

Lesson 6. A General Stochastic Process Model

1 A general stochastic process model

- Let's generalize the Bit Bucket example from last time
- Notation: $\mathbf{S}_n = \begin{pmatrix} S_{1,n} \\ \vdots \\ S_{m,n} \end{pmatrix}$ is a vector of m random variables
- $\{\mathbf{S}_n; n = 0, 1, 2, \dots\}$ is the **state-change process**
 - Represents all relevant information about system status
- $\{T_n; n = 0, 1, 2, \dots\}$ is the **event-epoch process**
 - T_n is the time of the n th system event
- $\{\mathbf{Y}_t; t \geq 0\}$ is the **output process**, defined by $\mathbf{Y}_t \leftarrow \mathbf{S}_n$ for $t \in [T_n, T_{n+1})$
 - Connects state changes with times that they occur
- **System events** e_1, e_2, \dots, e_k
 - Update the new system state \mathbf{S}_{n+1} from previous system state \mathbf{S}_n
 - Reset **clocks** $\mathbf{C} = (C_1, C_2, \dots, C_k)$ if necessary
- Initial system event e_0
- **Simulation algorithm**

algorithm Simulation:

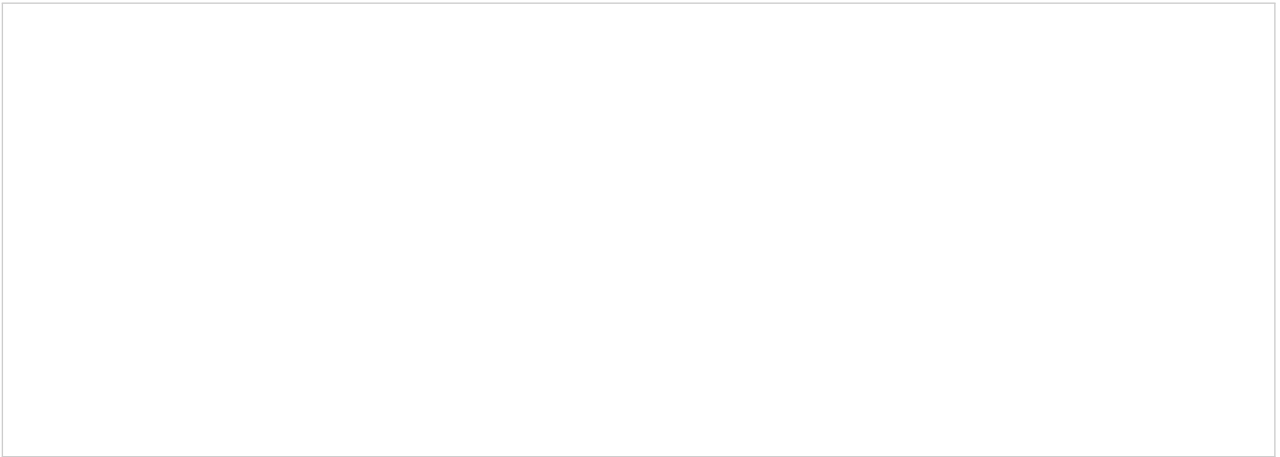
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|---|---|
| 1: $n \leftarrow 0$ | (initialize system event counter) |
| $T_0 \leftarrow 0$ | (initialize event epoch) |
| $e_0()$ | (execute initial system event) |
| 2: $T_{n+1} \leftarrow \min\{C_1, \dots, C_k\}$ | (advance time to next pending system event) |
| $I \leftarrow \arg \min\{C_1, \dots, C_k\}$ | (find index of next system event) |
| 3: $\mathbf{S}_{n+1} \leftarrow \mathbf{S}_n$ | (temporarily maintain previous state) |
| $C_I \leftarrow \infty$ | (event I no longer pending) |
| 4: $e_I()$ | (execute system event I) |
| $n \leftarrow n + 1$ | (update event counter) |
| 5: go to line 2 | |

- $\mathbf{S}_{n+1} \leftarrow \mathbf{S}_n$ in Step 3 is for convenience
 - ◊ With this, system event functions only need to specify changes in system state
- A **stochastic process** is a model describing a collection of time-ordered random variables that represent possible sample paths
- A **sample path** is a collection of time-ordered data describing how a stochastic process actually behaved in one instance

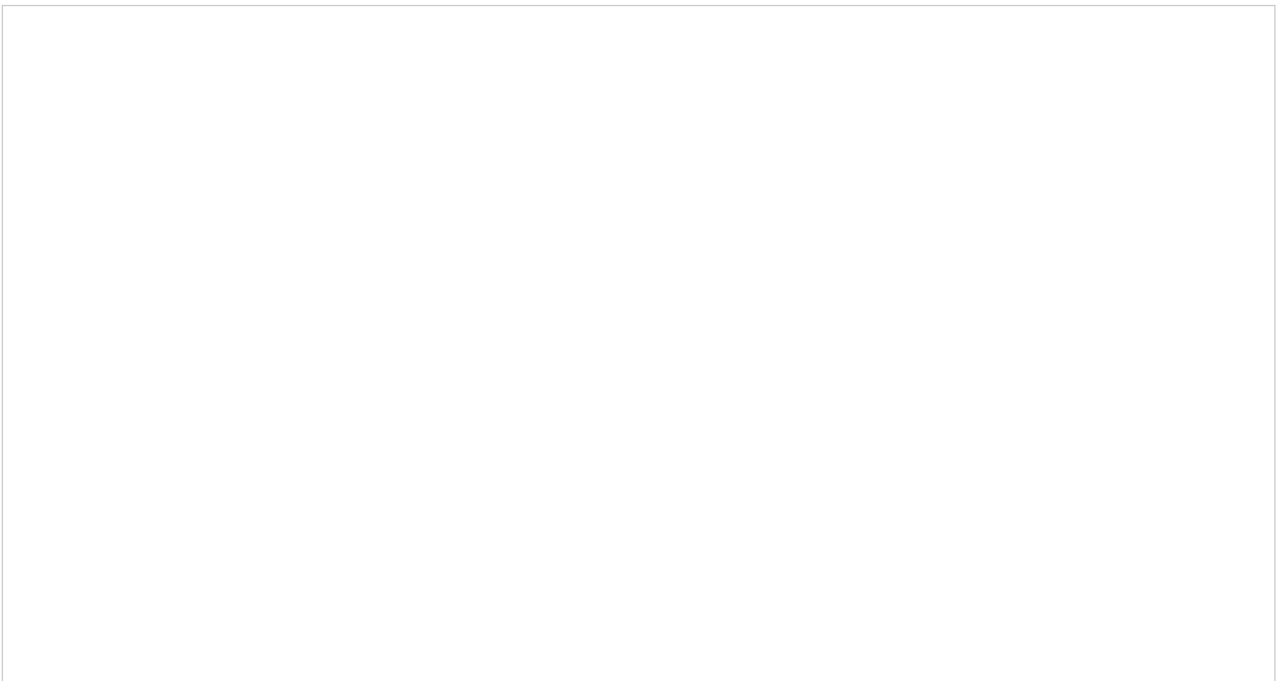
2 The Case of Copy Enlargement, revisited

The Darker Image, a national chain of small photocopying shops, currently configures each store with one photocopying machine and one clerk. Arriving customers stand in a single line to wait for the clerk. The clerk completes the customers' photocopying jobs one at a time, first-come-first-served, including collecting payment for the job.

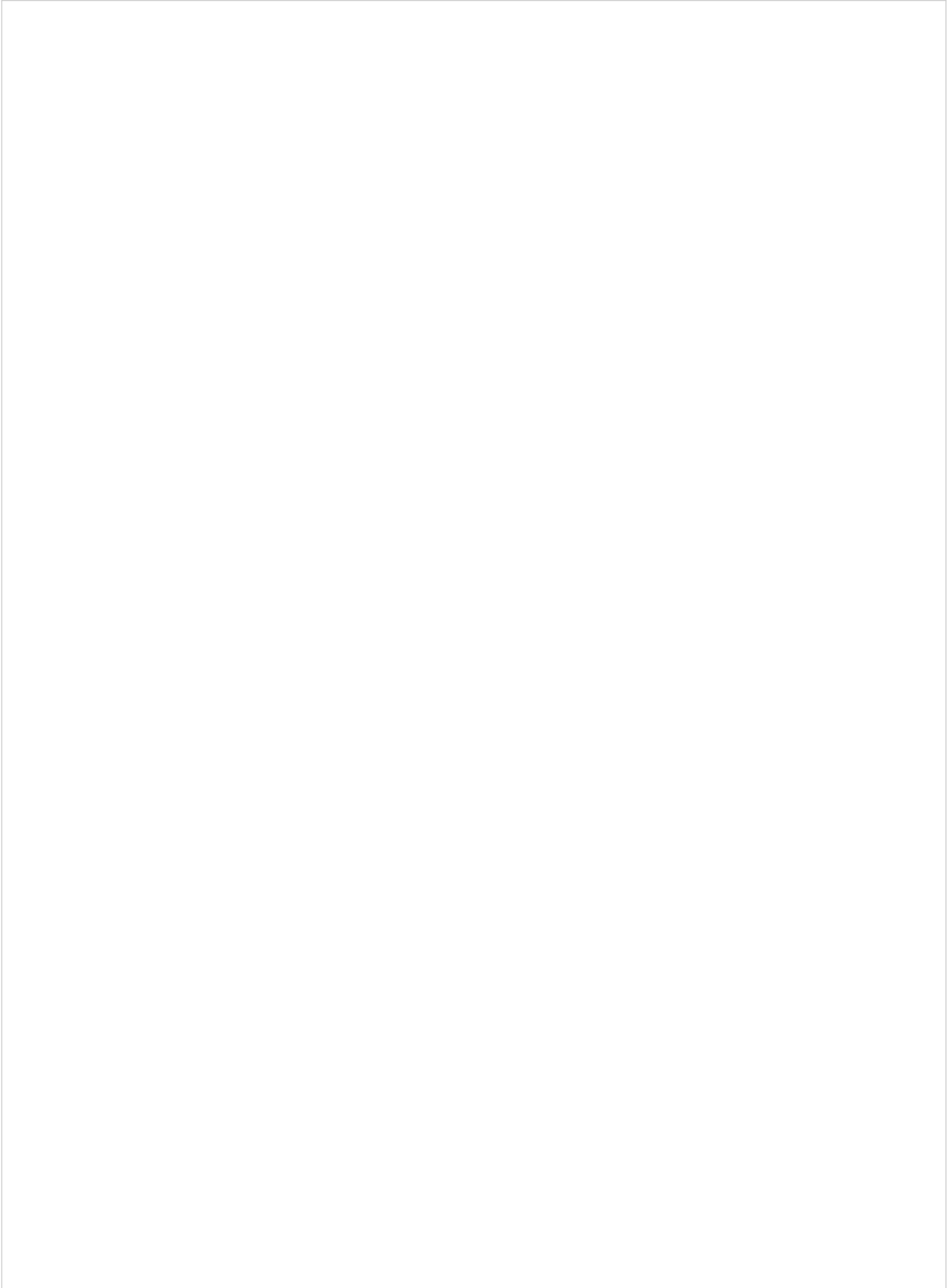
- Let's formulate a stochastic process model for the copy shop as it currently operates
- Assumptions:
 - Interarrival times are independent with common cdf F_G
 - Service times are independent with common cdf F_X
 - Interarrival times and service times are independent
- System events:



- System state:



- System event algorithms:



- Output process:

- Time-average number of customers waiting for service over the first 6 hours:

- Time-average number of copiers in use – the **utilization** of the copier – over the first 6 hours:

- In words, what is $\int_0^6 Y_{1,t} dt$?