ECSE 493 - Lab 2 Report

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1 Question 1

1.1 Part A

We take the original provided equation

$$G(s) = \frac{X(s)}{V(s)} = \frac{1}{L_s \frac{m_c r_g}{k_m k_g} s^2 + R_a \frac{m_c r_g}{K_m k_g} s^2 + \frac{k_e k_g}{r_g} s} \tag{1}$$

Substitute the known constants

$$\frac{X(s)}{V(s)} = \frac{1}{180\mu \frac{0.526*0.0064}{0.0077*3.7)} s^3 + 2.6 \frac{0.526*0.0064}{0.0077*3.7} s^2 + \frac{0.0077*3.7}{0.0064} s} \tag{2}$$

Evaluate with respect to velocity instead of position, and evaluate constants

$$\frac{\frac{dX(s)}{dt}}{V(s)} = \frac{1}{2.126 * 10^{-5} s^2 + 0.30721 s + 4.4515} \tag{3}$$

Seeing s it is typically customary to leave out the second order term as the inductance is small, we finally get

$$\frac{\frac{dX(s)}{dt}}{V(s)} = \frac{1}{0.30721 s + 4.4515} \tag{4}$$

1.2 Part B

Using the schematic shown below in Figure 1

We plot the following input and step response in Figures 2 and 3 respectively

And given the time constant of approximately 0.106, and the Gain of 0.1539 (using the conversion constant of $2.28 * 10^{-5}$, we utilize the equation

$$\frac{Gain}{Timeconstant * s + 1} = \frac{0.1539}{0.106 s + 1} \tag{5}$$

This equals

$$\frac{1}{0.689 s + 6.47} \tag{6}$$

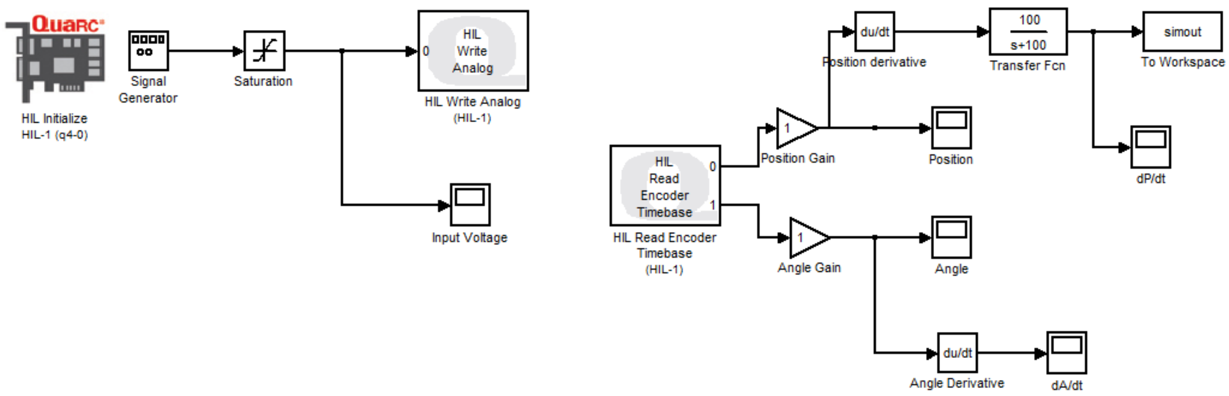

Figure 1: Simulink schematic for Question 1

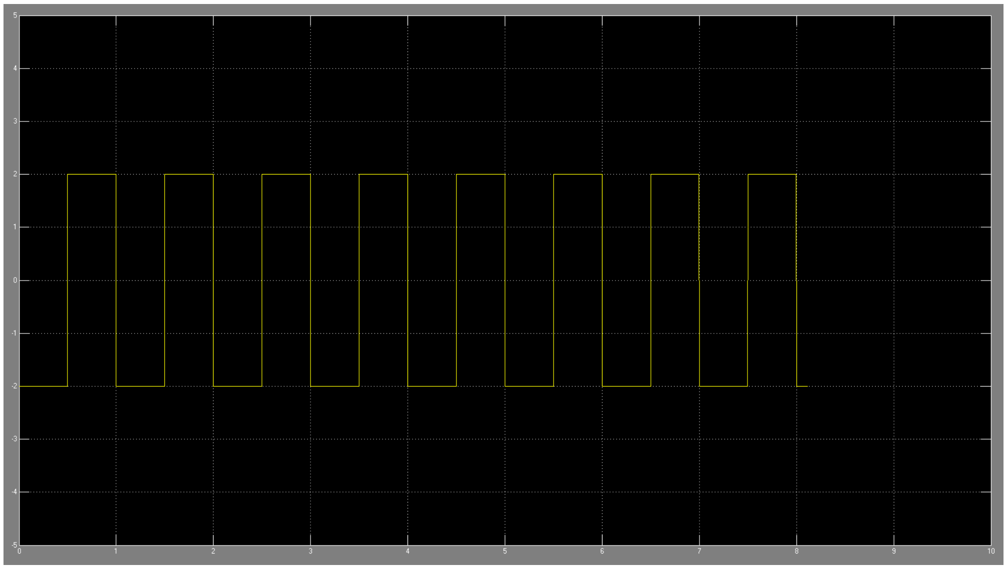

Figure 2: Input Signal

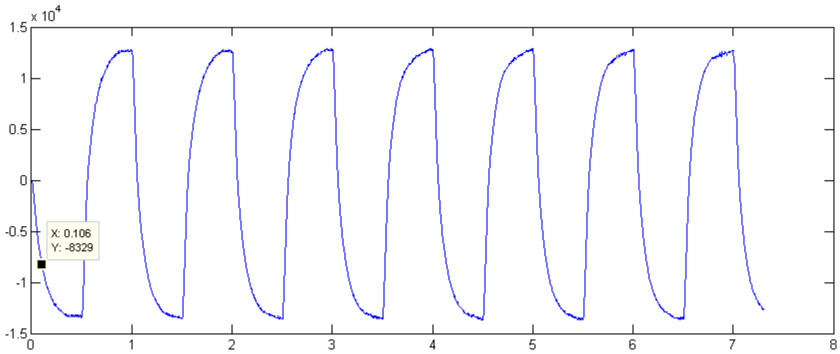

Figure 3: Step Response for Q1

1.3 Part C

The plot comparing the two step responses is shown below in figure 4

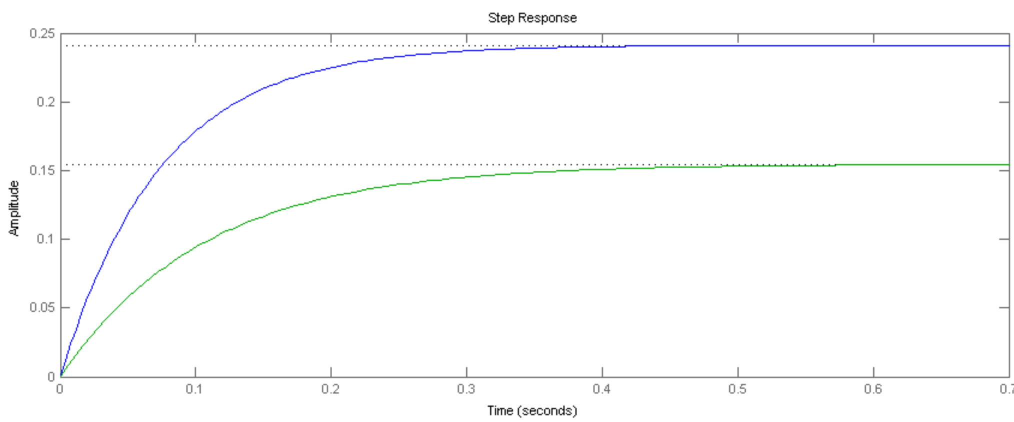

Figure 4: Comparing both step responses.

We see that these two values are different primarily in gain. Since the theoretical transfer function was calculated while neglecting friction, it seems that the disparity can be best explained by the introduction of friction to the system. It seems that with friction, the cart's steady state velocity is lower, which is intuitive as the amount of voltage, i.e. energy supplied to the system is the same, yet there is more opposing force than initially assumed.

2 Question 2

2.1 Part A

We take the definition of the closed loop transfer function from the lecture notes

$$T(s) = \frac{1}{\frac{R_a m_c r_g}{K_p k_m k_g} s + \frac{K_m k_g}{K_p r_g} + 1} \tag{7}$$

so we know that

$$\begin{aligned} \omega^2 = \frac{K_p k_m k_g}{R_a m_c r_g} = \frac{0.0077 * 3.7 * K_p}{2.6 * 0.526 * 0.0064} \\ \omega_n^2 = 3.255 K_p \\ \omega_n = \sqrt{3.255 K_p} \end{aligned} \tag{8}$$

And we also know that

$$\begin{aligned} \frac{2\zeta}{\omega_n} = \frac{K_m k_g}{K_p r_g} = \frac{0.0077 * 3.7}{K_p * 0.0064} \\ \frac{2\zeta}{\omega_n} = \frac{4.4516}{K_p} \\ 2\zeta = \frac{4.4516}{K_p} * \sqrt{3.255 K_p} \\ \zeta = \frac{8.031}{\sqrt{K_p}} \end{aligned} \tag{9}$$

So we can see that as K_p is increase, ω_n increases, indicating a faster response time, but ζ decreases, indicating lower damping.

2.2 Part B

We show the corresponding plots for 20 gain (figs 5 and 6, 100 gain (figs 7 and 8), and 200 gain (figs 9 and 10)

We can see that in theory, the agent reaches the steady state quicker for 20 gain, and overshoots further for 100 and 200 gain than their experimental counterparts. Both these results come from the fact that the theoretical results are derived from the assumption that there is no friction in the environment. As friction does exist when we are running the control system experimentally, we can see that the opposing forces create the aforementioned effects. Finally, since we can see that even in practice, 100 gain overshoots and has to adjust to return to steady state, we can assume that the critical gain is somewhere between 20 gain and 100 gain.

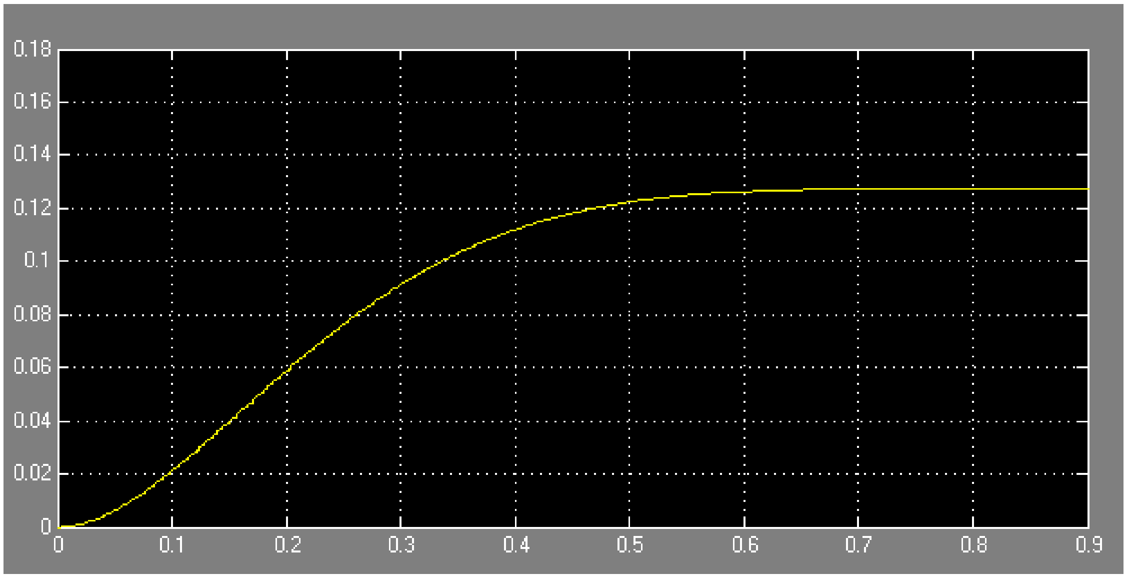

Figure 5: With 20 gain in practice

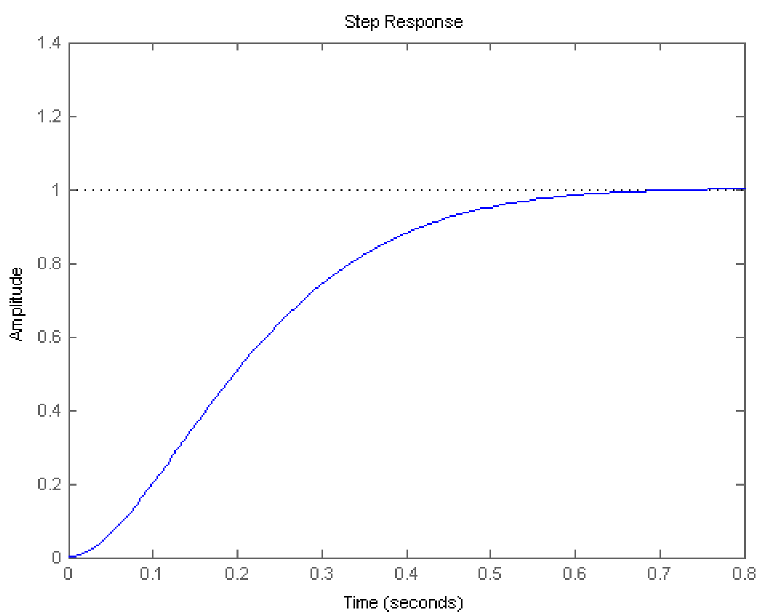

Figure 6: With 20 gain in theory

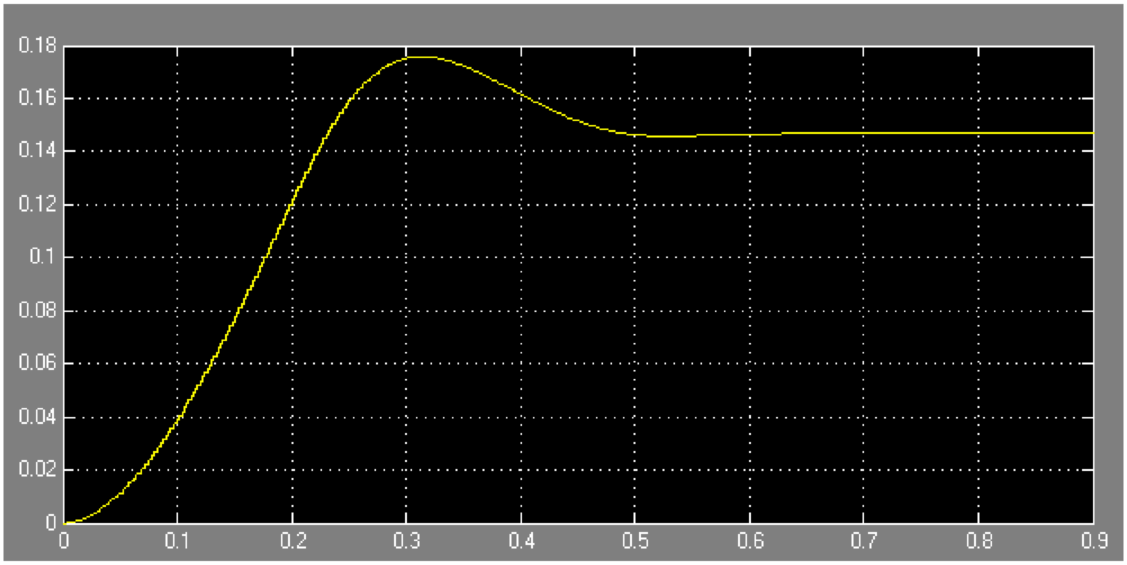

Figure 7: With 100 gain in practice

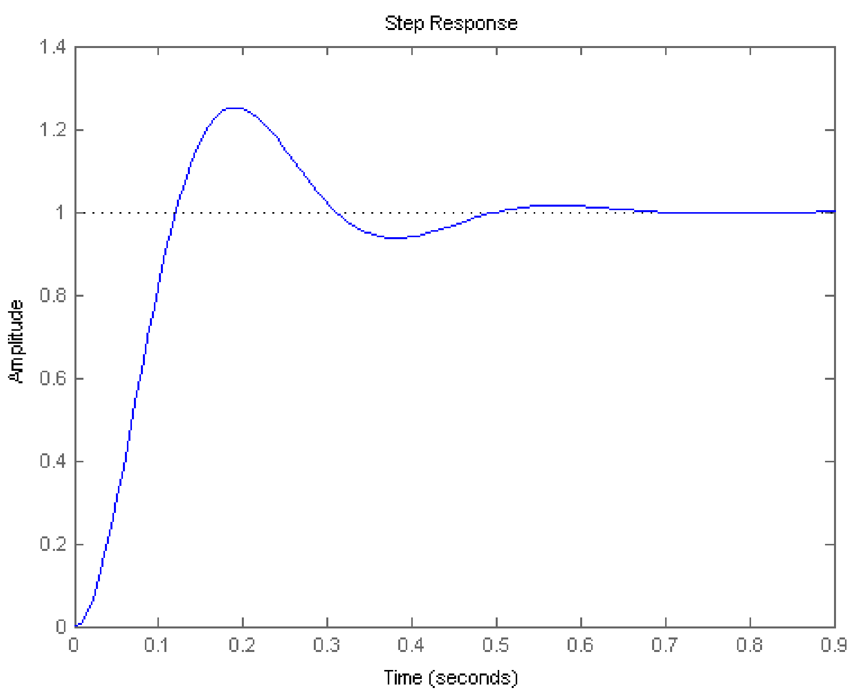

Figure 8: With 100 gain in theory

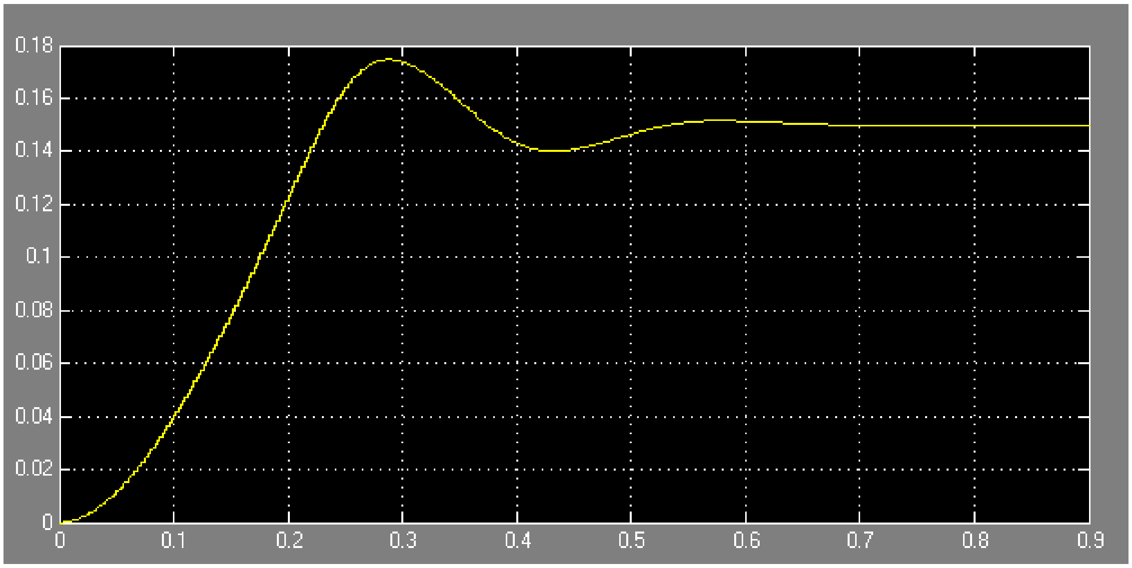

Figure 9: With 200 gain in practice

3 Question 3

3.1 Part A

We take the practical values and theoretical values of 0.1Hz and a gain of 20 in Figures 11 and 12

Where we can see that with 0.1Hz and a gain of 20, our practical gain (peak to peak) of $\approx 20log(\frac{0.26}{0.30}) = -1.243dB$ versus the theoretical gain of -0.00173dB and a practical phase shift of -10 degrees and a practical phase shift of -1.7 degrees.

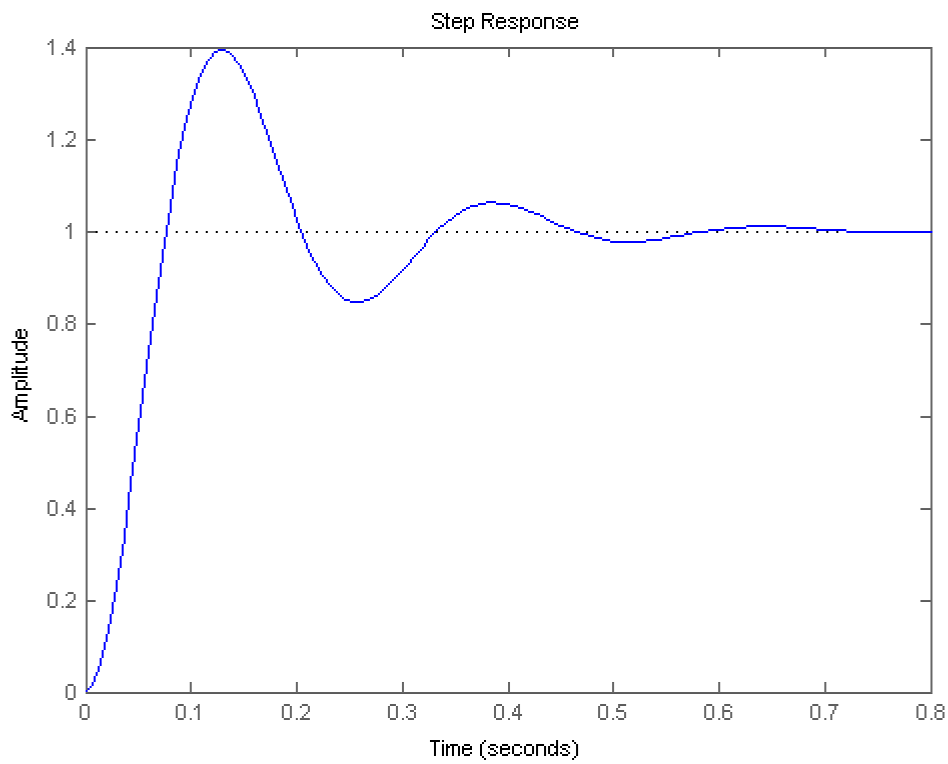

Figure 10: With 200 gain in theory

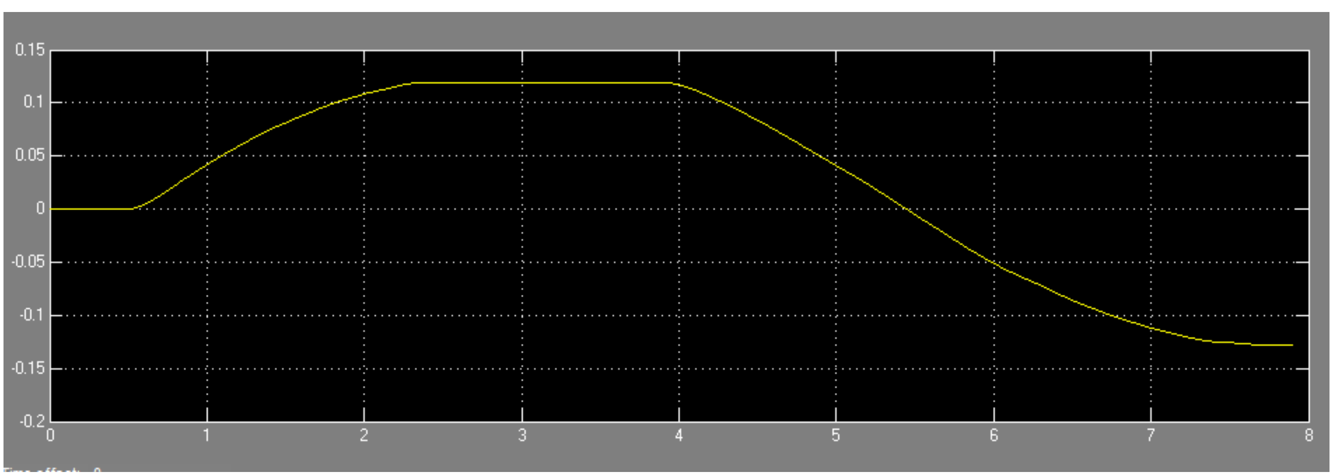

Figure 11: Practical values of 0.1Hz and a gain of 20

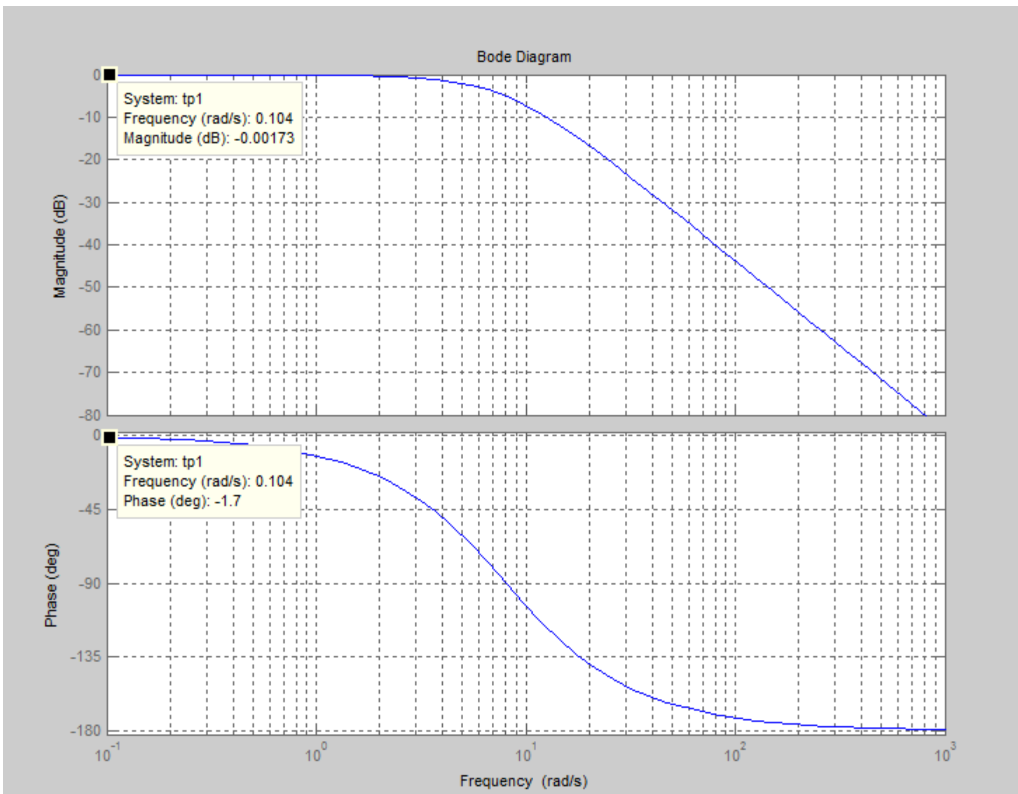

Figure 12: Theoretical values of 0.1Hz and a gain of 20

We then take the practical values of 0.5Hz, 1Hz, both at a gain of 20 and compare it to the theoretical values in figures 13, 14, and 15.

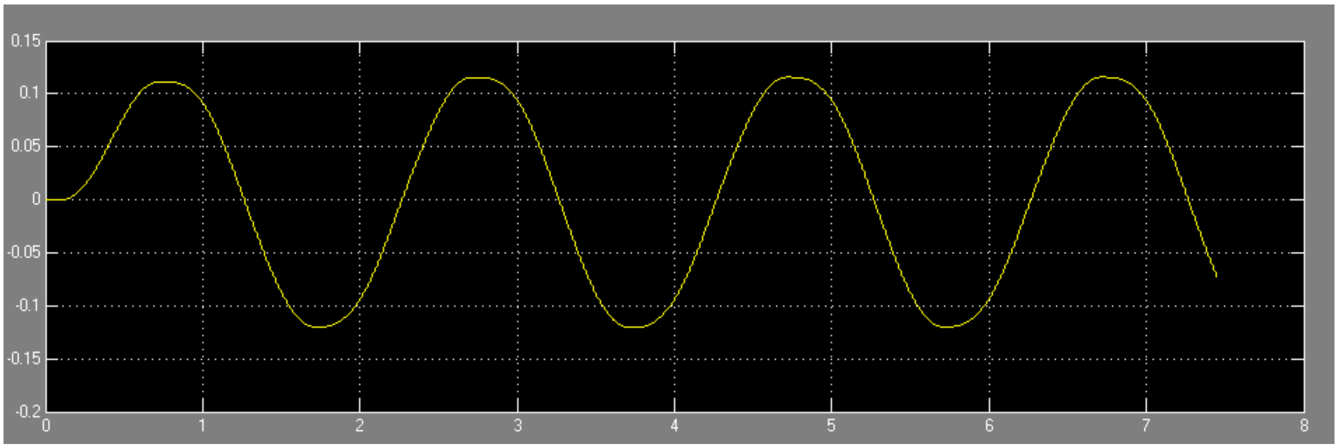

Figure 13: Practical values of 0.5Hz and a gain of 20

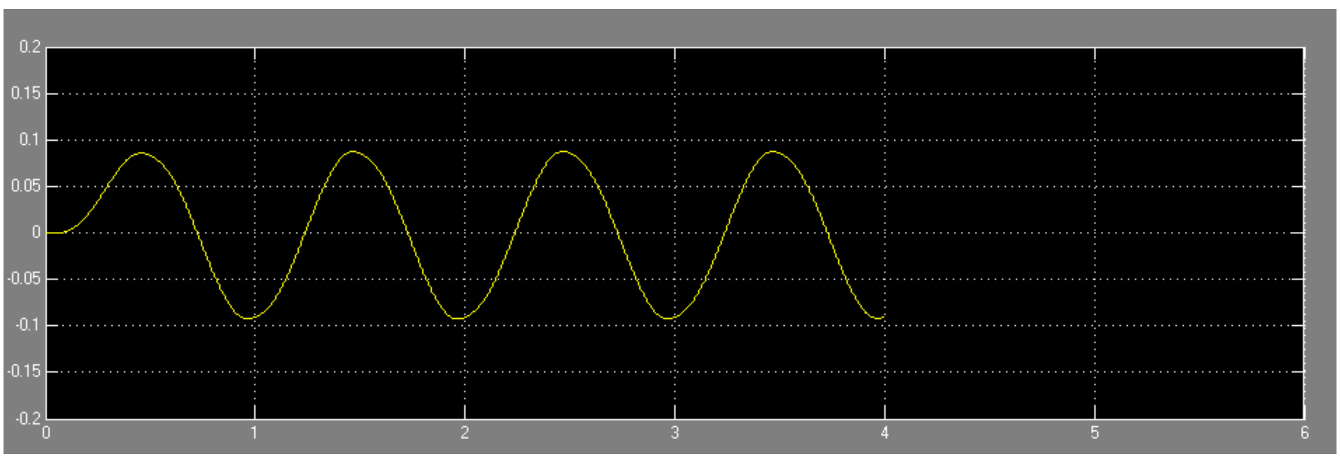

Figure 14: Practical values of 1Hz and a gain of 20

For 0.5Hz, we can see that the practical gain is $20log(\frac{0.24}{0.30}) = -1.938dB$ and the practical gain for 1Hz is $20log(\frac{0.16}{0.30} = -5.46dB$, and the theoretical gains are -0.0213dB and -0.0862dB respectively. Practically, the phase lags were -13 degrees and -17 degrees vs. their theoretical counterparts of -6.4 degrees and -13 degrees respectively.

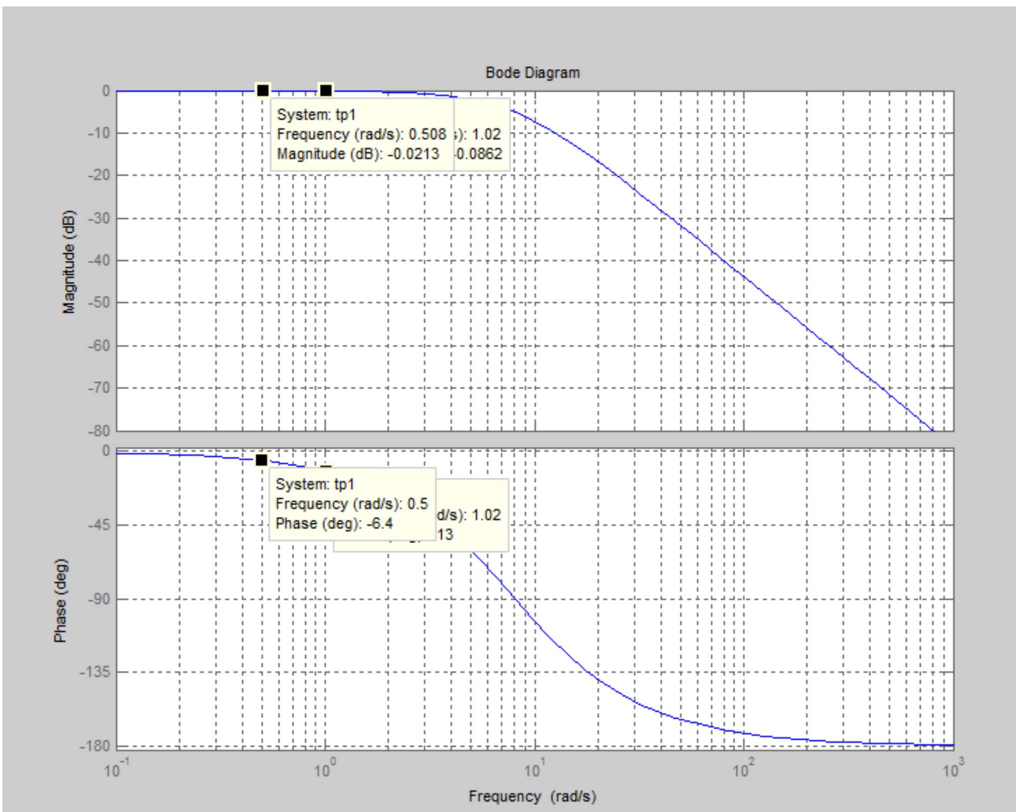

Figure 15: Theoretical values of 0.5, 1Hz both at 20 gain

We then take the practical values of 2Hz at a gain of 20 and compare it to its respective theoretical values in Figures 16 and 17

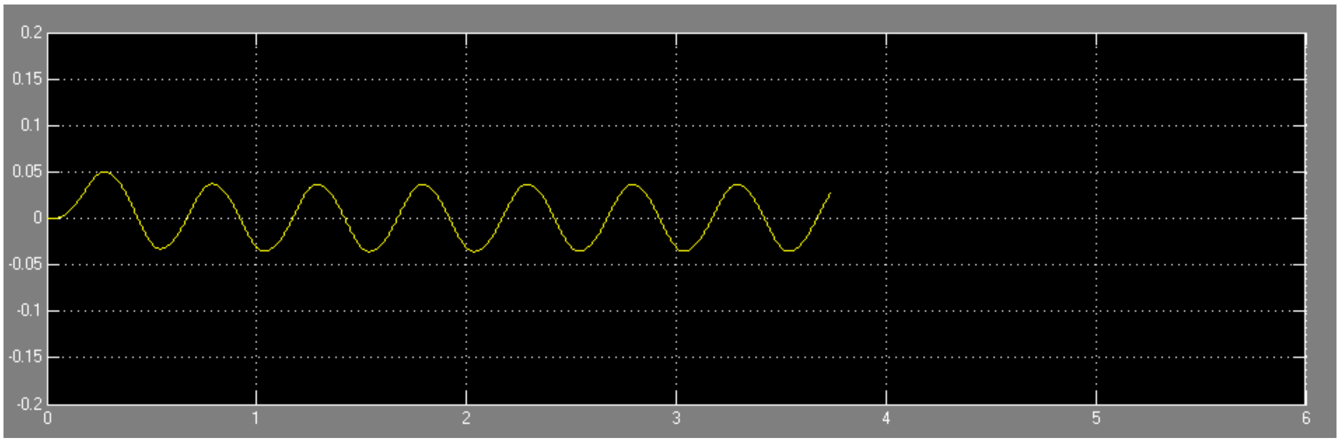

Figure 16: Practical values at 2Hz and 20 gain

For 2Hz, we can see that the practical gain is $20log(\frac{0.09}{0.30}) = -10.458dB$ versus the practical value of -0.342dB, we can also compare the practical phase values of -32 degrees to the theoretical values of -25.5 degrees

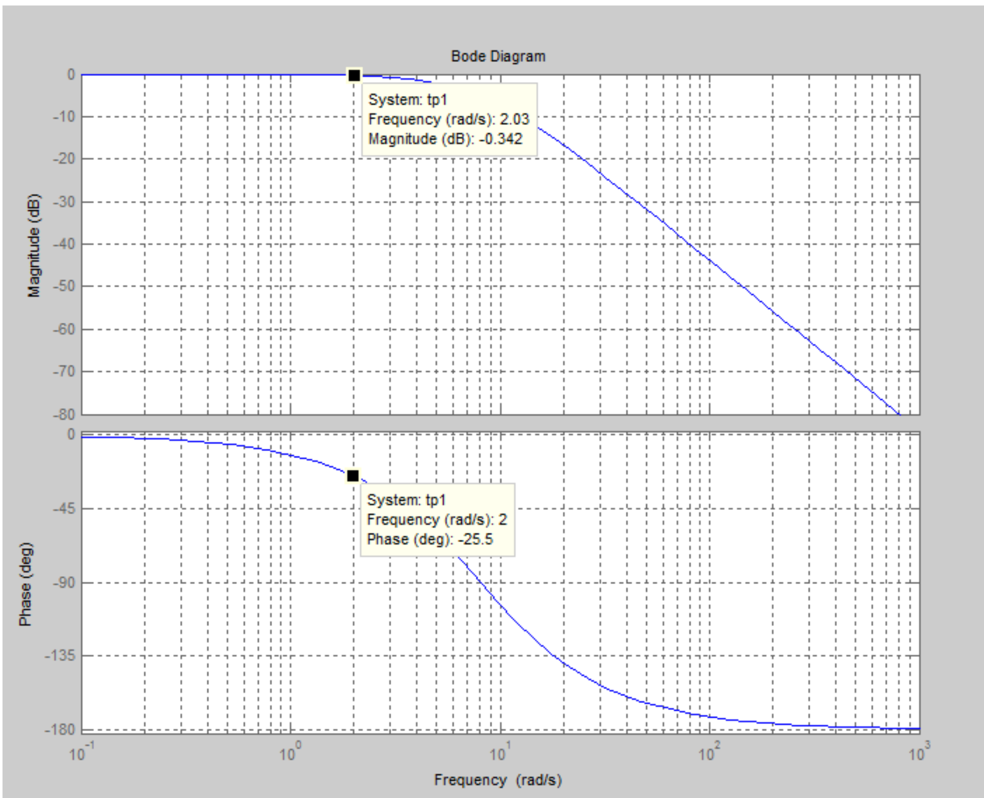

Figure 17: Theoretical values of 2Hz at 20 gain

We then take the practical values of 5Hz and 10Hz and compare it to their respective theoretical values in Figures 18, 19, and 20.

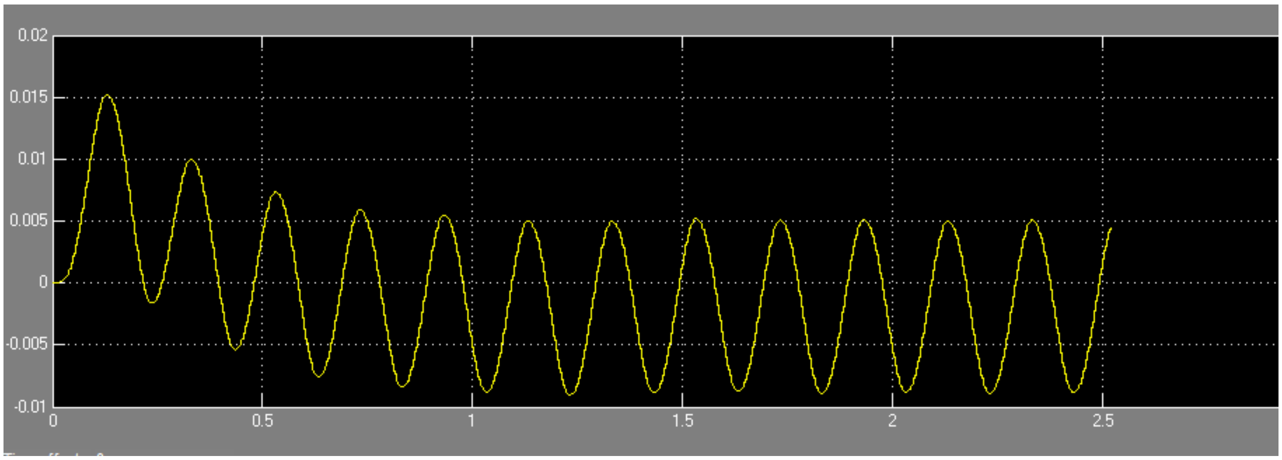

Figure 18: Practical values at 5Hz at 20 gain

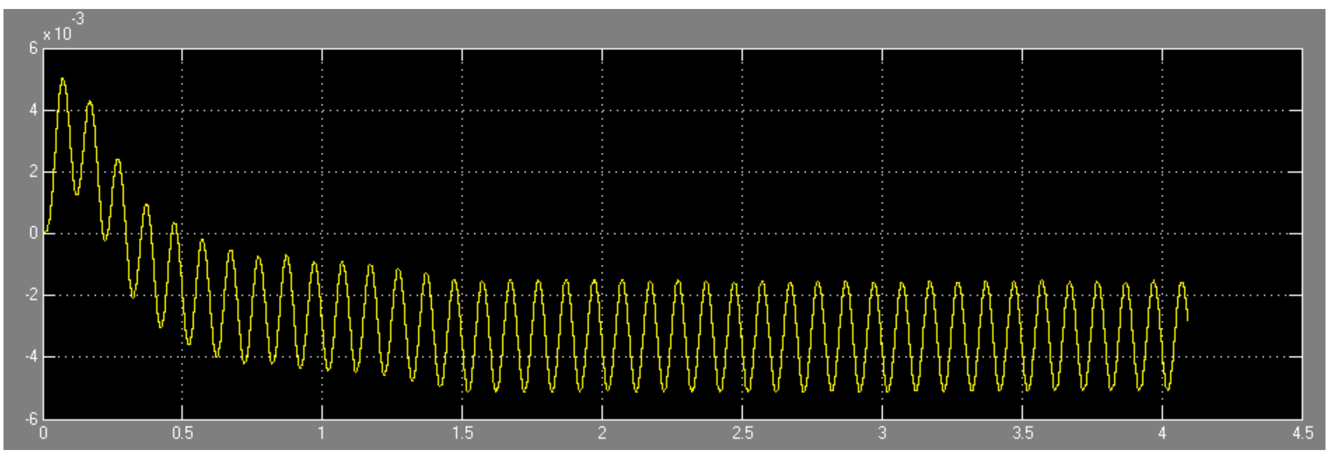

Figure 19: Practical values at 10Hz and 20 gain

For 5Hz, we can see the practical gain of $20log(\frac{0.03}{0.30} = -20$ and for 10Hz, we see a practical gain $20log(\frac{0.008}{0.30} = -31.5$, and we compare it to their theoretical gains of -2.1 and -7.32 respectively. We also compare the practical phase gains of -80.4 degrees and -120 degrees respectively for 5Hz and 10Hz

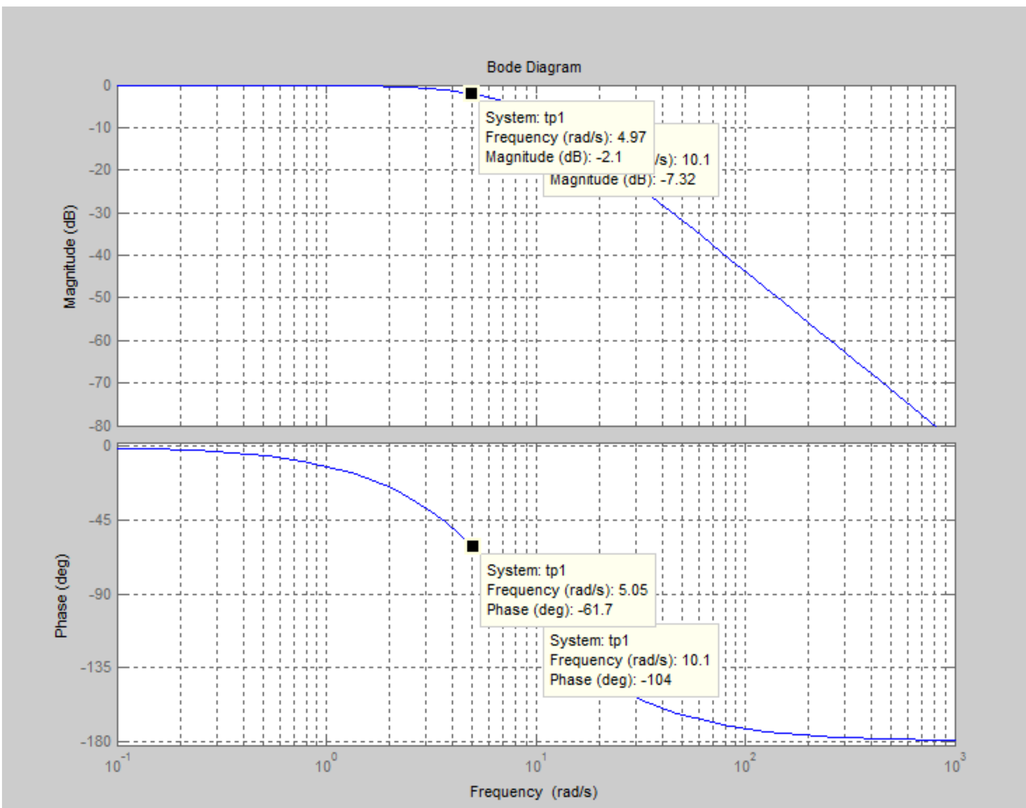

Figure 20: Theoretical values at 5-10Hz and 20 gain

We now take the practical values of 20Hz and 30Hz at 20 gain and compare it to their respective theoretical values in Figures 21, 22, and 23

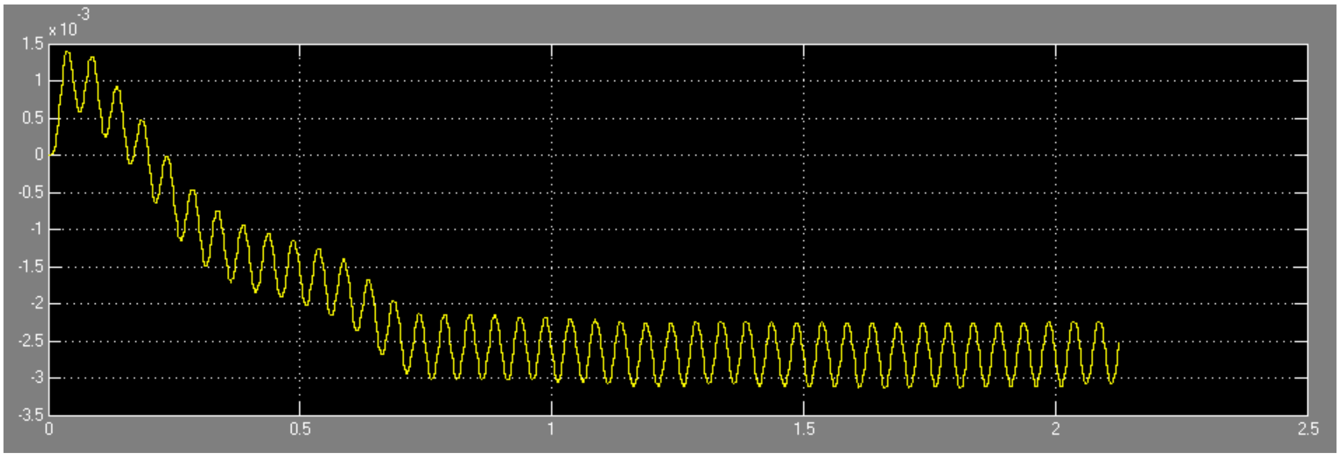

Figure 21: Practical values at 20Hz at 20 gain

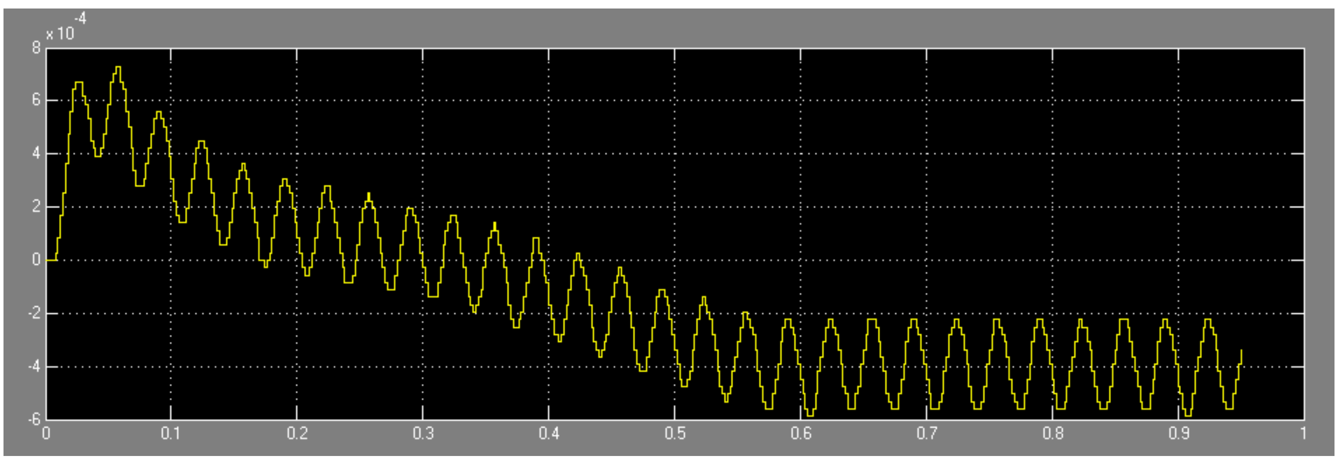

Figure 22: Practical values at 30Hz at 20 gain

We can see the practical gain values of -42.5dB for 20Hz, and -54.0dB for 30Hz for 20 gain and compare it to the theoretical values of -16.6dB and -23.3dB for 20 and 30Hz respectively. We also see the practical phase values of -150 degrees and -163 degrees compared to its theoretical values of -139 degrees and -152 degrees for 20 and 30Hz respectively

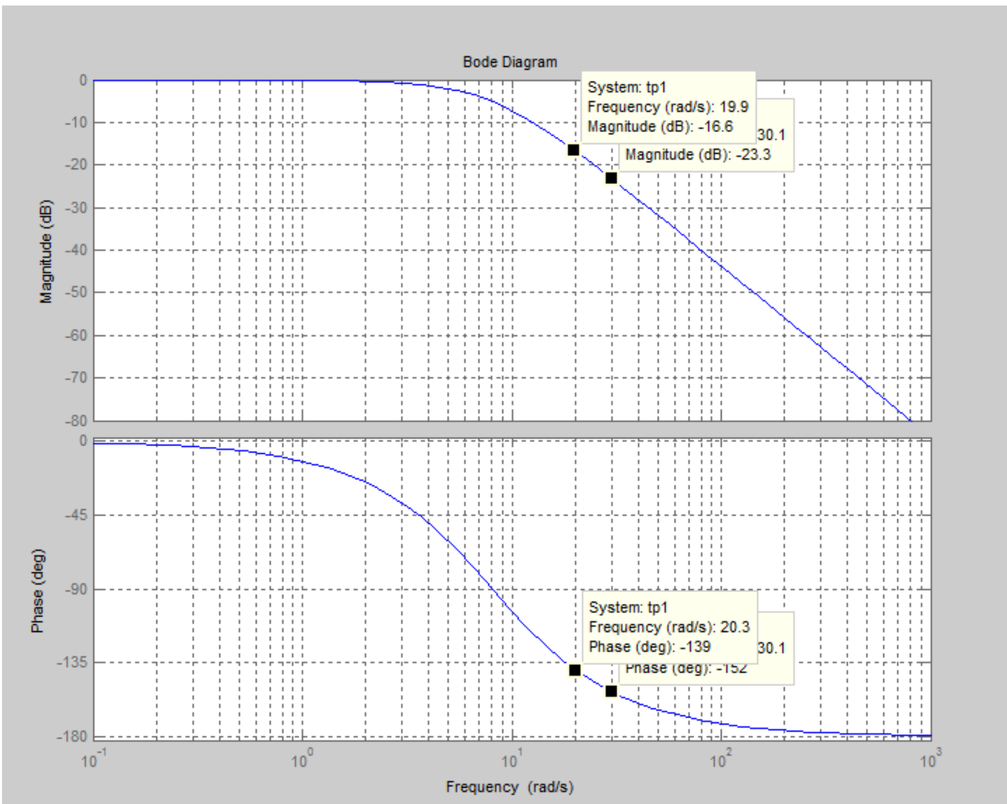

Figure 23: Theoretical values at 20-30Hz at 20 gain

We now take the practical values of 0.1Hz at 200 gain and compare it to their respective theoretical values in Figures 24 and 25

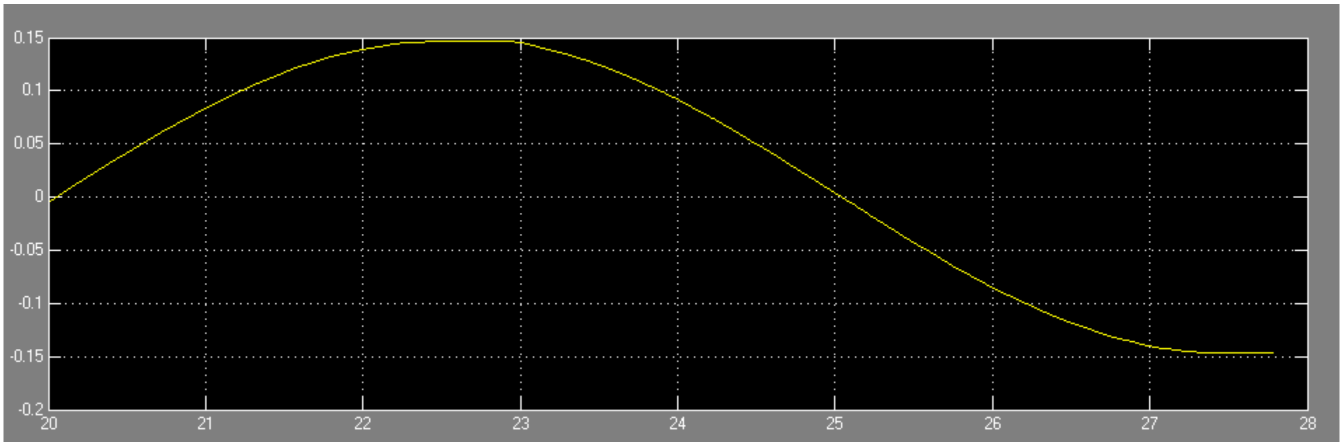

Figure 24: Practical values at 0.1Hz at 200 gain

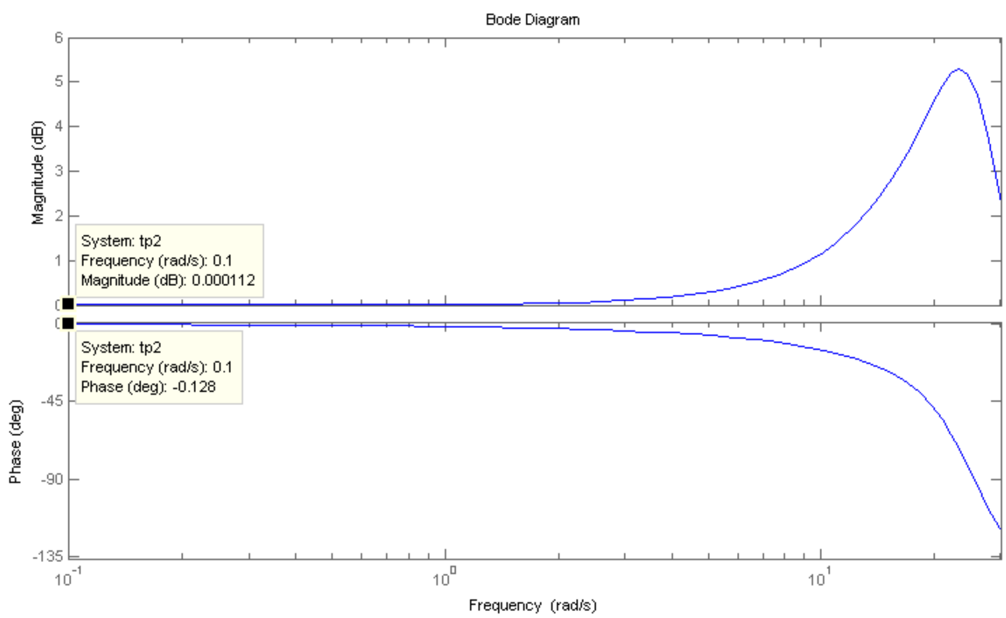

Figure 25: Practical values at 0.1Hz at 200 gain

We can now see that the practical gain values of 0dB for 0.01Hz, and compare it to its theoretical value of 0.000112Hz, we see the practical phase shift of -1.8 degrees versus its theoretical values of -1.7 degrees.

We now take the practical values of 0.5Hz-1Hz at 200 gain and compare it to their respective theoretical values in Figures 26, 27, and 28

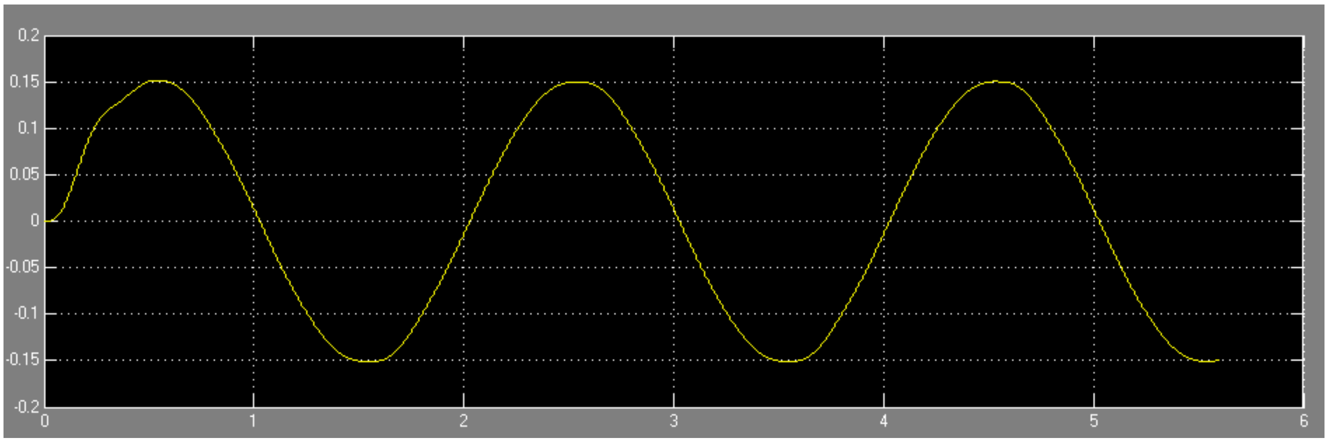

Figure 26: Practical values at 0.5Hz at 200 gain

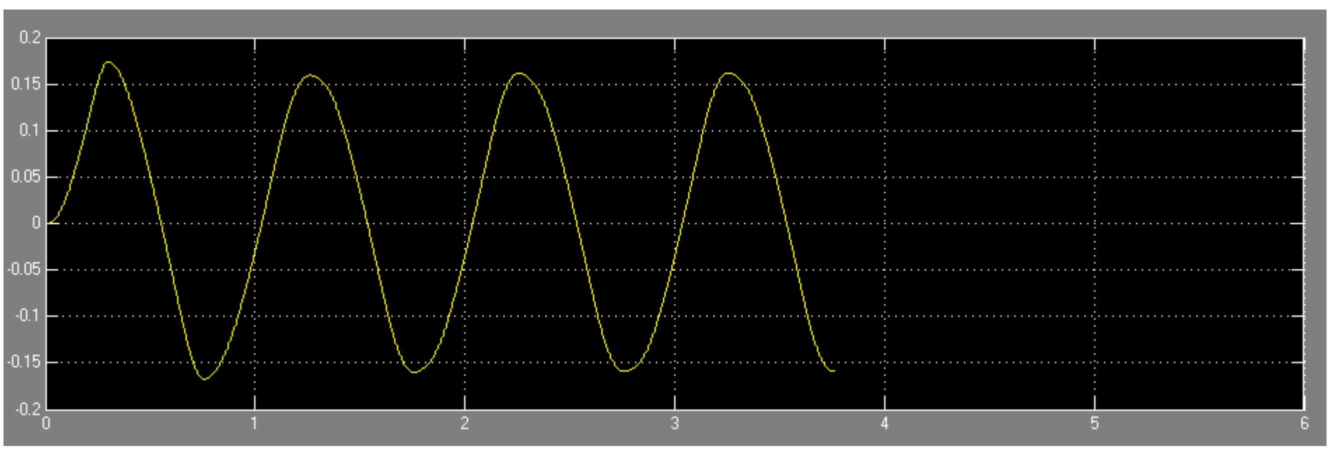

Figure 27: Practical values at 1Hz at 200 gain

We can now see that the practical values of 0dB and 0.52dB for 0.5Hz and 1Hz respectively, and compare it to their theoretical values of 0.000291dB and 0.0113dB respectively. The phase values are 0.9 degrees and -1.6 degrees for 0.5Hz and 1Hz respectively

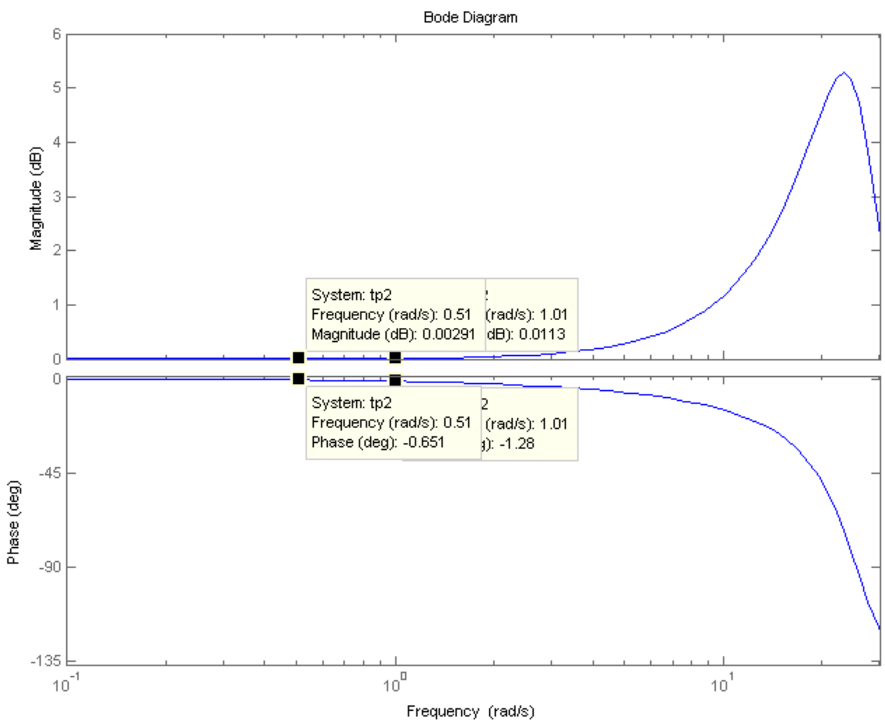

Figure 28: Theoretical values at 0.5-1Hz at 200 gain

We now take the practical values of 2-5Hz at 200 gain and compare it to their respective theoretical values in Figures 30, 30, and 31

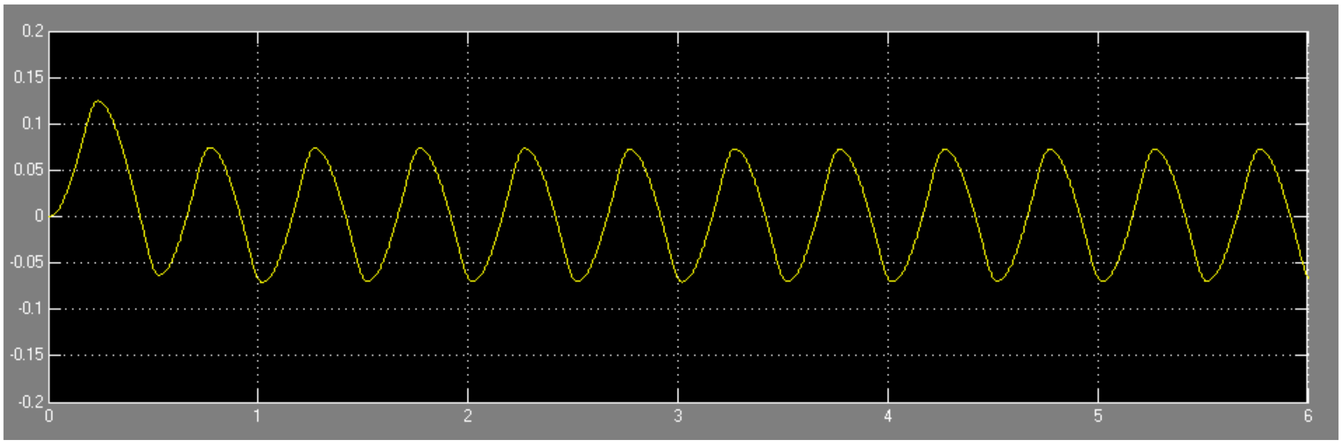

Figure 29: Practical values at 2Hz at 200 gain

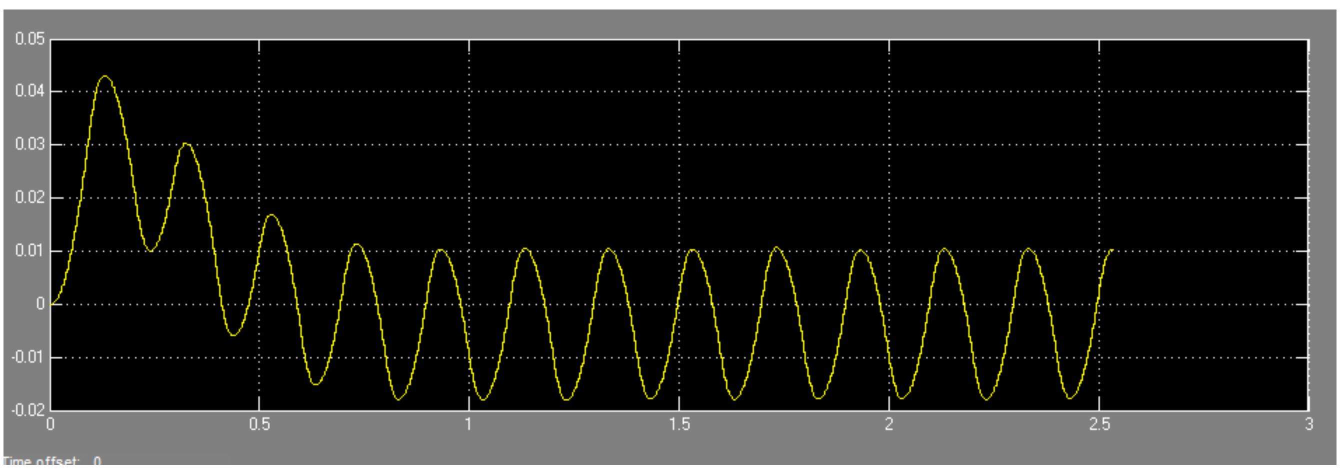

Figure 30: Practical values at 5Hz at 200 gain

We can now see that the practical values of 3.96dB and 12.64dB for 2Hz and 5Hz respectively and compare it to their theoretical values of 0.0463dB and 0.29dB respectively. We also take the practical phase lags of -4.2 degrees and -10.3 degrees respectively and compare it to their theoretical values of -2.63 degrees and -6.63 degrees respectively.

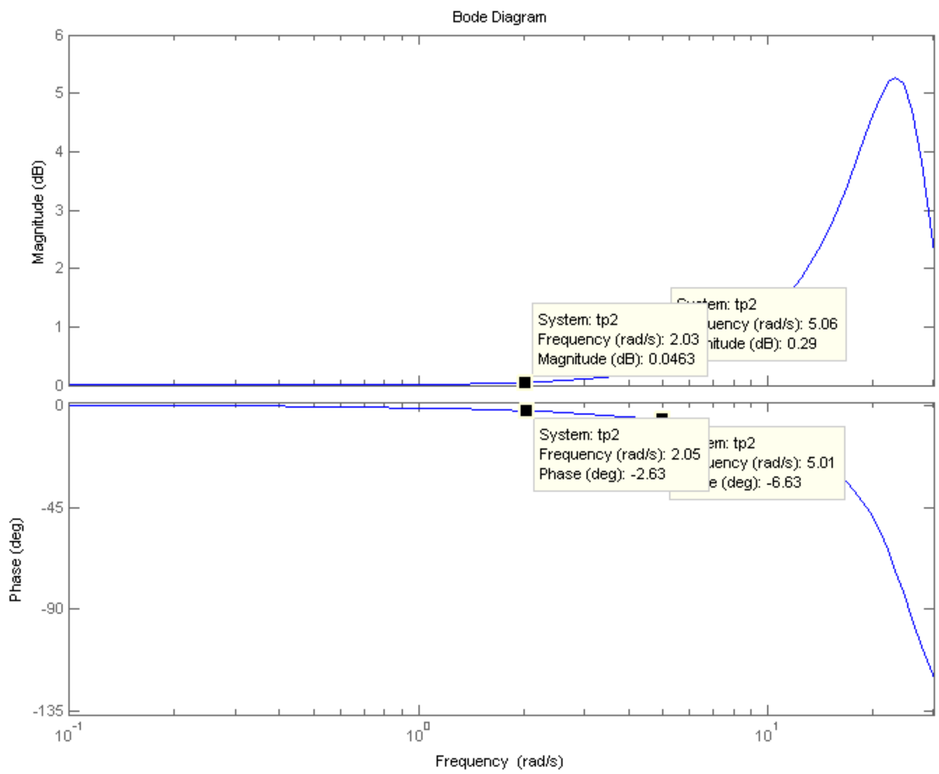

Figure 31: Theoretical values of 2-5Hz at 200 gain

We now take the practical values at 10 and 20Hz at 200 gain and compare it to their theoretical values in Figures 32, 33, and 34

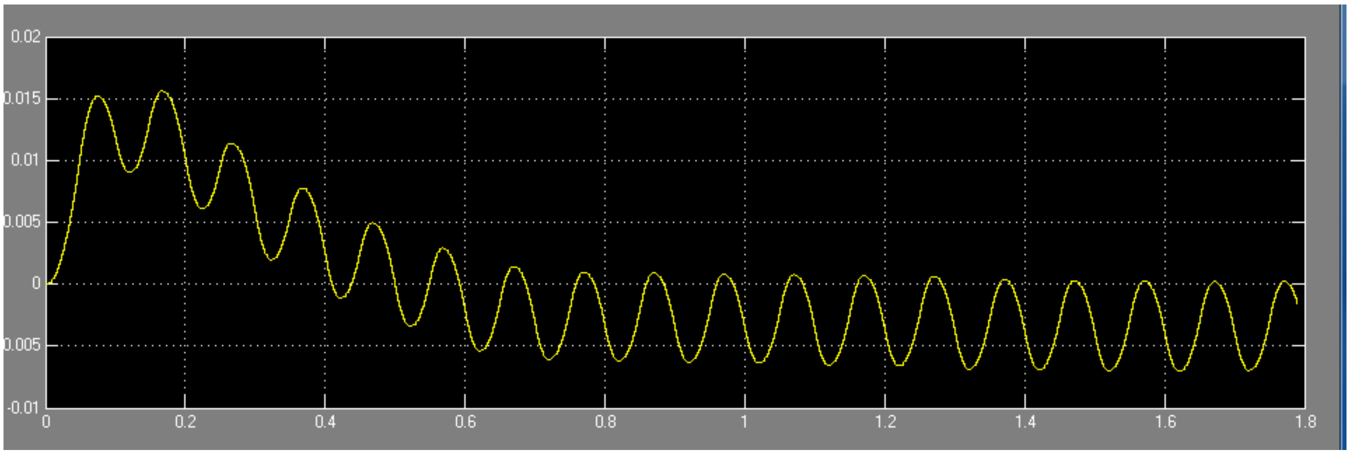

Figure 32: Practical values at 10Hz at 200 gain

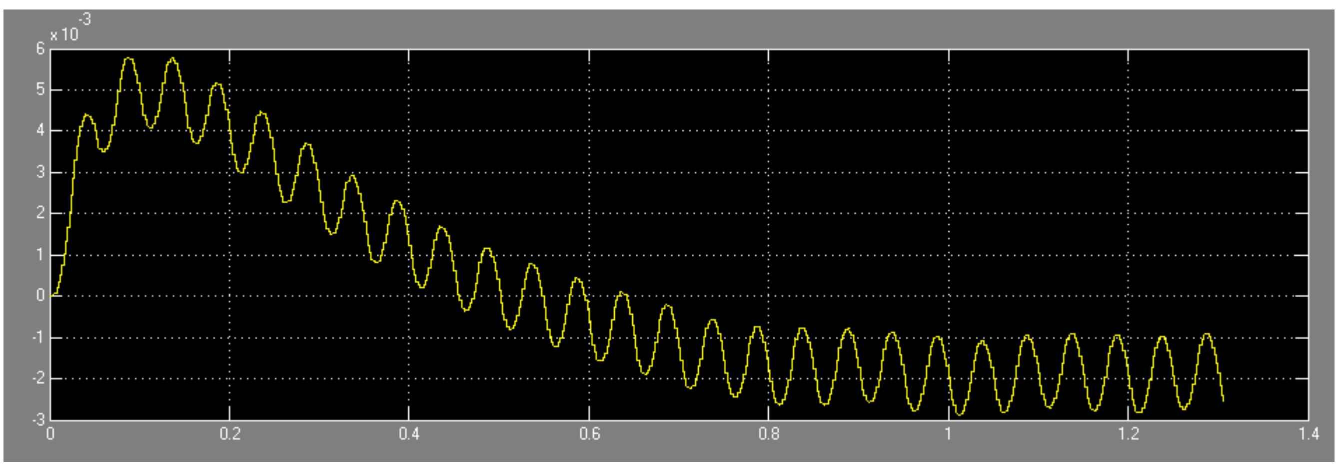

Figure 33: Practical values at 20Hz at 200 gain

We can now see that the practical gain of 34.0dB and 37.5dB respectively, and compare it to the theoretical values of of 1.15dB and 4.56dB respectively. We also compare the physical phase values of -25 degrees and -62.8 degrees to its theoretical values of -15 degrees and -47.6 degrees respectively.

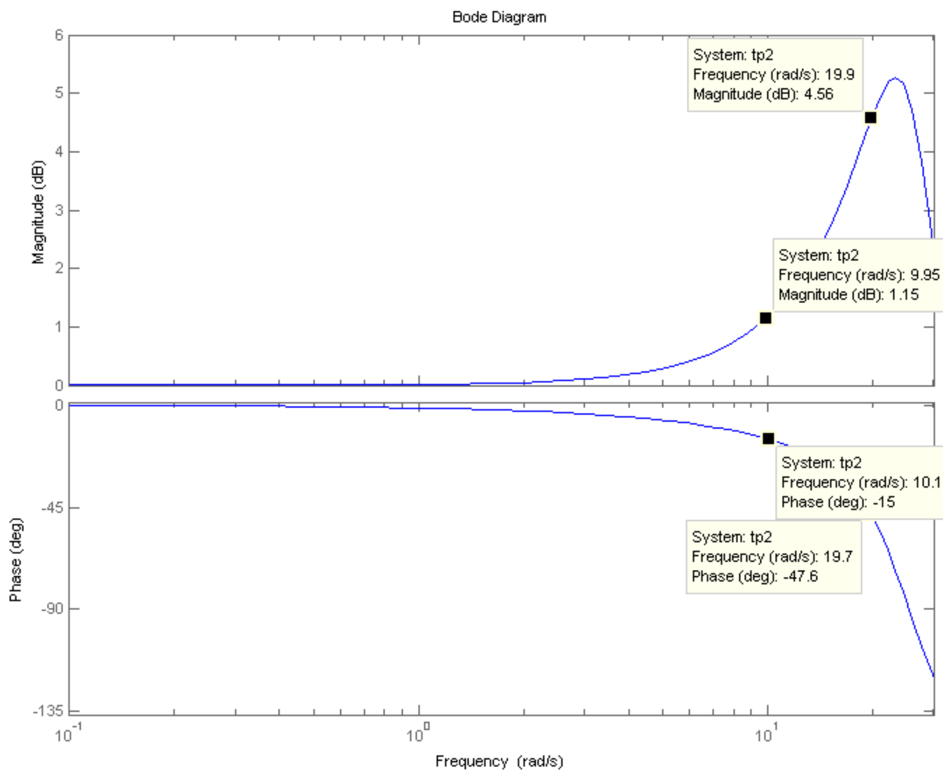

Figure 34: Theoretical values at 10 and 20Hz at 200 gain

We now take the practical values at 30Hz at 200 gain and compare it to their theoretical values in Figures

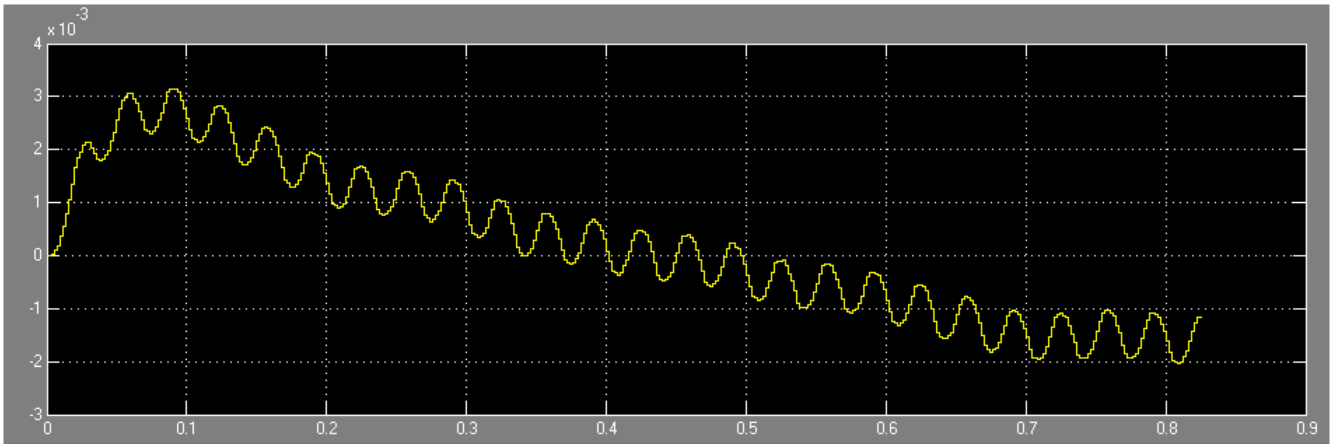

Figure 35: Practical values at 30Hz at 200 gain

We can now see that the practical gain value is 43.5 and compare it to its theoretical value of 2.32. We also compare its phase lag of -138 degrees to -119 degrees.

3.2 Part B

We can see that through the board, the gain is significantly lower in the practical case than it is in the theoretical case, and the phase lag is also slightly behind in the practical case when compared to the theoretical case. These two traits can both be attributed to friction, which would cost the agent some energy that is not accounted for in the theoretical case. This would both reduce the velocity (and since time is constant, the position - hence the gain) and its phase lag as it would have to overcome static friction in that timeframe as well.

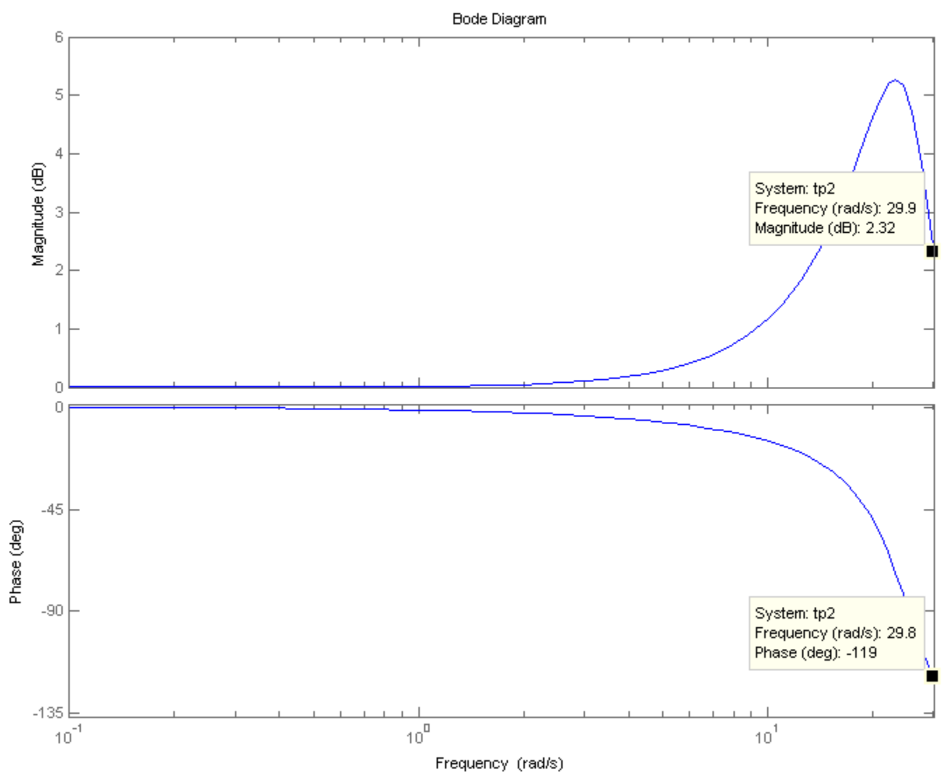

Figure 36: Theoretical values at 30Hz at 200 gain