SA402 – Dynamic and Stochastic Models
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Fall 2016

Standards-Based Grading – Rubric Examples

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Here are a few examples of rubrics for past assessments, to give you an idea of how your work translates to a score.

These examples are not meant to be definitive. Different rubrics may be appropriate for assessments covering different standards because of the varying topics and skills involved. Different rubrics may even be appropriate for assessments covering the same standard: assessments can vary in difficulty, and there are often several ways to assess the same standard.

1 Example: Standard B1, Exam 1 on 21 September

Problem. Suppose X is a random variable with pdf

$$f_X(a) = \begin{cases} 0 & \text{if } a < 2, \\ \frac{9}{2}a - \frac{3}{4}a^2 - 6 & \text{if } 2 \le a < 4, \\ 0 & \text{if } a \ge 4. \end{cases}$$

a. What is the cdf F_X of X?

$F_X(a) = \int_{-\infty}^{a} f_X(x)\,dx$:

$$\text{when } a < 2: \quad F_X(a) = \int_{-\infty}^{a} 0\,dx = 0$$

$$\text{when } 2 \le a < 4: \quad F_X(a) = \int_{-\infty}^{2} 0\,dx + \int_{2}^{a} \left(\frac{9}{2}x - \frac{3}{4}x^2 - 6\right) dx = 5 - 6a + \frac{9}{4}a^2 - \frac{1}{4}a^3$$

$$\text{when } a \ge 4: \quad F_X(a) = \int_{-\infty}^{2} 0\,dx + \int_{2}^{4} \left(\frac{9}{2}x - \frac{3}{4}x^2 - 6\right) dx + \int_{2}^{a} 0\,dx = 1$$

$$\underbrace{\Rightarrow F_X(a) = \begin{cases} 0 & \text{if } a < 2 \\ 5 - 6a + \frac{9}{4}a^2 - \frac{1}{4}a^3 & \text{if } 2 \le a < 4 \\ 1 & \text{if } a \ge 4 \end{cases}}_{\text{final answer}}$$

b. What is the expected value of X?

$$E[X] = \int_{-\infty}^{\infty} a f_x(a)\,da = \int_{2}^{4} a\left(\frac{9}{2}x - \frac{3}{4}x^2 - 6\right) da = \underbrace{3}_{\text{final answer}}$$

Score	General description	Description for this problem
M	**Mastery.** Your solution is complete and correct. Any error is trivial. You have provided sufficient justification and your arguments are easy to follow.	Your solutions to parts a and b are completely correct, with work that clearly shows the steps you used to obtain your final answers.
P	**Proficiency.** One of the following holds: (1) Your solution demonstrates understanding of the relevant topics, but there are some minor errors. (2) Your answer is correct, but you have not provided sufficient justification or your arguments are difficult to follow.	One of the following holds: (a) You have correct final answers for parts a and b, but your work does not clearly show the steps you used to obtain your final answers. (b) For part a, you set up the integrals correctly. In addition, your cdf has a value of 0 for $a < 2$ and 1 for $a \geq 4$. (You should have been able to determine these values without evaluating the integrals.) For part b, you set up the integral correctly.
B	**Basic ability.** Your solution demonstrates a partial understanding of the relevant topics, but has significant gaps. The gaps in your solution may be related to a significant error using a concept or skill, inadequately communicating your reasoning, or an inability to solve the problem completely.	Your solution does not meet the criteria for M, P, or N.
N	**Novice ability or no basis for assessment.** One or more of the following holds: (1) Your solution uses inappropriate skills or concepts. (2) Your solution contains too many errors to correct each one individually. (3) You provided an insubstantial response or no response.	You did not attempt the problem, or you exclusively used concepts unrelated to the problem.

2 Example: Standard D1, Quiz on 5 October

Problem. Erlang's Eatery serves passengers driving down Route 314 from 6 a.m. to 3 p.m. During this time period, cars pass Erlang's Eatery according to a Poisson process with an arrival rate of 10 per hour.

a. If 12 cars have passed the restaurant by 8 a.m., what is the probability that at most 36 cars arrive at the restaurant between 8 a.m. and 12 p.m.?

$$\begin{aligned}
\underbrace{\Pr\{Y_6 - Y_2 \le 36 \mid Y_2 = 12\}}_{\text{starting probability statement}} &= \Pr\{Y_6 - Y_2 \le 36\} && \text{(by independent increments)} \\
&= \underbrace{\Pr\{Y_4 \le 36\}}_{\text{simplified probability statement}} && \text{(by stationary increments)} \\
&= \sum_{j=0}^{36} \frac{e^{-10\cdot 4}(10\cdot 4)^j}{j!} \\
&\approx \underbrace{0.2963}_{\text{numerical answer}}
\end{aligned}$$

b. If 75 cars have passed the restaurant by 12 p.m., what is the probability that the 100th car passes the restaurant before it closes?

$$\begin{aligned}
\underbrace{\Pr\{T_{100} \le 9 \mid Y_6 = 75\}}_{\text{starting probability statement}} &= \underbrace{\Pr\{Y_9 \ge 100 \mid Y_6 = 75\}}_{\text{also a valid starting probability statement}} && \text{(because } \{T_n \le t\} \Leftrightarrow \{Y_t \ge n\}\text{)} \\
&= \Pr\{Y_9 - Y_6 \ge 25 | Y_6 = 75\} \\
&= \Pr\{Y_9 - Y_6 \ge 25\} && \text{(by independent increments)} \\
&= \underbrace{\Pr\{Y_3 \ge 25\}}_{\text{simplified probability statement}} && \text{(by stationary increments)} \\
&= 1 - \sum_{j=0}^{24} \frac{e^{-10\cdot 3}(10\cdot 3)^j}{j!} \\
&\approx \underbrace{0.8428}_{\text{numerical answer}}
\end{aligned}$$

Score	General description	Description for this problem
M	**Mastery.** Your solution is complete and correct. Any error is trivial. You have provided sufficient justification and your arguments are easy to follow.	Your solutions to parts a and b are completely correct, with work that clearly shows the steps you used to obtain your numerical answers. Rounding errors are OK.
P	**Proficiency.** One of the following holds: (1) Your solution demonstrates understanding of the relevant topics, but there are some minor errors. (2) Your answer is correct, but you have not provided sufficient justification or your arguments are difficult to follow.	One of the following holds: (a) You have correct numerical answers for parts a and b, but your work does not clearly show the steps you used to obtain your numerical answers. (b) You have correct numerical answers for parts a and b, but your notation is incorrect. (c) Your solution to one of the parts is completely correct, with work that clearly shows the steps you used to obtain your numerical answer. Your solution to the other part has the correct starting probability statement. (d) For both parts, you correctly apply independent and stationary increments to obtain the correct simplified probability statements. Your numerical answers are incorrect.
B	**Basic ability.** Your solution demonstrates a partial understanding of the relevant topics, but has significant gaps. The gaps in your solution may be related to a significant error using a concept or skill, inadequately communicating your reasoning, or an inability to solve the problem completely.	Your solution does not meet the criteria for M, P, or N.
N	**Novice ability or no basis for assessment.** One or more of the following holds: (1) Your solution uses inappropriate skills or concepts. (2) Your solution contains too many errors to correct each one individually. (3) You provided an insubstantial response or no response.	You did not attempt the problem, or you exclusively used concepts unrelated to the problem.