Bode plots contain the same information as the nonlogarithmic plots discussed in the previous section, but they are much easier to construct, as we shall see shortly.

The transfer function can be written as

$$\mathbf{H} = H\underline{/\phi} = He^{j\phi} \tag{14.12}$$

Taking the natural logarithm of both sides,

$$\ln \mathbf{H} = \ln H + \ln e^{j\phi} = \ln H + j\phi \tag{14.13}$$

Thus, the real part of $\ln \mathbf{H}$ is a function of the magnitude while the imaginary part is the phase. In a Bode magnitude plot, the gain

$$\boxed{H_{\text{dB}} = 20 \log_{10} H} \tag{14.14}$$

is plotted in decibels (dB) versus frequency. Table 14.2 provides a few values of H with the corresponding values in decibels. In a Bode phase plot, ϕ is plotted in degrees versus frequency. Both magnitude and phase plots are made on semilog graph paper.

A transfer function in the form of Eq. (14.3) may be written in terms of factors that have real and imaginary parts. One such representation might be

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots} \tag{14.15}$$

which is obtained by dividing out the poles and zeros in $\mathbf{H}(\omega)$. The representation of $\mathbf{H}(\omega)$ as in Eq. (14.15) is called the *standard form.* $\mathbf{H}(\omega)$ may include up to seven types of different factors that can appear in various combinations in a transfer function. These are:

1. A gain K
2. A pole $(j\omega)^{-1}$ or zero $(j\omega)$ at the origin
3. A simple pole $1/(1 + j\omega/p_1)$ or zero $(1 + j\omega/z_1)$
4. A quadratic pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$ or zero $[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$

TABLE 14.2

Specific gain and their decibel values.*

Magnitude H	$20 \log_{10} H$ (dB)
0.001	−60
0.01	−40
0.1	−20
0.5	−6
$1/\sqrt{2}$	−3
1	0
$\sqrt{2}$	3
2	6
10	20
20	26
100	40
1000	60

* Some of these values are approximate.

The origin is where $\omega = 1$ or $\log \omega = 0$ and the gain is zero.

In constructing a Bode plot, we plot each factor separately and then add them graphically. The factors can be considered one at a time and then combined additively because of the logarithms involved. It is this mathematical convenience of the logarithm that makes Bode plots a powerful engineering tool.

We will now make straight-line plots of the factors listed above. We shall find that these straight-line plots known as Bode plots approximate the actual plots to a reasonable degree of accuracy.

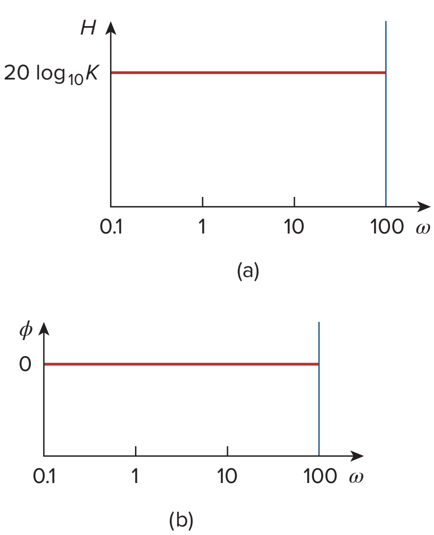


Figure 14.9
Bode plots for gain K: (a) magnitude plot, (b) phase plot.

Constant term: For the gain K, the magnitude is $20 \log_{10} K$ and the phase is 0°; both are constant with frequency. Thus, the magnitude and phase plots of the gain are shown in Fig. 14.9. If K is negative, the magnitude remains $20 \log_{10} |K|$ but the phase is $\pm 180°$.

Pole/zero at the origin: For the zero $(j\omega)$ at the origin, the magnitude is $20 \log_{10} \omega$ and the phase is 90°. These are plotted in Fig. 14.10, where we notice that the slope of the magnitude plot is 20 dB/decade, while the phase is constant with frequency.

The Bode plots for the pole $(j\omega)^{-1}$ are similar except that the slope of the magnitude plot is −20 dB/decade while the phase is −90°. In general, for $(j\omega)^N$, where N is an integer, the magnitude plot will have a slope of $20N$ dB/decade, while the phase is $90N$ degrees.

Simple pole/zero: For the simple zero $(1 + j\omega/z_1)$, the magnitude is $20\log_{10}|1 + j\omega/z_1|$ and the phase is $\tan^{-1}\omega/z_1$. We notice that

$$H_{\text{dB}} = 20\log_{10}\left|1 + \frac{j\omega}{z_1}\right| \quad \Rightarrow \quad 20\log_{10} 1 = 0 \qquad \text{as} \quad \omega \to 0 \tag{14.16}$$

$$H_{\text{dB}} = 20\log_{10}\left|1 + \frac{j\omega}{z_1}\right| \quad \Rightarrow \quad 20\log_{10} \frac{\omega}{z_1} \qquad \text{as} \quad \omega \to \infty \tag{14.17}$$

showing that we can approximate the magnitude as zero (a straight line with zero slope) for small values of ω and by a straight line with slope 20 dB/decade for large values of ω. The frequency $\omega = z_1$ where the two asymptotic lines meet is called the *corner frequency* or *break frequency*. Thus, the approximate magnitude plot is shown in Fig. 14.11(a), where the actual plot is also shown. Notice that the approximate plot is close to the actual plot except at the break frequency, where $\omega = z_1$ and the deviation is $20\log_{10}|(1 + j1)| = 20\log_{10}\sqrt{2} \simeq 3$ dB.

The phase $\tan^{-1}(\omega/z_1)$ can be expressed as

$$\phi = \tan^{-1}\left(\frac{\omega}{z_1}\right) = \begin{cases} 0, & \omega = 0 \\ 45^\circ, & \omega = z_1 \\ 90^\circ, & \omega \to \infty \end{cases} \tag{14.18}$$

As a straight-line approximation, we let $\phi \simeq 0$ for $\omega \le z_1/10$, $\phi \simeq 45^\circ$ for $\omega = z_1$, and $\phi \simeq 90^\circ$ for $\omega \ge 10z_1$. As shown in Fig. 14.11(b) along with the actual plot, the straight-line plot has a slope of 45° per decade.

The Bode plots for the pole $1/(1 + j\omega/p_1)$ are similar to those in Fig. 14.11 except that the corner frequency is at $\omega = p_1$, the magnitude has a slope of −20 dB/decade, and the phase has a slope of −45° per decade.

Quadratic pole/zero: The magnitude of the quadratic pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$ is $-20\log_{10}|1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2|$ and the phase is $-\tan^{-1}(2\zeta_2\omega/\omega_n)/(1 - \omega^2/\omega_n^2)$. But

$$H_{\text{dB}} = -20\log_{10}\left|1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right| \quad \Rightarrow \quad 0 \qquad \text{as} \quad \omega \to 0 \tag{14.19}$$

A decade is an interval between two frequencies with a ratio of 10; e.g., between ω_0 and $10\omega_0$, or between 10 and 100 Hz. Thus, 20 dB/decade means that the magnitude changes 20 dB whenever the frequency changes tenfold or one decade.

The special case of dc ($\omega = 0$) does not appear on Bode plots because log $0 = -\infty$, implying that zero frequency is infinitely far to the left of the origin of Bode plots.

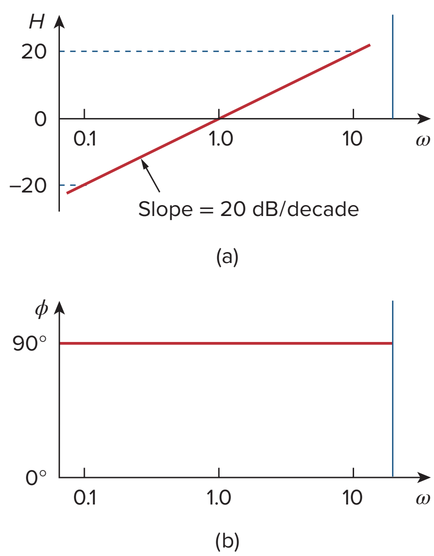


Figure 14.10
Bode plot for a zero $(j\omega)$ at the origin: (a) magnitude plot, (b) phase plot.

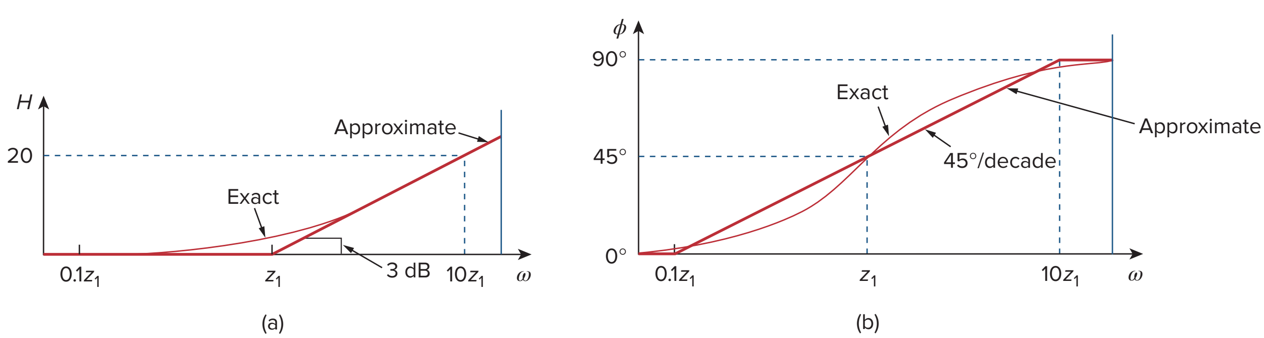


Figure 14.11
Bode plots of zero $(1 + j\omega/z_1)$: (a) magnitude plot, (b) phase plot.

and

$$H_{\mathrm{dB}} = -20 \log_{10}\left|1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right| \quad \Rightarrow \quad -40 \log_{10}\frac{\omega}{\omega_n} \quad \text{as} \quad \omega \to \infty \tag{14.20}$$

Thus, the amplitude plot consists of two straight asymptotic lines: one with zero slope for $\omega < \omega_n$ and the other with slope -40 dB/decade for $\omega > \omega_n$, with ω_n as the corner frequency. Figure 14.12(a) shows the approximate and actual amplitude plots. Note that the actual plot depends on the damping factor ζ_2 as well as the corner frequency ω_n. The significant peaking in the neighborhood of the corner frequency should be added to the straight-line approximation if a high level of accuracy is desired. However, we will use the straight-line approximation for the sake of simplicity.

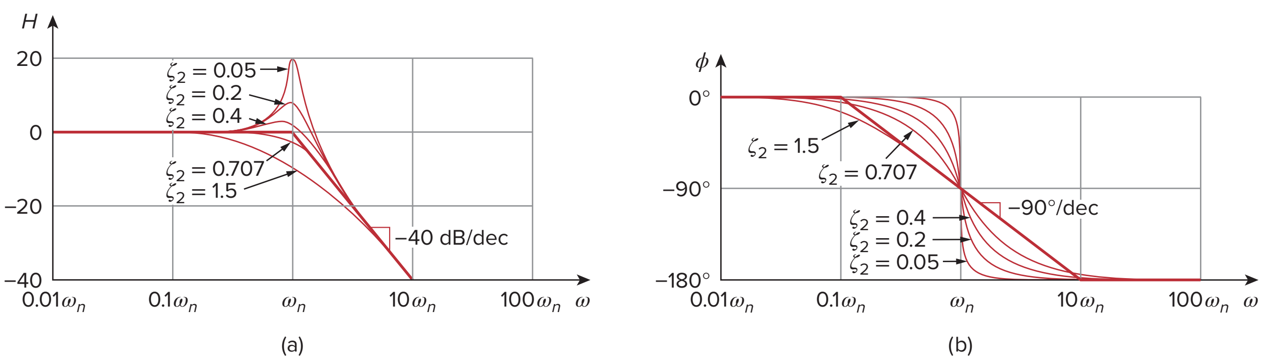


Figure 14.12
Bode plots of quadratic pole $[1 + j2\zeta\omega/\omega_n - \omega^2/\omega_n^2]^{-1}$: (a) magnitude plot, (b) phase plot.

There is another procedure for obtaining Bode plots that is faster and perhaps more efficient than the one we have just discussed. It consists in realizing that zeros cause an increase in slope, while poles cause a decrease. By starting with the low-frequency asymptote of the Bode plot, moving along the frequency axis, and increasing or decreasing the slope at each corner frequency, one can sketch the Bode plot immediately from the transfer function without the effort of making individual plots and adding them. This procedure can be used once you become proficient in the one discussed here.

Digital computers have rendered the procedure discussed here almost obsolete. Several software packages such as *PSpice, MATLAB,* Mathcad, and Micro-Cap can be used to generate frequency response plots. We will discuss *PSpice* later in the chapter.

The phase can be expressed as

$$\phi = -\tan^{-1}\frac{2\zeta_2\omega/\omega_n}{1-\omega^2/\omega_n^2} = \begin{cases} 0, & \omega = 0 \\ -90^\circ, & \omega = \omega_n \\ -180^\circ, & \omega \to \infty \end{cases} \tag{14.21}$$

The phase plot is a straight line with a slope of $-90°$ per decade starting at $\omega_n/10$ and ending at $10\omega_n$, as shown in Fig. 14.12(b). We see again that the difference between the actual plot and the straight-line plot is due to the damping factor. Notice that the straight-line approximations for both magnitude and phase plots for the quadratic pole are the same as those for a double pole, that is, $(1 + j\omega/\omega_n)^{-2}$. We should expect this because the double pole $(1+j\omega/\omega_n)^{-2}$ equals the quadratic pole $1/[1+j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$ when $\zeta_2 = 1$. Thus, the quadratic pole can be treated as a double pole as far as straight-line approximation is concerned.

For the quadratic zero $[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$, the plots in Fig. 14.12 are inverted because the magnitude plot has a slope of 40 dB/decade while the phase plot has a slope of 90° per decade.

Table 14.3 presents a summary of Bode plots for the seven factors. Of course, not every transfer function has all seven factors. To sketch the Bode plots for a function $\mathbf{H}(\omega)$ in the form of Eq. (14.15), for example, we first record the corner frequencies on the semilog graph paper, sketch the factors one at a time as discussed above, and then combine additively

TABLE 14.3

Summary of Bode straight-line magnitude and phase plots.

Factor | **Magnitude** | **Phase**

K

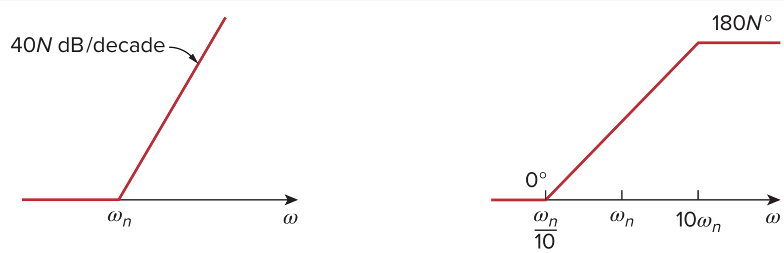
$(j\omega)^N$

$\dfrac{1}{(j\omega)^N}$

(1+jw/z) N

$\dfrac{1}{(1 + j\omega/p)^N}$

$\left[1 + \dfrac{2j\omega\zeta}{\omega_n} + \left(\dfrac{j\omega}{\omega_n}\right)^2\right]^N$



$\dfrac{1}{[1 + 2j\omega\zeta/\omega_k + (j\omega/\omega_k)^2]^N}$