

- Matrix Inverse

Matrix B is an inverse of $n \times n$ matrix A such that

$$AB = BA = I$$

If matrix A has an inverse then we say that A is invertible.

Theorem. *If an inverse of a matrix exists it is unique.*

- Procedure to Find the Inverse Matrix A Form the Matrix $[A|I]$ and perform elementary row operations to obtain $[C|D]$ such that matrix $C = I$

Theorem. *If A and B are $n \times n$ invertible matrix then AB is also invertible and*

$$(AB)^{-1} = B^{-1}A^{-1}$$

Theorem. *For an $n \times n$ matrix A, the following statement are equivalent*

- *A is invertible*
- *A is row equivalent to I*
- *For every n vector b , the system $Ax = b$ has a unique solution*
- *the homogeneous system $Ax = 0$ has only trivial solution*

1. Determine whether matrix A is an inverse of B?

a) $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} B = \begin{bmatrix} 1 & 3 \\ 1 & 6 \end{bmatrix}$

b) $A = \begin{bmatrix} -4 & 0 & -3 \\ 0 & 1 & 2 \\ 7 & 0 & 5 \end{bmatrix} B = \begin{bmatrix} 5 & 0 & 3 \\ 14 & 1 & 8 \\ -7 & 0 & 4 \end{bmatrix}$

2. Find the inverse of each matrix

a) $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. Prove or provide a counter example to the following statement

a) If A is invertible and B is invertible then $A + B$ is invertible

b) If A is invertible and B is invertible then AB is also invertible