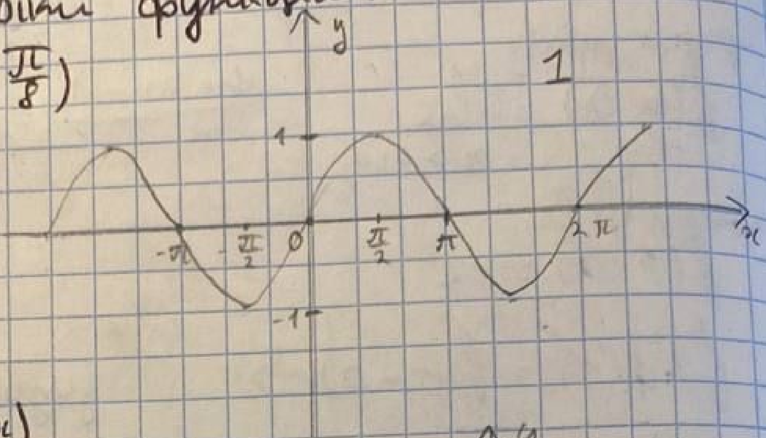
ТРР варіант 27 Скрипець

① побудувати графіки функцій:

1) $y = -2\sin\left(\frac{1}{2}x - \frac{\pi}{8}\right)$

1. $y = \sin x$
2. $y = 2\sin x$
3. $y = -2\sin x$
4. $y = -2\sin\left(\frac{1}{2}x\right)$
5. $y = -2\sin\left(\frac{1}{2}x - \frac{\pi}{8}\right) = 2\sin\frac{1}{2}\left(x - \frac{\pi}{4}\right)$

1



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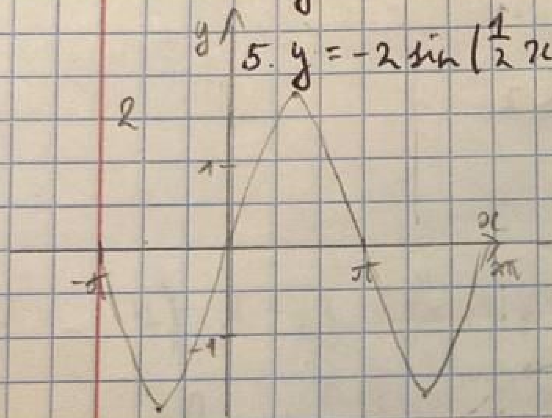

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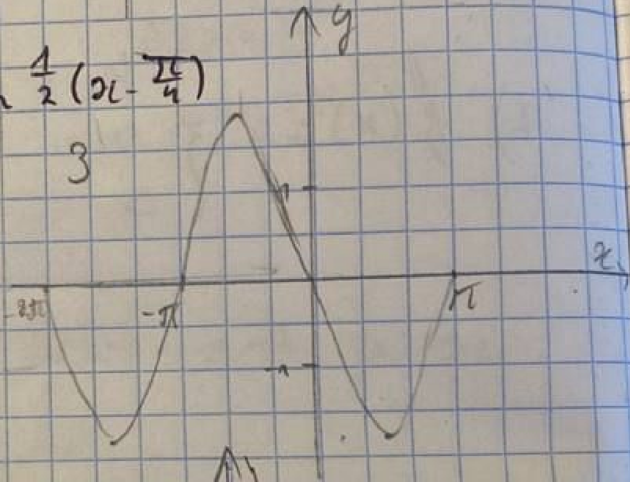

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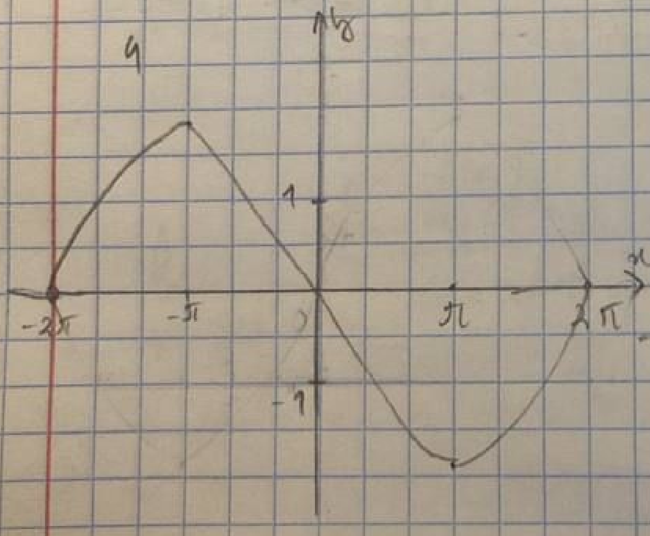

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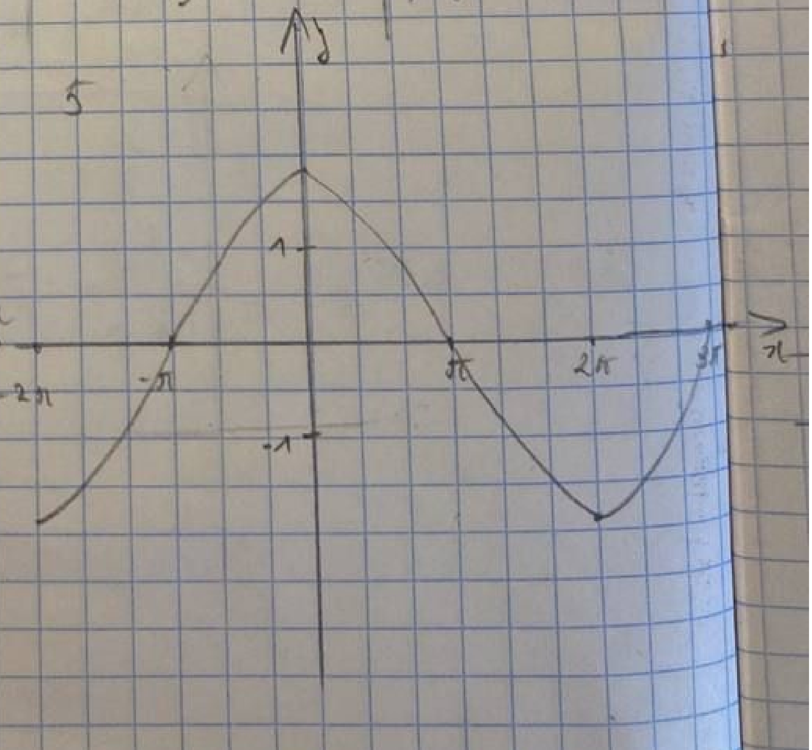


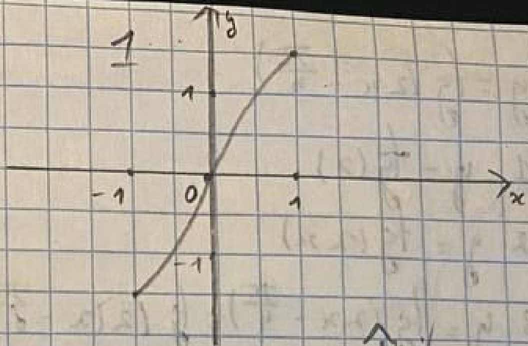
2) $y = 3\arcsin\left(x + \frac{1}{2}\right)$

1. $y = \arcsin(x)$

2. $y = 3\arcsin(x)$

3. $y = 3\arcsin\left(x + \frac{1}{2}\right)$

1



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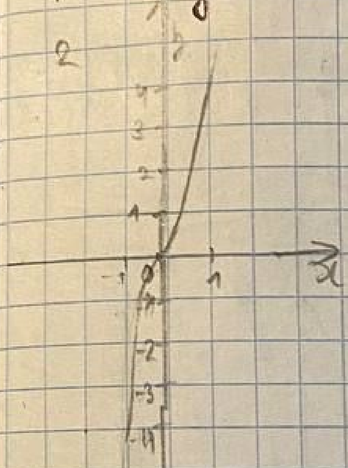

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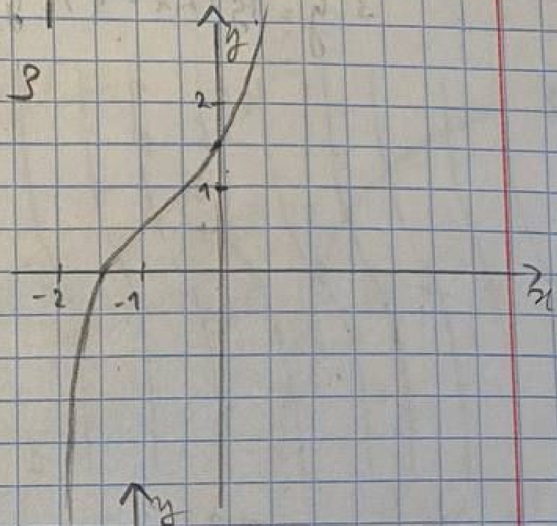


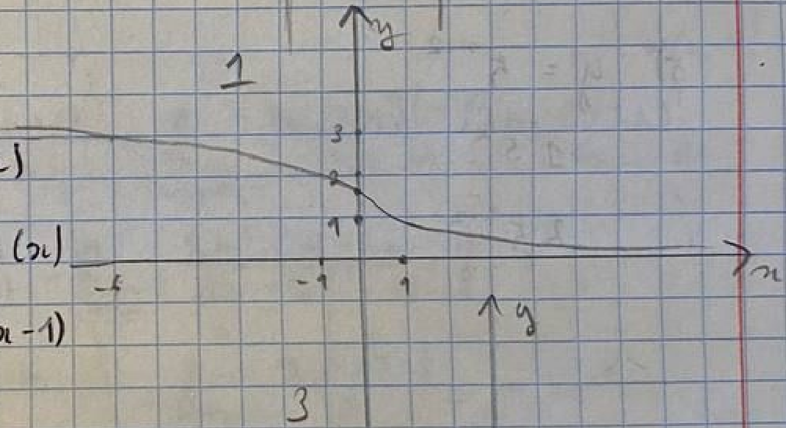
3) $y = \frac{1}{2}\operatorname{arcctg}(x-1)$

1. $y = \operatorname{arcctg}(x)$

2. $y = \frac{1}{2}\operatorname{arcctg}(x)$

3. $y = \frac{1}{2}\operatorname{arcctg}(x-1)$

1



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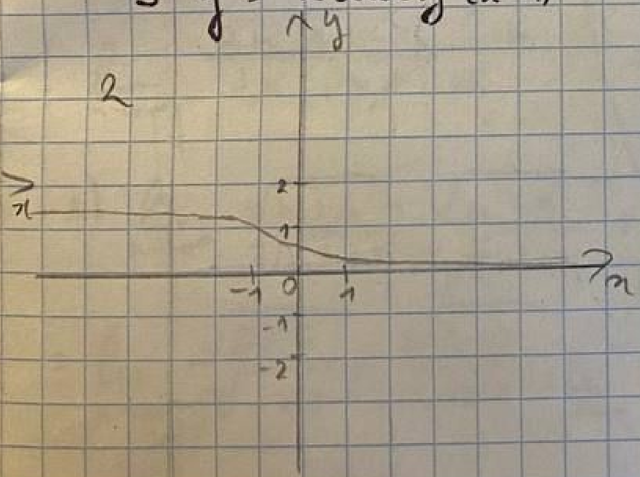

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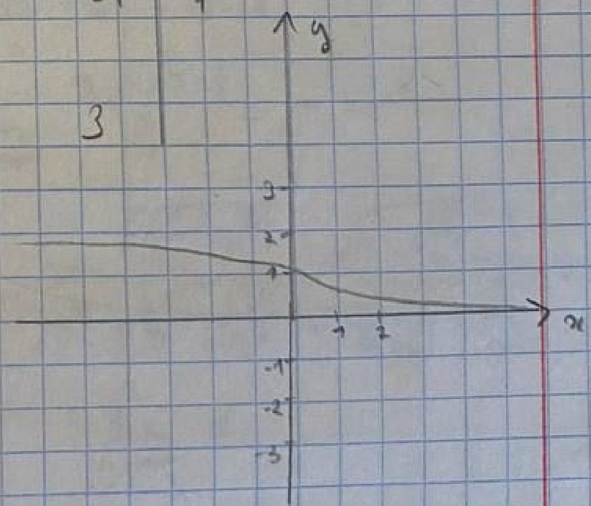
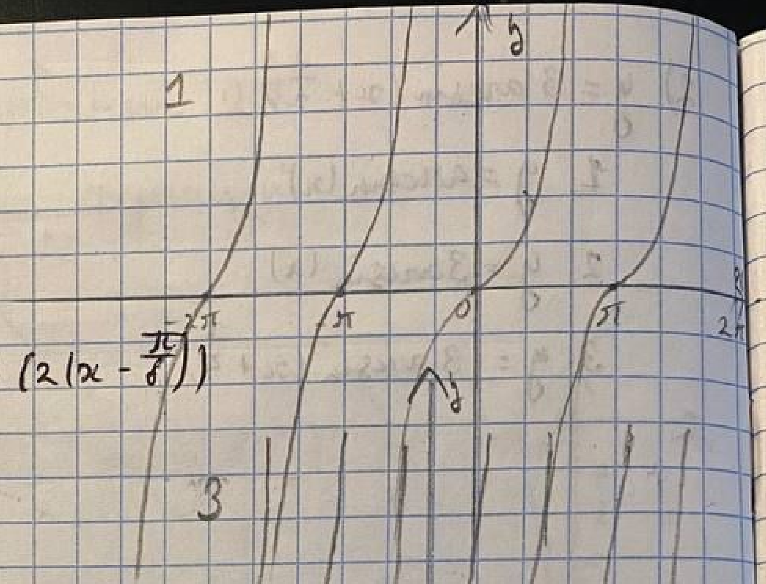


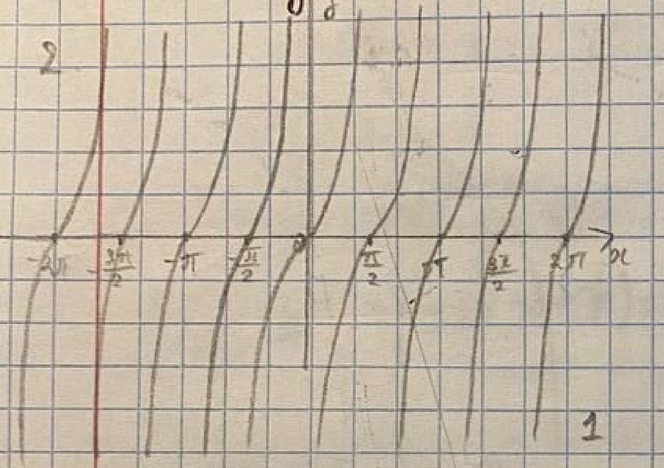
4) $y = tg\left(2x - \frac{\pi}{4}\right)$

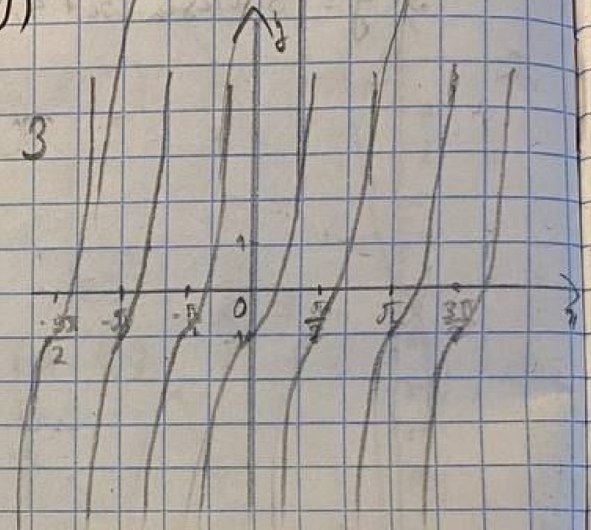
1 $y = tg(x)$

2 $y = tg(2x)$

3 $y = tg\left(2x - \frac{\pi}{4}\right) = tg\left(2\left(x - \frac{\pi}{8}\right)\right)$



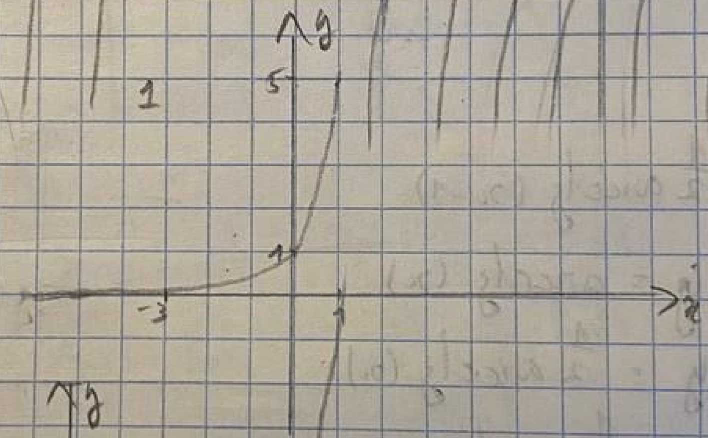


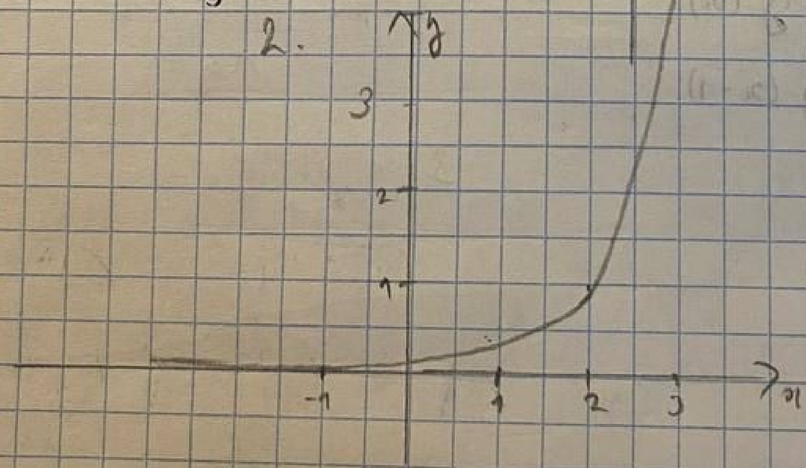


5) $y = 5^{x-2}$

1 5^x

2 5^{x-2}





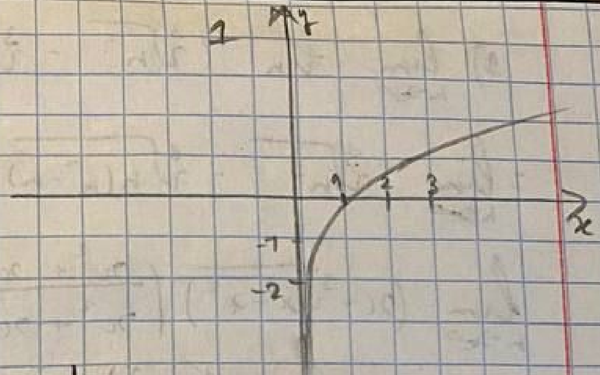
б) $y = 2\ln(2x-5)$

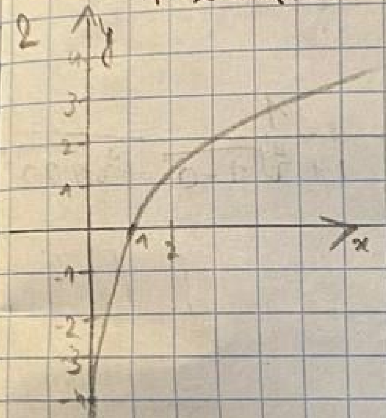
1 $\ln(x)$

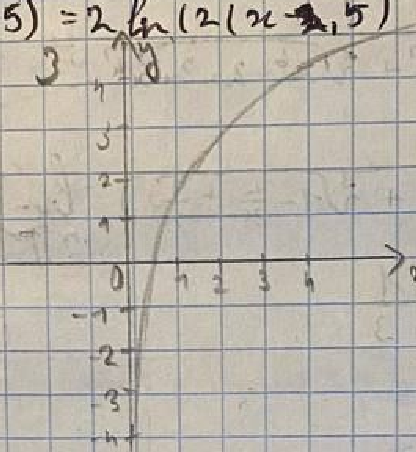
2 $2\ln(x)$

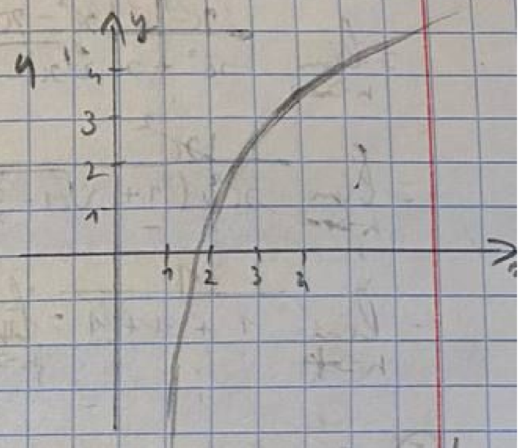
3 $2\ln(2x)$

4 $2\ln(2x-5) = 2\ln(2(x-2,5))$









$n! = n(n-1)!$

⑨ знайти границі

1) $$\lim_{n\to\infty} \frac{n(2n)! - (2n+1)!}{n(2n-1)! + (2n)!} = \lim_{n\to\infty} \frac{n(2n)! - (2n+1)(2n)!}{n(2n-1)! + (2n)(2n-1)!} =$$

$$= \lim_{n\to\infty} \frac{2n!(n-(2n+1))}{(2n-1)!(n+2n)} = \lim_{n\to\infty} \frac{2n!(n-2n-1)}{3n(2n-1)!} =$$

$$= \lim_{n\to\infty} \frac{2\cancel{n}(2\cancel{n}-1)!(-n-1)}{3\cancel{n}(2\cancel{n}-1)!} = \lim_{n\to\infty} \frac{2(-n-1)}{3} = -\infty$$

2) $$\lim_{n\to\infty} \frac{\sqrt{n+6} - \sqrt{n^2-5}}{\sqrt[3]{n^3+3} + \sqrt[4]{n^3+1}} = \lim_{n\to\infty} \frac{\cancel{n}\left(\sqrt{\frac{1}{n}+\frac{6}{n^2}} - \sqrt{1-\frac{5}{n^2}}\right)}{\cancel{n}\left(\sqrt[3]{1+\frac{3}{n^3}} + \sqrt[4]{\frac{1}{n}+\frac{1}{n^4}}\right)} =$$

$$= \cancel{\lim_{n\to\infty}} \frac{\sqrt{0+0} - \sqrt{1-0}}{\sqrt[3]{1+0} + \sqrt[4]{0+0}} = \cancel{\lim_{n\to\infty}} \frac{-\sqrt{1}}{\sqrt[3]{1}} = \cancel{\lim_{n\to\infty}} -1$$

3) $\lim\limits_{n\to\infty} \sqrt[3]{n}\left(\sqrt[3]{n^2} - \sqrt[3]{n(n-1)}\right) = \lim\limits_{n\to\infty} \sqrt[3]{n}\left(\sqrt[3]{n^2} - \sqrt[3]{n^2-n}\right) =$

$= \lim\limits_{n\to\infty} \sqrt[3]{n^3} - \sqrt[3]{n(n^2-n)} = \lim\limits_{n\to\infty} n - \sqrt[3]{n^3-n^2} = \left[\frac{\infty}{\infty}\right]$

$\lim\limits_{n\to\infty} \left(x - \sqrt[3]{x^3-x^2}\right)\left(\frac{x^2 + x\sqrt[3]{x^3-x^2} + \sqrt[3]{(x^3-x^2)^2}}{x^2 + x\sqrt[3]{x^3-x^2} + \sqrt[3]{(x^3-x^2)^2}}\right) =$

$= \lim\limits_{n\to\infty} \frac{\cancel{x^3} - (\cancel{x^3} - x^2)}{x^2 + x\sqrt[3]{x^3-x^2} + \sqrt[3]{x^6-2x^5+x^4}} =$

$= \lim\limits_{n\to\infty} \frac{\cancel{x^2}}{\cancel{x^2}\left(1 + \sqrt[3]{1-\frac{1}{x}} + \sqrt[3]{1-\frac{2}{x}+\frac{1}{x^2}}\right)} = \cancel{\lim\limits_{n\to\infty}} \frac{1}{1+\sqrt[3]{1-0}+\sqrt[3]{1-0+0}} =$

$= \cancel{\lim\limits_{n\to\infty}} \frac{1}{1+1+1} = \cancel{\lim\limits_{n\to\infty}} \frac{1}{3}$

(5) знайти границі

1) $\lim\limits_{x\to-5} \frac{x^2-2x-35}{2x^2+11x+5} = \left[\frac{25+10-35}{2\cdot25-55+5} = \frac{0}{0}\right]$

$\lim\limits_{x\to-5} \left(\frac{x^2+5x-7x-35}{2x^2+10x+x+5}\right) = \lim\limits_{x\to-5} \frac{x(x+5)-7(x+5)}{2x(x+5)+x+5} =$

$= \lim\limits_{x\to-5} \frac{\cancel{(x+5)}(x-7)}{\cancel{(x+5)}(2x+1)} = \lim\limits_{x\to-5} \frac{x-7}{2x+1} = \cancel{\lim\limits_{x\to-5}} \frac{-5-7}{2(-5)+1} = \cancel{\lim\limits_{x\to-5}} \frac{4}{3}$

2) $\lim\limits_{x\to6} \frac{2x^2-11x-6}{3x^2-20x+12} = \left[\frac{36\cdot2-66-6}{3\cdot36-20\cdot6+12} = \frac{0}{0}\right]$

$\lim\limits_{x\to6} \frac{2x^2+x-12x-6}{3x^2-2x-18x+12} = \lim\limits_{x\to6} \frac{x(2x+1)-6(2x+1)}{x(3x-2)-6(3x-2)} =$

$= \lim\limits_{x\to6} \frac{(2x+1)\cancel{(x-6)}}{(3x-2)\cancel{(x-6)}} = \lim\limits_{x\to6} \frac{2x+1}{3x-2} = \cancel{\lim\limits_{x\to6}} \frac{2\cdot6+1}{3\cdot6-2} = \frac{13}{16}$

3) $\lim\limits_{x\to\infty} \dfrac{4-5x^2-3x^5}{x^5+6x+8} = -3$

4) $\lim\limits_{x\to-\infty} \dfrac{7x^3-2x+4}{2x^3+x-5} = -\infty$

5) $\lim\limits_{x\to\infty} \dfrac{2x-13}{-3x^5-4x} = 0$

6) $\lim\limits_{x\to-4} \dfrac{\sqrt{x+20}-4}{x^3+64} = \left[\dfrac{\sqrt{-4+20}-4}{(-4)^3+64} = \dfrac{0}{0}\right]$

$$\lim_{x\to-4} \frac{(\sqrt{x+20}-4)(\sqrt{x+20}+4)}{(x^3+64)(\sqrt{x+20}+4)} = \lim_{x\to-4} \frac{x+20-16}{(x+4)(x^2-4x+16)(\sqrt{x+20}+4)} =$$

$$= \lim_{x\to-4} \frac{x+4}{(x+4)(x^2-4x+16)(\sqrt{x+20}+4)} = \frac{-4+4}{(-4+4)((-4)^2-4(-4)+16)(\sqrt{-4+20}+4)} =$$

$$= \frac{1}{384}$$

7) $\lim\limits_{x\to\infty} \left(\dfrac{1+2x}{3+2x}\right)^{1-2x} = \lim\limits_{x\to\infty} \left(\dfrac{2x+3-2}{3+2x}\right)^{1-2x} =$

$$= \lim_{x\to\infty} \left(\left(1+\left(-\frac{2}{2x+3}\right)\right)^{-\frac{2x+3}{2}}\right)^{\left(-\frac{2}{2x+3}\right)\cdot(1-2x)} =$$

$$= \left\{\begin{array}{l} \text{Заміна} \\ t = -\frac{2}{2x+3} \\ \frac{1}{t} = -\frac{2x+3}{3} \\ t \to 0 \end{array}\right\} = \left(\lim_{t\to0}(1+t)^{\frac{1}{t}}\right)^{\lim\limits_{x\to\infty}\frac{2-4x}{2x+3}} =$$

$$= e^{\lim\limits_{x\to\infty}\frac{4x-2}{2x+3}} = e^{\lim\limits_{x\to\infty}\frac{4x\left(1-\frac{1}{2x}\right)}{2x\left(1+\frac{3}{2x}\right)}} = e^{\lim\limits_{x\to\infty}\frac{4x}{2x}} = e^{-2}$$

8) $\lim\limits_{x\to\infty}\left(\frac{3x-1}{2x+5}\right)^{3x-\infty} = 0$

⑥ знайти границі

1) $\lim\limits_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2} = \left[\frac{0}{0}\right] \left\{ \begin{matrix} \text{Заміна} \\ y = x - \frac{\pi}{2},\ y\to 0,\ \text{якщо } x\to\frac{\pi}{2} \\ x = y + \frac{\pi}{2} \end{matrix} \right\} =$

$= \lim\limits_{y\to 0} \frac{1-\sin\left(y+\frac{\pi}{2}\right)}{\left(\cancel{\frac{\pi}{2}} - y - \cancel{\frac{\pi}{2}}\right)^2} = \lim\limits_{y\to 0} \frac{1-\cos y}{y^2} = \left| 1-\cos y \sim \frac{y^2}{2},\ y\to 0 \right| =$

$= \lim\limits_{y\to 0} \frac{\frac{y^2}{2}}{y^2} = \frac{1}{2}$

2) $\lim\limits_{x\to 3} \frac{\sin(x-3)}{x^3-27} = \lim\limits_{x\to 3} \left[\frac{\sin 3-3}{3^3-27} = \frac{0}{0}\right] =$

перша визначна границя

$\lim\limits_{x\to 3} \frac{\sin(x-3)}{(x-3)(x^2+3x+9)} = \lim\limits_{x\to 3} \frac{\overbrace{\frac{\sin(x-3)}{x-3}}^{=1}}{(x^2+3x+9)} = \frac{1}{9+9+9} = \frac{1}{27}$

3) $\lim\limits_{x\to 0} \frac{\operatorname{tg} x - \sin x}{x(1-\cos 2x)} = \left[\frac{\frac{0}{1}+0}{0(1-\cos 2\cdot 0)} = \frac{0}{0}\right] =$

$\lim\limits_{x\to 0} \frac{\sin x\left(\frac{1}{\cos x}-1\right)}{x(1-\cos 2x)} = \lim\limits_{x\to 0} \frac{\left(\frac{\sin x}{x}\right)\left(\frac{1-\cos x}{\cos x}\right)}{2\sin^2 x} =$

$= \lim\limits_{x\to 0} \frac{\cancel{2}\cancel{\sin^2\frac{x}{2}}}{\cos x\cdot\cancel{2}\cdot 4\,\cancel{\sin^2\frac{x}{2}}\cos^2\frac{x}{2}} = \lim\limits_{x\to 0} \frac{1}{4\cos x\cdot\cos^2\frac{x}{2}} = \frac{1}{4}$

4) $\lim\limits_{x\to 1} \frac{1-x^2}{\sin\pi x} = \left[\frac{0}{0}\right]$

$\lim\limits_{x\to 1} -\frac{(x-1)(x+1)}{\sin\pi x} = -2\lim\limits_{x\to 1} \frac{x-1}{\sin\pi x} \left\{ \begin{matrix} \text{Заміна} \\ x = t+1 \quad t = x-1 \\ x\to 1 \\ t\to 0 \end{matrix} \right\}$

$-2\lim\limits_{x\to 0} -\frac{t}{\sin(\pi t)} = 2\lim\limits_{t\to 0} \frac{1}{\pi\underbrace{\frac{\sin(\pi t)}{\pi t}}_{=1}} = 2\lim\limits_{t\to 0} \frac{1}{\pi} = \frac{2}{\pi}$

5) $\lim\limits_{x\to a} \dfrac{a^{x^2-a^2}-1}{tg \ln \frac{x}{a}} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to a} \dfrac{(x^2-a^2)\ln a}{\ln \frac{x}{a}} =$

$= \lim\limits_{x\to a} \dfrac{(x-a)(x+a)\ln a}{\ln(1+(\frac{x}{a}-1))} = \lim\limits_{x\to a} \dfrac{(x+a)(x-a)\ln a}{\frac{x}{a}-1} =$

$= \lim\limits_{x\to a} \dfrac{(x-a)(x+a)\ln a}{\frac{x-a}{a}} = \lim\limits_{x\to a} (x+a)\ln a \cdot a = (a+a)a\ln a = 2a^2\ln a$

6) $\lim\limits_{x\to 0} \dfrac{3^{5x}-2^{-7x}}{2x - tg\,x} = \lim\limits_{x\to 0} \dfrac{3^{5x}-2^{-7x}}{2x\cdot tg\,x} = \lim\limits_{x\to 0} \dfrac{3^{5x}-2^{-7x}}{2x\cdot tg\,x} =$

$= \lim\limits_{x\to 0} \dfrac{1-3^{5x}\cdot 2^{7x}}{tg(x)-2x} = \lim\limits_{x\to 0} \dfrac{1-3^{5x}\cdot 2^{7x}}{\frac{\sin(x)}{\cos(x)}-2x} = \lim\limits_{x\to 0} - \dfrac{(3^{5x}\cdot 2^{7x}-1)\cos x}{\sin(x)-2x\cos x} =$

$= \lim\limits_{x\to 0} - \dfrac{(3^{5x}\, 2^{7x}-1)\cos x}{x\frac{\sin x}{x} - 2x\cos x} = \lim\limits_{x\to 0} \dfrac{3^{5x}\, 2^{7x}-1}{2x\cos x - x} =$

$= \lim\limits_{x\to 0} \dfrac{\left(5x\,\frac{3^{5x}-1}{5x}+1\right)\left(7x\,\frac{2^{7x}-1}{7x}+1\right)-1}{2x\cos x - x} =$

$\lim\limits_{x\to 0} \dfrac{a^x-1}{x} = \ln(a)$

$= \lim\limits_{x\to 0} \dfrac{35\ln(2)\ln(3)x + 5\ln(3) + 7\ln(2)}{2\cos(x)-1} =$

$= \dfrac{35\ln(2)\ln(3)\cdot 0 + 5\ln(3) + 7\ln(2)}{2\cos(0)-1} = 5\ln(3) + 7\ln(2)$ $\qquad \dfrac{1-\cos 2x}{2} = \sin^2 x$

7) $\lim\limits_{x\to 0} \dfrac{\sqrt{\cos x}-1}{\sin^2 2x} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 0} \dfrac{\cos x - 1}{(\sqrt{\cos x}+1)\sin^2 2x} =$

$= \lim\limits_{x\to 0} \dfrac{2\sin^2\left(\frac{x}{2}\right)}{(\sqrt{\cos x}+1)\sin^2(2x)} = \lim\limits_{x\to 0} - \dfrac{\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}}{8(\sqrt{\cos x}+1)\left(\frac{\sin(2x)}{2x}\right)^2} =$ $= 1$ перша визначна границя

$= \lim\limits_{x\to 0} - \dfrac{1}{8(\sqrt{\cos x}+8} = - \dfrac{1}{8\sqrt{\cos(0)}+8} = - \dfrac{1}{8+8} = - \dfrac{1}{16}$

8) $\lim\limits_{x\to0}\left(\dfrac{1+x\,3^x}{1+x7^x}\right)^{ctg^2 x} = \left[1^\infty\right] =$

$= \lim\limits_{x\to0}\left(\dfrac{1+x\cdot7^x - x\cdot7^x + x\cdot3^x}{1+x\cdot7^x}\right)^{\frac{1}{tg^2 x}} =$

$= \lim\limits_{x\to0}\left(1+\dfrac{x(3^x-7^x)}{1+x\cdot7^x}\right)^{\frac{1+x\cdot7^x}{x(3^x-7^x)}\cdot\frac{x(3^x-7^x)}{1+x\cdot7^x}\cdot\frac{1}{tg^2x}} = e$

$a^x-1 \sim x\ln a$

$= e^{\lim\limits_{x\to0}\frac{x\cdot3^x\left(1-\left(\frac{7}{3}\right)^x\right)}{(1+x\cdot7^x)\,x^2}} = e^{\lim\limits_{x\to0}\frac{-3^x\cdot x\cdot\ln\frac{7}{3}}{(1+x\cdot7^x)x}} =$

$= e^{-\ln\frac{7}{3}} = e^{\ln\frac{3}{7}} = \dfrac{3}{7}$

8 Визначити порядок і головну частину розкладу $\alpha(x)$ відносно $\beta(x)$

1) $\alpha(x) = e^{x^5} - \cos x^3$, $\beta(x) = \sin x$, $x\to0$

$\lim\limits_{x\to0}\dfrac{\alpha(x)}{(\beta(x))^k} = \dfrac{e^{x^5}-\cos x^3}{(\sin x)^k} = \left[\dfrac{0}{0}\right]$

$\lim\limits_{x\to0}\dfrac{e^{x^5}-\cos x^3}{x^k} = \lim\limits_{x\to0}\dfrac{(e^{x^5}-1)+1-\cos x^3}{x^k} = \lim\limits_{x\to0}\dfrac{x^5(1-\cos x^3)}{x^k} =$

$= \lim\limits_{x\to0}\dfrac{x^5\cdot\frac{x^6}{2}}{x^k} = \lim\limits_{x\to0}\dfrac{x^{11}}{x^k} = 1$, $k=11$

головна частина $= 1(\sin x)^{11}$

2) $\alpha(x) = \ln(2x^2-2x-3)$, $\beta(x) = x-2$, $x\to2$

$\lim\limits_{x\to2}\dfrac{\ln(2x^2-2x-3)}{(x-2)^k} = \left\{\begin{array}{l}\text{заміна}\\ t = x-2\\ t\to0 \text{ при } x\to2\end{array}\right\} =$

$= \lim\limits_{t\to0}\dfrac{\ln((t+2)^2-2(t+2)-3)}{t^k} =$

$= \lim\limits_{t\to0}\dfrac{\ln(2t^2+6t+1)}{t^k} = \lim\limits_{t\to0}\dfrac{2t^2+6t}{t^k} = \lim\dfrac{t(2t+6)}{t^k} = 6$, при $k=1$.

головна частина $= 6\cdot(x-2)^1$

3) $\alpha(x) = \ln(x^8+1)$, $\beta(x) = \sin\sqrt{x}$, $x \to 0$

$$\lim_{x\to 0} \frac{\ln(x^8+1)}{(\sin\sqrt{x})^k} = \frac{x^8}{(x^{\frac{1}{2}})^k} = 1, \quad k = 16$$

головна частина $= 1\cdot(\sin\sqrt{x})^{16}$

(9) дослідити функцію на неперервність

1) $f(x) = \dfrac{\sin 2x}{x}$

$x = 0$ $\quad \nexists\, f(0)$ - точка розриву

$$f(0+0) = \lim_{x\to 0+0} \frac{\sin 2x - 1}{x} = \lim_{x\to 0+0} \frac{2x-1}{x} = 2$$

$f(0-0) = 2$

$f(0+0) = f(0-0) \Rightarrow$ усувне

2)
$$f(x) = \begin{cases} \sin x, & x < 0 \\ x, & 0 \le x \le 2 \\ 0, & x > 2 \end{cases}$$

$$f(0-0) = \lim_{x\to 0-0} \sin x = 0$$

$$f(0) = f(0+0) = \lim_{x\to 0+0} x = 0$$

$$f(2) = f(2-0) = \lim_{x\to 2-0} x = 2$$

$f(2+0) = 0$

у т. $x = 0$ функція неперервна

$x = 2$ точка розриву I роду

3) $f(x) = \frac{1}{3^{1-x}}$ у точках $x_1 = 1$ $x_2 = 2$

$f(1+0) = \lim_{x \to 1+0}$ не визначено

$f(1-0) = \lim_{x \to 1-0}$ не визначено

$f(2+0) = \lim_{x \to 2+0}$ $3^{-1} = \frac{1}{3}$

$f(2-0) = \lim_{x \to 2-0}$ $3^{-1} = \frac{1}{3}$

$f(2+0) = f(2-0) \Rightarrow x = 2$ - т. неперервності

$f(1+0) = \lim_{x \to 1+0} \frac{1}{3^{+1}} = 9^{\frac{1}{0}} = \infty$

$f(1-0) = \lim_{x \to 1-0} 9^{\frac{1}{0}} = -\infty$

$f(1+0) \neq f(1-0)$

$\Downarrow$

$x_0 = 2$ т. розриву II роду