MAE 6245 (Spring 2020)

Robotic Systems

Assignment # 2 Solutions

1) A frame {B} is located initially coincident with {A}. We rotate {B} about $\hat{Z}_B$ by θ degrees, and then we rotate the resulting frame about $\hat{X}_B$, by φ degrees. Give the rotation matrix that will change the descriptions of vectors from ${}^{B}P$ to ${}^{A}P$. [3 points]

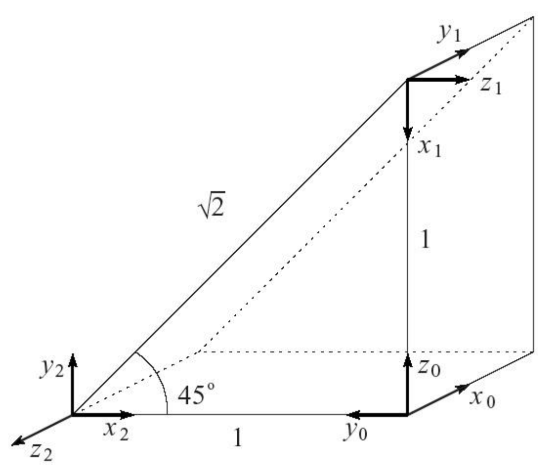
Solution

$$R = R(\hat{z},\theta)\, R(\hat{x},\phi)$$

$$= \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

$$= \begin{bmatrix} c\theta & -s\theta c\phi & s\theta s\phi \\ s\theta & c\theta c\phi & -c\theta s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

2) In the diagram shown below, find the homogenous transformations between each pair of frames (${}^{0}_{1}T$, ${}^{0}_{2}T$, ${}^{1}_{2}T$). [6 points]



Solution

Use the same procedure of expressing axes in old frame to get rotation matrices. Also look at translation vector

$$ {}^{A}_{B}T = \begin{bmatrix} & {}^{A}_{B}R & & {}^{A}P_{Borg} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \therefore {}^{0}_{1}T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{2}T = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3) Find homogeneous transformation matrix representing a rotation by angle α about the current x-axis, followed by translation of b units along current x-axis, followed by translation of d units along current z-axis, followed by rotation of angle θ about current z-axis. [4 points]

Solution

$$T = R_{x,\alpha}\ Trans_{x,b}\ Trans_{z,d}\ R_{z,\theta}$$

$$= \begin{bmatrix} c\theta & -s\theta & 0 & b \\ c\alpha s\theta & c\alpha c\theta & -s\alpha & -ds\alpha \\ s\alpha s\theta & s\alpha c\theta & c\alpha & dc\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4) (a) Using Z-Y-X (α-β-γ) Euler angle convention, write a MATLAB program to calculate the rotation matrix ${}^{A}_{B}R$ when the user enters the Euler angles. Test for two examples:

   i) α = 10 deg, β = 20 deg, γ = 30 deg
   ii) α = 30 deg, β = 90 deg, γ = -55 deg

For case (i), demonstrate the six constraints for orthonormal rotation matrices. Also demonstrate that ${}^{B}_{A}R = {}^{A}_{B}R^{-1} = {}^{A}_{B}R^{T}$.

(b) Write a MATLAB program to calculate the Euler angles when the user enters the rotation matrix ${}^{A}_{B}R$ (the inverse problem). Calculate both possible solutions. Demonstrate this inverse solution for the two cases from part (a). Use a circular check to verify your results – enter Euler angles and compute rotation matrix using the code above. Then use the rotation matrix and recalculate Euler angles. You will get two answers, one should match the original input and the other set can be re-run in the code in part (a) to get the same rotation matrix. [15 points]

Solution

Part (a) is straightforward

Use $\beta = \tan^{-1}\left(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right)$

$\alpha = \tan^{-1}\left(r_{21}/c\beta,\ r_{11}/c\beta\right)$

$\gamma = \tan^{-1}\left(r_{32}/c\beta,\ r_{33}/c\beta\right)$

2nd soln $\rightarrow$ $\beta = \tan^{-1}\left(-r_{31}, -\sqrt{r_{11}^2 + r_{21}^2}\right)$

5) A velocity vector is given by

$$^{B}v = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$$

Given,

$$^{A}_{B}T = \begin{bmatrix} 0.866 & -0.5 & 0 & 11 \\ 0.5 & 0.866 & 0 & -3 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Compute ^{A}v. [3 points]

Solution

Free vectors are not affected by ~~rotation~~ translation. So we consider only the rotation part

$$^{A}v = {}^{A}_{B}R \, {}^{B}v = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$$

$$= \begin{bmatrix} -1.34 \\ 22.32 \\ 30 \end{bmatrix}$$