1, r_2 \in \mathbb{I}$

$$y(x) = e^{\alpha x}(c_1 \cos(\beta x) + c_2 \sin(\beta x))$$

Example Problem

Determine the type of the ordinary differential equation and find the general solution

1. $y" - y" + y' + y = 0$
   First order because y" - y" = 0 and let the integrating factor be $e^{\int 1dx} = e^x$ therefore, integrating the left side we have that

$$e^x y' + e^x y = \int \frac{d}{dx} e^x y = e^x y$$

   and integrating the right side we have that

$$\int 0dx = k$$

   such that k is a constant thus as a result we have that

$$y = \frac{k}{e^x}$$

2. $y" - y = 0$
   This is a second order differential equation then let $r^2 = y"$ and $y = r^0$ so we have

$$r^2 - 1 = (r+1)(r-1)$$

   Thus, we have two real roots such that $r = \pm 1$ thus the general solution is

$$y(x) = c_1 e^x + c_2 e^{-x}$$

Practice Problems

Determine the type and find the general form of the ordinary differential equations

1. $y' = 2x$

2. $\cos(\theta) \frac{dr}{d\theta} - 2r\sin(\theta) = 0$