ECSE 493 - Lab 1 Report

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1 Question 1

1.1 Part A

The equation of the DC-motor that is given in the description is described by

$$J_m\ddot{\theta} + (b + \frac{K_t K_m}{R_a})\dot{\theta} = \frac{K_t}{R_a}v_a \tag{1}$$

And the coefficients of each of the values are defined as

$$J_m = 0.01, b = 0.01, K_e = K_t = 0.02, R_a = 10 \tag{2}$$

Substituting this into the equations, we get

$$0.01\ddot{\theta} + 0.00104\dot{\theta} = 0.002v_a \tag{3}$$

Applying a laplace transform, we get the following

$$0.01s^2 + 0.00104s = 0.002 \tag{4}$$

Or equivalently, the transfer function would be

$$\frac{\dot{\theta}}{V} = \frac{0.002}{0.01s + 0.00104} \tag{5}$$

1.2 Part b

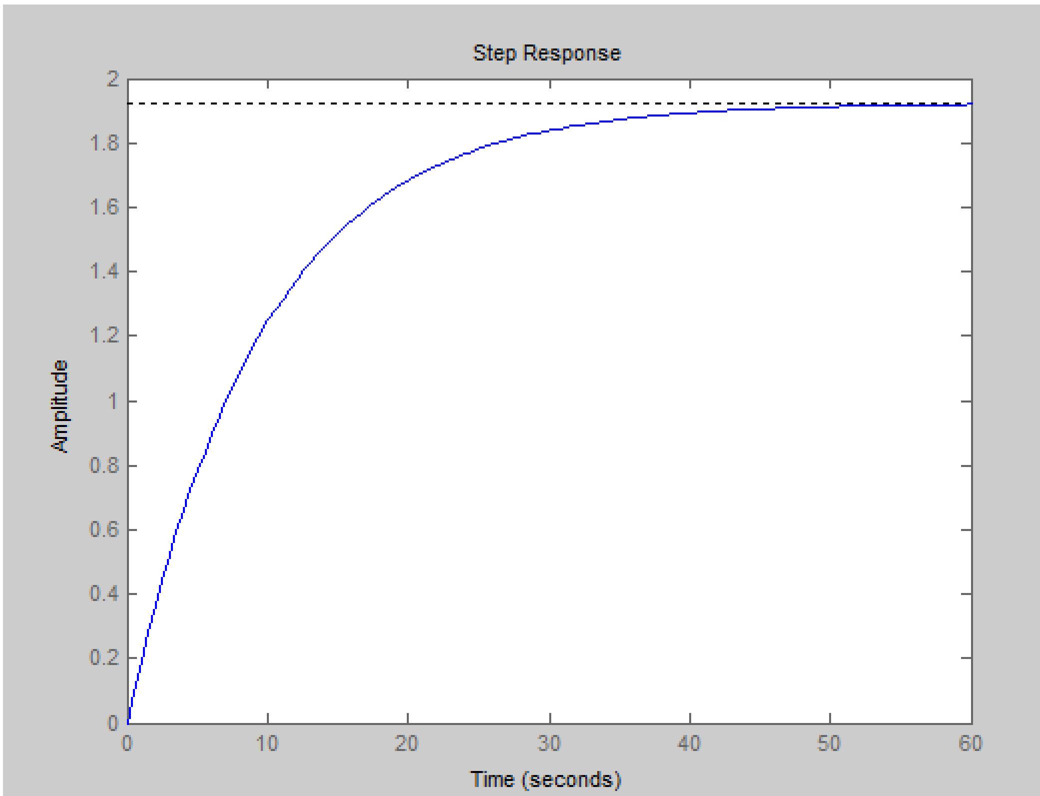

Figure 1: Plot of Steady State Amplitude

Steady state amplitude (i.e. speed of the motor) arrives at approximately 1.93rad/sec.

1.3 Part C

99% of the amplitude of 1.93rad/sec would be approximately 1.91. And we can see the graaph reaching approximately that point at approximately 45 seconds.

1.4 Part D

The final value theorem states

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) \tag{6}$$

Applying the RHS to compute the steady state, what we get is

$$\lim_{s \to 0} s \frac{1}{s} \frac{0.002}{0.01s + 0.00104} = \frac{0.002}{0.00104} = 1.923 \tag{7}$$

1.923 rad/s is the true steady state velocity, so our estimation of 1.93 rad/s is very close, and is off by 0.36%

1.5 Part E

The Transfer function is defined as follows

$$G(S) = \frac{\theta}{V} = \frac{1}{s} \frac{0.002}{0.01s + 0.00104} \tag{8}$$

1.6 Part F

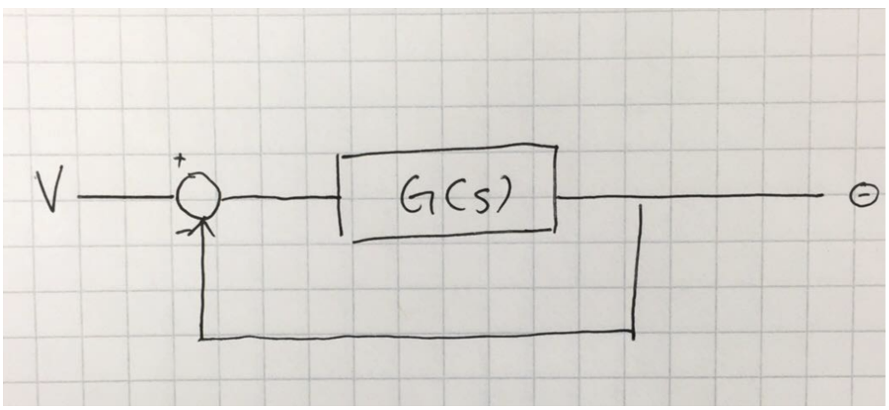

Figure 2: Model with feedback

and the corresponding transfer function would be

$$H(s) = \frac{G(s)}{1 + G(s)} \tag{9}$$

Adding a gain of K to this feedback system, the model we would get is
And the corresponding transfer function would be

$$H(s) = \frac{kG(s)}{1 + kG(s)} \tag{10}$$

1.7 Part G

K as a gain scaling unit, would have the units V/V, and would therefore be considered dimension-less

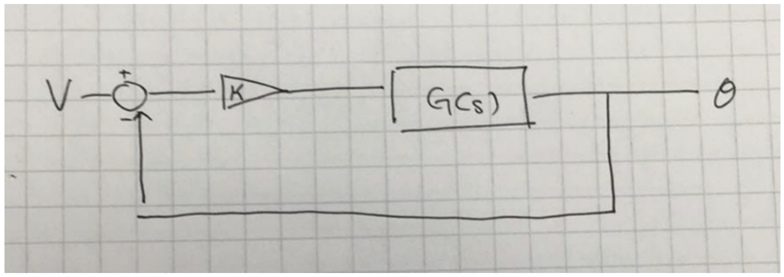

Figure 3: Model with feedback and gain

1.8 Part H

Taking expressions (drawn from the diagram), we get

$$\begin{aligned} e(s) &= V(s) - \theta(s) \\ \theta(s) &= ke(s) * G(s) \end{aligned} \tag{11}$$

We are looking for ke(s), where k is some arbitrary constant, so given 2 equations and 2 unknowns, we begin approaching this problem

$$\begin{aligned} \theta(s) &= V(s) - e(s) \\ V(s) - e(s) &= ke(s) * G(s) \\ V(s) &= ke(s) * G(s) + e(s) \\ V(s) &= e(s)[kG(s) + 1] \\ e(s) &= \frac{V(s)}{kG(s) + 1} \end{aligned} \tag{12}$$

and trivially, the end result is

$$ke(s) = \frac{kV(s)}{kG(s) + 1} \tag{13}$$

1.9 Part I

Taking the current transfer function

$$H(s) = \frac{kG(s)}{1 + kG(s)} \tag{14}$$

And substituting $G(s) = \frac{1}{s}\frac{0.002}{0.01s+0.00104}$, we do the following computation

$$
\begin{aligned}
H(s) &= \frac{k(\frac{1}{s}\frac{0.002}{0.01s+0.00104})}{1+k(\frac{1}{s}\frac{0.002}{0.01s+0.00104})} \\
&= \frac{\frac{k0.002}{0.01s^2+0.00104s}}{\frac{k*0.002+0.01s^2+0.00104s}{0.01s^2+0.00104s}} \\
&= \frac{k*0.002(0.01s^2+0.00104s)}{(0.01s^2+0.0014s)(k*0.002+0.01s^2+0.00104s)} \\
&= \frac{k*0.002}{k*0.002+0.01s^2+0.00104s}
\end{aligned} \tag{15}
$$