Name:

SA402 – Dynamic and Stochastic Models
Assoc. Prof. Nelson Uhan

Fall 2016

Quiz – 26 October 2016

Instructions. You have 20 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Standard	Problems	Score
E2	1a, 1b	
E3	2a	
E4	2b, 2c	

Problem 1. The law firm of Primal and Dual employs three types of lawyers: junior lawyers, senior lawyers, and partners. Some of these lawyers eventually leave as non-partners, others leave as partners.

Consider a Markov chain that models the career path of a lawyer at Primal and Dual with five states: Junior (1), Senior (2), Partner (3), Leave as non-partner (4), and Leave as partner (5). Each time step represents one year. The one-step transition probability matrix and initial state probability vector is

$$\mathbf{P} = \begin{bmatrix} 0.80 & 0.15 & 0 & 0.05 & 0 \\ 0.05 & 0.65 & 0.20 & 0.10 & 0 \\ 0 & 0 & 0.95 & 0 & 0.05 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} 0.75 \\ 0.20 \\ 0.05 \\ 0 \\ 0 \end{bmatrix}$$

Note that a senior lawyer can be demoted to a junior lawyer.

a. Suppose that you are a junior lawyer this year. What is the probability that you are a partner 5 years from now (and still at the firm)?

b. Again, suppose that you are a junior lawyer this year. What is the probability that you spend the next 5 years as either a junior lawyer or senior lawyer, and then leave as a non-partner in your 6th year?

Problem 2. Consider the Markov chain defined by the following one-step transition matrix:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

There are two irreducible sets of states: $\{2, 3\}$ and $\{5\}$.

a. Is state 3 transient or recurrent? Why?

b. Suppose the Markov chain has reached state 3. What is the steady-state probability of being in state 2?

c. Suppose the Markov chain starts in state 4. What is the probability that the Markov chain is absorbed into state 5?