MAE 6245 (Spring 2020)

Robotic Systems

Assignment # 3 Solutions

32 Points

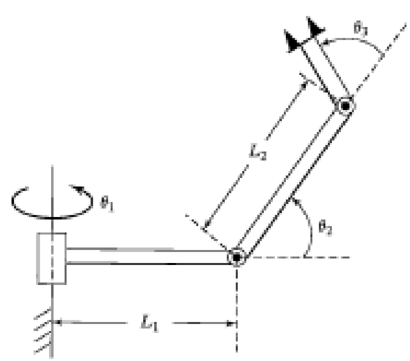
1) State True or False for the following:
   a) Configuration space for any manipulator is unique.
   b) There are 12 basic parameters characterizing a link (also called link parameters).
   c) Parallel robots are the way to go for high reachability.
   d) Each joint can physically have only one joint variable associated with it.
   e) Joint variables are always angles.

[5 points]

Solution

a) False
b) False
c) False
d) False
e) False

2) For the manipulator shown here, do the following:



   a) Derive the link parameters (to find the transformation from wrist to base frame).
   b) In MATLAB, First create a function to determine the transformation matrix (T_i^{i-1}) based on the values of the 4 link parameters. First create a function to determine the

transformation matrix (T_i^{i-1}) based on the values of the 4 link parameters (you can use the same function developed in class for the simple manipulator).

c) Use this function to calculate all matrices required to go from tool frame to base frame. [Try to do this in a recursive fashion by writing a script that calls the function automatically multiple times, for different values of link parameters.] [Assume L1 = 10m, L2 = 10m, L3 = 0m, and $\theta_1 = 45°$, $\theta_2 = 30°$, $\theta_3 = -30°$, or any arbitrary values for angles are also acceptable].

d) Use this further to create a function to find the end effector position for given joint angles [Hint: End effector position is only the translational part of the homogeneous matrix].

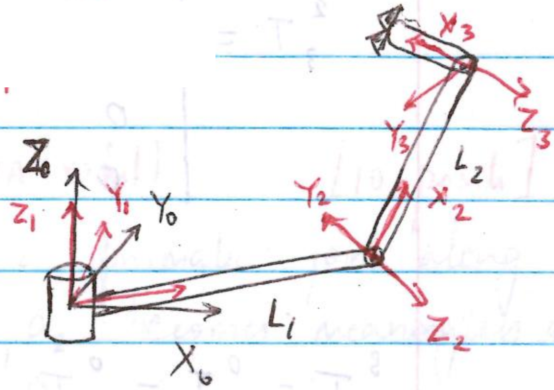
e) Plot the workspace for the robot, for $\theta_1 \in [0°, 90°]$, $\theta_2 \in [0°, 90°]$, $\theta_3 \in [-45°, 45°]$.

f) For a particular configuration, $\theta_1 = 45°$, $\theta_2 = 30°$, $\theta_3 = -30°$ plot the robot links. [Hint: It doesn't have to be a fancy plot. Just find the end points of each link and join them with lines. You can use "plot3" command in MATLAB, and use the option "LineWidth" to control the thickness of the line. Or use the "quiver" command used before, or use anything of choice]

[15 points]

Solution

Attach frames appropriately and find link parameters



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$90°$	L_1	0	θ_2
3	0	L_2	0	θ_3

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$
{}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{B}_{W}T = {}^{0}_{3}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T
$$

$$
= \begin{bmatrix} c\theta_1 c(\theta_2+\theta_3) & -c\theta_1 s(\theta_2+\theta_3) & s\theta_1 & L_1 c\theta_1 + L_2 c\theta_1 c\theta_2 \\ s\theta_1 c(\theta_2+\theta_3) & s\theta_1 s(\theta_2+\theta_3) & -c\theta_1 & L_1 s\theta_1 + L_2 s\theta_1 c\theta_2 \\ s(\theta_2+\theta_3) & c(\theta_2+\theta_3) & 0 & L_2 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The code could be as follows:

```
function [T] = characteristicTransformation(alpha,a,d,theta)
    T = [cos(theta), -sin(theta), 0, a;
         sin(theta)*cos(alpha), cos(theta)*cos(alpha), -sin(alpha), -
         d*sin(alpha);
         sin(theta)*sin(alpha), cos(theta)*sin(alpha), cos(alpha),
         d*cos(alpha);
         0, 0, 0, 1];
```

c) The results are:

T =

0.7071	-0.7071	0	0
0.7071	0.7071	0	0
0	0	1.0000	0
0	0	0	1.0000

T =

$$\begin{bmatrix} 0.8660 & -0.5000 & 0 & 10.0000 \\ 0.0000 & 0.0000 & -1.0000 & 0 \\ 0.5000 & 0.8660 & 0.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

T =

$$\begin{bmatrix} 0.8660 & 0.5000 & 0 & 10.0000 \\ -0.5000 & 0.8660 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

d) The final homogeneous matrix is:

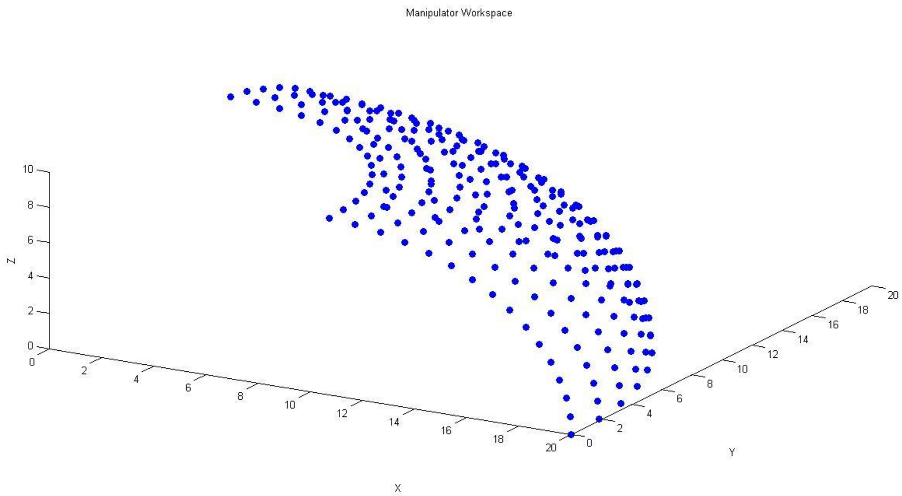
T_WB =

$$\begin{bmatrix} 0.7071 & -0.0000 & 0.7071 & 13.1948 \\ 0.7071 & 0 & -0.7071 & 13.1948 \\ 0 & 1.0000 & 0.0000 & 5.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

End Effector Position:

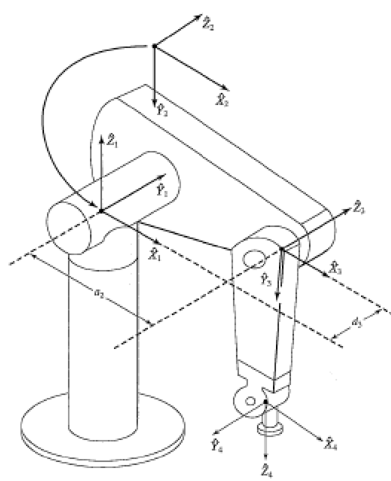
$x = 13.19$, $y = 13.19$, $z = 5$

e) The plot looks like the following:



3) Imagine an arm like the PUMA 560, except that joint 3 is replaced with a prismatic joint (Note that PUMA 560 is discussed in one of the examples in the textbook). Assume the prismatic joint slides along the direction of X_1 in the figure. However, there is still an offset equivalent to d_3 to be accounted for. Make any additional assumptions needed. Derive the kinematic equations.

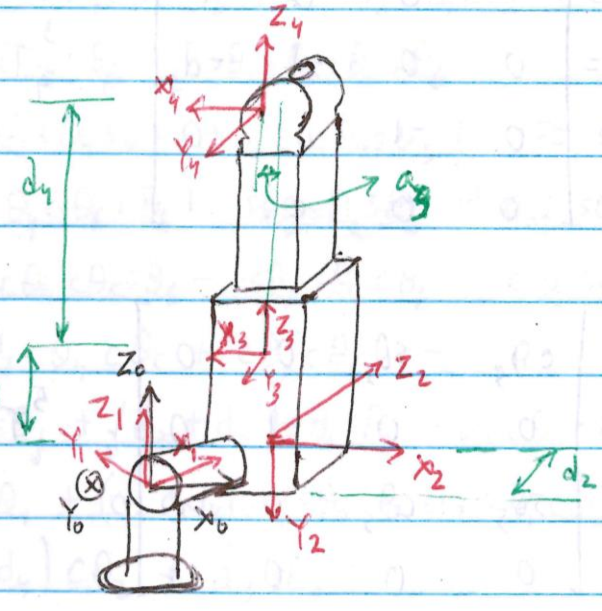
[10 points]



Solution

If we replace joint 3 by a prismatic joint along direction of $\hat{x}_1$, then the 'a_2' becomes meaningless and we can assume that $a_2 = 0$.
So when $d_3 = 0$, the origin of frame 2 and 3 will coincide.
Otherwise they are separated by 'd_3', the joint offset

Note that there is a little bit of abuse of notation in terms of a's and d's and numbering



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	d_2	θ_2
3	$90°$	0	d_3	$180°$
4	0	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$\therefore \; {}^{0}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where,

$$r_{11} = -c\theta_1 c\theta_2 c\theta_4 c\theta_5 c\theta_6 + c\theta_1 c\theta_2 s\theta_4 s\theta_6 + s\theta_1 s\theta_4 c\theta_5 c\theta_6$$
$$+ s\theta_1 c\theta_4 s\theta_6 + c\theta_1 s\theta_2 s\theta_5 s\theta_6$$

$$r_{12} = c\theta_1 c\theta_2 c\theta_4 c\theta_5 s\theta_6 + c\theta_1 c\theta_2 s\theta_4 c\theta_6 - s\theta_1 s\theta_4 c\theta_5 s\theta_6$$
$$+ s\theta_1 c\theta_4 c\theta_6 - s\theta_1 s\theta_2 s\theta_5 s\theta_6$$

$$r_{13} = c\theta_1 c\theta_2 c\theta_4 s\theta_5 - s\theta_1 s\theta_4 s\theta_5 + c\theta_1 s\theta_2 c\theta_5$$

$$r_{21} = -s\theta_1 c\theta_2 c\theta_4 c\theta_5 c\theta_6 + s\theta_1 c\theta_2 s\theta_4 s\theta_6 - c\theta_1 s\theta_4 c\theta_5 c\theta_6$$
$$- c\theta_1 c\theta_4 s\theta_6 + s\theta_1 s\theta_2 s\theta_5 c\theta_6$$

$$r_{22} = s\theta_1 c\theta_2 c\theta_4 c\theta_5 s\theta_6 + s\theta_1 c\theta_2 s\theta_4 c\theta_6 + c\theta_1 s\theta_4 c\theta_5 s\theta_6$$
$$- c\theta_1 c\theta_4 c\theta_6 - s\theta_1 s\theta_2 s\theta_5 s\theta_6$$

$$r_{23} = s\theta_1 c\theta_2 c\theta_4 s\theta_5 + c\theta_1 s\theta_4 s\theta_5 + s\theta_1 s\theta_2 c\theta_5$$

$$r_{31} = s\theta_2 c\theta_4 c\theta_5 c\theta_6 - s\theta_2 s\theta_4 s\theta_6 + c\theta_2 s\theta_5 c\theta_6$$

$$r_{32} = -s\theta_2 c\theta_4 c\theta_5 s\theta_6 - s\theta_2 s\theta_4 c\theta_6 - c\theta_2 s\theta_5 s\theta_6$$

$$r_{33} = -s\theta_2 c\theta_4 c\theta_5 + c\theta_2 c\theta_5$$

$$P_x = -d_2 s\theta_1 + (d_3 + d_4) c\theta_1 s\theta_2 - a_3 c\theta_1 c\theta_2$$

$$P_y = d_2 c\theta_1 + (d_3 + d_4) s\theta_1 s\theta_2 - a_3 s\theta_1 c\theta_2$$

$$P_z = (d_3 + d_4) c\theta_2 + a_3 s\theta_2$$