Key Concepts

Definition Let A be n x n matrix. A nonzero n vector v is called an eigenvector of A if

$$Av = \lambda v$$

for some scalar λ which is called an eigenvalue

Procedure

- Solve the characteristic polynomial of matrix A

$$\det(\lambda I - A) = 0$$

- From algebra the characteristics polynomial of n x n matrix has n polynomials so therefore this includes
  - $(\lambda - 3)^2 = 0$ has a double root 3 so 3 would be counted twice
  - complex roots: e.g. $\lambda^2 + 16 = 0$ has two complex roots $\pm 4i$
- For each eigenvalue of A solve the homogeneous system

$$(\lambda I - A)w = 0$$

Practice Problem

Find all the eigenvalues and bases for the corresponding eigenspaces for the given matrix

1. $\begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 3 & -1 & 1 \\ -2 & 3 & 0 & 0 \end{bmatrix}$

4. Prove or provide the counter example of the following statements

   a) If A is a zero $n \times n$ matrix then the only eignenvalue of A is zero.

   b) If A is an $n \times n$ matrix such that Au $=$ u then $\lambda = 1$ is an eigenvalue of A.