Key Terms

- Well Ordering Axiom
  every nonempty subset of the set of positive integers contains smallest element
- Prime
  an integer is considered prime if the only divisors are ± 1 and $\pm$ itself
- Relatively Prime
  two integers whose GCD is 1

Theorems

Theorem. *Division Algorithm*
Let a,b be integers with $b > 0$. Then there $\exists q$ and $r \in \mathbb{Z}$ such that $a = bq + r$. where $0 \geq r < b$

Theorem. *Fundamental Theorem of Arithmetic*
$\forall n \in \mathbb{Z}$ except 0 is a product of primes

Theorem. *Let $n > 1$. If n has no positive prime factors less than or equal to $\sqrt{n}$ then n is prime*

Practice Problems

1. Find the quotient q and remainder r when a is divided by b w/o the usage of technology
   (a) a = 17 b = 4
   $$17/4 = 4R1$$
   Therefore q = 4 and r = 1 then we have that 17 = 4(4) + 1
   (b) a = -51 and b = 6
   $$-51/6 = -8R3$$
   Since - 8 is negative then q = -9 and r = 3 so therefore,
   $$-51 = -9(6) + 3$$
2. Let a be any integer and let b and c be any integer divided by b, the quotient be q,and the remainder be r, so that
   $$a = bq + r; 0 \geq r < b$$
   Consider a, b, and c be any integer and
3. Find the GCD
   (a) (56,72)
   $$72 = 56(1) + 16$$
   $$56 = 16(3) + 8$$
   $$16 = 8(2)$$
   Hence, the (56,72) = 8

(b) (143, 231)

$$231 = 143(1) + 88$$
$$143 = 88(1) + 55$$
$$55 = 33(1) + 22$$
$$33 = 22(1) + 11$$
$$22 = 11(2)$$

(143,231) = 11

4. Express the numbers as a product of primes

   (a) 5040

$$5040 = 2^4 \times 7 \times 9$$

   (b)

5. Which of the following are prime

   a) $2^5 - 1$

$$2^5 - 1 = 32 - 1 = 31$$

   Let n = 31 and let us take the square root of n then

$$\sqrt{31} \approx 5.6$$

   There does not exist prime numbers less than 5 that 31 is divisible by, so therefore, 31 is prime.

   b) 1951
   The prime factorization of 1951 is $\pm 1, \pm 1951$ therefore, by definition of prime then 1951 is prime.