Lesson 8a. Poisson Arrival Processes, cont.

Example 1. The Markov Company has a manufacturing cell that processes jobs during a 12-hour shift starting at 6 a.m. and ending at 6 p.m. Jobs leave the cell according to a Poisson process with rate $\lambda = 8$ per hour.

a. If the cell has processed exactly 10 jobs by 8 a.m., what is the probability that the cell will have processed more than 30 jobs by 10 a.m.?
b. What is the probability that the cell will have processed its 50th job before 12 p.m.?
c. If the cell has processed exactly 40 jobs by 12 p.m., what is the probability that the cell will have processed its 100th job by the end of the shift?
d. What is the total expected waiting time of the first 4 jobs to be processed? (Assume all jobs are available starting at 6 a.m.)

Example 2. (Nelson 5.15) This problem concerns capacity planning for a manufacturing company. The company has two salespersons, John and Louise, who each cover one half of the United States. At the end of each week, the salespersons report their sales to the company, which then manufactures the products that have been ordered.

The company has three products, which it calls A, B and C for simplicity. Each salesperson obtains 10 orders per week, on average, of which approximately 20% are for A, 70% are for B, and 10% are for C. In terms of capacity, it takes 25 person-hours to produce one A, 15 person-hours to produce one B, and 40 person-hours to produce one C.

Help the company do its capacity planning by answering the following questions. You may assume that the arrival of orders to each salesperson can be well approximated as a Poisson process.

a. A Poisson process is time stationary. What is the physical interpretation of "time stationary" in this situation?
b. What is the probability that the total sales for 1 week will be more than 30 products?
c. Capacity can only be changed on a monthly basis. What is the expected number of person-hours the company will need over a 1-month period?
d. What is the probability that Louise will sell more than 5 product Bs on each of 2 consecutive weeks?