

Definition of Homogeneous Function

Definition

Let $z = f(x,y)$ define z as a function of x and y in a region R . Then the function is said to be homogeneous of order n if it can be written as

$$f(x, y) = x^n g(v)$$

Such that $v = \frac{y}{x}$ and $g(v)$ is a function of v

Solution of Differential Equation in which the Coefficient dx and dy are each homogeneous functions of same order

Definition First Order Differential Equation with Homogeneous Coefficient

$$P(x, y)dx + Q(x, y)dy = 0$$

where $P(x,y)$ and $Q(x,y)$ are each homogeneous functions of order n

Theorem. *If the coefficient are each homogeneous functions of order n then the substitution of*

$$y = vx, \frac{dy}{dx} = v + x \frac{dv}{dx}$$

will lead to an equation in which the variables are separable.

Example Problems

1. Find a 1-parameter family of solutions of $2xydx + (x^2 + y^2)dy = 0$