Name:

SA402 – Dynamic and Stochastic Models Fall 2016
Assoc. Prof. Nelson Uhan

Quiz – 14 November 2016

Instructions. You have 25 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Standard	Problems	Score
E5	2	
E6	3	
F1	1a, 1b	
F2	1c	

Problem 1. Simplexville Pizza has two mixers. The manager has modeled the operation of these two mixers as a Markov process with three states – 0, 1, and 2 – that represent the number of broken mixers. The generator matrix of this Markov process is:

$$\mathbf{G} = \begin{bmatrix} -0.02 & 0.02 & 0 \\ 0.04 & -0.06 & 0.02 \\ 0 & 0.04 & -0.04 \end{bmatrix}$$

a. What is the rate at which this process goes from zero broken mixers to one broken mixer?

b. When there are two broken mixers, what is the expected time until one of the mixers is repaired and working?

c. What is the long-run fraction of time that both mixers are working?

Problem 2. Primal Pizza, the main competitor of Simplexville Pizza, has an unreliable mixer. On any given day, the mixer is either working or broken. If it is working, it will break down the next day 5% of the time. If it is broken, it will be repaired and working the next day 80% of the time. If the mixer remains broken for 2 days, it is simply replaced with a new working mixer the next day.

Model this mixer as a Markov chain by defining:

- the state space and the meaning of each state in the setting's context,
- the meaning of one time step in the setting's context, and
- the one-step transition probabilities.

Hint. You need to separately consider when the mixer has been broken for 1 day and when the mixer has been broken for 2 days.

Problem 3. Consider a model that tracks the geographic location of an American family: the state of the system classifies whether the family lives in an urban, rural, or suburban location, and each time step corresponds to one year. Describe what assumptions need to be made in order for the Markov property and the time-stationarity property to hold in this model.