Name: Feedback

SA402 · Dynamic and Stochastic Models Fall 2023 · Uhan

Quiz 3 – 9/13/2023

Instructions. You have 25 minutes to complete this quiz. You may use your plebe-issue TI-36X Pro calculator. You may not use any other materials.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
1d	1	
2a	1	
2b	1	
2c	1	
Total		/ 70

Problem 1. You are a consultant for a political pollster in Simplexville. Each year, the citizens of Simplexville vote for one of three parties: (1) the Optimal Party, (2) the Unbounded Party, or (3) the Infeasible Party.

Based on historical data, you have determined that voting behavior in Simplexville can be modeled as a Markov chain with states $\mathcal{M} = \{1, 2, 3\}$ (1 = Optimal, 2 = Unbounded, 3 = Infeasible), and with each time step corresponding to one year. The one-step transition matrix is

$$\mathbf{P} = \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.10 & 0.80 & 0.10 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

For example, of those that voted for the Unbounded Party in this year's election, 10% will vote Optimal next year, 80% will vote Unbounded, and 10% will vote Infeasible.

Suppose in this year's election, 45% voted Optimal, 50% voted Unbounded, and 5% voted Infeasible.

a. Note that the diagonal entries of $\mathbf{P}$ are larger than the off-diagonal entries. What does that mean in this setting?

Take a look at Problem 1a from the Lesson 5 Exercises for a similar example.

Here is the one-step transition matrix from the previous page, for your convenience:

$$\mathbf{P} = \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.10 & 0.80 & 0.10 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

b. Suppose this year corresponds to time step $n = 0$. What is the probability that a citizen votes for the Unbounded Party 4 years from now ($n = 4$), given that the citizen voted for the Unbounded Party this year?

For similar examples, take a look at Example 2 from Lesson 5, as well as Problem 2b or Problem 3c from the Lesson 5 Exercises.

c. Again, suppose this year corresponds to time step $n = 0$. What is the probability that a randomly selected citizen votes for the Optimal Party 4 years from now ($n = 4$)?

For similar examples, take a look at Example 3 from Lesson 5, as well as Problem 2c or Problem 3d from the Lesson 5 Exercises.

Note that the problem gives you initial state probabilities: 45% voted Primal, 50% voted Dual, and 5% voted Infeasible in this year's election.

d. What is the probability that a citizen votes for the Unbounded Party this year, votes either Unbounded or Infeasible for the next 3 years, and then votes for the Optimal Party 4 years from now?

For similar examples, take a look at Example 4 from Lesson 5, or Problem 2d or 3e from the Lesson 5 Exercises.

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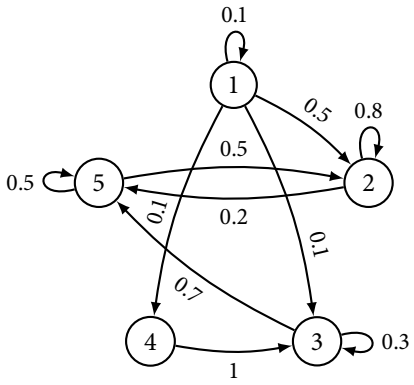
Problem 2. Consider a Markov chain with state space $\mathcal{M} = \{1, 2, 3, 4, 5\}$ and transition probabilities defined by the matrix and diagram below:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.5 & 0.1 & 0.1 & 0.2 \\ 0 & 0.8 & 0 & 0 & 0.2 \\ 0 & 0 & 0.3 & 0 & 0.7 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0.5 \end{bmatrix}$$



a. Is the set $\mathcal{R} = \{2, 5\}$ a recurrent class? Briefly explain.

For a similar example, take a look at Example 2 from Lesson 6. In addition, take a look at Problem 2b from the Lesson 6 Exercises, which is about the same 5-state Markov chain.

Also, recall the definition of a recurrent class from Lesson 6.

- From the top of page 3: a subset of states $\mathcal{R}$ is a **recurrent class** if (i) $\mathcal{R}$ forms a self-contained Markov chain and (ii) no proper subset of $\mathcal{R}$ also forms a Markov chain.
- Equivalently, from page 4, right before Example 3: $\mathcal{R}$ is a **recurrent class** if (i) $\mathcal{R}$ forms a self-contained Markov chain and (ii) all states in $\mathcal{R}$ communicate with each other.

Note that with either characterization of a recurrent class, you need to check for two conditions!

b. Is state 5 transient or recurrent? Briefly explain.

See Problem 2b from the Lesson 6 Exercises.

c. Is state 1 transient or recurrent? Briefly explain.

See Problem 2b from the Lesson 6 Exercises.