Name:

SA402 – Dynamic and Stochastic Models
Assoc. Prof. Nelson Uhan

Fall 2016

Quiz – 16 September 2016

Instructions. You have 25 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Standard	Problems	Score
B1	3ab	
B2	3cd	
C1	1ab	
C2	2ab	

Problem 1. Suppose X is a random variable with cdf

$$F_X(a) = \begin{cases} 0 & \text{if } a < 3, \\ \frac{1}{2}(a-3) & \text{if } 3 \leq a < 5, \\ 1 & \text{if } a \geq 5. \end{cases}$$

a. Construct a random variate generator for X. Your solution should be in the form: "$X = \cdots$ where $U \sim \text{Uniform}[0,1]$."

b. Suppose you have access to a function `random()` that generates random variates of Uniform$[0,1]$. Briefly describe how you would use your random variate generator in part a to generate random variates of X.

Problem 2. The Markov Butcher Shop sells two types of meat: beef and pork. Customers arrive at the butcher shop and form a single queue. There is one butcher who serves customers from the queue on a first-come-first-served basis.

Based on historical data, 60% of the customers want beef, and 40% of the customers want pork. The interarrival time between customers is modeled by a random variable G. The service times for customers who want beef or pork are modeled by random variables B and P, respectively. The interarrival times and service times are assumed to be independent.

Professor I. M. Wright is consulting for the Markov Butcher Shop, and has started to model the shop as a stochastic process using the algorithmic approach we discussed in class, as follows:

- System events:

$$\begin{aligned} e_0 &= \text{shop opens} \\ e_1 &= \text{customer arrives at shop} \\ e_2 &= \text{customer finishes being served and departs shop} \end{aligned}$$

- State variables:

$$\begin{aligned} Q_n &= \text{number of customers in the queue after the } n\text{th system event} \\ A_n &= \begin{cases} 0 & \text{if butcher is available} \\ 1 & \text{if butcher is busy} \end{cases} \quad \text{after the } n\text{th system event} \\ \mathbf{S}_n &= (Q_n, A_n) \end{aligned}$$

- Subroutine for e_0:

$e_0()$:
1: $Q_0 \leftarrow 0$
2: $A_0 \leftarrow 0$
3: $C_1 \leftarrow F_G^{-1}(\texttt{random()})$
4: $C_2 \leftarrow \infty$

The algorithm Simulation is provided below for your reference:

algorithm Simulation:

1:	$n \leftarrow 0$	(initialize system event counter)
	$T_0 \leftarrow 0$	(initialize event epoch)
	$e_0()$	(execute initial system event)
2:	$T_{n+1} \leftarrow \min\{C_1, \dots, C_k\}$	(advance time to next pending system event)
	$I \leftarrow \arg\min\{C_1, \dots, C_k\}$	(find index of next system event)
3:	$\mathbf{S}_{n+1} \leftarrow \mathbf{S}_n$	(temporarily maintain previous state)
	$C_I \leftarrow \infty$	(event I no longer pending)
4:	$e_I()$	(execute system event I)
	$n \leftarrow n + 1$	(update event counter)
5:	go to line 2	

a. Professor Wright has written a subroutine for e_2 below. Annotate Professor Wright's subroutine, explaining what each line does.

$e_2()$:

if $\{Q_n = 0\}$ **then**
    $A_{n+1} \leftarrow 0$
else
    $Q_{n+1} \leftarrow Q_n - 1$
    if $\{\texttt{random()} \leq 0.6\}$ **then**
        $C_2 \leftarrow T_{n+1} + F_B^{-1}(\texttt{random()})$
    else
        $C_2 \leftarrow T_{n+1} + F_P^{-1}(\texttt{random()})$
    end if
end if

b. Write a subroutine for e_1. Annotate your code line-by-line.

Problem 3. Consider a random variable X with pdf

$$f_X(a) = \begin{cases} 0 & \text{if } a < 0, \\ \dfrac{1}{4\sqrt{a}} & \text{if } 0 \leq a \leq 4, \\ 0 & \text{if } a > 4. \end{cases}$$

a. What is the cdf F_X of X? Make sure to define $F_X(a)$ for <u>all</u> $a \in (-\infty, \infty)$.

b. The expected value of X is $E[X] = \frac{4}{3}$. What is the variance of X?

c. What is the maximum value that X can take? Why?

d. For this random variable, which is more likely: a value near 1 or a value near 3? Why?