$$E + S \underset{K_2}{\overset{K_1}{\rightleftharpoons}} ES \xrightarrow{K_3} E + P$$

Solution:

Assume the weight of E is me, P is mp

S is ms, ES is mes

1. We can find four equations:

$$\begin{cases} \dfrac{\partial mes}{\partial t} = K_1 me \cdot ms - K_2 mes - K_3 mes \\ \dfrac{\partial ms}{\partial t} = K_2 mes - K_1 me \cdot ms \\ \dfrac{\partial mp}{\partial t} = K_3 me \cdot ms \\ \dfrac{\partial me}{\partial t} = K_2 mes + K_3 mes - K_1 me \cdot ms \end{cases}$$

2. (i)

$$\frac{\partial mes}{\partial t} = K_1 me \cdot ms - K_2 mes - K_3 mes = 10^3$$

$$\frac{\partial ms}{\partial t} = K_2 mes - K_1 me \cdot ms = -10^3 \mu m/min$$

$$\frac{\partial mp}{\partial t} = K_3 me \cdot ms = 0$$

$$\frac{\partial me}{\partial t} = K_2 mes + K_3 mes - K_1 me \cdot ms = -10^3$$

rate of p is 0, rate of interenediate ES is 10^3 μM/min which saticifies the fourth order Runge-kutta equation.

3. Since $M = m_e + m_{es}$, $m_e = M - m_{es}$

and $\frac{\partial m_{es}}{\partial t} = 0$

so $k_1 m_e \cdot m_s = k_2 m_{es}$

$$k_1 (M - m_{es}) m_s = k_2 m_{es}$$

Then $m_{es} = \frac{k_1 M \cdot m_e}{k_2 + k_1 \cdot m_s}$

$$= \frac{M \cdot m_e}{\frac{k_2}{k_1} + m_s}$$

so $v = \frac{m_p}{m_t} = k_3 m_{es} = \frac{k_3 M \cdot m_s}{\frac{k_2}{k_1} + m_s}$

$$= V_{max} \cdot \frac{m_s}{\frac{k_2}{k_1} + m_s}$$

The we can find $V_{max} = k_3 M$

Now we can plot the function

$$v = V_{max} \frac{m_s}{\frac{k_2}{k_1} + m_s} = k_3 M \frac{m_s}{\frac{k_2}{k_1} + m_s}$$