Review Problems for Final Exam

These review problems cover the material in Lessons 13-17. Take a look at the review problems for Exam 1 and Exam 2 as well.

Problem 1. The meteorologist at the Simplexville Observatory is trying to come up with a simple model for Simplexville's weather that can be used for some quick, preliminary analysis. According to his data, if the previous two days were sunny, then there is a 75% chance that the next day is also sunny. In a similar vein, if the previous two days were rainy, then there is a 35% chance that the next day is rainy. Otherwise, there is a 60% chance that the next day is sunny.

a. Model Simplexville's weather as a Markov chain by defining:
   - the state space and the meaning of each state in the setting's context,
   - the meaning of one time step in the setting's context, and
   - the one-step transition probabilities.
b. What is the long-run probability that the previous two days were sunny?

Problem 2. Customers arrive at Fantastic Dan's hair salon according to a Poisson process at a rate of 5 customers per hour. There are only 3 chairs provided for customers to wait for Dan. The time for Fantastic Dan to cut one customer's hair is exponentially distributed with a mean of 10 minutes. Since Fantastic Dan's customers are rather impatient, they may renege: the time a customer is willing to spend waiting before starting his or her haircut is exponentially distributed with a mean of 15 minutes.

a. Model this queueing system as a birth-death process by defining
   - the state space and what each state means,
   - the arrival rate in each state, and
   - the service rate in each state.
b. Over the long run, what is the expected number of customers in the hair salon?
c. Over the long run, what is the expected time a customer spends in the hair salon?

Problem 3. The Simplexville Community Center has 5 tennis courts. Pairs of players arrive at the courts according to a Poisson process with rate of one pair per 10 minutes, and use a court for an exponentially distributed time with mean 40 minutes.

a. What standard queueing model fits this setting best?
b. What is the traffic intensity in this queueing system?
c. What is the long-run expected fraction of time that all courts are empty?
d. Over the long run, how many pairs of players are in the queue to get a court on average?
e. What is the long-run expected delay time to get a court?

Problem 4. You have been put in charge of redesigning Turingtown's emergency call center. Based on historical data, you have estimated that calls arrive at a rate of 12 per hour, and that each call takes 1 operator 15 minutes on average. If all operators are busy when a call arrives, that call is put on hold. Model the call center as an M/M/∞ queue and determine the minimum number of operators needed to ensure that no calls are put on hold 99% of the time in the long run.