Review – 16 September 2016

Problem 1. Suppose X is a **Poisson random variable** with parameter λ. It has the following pmf:

$$p_X(a) = \frac{e^{-\lambda}\lambda^a}{a!} \quad \text{for } a = 0, 1, 2, \ldots$$

a. Explain why X is a discrete random variable.
b. Let $\lambda = 3$. What is the probability that X is less than or equal to 2?
c. Let $\lambda = 3$ again. Give an algorithm that outputs random variates of X. As usual, you have access to `random()`, a function that can output random variates of Uniform$[0,1]$.

Problem 2. Patients arrive at the Simplexville Hospital Emergency Room in one of three ways. Last year, 43% arrived as walk-ins, 53% arrived by ambulance (either air or ground), and 4% arrived by a public service vehicle (e.g. police car, social service vehicle). 73% of the patients who arrived by ambulance were given an MRI, compared with 63% of walk-ins and 59% of those who arrived by a public service vehicle. 11% of the patients who arrived by ambulance were admitted to the intensive care unit (ICU), compared with 0.2% of walk-ins and 6% of those who arrived by a public service vehicle. Select one of last year's patients at random.

a. What is the probability that this patient arrived as a walk-in and was given an MRI?
b. What is the probability that this patient was admitted to the ICU?

Problem 3. (Based on Nelson 2.9, 4.5.) The Orange Company is considering the following design for an automated manufacturing cell to produce its very popular mobile phones. A new phone will arrive at the cell at precisely 30 minute intervals, and phones will be processed one at a time, first come first served. There are three types of phones: let T be a random variable that represents the type of the arriving phone (i.e., $T \in \{1, 2, 3\}$). In addition, each phone type requires a different amount of (random) processing time: let P_i be a random variable that represents the processing time for a type i phone. Not all phones can be processed in 30 minutes, so there may be a queue of waiting phones.

Formulate a stochastic process model for this system by specifying

- the system events,
- the system state variables,
- a subroutine for each system event.