

# Assignment 3: The rectilinear traveling salesperson problem

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## [Improved Nearest Neighbor Algorithm](#)

INPUT: a positive integer  $n$  and a list  $P$  of  $n$  distinct points representing vertices of a Euclidean graph

OUTPUT: a list of  $n$  points from  $P$  representing a Hamiltonian cycle of relatively minimum total weight

## [Exhaustive Optimization Algorithm](#)

INPUT: a positive integer  $n$  and a list  $P$  of  $n$  distinct points representing vertices of a rectilinear graph

OUTPUT: a list of  $n$  points from  $P$  representing a Hamiltonian cycle of minimum total weight for the graph.

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## Pseudocode

### Improved Nearest Neighbor Algorithm

farthest\_point

```
for (i = 0 to n - 1 do) {  
    for (j = i to n do) {  
        dist1 = abs(P[i].x - P[j].x);  
  
        dist2 = abs(P[i].y -  
P[j].y);  
  
        dist = dist1 + dist2;  
        if (dist > max) {  
            max = dist;  
            A = i;  
        }  
    }  
}  
endfor
```

```
- ((n-1-0)/1 + 1) = n  
- ((n-i)/1 + 1) = n+i+1  
-2  
-2  
-2  
-1 + Max(2 , 0 )
```

$$S.C = \sum_{i=0}^{n-1} (6(n+i+1)) = 3n^2 + 4n - 1 = O(n^2)$$

## Nearest Point

```
for (int i = 0 to n ) {  
    if (Visited[i] == false) {  
        dist1 = abs(P[i].x - P[A].x);  
        dist2 = abs(P[i].y - P[A].y);  
        dist = dist1 + dist2;  
        if (dist < nearest) {  
            nearest = dist;  
            B = i;  
        }  
    }  
}  
endfor
```

```
- ((n-1-0)/1 + 1) = n  
-1 + max(6,3) = 7  
-2  
-2  
-2  
-1 + max(2,0) = 3  
-1  
-1
```

$$\mathbf{S.C} = ((n-1-0)/1 + 1) + 1 + \max(6,3) = 7n = O(n)$$

## Exhaustive Optimization Algorithm

<pre> if (n == 1) {     for (i = 0; i &lt; sizeA - 1; i++)     {         float index = abs(P[A[i]].x - P[A[i + 1]].x) + abs(P[A[i]].y - P[A[i + 1]].y);         dist = dist + index;     }     float index = abs(P[A[0]].x - P[A[sizeA - 1]].x) + abs(P[A[0]].y - P[A[sizeA - 1]].y);     dist = dist + index;     if (dist &lt; bestDist) {         bestDist = dist;         for (int i = 0; i &lt; sizeA; i++)         {             bestSet[i] = A[i];         }     } } </pre>	<pre> -1+max(8n+8+2+ n , 0) =9n + 11  -n *(6+2) -6 -2  -6 -2  -1+max(1+n,0) = 2 + n -1 -n </pre>
--	--

**S.C. (since we are only considering the  $n=1$  if branch, then I am assuming the else part in  $\max(\text{if}, \text{else})$  is 0.**

$$\begin{aligned}
 &= 1 + \max( [n*(6+2)] + [6+2] + [1 + \max(1 + n, 0)] ) \\
 &= 1 + \max( 8n + 8 + 1 + 1 + n ) = 1 + \max( 9n + 10, 0 ) \\
 &= 1 + 9n + 10 \\
 &= 9n + 11
 \end{aligned}$$

=  $O(n)$

## Output

### Improved Nearest Neighbor Algorithm

#### Example 1

```
CPSC 335-x - Programming Assignment #3
Rectilinear traveling salesperson problem: INNI algorithm
Enter the number of vertices (>2)
4
Enter the points; make sure that they are distinct
x=2
y=0
x=1
y=1
x=3
y=1
x=0.1
y=0
The Hamiltonian cycle of a relative minimum length
(3, 1) (2, 0) (0.1, 0) (1, 1)
The relative minimum length is 7.8
elapsed time: 0 seconds
계속하려면 아무 키나 누르십시오 . . .
```

#### Example 2

```

Rectilinear traveling salesperson problem: INNI algorithm
Enter the number of vertices (>2)
8
Enter the points; make sure that they are distinct
x=0
y=4
x=2
y=1
x=1
y=6
x=2
y=2
x=3
y=5
x=3
y=3
x=5
y=2
x=5
y=6
x=6
y=5
The Hamiltonian cycle of a relative minimum length
(2, 1) (3, 2) (5, 2) (6, 5) (3, 5) (1, 6) (2, 7) (0, 4)
The relative minimum length is 26
elapsed time: 0 seconds
계속하려면 아무 키나 누르십시오 . . .

```

## Exhaustive Optimization Algorithm

### Example 1

```

CPSC 335-x - Programming Assignment #3
Rectilinear traveling salesperson problem: exhaustive optimization algorithm
Enter the number of vertices (>2)
4
Enter the points; make sure that they are distinct
x=2
y=0
x=1
y=1
x=3
y=1
x=0.1
y=0
The Hamiltonian cycle of the minimum length
(2, 0) (3, 1) (1, 1) (0.1, 0) (2, 0)
Minimum length is 7.8
elapsed time: 0 seconds
계속하려면 아무 키나 누르십시오 . . .

```

### Example 2

```

Rectilinear traveling salesperson problem: exhaustive optimization algorithm
Enter the number of vertices (>2)
8
Enter the points; make sure that they are distinct
x=0
y=4
x=2
y=1
x=1
y=6
x=3
y=7
x=3
y=5
x=3
y=3
x=3
y=3
x=5
y=2
x=6
y=5
The Hamiltonian cycle of the minimum length
(3, 5) (2, 7) (1, 6) (0, 4) (2, 1) (3, 2) (5, 2) (6, 5) (3, 5)
Minimum length is 24
elapsed time: 0.049554 seconds
계속하려면 아무 키나 누르십시오 . . .

```

## Code

### #Improved Nearest Neighbor Algorithm

```

#include <iostream>
#include <iomanip>
#include <cstdlib>
#include <string>
#include <chrono>
#include <cmath>
using namespace std;

struct point2D {
    float x; // x coordinate
    float y; // y coordinate
};

void print_cycle(int, point2D*, int*);
// function to print a cyclic sequence of 2D points in 2D plane, given the
// number of elements and the actual sequence stored as an array of 2D points

```

```

// SAME AS IN THE PREVIOUS PROGRAM
// YOU NEED TO IMPLEMENT THIS FUNCTION

int farthest_point(int, point2D*);
// function to return the index of a point that is furthest apart from some other point
// YOU NEED TO IMPLEMENT THIS FUNCTION

int nearest(int, point2D*, int, bool*);
// function to calculate the nearest unvisited neighboring point
// YOU NEED TO IMPLEMENT THIS FUNCTION

int main() {
    point2D *P;
    int *M;
    bool *Visited;
    int i, n;
    float dist;
    int A, B;

    // display the header
    cout << endl << "CPSC 335-x - Programming Assignment #3" << endl;
    cout << "Rectilinear traveling salesperson problem: INNI algorithm" << endl;
    cout << "Enter the number of vertices (>2) " << endl;

    // read the number of elements
    cin >> n;

    // if less than 3 vertices then terminate the program
    if (n < 3)
        return 0;

    // allocate space for the sequence of 2D points
    P = new point2D[n];

```



```

// read the sequence of distinct points
cout << "Enter the points; make sure that they are distinct" << endl;
for (i = 0; i < n; i++) {
    cout << "x=";
    cin >> P[i].x;
    cout << "y=";
    cin >> P[i].y;
}

// allocate space for the INNA set of indices of the points
M = new int[n];
// set the best set to be the list of indices, starting at 0
for (i = 0; i < n; i++)
    M[i] = i;

// Start the chronograph to time the execution of the algorithm
auto start = chrono::high_resolution_clock::now();

// allocate space for the Visited array of Boolean values
Visited = new bool[n];
// set it all to False
for (i = 0; i < n; i++)
    Visited[i] = false;

// calculate the starting vertex A
A = farthest_point(n, P);
// add it to the path
i = 0;
M[i] = A;

// set it as visited
Visited[A] = true;

for (i = 1; i < n; i++) {

```

```

        // calculate the nearest unvisited neighbor from node A
        B = nearest(n, P, A, Visited);
        // node B becomes the new node A
        A = B;
        // add it to the path
        M[i] = A;
        Visited[A] = true;
    }

    // calculate the length of the Hamiltonian cycle
    dist = 0;
    for (i = 0; i < n - 1; i++)
        dist += abs(P[M[i]].x - P[M[i + 1]].x) + abs(P[M[i]].y - P[M[i + 1]].y);

    dist += abs(P[M[0]].x - P[M[n - 1]].x) + abs(P[M[0]].y - P[M[n - 1]].y);

    // End the chronograph to time the loop
    auto end = chrono::high_resolution_clock::now();

    // after shuffling them
    cout << "The Hamiltonian cycle of a relative minimum length " << endl;
    print_cycle(n, P, M);
    cout << "The relative minimum length is " << dist << endl;

    // print the elapsed time in seconds and fractions of seconds
    int microseconds =
        chrono::duration_cast<chrono::microseconds>(end - start).count();
    double seconds = microseconds / 1E6;
    cout << "elapsed time: " << seconds << " seconds" << endl;

    // de-allocate the dynamic memory space
    delete[] P;
    delete[] M;

```

```

        system("pause");
        return EXIT_SUCCESS;
    }

int farthest_point(int n, point2D *P)
// function to calculate the furthest distance between any two 2D points
// YOU NEED TO IMPLEMENT THIS FUNCTION
{
    float max = 0, dist = 0, dist1 = 0, dist2 = 0;
    int A = 0;
    for (int i = 0; i < n - 1; i++) {
        for (int j = i; j < n; j++) {
            dist1 = abs(P[i].x - P[j].x);
            dist2 = abs(P[i].y - P[j].y);
            dist = dist1 + dist2;
            if (dist > max) {
                max = dist;
                A = i;
            }
        }
    }
    return A;
}

int nearest(int n, point2D *P, int A, bool *Visited)
// function to calculate the nearest unvisited neighboring point
// YOU NEED TO IMPLEMENT THIS FUNCTION
{
    float dist = 0, dist1 = 0, dist2 = 0, nearest = numeric_limits<float>::max();

    int B = 0;
    for (int i = 0; i < n; i++) {

```

```

        if (Visited[i] == false) {
            dist1 = abs(P[i].x - P[A].x);
            dist2 = abs(P[i].y - P[A].y);
            dist = dist1 + dist2;
            if (dist < nearest) {
                nearest = dist;
                B = i;
            }
        }
    }
    return B;
}

void print_cycle(int n, point2D *P, int *seq)
// YOU NEED TO IMPLEMENT THIS FUNCTION
{
    for (int i = 0; i < n; i++) {
        cout << "(" << P[seq[i]].x << ", " << P[seq[i]].y << ")    ";
    }
    cout << endl;
}

```

### #Exhaustive Optimization Algorithm

```

#include <iostream>
#include <iomanip>
#include <cstdlib>
#include <string>
#include <chrono>
#include <cmath>
using namespace std;

struct point2D {

```

```

        float x; // x coordinate
        float y; // y coordinate
    };

void print_cycle(int, point2D*, int*);
// function to print a cyclic sequence of 2D points in 2D plane, given the
// number of elements and the actual sequence stored as an array of 2D points
// YOU NEED TO IMPLEMENT THIS FUNCTION

float farthest(int, point2D*);
// function to calculate the furthest distance between any two 2D points

void print_perm(int, int *, int, point2D*, int *&, float &);
// function to generate the permutation of indices of the list of points

int main() {
    point2D *P;
    int *bestSet, *A;
    int i, n;
    float bestDist, Dist;

    // display the header
    cout << endl << "CPSC 335-x - Programming Assignment #3" << endl;
    cout << "Rectilinear traveling salesperson problem: exhaustive optimization algorithm" <<
endl;
    cout << "Enter the number of vertices (>2) " << endl;

    // read the number of elements
    cin >> n;

    // if less than 3 vertices then terminate the program

```

```

if (n < 3)
    return 0;
// allocate space for the sequence of 2D points
P = new point2D[n];

// read the sequence of distinct points
cout << "Enter the points; make sure that they are distinct" << endl;
for (i = 0; i < n; i++) {
    cout << "x=";
    cin >> P[i].x;
    cout << "y=";
    cin >> P[i].y;
}

// allocate space for the best set representing the indices of the points
bestSet = new int[n];
// set the best set to be the list of indices, starting at 0
for (i = 0; i < n; i++)
    bestSet[i] = i;

// Start the chronograph to time the execution of the algorithm
auto start = chrono::high_resolution_clock::now();

// calculate the farthest pair of vertices
Dist = farthest(n, P);
bestDist = n*Dist;

// populate the starting array for the permutation algorithm
A = new int[n];
// populate the array A with the values in the range 0 .. n-1
for (i = 0; i < n; i++)

```

```

        A[i] = i;

// calculate the Hamiltonian cycle of minimum weight
print_perm(n, A, n, P, bestSet, bestDist);

// End the chronograph to time the loop
auto end = chrono::high_resolution_clock::now();

// after shuffling them
cout << "The Hamiltonian cycle of the minimum length " << endl;
print_cycle(n, P, bestSet);
cout << "Minimum length is " << bestDist << endl;

// print the elapsed time in seconds and fractions of seconds
int microseconds =
    chrono::duration_cast<chrono::microseconds>(end - start).count();
double seconds = microseconds / 1E6;
cout << "elapsed time: " << seconds << " seconds" << endl;

// de-allocate the dynamic memory space
delete[] P;
delete[] A;
delete[] bestSet;

return EXIT_SUCCESS;
}

void print_cycle(int n, point2D *P, int *seq)
// function to print a sequence of 2D points in 2D plane, given the number of elements and the
actual
// sequence stored as an array of 2D points

```

```

// n is the number of points
// seq is a permutation over the set of indices
// P is the array of coordinates
// YOU NEED TO IMPLEMENT THIS FUNCTION
{
    int i;
    for (i = 0; i < n; i++) {
        cout << "(" << P[seq[i]].x << ", " << P[seq[i]].y << ") ";
    }
    cout << "(" << P[seq[0]].x << ", " << P[seq[0]].y << ") ";

    cout << endl;

}

float farthest(int n, point2D *P)
// function to calculate the furthest distance between any two 2D points
{
    float max_dist = 0;
    int i, j;
    float dist;

    for (i = 0; i < n - 1; i++)
        for (j = 0; j < n; j++) {
            dist = abs(P[i].x - P[j].x) + abs(P[i].y - P[j].y);
            if (max_dist < dist)
                max_dist = dist;
        }
    return max_dist;
}

```



```

void print_perm(int n, int *A, int sizeA, point2D *P, int *&bestSet, float &bestDist)
// function to generate the permutation of indices of the list of points
{
    int i;
    float dist = 0;

    if (n == 1) {
        // we obtain a permutation so we compare it against the current shortest
        // Hamiltonian cycle
        // YOU NEED TO COMPLETE THIS PART
        //adding distance from the last node back to the first node
        for (i = 0; i < sizeA - 1; i++)
        {
            float index = abs(P[A[i]].x - P[A[i + 1]].x) + abs(P[A[i]].y - P[A[i + 1]].y);
            dist = dist + index;

        }
        float index = abs(P[A[0]].x - P[A[sizeA - 1]].x) + abs(P[A[0]].y - P[A[sizeA - 1]].y);
        dist = dist + index;
        if (dist < bestDist) {
            bestDist = dist;
            for (int i = 0; i < sizeA; i++)

                {
                    bestSet[i] = A[i];

                }
        }
    }
    else {
        for (i = 0; i < n - 1; i++) {

```

```

print_perm(n - 1, A, sizeA, P, bestSet, bestDist);
if (n % 2 == 0) {
    // swap(A[i], A[n-1])
    int temp = A[i];
    A[i] = A[n - 1];
    A[n - 1] = temp;
}
else
{
    // swap(A[0], A[n-1])
    int temp = A[0];
    A[0] = A[n - 1];
    A[n - 1] = temp;
}
}
print_perm(n - 1, A, sizeA, P, bestSet, bestDist);
}
}

```