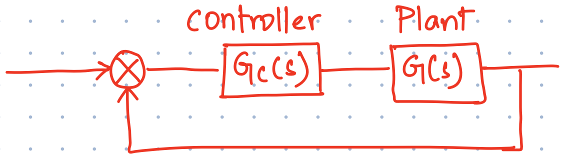
Lead-Lag compensator:



General form of $G_c(s) = K \dfrac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\alpha T}\right)}$

$G(s)$ - given (fixed)

$G_c(s)$ - designable

zeros:

$s = \dfrac{-1}{T}$

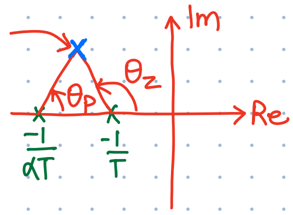
poles:

$s = -\dfrac{1}{\alpha T}$

Lead compensator

$0 < \alpha < 1$

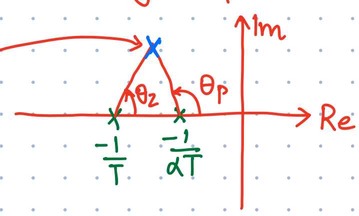
Desired dominant closed loop pole



$\alpha > 1$

Lag compensator

Desired dominant closed loop pole



$\angle G_c(s) = \angle \dfrac{(s+1/T)}{(s+1/\alpha T)}$

$\angle G_c(s) = \angle s + 1/T - \angle s + \dfrac{1}{\alpha T}$

$\angle G_c(s) = \theta_z - \theta_p$

* For lead compensator: $\theta_z - \theta_p > 0$
* For lag compensator: $\theta_z - \theta_p < 0$

A point "s" is said to be on root locus (conditions):

(1) Angle / Phase condition:

$\angle G_c \; G(p') = \pi \pm 2n\pi \quad n = 0, 1, 2 \ldots$

$p' \rightarrow$ desired closed loop poles

(2) Magnitude condition:

$|G_c \; G(p')| = 1$

Steps for lead-lag compensator design:

1. Determine the desired location of the dominant closed loop poles (GIVEN)

2. Evaluate $G(s)$ at the desired pole

3. Determine angle deficiency ϕ.

   If $\phi > 0$ = lead compensator

   else lag compensator

4. Determine $-1/T$, $-1/\alpha T$:

   * Draw a horizontal line parallel to real axis passing through desired pole.
   * Join the pole p' with origin
   * Bisect the angle θ
   * Draw lines of $\phi/2$
   * Intersection with (-ve) real axis are desired points.

Lead compensator:

* Improves stability

Lag compensator:

* reduces steady state error

from below example: desired poles

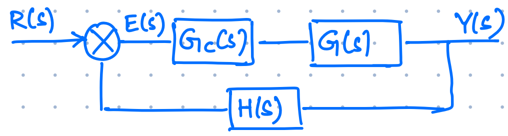
$$(s-p_1')(s-p_2')=0$$

$$s^2+3s+9=0$$

$2\zeta\omega_n=3$ $\qquad$ $\omega_n^2=9$

$\omega_n=3$

Example:



$* \; G_c(s) = K \dfrac{(s + 1/T)}{\left(s + \frac{1}{\alpha T}\right)}$

$* \; G(s) = \dfrac{10}{s(s+1)}$ $\quad * \; H(s) = 1$

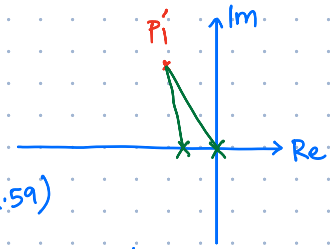
① Desired dominant closed loop poles:

$$p' = 3(-\cos 60^\circ \pm j \sin 60^\circ)$$

$$p' = -3\cos 60^\circ \pm j\, 3 \sin 60^\circ$$

$$\boxed{p' = \frac{-3}{2} \pm j\, \frac{3\cdot\sqrt{3}}{2}} \simeq -1.5 \pm j\, 2.59$$

$\sqrt{\frac{01234}{4}}$

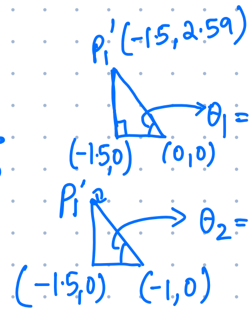


② Evaluate $\angle G(s)$ at desired poles

$$\angle G(p') = \angle 10 - \left(\angle P_1' + \angle P_1' + 1\right)$$

$$\angle G(p') = -\left(120^\circ + 100.93\right) = -220.93^\circ$$

Doesnot satisfy phase condition



$\theta_1 = \tan^{-1}\left(\dfrac{2.59}{1.5}\right) \approx 60^\circ$

$\theta_2 = \tan^{-1}\left(\dfrac{2.59}{0.5}\right) \approx 79.07^\circ$

40.93°

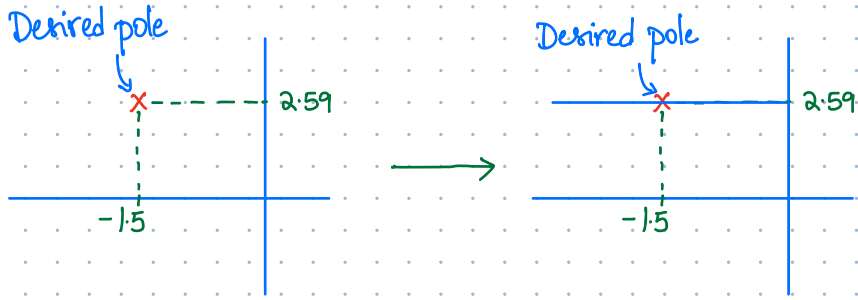
* Desired closed loop poles are not on the root locus. So simple gain adjusted will not work. So introduce a compensator in such a way that root locus passes through desired poles.

③ Calculate Angle deficiency:

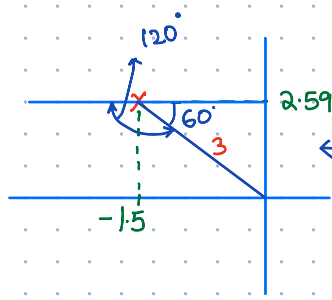
It should be -180° but it actually is -220.93°. Therefore angle compensation of 40.93° is required

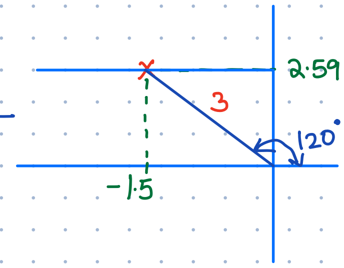
$$\boxed{\phi = 40.93^\circ}$$ $\therefore$ Lead compensator.

④



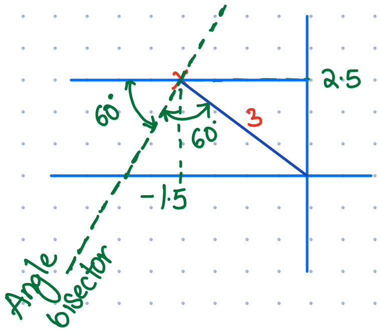
Draw horizontal line parallel to the real axis passing through desired pole

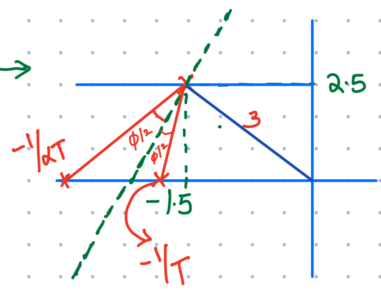




Join the pole & origin

Draw angle bisector of $\theta = 120°$





Angle required to compensate : ϕ

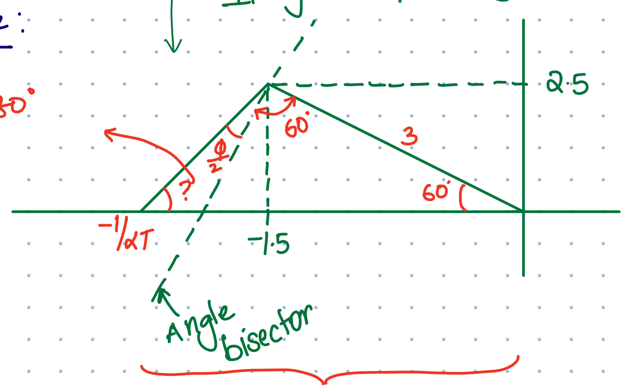
Take $\frac{\phi}{2} = \frac{40.93}{2} = 20.465°$

It's just Geometry

* Determine the position of pole :

$? + \left(\frac{\phi}{2} + 60°\right) + 60° = 180°$

$\boxed{? = 39.535°}$



X ← what is x?

Law of Sines

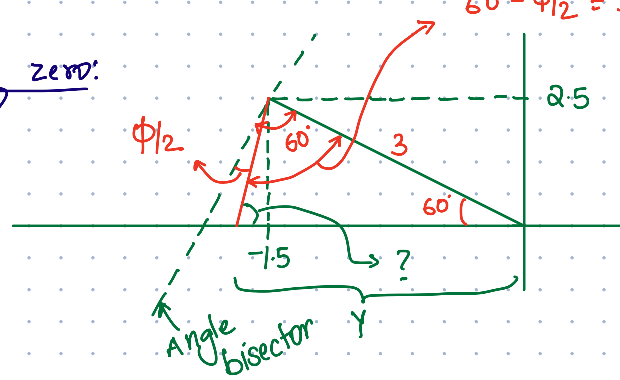
$$\frac{x}{\sin(80.465)} = \frac{3}{\sin(39.535°)}$$

$\boxed{x = 4.647}$

$\therefore \boxed{\frac{1}{\alpha T} = 4.647}$

$60° - \phi/2 = 39.535°$

* Determine the position of zero:



$? + 39.535° + 60° = 180°$

$? = 80.465°$

Law of sines:

$$\frac{3}{\sin 80.465} = \frac{y}{\sin 39.535}$$

$$\boxed{y = 1.9364}$$

$$\therefore \frac{1}{T} = 1.9364$$

$$\boxed{T = 0.5164}$$

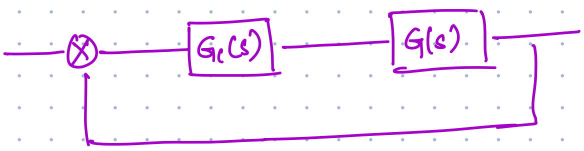
$$\frac{1}{\alpha T} = 4.647$$

$$\alpha = \frac{1}{4.647 \times T} = 0.4167$$

$$\boxed{\alpha = 0.4167}$$

Compensation Transfer function:

$$G_c(s) = K \frac{(s+1.9364)}{(s+4.647)}$$



Determine K:

Using magnitude condition

$$|G_c(s)\, G(s)| = 1$$

$$\left| K \frac{(s+1.9364)}{(s+4.647)} \cdot \frac{10}{s(s+1)} \right| = 1$$

$@\, s = -1.5 \pm j2.59$

$$K = \left| \frac{(s+4.647)\, s\, (s+1)}{10\,(s+1.9364)} \right|_{@\, s = -1.5 \pm j2.59}$$

$$K = \left| \frac{(3.147 + j2.59)(-1.5 + j2.59)(-0.5 + j2.59)}{10\,(0.4364 + j2.59)} \right|$$

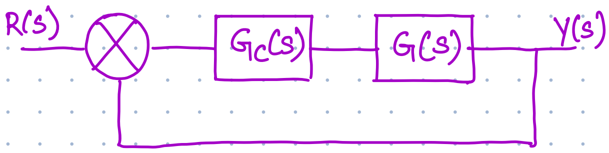
$$\boxed{K = 1.2251}$$

* Lead compensator T.F $G_c(s) = 1.2251 \dfrac{(s+1.9364)}{(s+4.647)}$

* Open loop compensated system : $G_c(s)\,G(s) = \dfrac{1.2251\,(s+1.9364)}{(s+4.647)} \cdot \dfrac{10}{s(s+1)}$

* closed loop compensated system: $\dfrac{Y(s)}{R(s)} = \dfrac{12.251\,(s+1.9364)}{s(s+1)(s+4.647) + 12.251(s+1.9364)}$

QUIZ:



$$G(s) = \frac{4}{s(s+2)}$$

If $G_c(s) = 1$

$$\frac{Y}{R} = \frac{G(s)}{1+G(s)} = \frac{\frac{4}{s(s+2)}}{1+\frac{4}{s(s+2)}}$$

$$\frac{Y}{R} = \frac{\frac{4}{\cancel{s(s+2)}}}{\frac{s(s+2)+4}{\cancel{s(s+2)}}} = \frac{4}{s^2+2s+4}$$

$2\zeta\omega_n = 2$ $\quad$ $\omega_n^2 = 4$

$\boxed{\zeta = 0.5}$ $\quad$ $\boxed{\omega_n = 2}$

* Specifications of the uncompensated system

* Required specifications of the compensated system

$\boxed{\omega_n = 4}$ $\quad$ $\boxed{\zeta = 0.5}$

$$s^2 + 2\zeta\omega_n + \omega_n^2 = 0$$

desired poles can be computed from this equation