1. a. $\Pr\{Y=0\} = p_{XY}(1,0) + p_{XY}(2,0) + p_{XY}(3,0)$

$$= \frac{1}{3} + \frac{1}{4} + \frac{3}{16} = \frac{37}{48} \approx 0.7708$$

b. $$\Pr\{Y=1 \mid X=2\} = \frac{\Pr\{Y=1 \text{ and } X=2\}}{\Pr\{X=2\}} = \frac{p_{XY}(2,1)}{p_{XY}(2,1) + p_{XY}(2,2) + p_{XY}(2,3)}$$

$$= \frac{1/12}{1/4 + 1/12 + 0} = \frac{1}{4}$$

c. This probability is 0 because Professor Right can't answer more questions than he is asked.

2. a. $p_M(1) = \Pr\{M=1\} = 0.2, \quad p_M(2) = 0.3, \quad p_M(3) = 0.5$

b. $\Pr\{D=1 \mid M=1\} = 0.01 \qquad \Pr\{D=1 \mid M=2\} = 0.02 \qquad \Pr\{D=1 \mid M=3\} = 0.03$

c. $$\Pr\{D=1\} = \sum_{m=1}^{3} \Pr\{D=1 \mid M=m\} \underbrace{\Pr\{M=m\}}_{p_M(m)}$$

Law of total probability

$$= 0.01(0.2) + 0.02(0.3) + 0.03(0.5) = 0.023$$

3. a. $\Pr\{Z=2\} = p_{ZM}(2,0) + p_{ZM}(2,1) + p_{ZM}(2,2) = 0.25$

$$p_{M|Z=2}(0) = \frac{\Pr\{M=0, Z=2\}}{\Pr\{Z=2\}} = 0.4 \qquad p_{M|Z=2}(1) = 0.32 \qquad p_{M|Z=2}(2) = 0.28$$

b. $$E[M \mid Z=2] = \sum_{m=0}^{2} m \cdot p_{M|Z=2}(m) = 0(0.4) + 1(0.32) + 2(0.28) = 0.88$$

c. M and Z are not independent because the probability of $M=1$ changes when we know that $Z=3$.

One can also check the definition of independence with M and Z. It turns out that $\Pr\{M=1\} = 0.35$, $\Pr\{Z=3\} = 0.19$ and

$$\Pr\{M=1 \text{ and } Z=3\} = 0.07 \neq (0.35)(0.19) = \Pr\{M=1\}\Pr\{Z=3\}$$

Therefore, M and Z are not independent.