Concept

Matrix Addition

- The sum of two m x n matrix A = $[a_{ij}]$ and B = $[b_{ij}]$ is the m x n matrix A + B = $[c_{ij}]$ such that
$$c_{ij} = a_{ij} + b_{ij} \forall i \& j$$
- if A and B have different sizes they cannot be added

Transpose

- transpose of a m x n matrix A = $[a_{ij}]$ is the n x m matrix $A^T = [b_{ij}]$ with
$$b_{ij} = a_{ji}$$
- Example: If B = $\begin{bmatrix} 4 & 2 \\ 0 & 2 \end{bmatrix}$ then $B^T = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

  Theorem. *The Properties of Transpose*
  - $(A + B)^T = A^T + B^T$
  - $(cA)^T = cA^T$
  - $(A^T)^T = A$
  - $(AB)^T = B^T A^T$
  - *A matrix is symmetric if* $A = A^T$

Matrix Multiplication

- Let m x n matrix A = $[a_{ij}]$ and n x p matrix B = $[b_{ij}]$ then the product of A and B is m x p matrix AB = $[c_{ij}]$ such that
$$[c_{ij}] = (row_i A)^T \times col_j B$$

Problems

1. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ Then if possible do these following operations

   a) A + B

   b) 3A - 2C

   c) AC

   d) CA

Perform the following matrix operation if possible.

2. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$

3. $\begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} =$

4. $\begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \end{bmatrix} =$

5. Find the transpose of the following matrix and determine if they are symmetric

   a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

   b) $B = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$