READINGS: Chapter 1 of required text.

Introduction to Control Systems

* What is "control"? : make some system behave as we desire
* Different kinds of control theory:
  1) Classical control theory [a.k.a conventional control theory]
  2) Modern control theory
  3) Robust control theory

* Why study control theory? {consequently Automatic control}
  - essential in any field of engineering and science
  - Applications include space-vehicle systems, robotic systems, modern manufacturing systems, any industrial operations involving control of temperature, pressure, humidity, flow etc.
  - convenient

* Feedback control systems:

  "A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control"

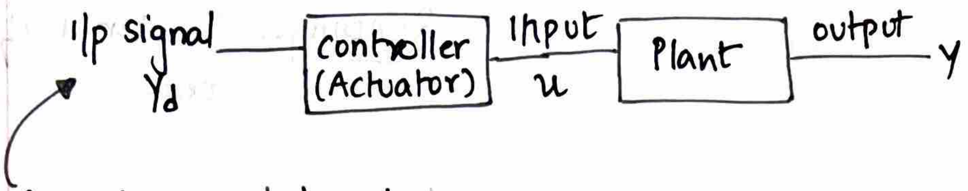
  Examples:

  - Room Temperature control system: By measuring the actual room temperature and comparing it with the reference temperature, the thermostat turns the heating or cooling equipment on or off in such a way as to ensure that the room temperature remains at a comfortable level.

* Closed-loop control systems:

  "The actuating error signal, which is the difference b/w the i/p signal and the feedback signal (which may be the output signal itself) is fed to the controller so as to reduce the error and bring the o/p of the system to a desired value."

  - used interchangeably with feedback control



* Open-loop control system:

"The system in which the o/p has no effect on the control action. In other words, the output is neither measured nor fed back for comparison with the input."

Example:

- Washing machine: Soaking, washing and rinsing in the washer operate on a time basis. The machine doesnot measure the output signal, i.e. the cleanliness of the clothes.

* Advantages of ~~closed~~ open loop control system:

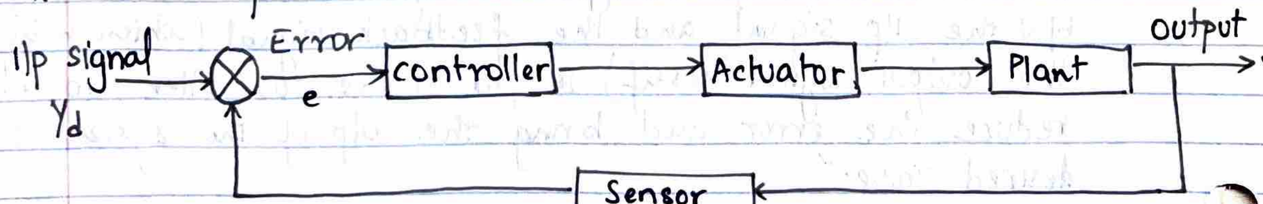
(i) Simple construction
(ii) ease of maintenance
(iii) less expensive than a corresponding closed loop system
(iv) There is no stability problem
(v) convenient when o/p is hard to measure

* Disadvantages of open loop control system:

(i) disturbances and changes in calibration cause errors and the o/p may be different from what is desired
(ii) To maintain the required quality in the output, recalibration is necessary from time to time.

* Calibration is key!!! in open-loop control system.

* Closed-loop (Feedback) control:



- Sensor and Actuator are key elements

* A tool in the belt for mathematical modeling of control systems.

* One of the most important mathematical tools in this course

Preliminaries : Laplace Transform

Let us define :

s = complex variable

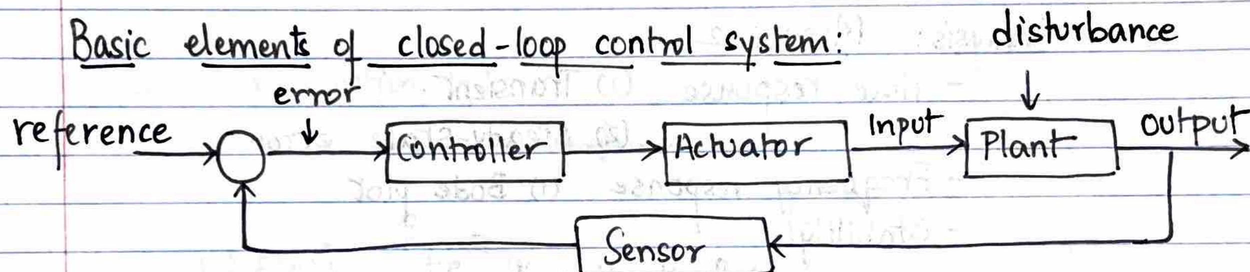
$f(t)$ = a function of time t s.t $f(t)=0$ for $t<0$

$F(s)$ = Laplace Transform of $f(t)$

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$$

—X— To be continued

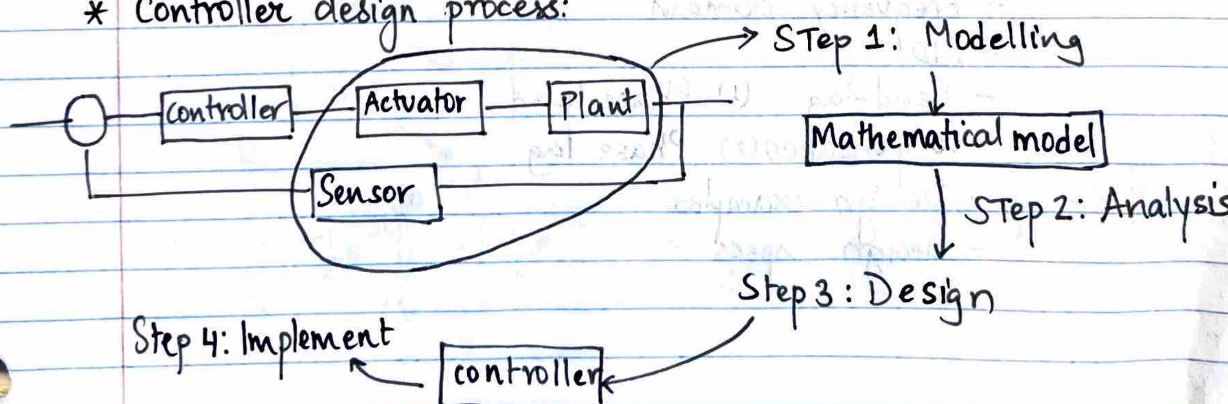
Basic elements of closed-loop control system:



* control system design objective:

"To design the controller s.t. the output follows the reference in a satisfactory manner even in the face of disturbances.

* Controller design process:



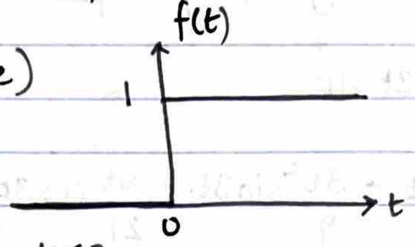
- Modeling : MODULE 1
  - Laplace Transform → (1) Laplace Transform, (2) Inverse Laplace transform
  - Models for systems
    (Focus on mechanical systems)
  - Transfer functions
  - Block diagram
  - Linearization
  - State space representation of dynamical systems
    (1) 1st order system (2) 2nd order system

- Analysis : MODULE 2
  - Time response (1) Transient
    (2) steady-state error
  - Frequency response (1) Bode plot
  - Stability
    (1) Routh - Hurwitz
    (2) Nyquist stability criterion

- Design : MODULE 3
  - Root Locus
  - Frequency domain
  - PID
  - Lead-lag compensation (1) Phase lead
    (2) Phase lag
  - Design examples
  - Design specs

Continuation of Laplace Transform:

Examples: (MEMORIZE These)

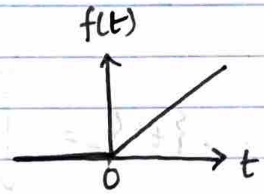


★ ① Unit step function:

$$f(t) = u_s(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$$

$$F(s) = L\{f(t)\} = \int_0^\infty u_s(t) e^{-st} dt = \int_0^\infty e^{-st} dt = -\frac{1}{s}\left[e^{-st}\right]_0^\infty = \frac{1}{s}$$

★ ② Unit ramp function:



$$f(t) = \begin{cases} 0 & t<0 \\ t & t \geq 0 \end{cases}$$

$$F(s) = L\{f(t)\} = \int_0^\infty t e^{-st} dt$$

Use integral formula.
(In future, we will find easier ways)

m=1

$$\begin{array}{ccc} \frac{d}{dt}(t) & & \int e^{-st} \\ & + & \\ 1 & & -\frac{1}{s} e^{-st} \\ & - & \\ 0 & & \frac{1}{s^2} e^{-st} \end{array}$$

$$F(s) = \left(t\left(\frac{-1}{s} e^{-st}\right)\right)_0^\infty - \left(\frac{1}{s^2} e^{-st}\right)_0^\infty = \frac{1}{s^2}$$

INTegral formula

If $\int t^m \begin{bmatrix} e^{at} \\ \sin at \\ \cos at \end{bmatrix} dt$, then use it

(OR)

A.K.A

Integration by parts

$$\int (uv)' dt = \int u'v dt + \int uv' dt$$

(or)

$$\int_a^b uv' dt = \left[uv\right]_a^b - \int_a^b u'v dt$$

problems using integral formula:

① $\int t^3 \sin 3t \, dt$

$$= -\frac{t^3}{3}\cos 3t + \frac{3t^2}{9}\sin 3t + \frac{6t}{27}\cos 3t - \frac{6}{81}\sin 3t$$

d/dt		$\int dt$
t^3		$\sin 3t$
$3t^2$	$(+)$	$\frac{-1}{3}\cos 3t$
$6t$	$(-)$	$\frac{-1}{9}\sin 3t$
6	$(+)$	$\frac{+1}{27}\cos 3t$
0	$(-)$	$\frac{1}{81}\sin 3t$

② $f(t) = t^2$

$$F(s) = L\{f(t)\} = L\{t^2\} = \int_0^\infty t^2 e^{-st} \, dt$$

$$F(s) = \left[-\frac{t^2}{s} e^{-st} - \frac{2t}{s^2} e^{-st} - \frac{2}{s^3} e^{-st} \right]_0^\infty$$

$$F(s) = \frac{2}{s^3}$$

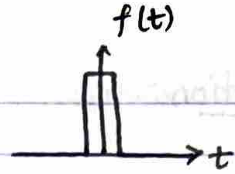
d/dt		$\int dt$
t^2		e^{-st}
$2t$	$(+)$	$\frac{-1}{s} e^{-st}$
2	$(-)$	$\frac{1}{s^2} e^{-st}$
0	$(+)$	$\frac{-1}{s^3} e^{-st}$

★ $\boxed{L\{t^n\} = \frac{n!}{s^{n+1}}}$ Memorize this

Remark:

Instead of computing Laplace transform for each function, and/or memorizing complicated Laplace transform use Laplace Transform table!!!

★ ③ Unit Impulse function:



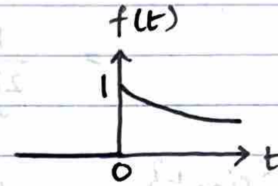
width = 0
height = ∞
Area = 1

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t) g(t)\, dt = g(0)$$

$$F(s) = L\{\delta(t)\} = \int_0^{\infty} \delta(t) e^{-st}\, dt = 1$$

★ ④ Exponential function:



$$f(t) = \begin{cases} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$F(s) = \int_0^{\infty} e^{-at} e^{-st} = \int_0^{\infty} e^{-(s+a)t} = \frac{-1}{(s+a)} \left[e^{-(s+a)t} \right]_0^{\infty} = \frac{1}{s+a}$$

⑤ Cosine function: $f(t) = \cos at = \dfrac{e^{jat} + e^{-jat}}{2}$

↑ Taylor series expansion

$$F(s) = L\{\cos at\} = \frac{1}{2} L\{e^{jat}\} + \frac{1}{2} L\{e^{-jat}\}$$

$$= \frac{1}{2}\left[\frac{1}{s-ja}\right] + \frac{1}{2}\left[\frac{1}{s+ja}\right]$$

$$= \frac{1}{2}\left[\frac{s+ja+s-ja}{s^2+a^2}\right] = \frac{s}{s^2+a^2}$$

$$\boxed{L\{\cos at\} = \frac{s}{s^2+a^2}}$$

⑥ Sine function $f(t) = \sin at = \dfrac{e^{jat} - e^{-jat}}{2j}$

$$F(s) = L\{\sin at\} = \frac{1}{2j} L\{e^{jat}\} - \frac{1}{2j} L\{e^{-jat}\}$$

$$= \frac{1}{2j}\left[\frac{1}{s-ja}\right] - \frac{1}{2j}\left[\frac{1}{s+ja}\right]$$

$$= \frac{1}{2j}\left[\frac{\cancel{s}+ja-\cancel{s}+ja}{s^2+a^2}\right]$$

$$F(s) = L\{\sin at\} = \frac{a}{s^2+a^2}$$

⑦ Hyperbolic cosine function:

$$f(t) = \cosh at = \frac{e^{at} + e^{-at}}{2}$$

$$F(s) = L\{f(t)\} = \frac{s}{s^2-a^2}$$

⑧ Hyperbolic sine function:

$$f(t) = \sinh at = \frac{e^{at} - e^{-at}}{2}$$

$$F(s) = L\{f(t)\} = \frac{a}{s^2-a^2}$$

Components of a control system: Terminology

* the plant is the system being controlled
* the sensors measure the quantity that is subject to control
* the actuators act on the plant
* the controllers processes the sensor signals and drives the actuators
* the control law is the rule for mapping sensor signals to actuator signals.

Properties of Laplace Transform:

① LINEARITY:

$$L\{\alpha_1 f_1(t) + \alpha_2 f_2(t)\} = \alpha_1 F_1(s) + \alpha_2 F_2(s)$$

Proof:

$$L\{\alpha_1 f_1(t) + \alpha_2 f_2(t)\} = \int_0^\infty [\alpha_1 f_1(t) + \alpha_2 f_2(t)] e^{-st} dt$$

$$= \alpha_1 \underbrace{\int_0^\infty f_1(t) e^{-st} dt}_{F_1(s)} + \alpha_2 \underbrace{\int_0^\infty f_2(t) e^{-st} dt}_{F_2(s)}$$

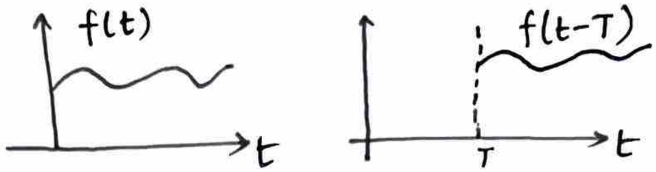
$$= \alpha_1 F_1(s) + \alpha_2 F_2(s)$$

Example:

$$L\{5u_s(t) + 3e^{-2t}\} = 5L\{u_s(t)\} + 3L\{e^{-2t}\}$$

$$= \frac{5}{s} + \frac{3}{s+2}$$

delayed by T seconds



2. TIME DELAY: $L\{f(t-T)u_s(t-T)\} = e^{-Ts}F(s)$

Proof:

$$L\{f(t-T)u_s(t-T)\} = \int_0^\infty \left(f(t-T)u_s(t-T)\right)e^{-st}\,dt$$

$$u_s(t-T) = \begin{cases} 0 & t<T \\ 1 & t \geq T \end{cases}$$

$$= \int_T^\infty f(t-T)e^{-st}\,dt$$

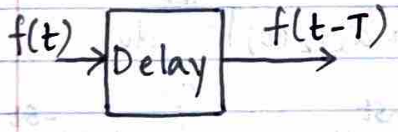
Let $t-T=\tau$

$t = \tau + T$

$$= \int_T^\infty f(\tau)e^{-s(\tau+T)}\,d\tau$$

$$= e^{-sT}F(s)$$

Example: $L\{e^{-0.5(t-4)}u_s(t-4)\} = \dfrac{e^{-4s}}{s+0.5}$



t-domain

s-domain

3. DIFFERENTIATION: $\boxed{L\{f'(t)\} = sF(s) - f(0)}$

Proof:

$$L\{f'(t)\} = \int_0^\infty f'(t)e^{-st}\,dt = \left[f(t)e^{-st}\right]_0^\infty + s\int_0^\infty f(t)e^{-st}\,dt$$

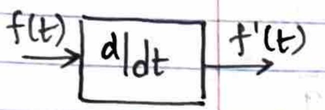
$$= sF(s) - f(0)$$

Example:

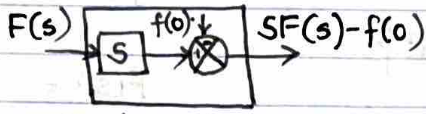
- $L\{(\cos 2t)'\} = s\,L\{\cos 2t\} - \cos 0$

$$= \frac{s^2}{s^2+4} - 1$$

- $L\{3\cos 3t\} = L\{(\sin 3t)'\} = s\,L\{\sin 3t\} - \cancel{\sin 0}^{\,0}$



Time domain



S-domain

$$= s \cdot \frac{3}{s^2+9}$$

$$L\{3\cos 3t\} = \frac{3s}{s^2+9}$$

Higher-order derivatives : Applying derivative formula twice yields

$$L\{f''(t)\} = s\,L\{f'(t)\} - f'(0)$$

$$= s\,[sF(s) - f(0)] - f'(0)$$

$$\boxed{L\{f''(t)\} = s^2F(s) - sf(0) - f'(0)}$$

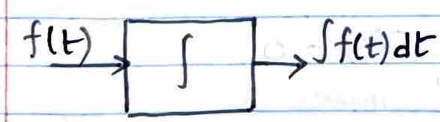
④ INTEGRATION : $L\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$

Proof: $L\left[\int_0^t f(\tau)d\tau\right] = \int_0^\infty \left[\int_0^t f(\tau)d\tau\right] e^{-st}dt$

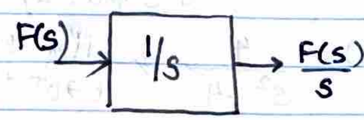
$$= -\frac{1}{s}\left[\left(\int_0^t f(\tau)d\tau\right)e^{-st}\right]_0^\infty + \frac{1}{s}\int_0^\infty f(t)e^{-st}dt$$

$$= \frac{F(s)}{s}$$

Time-domain



S-domain



⑤ $L\{e^{at} f(t)\} = F(s-a)$

$f(t) = \sin 3t$

Example: $L\{e^{5t} \sin 3t\}$

$F(s) = \frac{3}{s^2+9}$

$L\{e^{5t} \sin 3t\} = \frac{3}{(s-5)^2+9}$

⑥ FINAL VALUE THEOREM: $\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$

if all the poles of $sF(s)$ are in the left half plane

Example:

- $F(s) = \frac{5}{s(s^2+s+2)} \Rightarrow sF(s) = \frac{5}{s^2+s+2}$

poles of $sF(s)$: $-0.50 \pm j1.3229$

are in the left-half plane. Therefore, apply final value theorem:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} \frac{5}{s^2+s+2} = \frac{5}{2}$$

- $F(s) = \frac{4}{s^2+4} \Rightarrow sF(s) = \frac{4s}{s^2+4}$

poles: $\pm 2j$

are on the complex axis.

Therefore we cannot apply final value theorem.

⑦. INITIAL VALUE THEOREM: $\boxed{\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)}$ if the limit exists

* It doesnot matter if pole location is in LHS or not.

Example:

$F(s) = \frac{5}{s(s^2+s+2)} \Rightarrow \lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s) = 0$

$F(s) = \frac{4}{s^2+4} \Rightarrow \lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s) = 0$

In control systems, convolution plays an important role

$$U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s) \qquad Y(s) = U(s)\,G(s)$$

⑧ CONVOLUTION: Let $F_1(s) = L\{f_1(t)\}$

$F_2(s) = L\{f_2(t)\}$

Then

$$F_1(s)F_2(s) = L\{f_1(t) * f_2(t)\} = F_1(s)\,F_2(s) = L\left\{\int_0^t f_1(\tau)f_2(t-\tau)\,d\tau\right\}$$

$$= L\left\{\int_0^t f_1(t-\tau)\,f_2(\tau)\,d\tau\right\}$$

Note: $\boxed{F_1(s)\,F_2(s) = L\{f_1(t)\,f_2(t)\}}$

Example: $f_1(t) = t \qquad f_2(t) = \sin 4t$

$$f_1(t) * f_2(t) = \int_0^t f_1(\tau)\,f_2(t-\tau)\,d\tau$$

$$= \int_0^t \tau \sin 4(t-\tau)\,d\tau$$

$$= \left[\frac{\tau}{4}\cos 4(t-\tau) + \frac{1}{16}\sin 4(t-\tau)\right]_0^t$$

$$= \frac{t}{4} - \frac{\sin 4t}{16}$$

$$\begin{array}{c|cc}
 & \dfrac{d}{d\tau} & \int d\tau \\
 & \tau & \sin 4(t-\tau) \\
(+) & 1 & \frac{1}{4}\cos 4(t-\tau) \\
(-) & 0 & -\frac{1}{16}\sin 4(t-\tau)
\end{array}$$

$$F_1(s)\,F_2(s) = L\{f_1(t) * f_2(t)\} = L\left\{\frac{t}{4}\right\} - L\left\{\frac{\sin 4t}{16}\right\}$$

$$= \frac{1}{4}\cdot\frac{1}{s^2} - \frac{1}{\cancel{16}_{\,4}}\cdot\frac{\cancel{4}}{s^2+16}$$

$$= \frac{1}{4}\left[\frac{1}{s^2} - \frac{1}{s^2+16}\right]$$

$$= \frac{4}{s^2(s^2+16)}$$

ADVANTAGE of LAplace Transform:

$$\begin{tikzcd}
\boxed{\text{ODE}} \arrow[r, "① \mathcal{L}"] & \boxed{\text{AE}} \arrow[d, "②"] \\
\boxed{\text{Solution to ode}} & \boxed{\text{Partial fraction expansion}} \arrow[l, "③ \mathcal{L}^{-1}"']
\end{tikzcd}$$

Time domain | s-domain

LAPLACE Inverse Transform:

Given $Y(s) = \frac{A(s)}{B(s)}$ where

$$A(s) = a_0 s^m + a_1 s^{m-1} + \dots + a_m$$

$$B(s) = s^n + b_1 s^{n-1} + \dots + b_n$$

$$n \geq m$$

Obtain: $y(t)$ where $L\{y(t)\} = Y(s)$

Case (1) Let $B(s)$ have non-repeated real roots, then $B(s)$ can be expressed as

$$B(s) = (s+p_1)(s+p_2) \dots (s+p_n)$$

$p_1, p_2, \dots p_n$ are all real numbers

Then $Y(s)$ can be expressed as

$$Y(s) = \frac{c_1}{(s+p_1)} + \frac{c_2}{(s+p_2)} + \dots + \frac{c_n}{(s+p_n)}$$ where

METHOD 1:

$$\rightarrow c_i = \lim_{s \to -p_i} (s+p_i) Y(s) \quad \text{for } i = 1, \dots n$$

$$\boxed{y(t) = \sum_{i=1}^{n} c_i e^{-p_i t}}$$

Example:

$$Y(s) = \frac{s}{(s^2+3s+2)} \qquad s^2+3s+2 = (s+1)(s+2)$$

$$y(t) = L^{-1}\left\{\frac{s}{s^2+3s+2}\right\} = L^{-1}\left\{\frac{c_1}{s+1}\right\} + L^{-1}\left\{\frac{c_2}{s+2}\right\}$$

$$c_1 = \lim_{s \to -1} \cancel{(s+p_1)} \frac{s}{\cancel{(s+1)}(s+2)} = \frac{-1}{1} = -1$$

$$C_2 = \lim_{s \to -2} \cancel{(s+p_2)} \frac{s}{(s+1)\cancel{(s+2)}} = \frac{+2}{+1} = 2$$

$$Y(t) = L^{-1}\left\{\frac{-1}{s+1}\right\} + L^{-1}\left\{\frac{2}{s+2}\right\} = -e^{-t} + 2e^{-2t}$$

Case (2): $B(s)$ has real roots with some repeated roots. Then $B(s)$ can be rewritten as

$$B(s) = (s+p_1)^b (s+p_2) \ldots\ldots (s+p_{n-b+1})$$

Then

$$Y(s) = \frac{c_1}{s+p_1} + \ldots\ldots + \frac{c_b}{(s+p_1)^b} + \frac{c_{b+1}}{(s+p_2)} + \ldots\ldots + \frac{c_n}{(s+p_{n-b+1})}$$

$f(t)$ becomes $c_1 e^{-p_1 t} + \ldots\ldots + \dfrac{c_b e^{-p_1 t} t^{b-1}}{(b-1)!} + c_{b+1} e^{-p_2 t} + \ldots\ldots + c_n e^{-p_{n-b+1} t}$

Example:

1. $Y(s) = \dfrac{1}{s^n}$ According to case 2: $\quad L\{t^n\} = \dfrac{n!}{s^{n+1}}$

$$y(t) = L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{1}{s^n}\right\} \qquad L\{t^{n-1}\} = \frac{(n-1)!}{s^n}$$

$$= \frac{t^{n-1}}{(n-1)!}$$

~~2. $Y(s) = \frac{1}{(s+p_1)^n}$, Then $y(t) = L^{-1}\{Y(s)\} = e^{-p_1 t}\, y(t)$~~

2. $Y(s+p_1) = \dfrac{1}{(s+p_1)^n}$. Then $y(t) = L^{-1}\{Y(s+p_1)\} = \dfrac{e^{-p_1 t}\, t^{n-1}}{(n-1)!}$

(Think of Time delay)

Example of feedback control system

1. Cruise control:
   - common feedback system encountered in everyday life.

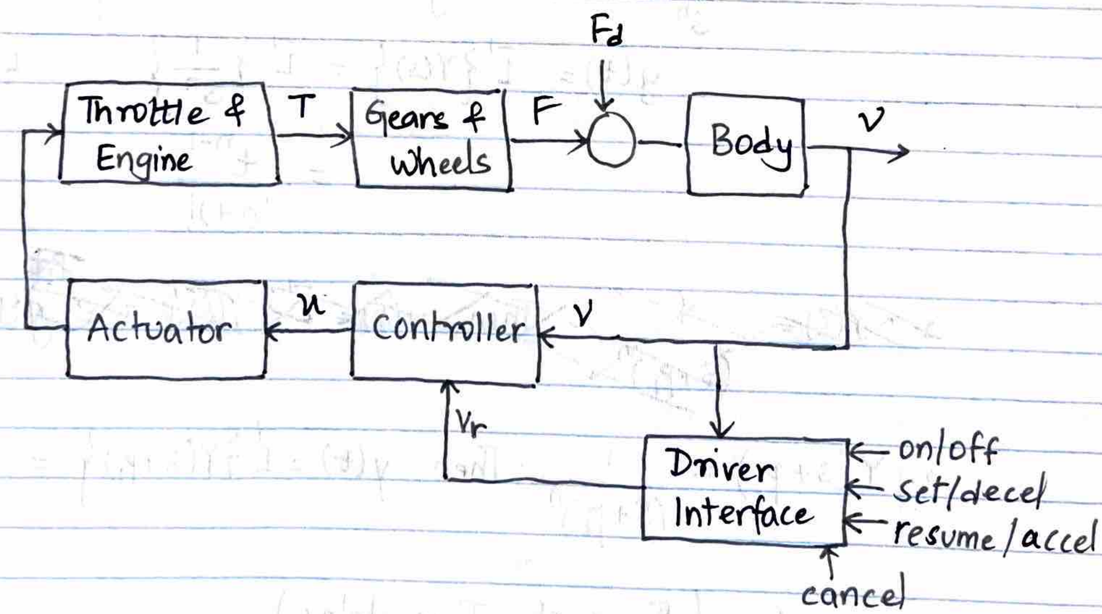
AIM: maintain a constant velocity in the presence of disturbances (primarily caused by changes in the slope of a road)

How:

Design the controller that compensates for these unknowns by measuring the speed of the car and adjusting the throttle appropriately.

Block diagram:

v : speed of the car
v_r : desired (reference) speed



* The controller (typically PI) receives the signals v and v_r and generates a control signal u that is sent to the actuator that controls the throttle position

* The throttle in turn controls the torque T delivered by the engine, which is transmitted through the gears and the wheels, generating a force F that moves the car.

* There are disturbances (F_d) due to variations in the slope of the road, the rolling resistance and aerodynamic forces.

* The cruise controller also has a human-machine interface that allows the driver to set and modify the desired speed.

3. $F(S) = \dfrac{(S+5)}{(S+1)^2}$ Apply partial fractions

$$\frac{S+5}{(S+1)^2} = \frac{c_1}{S+1} + \frac{c_2}{(S+1)^2}$$

$$\frac{S+5}{(S+1)^2} = \frac{c_1(S+1) + c_2}{(S+1)^2}$$

Compare coefficients : $S+5 = c_1 S + (c_1 + c_2)$

$c_1 = 1$

$c_1 + c_2 = 5$

$c_2 = 4$

$$\therefore F(s) = \frac{1}{S+1} + \frac{4}{(S+1)^2}$$

Taking inverse laplace

$$f(t) = e^{-t} + 4te^{-t}$$

4. Given $F(s) = \dfrac{s^2+2s+5}{(s+3)^2(s+5)}$ Find $f(t)$

$$\frac{s^2+2s+5}{(s+5)(s+3)^2} = \frac{A}{s+5} + \frac{B}{s+3} + \frac{C}{(s+3)^2}$$

$$\Rightarrow \frac{s^2+2s+5}{(s+5)(s+3)^2} = \frac{A(s+3)^2 + B(s+5)(s+3) + C(s+5)}{(s+5)(s+3)^2}$$

Comparing coefficients

$$\Rightarrow s^2+2s+5 = A(s^2+6s+9) + B(s^2+8s+15) + C(s+5)$$

$A + B = 1$

$A = 1 - B$

$\boxed{A = 5}$

$6A + 8B + C = 2$

$6(1-B) + 8B + C = 2$

$6 - 6B + 8B + C = 2$

$2B + C = -4$

$C = -4 - 2B$

$\boxed{C = 4}$

$9A + 15B + 5C = 5$

$9(1-B) + 15B + 5(-4-2B) = 5$

$9 - 9B + 15B - 20 - 10B = 5$

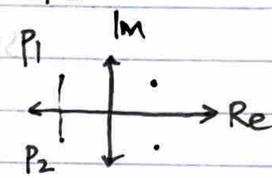
$-4B = 16$

$\boxed{B = -4}$

Therefore, $\dfrac{s^2+2s+5}{(s+5)(s+3)^2} = \dfrac{5}{s+5} - \dfrac{4}{(s+3)} + \dfrac{4}{(s+3)^2}$

Taking inverse Laplace

$$f(t) = L^{-1}\{F(s)\} = 5e^{-5t} - 4e^{-3t} + 4te^{-3t}$$



CASE (3): $B(s)$ has non-repeated complex roots
(The roots of $B(s)$ are conjugates)

Then $(s-p_1)(s-p_2) = s^2 + 2as + (a^2+b^2)$

Let $p_1 = -a+jb$
$p_2 = -a-jb$

Example: $F(s) = \dfrac{5}{s^2+2s+2} = \dfrac{A(s)}{B(s)}$

Roots of $s^2+2s+2 = $ $p_1 : -1+j$
$p_2 : -1-j$

$F(s) = \dfrac{5}{(s+1)^2+1}$

[check Laplace transform of
$L\{e^{-at}\sin wt\} = \dfrac{w}{(s+a)^2+w^2}$]

$\therefore f(t) = L^{-1}\{F(s)\} = 5e^{-t}\sin t$

Completing the SQUARES:

If $F(s) = \dfrac{1}{s^2+as+b}$

Step 1: Ensure s^2+as+b has complex roots

Step 2: convert and write $F(s)$ in the following form

$$F(s) = \frac{1}{\left(s+\frac{a}{2}\right)^2 + \left(b - \frac{a^2}{4}\right)}$$