Name: Notes

SA402 · Dynamic and Stochastic Models Fall 2019 · Uhan

Quiz 4 – 2 October 2019

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
5	1	
Total		/ 50

Instructions. You have 15 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

For Problems 1-5, consider the following setting.

Patients arrive at the Simplexville Hospital emergency room at a rate of 5 per hour. A doctor works an 8-hour shift from 12 a.m. until 8 a.m. Suppose the arrivals follow a Poisson process.

Problem 1. If the doctor has seen exactly 5 patients by 2 a.m., what is the probability that the doctor will see a total of 20 patients or more by 4 a.m.?

- Take a look at Example 2 in Lesson 7 to see how to approach a similar problem.

Problem 2. What is the expected number of patients the doctor will see during her shift, if she sees exactly 30 patients by 4 a.m.?

- Take a look at Example 3 in Lesson 7 to see how to approach a similar problem.

Problem 3. What is the probability that the doctor will see her 17th patient before or at 3 a.m.?

- Take a look at Problem 5.3d from the textbook, assigned for homework, to see how to approach a similar problem.

Problem 4. 12% of the patients admitted to the emergency room are classified as urgent. What is the probability that the doctor will see 5 or fewer urgent patients during her shift?

- Take a look at Problem 5.3e from the textbook, assigned for homework, to see how to approach a similar problem.

Problem 5. The hospital also has a walk-in clinic to handle minor problems. Patients arrive at this clinic at a rate of 10 per hour. What is the probability that the total number of patients arriving at both the emergency room and the clinic from 12 a.m. to 3 a.m. will be greater than or equal to 50?

- Take a look at Problem 5.3f from the textbook, assigned for homework, to see how to approach a similar problem.

Exponential random variable with parameter λ:	cdf $F(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value = $1/\lambda$
Erlang random variable with parameter λ and n phases:	cdf $F(a) = \begin{cases} 1 - \displaystyle\sum_{j=0}^{n-1} \frac{e^{-\lambda a}(\lambda a)^j}{j!} & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value = n/λ
Poisson random variable with parameter λt:	pmf $p(n) = \dfrac{e^{-\lambda t}(\lambda t)^n}{n!}$ for $n = 0, 1, 2, \ldots$	expected value = λt