<Est. of Dyn. Sys>

- prediction

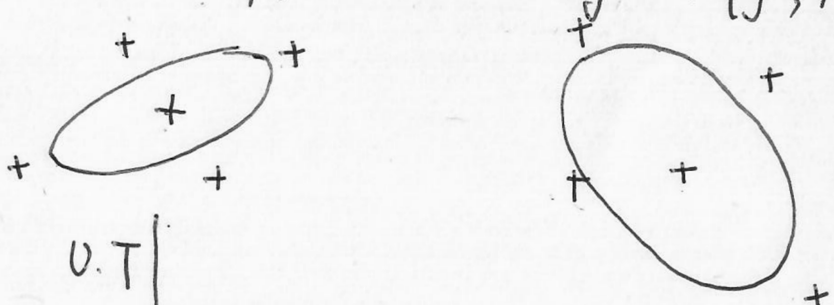
$x_k \sim N(\bar{x}_k, P_k) \Rightarrow x_{k+1} \sim N(\bar{x}_{k+1}, P_{k+1})$

- correction

$x^- \sim N(\bar{x}^-, P^-) \Rightarrow x|z = x^+ \sim N(x^+, P^+)$

- K.F : linear Gaussian.
- E.K.F : nonliear.
  using linearization. x_{k+1}, $x|z$ Gausslan
- Unscented Transform : use samples.

$x \sim N(\bar{x}, P) \xRightarrow{f(x)} y \overset{?}{\sim} N(\bar{y}, P_y)$



U.T ↓ ↑ UT^{-1} recover

$\{x^0, x^1, \cdots\} \xrightarrow{f(x)} \{y^0, y^1, \cdots\}$

EKF : linearization

$\frac{\partial f}{\partial x}$

first order approx.

UKF : sample points : σ-points (?

no jacobian required.

second order.

- Suppose $x \sim N(\bar{x}, P)$

We should choose σ-points, and weights s.t (x^i, w^i)

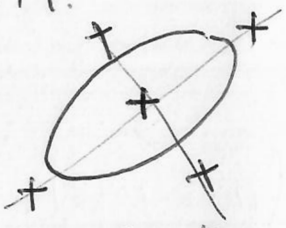
$\sum_i w^i = 1$

$\bar{x} = \sum_i w^i x^i$

$P = \sum_i w^i (x^i - \bar{x})(x^i - \bar{x})^T$

There is no unique sol.

• a common choice. $2n+1$.



$n=2$. 5 σ-points

$$x^0 = \bar{x}$$

$$x^i = \bar{x} + \left[\sqrt{(n+\lambda)P}\right]_i \qquad i = 1, 2, \dots n$$

$$x^i = \bar{x} - \left[\sqrt{(n+\lambda)P}\right]_{i-n} \qquad i = n+1, \dots, 2n$$

matrix sqrt. / dim / param / cov / $i-n$ th column

* sqrtm( )

• Let (λ_i, v_i) be the e-val/vec of P

$$P v_i = \lambda_i v_i$$

Define

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \end{bmatrix} \quad V = \begin{bmatrix} v_1 & v_2 & \cdots \end{bmatrix}$$

$$\Rightarrow PV = V\Lambda \Rightarrow P = V\Lambda V^{-1} = V\Lambda V^T$$

$$= (V\Lambda^{1/2} \underbrace{V^T)(V}_{I}\Lambda^{1/2} V^T) = \sqrt{P}$$

$$= V\Lambda V^T$$

$$\Lambda^{1/2} = \begin{bmatrix} \sqrt{\lambda_1} & & \\ & \sqrt{\lambda_2} & \\ & & \ddots \end{bmatrix}$$

* σ-points may not be along the principal axes.

• Weight. (W_m : for mean ; W_v : cov)

$$W_m^0 = \frac{\lambda}{n+\lambda}$$

$$W_v^0 = W_m^0 + (1-\alpha^2+\beta^2)$$

$$W_m^i = W_v^i = \frac{1}{2(n+\lambda)} \qquad i = 1 \dots 2n$$

• free parm.

$$\lambda = \alpha^2(n+\kappa) - n$$

$\kappa \geq 0$, $0 \leq \alpha < 1$ } how far σ-points are from $\bar{x}$.

$\beta = 2$: opt. for Gaussian.

These yield. U.T

$$x \sim N(\bar{x}, P) \Rightarrow \{(x^i, w^i)\}_{i=0}^{2n}$$

$2n+1$.

$y=f(x)$

- Inverse U.T. (recover Gau).

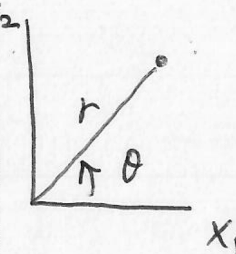
$$\{(y^i, w^i)\} \longrightarrow N(\bar{y}, P_y)$$

$$\bar{y} = \sum_{i=0}^{2n} w_m^i y^i$$

$$P_y = \sum_{i=0}^{2n} w_v^i (y^i - \bar{y})(y^i - \bar{y})^T$$

ex) $x \sim N(\bar{x}, P)$

$$y = \begin{bmatrix} r \\ \theta \end{bmatrix} = \begin{bmatrix} \sqrt{x_1^2 + x_2^2} \\ \tan^{-1} \frac{x_2}{x_1} \end{bmatrix}$$



$y \sim ?$

<UKF> $x_{k+1} = f(x_k, u_k) + w_k$

- prediction. $(\bar{x}_k, P_k) \longrightarrow (\bar{x}_{k+1}, P_{k+1})$

① choose σ points. x_k^i

② propagate x_k^i with $w_k = 0$

$$x_{k+1}^i = f(x_k^i, u_k)$$

③ find $\bar{x}_{k+1}, P_{k+1}$ from inv U.T.

④ $P_{k+1} = P_{k+1} + Q_k$

* Recall EKF.

$$\bar{x}^+ = \bar{x}^- + K(z - \bar{z})$$

$$P^+ = (I - KH)P^-$$

$$K = P^- H^T (H P^- H^T + R)^{-1}$$

$$= P_{xz} \times P_z^{-1}$$

$$P^+ = P^- - K P_{xz}^T$$

$$= P^- - K P_z P_z^{-1} P_{xz}^T \quad = K^T$$

$$= P^- - K P_z K^T$$

- correction $(\bar{x}^-, P^-) \rightarrow (\bar{x}^+, P^+)$

① choose σ-points x^i

② transform $z^i = h(x^i)$

③ $\bar{z} = \sum w_m^i z^i$

$$P_z = \sum w_v^i (z^i - \bar{z})(z^i - \bar{z})^T$$

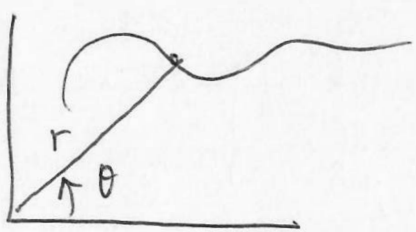
$$P_{xz} = \sum w_v^i (x^i - \bar{x})(z^i - \bar{z})^T$$

④ correction $K = P_{xz} P_z^{-1}$

$$\bar{x}^+ = \bar{x}^- + K(z - \bar{z})$$

$$P^+ = P^- - K P_z K^T$$

ex).



< Bayesian Est >.

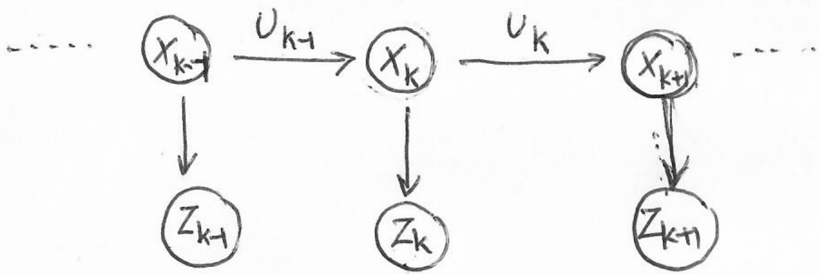
< Bayesian Estimation >

- most general framework.
- ① state transition prob.

$$p(x_{k+1} \mid x_k, u_k)$$

② sensor

$$p(z \mid x) \qquad z = h(x) + v$$



- Goal: $bel(x_k) = p(x_k \mid Z_k, U_{k-1})$

$Z_k = \{z_1, z_2, \cdots z_k\}$

$U_{k-1} = \{u_1, u_2 \cdots u_{k-1}\}$

ex) $x_{k+1} = f(x_k, u_k) + w_k \qquad w_k \sim N(0, Q_k)$

$$x_{k+1} \mid x_k, u_k \sim N(f(x_k, u_k), Q_k)$$

$$z = h(x) + v \qquad v \sim N(0, R_k)$$

$$z \mid x \sim N(h(x), R_k)$$

• Propagation

$$p(x_k \mid Z_k, U_{k-1}) \xrightarrow{u_k} p(x_{k+1} \mid \boxed{Z_k, U_k})$$

* Recall

$$p(x \mid \boxed{}) = \int_{-\infty}^{\infty} p(x, \overset{x_k}{y} \mid \boxed{})\, dy$$

$$= \int p(x \mid y, \boxed{})\, p(y \mid \boxed{})\, dy$$

$$\underbrace{p(x_{k+1} \mid \boxed{Z_k, U_k})}_{\text{propagated density}} = \int \underbrace{p(x_{k+1} \mid x_k, \boxed{u_k})}_{\text{state transition prob}}\, \underbrace{p(x_k \mid \boxed{Z_k, U_{k-1}})}_{\text{current belief.}}\, dx_k$$

• Correction.

$$p(x_{k+1} | \boxed{Z_k. U_k}) \xrightarrow{z_{k+1}} p(x_{k+1} | \underbrace{z_{k+1}, \boxed{Z_k. U_k}}_{Z_{k+1}})$$

* Bayes rule.

$$p(X|Y,\boxed{Z}) = \frac{p(Y|X,\boxed{Z})\, p(X|\boxed{Z})}{p(Y|\boxed{Z})}$$

$$p(x_{k+1}|z_{k+1},\boxed{\ }) = \frac{p(z_{k+1}|x_{k+1}, \cancel{\boxed{\ }})\, p(x_{k+1}|\boxed{\ })}{p(z_{k+1}|\boxed{\ })}$$

$$\propto \underbrace{p(z_{k+1}|x_{k+1})}_{\text{sensor model}}\, p(x_{k+1}|Z_k. U_k)$$

posterior by z_{k+1}

prior before z_{k+1}

⇒

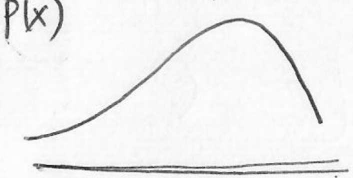
• Bayesian Est ⇒ Linear Gaussian sys

⇒ K.F

⟨Particle Filter⟩

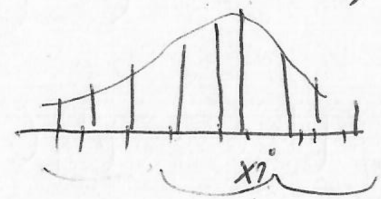
- non-parametric implementation of Bayesian Est.

p(x)



⇒ (w^i, x^i)

$\Sigma\, w^i \delta(x - x^i)$



• Prediction

$$p(x_k | Z_k. U_{k-1}) \approx (w^i_k, x^i_k)$$

Take a sample from S.T.P

$$x^i_{k+1} \Leftarrow p(x_{k+1} | x_k = x^i_k, u_k)$$

• Correction

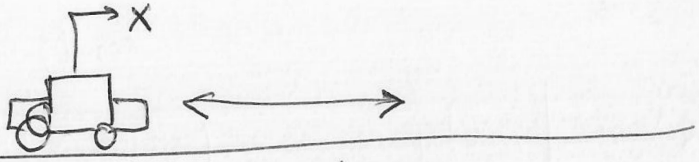
$$w^i_{k+1} = w^i_k \times p(z_{k+1} | x_{k+1} = x^i_{k+1})$$

$$\text{sumW} = \Sigma\, w^i_{k+1}$$

$$w^i_{k+1} = w^i_{k+1} / \text{sumW} \Rightarrow \Sigma_i w^i = 1$$

$$(w^i_k\ x^i_{k+1}) \Rightarrow (w^i_{k+1}, x^i_{k+1})$$

ex)



$$x_{k+1} = x_k + h \overset{\text{vel}}{u_k} + w_k \qquad w_k \sim N(0, Q_k)$$

time step $h = t_{k+1} - t_k$

$$z = x + v \qquad v \sim N(0, R)$$

$$x_0 \sim N(\bar{x}_0, P_0) \xrightarrow{\text{sampling}} (x_0^i, w_0^i)$$

- Prediction. (x_k^i, w_k^i)

$$p(x_{k+1} | x_k, u_k) = \underline{N(x_k + hu_k, Q_k)}$$

S.T.P

$$x_{k+1}^i \Leftarrow N(x_k^i + hu_k, Q_k)$$

- Correction.

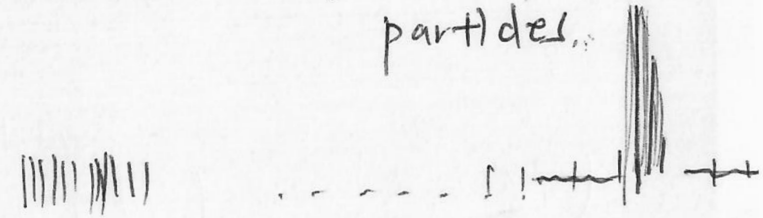
$$p(z|x) = N(x, R).$$

$$w_{k+1}^i = w_k^i \times \underbrace{p(z_{k+1} | x_{k+1} = x_{k+1}^i)}_{N(x_{k+1}^i, R)} = w_k^i \times \frac{1}{\sqrt{R}\sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(z_{k+1} - x_{k+1}^i)^2}{R}\right)$$

* after a few iterations, many particles may have negligible weight.

$$\bar{x} = \Sigma w^i x^i$$

⇒ reduce # of effective particles.



* SIR (Sampling Importance Resampl
filter.

: resample after each step with a prob $\propto w^i$