

Name:

SA402 – Dynamic and Stochastic Models
Assoc. Prof. Nelson Uhan

Fall 2016

Quiz – 5 October 2016

Instructions. You have 20 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Standard	Problems	Score
D1	1a, 1b	
D2	1c, 1d	
D3	1e, 1f	

Problem 1. Erlang's Eatery serves passengers driving down Route 314 from 6 a.m. to 3 p.m. During this time period, cars pass Erlang's Eatery according to a Poisson process with an arrival rate of 10 per hour.

- If 12 cars have passed the restaurant by 8 a.m., what is the probability that at most 36 cars arrive at the restaurant between 8 a.m. and 12 p.m.?
- If 75 cars have passed the restaurant by 12 p.m., what is the probability that the 100th car passes the restaurant before it closes?
- What is the expected time of the 30th car passing the restaurant?

- d. If 25 cars have passed the restaurant by 9 a.m., what is the expected number of cars that pass the restaurant before it closes?
- e. Suppose 25% of the cars passing Erlang's Eatery stop at the restaurant. What is the expected number of cars that stop at the restaurant between 11 a.m. and 1 p.m.?
- f. Suppose trucks pass Erlang's Eatery according to a Poisson process with an arrival rate of 5 per hour. What is the probability that at most 20 vehicles (cars and trucks) pass the restaurant between 11 a.m. and 1 p.m.?

You may find the following information useful:

Exponential random variable with parameter λ :	$\text{cdf } F(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value = $1/\lambda$
Erlang random variable with parameter λ and n phases:	$\text{cdf } F(a) = \begin{cases} 1 - \sum_{j=0}^{n-1} \frac{e^{-\lambda a} (\lambda a)^j}{j!} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value = n/λ
Poisson random variable with parameter λt :	$\text{pmf } p(n) = \frac{e^{-\lambda t} (\lambda t)^n}{n!} \text{ for } n = 0, 1, 2, \dots$	expected value = λt