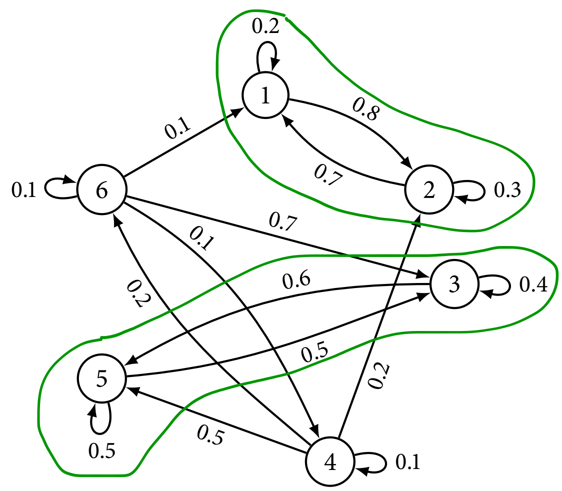
Problem 1. An autonomous UAV has been programmed to move between six regions to perform surveillance. The movements of the UAV follow a Markov chain with 6 states (1 for each region), and the following transition probability diagram:



a. There are two irreducible sets of states: $\{1, 2\}$ and $\{3, 5\}$. Briefly explain why these sets are irreducible.
b. Which states are transient? Which states are recurrent? Briefly explain.
c. Suppose the UAV starts in region 1. What is the long-run fraction of time that the UAV spends in region 1?
d. What is the probability that the UAV is absorbed into states 3 or 5, given that it starts in region 4?

a. Looking at the transition diagram, it is clear that $\{1,2\}$ and $\{3,5\}$ form self-contained Markov chains.

b. Transient states: $\{4,6\}$ Recurrent states: $\{1,2,3,5\}$

c. Let $\mathcal{R} = \{1,2\}$. To find π_1, let's find $\vec{\pi}_{\mathcal{R}}$:

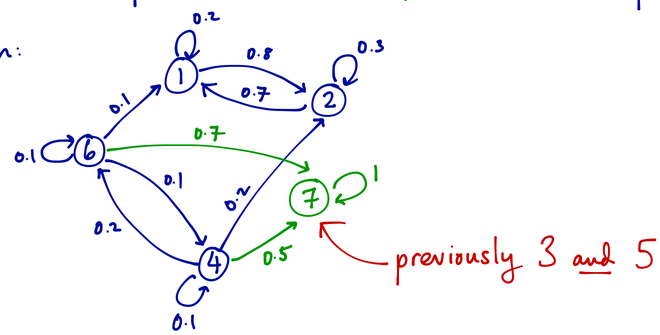
$$\mathbb{P}_{\mathcal{R}\mathcal{R}} = \begin{bmatrix} 0.2 & 0.8 \\ 0.7 & 0.3 \end{bmatrix} \Rightarrow \begin{matrix} \vec{\pi}_{\mathcal{R}}^T(\mathbb{I} - \mathbb{P}_{\mathcal{R}\mathcal{R}}) = \mathbb{0} \\ \vec{\pi}_{\mathcal{R}}^T \mathbb{1} = 1 \end{matrix} \iff \begin{matrix} 0.8\pi_1 - 0.7\pi_2 = 0 \\ -0.8\pi_1 + 0.7\pi_2 = 0 \\ \pi_1 + \pi_2 = 1 \end{matrix}$$

$$\Rightarrow \boxed{\pi_1 = \frac{7}{15}} \quad \pi_2 = \frac{8}{15}$$

Long-run fraction of time the UAV spends in region 1

d. This is a little tricky – our definition of an absorbing probability requires an absorbing state — an irreducible set of states with only one state.

Let's replace 3 and 5 with a "super state" called 7 – we end up with the following transition diagram:



Then, let $I = \{4,6\}$ and $R = \{7\}$. To find α_{47}, we compute α_{IR}:

$$\alpha_{IR} = (\mathbb{I} - \mathbb{P}_{II})^{-1}\mathbb{P}_{IR} = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}\right)^{-1}\begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix} \approx \begin{bmatrix} 0.747 \\ 0.861 \end{bmatrix}$$

$$\Rightarrow \alpha_{47} \approx 0.747.$$