$\boxed{1}$ a. X is discrete because it takes a countably infinite number of values.
Alternately, X is discrete because it does not take a continuum of values.

b. $$\Pr\{X=2\} = \Pr\{X=0\} + \Pr\{X=1\} + \Pr\{X=2\}$$
$$= p_X(0) + p_X(1) + p_X(2) = \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{2}2^2}{2!} \approx 0.4232$$

c. Following the inverse transform method for discrete random variables, in this case, here is a random variate generator for X:

$$X = a \quad \text{if} \quad F_X(a-1) < U \le F_X(a) \qquad \text{for } a = 0, 1, 2, \ldots \qquad (*)$$
where $U \sim \text{Uniform}[0,1]$.

To make this more concrete, we can find the cdf of X as:

$$F_X(0) = p_X(0) \approx 0.0498$$
$$F_X(1) = p_X(0) + p_X(1) \approx 0.1991$$
$$F_X(2) = p_X(0) + p_X(1) + p_X(2) \approx 0.4232$$
$$F_X(3) = p_X(0) + p_X(1) + p_X(2) + p_X(3) \approx 0.6472$$
$$F_X(4) = p_X(0) + p_X(1) + p_X(2) + p_X(3) + p_X(4) \approx 0.8153$$
... etc.

So $(*)$ becomes:

$$X = \begin{cases} 0 & \text{if} \quad 0 < U \le 0.0498 \\ 1 & \text{if} \quad 0.0498 < U \le 0.1991 \\ 2 & \text{if} \quad 0.1991 < U \le 0.4232 \\ 3 & \text{if} \quad 0.4232 < U \le 0.6472 \\ 4 & \text{if} \quad 0.6472 < U \le 0.8153 \\ \ldots & \text{etc.} \end{cases} \qquad (**)$$

We can then use random() to obtain a random variate u of $U \sim \text{Uniform}[0,1]$, and plug u into $(*)$ or $(**)$ to obtain a random variate of X.

2 Let $A = \begin{cases} 0 & \text{if walk-in} \\ 1 & \text{if ambulance} \\ 2 & \text{if public service vehicle} \end{cases}$ $\quad M = \begin{cases} 1 & \text{if MRI given} \\ 0 & \text{otherwise} \end{cases}$

$I = \begin{cases} 1 & \text{if admitted to ICU} \\ 0 & \text{otherwise} \end{cases}$

We are given: $\Pr\{A=0\} = 0.43 \quad \Pr\{A=1\} = 0.53 \quad \Pr\{A=2\} = 0.04$

$\Pr\{M=1 \mid A=0\} = 0.63 \quad \Pr\{M=1 \mid A=1\} = 0.73 \quad \Pr\{M=1 \mid A=2\} = 0.59$

$\Pr\{I=1 \mid A=0\} = 0.002 \quad \Pr\{I=1 \mid A=1\} = 0.11 \quad \Pr\{I=1 \mid A=2\} = 0.06$

a. $\Pr\{A=0 \text{ and } M=1\} = \Pr\{M=1 \mid A=0\} \Pr\{A=0\} = 0.2709$

b. $\Pr\{I=1\} = \sum_{a=0}^{2} \Pr\{I=1 \mid A=a\} \Pr\{A=a\}$

$= (0.002)(0.43) + (0.11)(0.53) + (0.06)(0.04) \approx 0.0616$

3 system events:

e_1 = phone arrival

e_2 = phone departure

e_0 = initialization.

state variables:

Q_n = # phones in queue after n^{th} system event

$A_n = \begin{cases} 0 & \text{if cell is available} \\ 1 & \text{otherwise} \end{cases}$ after n^{th} system event

$e_0()$:

$Q_n \leftarrow 0$ (start w/empty queue)

$A_n \leftarrow 0$ (cell available at start)

$C_1 \leftarrow 30$ (first arrival)

$C_2 \leftarrow +\infty$ (no pending departure)

$e_1()$:

if $\{A_n = 0\}$ (if cell available)

$i \leftarrow F_T^{-1}(\text{random}())$ (get phone type)

$C_2 \leftarrow T_{n+1} + F_{P_i}^{-1}(\text{random}())$ (set clock for next dep.)

$A_{n+1} \leftarrow 1$ (cell not available)

else

$Q_{n+1} \leftarrow Q_n + 1$ (add job to queue)

$C_1 \leftarrow T_{n+1} + 30$ (set clock for next arr.)

$e_2()$:

if $\{Q_n = 0\}$ (if queue empty)

$A_{n+1} \leftarrow 0$ (cell available)

else

$Q_{n+1} \leftarrow Q_n - 1$ (remove job from queue)

$i \leftarrow F_T^{-1}(\text{random}())$ (get next phone type)

$C_2 \leftarrow T_{n+1} + F_{P_i}^{-1}(\text{random}())$ (set clock for next dep.)