ECSE 493 - Lab 1 Report

Isaac Chan

September 2018

1 Question 1

1.1 Part A

The equation of the DC-motor that is given in the description is described by

$$J_m\ddot{\theta} + (b + \frac{K_t K_m}{R_a})\dot{\theta} = \frac{K_t}{R_a}v_a \tag{1}$$

And the coefficients of each of the values are defined as

$$J_m = 0.01, b = 0.01, K_e = K_t = 0.02, R_a = 10 \tag{2}$$

Substituting this into the equations, we get

$$0.01\ddot{\theta} + 0.00104\dot{\theta} = 0.002v_a \tag{3}$$

Applying a laplace transform, we get the following

$$0.01s^2 + 0.00104s = 0.002 \tag{4}$$

Or equivalently, the transfer function would be

$$\frac{\dot{\theta}}{V} = \frac{0.002}{0.01s + 0.00104} \tag{5}$$

1.2 Part b

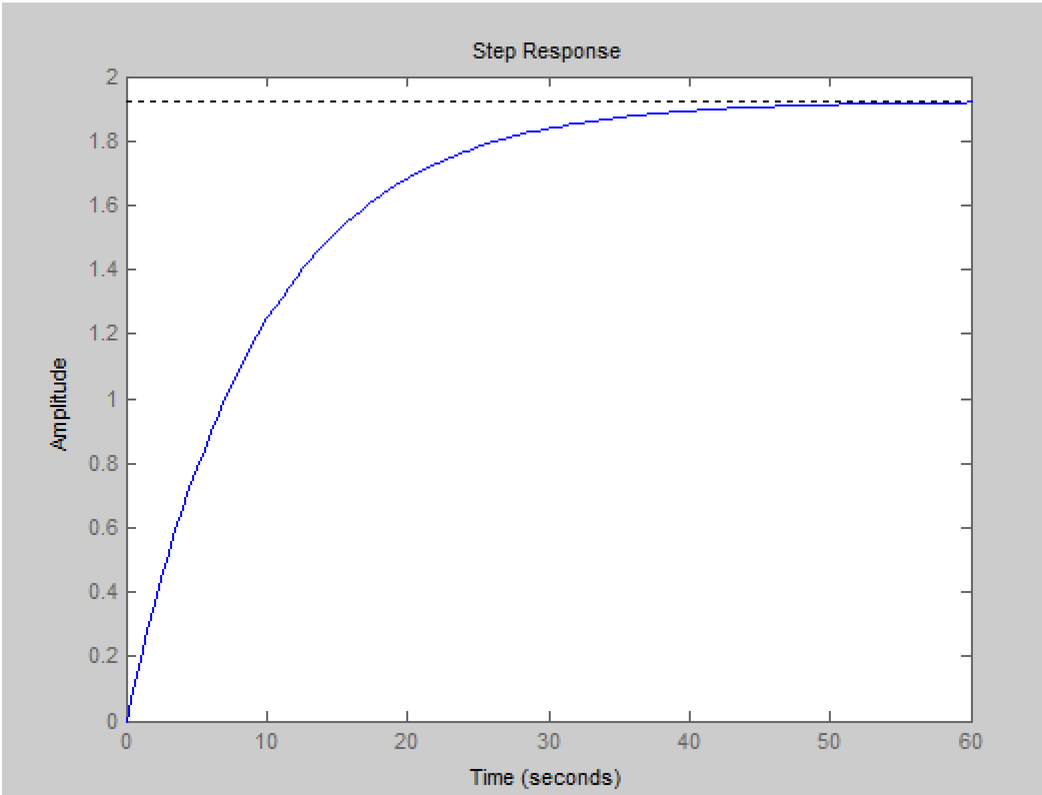

Figure 1: Plot of Steady State Amplitude

Steady state amplitude (i.e. speed of the motor) arrives at approximately 1.93rad/sec.

1.3 Part C

99% of the amplitude of 1.93rad/sec would be approximately 1.91. And we can see the graaph reaching approximately that point at approximately 45 seconds.

1.4 Part D

The final value theorem states

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \tag{6}$$

Applying the RHS to compute the steady state, what we get is

$$\lim_{s \to 0} s\frac{1}{s}\frac{0.002}{0.01s+0.00104} = \frac{0.002}{0.00104} = 1.923 \tag{7}$$

1.923 rad/s is the true steady state velocity, so our estimation of 1.93 rad/s is very close, and is off by 0.36%

1.5 Part E

The Transfer function is defined as follows

$$G(S) = \frac{\theta}{V} = \frac{1}{s}\frac{0.002}{0.01s+0.00104} \tag{8}$$

1.6 Part F

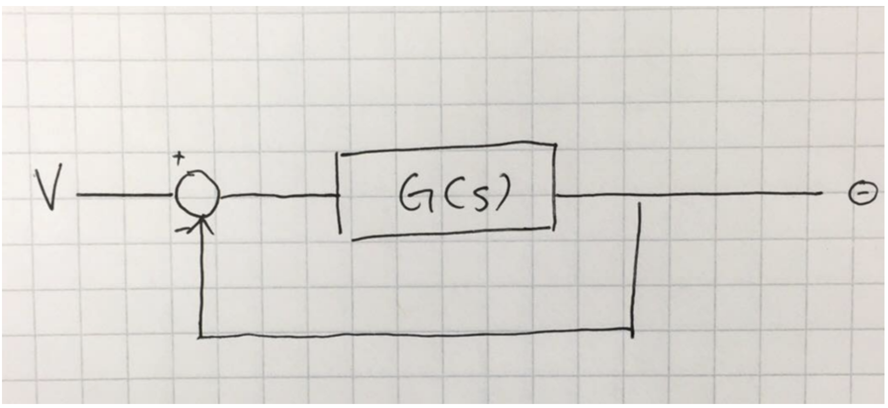

Figure 2: Model with feedback

and the corresponding transfer function would be

$$H(s) = \frac{G(s)}{1+G(s)} \tag{9}$$

Adding a gain of K to this feedback system, the model we would get is
And the corresponding transfer function would be

$$H(s) = \frac{kG(s)}{1+kG(s)} \tag{10}$$

1.7 Part G

K as a gain scaling unit, would have the units V/V, and would therefore be considered dimension-less

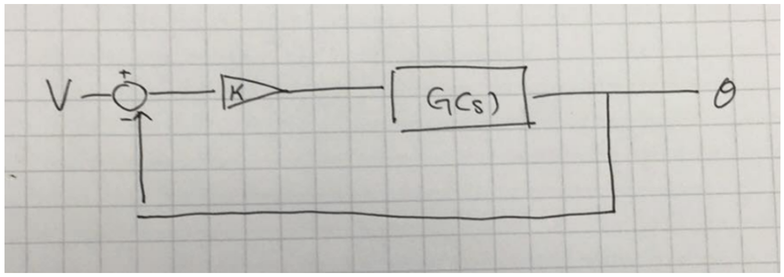

Figure 3: Model with feedback and gain

1.8 Part H

Taking expressions (drawn from the diagram), we get

$$\begin{aligned} e(s) &= V(s) - \theta(s) \\ \theta(s) &= ke(s) * G(s) \end{aligned} \tag{11}$$

We are looking for ke(s), where k is some arbitrary constant, so given 2 equations and 2 unknowns, we begin approaching this problem

$$\begin{aligned} \theta(s) &= V(s) - e(s) \\ V(s) - e(s) &= ke(s) * G(s) \\ V(s) &= ke(s) * G(s) + e(s) \\ V(s) &= e(s)[kG(s) + 1] \\ e(s) &= \frac{V(s)}{kG(s) + 1} \end{aligned} \tag{12}$$

and trivially, the end result is

$$ke(s) = \frac{kV(s)}{kG(s) + 1} \tag{13}$$

1.9 Part I

Taking the current transfer function

$$H(s) = \frac{kG(s)}{1 + kG(s)} \tag{14}$$

And substituting $G(s) = \frac{1}{s}\frac{0.002}{0.01s+0.00104}$, we do the following computation

$$
\begin{aligned}
H(s) &= \frac{k(\frac{1}{s}\frac{0.002}{0.01s+0.00104})}{1+k(\frac{1}{s}\frac{0.002}{0.01s+0.00104})} \\
&= \frac{\frac{k0.002}{0.01s^2+0.00104s}}{\frac{k*0.002+0.01s^2+0.00104s}{0.01s^2+0.00104s}} \\
&= \frac{k*0.002(0.01s^2+0.00104s)}{(0.01s^2+0.0014s)(k*0.002+0.01s^2+0.00104s)} \\
&= \frac{k*0.002}{k*0.002+0.01s^2+0.00104s}
\end{aligned} \tag{15}
$$

1.10 Part J

We show plots for values 0.1, 0.01, 0.05, and 0.063 in figures 4-7 respectively

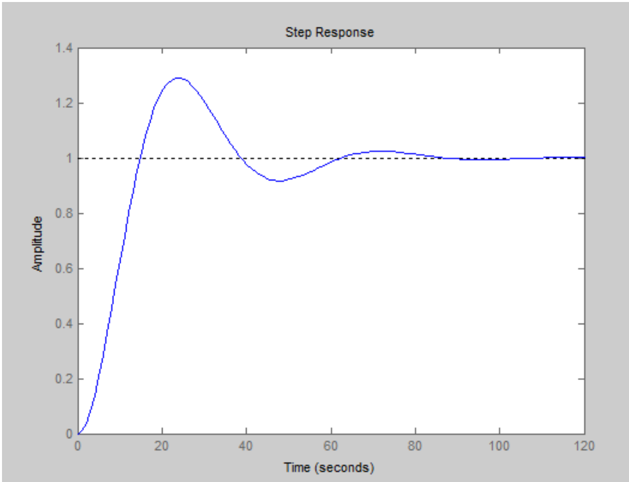

Figure 4: k = 0.1

And here we can see that when k=0.063, we can see that MP¡20%, where the amplitude maximizes at 1.19.

1.11 Part K

We can see from this graph that the value of 0.78 would get a rise time of 0.4s in figure 8

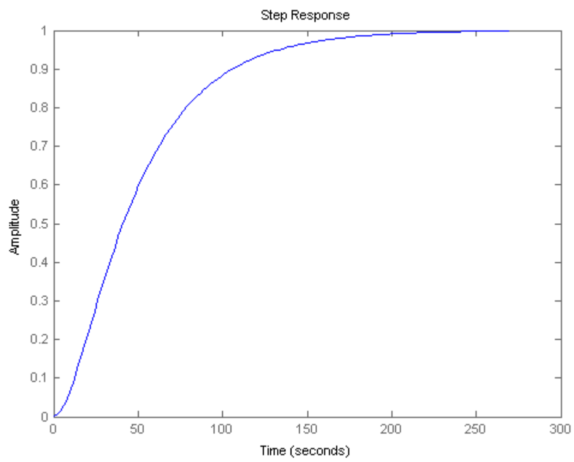

Figure 5: k = 0.01

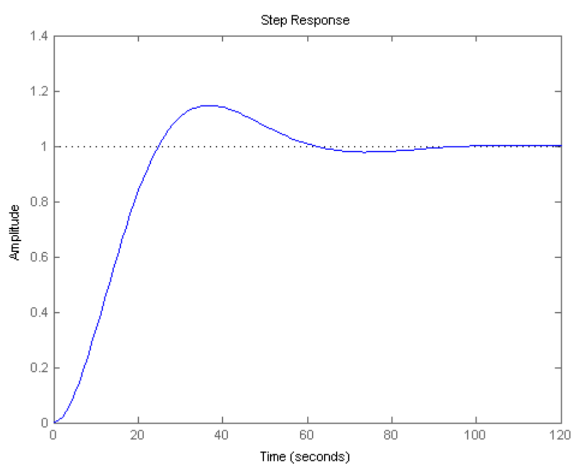

Figure 6: k = 0.05

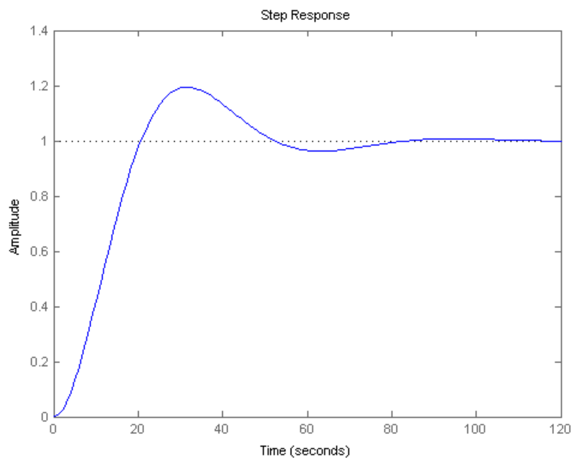

Figure 7: k=0.065

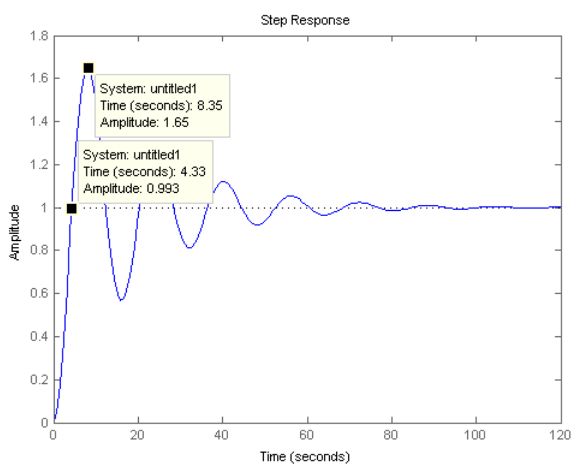

Figure 8: k=0.78

1.12 Part L

For k values for Figures 9-11

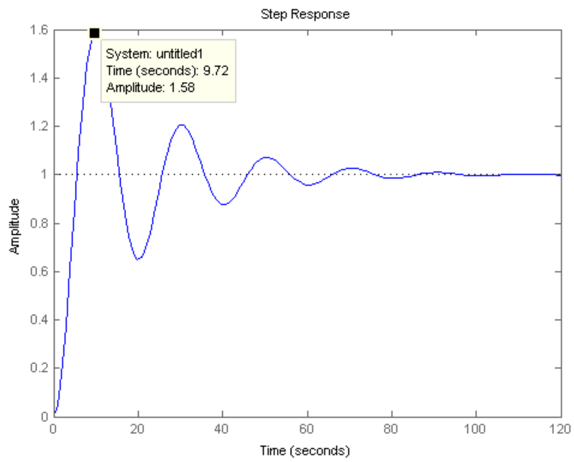

Figure 9: k=0.5

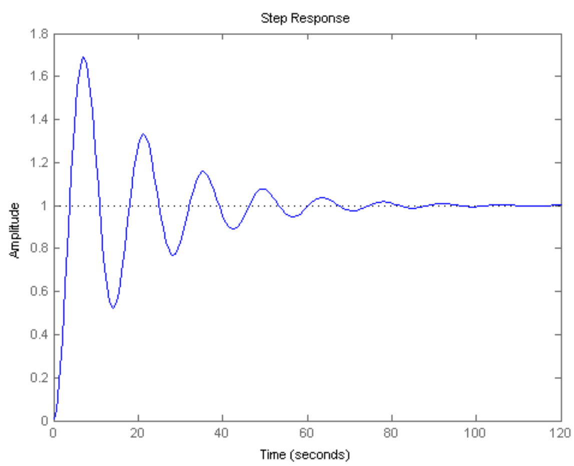

Figure 10: k=1.0

With a value of 0.5 for k has a rise-time 4.7 seconds with an overshoot of 60%. When we use a value of 1 for k, this has a rise time of 3.2 seconds and has an overshoot of 69%. When we use a value of 2, this has a rise of 2.5 seconds and an overshoot of 76%

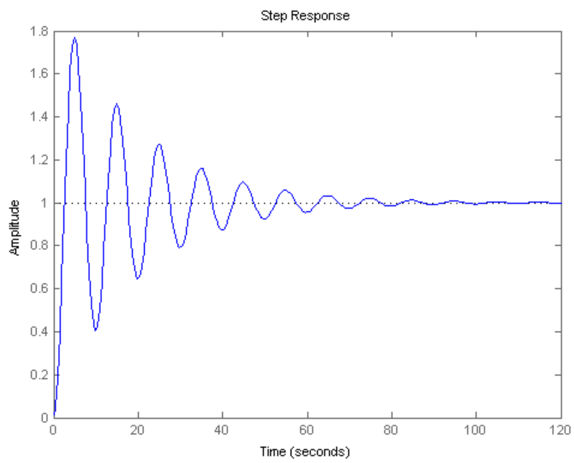

Figure 11: k=2.0

1.13 Part M

We find k to be a parameter that is directly proportional to overshooting and inversely proportional to rise time. This is due to the initial momentum it provides the function. So we see this to be really in line with the results from i, j, and k as they all reflect this conclusion.

1.14 Part N

The parameter k influences both the rise time and the overshooting. We can see that the rise time is inversely proportional to k, whereas the overshoot is directly proportional to the value of k. So higher k, when it overshoots, means that it's carrying more momentum, and therefore needs a longer stabilizing time before it reaches steady-state.

2 Question 2

2.1 Part A

This represents a transfer function where p is the numerator and q is the denominator such that

$$H(s) = \frac{s+1}{s^3 + 5s^2 + 6s} \tag{16}$$

2.2 Part B

We can see from figure 12 that this function has 3 poles and 1 zero. The three poles are denoted by the three colored branches and the pole is clearly denoted on -1 as a circle.

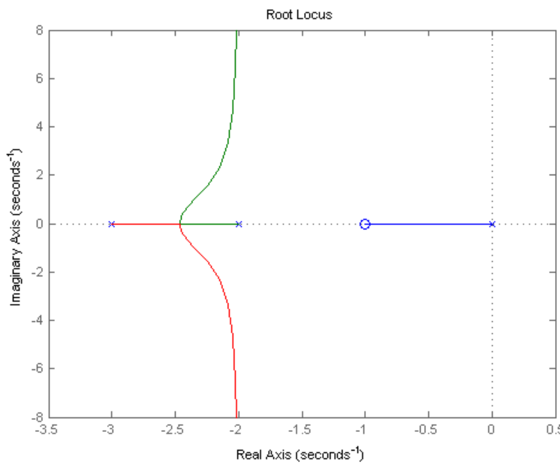

Figure 12: Root Locus Diagram

3 Question 3

Frequency response transfer function would be:

$$H(j\omega) = \frac{j\omega + 1}{(j\omega)^3 + 5(j\omega)^2 + 6j\omega} \tag{17}$$

3.1 Part A

We take the bode plot of this function in figure 13

First thing we can identify is that we have infinite gain stability as the phase never goes past 180, and we have approximately 90 degrees of phase stability. So we have absolute stability, which means that the values of k don't affect the stability of the transfer function. We derive the closed loop transfer function here

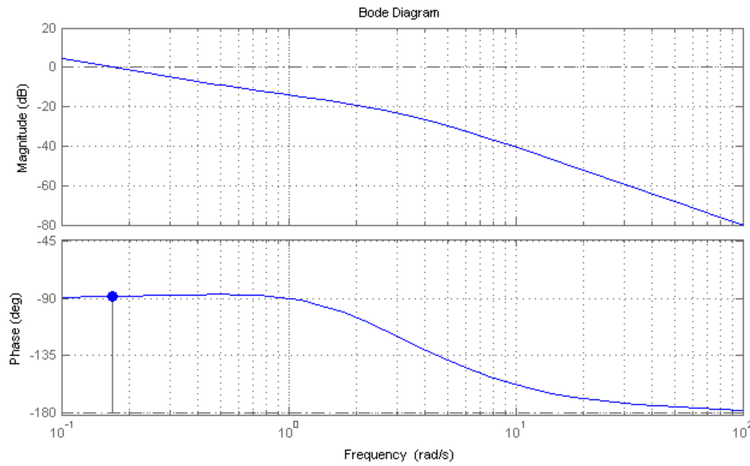

Figure 13: Bode Plot

$$H(s) = \frac{kG(s)}{1+kG(s)}$$

$$H(s) = \frac{\frac{k(s+1)}{s^3+5s^2+6s}}{1+\frac{k(s+1)}{s^3+5s^2+6s}} \tag{18}$$

$$H(s) = \frac{(k+1)s^4+(5k+5)s^3+12ks^2}{s^6+10s^5+(37+k)s^4+(61+5k)s^3+(41+6k)s^2+6s}$$

We compare the rising times of 2 drastically varying values of k (1, and 150) in figures 14 and 15

And we can see that k values are inversely proportional to rising time.

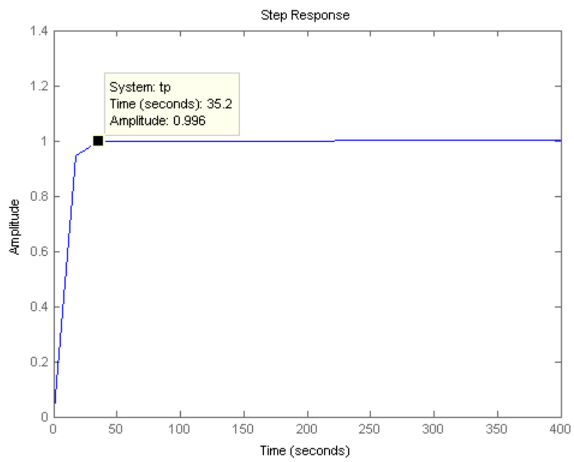

Figure 14: k=1

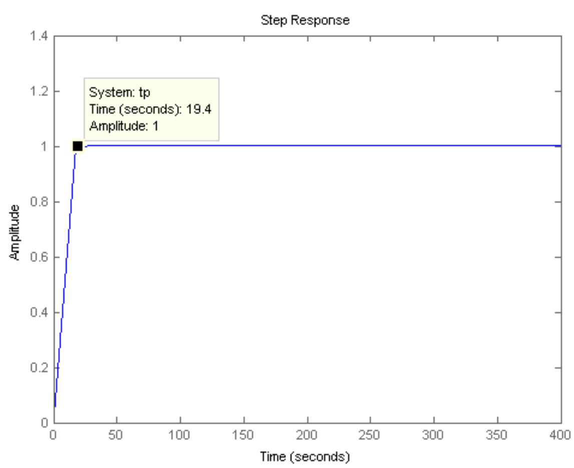

Figure 15: k=50

3.2 Part B

We take the same transfer function before, and apply basic lead and lag (and a combination of both) compensators and observe how they affect stability and gain of the function. We use the following lead compensator:

$$\frac{0.088s + 1}{0.022s + 1} \tag{19}$$

and the following lag compensator

$$\frac{10s + 1}{100s + 1} \tag{20}$$

And we will be using unit gain. We show the following figures from Figure 16-21 that correspond to bode plots and step plots of the original transfer function, transfer function with above lead, and transfer function with above lag respectively.

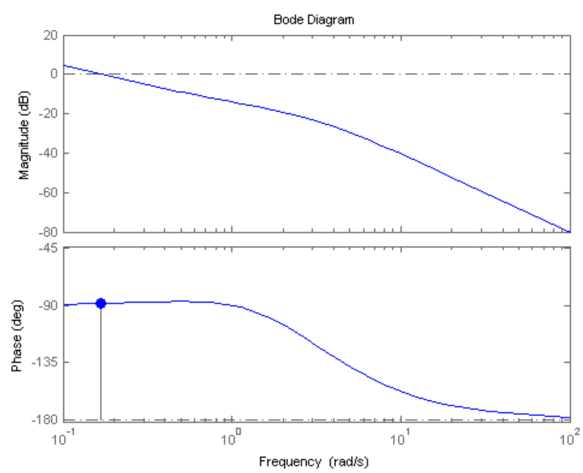

Figure 16: Transfer function bode plot without any lead or lag compensation

We can see with a lead compensator, since the zero is closer to the origin, we effectively increase the gain of the function at high frequencies. Conversely, when we use a lag compensator, since the pole is closer to the zero than the pole, then we can effectively decrease the gain of the function at high frequencies. Both of these exmaples have been somewhat destabilized from its previously very stable form. However, if we combine the lead and lag as we'll show in figure 22 and 23, we can see the system completely destabilizes.

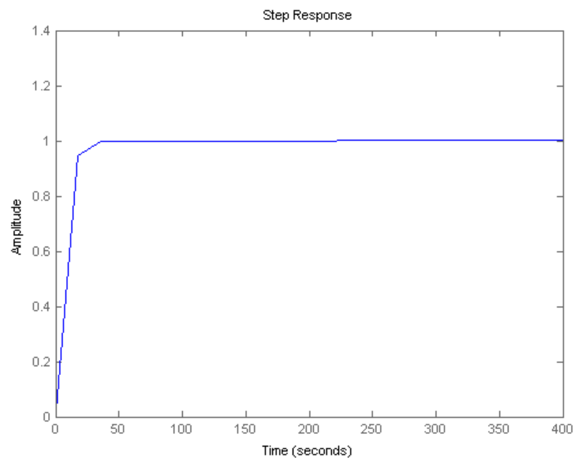

Figure 17: Transfer function step plot without any lead or lag compensation

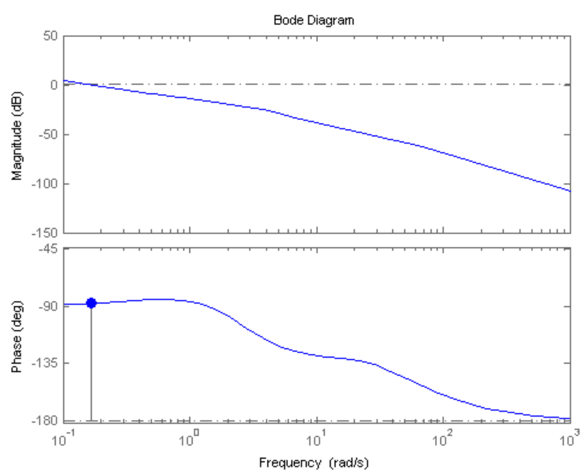

Figure 18: Transfer function bode plot with lead compensation

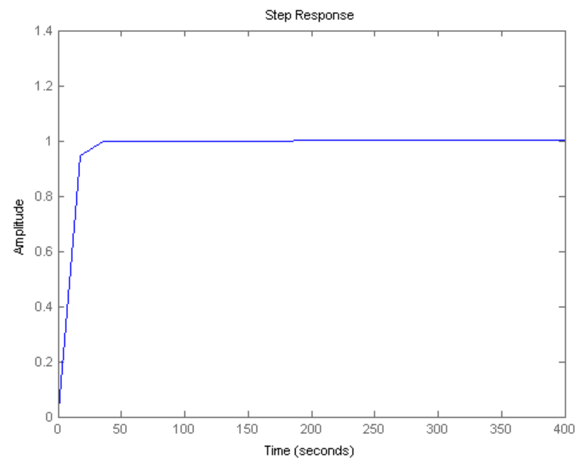

Figure 19: Transfer function step plot with lead compensation

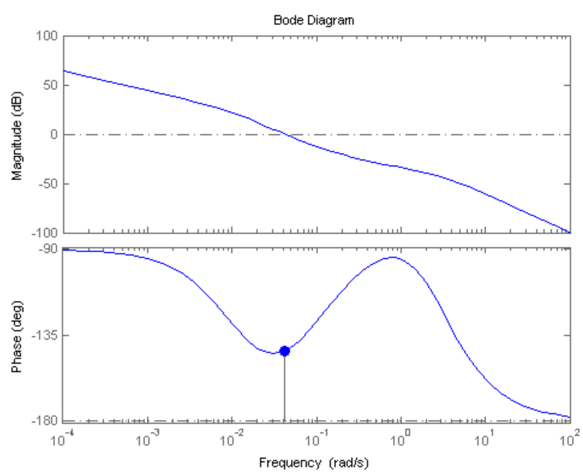

Figure 20: Transfer function bode plot with lag compensation

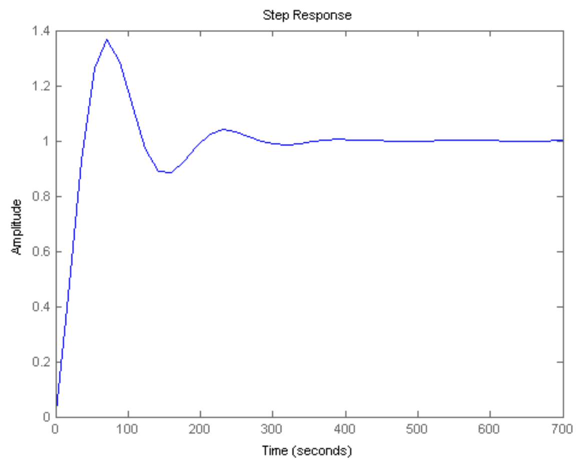

Figure 21: Transfer function step plot with lag compensation

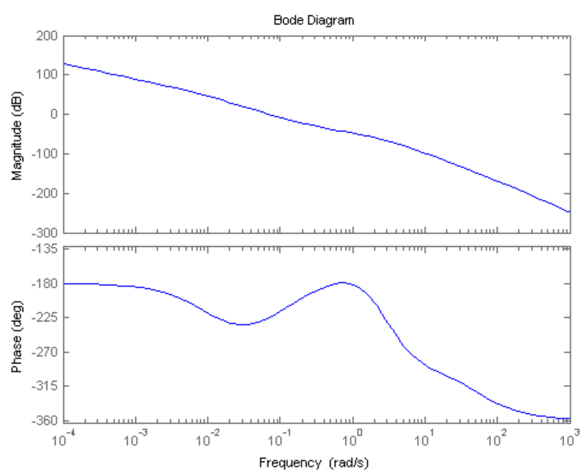

Figure 22: Transfer function bode plot with both lead and lag compensation

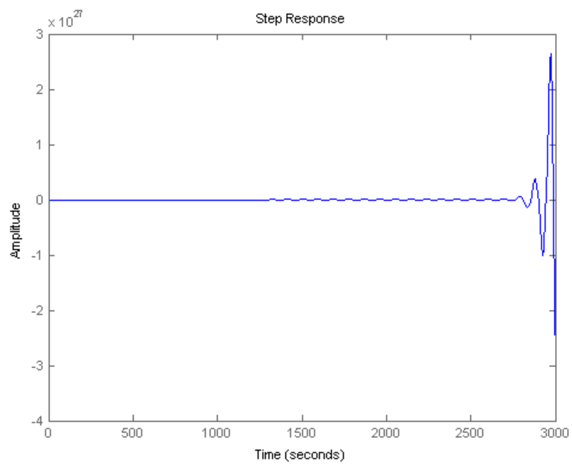

Figure 23: Transfer function step plot with lead and lag compensation