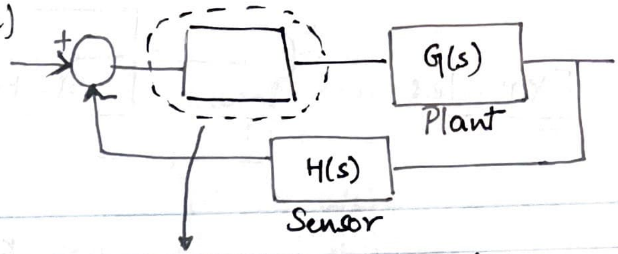
Control system design (big umbrella)
* selection of plant, sensors are important.



* Compensate for the deficiency in the plant
* we cannot change the plant and sensor (economic constraints)

Sketching a Root Locus

* Suitable controller so that all our performance requirements can be satisfied

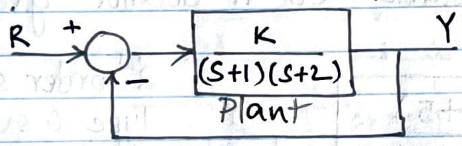
ROOT LOCUS METHOD:

Performance measures → $t_r, t_p, (\zeta, \omega_n), M_p, t_s, e_{ss}$ transient

(ζ, ω_n) gives complete performance of the system

* (ζ, ω_n) can be translated into desired locations of closed loop poles

Example:

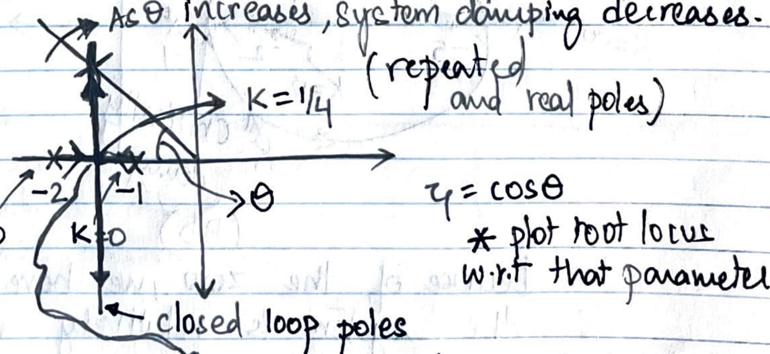


K − parameters subject to change.

$$\frac{Y(s)}{R(s)} = \frac{K}{s^2+3s+(2+K)}$$ closed loop T.F

$$s_{1,2} = -\frac{3}{2} \pm \frac{1}{2}\sqrt{1-4K}$$ ← closed loop poles are roots of the characteristic eq.

As $K \to 0$ to ∞



$\zeta = \cos\theta$

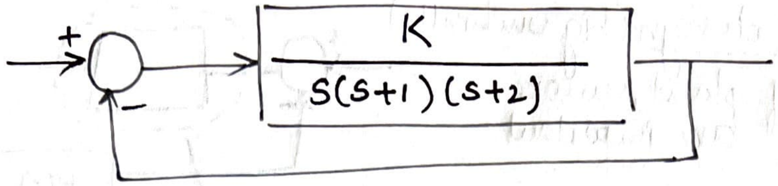
* Root locus plot is a complete description of a system w.r.t a specific design parameter

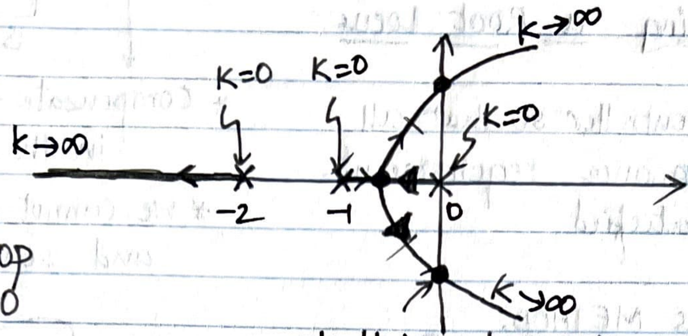
* plot root locus w.r.t that parameter.

overdamped → critical damped → underdamped

* for $K > \frac{1}{4}$, it becomes complex roots; one root start from −2 & another from −1
* we can deduce any inform about the system from root locus

Example:



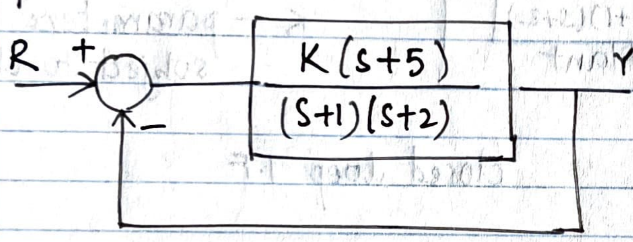


* system is moving towards unstable

* we are at open-loop pole point at K=0

at this value of K, system becomes oscillatory

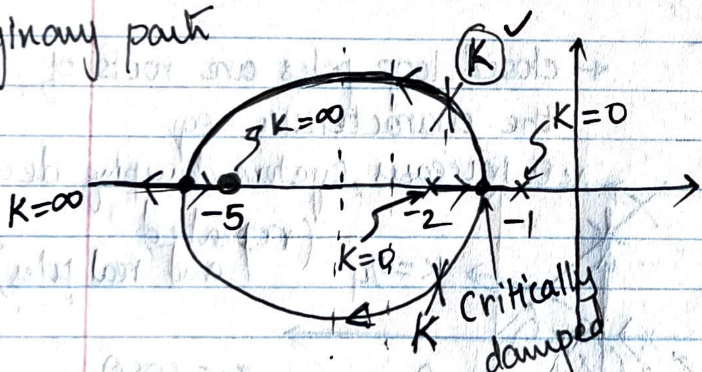
* Routh stability criterion gives us the range of K for which system is stable. But it doesnot give you the system poles.



2^{nd} order system
Type 0 system
PD controller

poles: x
zeros: o

imaginary path

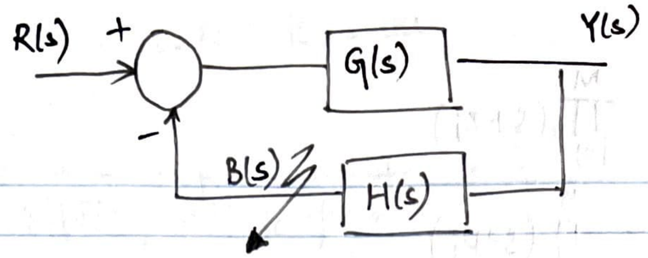


2^{nd} order system
- 2 closed loop poles
- 2 branches

* makes more stable

(PD)

- because of the zero, we have added to the system, the system is relatively more stable.
- because of PI, stability decreases.



closed loop T.F

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$\frac{B(s)}{E(s)}$ Open-loop T.F: $G(s)H(s)$

Characteristic Equation $\Rightarrow$ $1+G(s)H(s)=0$

roots of $=$ poles of system (will determine the transient behavior of the system)

Any form of $G(s)H(s)$ involves a gain parameter K

$$G(s)H(s) = \underbrace{\boxed{K\frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}}}_{F(s)}$$

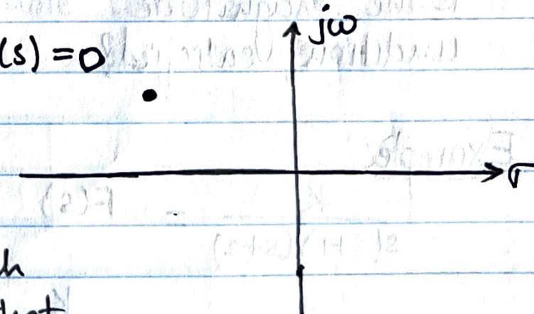
root locus gain

$z_i \rightarrow$ openloop zeros

$p_j \rightarrow$ openloop poles

* open-loop poles can be in RHP (can be unstable)
  closed-loop poles cannot be in RHP

characteristic Equation: $1+F(s)=0$



* Root locus method says
  scan for entire space ($\sigma - j\omega$)
  and find those point for which
  $1+F(s)=0$. Join those point, that
  is root locus plot

or $F(s) = -1$

$|F(s)| = 1$ $\qquad$ $\angle F(s) = \pm(2k+1)180^\circ \quad k=0,1,2\ldots$

magnitude.

$$F(s) = \frac{K \prod_{i=1}^{M} (s+z_i)}{\prod_{j=1}^{n} (s+p_j)}$$

Magnitude criterion: $0 \leq K < \infty$

$$\frac{K \prod_{i=1}^{m} |s+z_i|}{\prod_{j=1}^{n} |s+p_j|} = 1$$

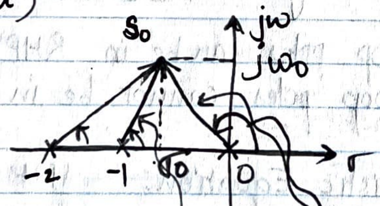
Angle condition

$$\sum_{i=1}^{M} \angle s+z_i - \sum_{j=1}^{n} \angle s+p_j = \pm(2k+1)180^\circ$$

* Any point that satisfies these conditions is a point on the root locus

(all points)

At what points in the entire s-plane is the magnitude condition satisfied?



$s_o = \sigma_o + j\omega_o$

$|s_o|$ = distance

$|s_o+1|$ = distance

$\angle s_o$

Example:

$$\frac{K}{s(s+1)(s+2)} = F(s)$$

$$\frac{K}{|s||s+1||s+2|} = 1$$

$$-\angle s - \angle s+1 - \angle s+2 = \pm(2k+1)180^\circ$$

Root locus gain

$$\frac{K}{|\sigma_0 + j\omega_0||\sigma_0 + j\omega_0 + 1||\sigma_0 + j\omega_0 + 2|} = 1$$

* If both these conditions are satisfied, we will say that s_0 fulfills both conditions and is a point on root locus

Example:

$$1 + \frac{K}{(s+1)(s+2)} = 0$$

$$1 + F(s) = 0$$

$$1 + K \frac{\prod_{i=1}^{m} (s+z_i)}{\prod_{j=1}^{n} (s+p_j)} = 0 \qquad K > 0 \qquad m \leq n$$

* Magnitude and Angle conditions

(1) Scan total s-plane where angle condition is satisfied.

* Any problem wherein the roots of the characteristic eq. are to be seen as a parameter is varied from 0 to ∞ can be represented in the following manner:

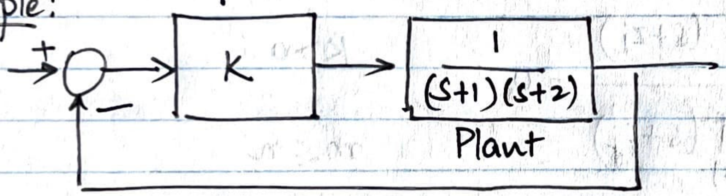
$$1 + F(s) = 0$$

where

$$F(s) = \frac{K \prod_{i=1}^{m} (s+z_i)}{\prod_{j=1}^{n} (s+p_j)}$$

K → root locus gain

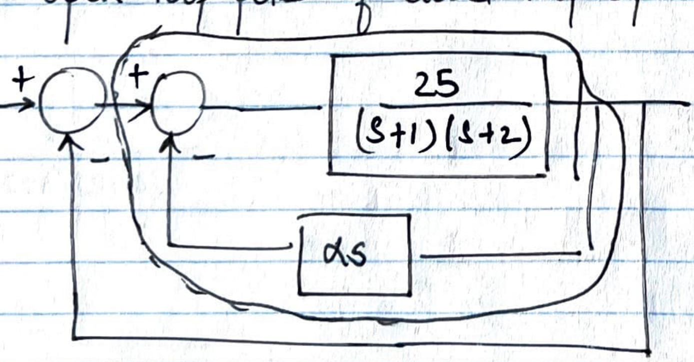
Example:

① 

characteristic eq: $1 + \underbrace{\frac{K}{(s+1)(s+2)}}_{F(s)} = 0$

$s = -1, -2$

* open loop poles are same as open loop poles of closed loop system

② 

minorloop:

$$\frac{\frac{25}{s^2+3s+2}}{1 + \frac{25\alpha s}{s^2+3s+2}}$$

$$\frac{25}{s^2+3s+2+25\alpha s}$$

characteristic equation of the system:

$$s^2 + 3s + 2 + 25\alpha s + 25 = 0$$

parameter

Find the roots using computer.

Root Locus expects (we need to write in this form) characteristic eq.

$$1 + F(s) = 0$$

$$F(s) = K \frac{\prod_i (s+z_i)}{\prod_j (s+p_j)}$$

$$1 + \frac{25\alpha s}{s^2+3s+27} = 0$$

root locus gain

$$1 + \frac{K(s+z_1)}{(s+p_1)(s+p_2)} = 0 \qquad z_1 = 0 \qquad p_1, p_2 \qquad K = 25\alpha$$

When we write F(s), the poles and zeros of F(s) are not necessarily the poles and zeros of open-loop system.

$$1 + F(s) = 0$$

z_i, p_j are zeros and poles of F(s)

$$1 + K \frac{\prod_{i=1}^{m} (s+z_i)}{\prod_{j=1}^{n} (s+p_j)} = 0$$

$K > 0$

$m \leq n$ (due to realizability requi)

$$K \frac{\prod_{i=1}^{m} |s+z_i|}{\prod_{j=1}^{n} |s+p_j|} = 1 \quad \& \quad \sum_{i=1}^{m} \angle(s+z_i) - \sum_{j=1}^{m} \angle(s+p_j) = \pm(2q+1)180^\circ$$

magnitude condition

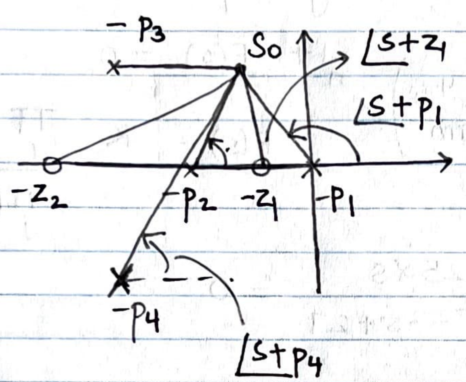
Angle condition

* Magnitude condition is satisfied at any point in s-plane. Substitute that value. Calculate magnitude. K = Inverse (   ), it will always be satisfied.

(1) Scan the total s-plane where angle condition is satisfied. (Join those points - gives me root locus plot)

(2) At

Brute force:



F(s): open loop poles & zeros

So: trial point

Check angle condition in this manner.

Guidelines: (for a rough root locus make an initial decision.)

Rule 1: Symmetry rule:

- Any root locus plot we make has to be symmetrical w.r.t real axis.
- Any complex roots which occur, they have to occur in complex conjugates.
- make the upper plot and take ~~sym~~ mirror image

Rule 2: Root Locus gain K varies — $F(s)$: m zeros, n poles

$0 \le K < \infty$

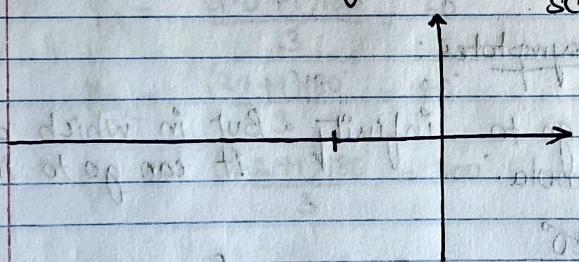
(starting point) $K=0$ is at open loop poles of $F(s)$

* number of root locus branches = No. of openloop poles of $F(s)$
(every root locus branch will start @ open-loop poles @ $K=0$)

$K=\infty$ (Terminal point)

① m zeros of $F(s)$
② $(n-m)$ branches will terminate @ infinity.

Rule 3: Real axis segments: scanning the total s-plane. scan the real-axis first



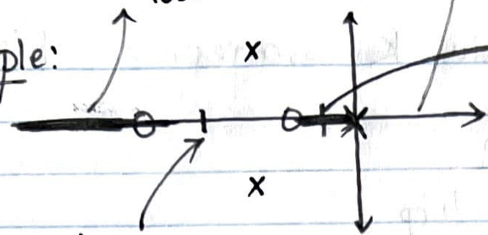
Any point on the real axis.
– If the number of real axis open-loop poles and zeros to the right of this plane is odd. Then that point lies on the root locus.

lies on the root locus

does not lie on root locus

real-axis

Example:

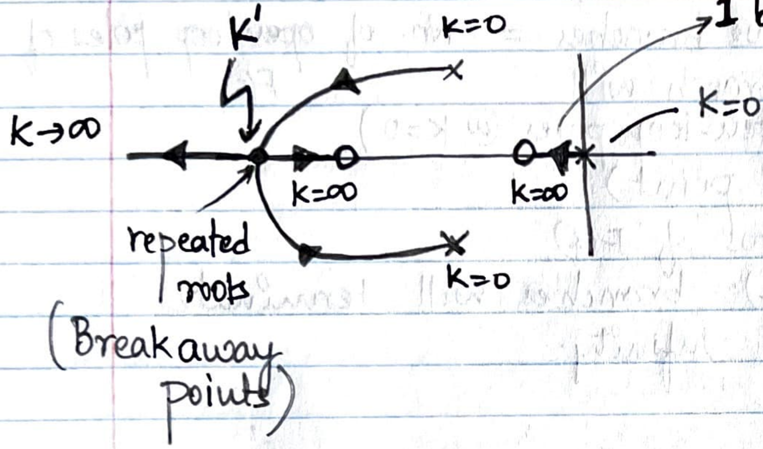


No. of open-loop poles and zeros = 1 (

∴ the total segment is root locus segment

not odd

does not lies on root locus



- 3 root locus branches
- 2 terminal points given by zeros
- 1 terminal point given by ∞

Root locus point gives closed loop poles

Rule 4: Directions of Asymptotes:

$(n-m)$ branches go to infinity. But in which directions it can go to infinity?

use this formula.

$$\phi_A = \frac{(2q+1)180^\circ}{n-m} \qquad q = 0, 1, \ldots, (n-m-1)$$

Rule 5: Asymptotes → centroid

from which point on the real-axis should we draw these asymptotes.

$$-\sigma_A = \frac{\Sigma(\text{real parts of poles}) - \Sigma(\text{real parts of zeros})}{n-m}$$

centroid

Example:

$$F(s) = \frac{K}{s(s+1)(s+2)}$$

$n=3 \quad m=0$

all root locus branches will go to ∞

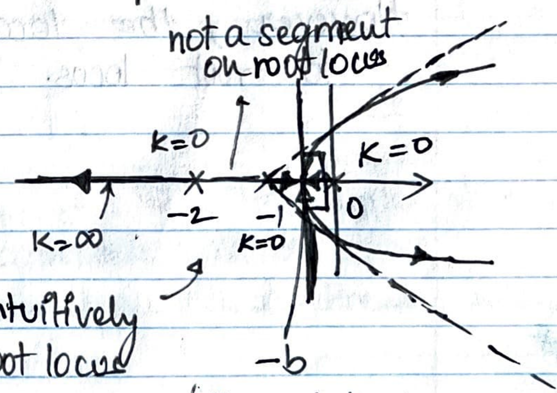
$$-\sigma_A = \frac{(-1-2)-0}{3} = -1$$

$$\phi_{A_1} = \frac{(2\cdot 0+1)180^\circ}{3} = 60^\circ$$

$$\phi_{A_2} = \frac{(2\cdot 1+1)180^\circ}{3} = 180^\circ$$

$$\phi_{A_3} = \frac{(2\cdot 2+1)180^\circ}{3} = 300^\circ$$

$q = 0, 1, 2, (3-0-1)$



(the point at which the 2 root locus branches break away)

Rule 6: Breakaway point:

$$1 + K \frac{\prod_{i=1} (s+z_i)}{\prod_{j=1} (s+p_j)} = 0$$

The value @ which the breakway point occurs will satisfy the equation $\boxed{\frac{dK}{ds} = 0}$

Ex:

$$1 + \frac{K}{s(s+1)(s+2)}$$

$$K = - s(s+1)(s+2) = -(s^3 + 3s^2 + 2s)$$

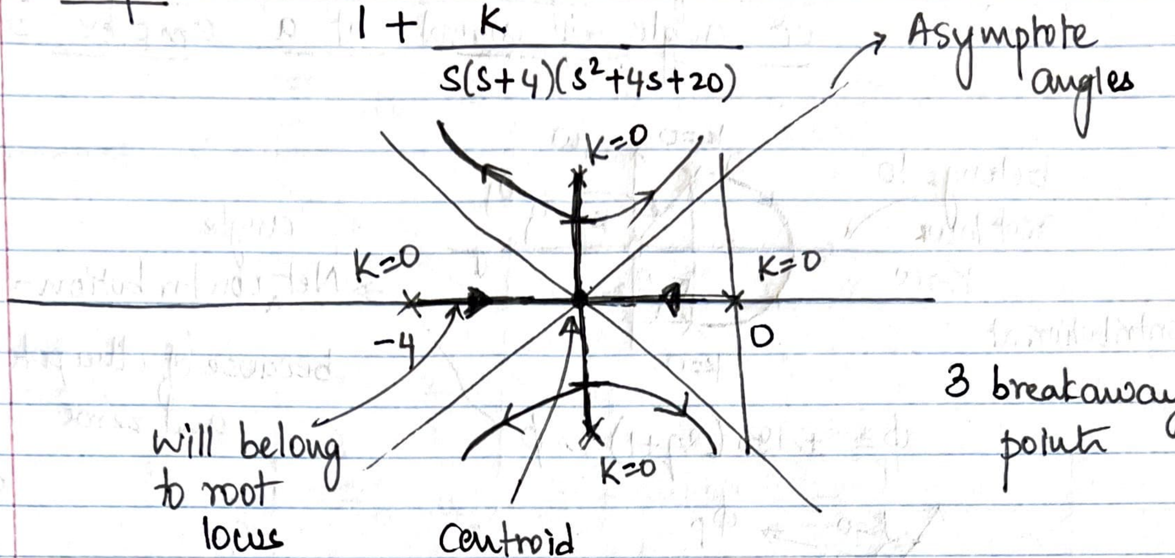
$$\frac{dK}{ds} = -\underbrace{(3s^2 + 6s + 2)}_{} = 0$$ (candidates of breakaway points)

$\hookrightarrow s_{1,2} = -0.423, -1.577$

However, the location b/w -1.577 does not belong to root locus.

Example:

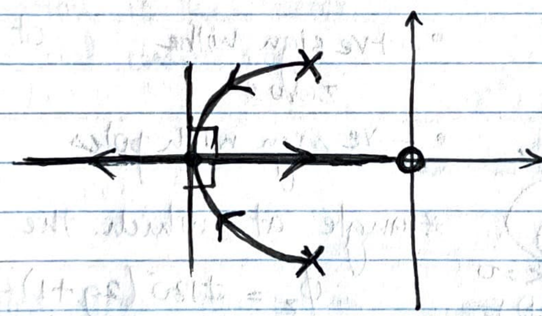
$$1 + \frac{K}{s(s+4)(s^2+4s+20)}$$



$$\frac{dK}{ds} = 0$$

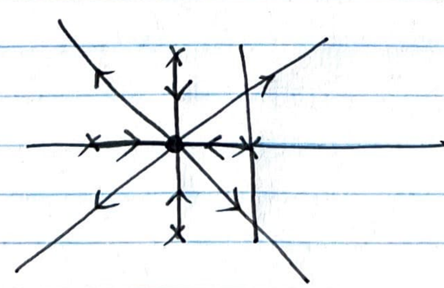
Example

$$1 + \frac{K s}{s^2+2\cancel{s}+27}$$



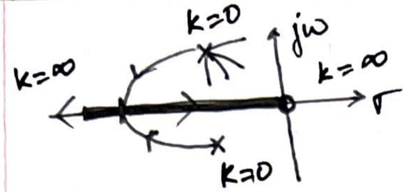
- 2 branches are approaching @ 90°, the angle at which the 2 branches which approach are 90° or the 2 branches after approaching and break away is 90°

Example



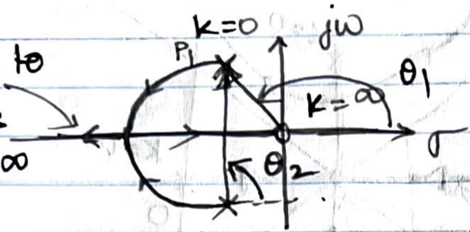
roots breakaway at an angle $\frac{180^\circ}{r}$

r = no. of branches which are meeting



Rule 7: Angle of departure from a complex pole or angle of arrival at a complex zero

belongs to root locus



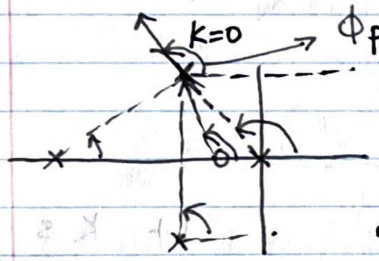
K→∞

Net contribution at P_1

$\phi_p = \pm 180(2q+1) + \phi$

Net angle contribution $= \theta_1 - \theta_2$
because of other poles and zeros

angle contribution of P_2 @ this P_1

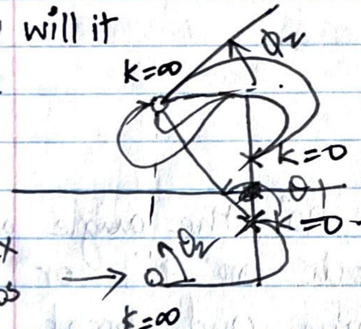


$\phi_p = \pm 180^\circ(2q+1) + \phi$

net angle contribution at pole under consideration.

- +ve sign with the zeros
- −ve sign with poles

what way will it approach a zero?



complex zeros

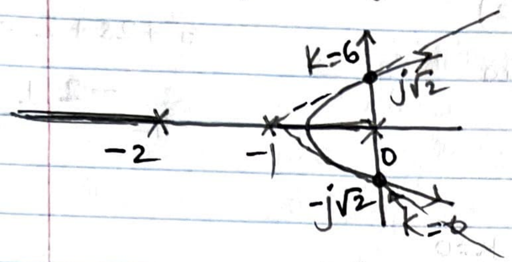
* angle at which the root locus branch will approach a complex zero

$\phi_z = \pm 180^\circ(2q+1) - \phi$

net angle contribution

$\phi_z = \pm 180^\circ - (\theta_2 - 2\theta_1)$

Rule 8: point of intersection with the imaginary axis



$$G(s) = \frac{K}{s(s+1)(s+2)}$$

characteristic eq

$$s^3 + 3s^2 + 2s + K = 0$$

Use routh-hurwitz criterion

s^3	1	2
s^2	3	K
s	$\cancel{\frac{6-K}{3}}$ 0	0
s^0	K	

$K > 0$

$\frac{6-K}{3} > 0$

$\boxed{K < 6}$

$3s^2 + 6 = A(s)$ → Auxiliary equation

at K=6, a row becomes 0

* Auxiliary equation is the factor of the original characteristic equation.

$$s^2 + 2 = 0$$

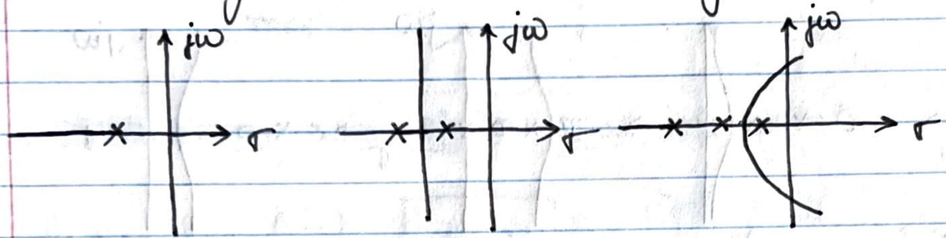
$$\boxed{s = \pm j\sqrt{2}}$$

All these rules give you a ~~rough~~ sufficient information to make a rough sketch.

Effect of the Addition of poles:

Addition of the pole to the open loop T.F has the effect of pulling the root locus to the right, tending to lower the system's relative stability and to slow down the settling of the response.

* Remember that the addition of integral control adds a pole at the origin, thus making the system less stable.



root locus plot of a single pole system

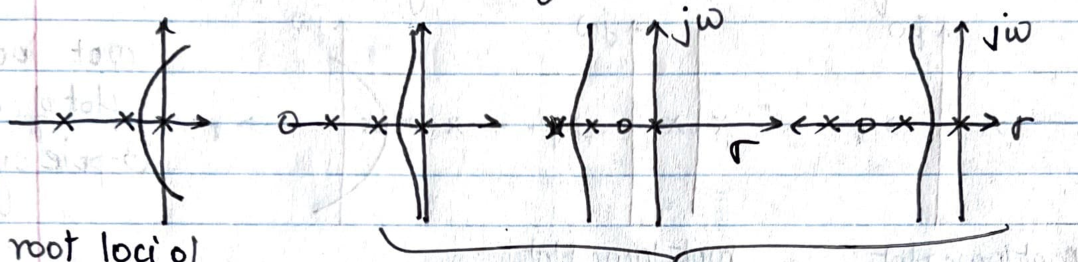
root locus plot of a 2pole system

root locus plot of a 3-pole system

Effects of the Addition of zeros

The addition of a zero to the open-loop transfer function has the effect of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response.

* Physically, the addition of a zero in a feedforward T.F means the addition of derivative control to the system



root loci of the system is stable for a small gain but unstable for large gain

* root locus plots for the system when a zero is added to the open-loop T.F.