Solutions to Problem 1.

a.

$$\begin{aligned}\Pr\{3 \le Y \le 5\} &= \Pr\{Y = 3\} + \Pr\{Y = 4\} + \Pr\{Y = 5\} \quad \text{(because } Y \text{ only takes on values } 0, 1, 2, \dots) \\ &= \frac{e^{-3}3^3}{3!} + \frac{e^{-3}3^4}{4!} + \frac{e^{-3}3^5}{5!} \approx 0.4929\end{aligned}$$

b.

$$\begin{aligned}\Pr\{3 \le Y \le 5\} &= \Pr\{2 < Y \le 5\} \quad \text{(because } Y \text{ only takes on values } 0, 1, 2, \dots) \\ &= F_Y(5) - F_Y(2) \\ &= \sum_{k=0}^{5} \frac{e^{-3}3^k}{k!} - \sum_{k=0}^{2} \frac{e^{-3}3^k}{k!} \\ &= \sum_{k=3}^{5} \frac{e^{-3}3^k}{k!} \approx 0.4929\end{aligned}$$

c.

$$\begin{aligned}\Pr\{Y > 5\} &= 1 - \Pr\{Y \le 5\} \quad \text{(because } Y \text{ only takes on values } 0, 1, 2, \dots) \\ &= 1 - F_Y(5) \\ &= 1 - \sum_{k=0}^{5} \frac{e^{-3}3^k}{k!} \approx 0.0839\end{aligned}$$

d. $E[Y] = 3$

e. $\operatorname{Var}(Y) = 3$

Solutions to Problem 2.

a.

$$\begin{aligned}\Pr\{2 \le G \le 4\} &= \int_2^4 \frac{1}{3} e^{-a/3}\, da \\ &= \Big[-e^{-a/3}\Big]_{a=2}^{4} \\ &= e^{-2/3} - e^{-4/3} \approx 0.2498\end{aligned}$$

b.

$$\begin{aligned}\Pr\{2 \le G \le 4\} &= \Pr\{2 < G \le 4\} \quad \text{(because } G \text{ is continuous)} \\ &= F_G(4) - F_G(2) \\ &= (1 - e^{-4/3}) - (1 - e^{-2/3}) \\ &= e^{-2/3} - e^{-4/3} \approx 0.2498\end{aligned}$$

c.

$$\begin{aligned}\Pr\{G > 4\} &= 1 - \Pr\{G \le 4\} \quad \text{(because } G \text{ is continuous)}\\ &= 1 - F_G(4)\\ &= 1 - (1 - e^{-4/3})\\ &= e^{-4/3} \approx 0.2636\end{aligned}$$

d. $E[G] = \frac{1}{1/3} = 3$

e. $\mathrm{Var}(G) = \frac{1}{(1/3)^2} = 9$

Solutions to Problem 3.

a.

$$\begin{aligned}\Pr\{1 \le T \le 2\} &= \Pr\{1 < T \le 2\} \quad \text{(because } T \text{ is continuous)}\\ &= F_T(2) - F_T(1)\\ &= \left[1 - \sum_{k=0}^{3} \frac{e^{-(2/5)(2)}((2/5)(2))^k}{k!}\right] - \left[1 - \sum_{k=0}^{3} \frac{e^{-(2/5)(1)}((2/5)(1))^k}{k!}\right]\\ &= \sum_{k=0}^{3} \frac{e^{-2/5}(2/5)^k}{k!} - \sum_{k=0}^{3} \frac{e^{-4/5}(4/5)^k}{k!} \approx 0.0083\end{aligned}$$

b. $\Pr\{T = 2\} = 0$ because T is continuous

c.

$$\begin{aligned}\Pr\{T > 2\} &= 1 - \Pr\{T \le 2\} \quad \text{(because } T \text{ is continuous)}\\ &= 1 - F_T(2)\\ &= 1 - \left[1 - \sum_{k=0}^{3} \frac{e^{-(2/5)(2)}((2/5)(2))^k}{k!}\right]\\ &= \sum_{k=0}^{3} \frac{e^{-4/5}(4/5)^k}{k!} \approx 0.9909\end{aligned}$$

d. $E[T] = \frac{4}{2/5} = 10$

e. $\mathrm{Var}(T) = \frac{4}{(2/5)^2} = 25$