Solutions to Example 1.

a. $$\begin{aligned}\Pr\{Y_4 > 30 \mid Y_2 = 10\} &= \Pr\{Y_4 - Y_2 > 20 \mid Y_2 = 10\}\\ &= \Pr\{Y_4 - Y_2 > 20\}\\ &= \Pr\{Y_2 > 20\}\\ &= 1 - \Pr\{Y_2 \le 20\}\\ &= 1 - \sum_{j=0}^{20} \frac{e^{-8(2)}(8(2))^j}{j!} \approx 0.1318\end{aligned}$$

b. $$\begin{aligned}\Pr\{T_{50} \le 6\} &= F_{T_{50}}(6) \quad (T_{50} \text{ is Erlang distributed with } n = 50 \text{ phases and parameter } \lambda = 8)\\ &= 1 - \sum_{j=0}^{49} \frac{e^{-8(6)}(8(6))^j}{j!} \approx 0.405\end{aligned}$$

Note. You should get the same answer if you computed $\Pr\{Y_6 \ge 50\}$ instead.

c. $$\begin{aligned}\Pr\{T_{100} \le 12 \mid Y_6 = 40\} &= \Pr\{Y_{12} \ge 100 \mid Y_6 = 40\}\\ &= \Pr\{Y_{12} - Y_6 \ge 60 \mid Y_6 = 40\}\\ &= \Pr\{Y_{12} - Y_6 \ge 60\}\\ &= \Pr\{Y_6 \ge 60\}\\ &= 1 - \sum_{j=0}^{59} \frac{e^{-8(6)}(8(6))^j}{j!} \approx 0.0523\end{aligned}$$

d. In scheduling theory, "total expected waiting time of the first 4 jobs to be processed" means to look at each of the first 4 jobs, measure (in expectation) how long you wait from time 0 (6 a.m.) until that job is completed, and then add up those times. For this problem, this translates to:

$$\begin{aligned}E\left[\sum_{n=1}^{4} T_n\right] &= \sum_{n=1}^{4} E[T_n] \quad (T_n \text{ is Erlang distributed with } n \text{ phases and parameter } \lambda = 8)\\ &= \sum_{n=1}^{4} \frac{n}{8}\\ &= \frac{5}{4}\end{aligned}$$

Note. If you had not seen the term "total expected waiting time" before, you might interpret it as the expected time until the 4th job is processed. In this case, you would get

$$E[T_4] = \frac{4}{8} = \frac{1}{2}$$

Solutions to Example 2.

a. In this situation, time-stationary means that demand is not time-dependent (i.e., no seasonal demand), and the market for A, B, C is not time-dependent (i.e., not increasing or decreasing).

b. Let Y_t = total sales up to week t. Y_t follows a Poisson process with arrival rate 10 + 10 = 20.

$$\begin{aligned}\Pr\{Y_1 > 30\} &= 1 - \Pr\{Y_1 \leq 30\} \\ &= 1 - \sum_{j=0}^{30} \frac{e^{-20(1)}(20(1))^j}{j!} \approx 0.013\end{aligned}$$

c. Let $Y_{A,t}$ = total sales of A up to week t, $Y_{B,t}$ = total sales of B up to week t, and $Y_{C,t}$ = total sales of C up to week t. $Y_{A,t}$ follows a Poisson process with arrival rate $20(0.2) = 4$, $Y_{B,t}$ follows a Poisson process with arrival rate $20(0.7) = 14$, and $Y_{C,t}$ follows a Poisson process with arrival rate $20(0.1) = 2$.

Expected sales in 1 month:

$$E[Y_{A,4}] = 4(4) = 16 \qquad E[Y_{B,4}] = 14(4) = 56 \qquad E[Y_{C,4}] = 2(4) = 8$$

Expected person-hours for 1 month:

$$25E[Y_{A,4}] + 15E[Y_{B,4}] + 40E[Y_{C,4}] = 25(16) + 15(56) + 40(8) = 1560$$

d. Let $Y_{B,t}^L$ = Louise's total sales of B up to week t. $Y_{B,t}^L$ follows a Poisson process with rate $10(0.7) = 7$.

$$\begin{aligned}\Pr\{Y_{B,2}^L - Y_{B,1}^L > 5 \text{ and } Y_{B,1}^L > 5\} &= \Pr\{Y_{B,2}^L - Y_{B,1}^L > 5\}\Pr\{Y_{B,1}^L > 5\} \quad \text{(independent increments)} \\ &= \Pr\{Y_{B,1}^L > 5\}\Pr\{Y_{B,1}^L > 5\} \quad \text{(stationary increments)} \\ &= \left(1 - \sum_{j=0}^{5} \frac{e^{-7(1)}(7(1))^j}{j!}\right)^2 \approx 0.4890\end{aligned}$$