Syllabus

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Course description. This course provides an introduction to modeling and analyzing systems that evolve dynamically over time and whose behavior is stochastic, or uncertain. This course focuses on models that are amenable to mathematical analysis, while using basic notions from simulation to develop intuition.

Course objectives. By the end of this course, students will be able to (1) formulate algorithmic models of real-world systems as general stochastic processes, (2) identify when a Poisson process, Markov chain, or birth-death queueing process is an appropriate model for a real-world system and construct such a model, and (3) analyze these models by computing and interpreting state probabilities and performance measures.

Textbook. B. Nelson. *Stochastic Modeling: Analysis and Simulation*. Dover, 2010.

Schedule. Here is a tentative schedule.

Unit	Day	Date	Topic	Readings	Homework
Introduction	M	8/22	Introduction	1	
			Sample paths	2.1-2.3	2.1
	W	8/24	Sample paths, cont.		
	F	8/26	Basic probability review	3.1.1-3.1.3	3.1, 3.2, 3.3, 3.5
	M	8/29	Basic probability review, cont.		
	W	8/31	Conditional probability review	3.1.4-3.1.5	3.6, 3.8, 3.9, 3.31
	F	9/2	Conditional probability review, cont.		
	T	9/6	Review		
	W	9/7	Random variate generation	3.3	3.17abc, 3.18abc, 3.19abc, 3.20abc
	F	9/9	Random variate generation, cont.		
			Introduction to stochastic processes	4.1-4.4	
	M	9/12	Introduction to stochastic processes, cont.		
	W	9/14	A general stochastic process model	4.5-4.6	4.6[1], 4.8, 4.4[2]
	F	9/16	Review		
	M	9/19	**Exam 1**		

[1] You'll need to keep track of the number of jobs waiting to be processed by CPU A and CPU B. Let B be a random variable that takes value 0 with probability 1/2, and 1 with probability 1/2. Map $B = 0$ to CPU A, and $B = 1$ to CPU B.

[2] Use Exercise 2.7 as a warm up if you're having trouble understanding how the system works; start by computing the demand for hamburgers each day using the given table and system logic. In your algorithm, let D be a random variable representing the number of hamburgers demanded on each day. You may assume that demand between days is identical and independent.

Unit	Day	Date	Topic	Readings	Homework
Poisson arrival processes	W	9/21	Introduction to arrival counting processes	5.1-5.4	
	F	9/23	Introduction to arrival counting processes, cont.		
			The Poisson arrival process	5.5, 5.8	5.1, 5.3abcd, 5.5, 5.6, 5.8, 5.14
	M	9/26	The Poisson arrival process, cont.		
	W	9/28	Decomposition and superposition of Poisson processes	5.6.1-5.6.2, 5.8	5.3ef, 5.10, 5.12, 5.13, 5.15, 5.17
	F	9/30	Decomposition and superposition of Poisson processes, cont.		
			Nonstationary Poisson processes	5.6.3	5.20
	M	10/3	Nonstationary Poisson processes, cont.		
	W	10/5	Lab: Spatial Poisson processes		
	F	10/7	Review		
	M	10/10	Holiday: Columbus Day		
Markov chains	W	10/12	Introduction to Markov chains	6.1-6.3, 6.4.1-6.4.3	6.2, 6.4ac[3], 6.5[4]
	F	10/14	Introduction to Markov chains, cont.		
			Time-dependent performance measures for Markov chains	6.5-6.6	6.4b, 6.17ab, 6.18[5]
	M	10/17	Time-dependent performance measures for Markov chains, cont.		
	W	10/19	Long-run performance measures for Markov chains	6.7	6.5[6], 6.6, 6.11, 6.17c, 6.8
	F	10/21	Modeling and assumptions with Markov chains	6.8	6.20, 6.21
	M	10/24	Lab: How to win at Monopoly		
	W	10/26	Review		
	F	10/28	**Exam 2**		
Queueing processes	M	10/31	A quick start guide to the Markov process	7.5, 7.6.4-7.6.5, 8.2.2	7.2, 7.10
	W	11/2	A quick start guide to the Markov process, cont.		
	F	11/4	Review		
	M	11/7	Introduction to queueing processes	8.3	
	W	11/9	The birth-death process	8.4.1, 8.5	8.4[7], 8.6a
	F	11/11	Holiday: Veteran's Day		
	M	11/14	The birth-death process, cont.		

[3] Assume $\Pr\{S_0 = 1\} = 1$.
[4] Draw the transition diagram only.
[5] Start by finding the probabilities of preferred beer brands in 1979 and 2016.
[6] Classify each state as transient or recurrent.
[7] Just model the system as a birth-death process.

Unit	Day	Date	Topic	Readings	Homework
	W	11/16	Performance measures for queueing processes[8]	8.4.2	8.4abc, 8.6bcd[9], 8.11
	F	11/18	Performance measures for queueing processes, cont.		
	M	11/21	Standard queueing models[10]	8.6, 8.7	8.5, 8.8, 8.10
	W	11/23	Standard queueing models, cont.		
	F	11/25	Holiday: Thanksgiving Break		
	M	11/28	Lab: Helicopter maintenance		
	W	11/30	Review		
	F	12/2	Review		
	M	12/5	Review		
	W	12/7	Review		

[8] For these homework problems, you may find it useful to write some code or setup a spreadsheet that can compute the performance measures for a birth-death process with a finite capacity.

[9] These quantities can be found analytically, by finding closed-form formulas for the involved infinite sums. Alternately, you can approximate these quantities by assuming that the system has some large capacity, say 50.

[10] For these homework problems, you may find it useful to write some code or setup a spreadsheet that can compute the performance measures for an $M/M/s$ queueing system.