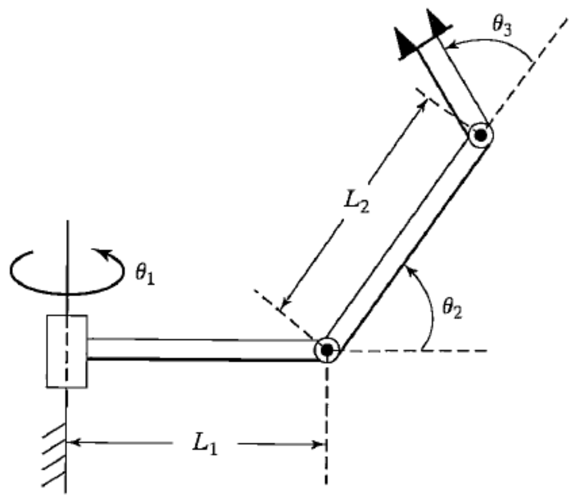
MAE 6245 (Spring 2020)

Robotic Systems

Assignment # 6 Solutions

[25 points]

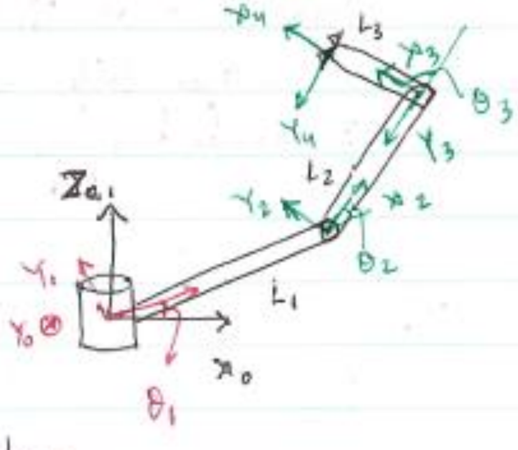
1) For the 3R nonplanar arm studied in one of the previous assignments, and shown below:



Find the velocity kinematic equations using any method of choice (not MATLAB) (recommended technique is the use of propagation equations discussed in class). Also find the Jacobian in the end effector frame (only translational velocity Jacobian is sufficient). **[15 points]**

Solution

We use the usual procedure to assign frames and find link parameters, but we also find $\dot{\theta}_i$ and $\dot{d}_i$ associated with each degree of freedom.



	i	α_{i-1}	a_{i-1}	d_i	θ_i	$\dot{\theta}_i$	$\dot{d}_i$
Link 0	1	0	0	0	θ_1	$\dot{\theta}_1$	0
Link 1	2	90°	L_1	0	θ_2	$\dot{\theta}_2$	0
Link 2	3	0	L_2	0	θ_3	$\dot{\theta}_3$	0
Link 3	4	0	L_3	0	0	0	0

We can use the general transformation to find the transformation matrices between frames (not shown here)

<u>Velocity Propagation Equation</u>

$$^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\, {}^{i}\omega_i + \dot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1}$$

$$^{i+1}v_{i+1} = {}^{i+1}_{i}R\left({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1}\right) + \dot{d}_{i+1}\, {}^{i+1}\hat{Z}_{i+1}$$

$$\therefore \; {}^1\omega_1 = {}^1_0R\,{}^0\omega_0 + \dot{\theta}\,{}^1\hat{Z}_1 = \dot{\theta}_1\,\hat{Z}_1$$

$${}^1v_1 = {}^1_0R\left({}^0v_0 + {}^0\omega_0 \times {}^0P_1\right) = [0,\ 0,\ 0]^T$$

$${}^2\omega_2 = {}^2_1R\,{}^1\omega_1 + \dot{\theta}_2\,{}^2\hat{Z}_2$$

$$= \begin{bmatrix} c\theta_2 & 0 & s\theta_2 \\ -s\theta_2 & 0 & c\theta_2 \\ 0 & -1 & c \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$$

$$= \left[s\theta_2\,\dot{\theta}_1\,;\ c\theta_2\,\dot{\theta}_1,\ \dot{\theta}_2\right]^T$$

$${}^2v_2 = {}^2_1R\left({}^1v_1 + {}^1\omega_1 \times {}^1P_2\right)$$

$$= \begin{bmatrix} c\theta_2 & 0 & s\theta_2 \\ -s\theta_2 & 0 & c\theta_2 \\ 0 & -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ 0 \\ -L_1\dot{\theta}_1 \end{bmatrix}$$

$${}^3\omega_3 = {}^3_2R\,{}^2\omega_2 + \dot{\theta}_3\,{}^3\hat{Z}_3$$

$$= \begin{bmatrix} c\theta_3 & s\theta_3 & 0 \\ -s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s\theta_2\,.\,\dot{\theta}_1 \\ c\theta_2\,\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} s(\theta_2+\theta_3)\dot{\theta}_1 \\ c(\theta_2+\theta_3)\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

$${}^{3}v_3 = {}^{3}_{2}R\left({}^{2}v_2 + {}^{2}\omega_2 \times {}^{2}P_3\right)$$

$$= {}^{3}_{2}R\left(\begin{bmatrix} 0 \\ 0 \\ -L_1\dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} s\theta_2\,\dot{\theta}_1 \\ c\theta_2\,\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \times \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} L_2\, s\theta_3\, \dot{\theta}_2 \\ L_2\, c\theta_3\, \dot{\theta}_2 \\ -L_1\dot{\theta}_1 - L_2 c\theta_2\, \dot{\theta}_1 \end{bmatrix}$$

$${}^{4}\omega_4 = {}^{4}_{3}R\, {}^{3}\omega_3 + 0 = {}^{3}\omega_3$$

$${}^{4}v_4 = {}^{4}_{3}R\left({}^{3}v_3 + {}^{3}\omega_3 \times {}^{3}P_4\right)$$

$$= {}^{4}_{3}R\left(\begin{bmatrix} L_2 s\theta_3 \dot{\theta}_2 \\ L_2 c\theta_3 \dot{\theta}_2 \\ -L_1\dot{\theta}_1 - L_2 c\theta_2 \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} s(\theta_2+\theta_3)\,\dot{\theta}_1 \\ c(\theta_2+\theta_3)\,\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix} \times \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} L_2\, s\theta_3\, \dot{\theta}_2 \\ L_2 c\theta_3\, \dot{\theta}_2 + L_3(\dot{\theta}_2 + \dot{\theta}_3) \\ -L_1\dot{\theta}_1 - L_2\, c\theta_2\, \dot{\theta}_1 - L_3\, c(\theta_2+\theta_3)\,\dot{\theta}_1 \end{bmatrix}$$

Now; we know that, ${}^{4}v_4 = {}^{4}J_v\left[\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3\right]^T$

$\therefore$ By inspection,

$${}^{4}J_v = \begin{bmatrix} 0 & L_2 s\theta_3 & 0 \\ 0 & L_2 c\theta_3 + L_3 & L_3 \\ -L_1 - L_2 c\theta_2 - L_3 c(\theta_2+\theta_3) & 0 & 0 \end{bmatrix}$$

${}^{4}J_{\omega}$ can be found using the procedure outlined in class. ~~where~~

But it is not required for this problem.

When we try to find Jacobian from the velocity kinematic equations, we typically refer to J_v

2) For the problem above, write functions in MATLAB to find the velocities of end effector in reference frame using recursive propagation of equations (use the general equations in the function, instead of hardcoding the closed form solution obtained above). Now, for $L_1 = 10$, $L_2 = 10$, $L_3 = 0$, $\theta_1 = 30$ degrees, $\theta_2 = 45$ degrees, $\theta_3 = 0$ degrees, find the end effector velocities using this code. Then compare it with direct computation of closed form solution above. Is the result the same? **[10 points]**

Solution

This is a simple exercise for coding up the velocity propagation equation (note that they require the forward kinematics solution as well) as a function (recursive).

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Then we can repeatedly call this function to compute the final velocities.