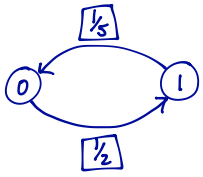
Problem 1. Each customer service representative at Jungle.com spends his or her time answering e-mails and taking phone calls. Phone calls receive first priority, so a representative must interrupt tending to his or her e-mail whenever the phone rings. The time between phone calls is exponentially distributed with a mean of 5 minutes. The length of each phone call is exponentially distributed with a mean of 2 minutes.

a. Model how a representative switches between his or her two tasks as a Markov process by (i) specifying the state space and defining what the states mean, and (ii) specifying the transition rates, either by drawing the transition rate diagram or defining the generator matrix.

b. What is the long-run fraction of time each customer service representative spends answering e-mail? Taking phone calls?

a. $\mathcal{M} = \{0, 1\}$ (0: on the phone; 1: answering e-mail)



$$\Rightarrow G = \begin{bmatrix} -1/2 & 1/2 \\ 1/5 & -1/5 \end{bmatrix}$$

b.
$$\begin{aligned} -\tfrac{1}{2}\pi_0 + \tfrac{1}{5}\pi_1 &= 0 \\ \tfrac{1}{2}\pi_0 - \tfrac{1}{5}\pi_1 &= 0 \\ \pi_0 + \pi_1 &= 1 \end{aligned} \Rightarrow \pi_0 = \frac{2}{7}, \quad \pi_1 = \frac{5}{7}$$

π_0: long-run fraction of time on the phone; π_1: long-run fraction of time answering e-mail