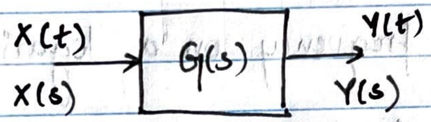
* Sinusoidal T.F is obtained by replacing 's' with 'jω' where ω is frequency in rad/s.

Control system analysis and design by frequency response

* frequency response: steady state response of a system to a sinusoidal input.
  → vary the frequency of the input signal over a certain range and study the resulting response.

Advantage: * we can use the data obtained from measurement on the physical system without deriving its mathematical model.

Obtain steady state o/p to sinusoidal inputs



replace $s \to j\omega$

* If the input is a sinusoidal signal, the steady-state o/p will also be a sinusoidal signal of the same frequency; but with possibly diff. magnitude and phase angle.

Check proof in T.B (Pages 399, 400)

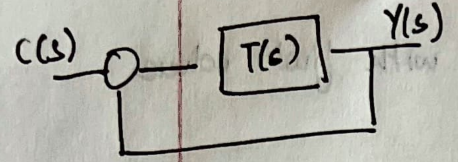
$$y_{ss}(t) = A\,|G(j\omega)|\sin(\omega t + \phi) \qquad x(t) = A\sin\omega t$$

$$\phi = \angle G(j\omega)$$

Bode plot:
The plot of $20\log_{10}|G(j\omega)|$ vs ω is called Bode plot of G

$$G(s) = \frac{s(2s+1)}{s^2+2s+5} \qquad \log_{10} G(s) = \log_{10}(s) + \log_{10}(2s+1) - \log_{10}(s^2+2s+5)$$

$$\log(s^2+2s+5)^{10} = 10\log_{10}(s^2+2s+5)$$



$G_{cl}(s) = \dfrac{T(s)}{1+T(s)}$ $\quad T(s) = -1 \leftarrow$ closed loop poles

Magnitude: $|T(s)| = 1$

$20\log_{10}|T(s)| = 20\log_{10} 1 = 0\,dB$

Angle: $\angle T(s) = \angle -1 = -180$

Basics of Bode plot:

* only applicable to minimum phase T.F (all poles & zeros are there in left half of s-plane)

$x(t) = A\sin\omega t \rightarrow$ [system] $\rightarrow y(t) = A'\sin(\omega t + \phi)$

(A' ↑ Gain, ϕ ↑ phase)

Consists of 2 diagrams:

$|G(j\omega)|$ vs ω — ① Amplitude plot (Gain) (logarithm of the magnitude of a sinusoidal T.F)

$\angle G(j\omega)$ vs ω — ② Phase plot (phase angle)

Plotted against the frequency on a logarithmic scale.

Procedure to plot Bode plot

① write given T.F in standard form [should only contain the following terms: s, $Ts+1$, $\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1$, K (gain)

$$G(s) = \frac{(s+a)(s+b)}{(s+p)(s+q)}$$

$$G(s) = \frac{a\left[\frac{s}{a}+1\right] b\left[1+\frac{s}{b}\right]}{p\left[1+\frac{s}{p}\right] q\left[1+\frac{s}{q}\right]} = \frac{ab}{pq}\left[\frac{(1+s/a)(1+s/b)}{(1+s/p)(1+s/q)}\right]$$

$\frac{ab}{pq}$ ↑ constant gain K

② Identify slope of 1st line for bode plot (based on poles & zeros @ origin) ex: if one pole @ origin → slope = −20 dB/decade

two poles @ origin → slope = −40 dB/decade

one zero @ origin → Slope = 20 dB/decade

two zeros @ origin → slope = 40 dB/decade

③ Gain of 1st line @ $\omega = 1$ rad/s

$$\text{gain}\Big|_{\omega = 1 \text{rad/s}} = 20 \log_{10} K \qquad \text{where } K = \frac{ab}{pq}$$

Practice example:

conversions:

$$(s+5) = 5\left(\frac{s}{5}+1\right) \qquad (s+6) = 6\left(\frac{s}{6}+1\right)$$

$$s^2+3s+4 = 4\left[\left(\frac{s}{2}\right)^2 + \frac{3}{4}s + 1\right]$$

$$G(s) = \frac{s \cdot 5\left(1+s/5\right)}{6\left(1+s/6\right)4\left\{\left(\frac{s}{2}\right)^2 + \frac{3}{4}s + 1\right\}}$$

dec: unit for measuring ratios on log scale

④ write all corner frequencies in ascending order and define slope for each line

2 poles (p, q) These frequencies are corner frequencies
2 zeros (a, b)

ex: $a > p > b > q$

ω	pole/zero	slope	resulting slope
a	zero	+20	+20
p	pole	-20	0
b	zero	+20	+20
q	pole	-20	0

⑤ Write phase equation and make a table of $\phi \to \omega$

$s = j\omega$

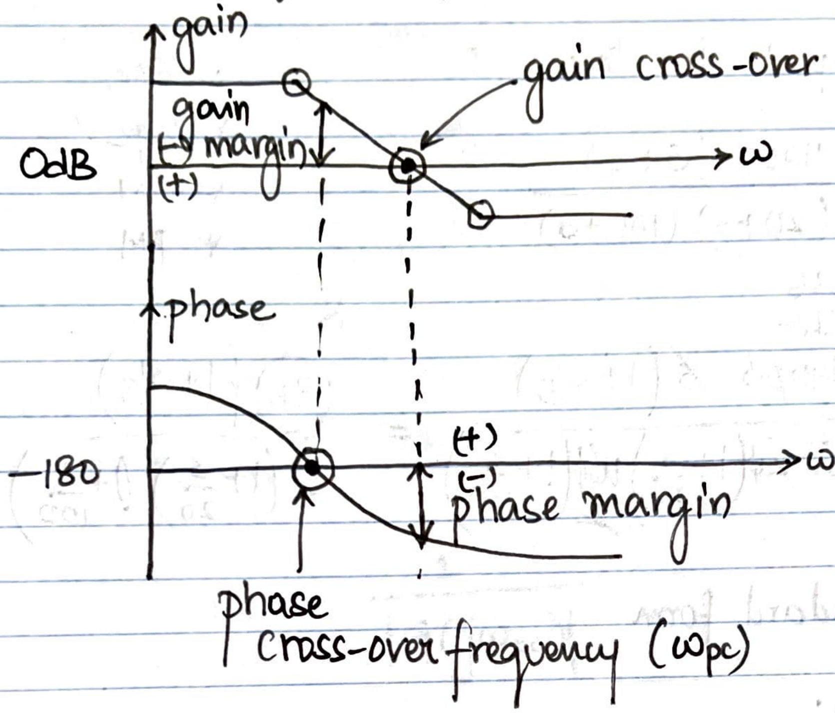
$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) + \tan^{-1}\left(\frac{\omega}{b}\right) - \tan^{-1}\left(\frac{\omega}{p}\right) - \tan^{-1}\left(\frac{\omega}{q}\right)$$

place values of ω write a table.

Advantages:

① we can identify stability of the system
② obtain GM, PM with minimum calculations

parameters of Bode plot:



$\omega_{gc} > \omega_{pc}$
: unstable

gain margin is (-) over OdB
(+) below OdB

phase margin is (+) above -180°
(-) below -180°

Stability by Bodeplot:

Using gain cross over frequency & phase cross over frequency

if $\omega_{pc} > \omega_{gc} \rightarrow$ system is stable

if $\omega_{pc} < \omega_{gc} \rightarrow$ unstable system

if $\omega_{pc} = \omega_{gc} \rightarrow$ marginally stable system