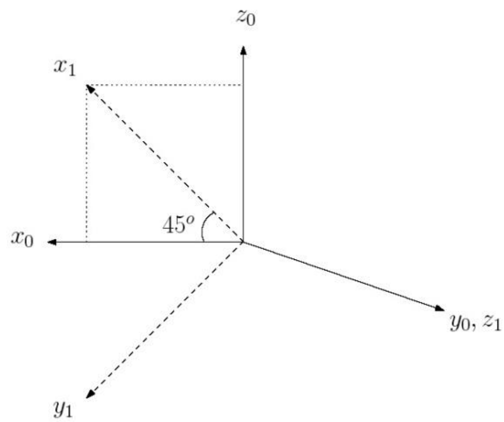
MAE 6245 (Spring 2020)

Robotic Systems

Assignment # 1 Solutions

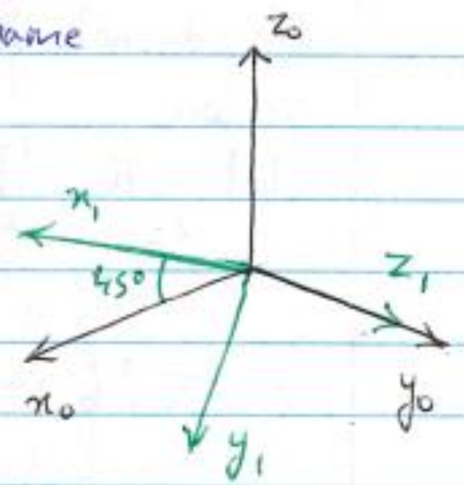
1) For the given frames (0 and 1), find the rotation matrix specifying the orientation of frame 1 relative to frame 0. [3 points]



Solution

Express new axes in old frame

$$
{}^0_1R = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}
$$



2) Show that the dot product of two free vectors does not depend on the choice of frames in which their coordinates are defined. [Hint: use the definition of the dot product (x^Tx)] [5 points]

Solution

We do not need to consider translation for free vectors.

Let ${}^1\bar{a}$, ${}^1\bar{b}$ be defined in frame $\{1\}$.

$$\therefore \; {}^2\bar{a} = {}^2_1R\,{}^1\bar{a} \quad ; \quad {}^2\bar{b} = {}^2_1R\,{}^1\bar{b}$$

$$\begin{aligned} \therefore \; {}^2\bar{a} \cdot {}^2\bar{b} &= \left({}^2_1R\,{}^1\bar{a}\right)^T \cdot \left({}^2_1R\,{}^1\bar{b}\right) \\ &= {}^1\bar{a}^T \, {}^2_1R^T \, {}^2_1R \, {}^1\bar{b} \\ &= {}^1\bar{a}^T \, {}^1\bar{b} \qquad \left(\because \; {}^2_1R^T = {}^2_1R^{-1}\right) \\ &= {}^1\bar{a} \cdot {}^1\bar{b} \end{aligned}$$

3) Imagine two unit vectors, v_1 and v_2, embedded in a rigid body. Note that, no matter how the body is rotated, the geometric angle between these two vectors is preserved (i.e., rigid-body rotation is an "angle-preserving" operation). Use this fact to give a concise (four- or five-line) proof that the inverse of a rotation matrix must equal its transpose and that a rotation matrix is orthonormal. [5 points]

Solution

Consider dot product, $\bar{v}_1^T \bar{v}_2 = \cos\theta$

This should remain the same even after rotation

$$\therefore \; (R\bar{v}_1)^T (R\bar{v}_2) = \bar{v}_1^T \bar{v}_2$$

$$\Rightarrow \; \bar{v}_1^T R^T R \, \bar{v}_2 = \bar{v}_1^T \bar{v}_2$$

$$\therefore \; R^T R = I$$

$$\Rightarrow \; R^{-1} = R^T$$

This proves orthonormality as well.

4) 2 points awarded to anybody who has either indicated their team member or indicated that they have been unable to find one, or indicated that they are going to go on their own.