One Independent Variables

Definitions

- Differential of γ
  Let y = f(x) define y as a function of x on an interval I. Then the differential is defined by

$$(dy)(x, \Delta x) = f'(x)\Delta x$$

Theorem. *If* $y = \widehat{x}$ *then*

$$(dy)(x, \Delta x) = f'(x)\Delta x = (d\widehat{x})(x, \Delta x) = f'(x)\Delta x = \Delta x$$

Two Independent Variables

Definition Let z = f(x,y) define z as a function of two independent variables. Then the differential of z is defined by

$$(dz)(x, y, \Delta x, \Delta y) = \frac{\delta f(x,y)}{\delta x}\Delta x + \frac{\delta f(x,y)}{\delta y}\Delta y$$

Theorem. *If z = f(x,y) =* $\widehat{x}$*, then*

$$(dz)(x, y, \Delta x, \Delta y) = (d\widehat{x})(x, y, \Delta x, \Delta y) = \Delta x$$

With Separable Variables

Definition In a form of

$$\int f(x)dx + \int g(y)dy = C$$

such that C is an arbitrary constant $C \in \mathbb{R}$

Examples

1. Find the 1 parameter family of solutions of $2xdx - 9y^2dy = 0$

$$x^2 - 3y^3 = Cs.t.C \in \mathbb{R}$$

2. Find a particular solution for which y(2) = 1 for $xy^2dx + (1-x)dy = 0$.
   If $y \neq 0$ and $x \neq 1$ we can get rewrite the equations as

$$(\frac{1}{1-x} - 1)dx + y^{-2}dy = 0 | x \neq 1, y \neq 0$$

   Then as a result from integration we have that

$$\ln|1-x| + x + \frac{1}{y} = 3, x \neq 1, y \neq 0$$