CH.1 Vector Space

Vector Space

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Definition a set V along with addition and multiplication properties on V with the following properties

- Commutative: $u + v = v + u; \forall u, v \in V$
- Associativity: $(u + v) + w = u + (v + w); \forall u, v, w \in V$
- Additive Identity: $\exists 0 \in V$ s.t. $v + 0 = v$
- Additive Inverse : $\forall v \in V, \exists w \in V$ s.t. $v + w = 0$
- Multiplication Identity: $1v = v; \forall v \in V$
- Distributive Property: $a(u+v) = au+av$ and (a + b)u = (au + bu) ; $\forall a, b \in \mathbb{F}; \forall u, v \in V$

Types of Vector Space

- Real Vector Space: vector space over $\mathbb{R}$
- Complex Vector Space: vector space over $\mathbb{C}$

Propositions

- A vector space has a unique additive identity
- Every element in a vector space has a unique additive inverse
- 0v = 0 $\forall v \in V$
- a0 = 0 $\forall a \in F$
- (-1)v = -v $\forall v \in V$

Subspace of Vector Spaces

Definition A subspace S of V "$S \subset V$" us a set $S \subseteq V$ s.t. the following properties hold

- $0 \in S$
- closed under addition: $u, v \in S \to U + V \in S$
- closed under scalar product: $u \in Sa \in F \to au \in S$

Examples of Subspace

- Trivial Subspace $\{0\}$
- All of V is always a subspace of vector space V

Sum of Vector Space
Definition If $U_1, U_2, ..., U_n$ are subspace of Vector Space V their sum is $\sum_{i=1}^{n} U_i := \{u_1 + ... + u_n | u_i \in U_i\}$
Direct Sum
Definition Where each element of $U_1 + ... + U_m$ can be written only one way as a sum of $u_1 + ... + u_m$
Determine if sum is direct sum

- A sum U + W is a direct sum iff U + W = V and $U \cap W = \{0\}$
- $V = U_1 + ... + U_n$
- the only way to write 0 as a sum $u_1 + ... + u_n$ where each $u_j \in U_j$ is by taking all the u_j equal to 0