ECSE 493 - Lab 1 Report

Isaac Chan

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1 Question 1

1.1 Part A

The equation of the DC-motor that is given in the description is described by

$$J_m\ddot{\theta} + (b + \frac{K_t K_m}{R_a})\dot{\theta} = \frac{K_t}{R_a}v_a \tag{1}$$

And the coefficients of each of the values are defined as

$$J_m = 0.01, b = 0.01, K_e = K_t = 0.02, R_a = 10 \tag{2}$$

Substituting this into the equations, we get

$$0.01\ddot{\theta} + 0.00104\dot{\theta} = 0.002v_a \tag{3}$$

Applying a laplace transform, we get the following

$$0.01s^2 + 0.00104s = 0.002 \tag{4}$$

Or equivalently, the transfer function would be

$$\frac{\dot{\theta}}{V} = \frac{0.002}{0.01s + 0.00104} \tag{5}$$

1.2 Part b

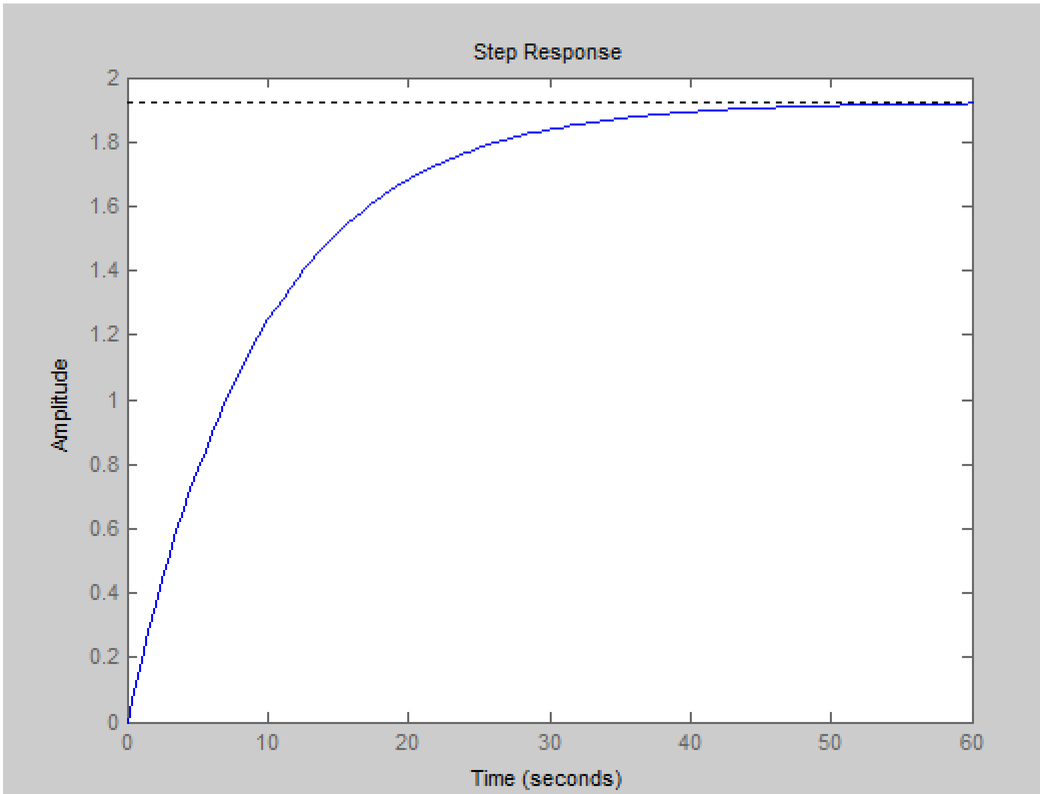

Figure 1: Plot of Steady State Amplitude

Steady state amplitude (i.e. speed of the motor) arrives at approximately 1.93rad/sec.

1.3 Part C

99% of the amplitude of 1.93rad/sec would be approximately 1.91. And we can see the graaph reaching approximately that point at approximately 45 seconds.

1.4 Part D

The final value theorem states

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \tag{6}$$

Applying the RHS to compute the steady state, what we get is

$$\lim_{s\to 0} s\frac{1}{s}\frac{0.002}{0.01s+0.00104} = \frac{0.002}{0.00104} = 1.923 \tag{7}$$

1.923 rad/s is the true steady state velocity, so our estimation of 1.93 rad/s is very close, and is off by 0.36%

1.5 Part E

The Transfer function is defined as follows

$$G(S) = \frac{\theta}{V} = \frac{1}{s}\frac{0.002}{0.01s+0.00104} \tag{8}$$

1.6 Part F

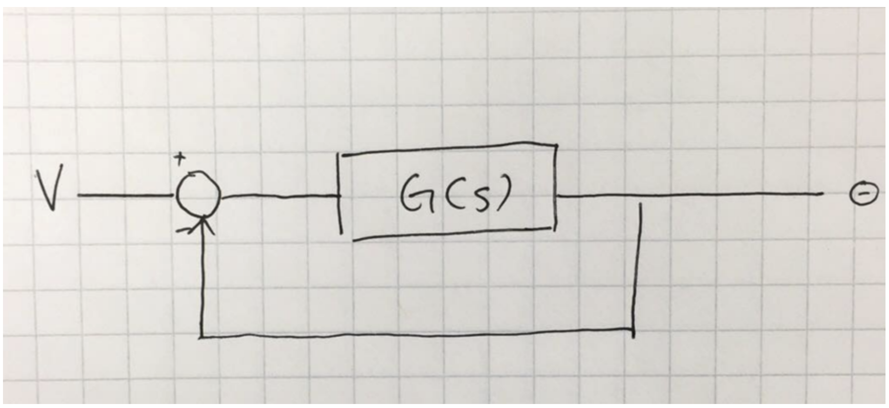

Figure 2: Model with feedback

and the corresponding transfer function would be

$$H(s) = \frac{G(s)}{1+G(s)} \tag{9}$$

Adding a gain of K to this feedback system, the model we would get is
And the corresponding transfer function would be

$$H(s) = \frac{kG(s)}{1+kG(s)} \tag{10}$$

1.7 Part G

K as a gain scaling unit, would have the units V/V, and would therefore be considered dimension-less

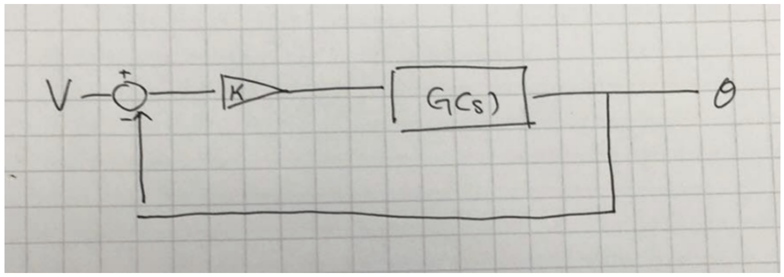

Figure 3: Model with feedback and gain

1.8 Part H

Taking expressions (drawn from the diagram), we get

$$
\begin{aligned}
e(s) &= V(s) - \theta(s) \\
\theta(s) &= ke(s) * G(s)
\end{aligned}
\tag{11}
$$

We are looking for ke(s), where k is some arbitrary constant, so given 2 equations and 2 unknowns, we begin approaching this problem

$$
\begin{aligned}
\theta(s) &= V(s) - e(s) \\
V(s) - e(s) &= ke(s) * G(s) \\
V(s) &= ke(s) * G(s) + e(s) \\
V(s) &= e(s)[kG(s) + 1] \\
e(s) &= \frac{V(s)}{kG(s) + 1}
\end{aligned}
\tag{12}
$$

and trivially, the end result is

$$ke(s) = \frac{kV(s)}{kG(s) + 1} \tag{13}$$

1.9 Part I

Taking the current transfer function

$$H(s) = \frac{kG(s)}{1 + kG(s)} \tag{14}$$

And substituting $G(s) = \frac{1}{s}\frac{0.002}{0.01s+0.00104}$, we do the following computation

$$
\begin{aligned}
H(s) &= \frac{k(\frac{1}{s}\frac{0.002}{0.01s+0.00104})}{1+k(\frac{1}{s}\frac{0.002}{0.01s+0.00104})} \\
&= \frac{\frac{k0.002}{0.01s^2+0.00104s}}{\frac{k*0.002+0.01s^2+0.00104s}{0.01s^2+0.00104s}} \\
&= \frac{k*0.002(0.01s^2+0.00104s)}{(0.01s^2+0.0014s)(k*0.002+0.01s^2+0.00104s)} \\
&= \frac{k*0.002}{k*0.002+0.01s^2+0.00104s}
\end{aligned} \tag{15}
$$

1.10 Part J

We show plots for values 0.1, 0.01, 0.05, and 0.063 in figures 4-7 respectively

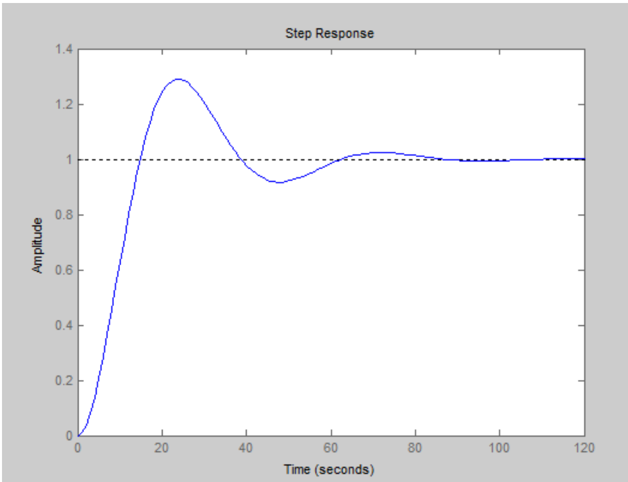

Figure 4: k = 0.1

And here we can see that when k=0.063, we can see that MP¡20%, where the amplitude maximizes at 1.19.

1.11 Part K

We can see from this graph that the value of 0.78 would get a rise time of 0.4s in figure 8

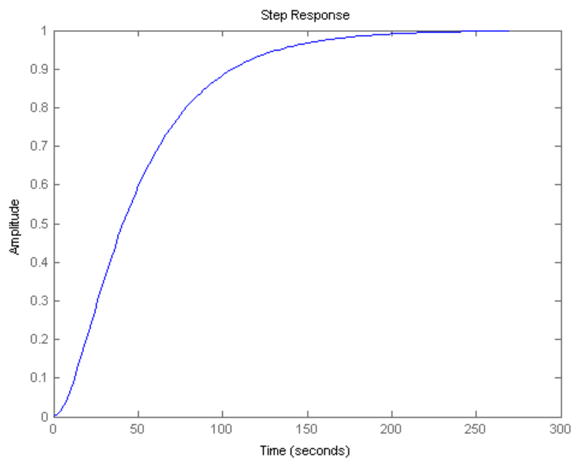

Figure 5: k = 0.01

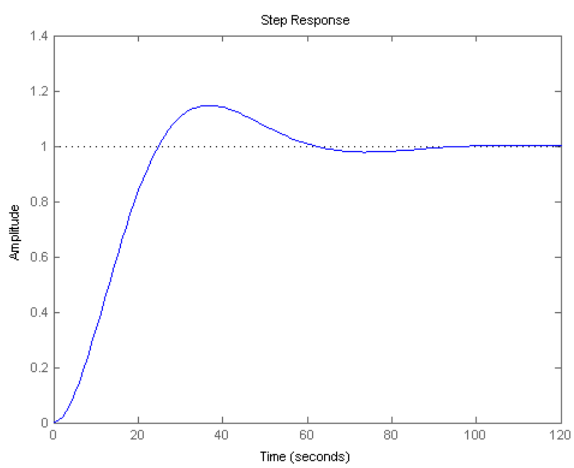

Figure 6: k = 0.05

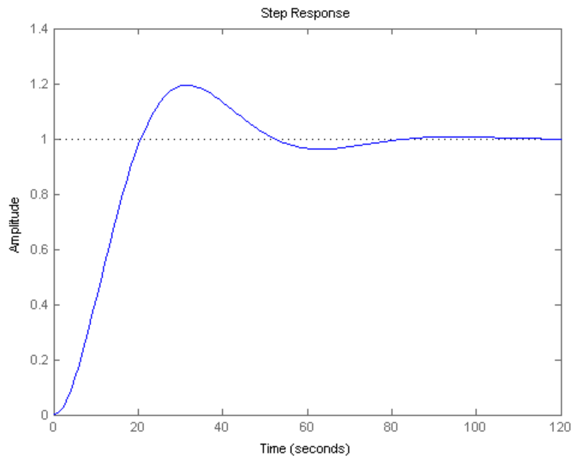

Figure 7: k=0.065

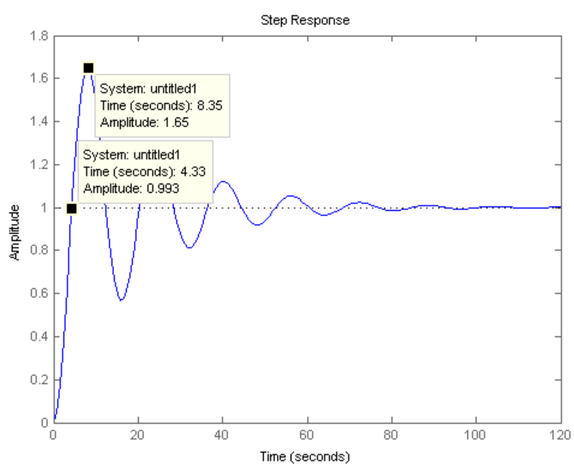

Figure 8: k=0.78

1.12 Part L

For k values for Figures 9-11

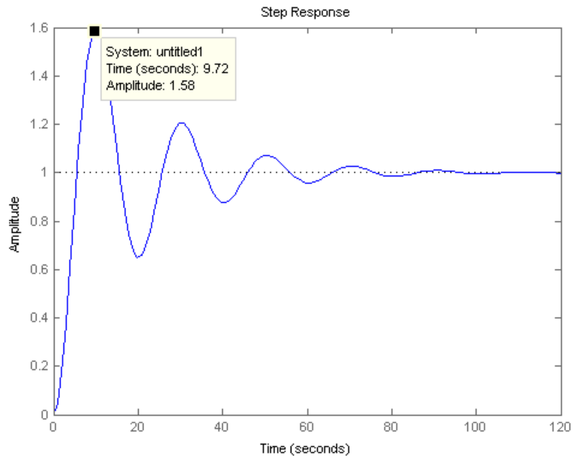

Figure 9: k=0.5

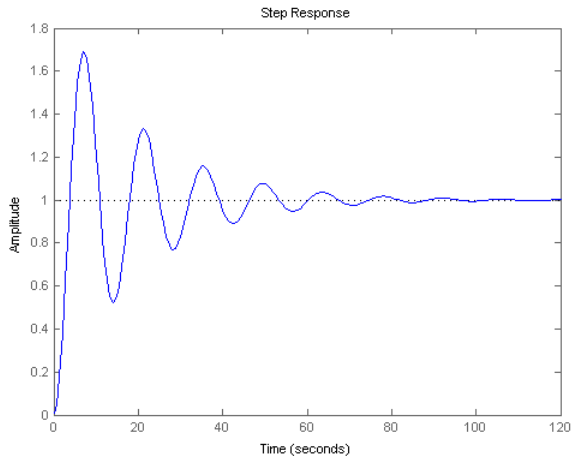

Figure 10: k=1.0

With a value of 0.5 for k has a rise-time 4.7 seconds with an overshoot of 60%. When we use a value of 1 for k, this has a rise time of 3.2 seconds and has an overshoot of 69%. When we use a value of 2, this has a rise of 2.5 seconds and an overshoot of 76%

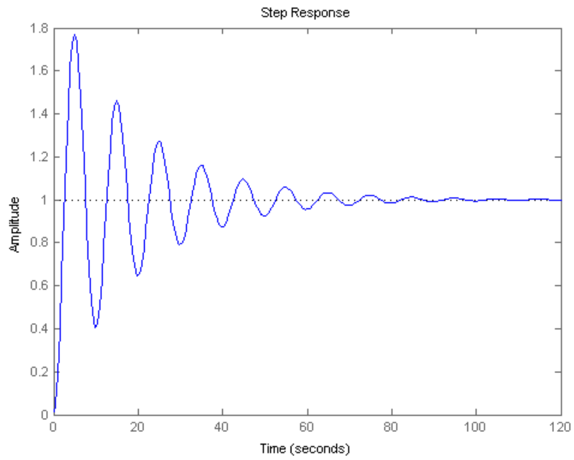

Figure 11: k=2.0

1.13 Part M

1.14 Part N

The parameter k influences both the rise time and the overshooting. We can see that the rise time is inversely proportional to k, whereas the overshoot is directly proportional to the value of k. So higher k, when it overshoots, means that it's carrying more momentum, and therefore needs a longer stabilizing time before it reaches steady-state.