SA402 · Dynamic and Stochastic Models   Fall 2019 · Uhan

Quiz 4 – 2 October 2019

Instructions. You have 15 minutes to complete this quiz. You may use your calculator. You may <u>not</u> use any other materials (e.g., notes, homework, books).

<u>Show all your work.</u> To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
5	1	
Total		/ 50

For Problems 1-5, consider the following setting.

Patients arrive at the Simplexville Hospital emergency room at a rate of 5 per hour. A doctor works an 8-hour shift from 12 a.m. until 8 a.m. Suppose the arrivals follow a Poisson process.

Problem 1. If the doctor has seen exactly 5 patients by 2 a.m., what is the probability that the doctor will see a total of 20 patients or more by 4 a.m.?

Problem 2. What is the expected number of patients the doctor will see during her shift, if she sees exactly 30 patients by 4 a.m.?

Problem 3. What is the probability that the doctor will see her 17th patient before or at 3 a.m.?

Problem 4. 12% of the patients admitted to the emergency room are classified as urgent. What is the probability that the doctor will see 5 or fewer urgent patients during her shift?

Problem 5. The hospital also has a walk-in clinic to handle minor problems. Patients arrive at this clinic at a rate of 10 per hour. What is the probability that the total number of patients arriving at both the emergency room and the clinic from 12 a.m. to 3 a.m. will be greater than or equal to 50?

Exponential random variable with parameter λ:	cdf $F(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value $= 1/\lambda$
Erlang random variable with parameter λ and n phases:	cdf $F(a) = \begin{cases} 1 - \sum_{j=0}^{n-1} \dfrac{e^{-\lambda a}(\lambda a)^j}{j!} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value $= n/\lambda$
Poisson random variable with parameter λt:	pmf $p(n) = \dfrac{e^{-\lambda t}(\lambda t)^n}{n!}$ for $n = 0, 1, 2, \ldots$	expected value $= \lambda t$