

1. Integrate the following

a) $\int \tan^3 \theta \sec \theta \, d\theta$ b) $\int \sin^2 x \cos^2 x \, dx$ c) $\int \frac{\sqrt{y^2 - 2}}{y} \, dy$ d) $\int \frac{x^2}{\sqrt{9 - x^2}} \, dx$

2. Write the partial fraction decomposition for the rational function, but do not solve for the constants.

$$\frac{x^2 + 2}{x^2(x^2 + 1)^2(x - 3)^3}$$

3. Find the partial fraction decomposition for the rational function (find the coefficients). Don't forget that PFD only applies to rational expressions where the degree of the denominator *strictly exceeds* the degree of the numerator.

$$\frac{x^2 + 1}{x^2 - 9x + 20}$$

4. Integrate the following.

a)

$$\int_4^8 \frac{y}{y^2 - 2y - 3} \, dy$$

b)

$$\int \frac{\sin(\theta)}{(\cos(\theta) - 1)(\cos^2(\theta) + 2\cos(\theta) + 1)} \, d\theta$$

5. Approximate $\int_{-2}^2 (x^2 + 1) dx$ using Trapezoid Rule with $n = 4$, and then using Simpson's Rule with $n = 4$. Compare both of these to the exact value of $\int_{-2}^2 (x^2 + 1) dx$. Why do you think this is?
6. Find an upper bound for the error when one uses Trapezoid rule with $n = 4$ to approximate $\int_{-1}^1 e^{x^2} \, dx$. **Note: you do not need to find the approximation, only the upper bound for the estimate.**

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}, \text{ where } |f''(x)| \leq K \text{ for all } x \text{ in } (a, b)$$

7. For the following integrals, evaluate or conclude divergence.

a)

$$\int_0^1 \frac{dx}{x^{3/2}}$$

b)

$$\int_{-\infty}^0 \theta e^{\theta} d\theta$$

8. For the following integrals, test for convergence or divergence. (Remember “ p -integrals!”)

a)

$$\int_3^{\infty} \frac{dx}{x^2 - 4}$$

b)

$$\int_1^{\infty} \frac{\arctan x}{x}$$

c)

$$\int_{\pi}^{\infty} \frac{1 + \sin x}{x^2} dx$$

d)

$$\int_1^{\infty} \frac{\sqrt{x^4 + 1}}{x^3} dx$$

e)

$$\int_2^{\infty} \frac{dt}{t - \sqrt{t}}$$

f)

$$\int_{12}^{\infty} \frac{dx}{x^{100} - 6x + 7}$$