Lesson 8. Cramer's Rule, Applications to Economic Models

0 Warm up

Example 1. Find the following determinants:

a.
$$\begin{vmatrix} 2 & 3 & 0 \\ 0 & 4 & 5 \\ 6 & 0 & 7 \end{vmatrix}$$

a.
$$\begin{vmatrix} 2 & 3 & 0 \\ 0 & 4 & 5 \\ 6 & 0 & 7 \end{vmatrix}$$
 b. $\begin{vmatrix} 8 & 3 & 0 \\ 3 & 4 & 5 \\ -1 & 0 & 7 \end{vmatrix}$ c. $\begin{vmatrix} 2 & 8 & 0 \\ 0 & 3 & 5 \\ 6 & -1 & 7 \end{vmatrix}$ d. $\begin{vmatrix} 2 & 3 & 8 \\ 0 & 4 & 3 \\ 6 & 0 & -1 \end{vmatrix}$

c.
$$\begin{vmatrix} 2 & 8 & 0 \\ 0 & 3 & 5 \\ 6 & -1 & 7 \end{vmatrix}$$

d.
$$\begin{vmatrix} 2 & 3 & 8 \\ 0 & 4 & 3 \\ 6 & 0 & -1 \end{vmatrix}$$

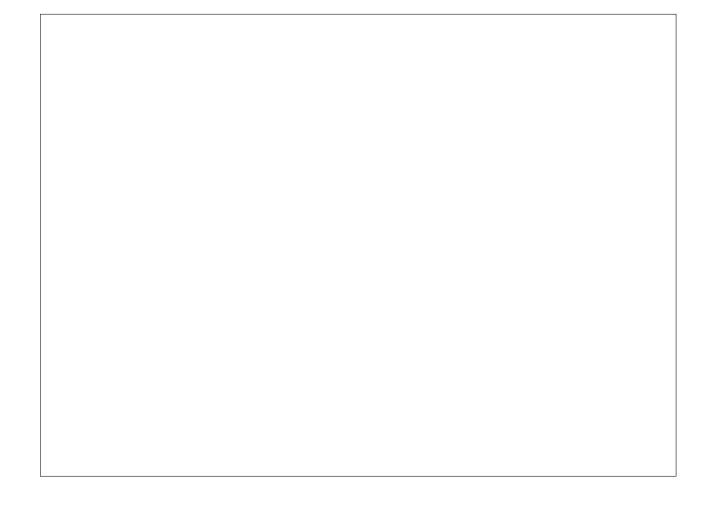
1 Cramer's rule

Suppose we want to solve a system of equations Ax = d for x, where A is n × n and d is n × 1
Quick check: x has dimension
Let A_j be the matrix A, but with the jth column replaced by d
Cramer's rule:

Example 2. Solve the following system of equations using Cramer's rule:

$$2x_1 + 3x_2 = 8$$

 $4x_2 + 5x_3 = 3$
 $6x_1 + 7x_3 = -1$



2 Two commodity partial market equilibrium

- Market with two products that are related to each other
- Variables:

 Q_{d1} = quantity demanded for product 1

 Q_{s1} = quantity supplied for product 1

 P_1 = price of product 1

 Q_{d2} = quantity demanded for product 2

 Q_{s2} = quantity supplied for product 2

 P_2 = price of product 2

• A general model with 6 variables and 6 equations:

$$Q_{d1} = Q_{s1}$$

$$Q_{d1} = a_0 + a_1 P_1 + a_2 P_2$$

$$Q_{s1} = b_0 + b_1 P_1 + b_2 P_2$$

$$Q_{d2} = Q_{s2}$$

$$Q_{d2} = \alpha_0 + \alpha_1 P_1 + \alpha_2 P_2$$

$$Q_{s2} = \beta_0 + \beta_1 P_1 + \beta_2 P_2$$

- Depending on the economic context, the parameters a_0 , a_1 , a_2 , b_0 , b_1 , b_2 , α_0 , α_1 , α_2 , β_0 , β_1 , β_2 will have particular signs, magnitudes or relationships between each other
 - Product 1 and product 2 are **substitutes** if:

• Product 1 and product 2 are **complements** if:

• Using the equilibrium conditions, we can simplify the above model into 2 variables and 2 equations:

 $(a_1 - b_1)P_1 + (a_2 - b_2)P_2 = -(a_0 - b_0)$ $(\alpha_1 - \beta_1)P_1 + (\alpha_2 - \beta_2)P_2 = -(\alpha_0 - \beta_0)$ \Leftrightarrow

• Using Cramer's rule, we can find the equilibrium prices:

• Using this closed form solution, we can <u>analytically</u> determine the effects of the parameters on the equilibrium prices

• Variables:

$$Y$$
 = national income

$$C = (planned)$$
 consumption expenditure

• Parameters:

$$I_0$$
 = investment expenditures

$$G_0$$
 = government expenditures

$$a =$$
 autonomous consumption expenditure

$$b =$$
marginal propensity to consume

• Model:

$$Y = C + I_0 + G_0$$

 $C = a + bY$ $(a > 0, 0 < b < 1)$

• How is consumption related to national income in this model?

Example 3.

- a. Rewrite the national income model above in matrix form, listing the variables in the order *Y*, *C*.
- b. Solve for variables *Y* and *C* using Cramer's rule.