Definitions to know

- increasing
- decreasing
- monotone
- limit
- continuous
- closed
- open
- compact
- Heine Borel Theorem

Chapter 2 Problems

- 2,7,11,12,15,19,22,24

Chapter 3 Problems

- 2,3,5,6,7,8,9,14,15,17,26,27,33,36

Past Exam Problems

1. Assume f: $D \to \mathbb{R}$ and $x_o \in D$ Define what it means for f to be continuous at x_0
2. Given an example of an open cover of the set [1,5) that has no finite subcover
3. State the Heine-Borel Theorem:
4. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Prove that the set A $= \{x \in \mathbb{R} | f(x) = 0\}$ is a closed subset of R?
5. True or False problems
   (a) If f: $\mathbb{R} \to \mathbb{R}$ is continuous and $E \subset \mathbb{R}$ is open, then f(E) is open
   (b) A union of any collection of closed sets of real numbers is a closed sets
   (c) Let f:[a,b] $\to \mathbb{R}$ be continuous. Then the image of f is a closed interval
   (d) If a set is not open, then it is closed
6. Give the defintion of a compact set (Do not state the Heine-Borel Theorem)
7. Give an example of an open cover of the set $[0, \infty]$ that has no finite subcover.

8. Are the following sets of real numbers compact.

   (a) $\{-1, 0, 1\}$

   (b) $\{0\} \cup (1, 4]$

   (c) $\{\frac{1}{n} : n \in \mathbb{N}\}$

9. Define a function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x\sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   (a) Explain why f is continuous at each point of $\mathbb{R} \setminus 0$

   (b) SHow that f is continuous at x = 0

10. Let E = $\{\frac{1}{n} : n \in \mathbb{N}\}$ and define f: $E \to \mathbb{R}$ by $f(\frac{1}{n}) = (-1)^n$ Is f continuous?

11. Let f: [0,2] $\to \mathbb{R}$ given by f(x) = $\frac{x}{1+x}$. Use $\epsilon\delta$ argument that f has a limit at x = 1.