

American style Options - Regression methods on Pricing

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Introduction

- ▶ simulation-based approximate dynamic programming method for pricing complex American-style options
- ▶ starting point the binomial option pricing model
- ▶ extended analysis by using regression methods
- ▶ methods involve the evaluation of value functions at a finite set, consisting of “representative” elements of the state space

American Style Option

- ▶ gives the holder the right to exercise at any time during the contract period
- ▶ holder of call(put) option may buy(sell) the underlying asset S at a prescribed price K (strike price)
- ▶ the exercise time τ can be represented as a stopping time
- ▶ option price determined by computing the discounted expected payoff of the option under a risk-neutral measure

Define Problem

The price of the option is given by:

$$\sup_{\tau \in [0, \mathcal{T}]} \mathbb{E}[e^{-r\tau} g(x_{\tau})]$$

where

- ▶ $\{x_t \in \mathbb{R}^d | 0 \leq t \leq \mathcal{T}\}$ - risk-neutral process, assumed to be Markov
- ▶ r - risk-free interest rate, assumed to be a known constant
- ▶ $g(x)$ - intrinsic value of the option when the state is x
- ▶ \mathcal{T} - expiration time, and the supremum is taken over stopping times that assume values in $[0, \mathcal{T}]$

Option Price

- ▶ Without loss of generality, assume \mathcal{T} equal to integer N , and that allowable exercise times separated by unit length time intervals.
- ▶ The price of this option is then:

$$\sup_{\tau} \mathbb{E}[\alpha^{\tau} g(x_{\tau})]$$

where $\alpha = e^{-r}$

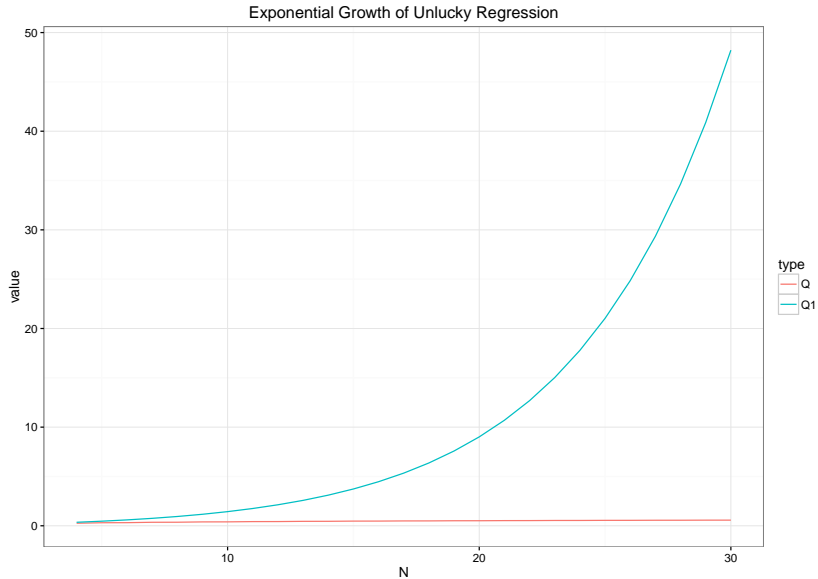
In this discrete-time and Markovian formulation, the dynamics of the risk-neutral process can be described by a transition operator P , defined by:

$$(PJ)(x) = \mathbb{E}[J(x_{n+1}) | x_n = x]$$

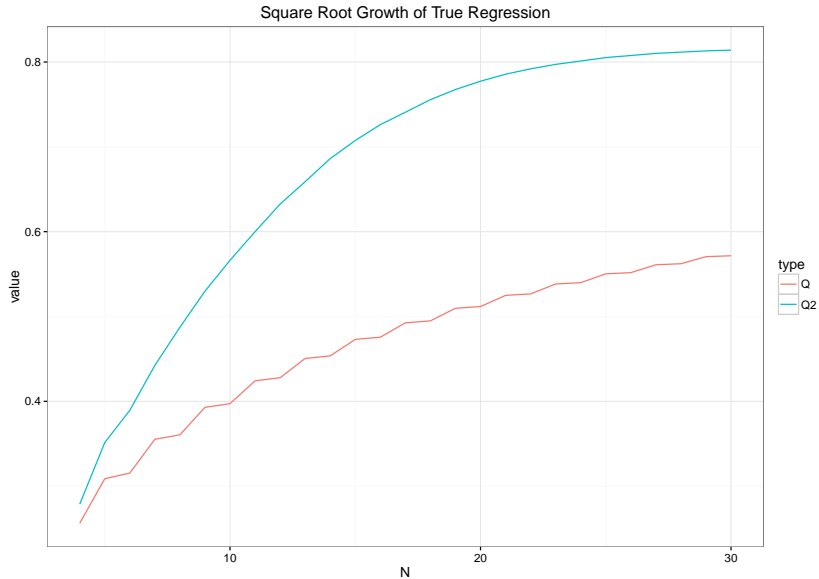
Dynamics

Approximations

Exponential Growth Rate



Square Root Growth Rate



Sampling Method

