HW 2 Solutions

1.

B-5-3

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2% criterion for settling time : $t_s = \frac{4}{\zeta\omega_n} = 2$

$$\zeta\omega_n = 2$$

Maximum overshoot: $e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} = 0.05$

Take natural logarithm on both sides

$$-\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)\pi = \ln(0.05)$$

$$\frac{-\zeta}{\sqrt{1-\zeta^2}} = \frac{\ln(0.05)}{\pi}$$

Taking squares on both sides

$$\frac{\zeta^2}{1-\zeta^2} = \left[\frac{\ln(0.05)}{\pi}\right]^2$$

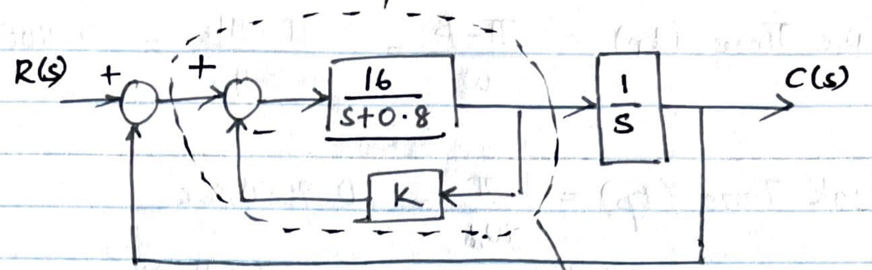
$$\zeta^2\left[1+\left(\frac{\ln(0.05)}{\pi}\right)^2\right] = \left(\frac{\ln(0.05)}{\pi}\right)^2$$

$$\zeta^2 = 0.4762 \Rightarrow \boxed{\zeta = 0.6901}$$

$$\omega_n = \frac{2}{0.69} = 2.90 \text{ rad/s}$$

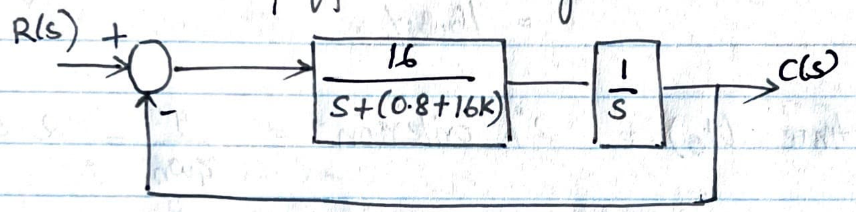
$$\frac{\frac{16}{s+0.8}}{1+\frac{16}{0.8+s}\cdot K} = \frac{16}{s+(0.8+16K)}$$

B-5-9 : Problem 2:

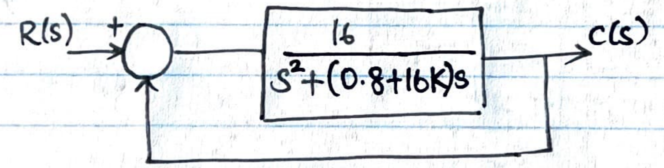


Damping ratio $\zeta = 0.5$

Simplify block diagram (1st inner block/loop)



Simplify



Simplify

$$\frac{C(s)}{R(s)} = \frac{16}{s^2+(0.8+16K)s+16}$$

Compare to standard form: $\omega_n^2 = 16$

$$\boxed{\omega_n = 4}$$

$$2\zeta\omega_n = 0.8+16K$$

$$2\times 0.5\times 4 = 0.8+16K \Rightarrow \boxed{K = 0.2}$$

$$3.2 = 16K$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 4\sqrt{1-0.5^2} = 3.4641$$

$$\beta = \sin^{-1}\frac{\omega_d}{\omega_n} = \sin^{-1}(0.866) = \frac{\pi}{3}$$

Rise Time $(t_r) = \frac{\pi - \beta}{\omega_d} = \frac{\pi - \pi/3}{3.46} = 0.6053$ sec

Peak Time $(t_p) = \frac{\pi}{\omega_d} = 0.9080$ sec

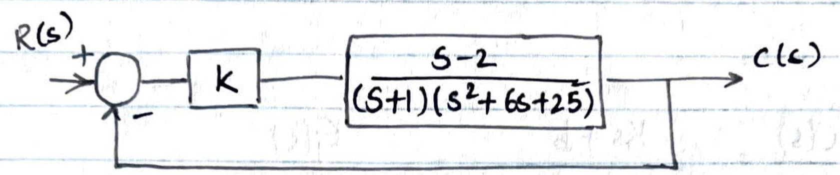
Maximum overshoot $(M_p) = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100\%$

$= 16.3\%$

Settling time $(t_s) =$ 2% criterion $= \frac{4}{\zeta\omega_n} = 2$ sec

5% criterion $= \frac{3}{\zeta\omega_n} = 1.5$ sec

Problem 4 : B-5-22: Assume $K>0$



Closed loop T.F

$$\frac{C(s)}{R(s)} = \frac{K(s-2)}{(s+1)(s^2+6s+25)+K(s-2)} = \frac{K(s-2)}{s^3+7s^2+(31+K)s+(25-2K)}$$

Use Routh stability criterion

s^3	1	$31+K$
s^2	7	$25-2K$
s^1	A_1	
s^0	A_2	

$$A_1 = \frac{(31+K)7-(25-2K)}{7}$$

$A_1>0$

$\therefore (31+K)7-(25-2K)>0$

$217+7K-25+2K>0$

$9K>-192$

$\boxed{K>0}$

$$A_2 = \frac{\cancel{A_1}(25-2K)}{\cancel{A_1}}$$

$A_2>0$

$25-2K>0 \quad \Rightarrow \quad \boxed{12.5>K}$

Range of K for stability $\boxed{12.5>K>0}$

Problem 5: B-5-26:

$$\frac{C(s)}{R(s)} = \frac{Ks+b}{s^2+as+b} = \frac{G(s)}{1+G(s)}$$

Closed loop

unity feedback.

$$(Ks+b)(1+G(s)) = G(s)(s^2+as+b)$$

$$(Ks+b) + (Ks+b)G(s) = G(s)(s^2+as+b)$$

$$(Ks+b) = G(s)(s^2+as+\cancel{b}-Ks-\cancel{b})$$

$$G(s) = \frac{Ks+b}{s(s+a-K)}$$

Steady state error in unit-ramp response

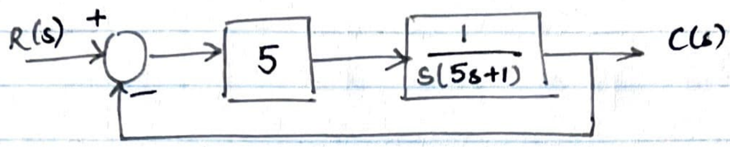
$$e_{ss} = \frac{1}{K_v} = \frac{1}{\lim_{s\to 0} sG(s)} = \frac{a-K}{b}$$

Problem 3: Ignore written notes & check matlab

Problem B-5-13:

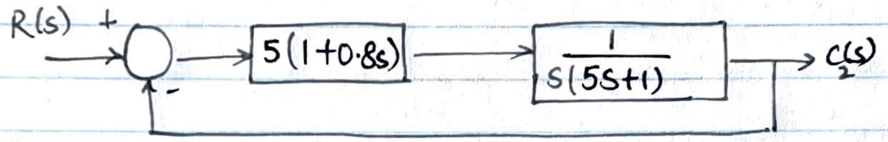
closed loop T.F

System I:



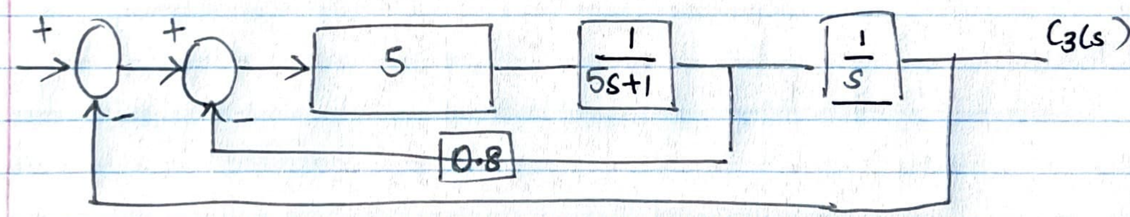
$$\frac{C(s)}{R(s)} = \frac{1}{s^2+0.2s+1}$$

System II:



$$\frac{C_2(s)}{R(s)} = \frac{1+0.8s}{s^2+s+1}$$

System III:



$$\frac{C_3(s)}{R(s)} = \frac{1}{s^2+s+1}$$

```
clc;
clearvars;
close all;
```

```
% System 1:
num1 = [0 0 1];
den1 = [1 0.2 1];
sys1 = tf(num1, den1)
```

```
sys1 =

         1
  ---------------
  s^2 + 0.2 s + 1

Continuous-time transfer function.
```

```
% System 2:
num2 = [0 0.8 1];
den2 = [1 1 1];
sys2 = tf(num2, den2)
```

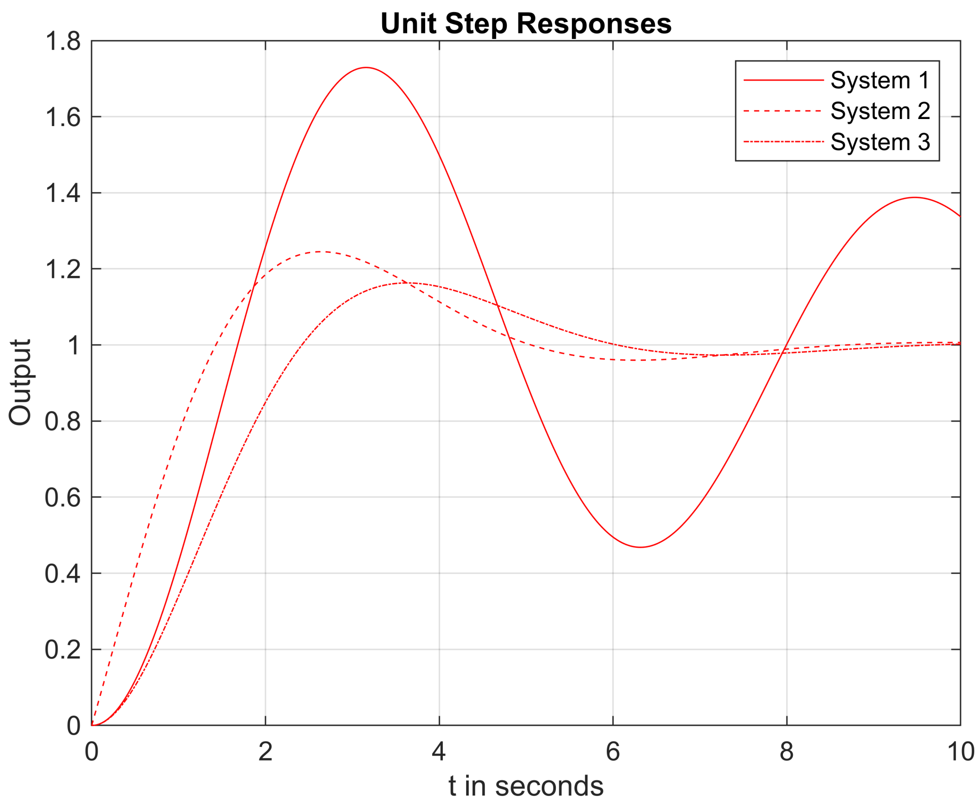
```
sys2 =

   0.8 s + 1
  -----------
  s^2 + s + 1

Continuous-time transfer function.
```

```
% System 3:
num3 = [0 0 1];
den3 = [1 1 1];
sys3 = tf(num3, den3)
```

```
sys3 =

       1
  -----------
  s^2 + s + 1

Continuous-time transfer function.
```

```
t = 0:0.01:10;
% Plot Step Response
c1 = step(sys1, t);
c2 = step(sys2, t);
c3 = step(sys3, t);

plot(t,c1,'r-');
hold on;
grid on;
```

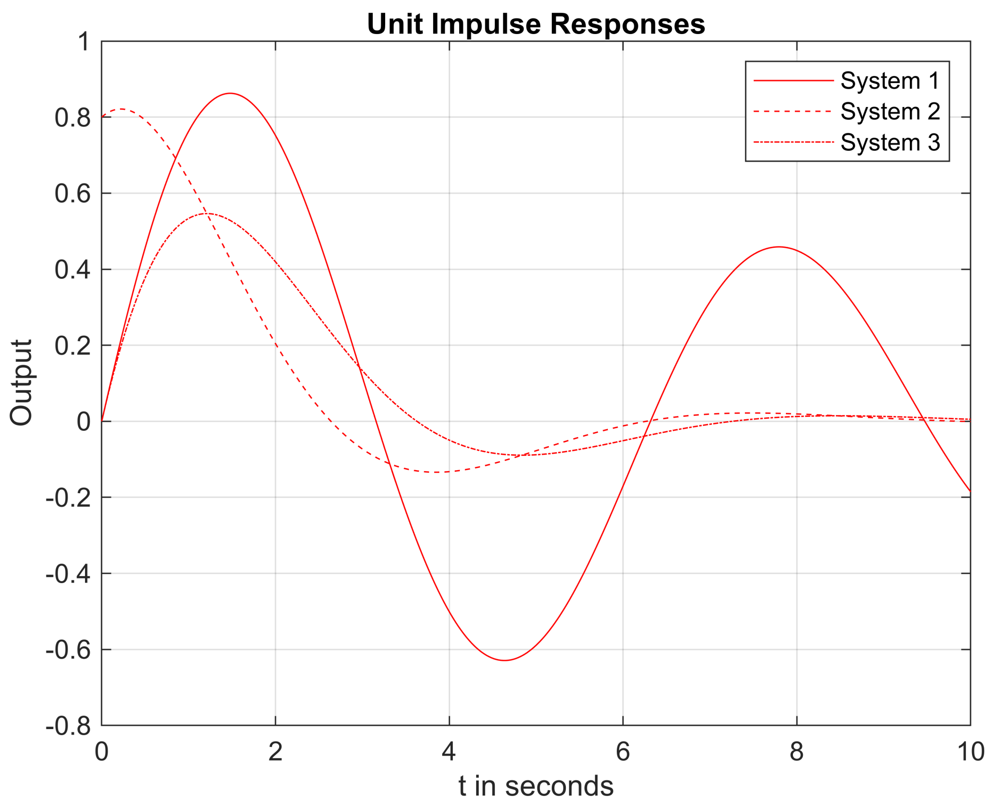
```
title("Unit Step Responses");
xlabel("t in seconds");
ylabel("Output");
plot(t,c2,'r--');
plot(t,c3,'r-.');
legend(["System 1", "System 2", "System 3"]);
hold off;
```



1. With respect to speed of response: System 2 exhibits the shortest rise time
2. With respect to maximum overshoot: System 3 exhibits least overshoot

```
t = 0:0.01:10;
% Plot Impulse Response
c1 = impulse(sys1, t);
c2 = impulse(sys2, t);
c3 = impulse(sys3, t);

plot(t,c1,'r-');
hold on;
grid on;
title("Unit Impulse Responses");
xlabel("t in seconds");
ylabel("Output");
plot(t,c2,'r--');
plot(t,c3,'r-.');
```

```
legend(["System 1", "System 2", "System 3"])
hold off;
```




```
% System 1:
num1 = [0 0 0 1];
den1 = [1 0.2 1 0];
sys1 = tf(num1, den1);
```

```
% System 2:
num2 = [0 0 0.8 1];
den2 = [1 1 1 0];
sys2 = tf(num2, den2);
```

```
% System 3:
num3 = [0 0 0 1];
den3 = [1 1 1 0];
sys3 = tf(num3, den3);
t = 0:0.01:10;
% Plot Ramp Response
c1 = step(sys1, t);
c2 = step(sys2, t);
c3 = step(sys3, t);

plot(t,c1,'r-');
```

```
hold on;
grid on;
title("Unit Ramp Responses");
xlabel("t in seconds");
ylabel("Output");
plot(t,c2,'r--');
plot(t,c3,'r-.');
legend(["System 1", "System 2", "System 3"])
hold off;
```