

**Problem 1.** The Kalman Theater Group (KTG) is building a movie theater mega-complex. They have decided that there will automatic ticket kiosks in front of a single first-come-first-served queue, but they still need to decide how many kiosks to include in the complex design. Based on data that they have collected from their other theaters in similar markets, they have estimated that customers arrive at the kiosks at a rate of 5 per minute, and customers can be served in 2 minutes on average. KTG's standard configuration for mega-complexes in similar markets is 12 kiosks. You have been asked to evaluate this standard configuration.

Assume that the interarrival times and the service times are exponentially distributed.

- What standard queueing model fits this setting best?
- What is the traffic intensity in this queueing system?
- What is the long-run expected fraction of time that all kiosks are unoccupied?
- Over the long-run, what is the expected time a customer waits in line?

a.  $M/M/12$  <sup>s</sup> with  $\lambda = 5$  customers/min and  $\mu = \frac{1}{2}$  customer/min

b.  $\rho = \frac{\lambda}{s\mu} = \frac{5}{12(\frac{1}{2})} = \frac{5}{6} \approx 0.833$

c. Want  $\pi_0$ :  $\rho = \frac{5}{6}$ ,  $s\rho = \frac{\lambda}{\mu} = 10$

$$\Rightarrow \pi_0 = \left[ \sum_{j=0}^{12} \frac{10^j}{j!} + \frac{12^{12} (5/6)^{13}}{12! (1 - 5/6)} \right]^{-1} \approx 0.000036$$

d.  $\pi_{12} = \frac{10^{12}}{12!} \pi_0 \approx 0.075 \Rightarrow \omega_q = \frac{\pi_{12} \rho}{\lambda (1 - \rho)^2} = 0.449$  minutes