Summer 2023

AME 455: Control System Design

Midterm Exam

Date: June 16, 2023

Student Name: ____

Instructions:

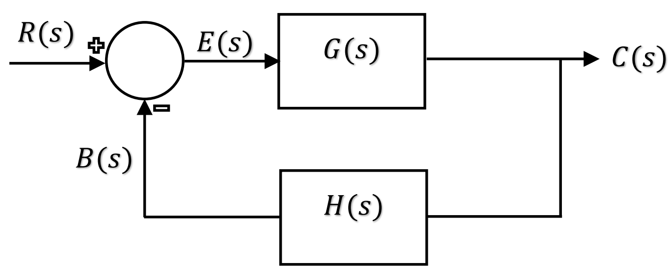
- This exam is closed book, and closed notes. A calculator with basic functions can be used.
- Please write your name clearly above.
- This is a 120-minute exam, and your proctor will keep time.

Problem 1 (10 points):

i. **(1 point)** Obtain $f(t) = \mathcal{L}^{-1}\{F(s)\}$ where $F(s) = \frac{5}{(s+1)(s+2)(s+3)}$

ii. **(1 point)** Obtain $F(s) = \mathcal{L}\{f(t)\}$ where $f(t) = t^2 e^{-3t}$

iii. **(3 points)** Based on the block diagram below and answer what kind of transfer function do the ratios belong to:

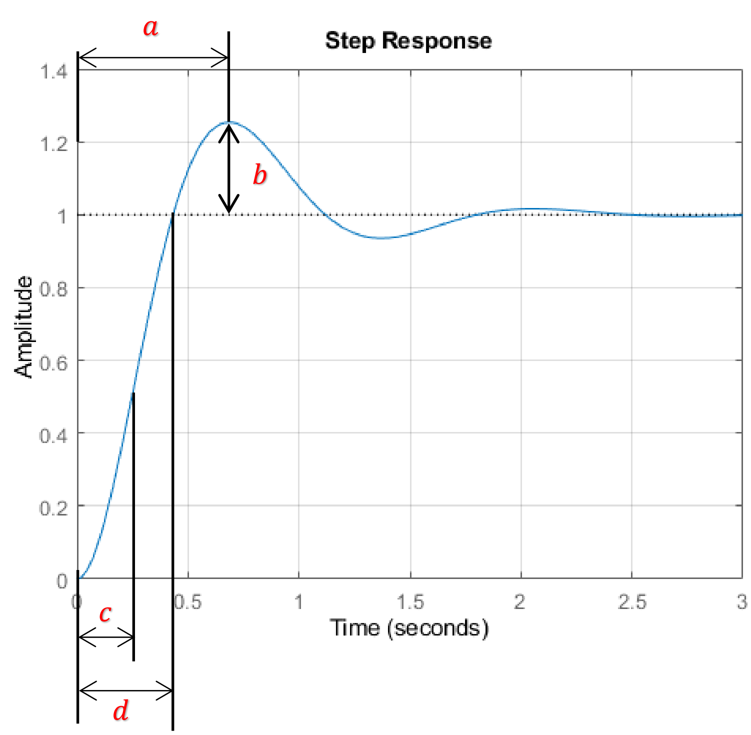


a. $\frac{B(s)}{E(s)} =$

b. $\frac{C(s)}{E(s)} =$

c. $\frac{C(s)}{R(s)} =$

iv. **(4 points)** Observe the following graph and assign the missing parameters.

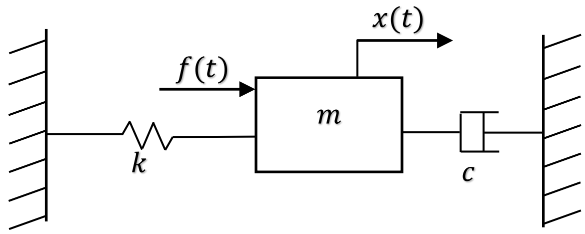


a –
b –
c –
d –

v. **(1 point)** At $t = T$, the exponential response curve $c(t)$, of a first-order system reaches _____ % of its final value.

Problem 2 (10 points):

Consider the mass-spring-dashpot system shown below. The block of mass m is attached to a spring on the left side while the dashpot is attached on the right side. A force $f(t)$ is applied to the mass m, which undergoes displacement $x(t)$.



Answer the following questions:

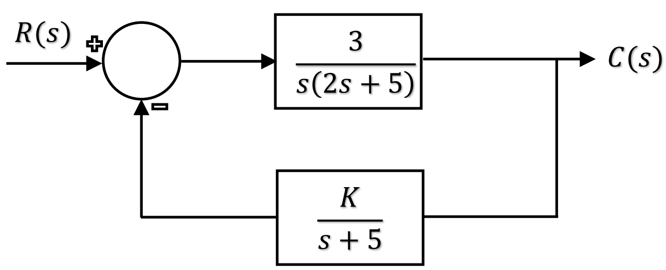
i. **(2 points)** Draw the free body diagram and use Newton's second law to determine the equation of motion.
ii. **(2 points)** Derive the transfer function from the input force $f(t)$ and the output displacement $x(t)$ i.e.,

$$G(s) = \frac{X(s)}{F(s)} = \frac{\mathcal{L}(x(t))}{\mathcal{L}(f(t))}$$

iii. **(1 point)** What is the order of the resulting system?
iv. **(5 points)** For $c = 2Ns/m, m = \frac{1}{3}kg, k = 3N/m, f(t) = 10N$ **step-input**, determine the time response $x(t)$?

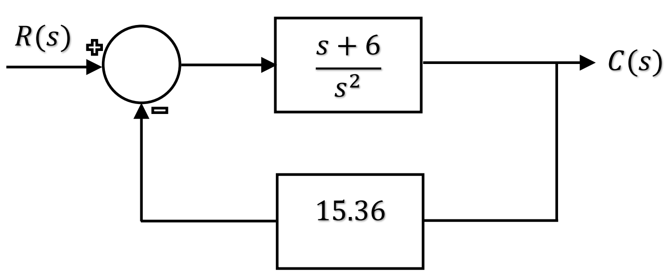
Problem 3 (10 points):

Determine the range of values of K, for which the following system is stable.



Problem 4 (10 points):

i. **(6 points)** For the following system:



a. **(2 points)** Convert the above block diagram into a closed loop system with unity-feedback.
b. **(2 points)** Determine steady-state error due to unit-step input?
c. **(2 points)** Determine steady-state error due to unit-ramp input?

ii. **(4 points)** For the following transfer function:

$$\frac{C(s)}{R(s)} = \frac{25}{s^2 + 4s + 25}$$

Determine

a. **(1 point)** Maximum Percent Overshoot
b. **(1 point)** Rise Time
c. **(1 point)** Peak Time
d. **(1 point)** 2% Settling Time

Use the following formulae if required:

1. $\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$
2. $\mathcal{L}\{t^n e^{-at}\} = \frac{n!}{(s+a)^{n+1}}$
3. $\mathcal{L}\{1\} = \frac{1}{s}$
4. Rise time $(t_r) = \frac{\pi-\beta}{\omega_d}$
5. Peak time $(t_p) = \frac{\pi}{\omega_d}$
6. 2% Settling time $(t_s) = \frac{4}{\zeta\omega_n}$
7. 5% Settling time $(t_s) = \frac{3}{\zeta\omega_n}$
8. Maximum % overshoot $(M_p) = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} \times 100\%$