

Solutions to Problem 1.

- a. • For $0 \leq t < 6$, $\Lambda(\tau) = \int_0^\tau 1 da = \tau$
- For $6 \leq t < 13$, $\Lambda(\tau) = \int_0^6 1 da + \int_6^\tau 2 da = 6 + (2\tau - 12) = 2\tau - 6$
- For $13 \leq t < 24$, $\Lambda(\tau) = \int_0^6 1 da + \int_6^{13} 2 da + \int_{13}^\tau \frac{1}{2} da = 6 + 14 + \left(\frac{1}{2}\tau - \frac{13}{2}\right) = \frac{1}{2}(\tau + 27)$

b. $Z_8 - Z_2 \sim \text{Poisson}(\Lambda(8) - \Lambda(2)) = \text{Poisson}(8)$

$$\begin{aligned} \Pr\{Z_8 - Z_2 > 12\} &= 1 - \Pr\{Z_8 - Z_2 \leq 12\} \\ &= 1 - \sum_{k=0}^{12} \frac{e^{-8}(8)^k}{k!} \approx 0.06 \\ E[Z_8 - Z_2] &= 8 \end{aligned}$$

c. $Z_4 - Z_2 \sim \text{Poisson}(\Lambda(4) - \Lambda(2)) = \text{Poisson}(2)$

$$\begin{aligned} \Pr\{Z_4 = 9 \mid Z_2 = 6\} &= \Pr\{Z_4 - Z_2 = 3 \mid Z_2 = 6\} \\ &= \Pr\{Z_4 - Z_2 = 3\} \\ &= \frac{e^{-2}(2)^3}{3!} \approx 0.18 \end{aligned}$$

d. $Z_{1/4} \sim \text{Poisson}(\Lambda(1/4)) = \text{Poisson}(1/4)$

$$\begin{aligned} \Pr\{Z_{1/4} > 0\} &= 1 - \Pr\{Z_{1/4} = 0\} \\ &= 1 - \frac{e^{-1/4}(1/4)^0}{0!} \approx 0.22 \end{aligned}$$

e. $Z_7 \sim \text{Poisson}(\Lambda(7)) = \text{Poisson}(8)$

$$\begin{aligned} \Pr\{Z_7 \geq 13\} &= 1 - \Pr\{Z_7 \leq 12\} \\ &= 1 - \sum_{k=0}^{12} \frac{e^{-8}(8)^k}{k!} \approx 0.06 \end{aligned}$$

Solutions to Problem 2.

- a. • For $0 \leq t < 1$, $\Lambda(\tau) = \int_0^\tau 144 da = 144\tau$
- For $1 \leq t < 2$, $\Lambda(\tau) = \int_0^1 144 da + \int_1^\tau 229 da = 144 + 229(\tau - 1) = 229\tau - 85$
- For $2 \leq t < 3$, $\Lambda(\tau) = \int_0^1 144 da + \int_1^2 229 da + \int_2^\tau 383 da = 373 + 383(\tau - 2) = 383\tau - 393$
- For $3 \leq t \leq 4$, $\Lambda(\tau) = \int_0^1 144 da + \int_1^2 229 da + \int_2^3 383 da + \int_3^\tau 96 da = 756 + 96(\tau - 3) = 96\tau + 468$

b. $Z_{3.4} - Z_{1.75} \sim \text{Poisson}(\Lambda(3.4) - \Lambda(1.75)) = \text{Poisson}(478.65)$

$$E[Z_{3.4} - Z_{1.75}] = \Lambda(3.4) - \Lambda(1.75) = 478.65$$

$$\begin{aligned}
\text{c. } \Pr\{Z_{3.4} - Z_{1.75} > 700\} &= 1 - \Pr\{Z_{3.4} - Z_{1.75} \leq 700\} \\
&= 1 - \sum_{k=0}^{700} \frac{e^{-478.65} (478.65)^k}{k!} \approx 0
\end{aligned}$$

Solutions to Problem 3.

a. Note that $\Lambda(22) - \Lambda(18) = 87 - 74 = 13$, so $Z_{22} - Z_{18} \sim \text{Poisson}(13)$. Therefore,

$$\Pr\{Z_{22} - Z_{18} \leq 12\} = \sum_{j=0}^{12} \frac{e^{-13} (13)^j}{j!} \approx 0.4631$$

b. Note that $\Lambda(24) - \Lambda(12) = 90 - 44 = 46$, so $Z_{24} - Z_{12} \sim \text{Poisson}(46)$. Therefore,

$$\begin{aligned}
\Pr\{Z_{24} \geq 80 \mid Z_{12} = 40\} &= \Pr\{Z_{24} - Z_{12} \geq 40 \mid Z_{12} = 40\} \\
&= \Pr\{Z_{24} - Z_{12} \geq 40\} \\
&= 1 - \Pr\{Z_{24} - Z_{12} \leq 39\} \\
&= 1 - \sum_{j=0}^{39} \frac{e^{-46} (46)^j}{j!} \approx 0.8307
\end{aligned}$$

c. $\Lambda(24) = 90$ is the expected number of phone calls to the dispatch over the course of the entire day.