Key Terms

- Partition
- Integrable
- Least Upper Bound Property
- supremum
- infrimum
- sequence
- convergence
- neighborhood
- cauchy sequence
- accumulation points
- Bolzano Weiestrass Theorem
- Sequential Limit Theorem
- sub-sequence
- monotone
- increasing
- decreasing
- limits
- continuity
- uniform continuity
- open
- closed
- compact
- Heine Borel Theorem
- Extreme Value Theorem
- Bolzano's Theormem
- connected

- Intermediate Value Theorem
- differentiable
- Chain Rule
- relative maximum
- Rolle's Theorem
- Mean Value Theorem
- Cauchy Mean Value Theorem
- L'Hospital's Rule
- partition
- integrable

Sample Problems

1. Let the sequence (a_n) converge to A and $(b_n - a_n)$ converge to 0. Using the ϵ and N argument show that (b_n) converges to A.
2. Using $\epsilon - N$ argument prove that the sequence $(\frac{n}{2n+1})_{n=1}^{\infty}$ converges and find its limit.
3. Define what it means for the sequence $\{a_n\}_{n=1}^{\infty}$ to converge to a real number A.
4. Suppose f: [a,b] $\to \mathbb{R}$ is a bounded function.
   - Define what it means for P to be a partition of [a,b]
   - Define a Lower Sum L(P,f)
   - Define a Upper Sum U(P,f)
   - Define the lower integral of f
   - Define the upper integral of f
   - Define what it means for a function to be integrable
5. State the following theorems
   - Mean Value Theorem
   - Extreme Value Theorem
   - Intermediate Value Theorem
   - Rolle's Theorem
6. Give an example of an open cover of the set [1,5) that has no finite subcover
7. Define f: $\mathbb{R} \to \mathbb{R}$ by f(x) = $x^2 - 5$. Use $\epsilon - \delta$ definition to prove that $\lim_{x\to 1} f(x) = -4$

8. State one of the Theorems that gives a necessary and sufficient condition for f to be Riemann integrable on the interval [a,b].

9. Suppose E $\subset \mathbb{R}$ is nonempty and that $E \cap [0,1] =$

   - Is it possible that the $\sup E = 0$
   - Is it possible that the $\sup E = 1$

10. Suppose f:[0,2] $\to \mathbb{R}$ is defined by f(x) = 1 - x^2

    - Explain how you can be sure that $f \in \mathbf{R}[0,2]$.
    - For P the partition of [0,2] given by P = $\{0, 0.5, 1, 2\}$ compute L(P,f)

11. Give the definitions of the following words

    - differentiable
    - uniformly continuous
    - continuous
    - closed open
    - compact

12. Prove that $g(x) = x^3 + x - 1$ has at least one root which lies in the open interval (0, 1).

13. Prove that if f: $[a, b] \to \mathbb{R}$ is continuous, then f is Riemann integrable on [a,b].

14. Prove that every Convergent Sequence is Cauchy

15. Answer these questions regarding to compact sets:

    - State the Heine Borel Theorem
    - $\{-1, 0, 1\}$: Is it compact?
    - $\{0\} \cup (1, 4]$ Is it compact?
    - $\{\frac{1}{n} : n \in \mathbb{N}\}$ Is it compact?

16. The following Statement is false. Explain why.
    There is a function $f \in \mathbf{R}(x)$ on [-1,1] and a partition P of [-1,1] such that L(P,f) = 1 and U(P,f) = 2 and $\int_{-1}^{1} f(x)dx = 3$

17. State True (T) or False (F) for the following:

    - If A is a non-empty and compact set of real numbers then A contains inf A and sup A.
    - If f:(2,10) $\to \mathbb{R}$ is uniformly continuous, then it is bounded
    - If A and B are compact sets of real numbers then so is $A \cup B$

- If A and B are open sets of real numbers then so is $A \cup B$
- Every monotone sequence of real numbers converges.

18. Define a function f: $\mathbb{R} \rightarrow \mathbb{R}$

$$\begin{cases} 0 & if x \in \mathbb{Q} \\ x^2 & if x \notin \mathbb{Q} \end{cases}$$

(a) Is f continuous at x = 0? Justify your answer. (Justification based on definition will receive the most points)
(b) Is f differentiable at x = 0? Justify your answer.