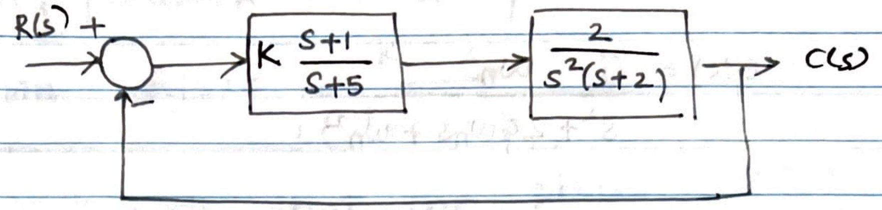
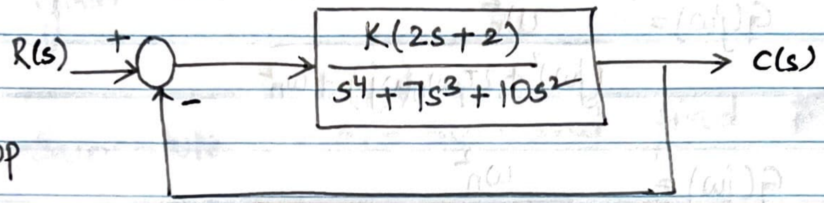
Problem 2:

B-6-7:



⇓



closed loop T.F

↓

$$\frac{C(s)}{R(s)} = \frac{\frac{K(2s+2)}{s^4+7s^3+10s^2}}{1+\frac{K(2s+2)}{s^4+7s^3+10s^2}} = \frac{K(2s+2)}{s^4+7s^3+10s^2+2Ks+2K}$$

Use Routh Hurwitz

s^4	1	10	2K
s^3	7	2K	
s^2	A_1	B_1	
s^1	A_2	0	
s^0	A_3		

$$A_1 = \frac{70-2K}{7}$$

$$B_1 = \frac{14K}{7} = 2K$$

$$A_2 = \frac{A_1(2K)-7B_1}{A_1}$$

$A_1 > 0$

$\Rightarrow \frac{70-2K}{7} > 0$

$\Rightarrow 70-2K > 0$

$\Rightarrow \boxed{35 > K}$

$B_1 > 0$

$\Rightarrow 2K > 0$

$\Rightarrow \boxed{K > 0}$

$A_2 > 0$

$A_1(2K) - 7B_1 > 0$

$A_3 = B_1$

$\frac{(70-2K)2K}{7} - 7(2K) > 0$

$(70-2K)2K - 49(2K) > 0$

$2K\{21-2K\} > 0$

$21 > 2K$

$\boxed{10.5 > K}$

$\therefore \underline{10.5 > K > 0}$

Range of K.

Problem 6: B-7-5

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Substitute $s = j\omega$

$$G(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(\omega j) + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{j^2\omega^2 + j\,2\zeta\omega\omega_n + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{(\omega_n^2 - \omega^2) + j(2\zeta\omega\omega_n)}$$

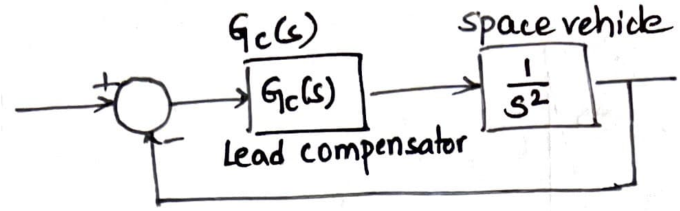
$$|G(j\omega)| = \frac{\omega_n^2}{\sqrt{\omega_n^4 + \omega^4 - 2\omega^2\omega_n^2 + 4\zeta^2\omega^2\omega_n^2}}$$

when $\omega = \omega_n$

$$|G(j\omega_n)| = \frac{\omega_n^2}{\sqrt{\cancel{\omega_n^4} + \cancel{\omega_n^4} - \cancel{2\omega_n^4} + 4\zeta^2\omega_n^4}} = \frac{\cancel{\omega_n^2}}{2\zeta\cancel{\omega_n^2}} = \frac{1}{2\zeta}$$

Solution

Q3 : B-6-18 :



① Desired dominant closed loop poles:

$$s_1 = -1 \pm j1 \qquad s_2 = -1 - j1$$

$(s-s_1)(s-s_2) = 0$ ← characteristic equation

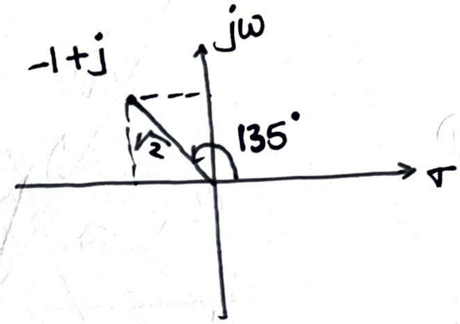
$$(s+1-j1)(s+1+j1) = 0$$

$$s^2 + s + \cancel{js} + s + 1 + \cancel{j} - \cancel{js} - \cancel{j} - j^2 = 0$$

$$\boxed{s^2 + 2s + 2 = 0}$$

$$\omega_n^2 = 2$$

$$\omega_n = \sqrt{2}$$



② Evaluate $\angle G(s)$ @ desired poles

$$\angle G(s_1) = \cancel{\angle 1}^{\,0} - 2\angle s_1$$

$$= -2 \angle -1+j$$

$$= -2 \tan^{-1}\left(\frac{1}{-1}\right)$$

$$= -2\,(135^\circ) = -270^\circ$$ ← doesnot satisfy phase condition

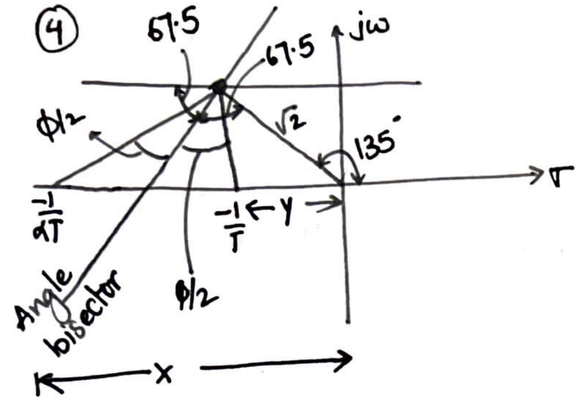
Desired closed loop poles are not on the root locus. So simple gain adjustment will not work. So introduce a compensator in such a way it passes the root locus through the desired poles.

③ Angle deficiency:

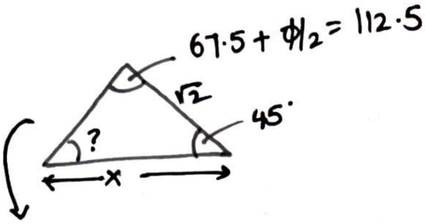
should be -180° instead of -270°. Therefore angle compensation required is $+90^\circ$

$$\boxed{\phi = 90^\circ}$$ Lead compensator

④



$\phi/2 = 45°$



Law of sines

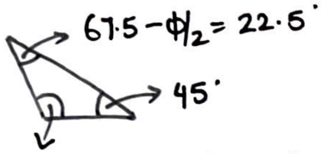
$$\frac{\sqrt{2}}{\sin 22.5°} = \frac{x}{\sin 112.5}$$

$$\boxed{x = 3.4142}$$

$? = 180° - 112.5 - 45°$

$\boxed{? = 22.5°}$

Similarily



$? = 180° - 45° - 22.5$

$\boxed{? = 112.5}$

Law of sines

$$\frac{\sqrt{2}}{\sin 112.5°} = \frac{y}{\sin 22.5}$$

$$\boxed{y = 0.5858}$$

$\frac{1}{T} = 0.5858 \Rightarrow \boxed{T = 1.7071}$

$\frac{1}{\alpha T} = 3.4142 \Rightarrow \boxed{\alpha = 0.1716}$

Lead compensator

$$G_c(s) = K\left(\frac{s + 0.5858}{s + 3.4142}\right)$$

Determine K using magnitude condition:

$$\left|G_c(s)G(s)\right|_{@ s=s_1} = 1$$

$$K = \left|\frac{s^2(s+3.4142)}{s+0.5858}\right| \Rightarrow \text{Use MATLAB}$$

$$\boxed{K = 4.8112}$$

* Lead compensator

$$\boxed{G_c(s) = 4.8112\left(\frac{s+0.5858}{s+3.4142}\right)}$$

* Feedforward T.F $= G_c(s)\,G(s) = \dfrac{4.8112s + 2.8184}{s^3 + 3.4142s^2}$

Problem 9:

$PM = 50^\circ = 180^\circ + \phi_{gc}$

$\boxed{\phi_{gc} = -130^\circ}$ → check the frequency (ω) @ -130° in the phase plot.

$\boxed{\omega_{gc} = 1.44024}$

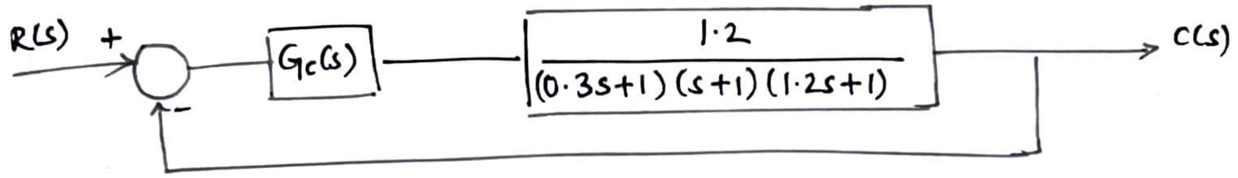
Obtain gain K:

$$|G(j\omega_{gc})| = 1$$

$$\left| K \left(\frac{j\omega_{gc} + 0.1}{j\omega_{gc} + 0.5} \right) \left(\frac{10}{j\omega_{gc}(j\omega_{gc}+1)} \right) \right| = 1$$

$\boxed{K = 0.2667}$

Problem 10 :



Open-loop transfer function of the system

$$G(s) = \frac{K(s+a)^2}{s} \frac{1.2}{(0.3s+1)(s+1)(1.2s+1)}$$

$$G(s) = \frac{1.2Ks^2 + 2.4Kas + 1.2Ka^2}{0.36s^4 + 1.86s^3 + 2.5s^2 + s}$$

closed-loop transfer function of the system

$$\frac{C(s)}{R(s)} = \frac{1.2Ks^2 + 2.4Kas + 1.2Ka^2}{0.36s^4 + 1.86s^3 + (2.5+1.2K)s^2 + (1.2+2.4Ka)s + 1.2Ka^2}$$

Maximum overshoot

$$0.02 < M_p < 0.1$$

Therefore, the output y should be within the range $1.02 < \max(y) < 1.1$

* HW problem 4: will not be graded

AME 455

Homework-7 Solutions

① Consider a unity feedback control system with open loop transfer function $G(s) = \frac{K}{s(s^2+s+4)}$. Determine the value of the gain K such that the phase margin is 50°. What is the gain margin with this gain K?

Sol: Step 1: Substitute P.M = 50° and find ~~|G(jω)|~~ $\angle G(j\omega_{gc})$

$$P.M = 180^\circ + \phi_{gc}$$

$$\boxed{\phi_{gc} = -130^\circ}$$

phase @ gain-cross over frequency

Step 2: Obtain gain cross-over frequency

$$\angle G(j\omega_{gc}) = \phi_{gc} = -130^\circ$$

$$\angle \left[\frac{K}{j\omega_{gc}\left((j\omega_{gc})^2 + j\omega_{gc} + 4\right)} \right] = -130^\circ$$

$$\angle \left\{ \frac{K}{-\omega_{gc}^2 + j(4\omega_{gc} - \omega_{gc}^3)} \right\} = -130^\circ$$

$$\cancel{\angle K}^{0} - \left\{ \angle -\omega_{gc}^2 + j(4\omega_{gc} - \omega_{gc}^3) \right\} = -130^\circ$$

$$+\angle -\omega_{gc}^2 + j(4\omega_{gc} - \omega_{gc}^3) = +130^\circ$$

$$\tan^{-1}\left\{ \frac{4\omega_{gc} - \omega_{gc}^3}{-\omega_{gc}^2} \right\} = 130^\circ$$

$$\frac{4\omega_{gc} - \omega_{gc}^3}{-\omega_{gc}^2} = -1.1917$$

$$4\cancel{\omega_{gc}} - \omega_{gc}^{\cancel{3}\,2} = 1.1917\,\omega_{gc}^{\cancel{2}}$$

$$\boxed{\omega_{gc}^2 + 1.1917\,\omega_{gc} - 4 = 0}$$

Use calculator to find ω_{gc}

$$\boxed{\omega_{gc} = 1.49}$$

gain cross-over frequency

Step 3: At gain cross-over frequency (ω_{gc}), we have $|G(j\omega_{gc})| = 1$

$$|G(j\omega_{gc})| = \left| \frac{K}{-(1.49)^2 + j(4(1.49) - (1.49)^3)} \right| = 1$$

$$\left| \frac{K}{-2.22 + j(2.652)} \right| = 1$$

$$\frac{K}{\sqrt{(2.22)^2 + (2.652)^2}} = 1$$

$$\boxed{K = 3.46}$$

Step 4: Obtain phase cross-over frequency (ω_{pc})

At phase cross-over frequency $\angle G(j\omega_{pc}) = -180^\circ$

$$\angle \left\{ \frac{K}{-\omega_{pc}^2 + j(4\omega_{pc} - \omega_{pc}^3)} \right\} = -180^\circ$$

$$\cancelto{0}{\angle K} - \angle -\omega_{pc}^2 + j(4\omega_{pc} - \omega_{pc}^3) = -180^\circ$$

$$+\tan^{-1}\left\{ \frac{4\omega_{pc} - \omega_{pc}^3}{-\omega_{pc}^2} \right\} = +180^\circ$$

$$\frac{4\omega_{pc} - \omega_{pc}^3}{-\omega_{pc}^2} = 0$$

$$\omega_{pc}(4 - \omega_{pc}^2) = 0$$

$\omega_{pc} = 0$ $\uparrow$ Trivial solution

$4 - \omega_{pc}^2 = 0$

$\boxed{\omega_{pc} = 2}$

Step 5: Compute magnitude @ $\omega = \omega_{pc}$

$$|G(j\omega_{pc})| = \left|\frac{K}{-\omega_{pc}^2 + j(4\omega_{pc} - \omega_{pc}^3)}\right|$$

$$= \left|\frac{3.46}{-2^2 + j(4(2) - 2^3)}\right|$$

$$|G(j\omega_{pc})| = \left|\frac{3.46}{-4}\right| = \frac{3.46}{4} = 0.865$$

Step 6: Compute gain margin (in dB)

$$\text{Gain margin (in dB)} = -20\log|G(j\omega_{pc})|$$

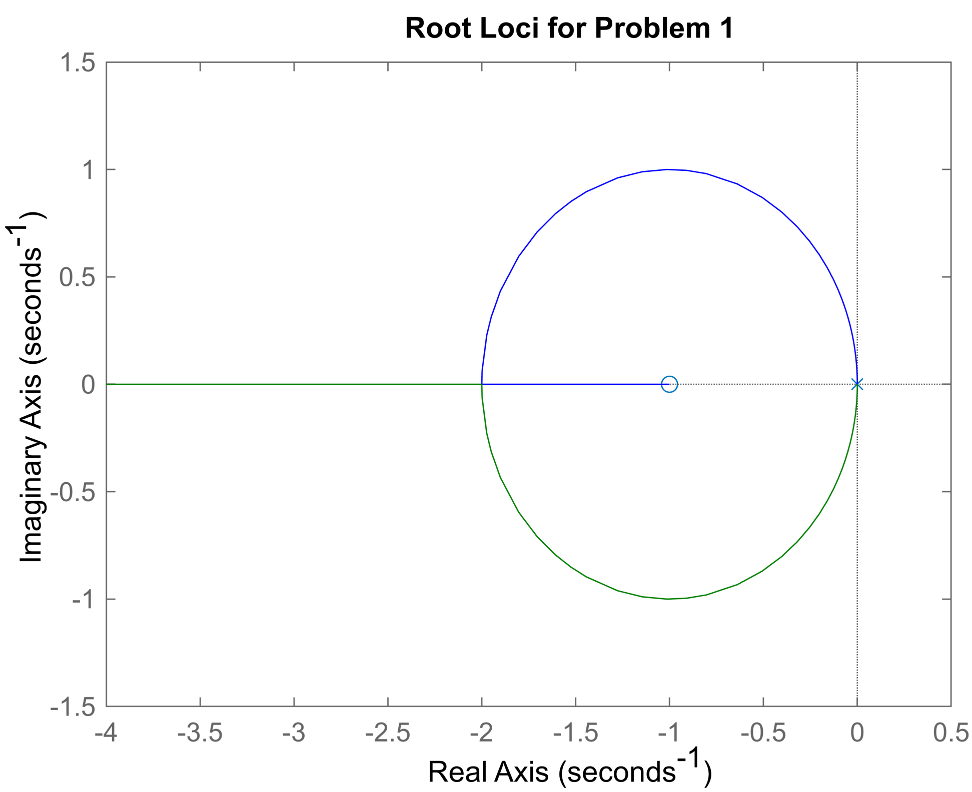
$$= -20\log(0.865)$$

$$\text{Gain margin (in dB)} = 1.26\text{ dB}$$

```
clc;
clearvars;
close all;
```

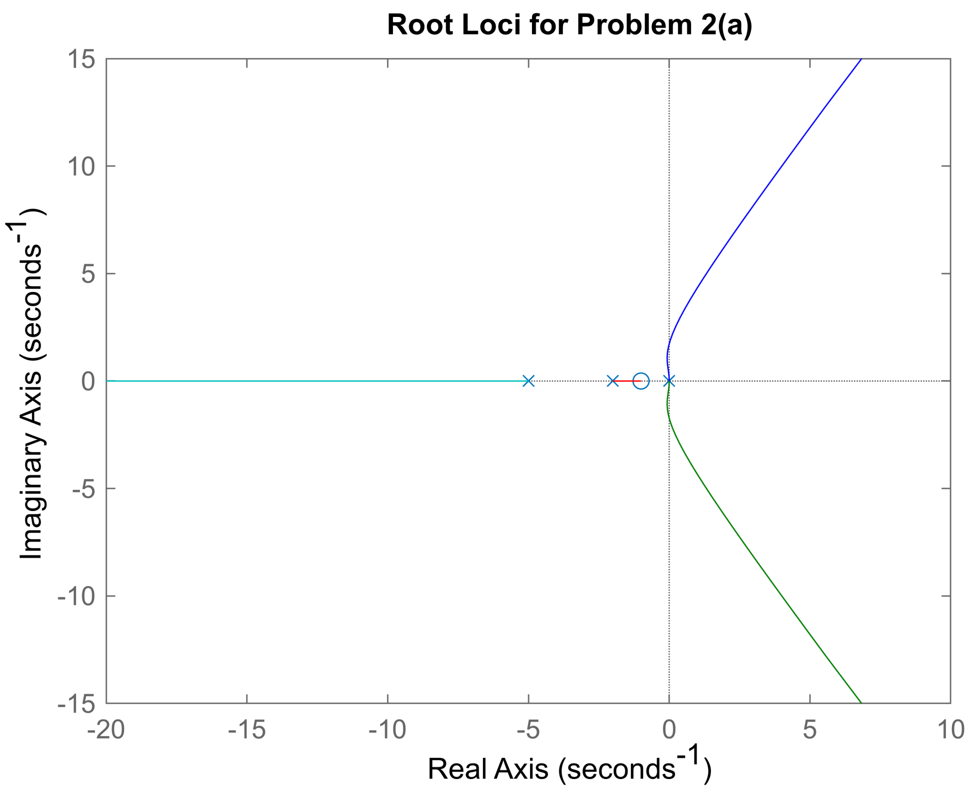
Problem 1: Root Locus

```
num = [1 1];
den = [1 0 0];
sys = tf(num, den);
rlocus(sys)
title("Root Loci for Problem 1")
```

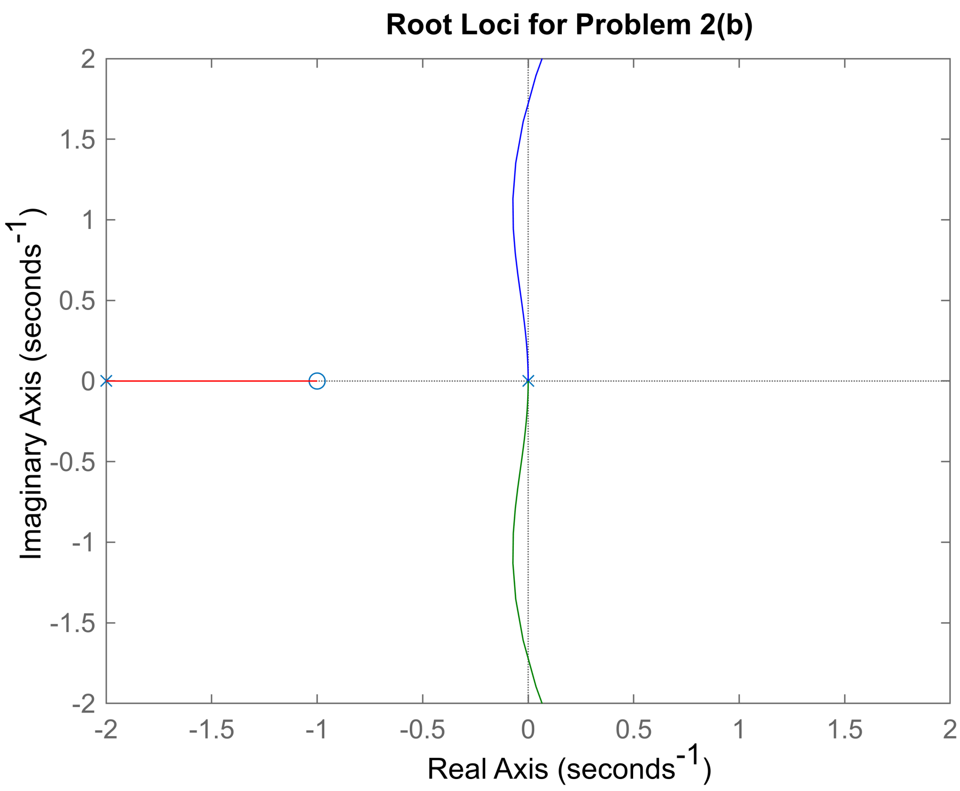


Problem 2: Root Locus

```
% Problem 2(a): Plot root loci
num = [0 0 0 2 2];
den = [1 7 10 0 0];
sys = tf(num, den);
rlocus(sys)
title("Root Loci for Problem 2(a)")
```

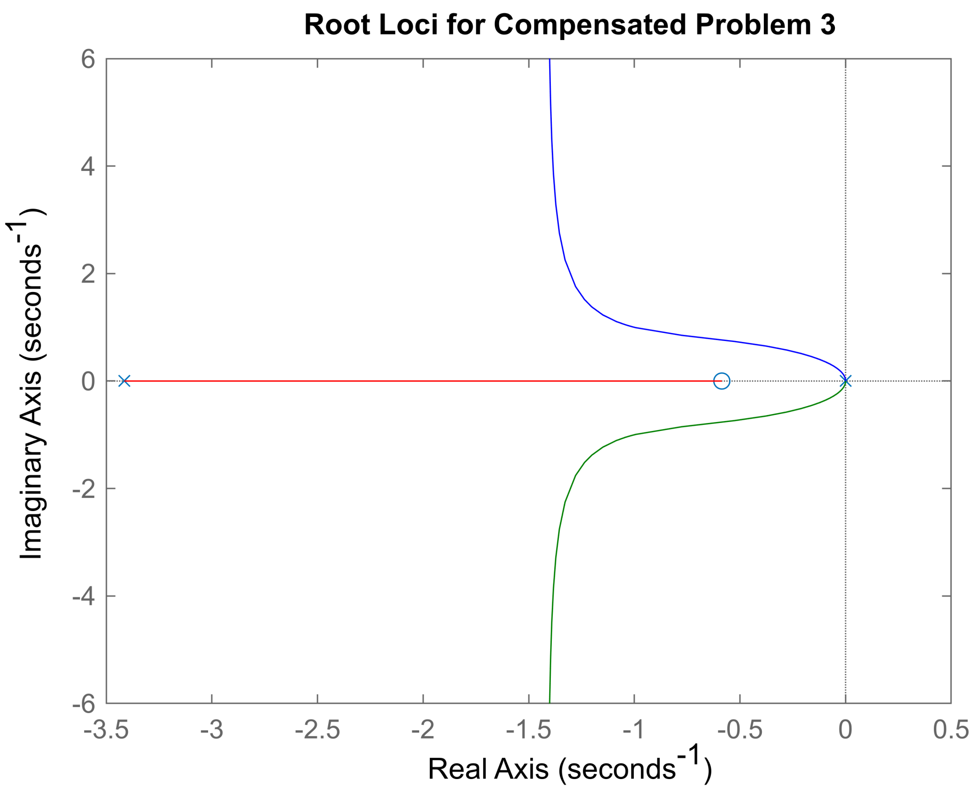



```
% Problem 2(b): Range of gain K
num = [0 0 0 2 2];
den = [1 7 10 0 0];
sys = tf(num, den);
rlocus(sys)
xlim([-2 2]);
ylim([-2 2]);
title("Root Loci for Problem 2(b)")
```



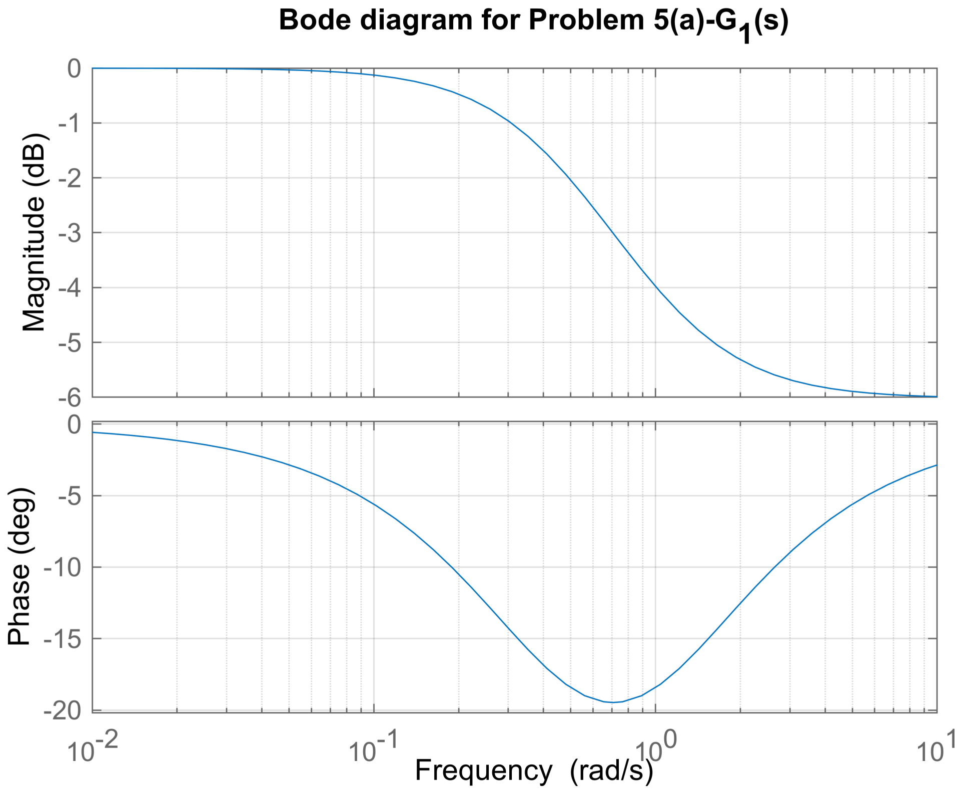
Problem 3: Compensator

```
% Problem 3: Plot root loci
num = [0 0 4.8112 2.8184];
den = [1 3.4142 0 0];
sys = tf(num, den);
rlocus(sys)
title("Root Loci for Compensated Problem 3")
```

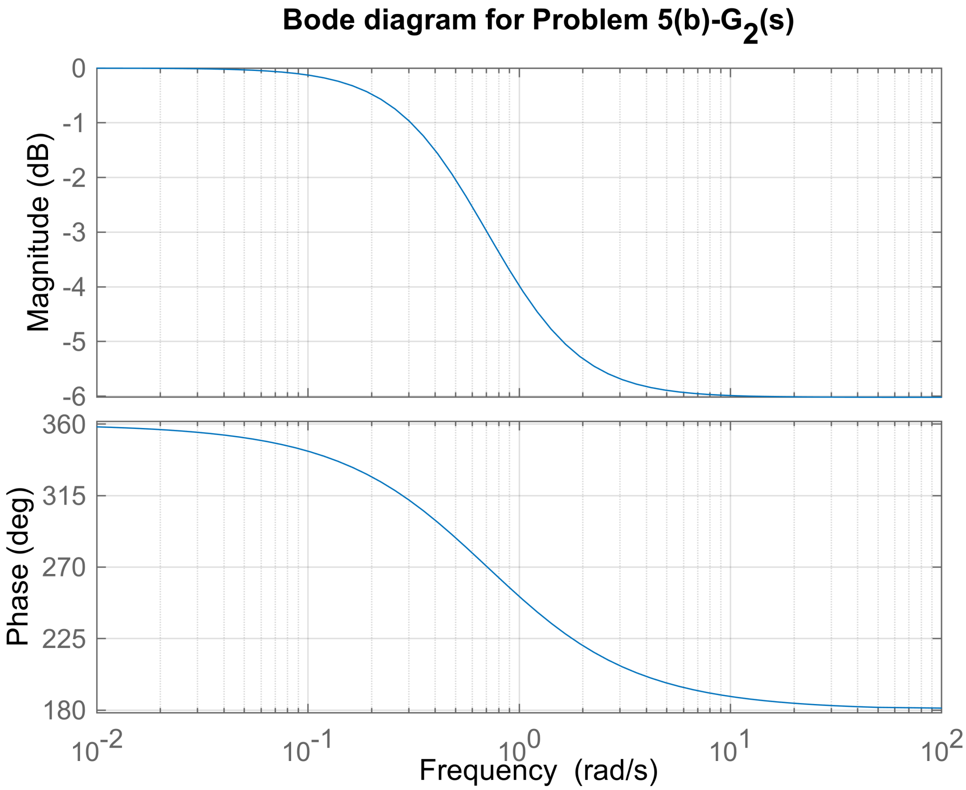


Problem 5: Bode Diagram

```
% Problem 5(a) - Minimum phase system
num = [1 1];
den = [2 1];
sys = tf(num, den);
bode(sys)
grid on;
title("Bode diagram for Problem 5(a)-G_1(s)")
```

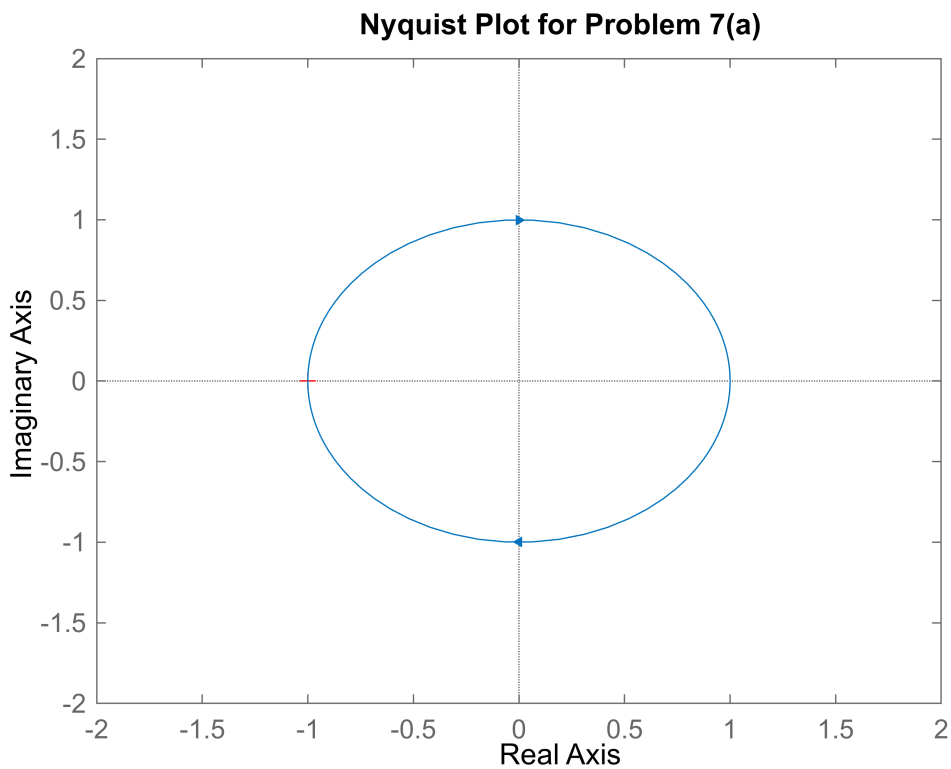



```
% Problem 5(b) - Non-minimum phase system
num = [-1 1];
den = [2 1];
sys = tf(num, den);
bode(sys)
grid on;
title("Bode diagram for Problem 5(b)-G_2(s)")
```



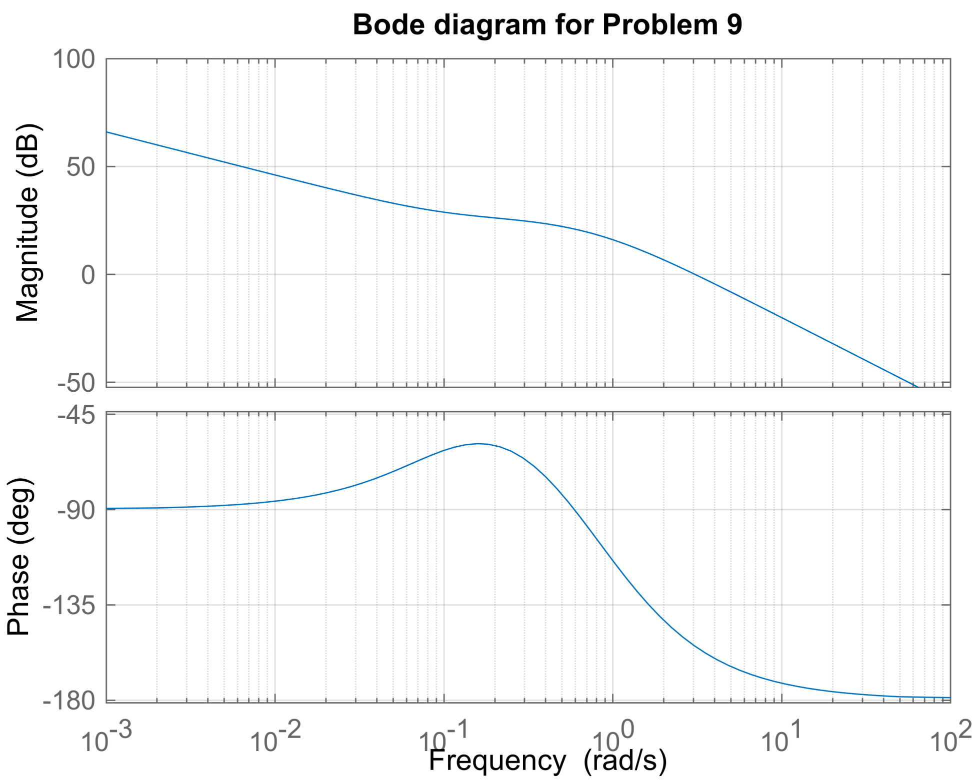
Problem 7: Nyquist Plot

```
num = [-1 1];
den = [1 1];
sys = tf(num, den);
nyquist(sys)
xlim([-2 2]);
ylim([-2 2]);
title("Nyquist Plot for Problem 7(a)")
```



Problem 9: Bode Diagram

```
num = [10, 1];
den = [1 1.5 0.5 0];
sys = tf(num, den);
bode(sys)
grid on;
title("Bode diagram for Problem 9")
```




```
omega_gc = 1.44024;
K = abs(((0.5+1i*omega_gc)*(1i*omega_gc)*(1+1i*omega_gc))/(10*(0.1+1i*omega_gc)))
```

```
K = 0.2667
```

Phase curve always lies above -180 degrees. Therefore, there exists no phase-cross over frequency. Hence, the gain margin is infinity.

Problem 10: PID Control

```
clc;
clearvars;
close all;
%
t = 0:0.01:10;
k = 0;
for K = 4:-0.05:1
    for a = 4:-0.05:0.4
        num = [0 0 1.2*K 2.4*K*a 1.2*K*a^2];
        den = [0.36 1.86 2.5+1.2*K 1+2.4*K*a 1.2*K*a^2];
        sys = tf(num, den);
        y = step(sys, t);
        m = max(y);
        if m < 1.1 && m > 1.02
```

```
            break;
        end
    end
    if m < 1.1 && m > 1.02
        break;
    end
end
% Solution:
disp("K = " + K);
```

```
K = 4
```

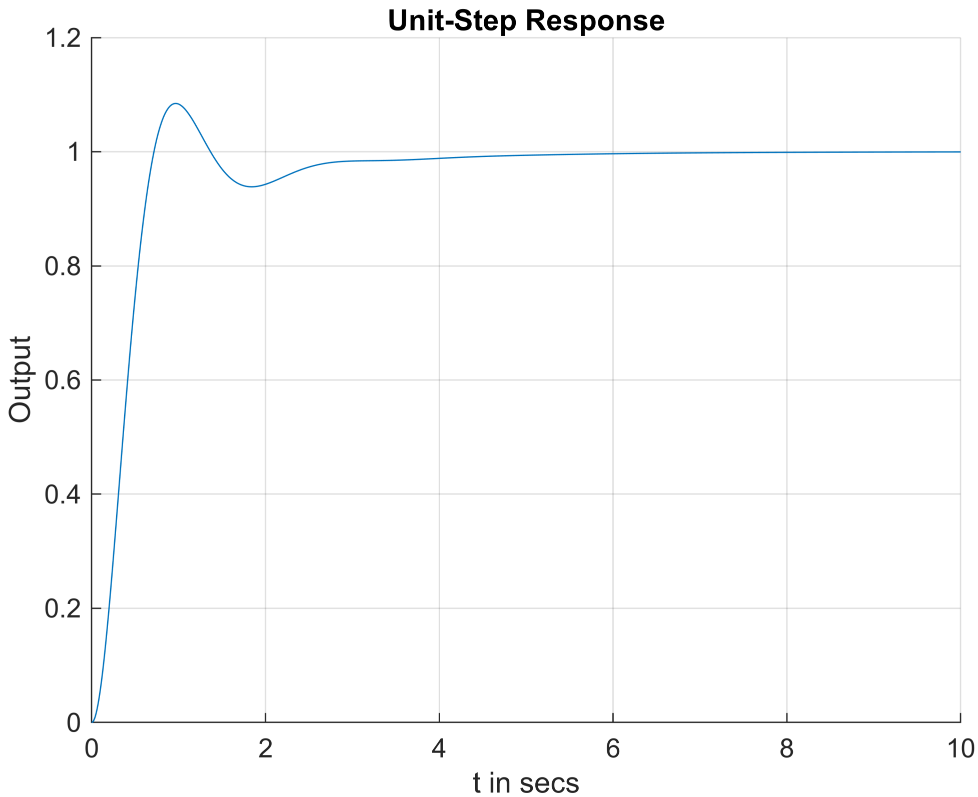
```
disp("a = " + a);
```

```
a = 0.7
```

```
disp("m = " + m);
```

```
m = 1.0846
```

```
% Plotting
figure(1);
hold on;
grid on;
title("Unit-Step Response");
xlabel("t in secs");
ylabel("Output");
plot(t,y)
```


Unit-Step Response
Output
t in secs
0
0.2
0.4
0.6
0.8
1
1.2
0
2
4
6
8
10