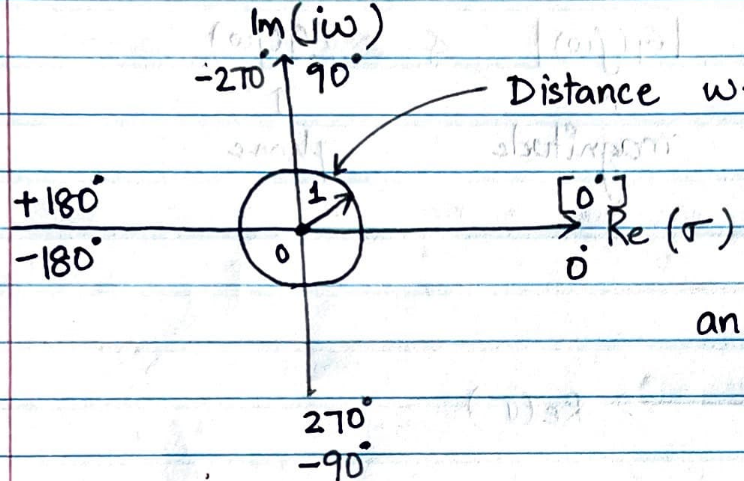
Basics and Advantages of polar plot

- polar plot is used for frequency response characteristics of the system
- polar plot is a plot of magnitude & phase by varying frequency from 0 to ∞.



Distance w.r.t center is magnitude $|G(j\omega)|$

phase = Angle w.r.t real axis $\angle G(j\omega)$

anti-clockwise → +ve
clockwise → -ve
} phase

Advantages:

1. polar plot explains frequency response characteristics by a single plot. It includes details of magnitude and phase w.r.t frequency.

2. Graphical study of stability is easy to determine by polar plot (compared to root locus & bode plot).

3. easy to determine ω_{gc} and ω_{pc} by polar plot.

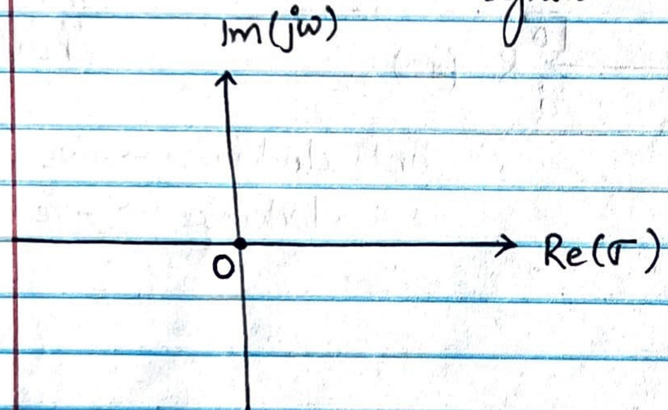
4. In frequency domain technique, we need CLTF but polar plot can be plotted by OLTF.

Polar plot and procedure to plot:

- it is a plot of magnitude and phase by varying frequency from 0 to ∞.

- Open Loop Transfer function of system as $G(s)$:

$$G(j\omega) = \underset{\uparrow \atop \text{magnitude}}{|G(j\omega)|} \;\; \& \;\; \underset{\uparrow \atop \text{phase}}{\angle G(j\omega)}$$



Procedure:

1. Determine Open Loop Transfer function $\rightarrow G(s)$
2. Substitute $s = j\omega$ and write $|G(j\omega)|$ & $\angle G(j\omega)$
3. Substitute $\omega = 0$ & $\omega = \infty$ and find $|G(j\omega)|$ & $\angle G(j\omega)$
4. Separate out real and imaginary parts of $G(j\omega)$

$$\rightarrow G(j\omega) = \text{Real}\,[G(j\omega)] + j\,\text{Imaginary}\,[G(j\omega)]$$

TIP: Separation of real and imaginary parts is done by multiplying & dividing complex conjugate of denominator of $G(j\omega)$

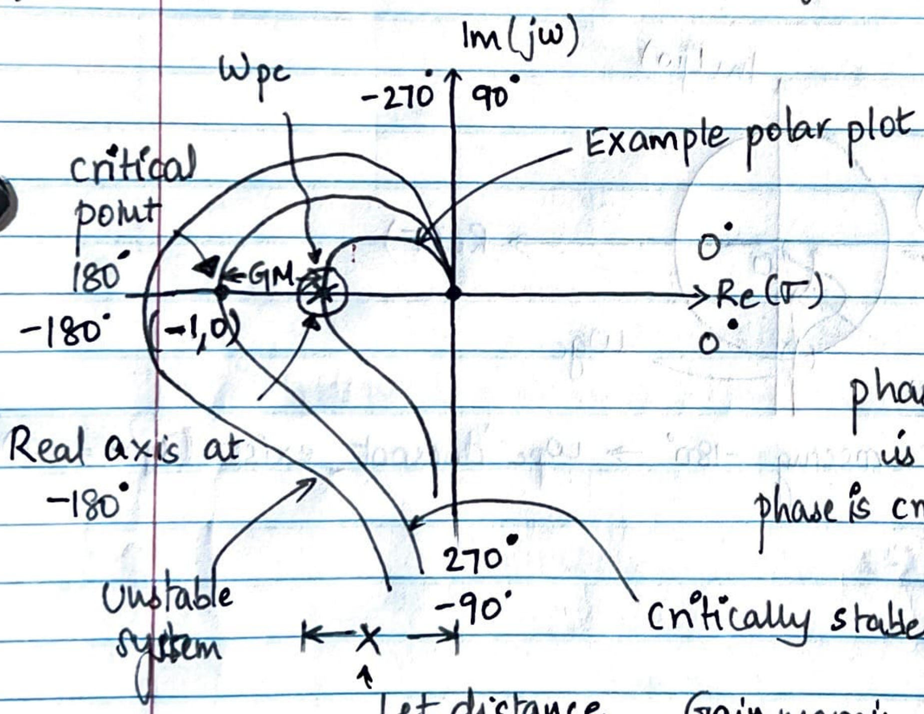
5. Intersection with real and imaginary axis

* Equate $Re[G(j\omega)] = 0 \Rightarrow$ intersection with imaginary axis

* Equate $Im[G(j\omega)] = 0 \Rightarrow$ intersection with real axis

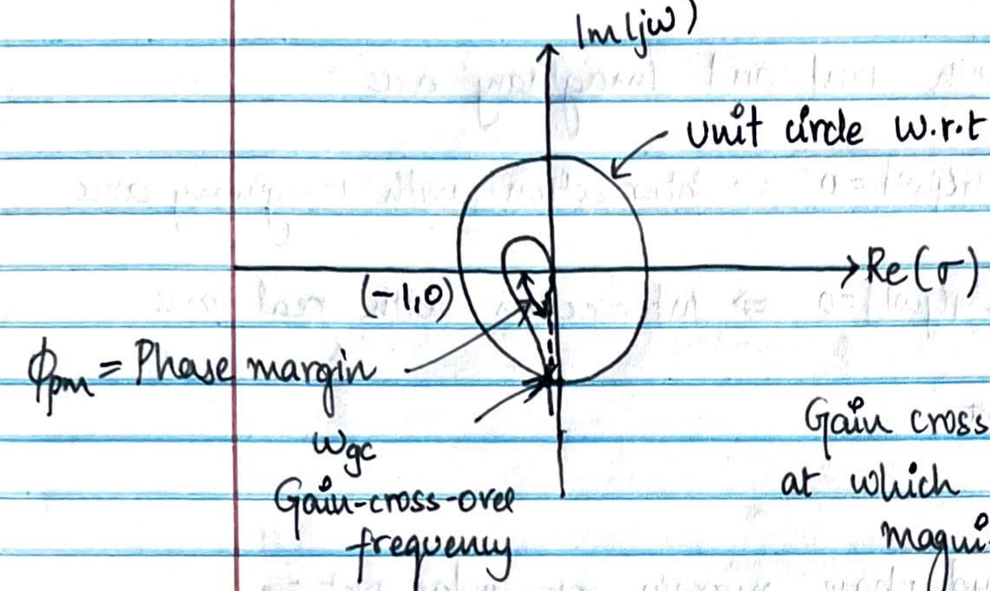
6. Connect the dots

Gain margin and phase margin on polar plot:



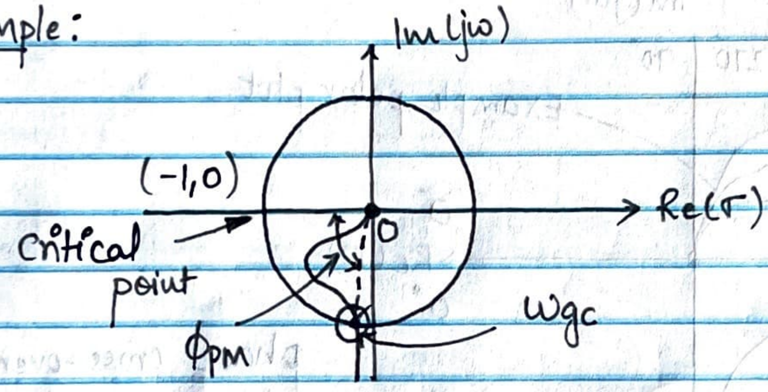
phase cross-over frequency is the frequency at which phase is crossing is -180°

Gain margin
$GM = 20\log\left(\frac{1}{x}\right)$

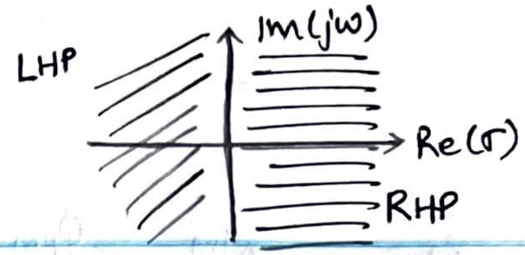


Gain cross over frequency is frequency at which gain crosses unit magnitude.

Example:



* graph is not crossing -180° → ω_{pc} doesnot exist (GM = ∞)
↳ polar plot

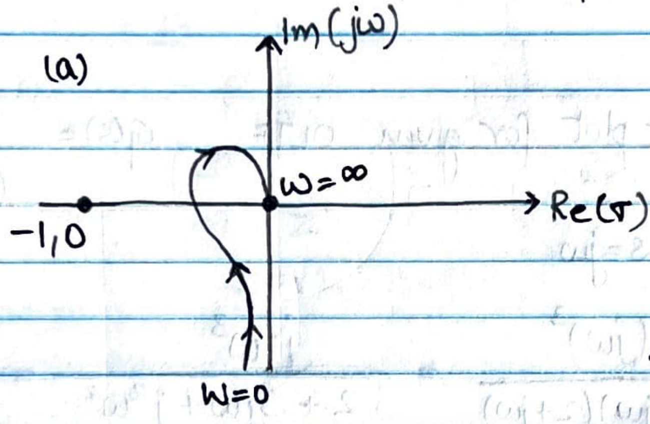


Stability using polar plot:

→ polar plot stability analysis is only applicable to minimum phase system
For minimum phase system, all the poles and zeros are in LHP.

→ To check stability, see critical point (-1,0). check enclosement around (-1,0) in polar plot

(a)

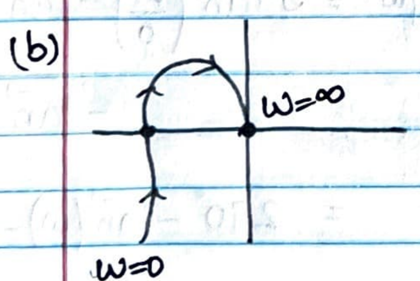


* see right side of the direction of this path ⇒ critical pt is not enclosed

∴ STABLE

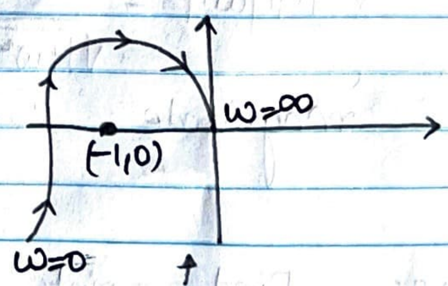
2 cases:
(i) if (-1,0) is enclosed ⇒ closed loop system is unstable
(ii) if (-1,0) is not enclosed ⇒ closed loop system is stable

(b)



intersecting
∴ critically stable

(c)

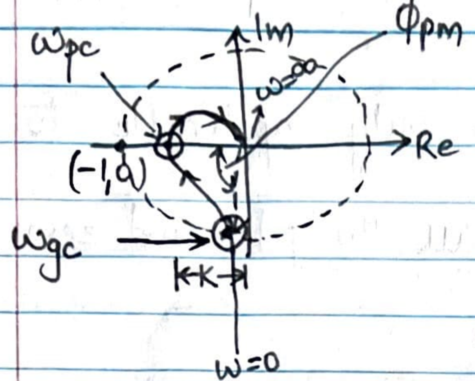


critical pt is enclosed
∴ UNSTABLE

→ $\omega_{pc} > \omega_{gc}$: STable system

$\omega_{pc} < \omega_{gc}$: UnsTable system

$\omega_{pc} = \omega_{gc}$: critically stable



$GM = 20 \log 1/K = [+ve]$

$\omega_{pc} > \omega_{gc}$: Stable system

Example:

Sketch polar plot for given OLTF $G(s) = \dfrac{s^3}{(s+1)(s+2)}$

Step 1: substitute $s = j\omega$

$$G(j\omega) = \frac{(j\omega)^3}{(1+j\omega)(2+j\omega)} = \frac{j^3\omega^3}{2+3j\omega+j^2\omega^2} = \frac{-j\omega^3}{2+3j\omega-\omega^2}$$

Step 2: write equation in standard form for polar plot

$$* \ |G(j\omega)| = \frac{\omega^3}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

magnitude

phase

$$\angle G(j\omega) = 3\tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$$= 270^\circ - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

Step 3: Find points @

$\omega = 0$ $\quad |G(j\omega)| = 0 \quad \angle G(j\omega) = 270^\circ$

$\omega = \infty$ $\quad |G(j\omega)| = \dfrac{\omega^3}{\omega^2\sqrt{\left(1+\frac{1}{\omega^2}\right)\left(\frac{4}{\omega^2}+1\right)}} \Rightarrow \infty \quad \angle G(j\omega) = 270^\circ - 90^\circ - 90^\circ$

$\angle G(j\omega) = 90^\circ$

Step 4: Separate real and imaginary parts

$$G(j\omega) = \frac{-j\omega^3}{(2-\omega^2)+j3\omega} \times \frac{(2-\omega^2)-j3\omega}{(2-\omega^2)-j3\omega} = \frac{j(\omega^2-2)\omega^3 - 3\omega^4}{(2-\omega^2)^2+(3\omega)^2}$$

$$= \frac{-3\omega^4}{(2-\omega^2)^2+(3\omega)^2} + j\frac{\omega^3(\omega^2-2)}{(2-\omega^2)^2+(3\omega)^2}$$

Step 5: Intersection points

with real axis: $Im(G(j\omega)) = 0$

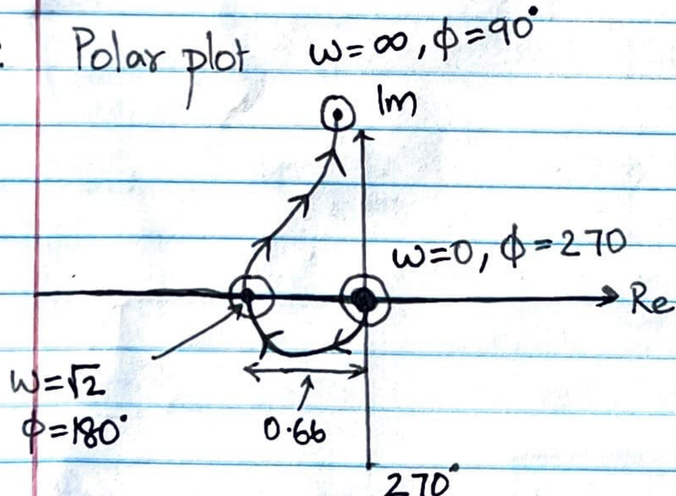
$\Rightarrow \omega = 0 \qquad \omega^2 = 2$

$\omega = \sqrt{2}$ rad/s

with imaginary axis: $Re(G(j\omega)) = 0$

$= \omega = 0$

Step 6: Polar plot



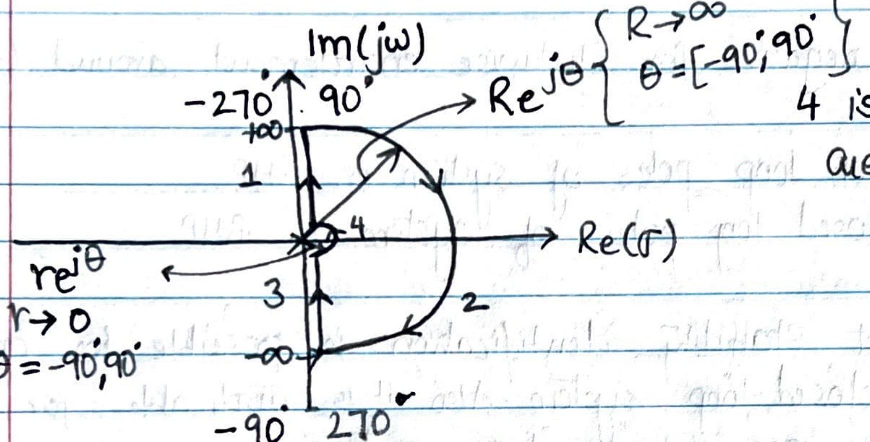
@ $\omega = \sqrt{2}$

$|G(j\omega)|_{\omega=\sqrt{2}} = 0.66$

$\angle G(j\omega) = +180°$ @ $\omega=\sqrt{2}$

Nyquist plot :

- It contains 3 major steps
  - (i) polar plot
  - (ii) inverse polar plot
  - (iii) Nyquist contour



Part 1) $s = j\omega$ polar plot $G(j\omega)\ H(j\omega)$

Part 2) $s = \lim_{R \to \infty} Re^{j\theta}$ $\boxed{\theta \to +90° \text{ to } -90°}$

Angle varies from +ve to -ve means clockwise motion of circle

Find $G(s)H(s) = \lim_{R \to \infty} G(Re^{j\theta})\, H(Re^{j\theta})$

Part 3) $s = -j\omega$ inverse polar plot

Part 4) $s = \lim_{r \to 0} re^{j\theta}$ $\boxed{\theta \to -90° \text{ to } 90°}$

When angle varies from -ve to +ve means circle is in anticlockwise direction

Find $G(s)H(s) = \lim_{r \to 0} G(re^{j\theta})\, H(re^{j\theta})$

applicable to both minimum and non-minimum phase systems

Stability using Nyquist plot

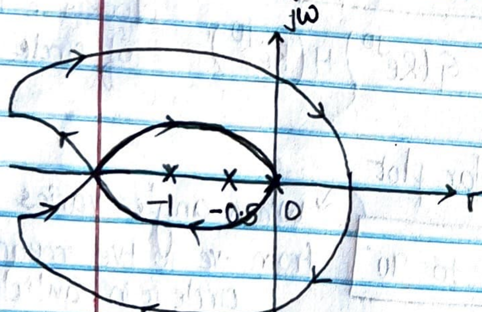
Based on this equation: $N = P - Z$

- N is positive for anti-clockwise encirclement around (-1,0)
- N is negative for clockwise encirclement around (-1,0)

P is open loop poles of system on RHP
Z is closed loop poles of system on RHP

Note
* Nyquist plot stability identification is possible for open-loop and closed-loop system. Also it is applicable for minimum phase and non-minimum phase system

Ex: $G(s)H(s) = \dfrac{(4s+1)}{s^2(s+1)(2s+1)}$



$N = P - Z$

$\boxed{P = 0}$

(-1,0) is enclosed by 2 circles
- smaller circle
- bigger circle

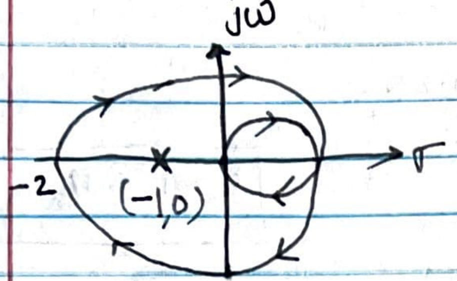
} clockwise

$N = -2$

$\boxed{Z = 2}$

2 poles of closed-loop system on RHP ⇒ unstable

Example: $G(s)H(s) = \dfrac{s-2}{(s+1)^2}$



$N = P - Z$ $\quad P = 0$

$N = -1$

$-1 = 0 - Z$

$\boxed{Z = 1}$ unstable system

~~**Example:** A unity feedback system has the open-loop T.F~~

~~$G(s) = \dfrac{1}{(s-1)(s+2)(s+3)}$~~

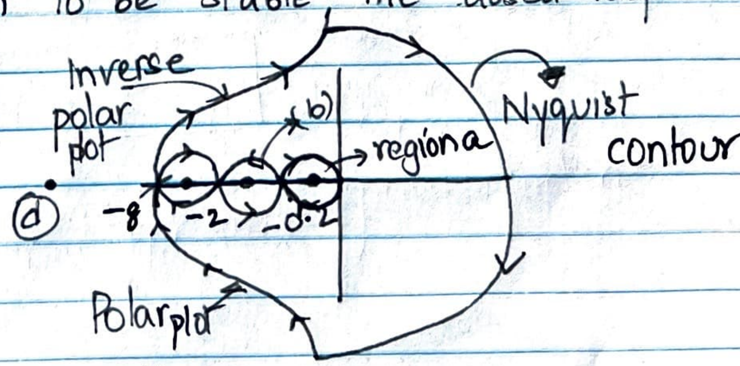
inverse polar plot is a mirror replica of polar pot

~~**Sol:**~~

~~$N = P - Z$~~

Ex: Nyquist plot for range of stability:

The polar diagram of a conditionally stable system for open-loop gain $K=1$ is shown in figure. The open-loop T.F of system is known to be stable. The closed loop system is stable for:



Sol:

$P = 0$

$N = \cancel{P}^{\,0} - Z$

$N = -Z$

$K = 1 \quad \therefore -0.2K, -2K, -8K$

In region (a) : 2 encirclements $\quad N = -2 \Rightarrow Z = 2 \quad$ unstable

region (b) : 2 encirclement $\left(\begin{smallmatrix}1 & \curvearrowright \\ 1 & \circlearrowleft\end{smallmatrix}\right)$ $N = 0 \Rightarrow Z = 0 \quad$ stable

region (c) : 2 encirclements $\quad N = -2 \Rightarrow Z = 2 \quad$ unstable region (d) $N=0 \Rightarrow Z=0 \Rightarrow$ stable system

For region (b)

$-2K < -1 < -0.2K$

$K > 1/2$ $\quad 5 > K$

$\boxed{5 > K > 1/2}$

For region (d)

$-8K > -1$

$K < 1/8$