Name:

SA402 – Dynamic and Stochastic Models
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Fall 2016

Quiz – 14 October 2016

Instructions. You have 15 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Standard	Problems	Score
D4	1a	
D5	1b	
D6	2	

Problem 1. Customers arrive at Erlang's Eatery between 7 a.m. and 3 p.m. according to a nonstationary Poisson arrival process with integrated rate function

$$\Lambda(\tau) = \begin{cases} 4\tau & \text{if } 0 \leq t < 2 \\ \tau + 6 & \text{if } 2 \leq t < 5 \\ 5\tau - 14 & \text{if } 5 \leq t \leq 8 \end{cases}$$

where τ is in hours, $\tau = 0$ corresponds to 7 a.m., and $\tau = 8$ corresponds to 3 p.m.

a. In words, briefly describe the meaning of $\Lambda(4) = 10$ in the context of this problem.

b. If exactly 20 customers have arrived by 10 a.m., what is the probability that at least 30 customers will arrive by 1 p.m.?

Problem 2. The Simplexville Electric Company is conducting a study of its power line along the busiest part of Main Street. Looking at its historical data, the company has observed that power surges occur at a rate of 12 per hour. It also has noticed that the power surges come in "waves": a large power surge is always followed by a smaller power surge exactly 1 minute later. The company is modeling the occurrence of power surges (both large and small) as a stationary Poisson process with an arrival rate of 12. Is this a good idea? Why or why not?

You may find the following information useful:

Exponential random variable with parameter λ: $\quad \text{cdf } F(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases} \quad$ expected value $= 1/\lambda$

Erlang random variable with parameter λ and n phases: $\quad \text{cdf } F(a) = \begin{cases} 1 - \displaystyle\sum_{j=0}^{n-1} \frac{e^{-\lambda a}(\lambda a)^j}{j!} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases} \quad$ expected value $= n/\lambda$

Poisson random variable with parameter λt: $\quad \text{pmf } p(n) = \dfrac{e^{-\lambda t}(\lambda t)^n}{n!}$ for $n = 0, 1, 2, \ldots \quad$ expected value $= \lambda t$