Key Concepts

- Calculate determinant of 2×2 matrix

  Theorem. *To calculate a determinant of a 2×2 matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then*

$$\det A = a_{11}a_{22} - a_{21}a_{12}$$

- Formula for 3 x 3 determinant

  Theorem. *Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ b_{21} & b_{22} & b_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$ then*

$$\det(A) = a_{11} \begin{bmatrix} b_{22} & b_{23} \\ c_{32} & c_{33} \end{bmatrix} - a_{12} \begin{bmatrix} b_{21} & b_{23} \\ c_{31} & c_{33} \end{bmatrix} + a_{13} \begin{bmatrix} b_{21} & b_{22} \\ c_{31} & c_{32} \end{bmatrix}$$

  Theorem. *For every n x n matrix A*

$$\det A = \det A^T$$

$$\det A^{-1} = \frac{1}{\det A}$$

  Theorem. *The following statements are equivalent for a n x n matrix A*

  - *A is invertible*
  - *A is row equivalent to I*
  - $detA \neq 0$
  - *Homogeneous system $Ax = 0$ has only trivial solutions*
  - *For n vector b the system $Ax = b$ has unique solution*

Directions Read the following directions.

1. Prove or disprove that: $\det(AB) = \det(A)\det(B)$

2. Prove or disprove that: $\det(A+B) = \det(A) + \det(B)$

3.

$$\det(\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix})$$

4.

$$\det(\begin{bmatrix} 2 & 4 & 0 \\ 1 & 1 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

5.

$$\det(\begin{bmatrix} 2 & 1 & 0 & 3 \\ 4 & 0 & 2 & 0 \\ 1 & -1 & 1 & 2 \\ 0 & 0 & -1 & 2 \end{bmatrix})$$