Problem 1. Consider a model of a consumer's preferences for toothpaste brands that defines the state of the system to be the brand that the consumer currently uses, and the time index to be the number of tubes of toothpaste purchased. Discuss what assumptions need to be made in order for the Markov property and time stationarity property hold, and whether these assumptions are plausible.

Markov

Next toothpaste brand purchased only depends on current toothpaste brand

Not reasonable – brand loyalty usually goes beyond 1 tube

Time-stationarity

The probabilities of moving between brands is constant over time

Not reasonable – tastes tend to change over time

Problem 2. Bit Bucket Computers specializes in installing and maintaining computer systems. Unfortunately, the reliability of their computer systems is somewhat questionable. One of its standard configurations is to install two computers. When both are working at the start of business, there is a 30% chance that one will fail by the close of business, and a 10% chance that both will fail. If only one computer is working at the start of business, then there is a 20% chance it will fail by the close of business. Computers that fail during the day are picked up the following morning, repaired, and then returned the next day after that, before the start of business. Suppose we start observing an office with this configuration on a day with two working computers at the start of business.

Model this setting as a Markov chain by defining:

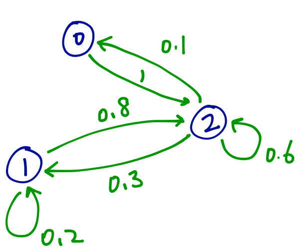
- the state space and the meaning of each state in the setting's context,
- the meaning of one time step in the setting's context,
- the meaning of the state visited in the nth time step in the setting's context, and
- the one-step transition probabilities and initial state probabilities.

State space: $\mathcal{M} = \{0, 1, 2\} \leftarrow$ # working computers at the beginning of the day

Time step: one day

State in n^{th} time step: S_n = # working computers at the beginning of the n^{th} day

One-step transition probabilities:



Initial state probabilities:

$p_2 = 1 \qquad p_0 = p_1 = 0$