5 Law of total probability

- We can write a joint probability as the product of a conditional probability and a marginal probability:

$$\Pr\{Y \in B \text{ and } X \in A\} = \Pr\{Y \in B \mid X \in A\} \Pr\{X \in A\}$$

- Using this, we can also decompose a marginal probability into the products of conditional and marginal probabilities
- **The law of total probability.** Suppose X is a discrete random variable taking values $a_1, a_2, \dots$. Then:

$$\Pr\{Y \in B\} = \sum_{\text{all } i} \Pr\{Y \in B \text{ and } X = a_i\} = \sum_{\text{all } i} \Pr\{Y \in B \mid X = a_i\} \Pr\{X = a_i\}$$

- We have a similar law when X is a continuous random variable

Example 6. In Example 1, the conditional pmf of W given that $V = 2$ is:

b	1	2	3
$p_{W\mid V=2}(b)$	1/12	8/12	3/12

Use this with your answer to Example 4 to find $\Pr\{W = 2\}$.

$$\Pr\{W=2\} = \Pr\{W=2 \mid V=1\}\Pr\{V=1\} + \Pr\{W=2 \mid V=2\}\Pr\{V=2\}$$
$$= \left(\frac{1}{4}\right)\left(\frac{2}{5}\right) + \left(\frac{8}{12}\right)\left(1 - \frac{2}{5}\right)$$
$$= \frac{1}{2}$$