Name: Notes

SA402 · Dynamic and Stochastic Models Fall 2019 · Uhan

Quiz 5 – 9 October 2019

Instructions. You have 15 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	**Weight**	**Score**
1	1	
2	1	
3	1	
4	1	
Total		/ 40

For Problems 1-3, consider the following setting.

The Simplexville Emergency Dispatch receives phone calls according to a nonstationary Poisson arrival process with integrated rate function

$$\Lambda(\tau) = \begin{cases} 3\tau & \text{if } 0 \le t < 8 \\ 5\tau - 16 & \text{if } 8 \le t < 20 \\ \frac{3}{2}\tau + 54 & \text{if } 20 \le t \le 24 \end{cases}$$

where τ is in hours and $\tau = 0$ corresponds to 0:00.

Problem 1. What is the probability that 12 or fewer phone calls have been received between 6:00 and 10:00?

- Let's follow page 3 of Lesson 9 to answer this problem.
- Z_τ = number of arrivals by time τ.
- $Z_{10} - Z_6$ is the number of arrivals between time 6 and time 10, and follows a Poisson distribution with parameter $\Lambda(10) - \Lambda(6) = 16$.
- Then,

$$\Pr\{Z_{10} - Z_6 \le 12\} = \sum_{n=0}^{12} \underbrace{\Pr\{Z_{10} - Z_6 = n\}}_{\text{pmf of Poisson with parameter 16 at } n} = \dots$$

Problem 2. If exactly 50 phone calls have been received between 0:00 and 12:00, what is the probability that 100 or fewer phone calls have been received over the course of the entire day (0:00 - 24:00)?

- Take a look at Example 2d of Lesson 9 for a similar problem.

Problem 3. In words, briefly describe the meaning of $\Lambda(12)$ in the context of this problem.

- Take a look at page 1 of Lesson 9 for an interpretation of the integrated rate function.
- Take a look at Example 2a of Lesson 9 as well.

Problem 4. The Simplexville Electric Company is conducting a study of its power line along the busiest part of Main Street. Looking at its historical data, the company has observed that power surges occur at a rate of 12 per hour, and this rate does not change over time. However, it also has noticed that the power surges come in "waves": a large power surge is always followed by a smaller power surge exactly 1 minute later.

Professor I. M. Wright is consulting for the electric company. He thinks that the occurrence of power surges (both large and small) satisfies the independent increments property. Is he correct? Briefly explain.

- Suppose a large power surge occurs at time 1 (in minutes).
- According to the problem, a small power surge then occurs at time 2.
- Consider the number of arrivals in the nonoverlapping time intervals $[0.5, 1.5)$ and $[1.5, 2.5)$. Are they independent?

Exponential random variable with parameter λ:	cdf $F(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value $= 1/\lambda$
Erlang random variable with parameter λ and n phases:	cdf $F(a) = \begin{cases} 1 - \displaystyle\sum_{j=0}^{n-1} \frac{e^{-\lambda a}(\lambda a)^j}{j!} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value $= n/\lambda$
Poisson random variable with parameter λt:	pmf $p(n) = \dfrac{e^{-\lambda t}(\lambda t)^n}{n!}$ for $n = 0, 1, 2, \ldots$	expected value $= \lambda t$