American Style Options - Regression methods on Pricing

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Introduction

Goal: simulation-based approximate dynamic programming method for pricing complex American-style options

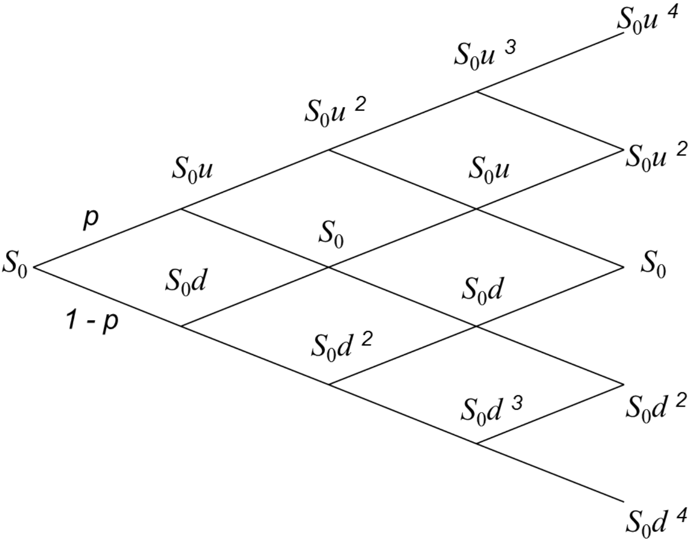
- starting point: the binomial option pricing model
- extended analysis by using regression methods
- methods involve the evaluation of value functions at a finite set, consisting of "representative" elements of the state space

$\Rightarrow$ deciding when to exercise an American option is an optimal stopping problem

American Style Option

- gives the holder the right to exercise at any time during the contract period
- holder of call(put) option may buy(sell) the underlying asset S at a prescribed price K (strike price)
- the exercise time τ can be represented as a stopping time
- option price determined by computing the discounted expectated payoff of the option under a risk-neutral measure

Binomial Tree



Define Problem

The price of the option is given by:

$$\sup_{\tau \in [0,\mathcal{T}]} \mathbb{E}[e^{-r\tau} g(x_\tau)]$$

where

- $\{x_\tau \in \Re^d | 0 \leq t \leq \mathcal{T}\}$ - risk-neutral process, assumed to be Markov
- r - risk-free interest rate, assumed to be a known constant
- $g(x)$ - intrinsic value of the option when the state is x
- $\mathcal{T}$ - expiration time, and the supremum is taken over stopping times that assume values in $[0, \mathcal{T}]$

Option Price

- Without loss of generality, assume $\mathcal{T}$ equal to integer N, and that allowable exercise times seperated by unit length time intervals.
- The price of this option is then:

$$\sup_{\tau} \mathbb{E}[\alpha^{\tau} g(x_{\tau})]$$

where $\alpha = e^{-r}$

In this discrete-time and Markovian formulation, the dynamics of the risk-neutral process can be described by a transition operator P, defined by:

$$(PJ)(x) = \mathbb{E}[J(x_{n+1})|x_n = x]$$

Approximations

A more convenient approach relies on single sample estimates of the desired expectation. Define for each $n = 0, \dots N-1$ a Q-function

$$Q_n = \alpha P J_{n+1}$$

where $Q_n(x)$ represents the expected discount payoff at time n conditioned on a decision not to exercise. This modification gives a new version of our dynamic pricing algorithm.

$$\tilde{Q}(\cdot, r_{N-1}) = \alpha \tilde{\Pi} \tilde{P} g$$

$$\tilde{Q}(\cdot, r_n) = \alpha \tilde{\Pi} \tilde{P} \max(g, \tilde{Q}(\cdot, rn+1))$$

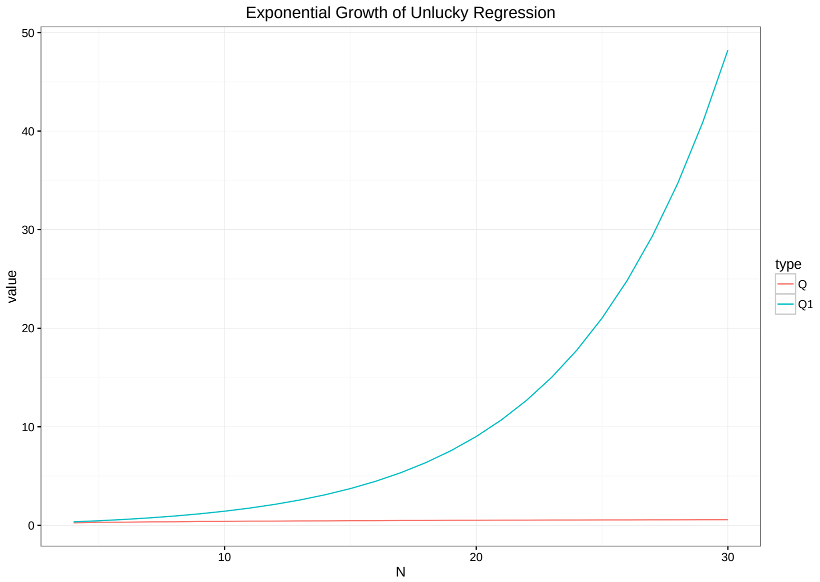
where $n = N-2, N-1, \dots, 0$

numerical example (high level Allele)

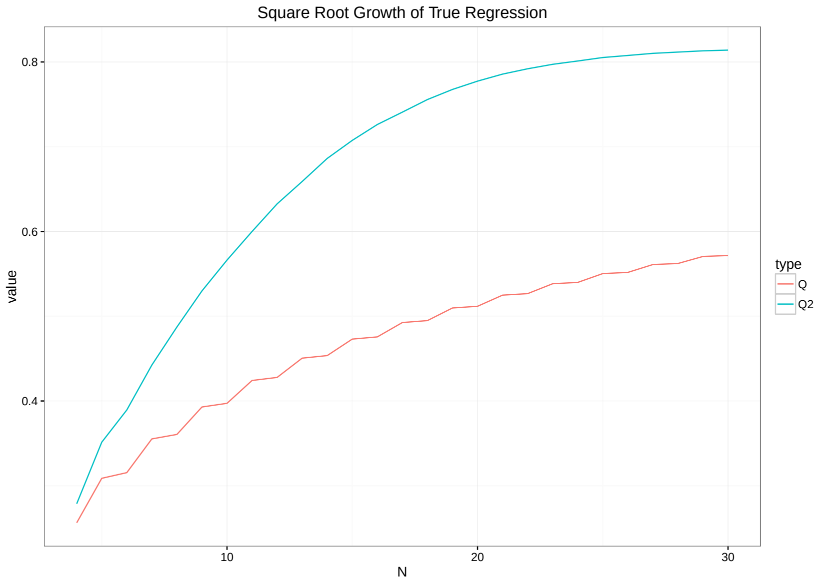
Let us consider stock price movements according to the tree structure seen earlier

strike	$\bar{x}$	1
high return	u	$3/2$
low return	d	$2/3$
probability of high return	p	0.4121
discount factor	α	0.99

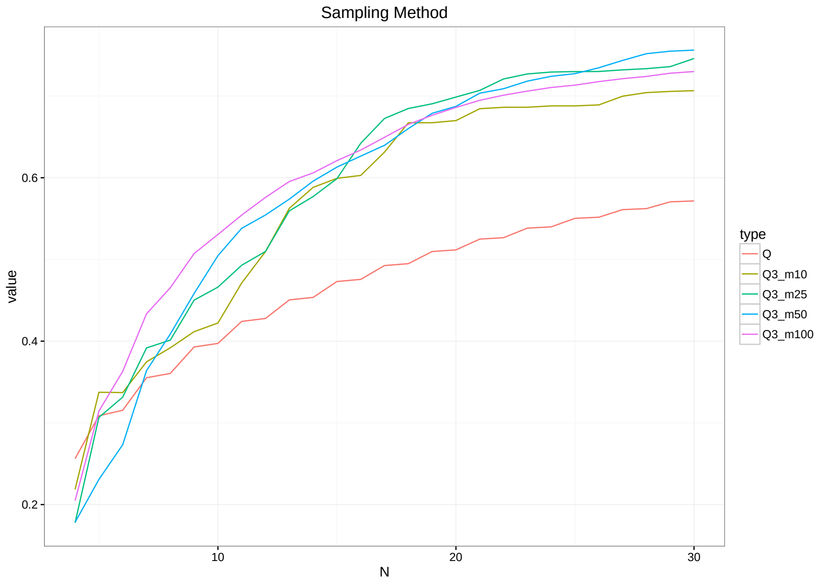
Exponential Growth Rate



Square Root Growth Rate



Sampling Method



Conclusion

Simple contracts, including “vanilla options” such as American puts and calls, the relevant optimal stopping problems can be solved efficiently by traditional numerical methods. However, the computational requirements become prohibitive as the number of uncertainties of a contract grows.

- We introduced certain simulation-based methods of the value iteration type for pricing complex American-style options.
- provided convergence results and error bounds that establish that such methods are viable, as long as state sampling is carried out by simulating the natural distribution of the underlying state process.
- This provides theoretical support for the apparent effectiveness of this particular form of state sampling.