Name:

SA402 – Dynamic and Stochastic Models — Fall 2016
Assoc. Prof. Nelson Uhan

Quiz – 28 September 2016

Instructions. You have 15 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Standard	Problems	Score
C2	1a, 1b	
D1	2a, 2b	
D2	2c, 2d	

Problem 1. You have been recently hired by the data analytics group at Jungle.com, an online retailer. They need a model that describes customer behavior on their website so that they can evaluate changes to the company's network architecture. They have identified four key types of customer actions: (1) searching for a product, (2) reading the main page of a product, (3) reading the reviews for a product, and (4) checking out.

Based on historical data, the group believes that

- the next customer action can be modeled as a random variable N with cdf F_N (note that $N \in \{1, 2, 3, 4\}$), and
- the time a customer spends in action type $i \in \{1, 2, 3, 4\}$ can be modeled as a random variable P_i with cdf F_{P_i}.

You have started modeling customer behavior as a stochastic process:

- System events:

$$e_0 = \text{the customer starts shopping on Jungle.com}$$
$$e_1 = \text{the customer starts his or her next action}$$

- State variable:

$$S_n = \text{the action that the customer is doing after the } n\text{th system event}$$

You assume that the customer starts by searching for a product (action type 1).

The algorithm Simulation is provided on the reverse side for your reference. You may abuse notation and use $F_X^{-1}(\texttt{random()})$ to represent generating a random variate of a random variable X, even if X is discrete.

a. Write a subroutine for e_0. Annotate your code line-by-line.

b. Write a subroutine for e_1. Annotate your code line-by-line.

algorithm Simulation:

1:	$n \leftarrow 0$	(initialize system event counter)
	$T_0 \leftarrow 0$	(initialize event epoch)
	$e_0()$	(execute initial system event)
2:	$T_{n+1} \leftarrow \min\{C_1, \ldots, C_k\}$	(advance time to next pending system event)
	$I \leftarrow \arg\min\{C_1, \ldots, C_k\}$	(find index of next system event)
3:	$\mathbf{S}_{n+1} \leftarrow \mathbf{S}_n$	(temporarily maintain previous state)
	$C_I \leftarrow \infty$	(event I no longer pending)
4:	$e_I()$	(execute system event I)
	$n \leftarrow n + 1$	(update event counter)
5:	go to line 2	

Problem 2. Patients arrive at a hospital emergency room at a rate of 4 per hour. A doctor works a 12-hour shift from 6 a.m. until 6 p.m. Suppose the arrivals follow a Poisson process.

a. If the doctor has seen exactly 5 patients by 9 a.m., what is the probability that the doctor will see a total of at most 15 patients by 12 p.m.?

b. What is the probability that the doctor will see her 6th patient before 8 a.m.?

c. What is the expected time after coming on duty until the doctor sees her second patient?

d. What is the expected number of patients the doctor will see during her shift, if she sees 1 patient by 8 a.m.?

You may find the following information useful:

Exponential random variable with parameter λ: cdf $F(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$ expected value $= 1/\lambda$

Erlang random variable with parameter λ and n phases: cdf $F(a) = \begin{cases} 1 - \displaystyle\sum_{j=0}^{n-1} \frac{e^{-\lambda a}(\lambda a)^j}{j!} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$ expected value $= n/\lambda$

Poisson random variable with parameter λt: pmf $p(n) = \dfrac{e^{-\lambda t}(\lambda t)^n}{n!}$ for $n = 0, 1, 2, \ldots$ expected value $= \lambda t$