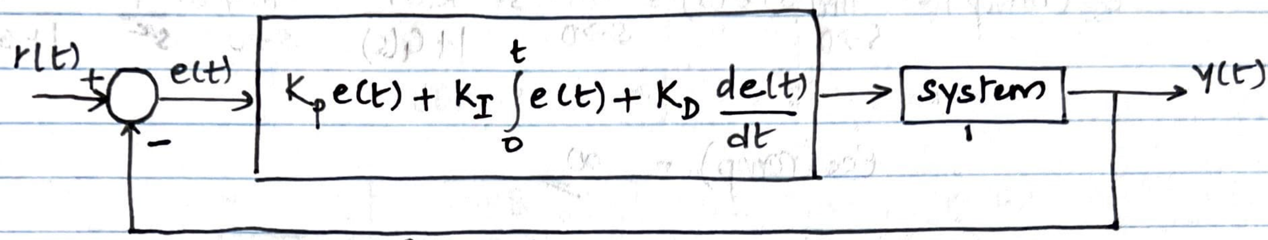
PID:

1922 — Nicolas Minorsky

"Directional Stability of Automatically Steered Bodies"



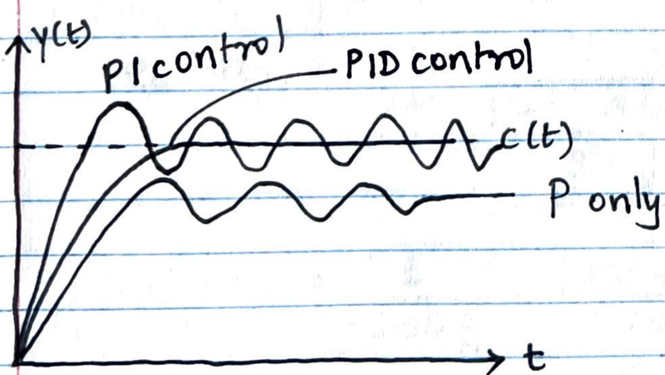
	Reaction	(Rise time)	overshoot	SSE	Settling Time
$K_p \uparrow$	Quicker ✓	decrease	Increase	decrease ✓	Small change
$K_I \uparrow$	Quicker	decrease	Increase	decrease (eliminate)	increase
$K_D \uparrow$	small change	small change	decrease ✓	small change NO change	decrease ✓

Advantages

✓ Easy to implement
✓ Easy to understand
✓ low order linear sys (1,2,3 order)
✓ Robustness to disturbances

Disadvantages

* High order linear sys
* Time-varying systems
* Non-linear systems
* Significant time-delays

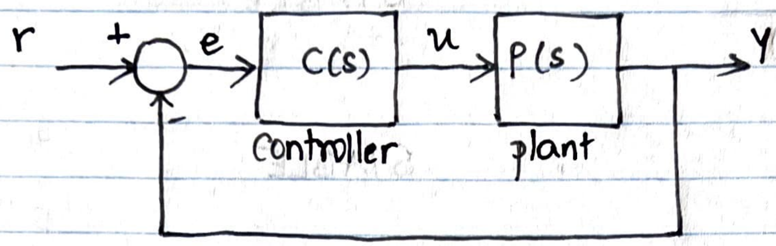


* Proportional - Integral - Derivative
* widely employed (easy to understand, quite effective)

PID control Fundamentals

K_P : proportional gain
K_I : integral gain
K_D : derivative gain

Consider a unity feedback system



output of a PID controller = control input to the plant

$$u(t) = K_P e(t) + K_I \int e(t)dt + K_D \frac{de}{dt}$$

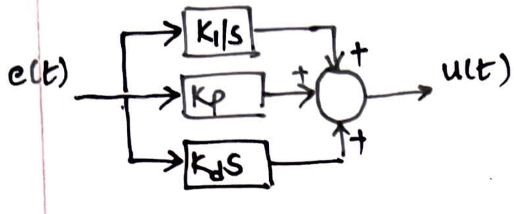
$e(t)$: Tracking error = $r(t) - y(t)$
Desired output - Actual output

* Error signal $e(t)$ is fed into the PID controller, the controller computes both the derivative and the integral of this error signal w.r.t time. The control

* The control signal $u(t)$ is fed to the plant and the new output ($y(t)$) is obtained.

* The new output y is fed back and compared to the reference to find new error signal $e(t)$.

* The controller takes this new error signal and computes an update of the control input. This process is repeated



Taking the Laplace transform, we get transfer function of PID controller

$$K_p + \frac{K_I}{s} + K_d s = \frac{K_D s^2 + K_I + K_P s}{s}$$

Characteristics of P, I, and D Terms

- Increasing K_p (proportional gain) has the effect of proportionally increasing the control signal for the same-level of error. Controller will "push" harder for a given level of error tends to cause the closed loop system to react more quickly, but also overshoot more.

- Another effect of increasing K_p is that it tends to reduce, but not eliminate steady state error.

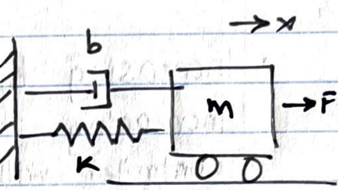
- The addition of an integral term (K_I) to the controller tends to help reduce steady state error. If there is a persistent, steady error, the integrator builds and builds, thereby increasing the control signal and driving the error down. A drawback of the integral term, is that it can make the system more sluggish and oscillatory since when the error signal changes sign, it may take a while for the integrator to unwind.

- The addition of a derivative term (K_D) adds the ability of the controller to "anticipate" error. With simple proportional control, if K_p is fixed, the only way control will increase is if the error increases.

With derivative control, the control signal can become large if the error begins sloping upward, even while the magnitude of the error is relatively small. This anticipation tends to add damping to the system, thereby decreasing overshoot.

Example:

mass-spring-damper



EOM:

$$m\ddot{x} + b\dot{x} + kx = F$$

Taking L.T

$$ms^2 X(s) + bs X(s) + kX(s) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

let

$m = 1kg$ $b = Ns/m$ $k = 20 N/m$ $f = 1$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 20}$$

GOAL:

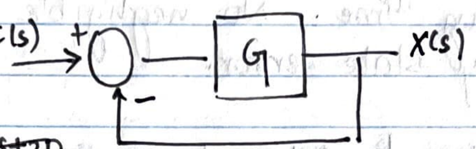
* Analyze how K_P, K_D, K_I contribute to
  - fast rise time
  - minimizing overshoot
  - zero steady-state error

Open-loop Step response

- DC gain of the plant transfer func is $1/20 = 0.05$ = final value of the output to a unit step input.

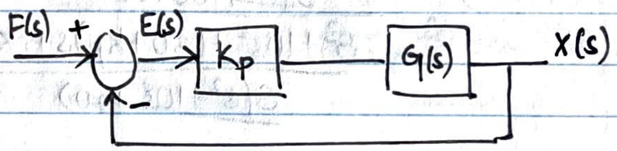
i.e. steady state error $= 0.95$ LARGE
Rise time $= 1.0s$.
Settling time $= 1.5s$

Unity-feedback



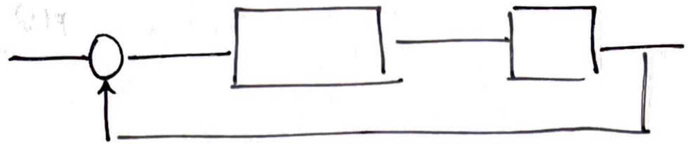
$$\frac{X(s)}{F(s)} = \frac{G}{1+G} = \frac{1/\cancel{s^2+10s+20}}{\frac{s^2+10s+21}{\cancel{s^2+10s+20}}} = \frac{1}{s^2+10s+21}$$

Proportional control



$$\frac{X(s)}{F(s)} = \frac{K_pG}{1+K_pG} = \frac{\frac{K_p}{s^2+10s+20}}{1+\frac{K_p}{s^2+10s+20}} = \frac{K_p}{s^2+10s+(20+K_p)}$$

proportional control reduced both the rise time and the steady-state error, increased the overshoot, decreased settling time by a small amount.



Proportional derivative control:

$$\frac{X(s)}{R(s)} = \frac{\dfrac{K_D s + K_p}{\cancel{s^2+10s+20}}}{\dfrac{s^2+(10+K_d)s+(20+K_p)}{\cancel{s^2+10s+20}}} = \frac{K_D s + K_p}{s^2+(10+K_D)s+(20+K_p)}$$

Addition of derivative term reduced both the overshoot and settling time. No negligible effect on rise time and the steady state error.

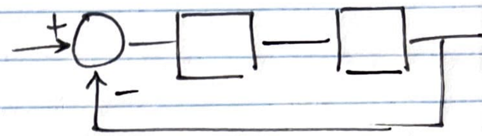
Proportional integral control:

$$\frac{X(s)}{R(s)} = \frac{\dfrac{K_p s + K_I}{\cancel{s(s^2+10s+20)}}}{\dfrac{s^3+10s^2+(20+K_p)s+K_I}{\cancel{s(s^2+10s+20)}}} = \frac{K_p s + K_I}{s^3+10s^2+(20+K_p)s+K_I}$$

Reduce K_p because K_I also ~~increases~~ reduces rise time and increases the over shoot.

Integral controller eliminated (or) decreased the steady-state error (

Proportional - Integral - Derivative controller



$$\frac{X(s)}{R(s)} = \frac{\dfrac{K_D s^2 + K_p s + K_I}{s(s^2+10s+20)}}{1 + \dfrac{K_D s^2 + K_p s + K_I}{s(s^2+10s+20)}} = \frac{K_D s^2 + K_p s + K_I}{s^3 + (10+K_D)s^2 + (20+K_p)s + K_I}$$

No overshoot, fast rise time no steady state error.

Tips To Design a PID controller:

When you are designing a PID controller, follow the following steps to obtain desired response.

1. Obtain an open-loop response and determine what needs to be improved.
2. Add a proportional control to improve rise-time.
3. Add a derivative control to reduce overshoot
4. Add an integral control to reduce steady-state error.
5. Adjust K_p, K_D, K_I until you obtain a desired overall response.