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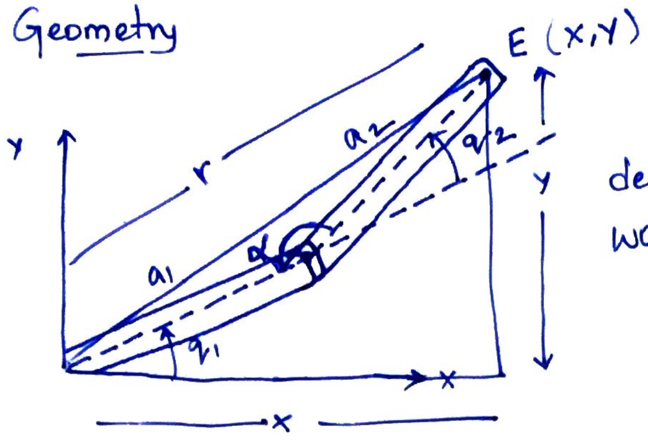
Assignment-5

April 14:

Problem 1

(a) Closed form Inverse Kinematic Solution:

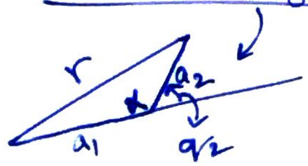
① Using Geometry



Tool tip (E) pose is described by (x,y) in the world coordinate frame

From geometry, we have $r^2 = x^2 + y^2$

From the triangle,



applying cosine rule, we have

$$r^2 = a_1^2 + a_2^2 - 2a_1a_2\cos\alpha$$

$$\cos\alpha = \frac{a_1^2 + a_2^2 - r^2}{2a_1a_2} = \frac{a_1^2 + a_2^2 - x^2 - y^2}{2a_1a_2}$$

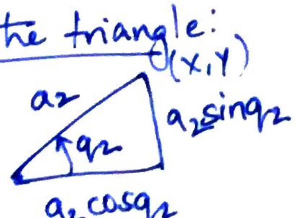
Also, we have

$$q_2 = \pi - \alpha$$

$$\cos q_2 = -\cos\alpha$$

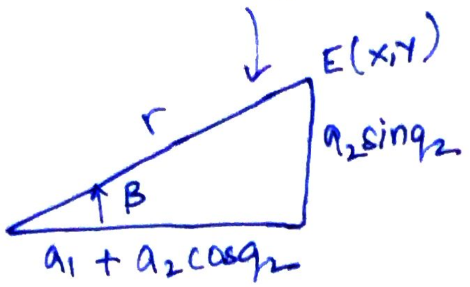
$$\cos q_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

Using the triangle:



$\rightarrow$ $\sin q_2 = \sqrt{1 - \cos^2 q_2}$

From the triangle



$$\tan\beta = \frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}$$

$$\beta = \tan^{-1}\left(\frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}\right)$$

Let $q_1 + \beta = \gamma$

$$\tan\gamma = \frac{y}{x}$$

$$\gamma = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\therefore\; q_1 = \gamma - \beta$$

$$\boxed{q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}\right)}$$

$$\boxed{q_2 = \cos^{-1}\left(\frac{x^2 + y^2 - a_1^2 - a_2^2}{2 a_1 a_2}\right)}$$

choosing only the positive angle for q_2

If q_2 is negative.

$$\boxed{q_2 = -\cos^{-1}\left(\frac{x^2 + y^2 - a_1^2 - a_2^2}{2 a_1 a_2}\right)}$$

Similarily, we have

$$\boxed{q_1 = \tan^{-1}\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}\right)}$$

```
% Problem 1(a) - Test
% Using Closed Form Inverse Kinematics Solution
clc;
clear all;
close all;

tic
x = 20/sqrt(2);
y = 20/sqrt(2);
[T1, T2] = getJointAngles(x,y);
disp('The value of theta1 : ')
```

```
The value of theta1 :
```

```
disp(T1)
```

```
   0.785398150492664
```

```
disp('The value of theta2 : ')
```

```
The value of theta2 :
```

```
disp(T2)
```

```
     2.580956827951785e-08
```

```
toc
```

```
Elapsed time is 0.077970 seconds.
```

```
% Problem 1(b) - Test
% Using unconstrained optimization
tic
x = 20/sqrt(2);
y = 20/sqrt(2);
[T1, T2] = getOptimizer(x,y);
```

```
Local minimum found.

Optimization completed because the size of the gradient is less than
the value of the optimality tolerance.

<stopping criteria details>
```

```
disp('The value of theta1 : ')
```

```
The value of theta1 :
```

```
disp(T1)
```

```
   0.785398152194258
```

```
disp('The value of theta2 : ')
```

```
The value of theta2 :
```

```
disp(T2)
```

```
2.240638097283920e-08
```

```
toc
```

```
Elapsed time is 0.205136 seconds.
```

Problem 1 - Part(c) - theta2 is really small.

```
% Problem 1(c) - Test
% Using Brute Force Approach
tic
x = 20/sqrt(2);
y = 20/sqrt(2);
final_result = bruteForceSearch(x,y);
disp('The value of theta1 & theta2 : ')
```

```
The value of theta1 & theta2 :
```

```
disp(final_result(1,:))
```

```
0.784611979049703   0.001572368695490
```

```
toc
```

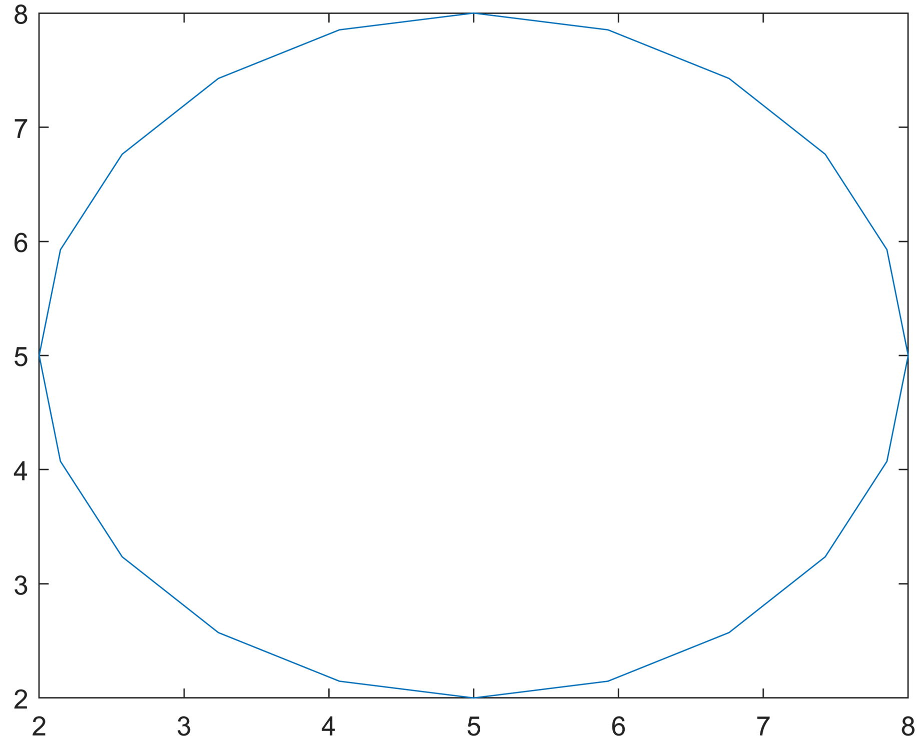
```
Elapsed time is 6.866541 seconds.
```

Problem 1 - Part (d)

I have chosen the circle to be centered at (5,5) which is in the workspace.

```
% Problem 1(d) - Test
clear;
clc;
close all

% Circle Trajectory to follow
circleCenter = [5 5]; % Center of the circle
radius = 3; % Radius of the circle
% Link Lengths
a1 = 10;
a2 = 10;
xc = circleCenter(1);
yc = circleCenter(2);

r = sqrt(xc^2+yc^2);

% test whether the value is in the workspace or not.
max_r = r + radius;
if max_r > (a1+a2)
    disp('The points on the circle are not reachable\n');
end
```

```
t = 0:2*pi/20:2*pi;
x = xc + radius*cos(t);
y = yc + radius*sin(t);

points = [x' y']; % Given 20 equidistant point on the circle
plot(x,y)
```

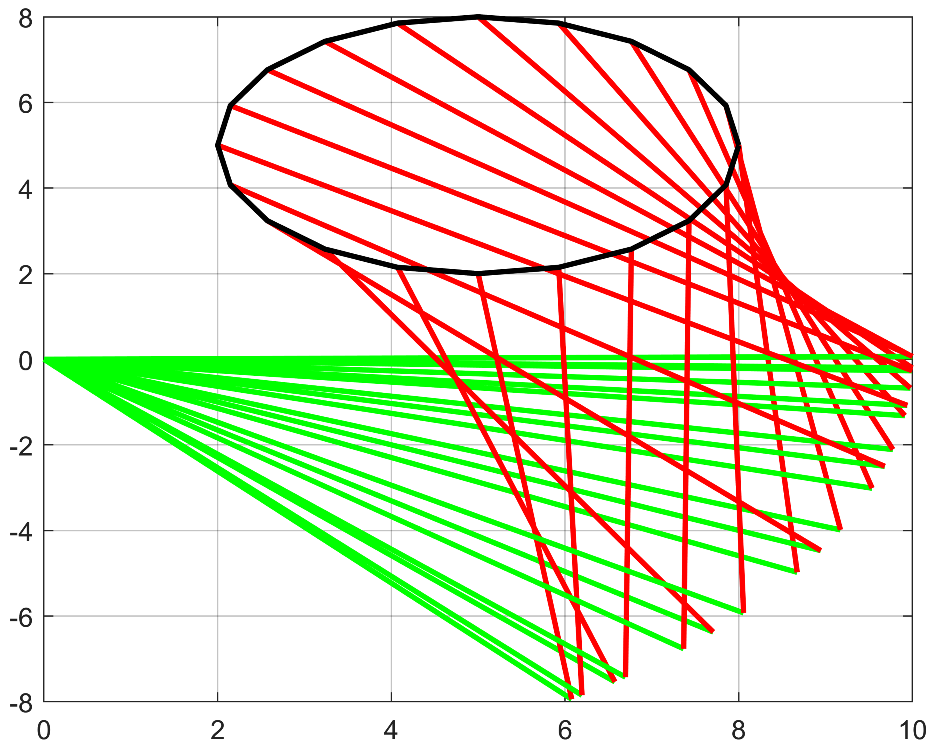



```
% Problem 1(e) - Test
% Plot robot configuration when given theta1 and theta2
```

The function getPlots() written along with other functions gives the robot configuration for every theta1 and theta2

```
% Problem 1(f) - Test

tic

a1 = 10; % given value of link 1
a2 = 10; % given value of link 2

for i = 1:20
    x(i) = points(i,1); % x coordinate of the end position
    y(i) = points(i,2); % y coordinate of the end position
    [t1, t2] = getJointAngles(x(i),y(i)); % Obtain theta1 and theta2
    [link1x(i,:), link1y(i,:), link2x(i,:), link2y(i,:)] = getPlots(t1, t2);
    plot(link1x(i,:),link1y(i,:),'g','linewidth',2);
```

```
    hold on;
    plot(link2x(i,:),link2y(i,:),'r','linewidth',2)

end
plot(x,y,'k','linewidth',2);grid
```




```
toc
```

```
Elapsed time is 0.263499 seconds.
```

```
% Problem 1(g) - Test
% Using Optimization
tic

a1 = 10;
a2 = 10;

for i = 1:20
    x(i) = points(i,1);
    y(i) = points(i,2);
    [t1, t2] = getOptimizer(x(i), y(i));
    [link1x(i,:), link1y(i,:), link2x(i,:), link2y(i,:)] = getPlots(t1, t2);
    plot(link1x(i,:), link1y(i,:),'g', 'LineWidth', 2);
    hold on;
    plot(link2x(i,:), link2y(i,:), 'r','LineWidth',2);
end
```

```
Local minimum possible.
```

```
fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
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along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
Local minimum possible.

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along the current search direction.

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Local minimum possible.

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along the current search direction.

<stopping criteria details>
Local minimum possible.

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along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
```

```
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
Local minimum possible.

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along the current search direction.

<stopping criteria details>
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along the current search direction.

<stopping criteria details>
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along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
Local minimum possible.

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along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
Local minimum possible.

fminunc stopped because it cannot decrease the objective function
along the current search direction.

<stopping criteria details>
```
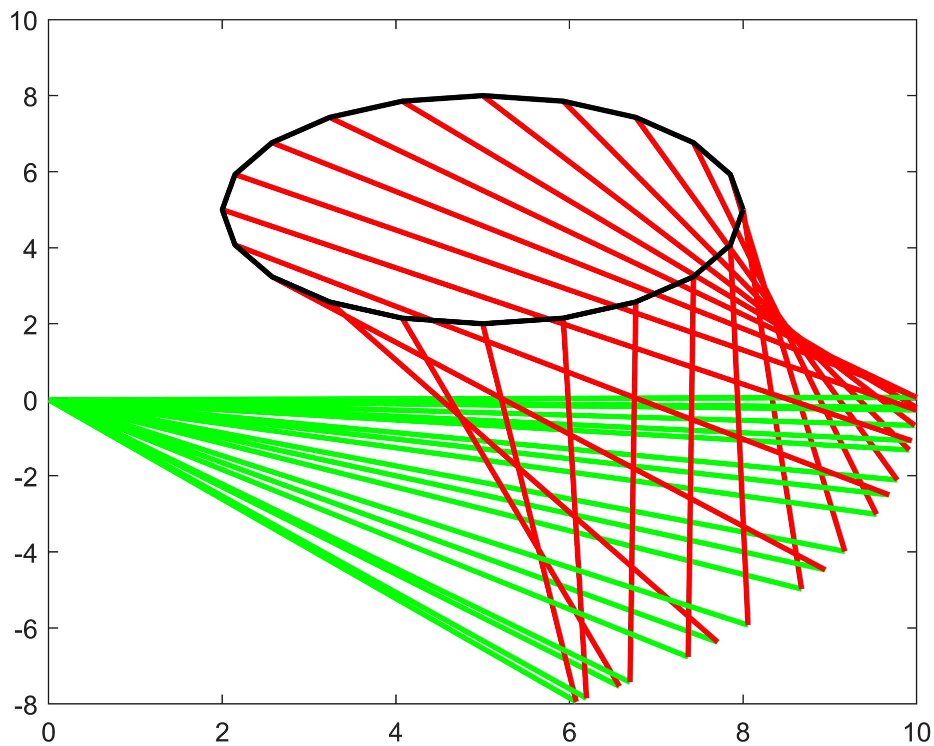
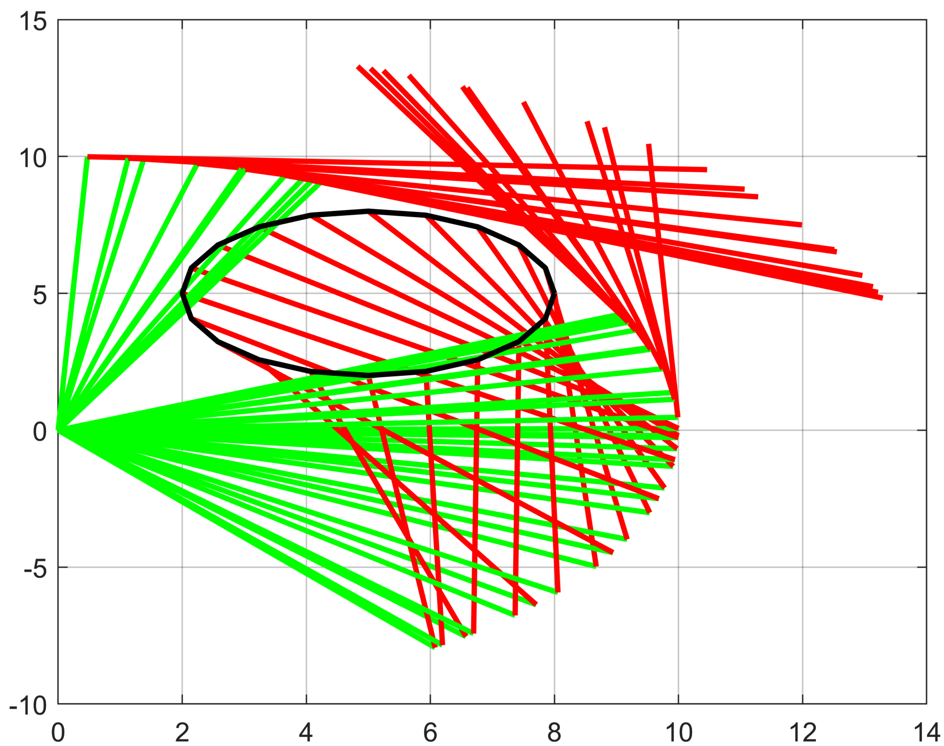

```
plot(x,y, 'k', 'LineWidth',2);grid
```




```
toc
```

```
Elapsed time is 0.439364 seconds.
```

```
% Problem 1(g) - Test
% Using BruteForceSearch
tic

a1 = 10; % given value of link 1
a2 = 10; % given value of link 2

for i = 1:20
    x(i) = points(i,1); % x coordinate of the end position
    y(i) = points(i,2); % y coordinate of the end position
    result = bruteForceSearch(x(i),y(i)); % Obtain theta1 and theta2
    t1 = result(1);
    t2 = result(2);
    [link1x(i,:), link1y(i,:), link2x(i,:), link2y(i,:)] = getPlots(t1, t2);
    plot(link1x(i,:),link1y(i,:),'g','linewidth',2);
    hold on;
    plot(link2x(i,:),link2y(i,:),'r','linewidth',2)

end
plot(x,y,'k','linewidth',2);grid
```




```
toc
```

```
Elapsed time is 165.693334 seconds.
```

As we can see from the above results:

The brute force approach takes a long time to come to an optimal solution whereas closed form inverse kinematics solution and optimization read the optimal solution quickly. So only closed form inverse kinematics solution and optimization can probably used(depending on the number of variables to actually optimize for) but brute force approach takes long time.

```
% Problem 2 - Test
clc;
clear ALL;
close ALL;
x = 20/sqrt(2);
y = 20/sqrt(2);
% Define initial values
theta1_init = 0.4;

% Define function handle
fun2 = @inverseEqn2;

% Compute inversse using optimization
[z, fval] = fmincon(fun2, theta1_init, [], []);
```

```
Local minimum found that satisfies the constraints.
```

```
Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
theta1 = z;
theta2 = acos((x - a1*cos(theta1))/a2) - theta1;

% Print values
disp('The value of theta1 : ')
```

```
The value of theta1 :
```

```
disp(theta1);
```

```
   0.785398160463945
```

```
disp('The value of theta2 : ')
```

```
The value of theta2 :
```

```
disp(theta2);
```

```
     5.867006214188564e-09
```

```
function [theta1, theta2] = getJointAngles(x,y) % problem 1(a)
a1 = 10;
a2 = 10;

theta2 = acos((x^2 + y^2 - a1^2 - a2^2)/(2*a1*a2));
if theta2 >= 0
    theta1 = atan(y/x) - atan(a2*sin(theta2)/(a1 + a2*cos(theta2)));
else
    theta1 = atan(y/x) + atan(a2*sin(theta2)/(a1 + a2*cos(theta2)));
end

end

% Problem 1(b)

function [theta1, theta2] = getOptimizer(x,y) % problem 1(b)

a1 = 10;
a2 = 10;

theta_init = 0.4;

f = @(theta1)getF(theta1, x, y, a1, a2);

[z, ~, exitflag] = fminunc(f, theta_init);
%if exitflag ~= 1
%    disp("Value not optimized")
%else
```

```
theta1 = z;
theta2 = acos((x - a1*cos(theta1))/a2) - theta1;
%end

end

function f = getF(theta1, x, y, a1, a2) % problem 1(b)
f = abs(a1*sin(theta1) + a2*sin(acos((x - a1*cos(theta1))/a2)) - y);
end


function result = bruteForceSearch(x, y) % Problem 1(c)
a1 = 10;
a2 = 10;

theta1 = linspace(0, pi/2, 1000);
theta2 = linspace(-pi/2, pi/2, 1000);

for i = 1:length(theta1)
    for j = 1:length(theta2)
        newX = a1*cos(theta1(i)) + a2*cos(theta1(i)+theta2(j));
        newY = a1*sin(theta1(i)) + a2*sin(theta1(i)+theta2(j));
        error(i,j) = (x - newX)^2 + (y - newY)^2;
    end
end

M = min(error);
minError = min(M);

[row, col] = find(error == minError);

final_theta1 = theta1(row);
final_theta2 = theta2(col);

result = [final_theta1', final_theta2'];
end

% This function calculates the coordinates of the links
function [link1x,link1y,link2x,link2y] = getPlots(theta1, theta2) % Problem 1(e)
a1 = 10;
a2 = 10;
link1x = [0:0.1:a1]*cos(theta1);
link1y = [0:0.1:a1]*sin(theta1);
link2x = [0:0.1:a2]*cos(theta1+theta2)+a1*cos(theta1);
link2y = [0:0.1:a2]*sin(theta1+theta2)+a1*sin(theta1);
end

% Problem 2
function f = inverseEqn2(theta1)
x = 20/sqrt(2);
y = 20/sqrt(2);
a1 = 10;
a2 = 10;
f= abs(a1*sin(theta1) + a2*sin(acos((x- a1*cos(theta1))/a2)) - y);
```

```
end
```