Lesson 4. Conditional Probability Review, cont.

Problem 1. Professor I. M. Right often has his facts wrong. Let X be a random variable that represents the number of questions he is asked during one class, and let Y be the number of questions that he answers incorrectly during one class. The joint pmf p_{XY} of X and Y is:

		Y		
	p_{XY}	0	1	2
	1	1/3	0	0
X	2	1/4	1/12	0
	3	3/16	1/8	1/48

a. What is the probability that Professor Right answers all questions correctly during one class?

b. What is the probability that Professor Right answers 1 question incorrectly during one class, given that he is asked two questions?

c. Explain why $p_{XY}(1, 2) = 0$.

Problem 2. The Simplex Company uses three machines to produce a large batch of similar manufactured items. 20% of the items were produced by machine 1, 30% by machine 2, and 50% by machine 3. In addition, 1% of the items produced by machine 1 are defective, 2% by machine 2 are defective, and 3% by machine 3 are defective. Suppose you select 1 item at random from the entire batch.

a. Define the random variable M as the machine used ($M \in \{1, 2, 3\}$) to produce this item. Write the pmf p_M of M.

b. Define another random variable D that is equal to 1 if this item is defective, and 0 otherwise. Find the probability that $D = 1$ given $M = m$, for $m = 1, 2, 3$.

c. Find the probability that $D = 1$; that is, the probability that the randomly selected item is defective.

Problem 3. Simplex Pizza sells pizza (of course) and muffins (that's weird). Let Z and M be random variables that represent the number of pizzas and muffins in one order, respectively. Based on historical data, the company has determined the joint pmf p_{ZM} for Z and M:

		M		
	p_{ZM}	0	1	2
	0	0	0.09	0.06
	1	0.25	0.11	0.05
Z	2	0.10	0.08	0.07
	3	0.08	0.07	0.04

a. What is the conditional pmf of M, given that $Z = 2$?

b. What is the expected number of muffins in an order, given that it contains 2 pizzas?

c. It turns out that $\Pr\{M = 1\} = 0.35$ and $\Pr\{M = 1 | Z = 3\} \approx 0.368$. Based on this information, are M and Z independent? Why or why not?