# Lesson 6. Reduced Row Echelon Form, cont.

### 0 Warm up

**Example 1.** Consider the following system of equations:

$$x + y - 2z = 1$$
$$-x + 10z = -1$$
$$2x + 3y + 4z = 2$$

- a. Form the augmented matrix for this system.
- b. Solve this system by putting its augmented matrix into RREF.

#### 1 Last time

• Elementary row operations and RREF

• To solve a system of linear equations, we

1. Form its augmented matrix

2. Find the RREF of the augmented matrix

3. Solve for the leading variables in terms of the free variables

• Three possible outcomes:

1. The system has a unique solution

2. The system has infinitely many solutions (if there is at least one free variable)

3. The system is inconsistent (if the RREF implies 0 = 1)

• What else can RREF tell us?

### 2 The rank of a matrix

• Let rref(*A*) denote the reduced row echelon form of matrix *A* 

• The rank of matrix A – denoted by r(A) – is the number of leading 1s in rref(A)

**Example 2.** Let 
$$A = \begin{bmatrix} 0 & -11 & -4 \\ 2 & 6 & 2 \\ 4 & 1 & 0 \end{bmatrix}$$
. We have that  $rref(A) = \begin{bmatrix} 1 & 0 & -1/11 \\ 0 & 1 & 4/11 \\ 0 & 0 & 0 \end{bmatrix}$ . What is  $r(A)$ ?

**Example 3.** What is the maximum possible rank of an  $m \times n$  matrix?

• r(A) is the maximum number of linearly independent rows that can be found in A

• Similarly, r(A) is the maximum number of linearly independent columns that can be found in A

• Recall that a square matrix *A* is nonsingular if and only if its rows/columns are linearly independent

2

<b>Example 4.</b> Is <i>A</i> in Example 2 nonsingular? Why?	
$\Rightarrow$ An $n \times n$ matrix is nonsingular if and only if its rank is	
3 Finding the inverse of a matrix	
• To find the inverse of an $n \times n$ matrix $A$ , compute rref( $\begin{bmatrix} A & I_n \end{bmatrix}$ )	
• If $\operatorname{rref}(\begin{bmatrix} A & I_n \end{bmatrix})$ has the form $\begin{bmatrix} I_n & B \end{bmatrix}$ , then A is invertible an	$\operatorname{ind} A^{-1} = B$
• If $\operatorname{rref}(\begin{bmatrix} A & I_n \end{bmatrix})$ has another form (i.e., the left side fails to be	
Example 5. Find the inverse of $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}$ .	

## 4 A two-commodity market equilibrium model

- Suppose we have a market with two commodities
- The unit prices of the two commodities are  $P_1$  and  $P_2$
- The quantities demanded  $Q_{d1}$  and  $Q_{d2}$  and the quantities supplied  $Q_{s1}$  and  $Q_{s2}$  of the two commodities are given by

$$Q_{d1} = 70 - 2P_1 + P_2$$

$$Q_{s1} = -14 + 3P_1$$

$$Q_{d2} = 105 + P_1 - P_2$$

$$Q_{s2} = -7 + 2P_2$$

• What is the relationship between the two commodities?

1		
1		

• Equilibrium condition – demand equals supply for each commodity:

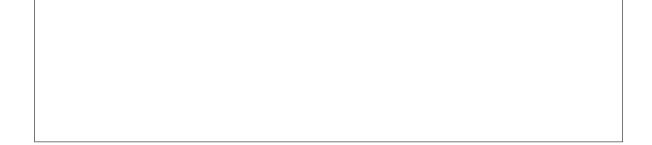
$$Q_{d1} = Q_{s1}$$

$$Q_{d2} = Q_{s2}$$

 $\bullet\,$  Using the equilibrium condition, we can simplify the two-commodity market equilibrium model:

- 1	
- 1	
- 1	
- 1	
- 1	
- 1	
- 1	
- 1	
- 1	
- 1	

• Equivalently, in matrix form:



<b>Example 6.</b> Solve for the equilibrium prices of this two-common RREF of the augmented matrix of the above system.	odity market equilibrium model by finding the
<b>Example 7.</b> Solve for the equilibrium prices of this two-commonwerse of the coefficient matrix of the above system.	odity market equilibrium model by finding the
inverse of the coefficient matrix of the above system.	
5 If we have time	
<b>Example 8</b> (Also a homework problem). What is the rank of	7 6 3 3 0 1 2 1 ? Is this matrix nonsingular?