Name: Notes

SA402 · Dynamic and Stochastic Models — Fall 2019 · Uhan

Quiz 7 – 20 November 2019

Problem	Weight	Score
1	1	
2	1	
3	2	
4	1	
Total		/ 50

Instructions. You have 20 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

For the problems on this page, consider the following setting.

Four Guys Burgers and Fries has 3 cashiers at its Simplexville location. Customers wait in a single queue and are served by the first available cashier, first-come first-served. The cashier area is relatively small, and can only hold 10 customers (including the 3 receiving their orders). Any customers that arrive when the cashier area is full simply go elsewhere. The average service time is 2 minutes per customer, and customers arrive at a rate of 24 per hour. Assume the interarrival times and the service times are exponentially distributed.

Model this setting as a birth-death process by answering Problems 1 and 2.

Problem 1. Define the arrival rate in each state, in terms of the number of customers per hour.

- Note that this is an example of limited capacity. See Section 4 of Lesson 15.

Problem 2. Define the service rate in each state, in terms of the number of customers per hour.

- Recall that the average service time given (2 minutes per customer) is for each cashier.
- Many of you wrote
$$\mu_i = 30(3) = 90 \quad \text{if } i = 1, 2, \ldots$$
What if there is only 1 customer in the restaurant? 2 customers?
- Note that this is an example of multiple identical servers. See Section 5 of Lesson 15.

For the problems on this page, consider the following setting.

Customers call the reservation desk at Fluttering Duck Airlines at a rate of 4 customers per hour. There is 1 agent working at the reservation desk at any given time, and each phone call takes an average of 20 minutes. The phone system can only handle 3 customers at a time (1 with an agent, 2 waiting) – any phone calls arriving when there are 3 customers are simply lost.

This setting can be modeled as a birth-death process with the following arrival and service rates:

$$\lambda_i = \begin{cases} 4 & \text{if } i = 0, 1, 2 \\ 0 & \text{if } i = 3, 4, \ldots \end{cases} \qquad \mu_i = 3 \quad \text{for } i = 1, 2, \ldots$$

Problem 3. Over the long run, what is the probability that there are n customers in the system? ($n = 0, 1, 2, 3$)

- Most of you had the right idea here. Be careful when using the formulas for the steady-state probabilities.
- See Example 1 of Lesson 16 for a similar problem.

Problem 4. Over the long run, what fraction of time is the agent busy?

- Most of you had the right idea here as well.
- See Example 1 of Lesson 16 for a similar problem.

Steady-state probabilities of a birth-death process:

$$\pi_j = \frac{d_j}{D} \quad \text{for } j = 0, 1, 2, \ldots \qquad \text{where} \qquad d_0 = 1 \qquad d_j = \frac{\lambda_0 \lambda_1 \cdots \lambda_{j-1}}{\mu_1 \mu_2 \cdots \mu_j} \quad \text{for } j = 1, 2, \ldots \qquad D = \sum_{i=0}^{\infty} d_i$$