Main Topics

- Span
- Bases
- Dimensions

Span

Linear Combination
a list of vectors in V is a vector of the form

$$a_1v_1 + a_2v_2 + ... + a_mv_m$$

where $a_1, ..., a_m \in F$
span
a set of all linear combination denoted as $span(v_1, ..., v_m) = \{a_1v_1 + ... + a_mv_m\}$

- span is the smallest containing subspace

Linear Independence
a list of vectors in $\vec{V}$ where $a_1v_1 + ... + a_mv_m = 0$ such that the only real numbers that satisfy the equation are $a_1 = ... = a_m = 0$.
Otherwise if not then it is said to be linearly dependent.
Examples

- (1,0,0,0) and (0,1,0,0) are L.I. in F^4
- (1,2,1,0) and (0,1,2,0) and (1,1,0,-1) are L.I.
- (2,3,1), (1,-1,2), (7,3,0) are L.D.

Bases

Definition
a list of vectors in V that is linearly independent and that spans V
Example

- Standard Basis: (1,0,...,0), (0,1,...,0), ... (0,0,...,1)
- (1,0,1) , (0,1,-1), and (1,1,1) form a basis for F^3

Key Concepts

- Every finite dimensional vector space has a basis
- every linearly independent list of a finite dimensional vector space can be extended to a basis

Dimension

definition
number of vectors in any basis for a vector space V and is denoted by $\dim V$

- $\dim R^n = n$
- $\dim P_n = n + 1$
- $\dim M_{mn} = mn$

Theorems

- Let S be an indexed set of vectors in the vector space V with $\dim V = n$ then
  (a) if S spans V then S is a basis of V
  (b) if S is L.I. then S is a basis for V
- Let V be a finite dimensional vector space with a subspace W
  (1) $\dim W \leq \dim V$
  (2) If $\dim W = \dim V$ then W = V