Lesson 11a. Markov Chains - Time-Dependent Performance Measures, cont.

Problem 1. An automated guided vehicle (AGV) transports parts between three locations: the release station, the machining station, and the output buffer. The movement of the AGV can be described as making trips from location to location based on requests to move parts. Consider a Markov chain in which the states 1, 2, 3 correspond to the release station, machining station, and output buffer, respectively, and each time step corresponds to one trip of the AGV. The one-step transition probability matrix for the AGV is:

$$\mathbf{P} = \begin{bmatrix} 1/5 & 3/5 & 1/5 \\ 1/4 & 1/2 & 1/4 \\ 1/5 & 2/5 & 2/5 \end{bmatrix}$$

a. Draw the transition-probability diagram for this Markov chain.
b. Suppose the AGV is at the release station. What is the probability that it will be back at the release station in 3 trips?
c. Suppose at the beginning of the day, the AGV is equally likely to be at any of the three locations. What is the probability that it will be at the output buffer in 3 trips?
d. Suppose the AGV is currently at the machining station. What is the probability that the AGV then travels between the release station and the machining station for 4 trips, and then finally visits the output buffer in the 5th trip?

Problem 2. The law firm of Primal and Dual employs three types of lawyers: junior lawyers, senior lawyers, and partners. Some of these lawyers eventually leave as non-partners, others leave as partners.

Consider a Markov chain that models the career path of a lawyer at Primal and Dual with five states: Junior (1), Senior (2), Partner (3), Leave as non-partner (4), and Leave as partner (5). Each time step represents one year. The one-step transition probability matrix and initial state probability vector is

$$\mathbf{P} = \begin{bmatrix} 0.80 & 0.15 & 0 & 0.05 & 0 \\ 0.05 & 0.65 & 0.20 & 0.10 & 0 \\ 0 & 0 & 0.95 & 0 & 0.05 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} 0.75 \\ 0.20 \\ 0.05 \\ 0 \\ 0 \end{bmatrix}$$

Note that a senior lawyer can be demoted to a junior lawyer.

For this problem, you'll need a way to quickly perform computations on matrices with 5 rows and 5 columns. Unfortunately, your TI-36X Pro calculators cannot do this. If you are comfortable with MATLAB or Python/NumPy, feel free to use them. If not, you can use an online calculator such as Symbolab (`https://www.symbolab.com`), which is free and pretty easy to use.

a. Draw the transition-probability diagram for this Markov chain.
b. Note that p_{44} and p_{55} are equal to 1. Why does this make sense in the context of the problem?
c. Suppose that you are a junior lawyer this year. What is the probability that you are a partner 5 years from now (and still at the firm)?
d. Randomly pick a lawyer at the firm today. (You would do this using the initial state probabilities given above.) What is the probability that this randomly chosen lawyer is a partner 5 years from now (and still at the firm)?
e. Again, suppose that you are a junior lawyer this year. What is the probability that you spend the next 5 years as either a junior lawyer or senior lawyer, and then leave as a non-partner in your 6th year?