

## Quiz 6 – 21 October 2019

**Instructions.** You have 20 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
Total		/ 40

For this quiz, consider the following setting.

You are a consultant for a political pollster in Simplexville. Each year, the citizens of Simplexville vote for one of three parties: (1) the Primal Party, (2) the Dual Party, or (3) the Infeasible Party.

Based on historical data, you have determined that voting behavior in Simplexville can be modeled as a Markov chain with states  $\mathcal{M} = \{1, 2, 3\}$  (1 = Primal, 2 = Dual, 3 = Infeasible), and with each time step corresponding to one year. The one-step transition matrix is

$$\mathbf{P} = \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.10 & 0.80 & 0.10 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

For example, of those that voted for the Dual Party in this year's election, 10% will vote Primal next year, 80% will vote Dual, and 10% will vote Infeasible.

Suppose in this year's election, 45% voted Primal, 50% voted Dual, and 5% voted Infeasible.

**Problem 1.** Note that the diagonal entries of  $\mathbf{P}$  are larger than the off-diagonal entries. What does that mean in this setting?

**Problem 2.** What is the probability that a citizen votes for the Primal Party 4 years from now, given that the citizen voted for the Primal Party this year?

Here is the one-step transition matrix from the previous page, for your convenience:

$$\mathbf{P} = \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.10 & 0.80 & 0.10 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

**Problem 3.** What is the probability that a randomly selected citizen votes for the Dual Party 4 years from now?

**Problem 4.** What is the probability that a citizen votes for the Primal Party this year, votes either Primal or Dual for the next 3 years, and then votes for the Infeasible Party 4 years from now?