MAE6292: Homework 5

Due date: April 24, 2020

Problem 1 In class we found that the Kalman gain and the a posteriori covariance are given by

$$K = P^- H^T (HP^- H^T + R)^{-1},$$
$$P^+ = (I - KH)P^-.$$

(a) Using the matrix inversion lemma, show that they can be rewritten as

$$K = P^+ H^T R^{-1},$$
$$P^+ = ((P^-)^{-1} + H^T R^{-1} H)^{-1}.$$

(b) The above expressions for P^+ may cause numerical issues: the first form is not symmetric, and the second form involves multiple matrix inversions. To avoid these, in practice, it is often calculated by as follows:

$$P^+ = (I - KH)P^-(I - KH)^T + KRK^T,$$

which is symmetric by definition, and it does not require any matrix inversion. The above is referred to as the *Joseph* form. Show that the Joseph form is equivalent to any of the above two expression for P^+.

Problem 2 The dual of the Kalman filter is referred to as the *information filter*, or IF. IF is equivalent to KF as both are developed for linear Gaussian systems. But, instead of parameterizing a Gaussian distribution via the mean and the variance, IF represents a Gaussian distribution via the canonical parameterization. More explicitly, the canonical parameterization of a Gaussian distribution with the mean $\bar{X}$ and the covariance P are given by

$$\xi = P^{-1}\bar{X}, \quad \Omega = P^{-1},$$

which are referred to as the *information vector* and the *information matrix*, respectively. For a given (ξ, Ω), the mean and the covariance are simply given by

$$\bar{X} = \Omega^{-1}\xi, \quad P = \Omega^{-1}.$$

(a) From the prediction step of KF, show that the prediction step of IF: $(\xi_k, \Omega_k) \to (\xi_{k+1}, \Omega_{k+1})$ can be written as

$$\Omega_{k+1} = (A_k \Omega_k^{-1} A_k^T + Q_k)^{-1},$$
$$\xi_{k+1} = \Omega_{k+1}(A_k \Omega_k^{-1} \xi_k + B_k u_k).$$

(b) From the correction step of KF, show that the correction step of IF: $(\xi^-, \Omega^-) \to (\xi^+, \Omega^+)$ can be written as

$$\Omega^+ = H^T R^{-1} H + \Omega^-,$$
$$\xi^+ = H^T R^{-1} z + \xi^-.$$

(c) Compare the prediction step of KF with IF, and compared the computational load required.

(d) Compare the correction step of KF with IF, and compared the computational load required.

(e) IF can be extended to nonlinear systems in the same way that KF is extended to EKF. Even though it is less popular, IF has several advantages over KF. Google IF and itemize several advantages of IF compared with KF.

Problem 3 Consider the 2D dynamics of a projectile. The state vector is given by $\mathbf{x} = [x, \dot{x}, y, \dot{y}] \in \mathbb{R}^4$. The discrete equations of motion are given by

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + Bu_k + w_k,$$

where

$$A = \begin{bmatrix} 1 & h & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ h^2/2 \\ h \end{bmatrix}, \quad u_k = -0.1.$$

There are two radar sites located at $(4, 0)$ and $(8, 0)$, and each can measure the range to the projectile. The corresponding measurement equation is given by

$$z = \begin{bmatrix} \sqrt{(x-4)^2 + y^2} \\ \sqrt{(x-8)^2 + y^2} \end{bmatrix} + v.$$

The mean of the initial estimate $\bar{\mathbf{x}}_0$, and the covariance matrices P_0, Q_k, R_k are specified at the Matlab file `HW5_prob3.m`.

(a) Develop a Matlab code for EKF.

(b) Plot the true trajectory and the estimated trajectory on the x–y plane.

(c) Plot the estimation error and the 3σ bounds with respect to t for x and y.

(d) Plot the 3σ Gaussian ellipsoids, the true position, and the estimated position on the x–y plane for `k=1:10:N`.