Review Problems for Exam 1

Problem 1. Suppose Y is a random variable with cdf

$$F_Y(a) = \begin{cases} 0 & \text{if } a < 1, \\ 0.2 & \text{if } 1 \leq a < 3, \\ 0.5 & \text{if } 3 \leq a < 5, \\ 0.6 & \text{if } 5 \leq a < 7, \\ 0.9 & \text{if } 7 \leq a < 9, \\ 1 & \text{if } a \geq 9 \end{cases}$$

a. Is Y discrete or continuous? Briefly explain why.
b. What is the pmf/pdf of Y?
c. What is the expected value of Y?
d. What is the variance of Y?
e. What is the maximum possible value of Y? Briefly explain why.

Problem 2. Suppose X is a continuous random variable with pdf

$$f_X(a) = \begin{cases} 0 & \text{if } a < 0, \\ a & \text{if } 0 \leq a \leq 1, \\ 2 - a & \text{if } 1 < a \leq 2, \\ 0 & \text{if } a > 2 \end{cases}$$

a. What is the cdf of X?
b. What is the expected value of X?
c. What is the variance of X?
d. What is the probability that X is in the interval $[1/2, 3/4]$?
e. What is the maximum possible value of X? Briefly explain why.

Problem 3. Suppose `random()` is a function that can output random variates of a uniformly distributed random variable on $[0, 1]$. Give an algorithm that outputs random variates for the random variable Y described in Problem 1.

Problem 4. Patients arrive at the Simplexville Hospital Emergency Room in one of three ways. Last year, 43% arrived as walk-ins, 53% arrived by ambulance (either air or ground), and 4% arrived by a public service vehicle (e.g. police car, social service vehicle). 73% of the patients who arrived by ambulance were given an MRI, compared with 63% of walk-ins and 59% of those who arrived by a public service vehicle. 11% of the patients who arrived by ambulance were admitted to the intensive care unit (ICU), compared with 0.2% of walk-ins and 6% of those who arrived by a public service vehicle. Select one of last year's patients at random.

a. What is the probability that this patient arrived as a walk-in and was given an MRI?
b. What is the probability that this patient was admitted to the ICU?

Problem 5. (Based on Nelson 2.9, 4.5.) The Orange Company is considering the following design for an automated manufacturing cell to produce its very popular mobile phones. A new phone will arrive at the cell at precisely 30 minute intervals, and phones will be processed one at a time, first come first served. There are three types of phones: let T be a random variable that represents the type of the arriving phone (i.e., $T \in \{1, 2, 3\}$). In addition, each phone type requires a different amount of (random) processing time: let P_i be a random variable that represents the processing time for a type i phone. Not all phones can be processed in 30 minutes, so there may be a queue of waiting phones.

Formulate a stochastic process model for this system by specifying

- the system events,
- the system state variables,
- a subroutine for each system event.