Case 6 (Reneging). When customers call Fluttering Duck Airline's toll-free number to make reservations, they may be placed in a "hold" queue until an agent is available. Some customers will hang up if they are on hold too long. This phenomenon should be a part of Fluttering Duck Airline's capacity-planning models.

- **Reneging** occurs when customers in a queueing system choose to leave the system prior to receiving service
- Reneging ⇒ increases service rate of the system
- Suppose:
  - the time a customer is willing to spend waiting in the queue prior to starting service ~ Exponential(β)
  - the service time ~ Exponential(μ)
  - s identical servers
- $i > s$ customers in system

  ⇒ $\boxed{s}$ customers receiving service and $\boxed{i-s}$ who might renege

⇒ Arrival rate into the system in state i:

$$\mu_i = \begin{cases} i\mu & \text{if } i = 0, 1, \dots, s \\ \underbrace{s\mu}_{\text{first to be served}} + \underbrace{(i-s)\beta}_{\text{first to renege}} & \text{if } i = s+1, s+2, \dots \end{cases}$$

6 Next time...

- Computing steady-state probabilities and using them to compute different performance measures

Problem 1 (Nelson 8.4, modified). A small ice-cream shop competes with several other ice-cream shops in a busy mall. If there are too many customers already in line at the shop, then potential customers will go elsewhere. Potential customers arrive at a rate of 20 per hour. The probability that a customer will go elsewhere is $j/5$ when there are $j \le 5$ customers already in the system, and 1 when there are $j > 5$ customers already in the system. The server at the shop can serve customers at a rate of 10 per hour. Approximate the process of potential arrivals as Poisson, and the service times as exponentially distributed.

Model the process of customer arrivals and departures at this ice-cream shop as a birth-death process (i.e. what are λ_i and μ_i for $i = 0, 1, 2, \dots$?).

Arrival rates:
$$\lambda_i = \begin{cases} 20(1 - i/5) & \text{if } i = 0, 1, \dots, 5 \\ 0 & \text{if } i = 6, 7, \dots \end{cases}$$

Service rates:
$$\mu_i = \begin{cases} 10 & \text{if } i = 0, 1, \dots, 5 \\ 0 & \text{if } i = 6, 7, \dots \end{cases}$$

← we never reach these states in this birth-death process