Homework-1 Solutions

Problem 1.

(a) $f(t) = e^{5t} + 5$

$$F(s) = L\{f(t)\} = L\{e^{5t}\} + 5L\{1\} = \frac{1}{s-5} + \frac{5}{s}$$

(b) $f(t) = \cos(3t) + 7\sin(3t)$

$$F(s) = L\{f(t)\} = L\{\cos 3t\} + 7L\{\sin 3t\}$$

$$= \frac{s}{s^2+9} + \frac{7\cdot 3}{s^2+9}$$

$$= \frac{s}{s^2+9} + \frac{21}{s^2+9}$$

(c) $f(t) = e^{-4t}\cosh(5t) + 6e^{-4t}\sinh(5t)$

$$F(s) = L\{f(t)\} = L\{e^{-4t}\cosh(5t)\} + L\{6e^{-4t}\sinh(5t)\}$$

$$= \frac{(s+4)}{(s+4)^2 - 25} + \frac{6\cdot 5}{(s+4)^2 - 5^2}$$

$$= \frac{s+4}{(s+4)^2 - 5^2} + \frac{30}{(s+4)^2 - 5^2}$$

Problem

2

(a) $\ddot{x} - 10\dot{x} + 25x = 24t^2 e^{5t}$

Initial conditions: $x(0) = -2 \quad \dot{x}(0) = -10$

Apply L.T

$$s^2 X(s) - s x(0) - \dot{x}(0) - 10[s X(s) - x(0)] + 25 X(s) = 24 \cdot \frac{2!}{(s-5)^3}$$

$$X(s)[s^2 - 10s + 25] + 2s - 10 = \frac{48}{(s-5)^3}$$

$$X(s)[s^2 - 10s + 25] = \frac{48}{(s-5)^3} - 2s + 10$$

$$X(s) = \frac{\frac{48}{(s-5)^3}}{(s-5)^2} - \frac{2s}{(s-5)^2} + \frac{10}{(s-5)^2}$$

Apply Inverse L.T

$$L^{-1}\{X(s)\} = L^{-1}\left\{\frac{\frac{48}{(s-5)^3}}{(s-5)^2}\right\} - L^{-1}\left\{\frac{2s}{(s-5)^2}\right\} + L^{-1}\left\{\frac{10}{(s-5)^2}\right\}$$

$$L^{-1}\{X(s)\} = L^{-1}\left\{\frac{48}{(s-5)^5}\right\} - L^{-1}\left\{\frac{2s}{(s-5)^2}\right\} + L^{-1}\left\{\frac{10}{(s-5)^2}\right\}$$

$$X(t) = 2t^4 e^{5t} - 2e^{5t} - e^{5t}\cancel{10t} + e^{5t}\cancel{10t}$$

$$\boxed{x(t) = -2e^{5t} + 2t^4 e^{5t}}$$

(b) $\ddot{y} - 3\dot{y} - 10y = 1$ Initial conditions:

$y(0) = -1 \quad \dot{y}(0) = 2$

Taking L.T

$$L\{\ddot{y}\} - 3L\{\dot{y}\} - 10L\{y\} = L\{1\}$$

$$s^2Y(s) - sy(0) - \dot{y}(0) - 3[sY(s) - y(0)] - 10Y(s) = \frac{1}{s}$$

$$s^2Y(s) + s - 2 - 3sY(s) - 3 - 10Y(s) = 1/s$$

$$Y(s)[s^2 - 3s - 10] + s - 5 = \frac{1}{s}$$

$$Y(s)[s^2 - 3s - 10] = \frac{1}{s} - s + 5$$

$$Y(s) = \frac{1}{s(s-5)(s+2)} - \frac{s}{(s-5)(s+2)} + \frac{5}{(s-5)(s+2)}$$

Take partial fractions and apply Inverse LT

$$L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{1}{s(s-5)(s+2)}\right\} - L^{-1}\left\{\frac{s}{(s-5)(s+2)}\right\} + L^{-1}\left\{\frac{5}{(s-5)(s+2)}\right\}$$

$$y(t) = \left[\frac{-1}{10} + \frac{e^{-2t}}{14} + \frac{e^{5t}}{35}\right] - \left[\cancel{\frac{5}{7}e^{5t}} + \frac{2}{7}e^{-2t}\right] + \left[\cancel{\frac{5}{7}e^{5t}} - \frac{5}{7}e^{-2t}\right]$$

$$\boxed{y(t) = \frac{-1}{10} - \frac{13}{14}e^{-2t} + \frac{1}{35}e^{5t}}$$

Initial conditione.

(C) $\ddot{y} + 16y = 1$ $\qquad y(0) = 1 \qquad \dot{y}(0) = 2$

Take L.T

$$s^2 Y(s) - s y(0) - \dot{y}(0) + 16 Y(s) = \frac{1}{s}$$

$$Y(s)\{s^2 + 16\} - s - 2 = \frac{1}{s}$$

$$(s^2+16)\,Y(s) = \frac{1}{s} + s + 2$$

$$Y(s) = \frac{1}{s(s-4i)(s+4i)} + \frac{s}{(s-4i)(s+4i)} + \frac{2}{(s-4i)(s+4i)}$$

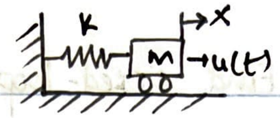
Taking partial fractions and apply inverse laplace transform

$$y(t) = \left(\frac{1}{16} - \frac{1}{16}\cos 4t\right) + (\cos 4t) + \frac{1}{2}\sin(4t)$$

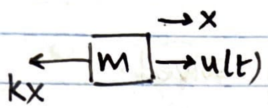
$$\boxed{y(t) = \frac{1}{16} + \frac{15}{16}\cos 4t + \frac{1}{2}\sin 4t}$$

Problem 3:

(a)



F.B.D



$$m\ddot{x} = -kx + u(t)$$

$$m\ddot{x} + kx = u(t)$$

Apply Laplace transform

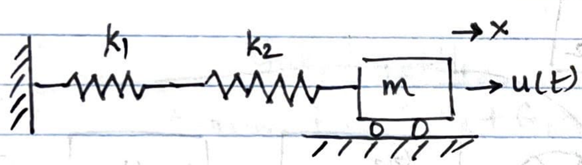
* Assume initial conditions are zero

$$ms^2X(s) + kX(s) = U(s)$$

$$X(s)\left[ms^2 + k\right] = U(s)$$

$$\boxed{\frac{X(s)}{U(s)} = \frac{1}{ms^2+k}}$$

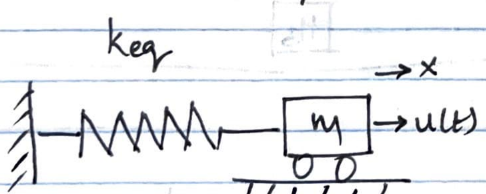
(b)



* Assume initial conditions are zero.

2 springs in series:

Equivalent spring constant



$$k_{eq} = \frac{k_1k_2}{k_1+k_2}$$

$$m\ddot{x} = -k_{eq}x + u(t)$$

Apply Laplace transform

$$\boxed{\frac{X(s)}{U(s)} = \frac{1}{ms^2+k_{eq}}}$$

$$ms^2X(s) = -k_{eq}X(s) + U(s)$$

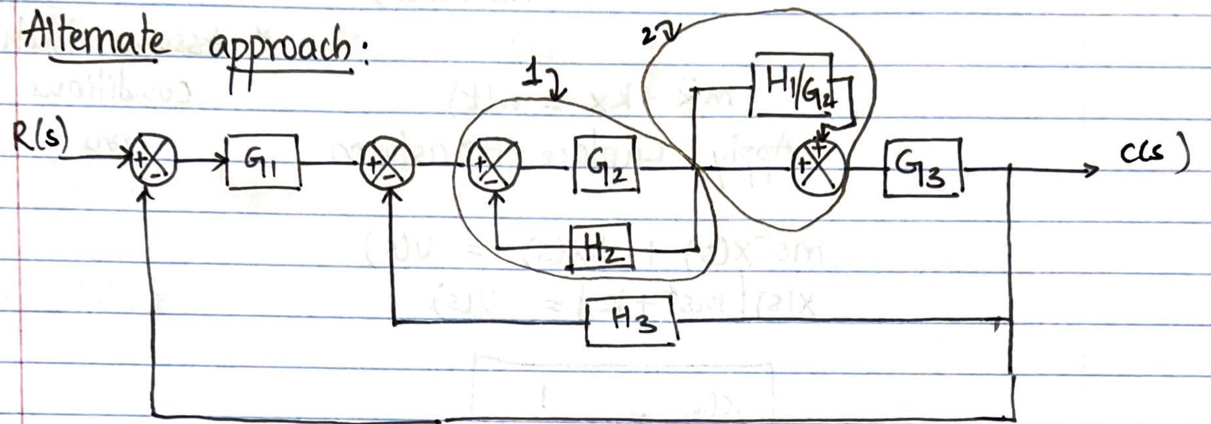
$$X(s)\left[ms^2 + k_{eq}\right] = U(s)$$

* Assume initial conditions are zero.

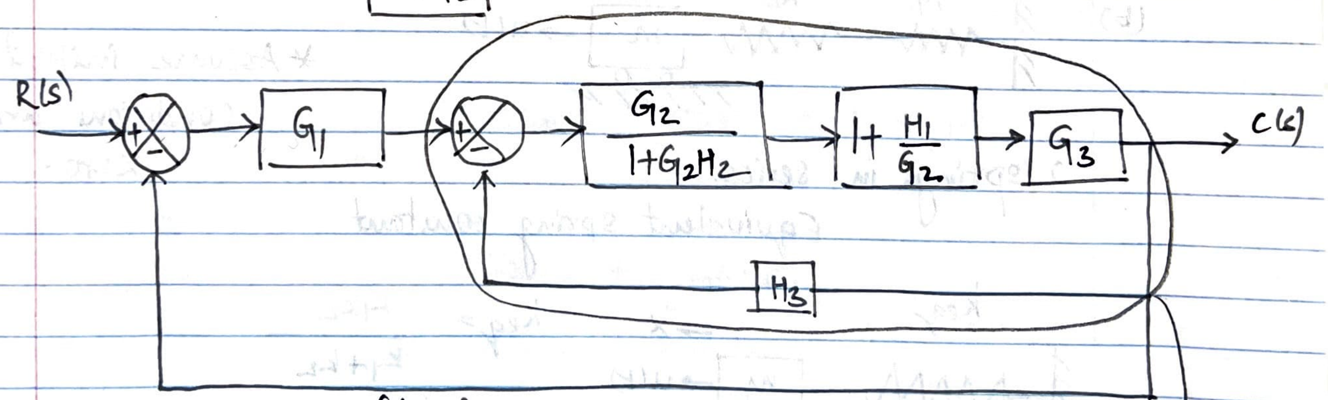
Problem (4): Find closed-loop T.F

– check Notes. Already solved.

Alternate approach:



Step 1: moving H_1 block behind the block. Hence new block $\boxed{H_1/G_2}$

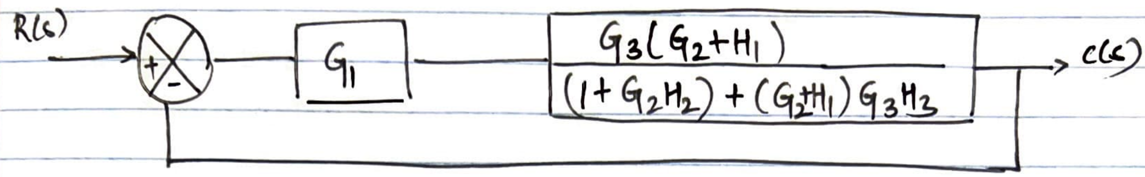


(1, 2)

Step 2: Solve the circle blocks individually and write the equivalent block.

$$\frac{G_3(G_2+H_1)}{1+G_2H_2+(G_2+H_1)(G_3H_3)} \Leftarrow \frac{\left(\frac{\cancel{G_2}G_3}{\cancel{1+G_2H_2}}\right)\left(\frac{G_2+H_1}{\cancel{G_2}}\right)}{1+\frac{\cancel{G_2}G_3H_3(G_2+H_1)}{\cancel{(1+G_2H_2)}\cancel{G_2}}}$$

Step 3:



$$R(s) \rightarrow \boxed{\dfrac{\dfrac{G_1G_3(G_2+H_1)}{(1+G_2H_2)+(G_2+H_1)G_3H_3}}{1+\dfrac{G_1G_3(G_2+H_1)}{(1+G_2H_2)+(G_2+H_1)G_3H_3}}} \rightarrow C(s)$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_2G_3H_3+G_3H_1H_3+G_1G_2G_3+G_1G_3H_1}$$

Solution.