MAE 6245 (Spring 2020)

Robotic Systems

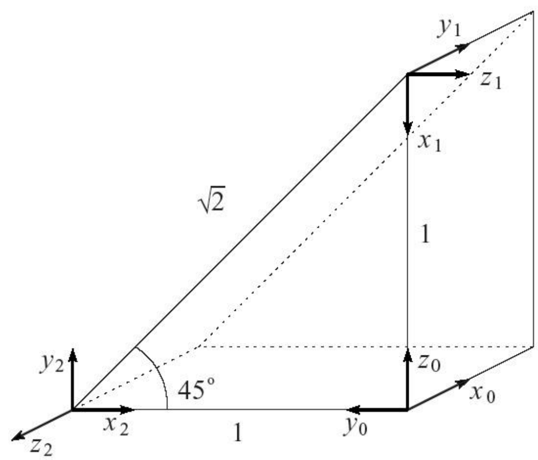
Assignment # 2

Total Points: 31

NOTE: For this assignment, it would be useful to look at Section 2.9 from Craig (3rd edition) (the section called "Transformation of Free Vectors")

1) A frame {B} is located initially coincident with {A}. We rotate {B} about $\hat{Z}_B$ by θ degrees, and then we rotate the resulting frame about $\hat{X}_B$, by φ degrees. Give the rotation matrix that will change the descriptions of vectors from ${}^{B}P$ to ${}^{A}P$. [3 points]

2) In the diagram shown below, find the homogenous transformations between each pair of frames (${}^{0}_{1}T$, ${}^{0}_{2}T$, ${}^{1}_{2}T$). [6 points]



3) Find homogeneous transformation matrix representing a rotation by angle α about the current x-axis, followed by translation of b units along current x-axis, followed by translation of d units along current z-axis, followed by rotation of angle θ about current z-axis. [4 points]

4) (a) Using Z-Y-X (α-β-γ) Euler angle convention, write a MATLAB program to calculate the rotation matrix ${}^{A}_{B}R$ when the user enters the Euler angles. Test for two examples:

   i. α = 10 deg, β = 20 deg, γ = 30 deg
   ii. α = 30 deg, β = 90 deg, γ = -55 deg

   For case (i), demonstrate the six constraints for orthonormal rotation matrices. Also demonstrate that ${}^{B}_{A}R = {}^{A}_{B}R^{-1} = {}^{A}_{B}R^{T}$.

   (b) Write a MATLAB program to calculate the Euler angles when the user enters the rotation matrix ${}^{A}_{B}R$ (the inverse problem). Calculate both possible solutions. Demonstrate this inverse solution for the two cases from part (a). Use a circular check to verify your results – enter Euler angles and compute rotation matrix using the code above. Then use the rotation matrix and recalculate Euler angles. You will get two answers, one should match the original input and the other set can be re-run in the code in part (a) to get the same rotation matrix. [15 points]

5) A velocity vector is given by

$$^{B}v = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$$

Given,

$$^{A}_{B}T = \begin{bmatrix} 0.866 & -0.5 & 0 & 11 \\ 0.5 & 0.866 & 0 & -3 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Compute ^{A}v. [3 points]