# **Flight Price Predictions**

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### **Abstract**

The primary goal of our project is to build a machine learning model that can estimate changes in future flight prices based on historical data by using regression techniques. We will investigate how factors such as time of departure, number of stops, and the choice of airline influence flight prices. The secondary objective is to analyze if certain trends can be linked to broader environmental, economic and/or policy factors. The dataset includes columns for departure and destination locations, total stops, travel duration, and price information. The model will be trained using machine learning techniques, with a focus on determining which features contribute most to price variations. The aviation industry is a critical component of the global transportation network, impacting not only the economy but also the environment due to its significant carbon footprint. By developing accurate flight price prediction models, we can contribute to better planning and optimization of air travel routes, which is essential for both transportation engineering and environmental sustainability. If airlines and passengers can anticipate future price trends, it enables more efficient scheduling, potentially increasing the efficiency of flight operation and possibly minimizing unnecessary emissions.

# **Proposal**

Our team plans to use a Kaggle flight prediction dataset to develop a machine learning model in Julia that predicts future flight prices for domestic routes in India. The primary goal is to build a machine learning model that can estimate changes in future flight prices based on historical data, using regression techniques. We will investigate how factors such as time of departure, number of stops, the choice of airline, among others, influence flight prices. The secondary objective is to analyze if certain trends can be linked to broader environmental or policy factors. While predicting flight prices may seem primarily economic, it intersects with transportation engineering by optimizing air traffic

and scheduling. Additionally, these predictions could indirectly inform decisions aimed at reducing the environmental impact of flights. By better understanding pricing trends, stakeholders can implement dynamic pricing strategies that encourage sustainable travel, such as offering discounts for off-peak flights or promoting direct routes to cut down on fuel consumption.

# **Dataset Description**

- Source: The dataset used for this project can be found on Kaggle, at this link: <u>Kaggle</u> It was used to help build a predictive model for flight price prediction using the data that will be explained below.
- Format: The dataset is in CSV format, which is commonly used for tabular data storage. Each row represents a specific data point, with columns detailing various features that might impact flight prices.
- Contents: The dataset serves as a basis for training machine learning models for predicion of flight costs. More specifically, the dataset includes the following columns:
  - 1. Airline: A String value representing the name of the Indian airline company included in the study
  - 2. Source: Another String value representing the city from which the airline departs
  - 3. Destination: Yet another String value representing the arrival city
  - 4. Total\_Stops: a ternary integer variable between 0 and 2 that represents the number of of stopd from the city of departure to the arrival
  - 5. Price: An integer variable presententing the cost, in rupees, for each ticket
  - 6. Day/Month/Year: Three columns containing integer variables representing the date when the flight took place. Note that the year column contains only the year 2019, so we may remove this column
  - 7. Dep\_hours/Dep\_min: Two columns containing integer numbers representing the hour, in military time, and minute at which the flight departed
  - 8. Arrival\_hours/Arrival\_min: Similar to the Dep\_hours/Dep\_min columns, but for the the arrival time of the flight
  - 9. Duration\_hours/Duration\_min: Two columns with integer values representing the number of hours and minuted a flight lasted

# Introduction

### The dynamics of flight price predictions

Flight pricing models have long been of interest to researchers, airline companies, and consumers alike <u>Sun et al.</u>. As global air travel expands, so do the complexities involved in predicting flight prices due to the dynamic nature of factors such as fuel costs, demand fluctuations, and the regulatory environment <u>Sun et al.</u>; <u>Belobaba et al.</u>; <u>International Air Transport Association</u>; <u>Borenstein et al.</u>. Predicting flight prices accurately allows both passengers and airlines to optimize travel schedules, with potential economic and environmental benefits <u>Belobaba et al.</u>. In recent years, machine learning techniques have been widely adopted in various industries, including aviation, to model complex relationships and predict outcomes such as pricing <u>Annabel et al.</u>; <u>Kalampokas et al.</u>; <u>Xu et al.</u>. For this project, we propose a machine learning-based predictive model designed to estimate future flight prices based on historical data from domestic flight routes to and from India.

### The importance of predicting flight prices

Accurately predicting flight prices has significant implications for multiple stakeholders. For consumers, anticipating price trends helps optimize travel costs, allowing for better budgeting and planning <u>International Air Transport Association</u>; <u>Borenstein et al.</u>; <u>Gillen & Morrison</u>. Airlines, on the other hand, can improve their revenue management strategies through dynamic pricing, ensuring that flights operate closer to capacity while adjusting pricing to meet seasonal demand shifts and

market competition <u>Belobaba et al.</u>; <u>Borenstein et al.</u>; <u>Gillen & Morrison</u>. Additionally, by integrating environmental factors such as carbon emissions into the pricing model, this research can help airlines design more sustainable routes, potentially leading to fewer emissions through better traffic management and efficient flight operations <u>Sun et al.</u>; <u>International Air Transport Association</u>; <u>Gillen & Morrison</u>; <u>Xiong et al.</u>. The aviation industry contributes significantly to the global economy but also poses challenges due to its substantial carbon footprint <u>Brueckner et al.</u>.

### Methodological approach and dataset

This project utilizes a Kaggle dataset on domestic flights in India, covering variables such as airline name, total stops, departure and destination locations, and flight prices [3–5, 8]. Using regression techniques within a machine learning framework, the researchers aim to identify which variables most significantly impact flight prices. Initial findings suggest that factors such as the number of stops, flight duration, and seasonal demand (correlating with major holidays in India) are key drivers of price variation. For instance, a notable spike in flight prices is observed around holidays in India, are insights that not only advance knowledge in transportation economics but also serve as a crucial decision-making tool for airline pricing strategies <u>Sun et al.</u>; <u>International Air Transport Association</u>; <u>Borenstein et al.</u>; <u>Gillen & Morrison</u>.

### Multidisciplinary implications

By developing this predictive model, the research bridges the gap between economics, machine learning, and environmental sustainability. Future applications of this work could extend beyond pricing into areas like route optimization and emission reduction strategies <a href="International Air Transport Association">International Air Transport Association</a>. As flight prices are a key variable in both economic and environmental calculations, understanding their underlying dynamics can contribute to more sustainable and cost-effective air travel in the future <a href="Belobaba et al.">Belobaba et al.</a>; <a href="International Air Transport Association">International Air Transport Association</a>; <a href="Gillen & Morrison">Gillen & Morrison</a>.

# Exploratory Data Analysis of Indian Domestic Flights (March - June 2019)

The dataset includes domestic flights of Indian airlines from March 2019 to June 2019, and is derived from <u>Kaggle</u>. Each column in the dataset corresponds to a specific variable, and each row represents an observation. The dataset is clean, with consistent measurement units and no missing values.

### **Dataset Variables:**

- Airlines: The name of the airline operating the flight.
- **Source and Destination**: Cities where the flights originate and land.
- **Total Stops**: The number of stops made by the flight.
- **Price**: The ticket price for the respective flight.
- Date, Month, and Year: The specific date on which the flight is scheduled.
- **Departure and Arrival Times**: Detailed departure and arrival hours and minutes.
- **Duration**: The total duration of the flight in hours and minutes.

# **Correlation Analysis:**

We explored possible correlations between variables in the dataset. One expected correlation is between flight price and flight duration. Using the correlation in Julia, we found a positive correlation of **0.51** between these two variables. Similarly, the correlation between the number of

stops and price is **0.60**. It makes sense that as the number of stops increases, the flight distance and, consequently, the price also increase.

The chart depicted in **Figure 1** illustrates that most flights in the dataset have ticket prices below 10,000 Rupees.



Figure 1: Distribution of Flight Prices (Positive Skew).

### **Seasonal Price Variations:**

To analyze seasonal price variations, we created a new column, Adjusted-Date, by combining the values from the Date, Month, and Year columns into a single date format. We then plotted the mean price over time using this adjusted date. As shown in **Figure 2**, flight prices fluctuate significantly over time, with notable peaks around the major Indian holidays.



Figure 2: Flight price trends over time.

These price variations can be correlated with the seasonal demand and cultural events during this period. Upon reviewing the price fluctuations, we explored the major holidays in India during this period to identify possible correlations between price peaks and holidays. Interestingly, many of the price peaks align with Indian holidays. For example: - In March, price spikes around March 4th and 21st coincide with **Maha Shivaratri** and **Holi**, respectively. - In April, a price increase occurs around April 13th and 14th, aligning with **Ram Navami**, **Baisakhi**, and **Tamil New Year/Vishu**. - In May, a price increase is observed around May 1st (coinciding with **May Day**) and May 18th (coinciding with **Buddha Purnima**). - High prices persist into early June, corresponding with **Eid-ul-Fitr** (June 4th) and **Ganga Dussehra** (June 12th).

# **Destination Analysis:**

We reviewed **10,684** flights during this period. **Cochin**, **Bangalore**, and **Delhi** were the top destinations, with Cochin being the most attractive, receiving the highest number of flights. The details of the top destinations are shown in **Table 1**.

**Table 1: Top Flight Destinations** 

Rank	Destination	Count
1	Cochin	4,537
2	Bangalore	2,871
3	Delhi	1,265
4	New Delhi	932
5	Hyderabad	697
6	Kolkata	381

# Origin-Destination (O/D) Pair Analysis:

We also identified the most frequent origin-destination pairs, as shown in **Table 2**.

**Table 2: Most Frequent Origin-Destination Pairs** 

Rank	Source	Destination	Count
1	Delhi	Cochin	4,537
2	Kolkata	Bangalore	2,871
3	Bangalore	Delhi	1,265
4	Bangalore	New Delhi	932
5	Mumbai	Hyderabad	697
6	Chennai	Kolkata	381

# **Airline Insights:**

Our analysis of the airlines provided the following insights:

# 1. Mean Price by Airline:

The table below (**Table 3**) shows the mean flight price for each airline, sorted from highest to lowest.

**Table 3: Mean Price by Airline** 

Rank	Airline	Mean Price (INR)
1	Jet Airways Business	58,359
2	Jet Airways	11,644
3	Multiple Carriers Premium	11,419
4	Multiple Carriers	10,903
5	Air India	9,611
6	Vistara Premium Economy	8,962
7	Vistara	7,796
8	GoAir	5,861
9	IndiGo	5,674
10	Air Asia	5,590
11	SpiceJet	4,338
12	Trujet	4,140

# 2. Airlines with the Most Number of Flights:

The table below (**Table 4**) lists the airlines with the most flights in the dataset.

**Table 4: Airlines with the Most Number of Flights** 

Rank	Airline	Number of Flights
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Rank	Airline	Number of Flights
1	Jet Airways	3,849
2	IndiGo	2,053
3	Air India	1,752
4	Multiple Carriers	1,196
5	SpiceJet	818
6	Vistara	479
7	Air Asia	319
8	GoAir	194
9	Multiple Carriers Premium	13
10	Jet Airways Business	6
11	Vistara Premium Economy	3
12	Trujet	1

# 3. Airlines Frequently Used in Long-Haul Flights:

The table below (**Table 5**) lists the airlines frequently used for long-haul flights (flights with a duration greater than 10 hours).

**Table 5: Airlines Frequently Used in Long-Haul Flights** 

Rank	Airline	Long-Haul Flights
1	Jet Airways	2,395
2	Air India	1,178
3	Multiple Carriers	625
4	IndiGo	231
5	Vistara	197

It is worth noting that there is limited data available for multiple-carrier flights, so further analysis of these flights is not possible.

# Singular Value Decomposition (SVD) and Principal Components Analysis (PCA):

Before beginning with SVD or even PCA, we must normalize the data. Since most of out variables are categorical, only two variables needed to be normalized. These two variables are Fuel\_Consumption\_normalized and CO2\_Emitted\_normalized, and their normalization values are shown in the bar chart below, in **Figure 3**. This provided a preview that perhaps SVD and PCA would not be needed, given the small number of numerical variables.

#### Normalized Numerical Data



**Figure 3:** Bar chat showing the normalization of our two numerical variables.

Since there are only two numerical variables to be analysed in this dataset, only two singular values were created, as can be seen below in **Figure 4**.



Figure 4: Line plot with two points representing two singular values.

As we only have 2 numerical variables, it makes sense that most of the data points are concentrated around the first and second principal components, because they correspond to the two numerical variables. This can be seen below in **Figure 5**. Of course, there are the only two principal components.

Because we have such few numerical variables, if we were to use PCA, we might lose valuable information. Hence, we will proceed with regression analysis in the next section of our project.

# 0.04 0.02 0.00 0.00 -0.02 -0.04 30 principal component 20 principal component 30 principal component 20 principal component 30 principal component

### PCA: First and Second Principal Components

**Figure 5:** First and second principal components.

# **Predictive Model Planning**

The project goal is to build a Machine learning-based predictive model. Linear regression will be used to establish the relationship between the datasets' features and prices. Gradient boosting algorithm of XGBoost Regressor or/and Random Forest Regressor will be used for capturing complex patterns and the predictive tasks. In the case of non-linearity and high interactions PCA or/and neural network will be used to enhance the performance. To measure the difference between actual and predicted prices, RMSE (Root Mean Square Error) or MSE(Mean Square Error) will be used. R-Squared (R2-Coefficient of determination – measure of goodness-of-fit of regression) will evaluate how well independent variable explains the variance in the flight prices. While building this model, we will be working with the packages such as DataFrame.jl, StatsPlots.jl, Statistics.jl, MLJ.jl, etc. from Julia library. If needed, ScikitLearn.jl (access to python's Scikit-learn models in julia) will be implemented.

Our model will be able to predict the seasonal spikes in flight prices because of the factors such as festivals, holidays or high demand periods so that airlines can adjust the pricing and offer more competitive pricing. Additionally, Expensive (with higher mean ticket prices) airlines can be benefited by understanding the price elasticity and regulating accordingly. Furthermore, we can explore the environmental sustainability factors such as carbon emission by adopting environmentally sustainable options like fewer layover routes, energy efficient airlines, etc.

# Implementing Linear Regression Models for Price Prediction

In this section, we first implemented a simple linear regression model for price prediction based on our dataset, to better understand the independent variables and their influence on the results. Then, we proceeded to enhance the model based on our findings.

To begin with, we developed a simple linear regression model using the LinearRegression module from the scikit-learn package. This package fits a linear model with coefficients ( $w = (w_1, ..., w_p)$ ) using the ordinary least squares (OLS) method to minimize the residual sum of squares between the observed targets in the dataset and the targets predicted by the linear approximation, as shown in Equation (1) (scikit-learn documentation).

### Equation (1):

$$\min_w \|\|Xw-y\|\|_2^2$$

Based on our exploratory analysis, we chose the number of stops as an independent variable, as this variable has a 60% correlation with price. For simplicity, we did not use cross-validation in our first step and simply divided the data into 75% for training and 25% for testing. This resulted in the model shown in Equation (2):

### Equation (2):

$$Price = 5669.74 + 4150.40 \times Total Stops$$

To evaluate the performance of the model, we calculated three different metrics: root mean squared error (RMSE), mean squared error (MSE), and ( $R^2$ ) coefficient, which were 3613.11, (1.3 e+07), and 0.36, respectively. As you can see, the model's performance is poor.

To enhance the model, we added another independent variable, flight duration in minutes, as this variable has a 50% correlation with price. This resulted in the model formulated in Equation (3), with RMSE, MSE, and (R^2) metrics of 3586.91, (1.28 e+07), and 0.37, respectively.

#### Equation (3):

$$Price = 5447.66 + 3496.11 \times Total Stops + 1.19 \times Flight Duration$$

Although the second model performs slightly better than the first one, it is still not satisfactory. Therefore, we adjusted the dataset to consider holidays as well. We added a binary variable named Holiday to the dataset, which equals 1 when the flight is on the following dates: Maha Shivaratri (March 4), Holi (March 21), Ram Navami (April 13), Baisakhi (April 14), May Day (May 1), Buddha Purnima (May 18), Eid-ul-Fitr (June 4), and Ganga Dussehra (June 12). In this way, we reached the model presented in Equation (4):

#### Equation (4):

$$\mathrm{Price} = 5534.45 + 3455.92 \times \mathrm{Total\ Stops} + 1.17 \times \mathrm{Flight\ Duration} - 2212.58 \times \mathrm{Holiday}$$

The RMSE, MSE, and (R^2) metric values for this model are 3571.72, (1.27 e+07), and 0.38, respectively. As you may notice from the above results and also **Figure 6**, all of the models are unable to provide good estimations of the price, and their performance is poor. This may be the result of not using cross-validation or the linear regression model being unsuitable for our purpose.

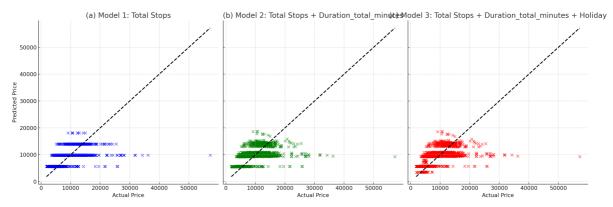


Figure 6: Performance of models 1(a), 2(b), and 3(c).

To test these theories, we first implemented cross-validation with 5 folds, as this is common and ensures both computational efficiency and performance balance. To do so, we used the third model, the best one we had so far. This led to a slight enhancement in the results. The metrics are presented in **Table 6**:

**Table 6: Performance Metrics for Model 4** 

Metric	Fold Values	Mean Value	SD
RMSE	[3731.48, 3732.26, 3636.08, 3468.79, 3627.01]	3639.12	96.31
MSE	[13,923,954.82, 13,929,781.75, 13,221,049.43, 12,032,485.40, 13,155,213.99]	13,252,497. 08	694,039. 12
(R^2)	[0.378, 0.369, 0.377, 0.380, 0.379]	0.377	0.004

As we can observe the performance of the Model 4 is quite low, So we decided to add one more feature "Attractive Destination" in binary form. The origin and the destination mentioned in the Table 2 will be 1 and other will be 0.

**Equation (5):** Price = 5867.98 + 4764.5065 x Total Stops - 900.40 x Holiday + 1.10 x Flight Duration - 636.365 x Attractive Destination

The performance metrics RMSE, MSE and  $\rm R^2$  observed from this model are 3443.25, (1.185 e+07), and 0.42 respectively. We can notice that the performance has slightly improved, Although we could not get the satisfactory results.

**Equation (6):** Price =  $4332.83 + 3652.17 \times \text{Total Stops} + 4.98 \times \text{Flight Duration} - 424.07 \times (\text{Total Stops})^2 - 0.001 \times (\text{Flight Duration})^2 - 1.43(\text{Noise})$ 

The relatively low performance of linear models suggests that the relationships between the variables in the dataset may be non-linear or involve complex interactions between variables. we evaluated the performance with the polynomial regression model. This time we eliminated the one feature "Holiday" as it has less correlation with the price and we divided the data into 80% training and 20% testing. The RSME, MSE and R^2 values we observed from this model are 3082.22, (9.50 e+06), and 0.48, respectively. You can observed that the model performance have improved than the last model but still it has low performance.

# **Decision Tree Analysis**

This section is dedicated to decision tree analysis. Because the dependent variable is not categorical, the DecisionTreeRegressor from scikit-learn was employed. After the first decision tree was created, using the original dataset (with Duration\_hours and Duration\_min combined into a single variable Total\_Duration), the  $R^2$  value was 0.999977. This value seemed suspuciously perfect. The effect of the high correlation value can also be seen in the figure below, which is a plot of the actual vs predicted value, and as can be seen the predicted values fall almost perfectly along the actual values.

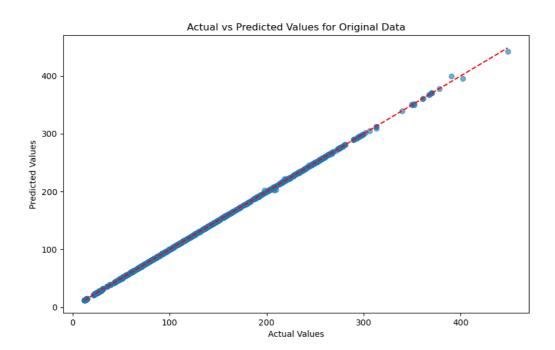


Figure 7: Correlation matrix created using the original dataset.

To understand the origins of this  $\mathbb{R}^2$  value, firstly, a correlation plot was created. The first correlation plot is seen below in Figure 8:

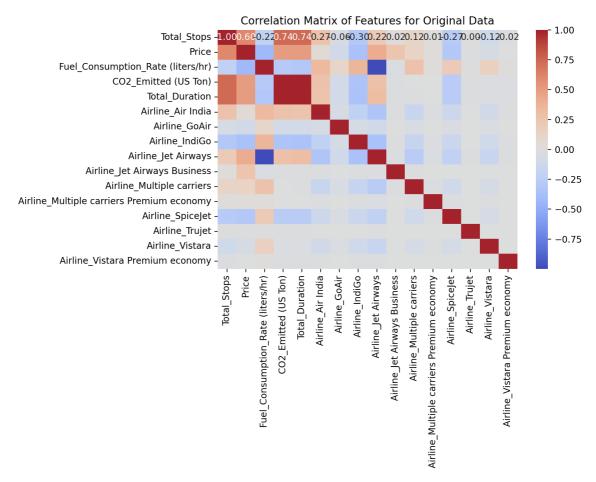


Figure 8: Correlation matrix created using the original dataset.

As can be seen from the figure above, the highest correlation appears between Total\_Duration and CO2\_Emitted (US Ton), the dependent variable. This makes sense, of course, because the longer the plane is in flight, the more  $CO_2$  will be emitted.

In order to question this highly suspicious result, we divided the original dependent variable, CO2\_Emitted (US Ton), by Total\_Duration to create a new dependent variable called CO2\_Emitted/Hour. Now we notice the appearance of two clusters, which indicates that there must be a variable(s) that is causing a binomial distribution.

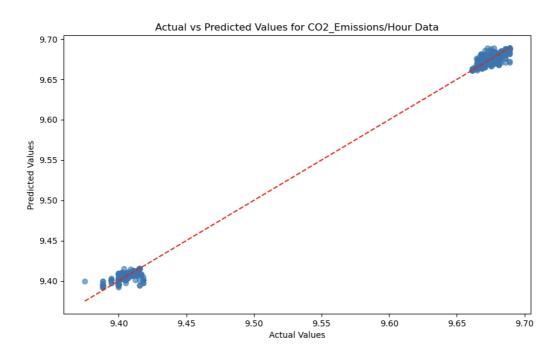


Figure 9: Actual vs predicted values using CO2\_Emitted per Hour as a dependent variable.

To figure out what variable could be causing this binomial distribution, we used RandomForestRegression from scikit-learn to see which features were the most important.



**Figure 10:** Most important features using CO2\_Emitted per Hour as a dependent variable.

We see that the fleet, Boeing 737 is the most important feature when using CO2\_Emitted per Hour as a dependent variable. Originally, we removed this variable to create the first correlation plot since it is a binary variable (its counterpart is Airbus242). However, we see, thanks to Figure 10, that it would be important to bring the Fleet variable back because it is causing the binomial distribution seen in Figure 9.

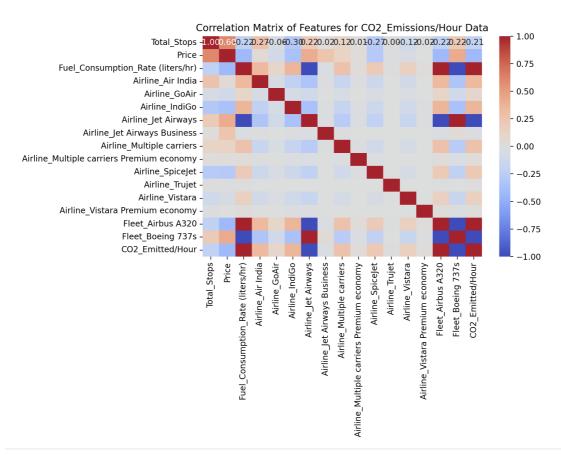


Figure 11: Correlation matrix created using CO2\_Emitted per Hour as a dependent variable.

Note the appearance of binomial data

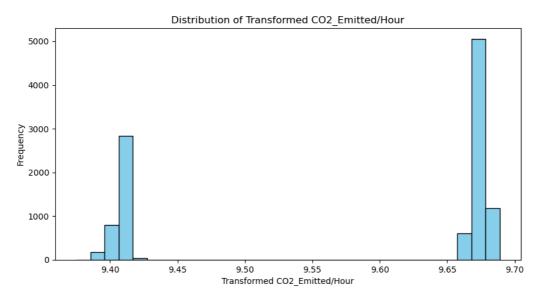


Figure 12: DISTRIBUTION using CO2\_Emitted/Hour as a dependent variable.

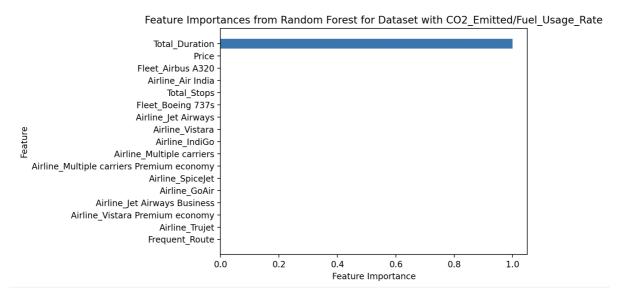


Figure 13: Most important features using CO2\_Emitted per Fuel Usage as a dependent variable.

Figure 13 shows us that when we use CO2\_Emitted per Fuel Usage as a dependent variable, we once again see that the most important feature is Total\_Duration, so clearly Total\_Duration and Fuel\_Usage\_Rate are variables that greatly affect the dependent variable.

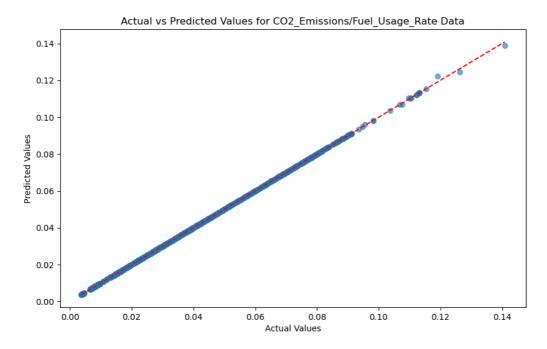


Figure 14: Actual vs predicted values using CO2\_Emitted/Fuel\_Usage\_Rate as a dependent variable.

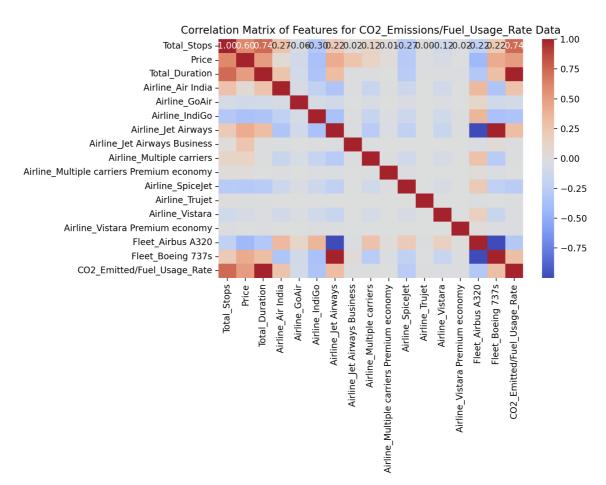


Figure 15: Correlation matrix created using CO2\_Emitted/Fuel\_Usage\_Rate as a dependent variable.

Note the appearance of binomial data

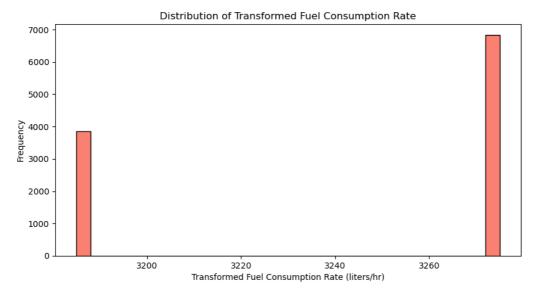


Figure 16: DISTRIBUTION of Fuel\_Usage\_Rate.

Data mimickry noting that Total\_Duration and Fuel\_Consumption\_Rate are most influential independent variables

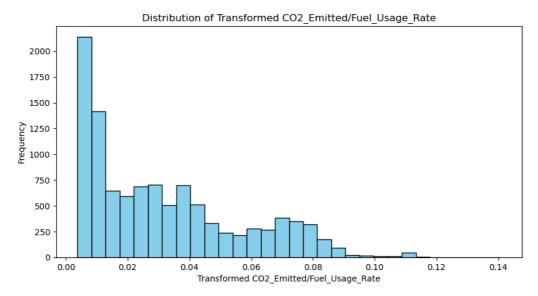


Figure 17: DISTRIBUTION using CO2\_Emitted/Hour as a dependent variable.



**Figure 18:** DISTRIBUTION of Total\_Duration.

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