Exam 3 Information

- Date: April 8, 2020
- Topics: Chapter 3 4

Key Terms and Concepts

- Uniform Continuity
- Closed
- Open
- Compact
- Heine Borel Theorem
- Extreme Value Theorem
- Bolzano's Theorem
- connected
- Modified Bolzano's Theorem
- Intermediate Value Theorem
- Differentiable
- Chain Rule
- Relative Extrema
- Rolle's Theorem
- Mean Value Theorem
- Cauchy Mean Value Theorem

Sample Problems

1. Give the definitions of a uniformly continuous function on its domain $D \subset \mathbb{R}$
2. Give the definition of a f to be differentiable at x_0.
3. State the following theorems:
    - The Mean Value Theorem
    - The Extreme Value Theorem
    - The Intermediate Value Theorem

4. Prove that the equation $x^3 + 3x + 1 = 0$ has exactly one root in the interval [-2,2].

5. Let f be defined by

$$f(x) = \begin{cases} \mathrm{x}^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   Show that f'(x) exists for all $x \in \mathbb{R}$ but the function $f' : \mathbb{R} \to \mathbb{R}$ is not continuous.

6. Prove that the curves $f(x) = 2x^3$ and g(x) $= 3x^2 - 2$ intersect on the interval [-1,1]. Justify your answers.

7. True or False

   - If f:(2,10) $\to \mathbb{R}$ is uniformly continuous then f is bounded
   - If a function is defined on (-1,1) and f is differentible at x $= 0$, then $\lim_{x\to 0} f(x) = 0$

8. Let f,g: $\mathbb{R} \to \mathbb{R}$ be bounded and uniformly continuous on $\mathbb{R}$. Prove that the product fg is uniformly continuous on $\mathbb{R}$