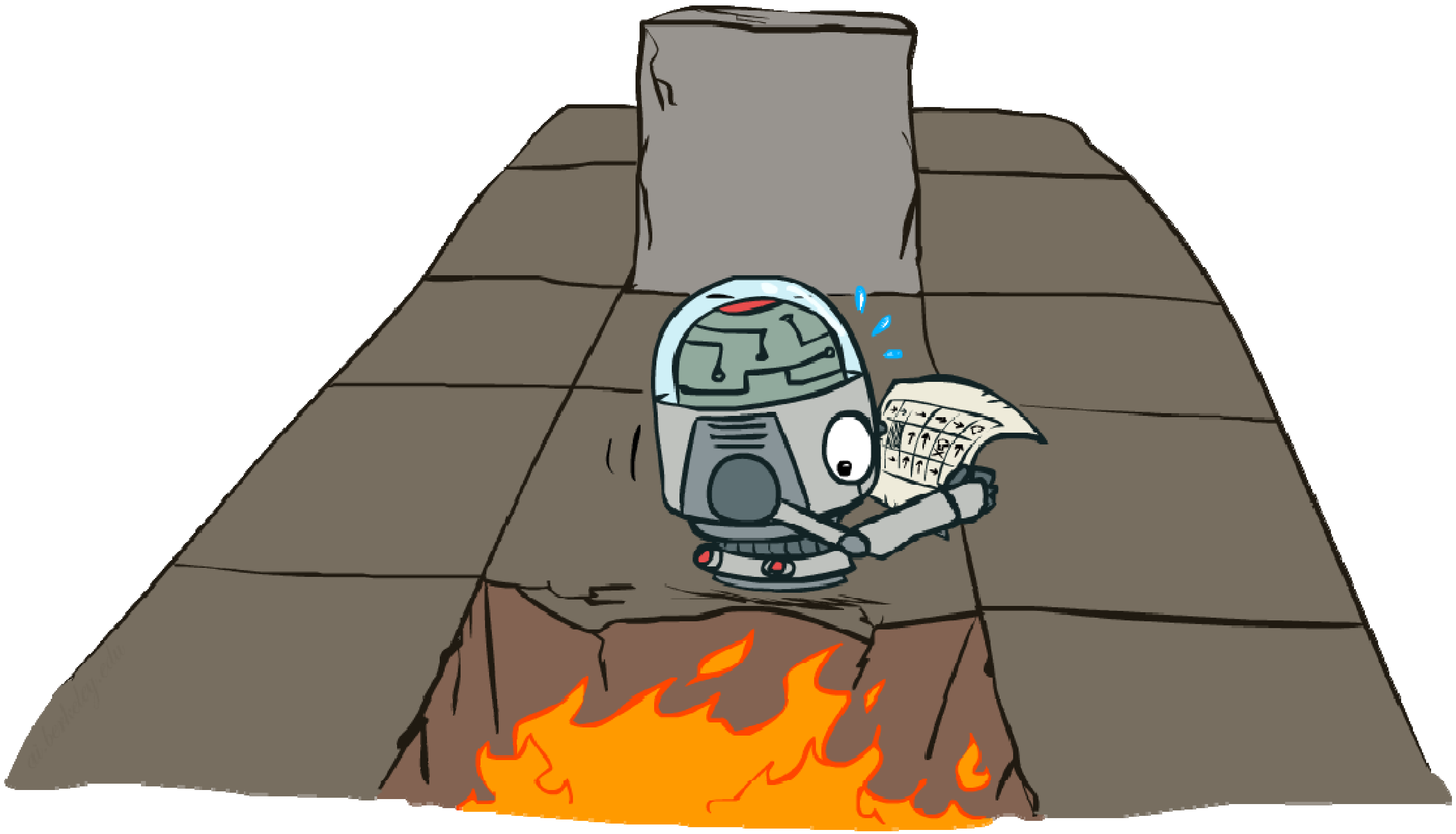
Artificial Intelligence

Markov Decision Processes II



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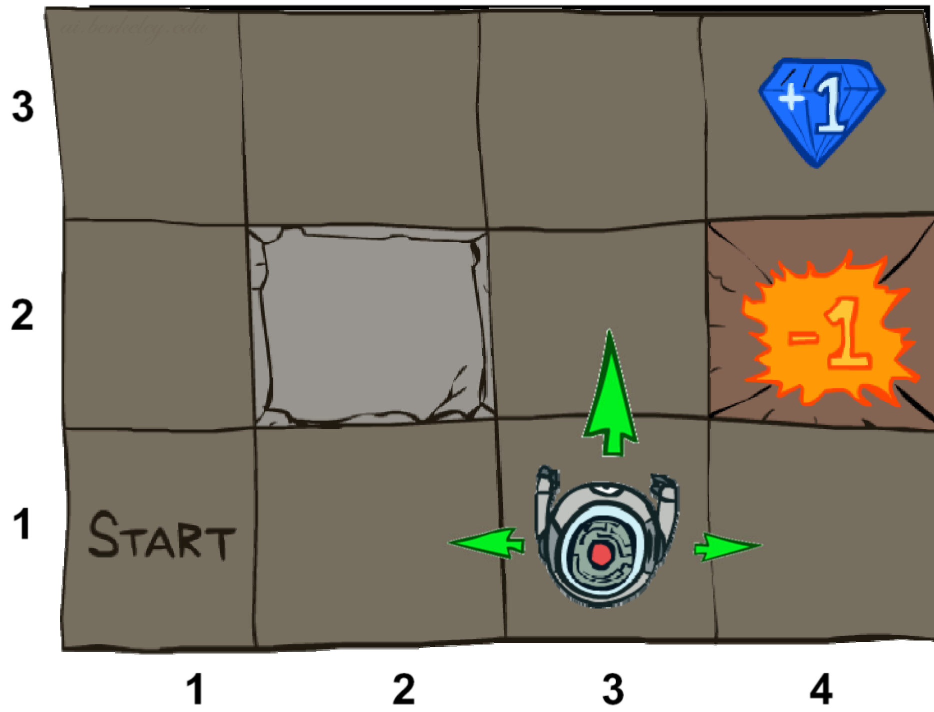
[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.]

Example: Grid World

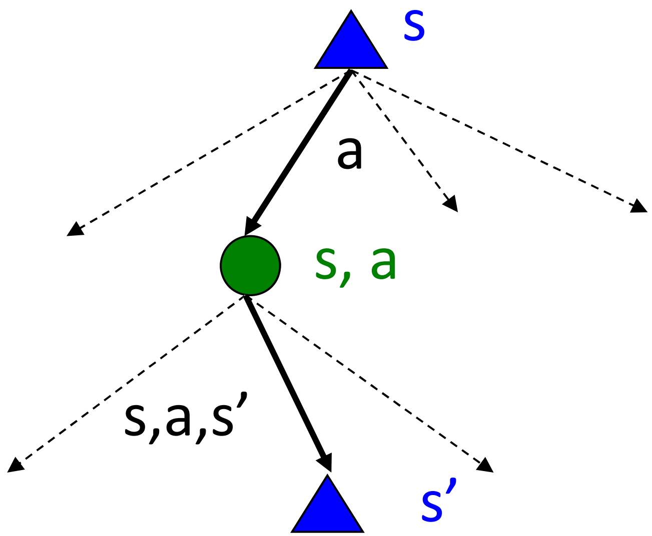
- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of (discounted) rewards



Recap: MDPs

Markov decision processes:

- States S
- Actions A
- Transitions P(s'|s,a) (or T(s,a,s'))
- Rewards R(s,a,s') (and discount γ)
- Start state s_0



Quantities:

- Policy = map of states to actions
- Utility = sum of discounted rewards
- Values = expected future utility from a state (max node)
- Q-Values = expected future utility from a q-state (chance node)

Optimal Quantities

The value (utility) of a state s:

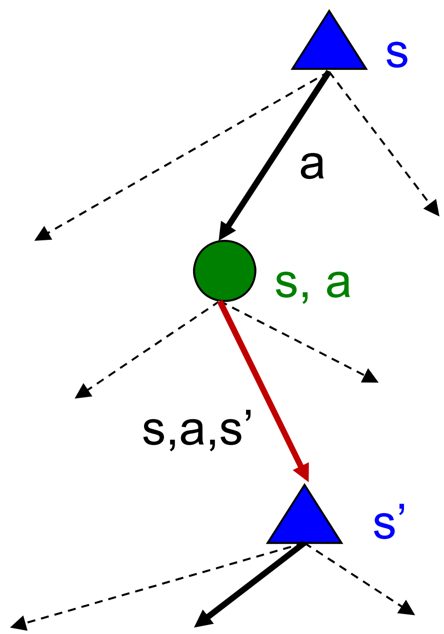
$V^*(s)$ = expected utility starting in s and acting optimally

The value (utility) of a q-state (s,a):

$Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

The optimal policy:

$\pi^*(s)$ = optimal action from state s



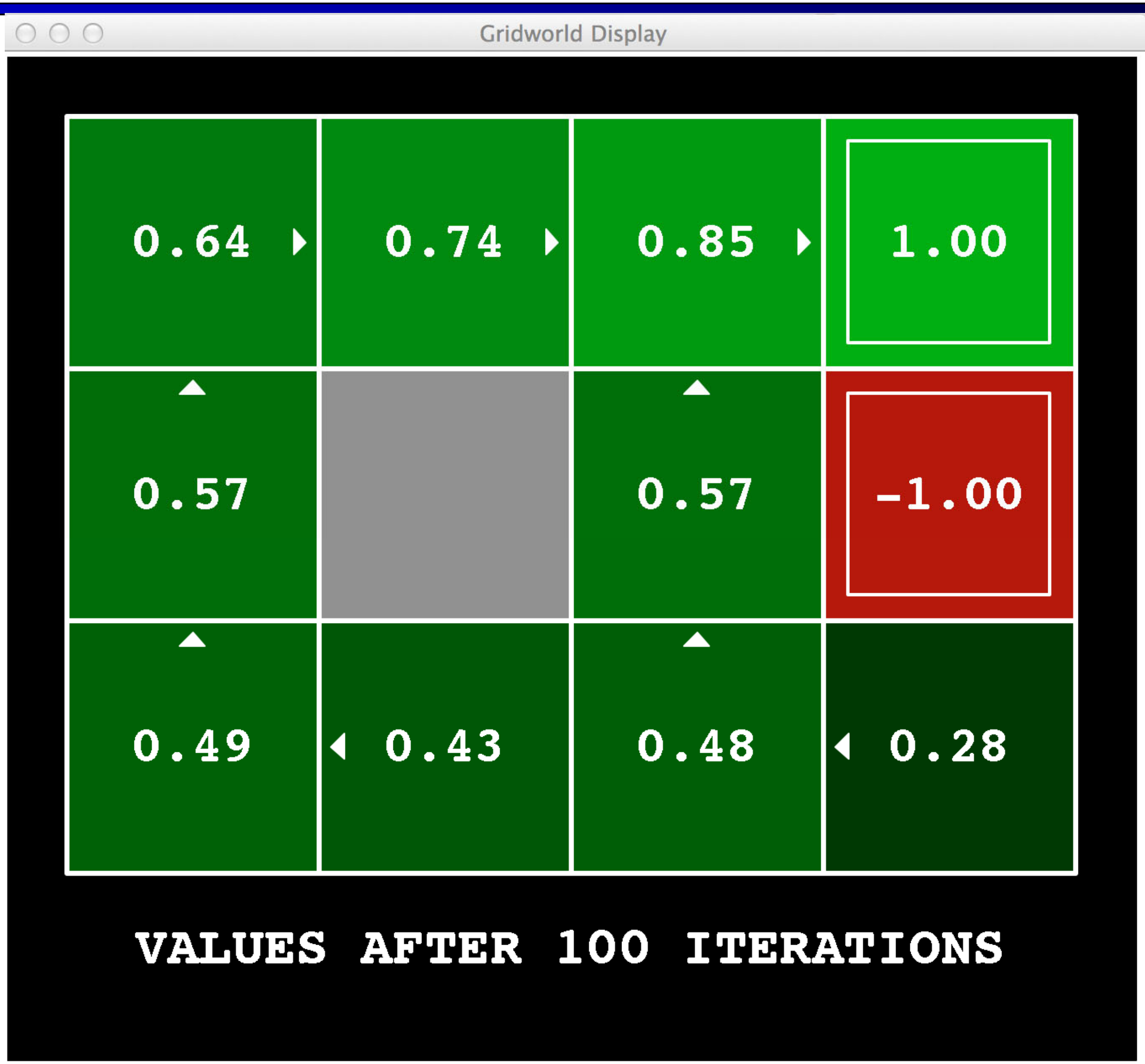
s is a *state*

(s, a) is a *q-state*

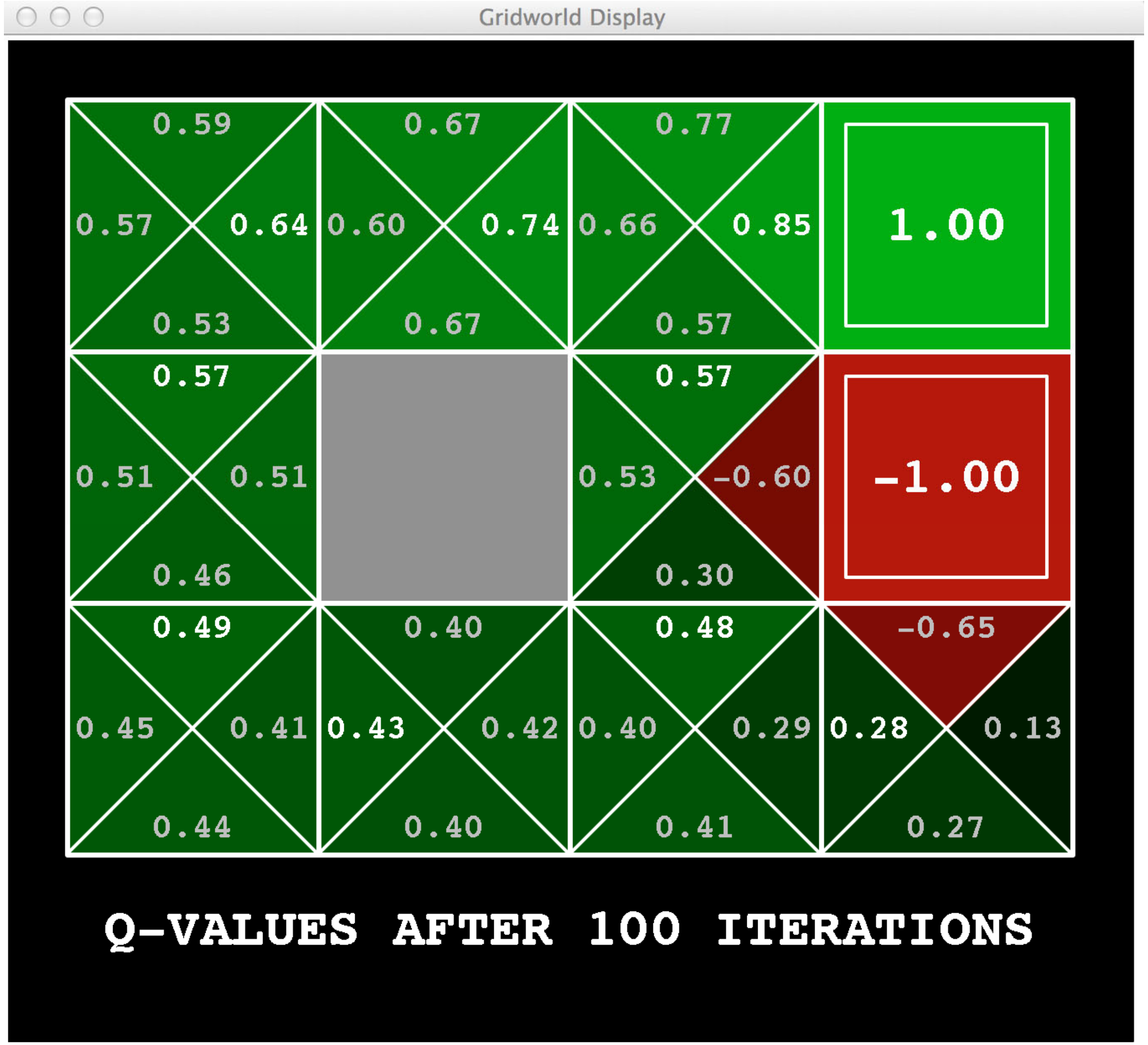
(s,a,s') is a *transition*

[Demo: gridworld values

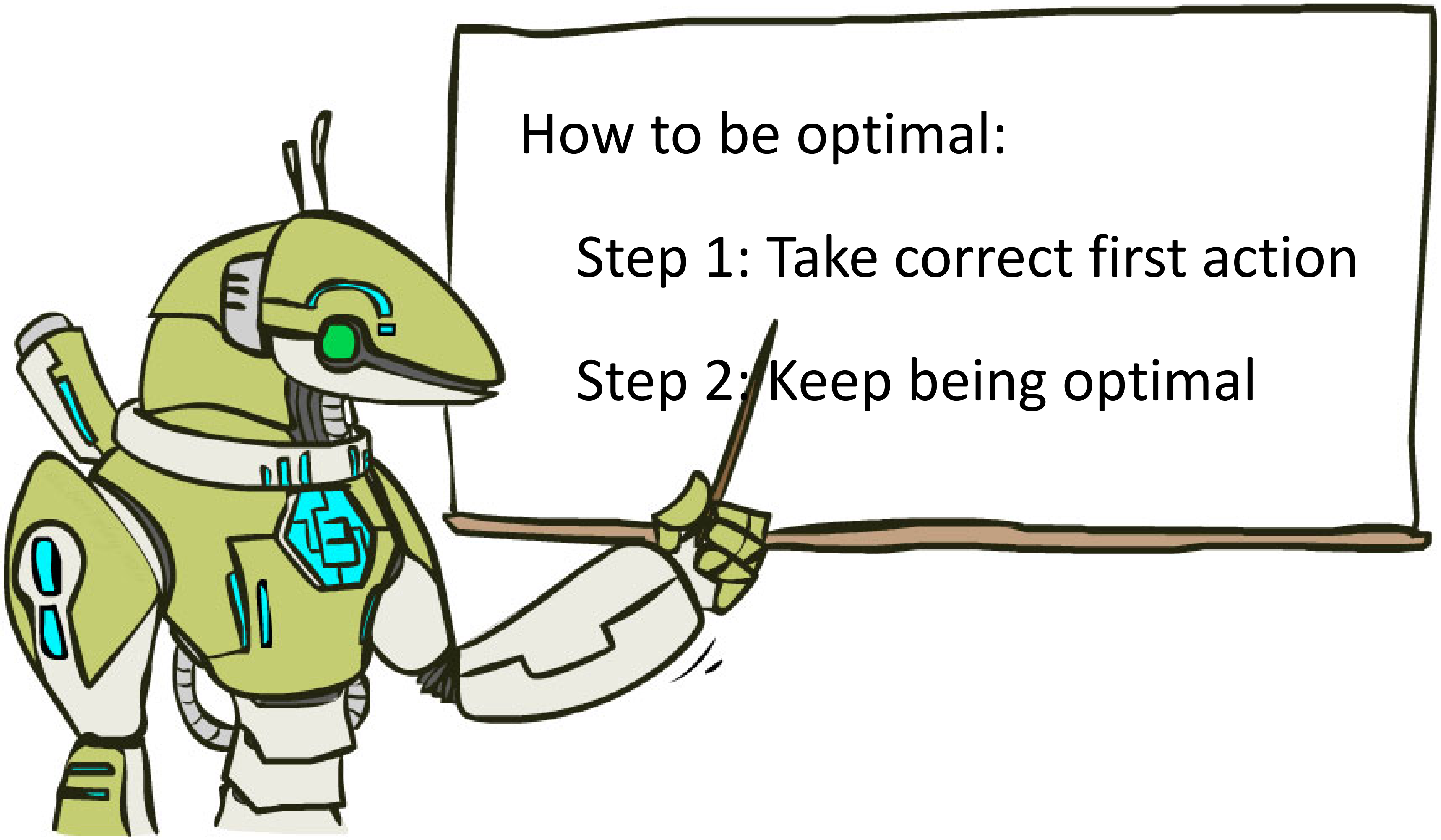
Gridworld Values V*



Gridworld: Q*

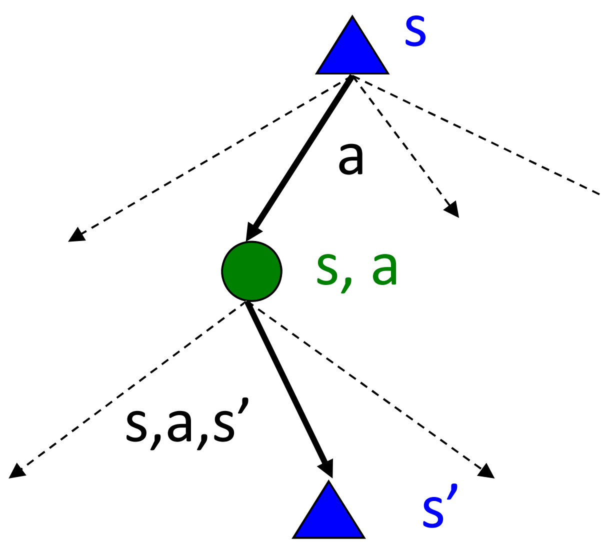

Gridworld Display
0.59
0.57
0.64
0.53
0.67
0.60
0.74
0.67
0.77
0.66
0.85
0.57
1.00
0.57
0.51
0.51
0.46
0.57
0.53
-0.60
0.30
-1.00
0.49
0.45
0.41
0.44
0.40
0.43
0.42
0.40
0.48
0.40
0.29
0.41
-0.65
0.28
0.13
0.27
Q-VALUES AFTER 100 ITERATIONS

The Bellman Equations



The Bellman Equations

Definition of “optimal utility” via expectimax recurrence gives a simple one-step lookahead relationship amongst optimal utility values



$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

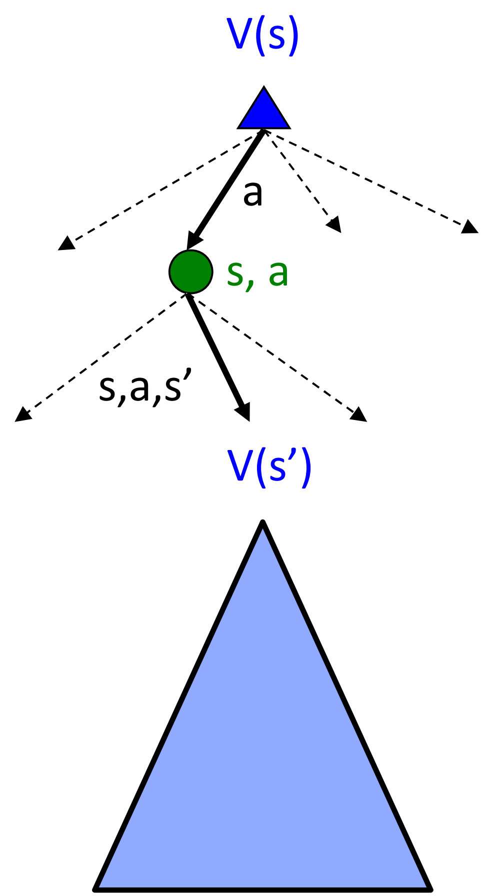
$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

These are the Bellman equations, and they characterize optimal values in a way we’ll use over and over

Value Iteration

Bellman equations characterize the optimal values:

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$



Value iteration computes them:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

Value iteration is just a fixed point solution method

- … though the V_k vectors are also interpretable as time-limited values

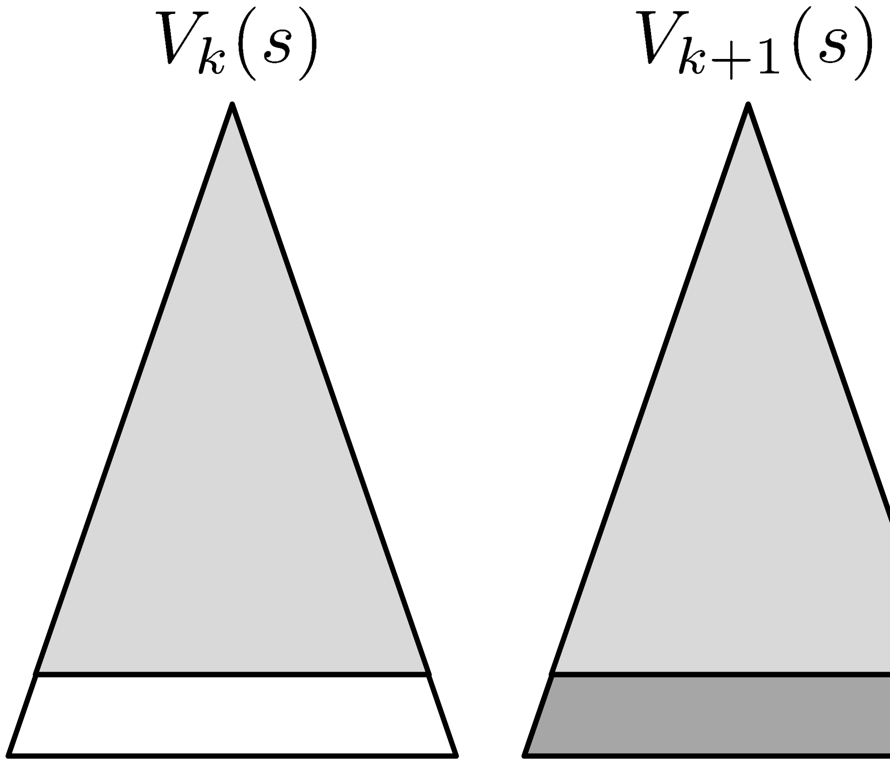
Convergence*

How do we know the V_k vectors are going to converge?

Case 1: If the tree has maximum depth M, then V_M holds the actual untruncated values

Case 2: If the discount is less than 1

- Sketch: For any state V_k and V_{k+1} can be viewed as depth k+1 expectimax results in nearly identical search trees
- The difference is that on the bottom layer, V_{k+1} has actual rewards while V_k has zeros
- That last layer is at best all R_{MAX}
- It is at worst R_{MIN}
- But everything is discounted by γ^k that far out
- So V_k and V_{k+1} are at most γ^k max|R| different
- So as k increases, the values converge



Question

Suppose that, there is a game that you repeatedly draw a card (with replacement) that is equally likely to be a 2, 3, or 4. You can either Draw or Stop if the total score of the cards you have dra is less than 6. If your total score is 6 or higher, the game ends, and you receive a utility of 0. W you Stop, your utility is equal to your total score (up to 5), and the game ends. When you Draw you receive no utility. There is no discount ($\gamma = 1$).

Formulate this problem as an MDP with the following states:

- 0, 2, 3, 4, 5 and a Done state, for when the game ends.

What is the transition function and the reward function for this MDP

Find the optimal values of each state for first 4 iterations

What is the optimal policy

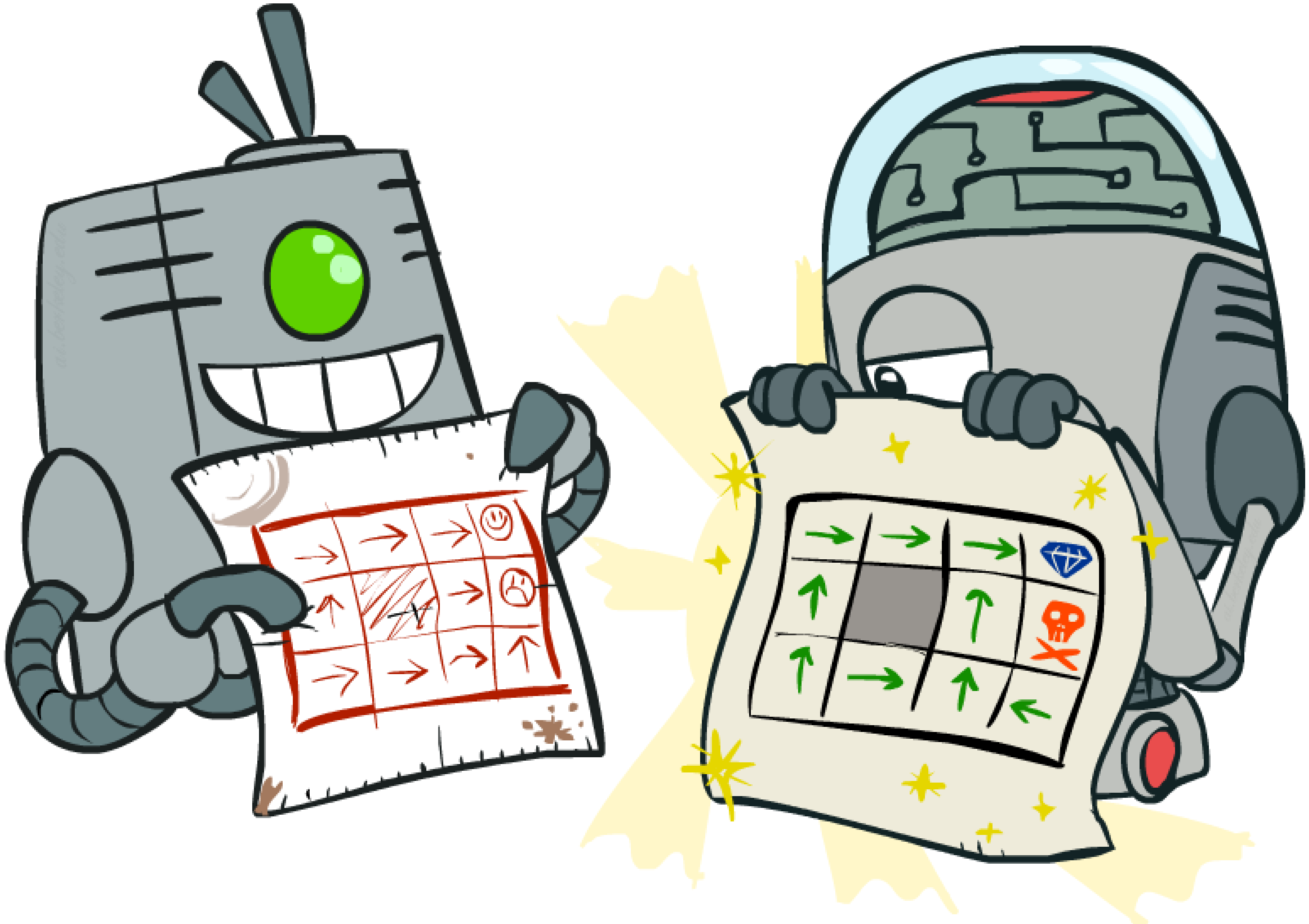
solution

ates	0	2	3	4	5
)	0	0	0	0	0
	0	2	3	4	5
2	3	3	3	4	5
3					
4					

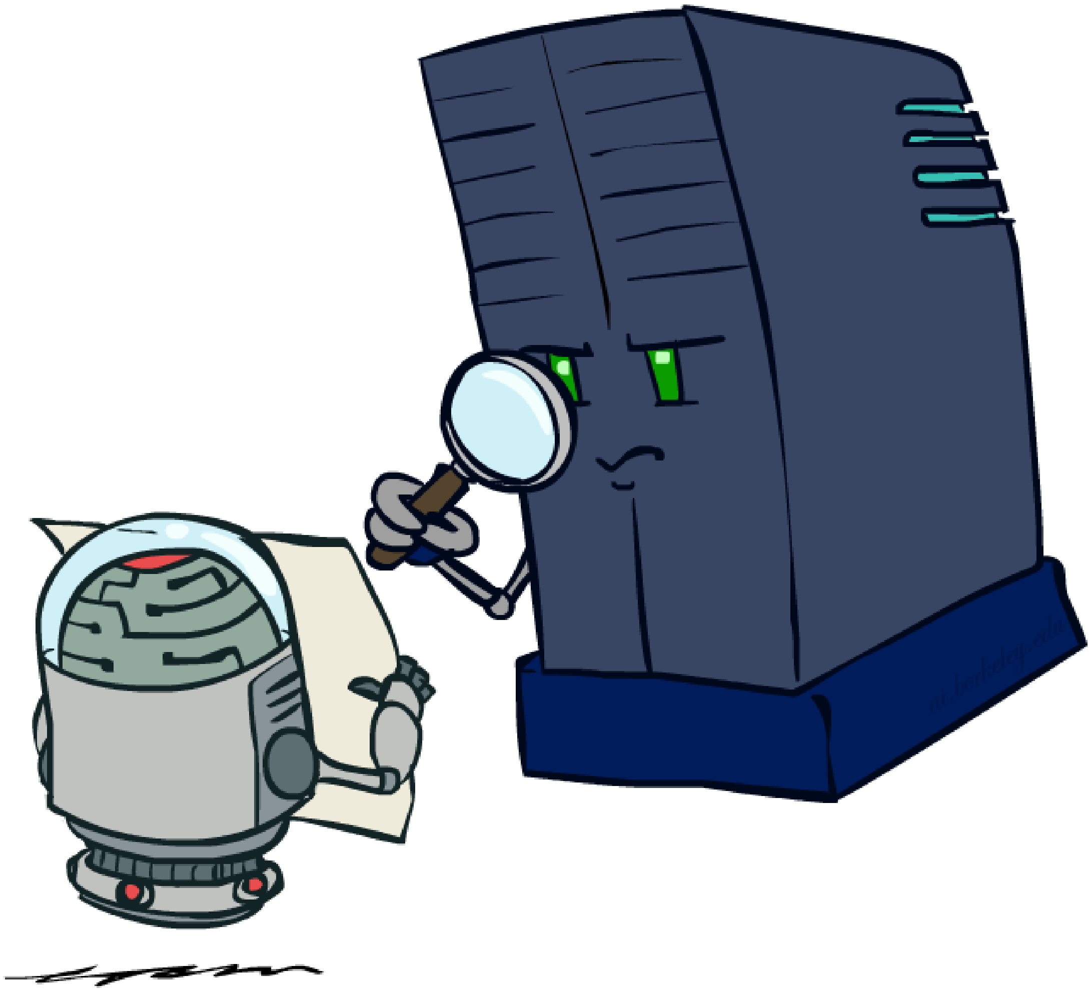
hat is the optimal policy for the MDP?

States	0	2	3	4	5
π^*	Draw	Draw	Stop	Stop	Stop

Policy Methods

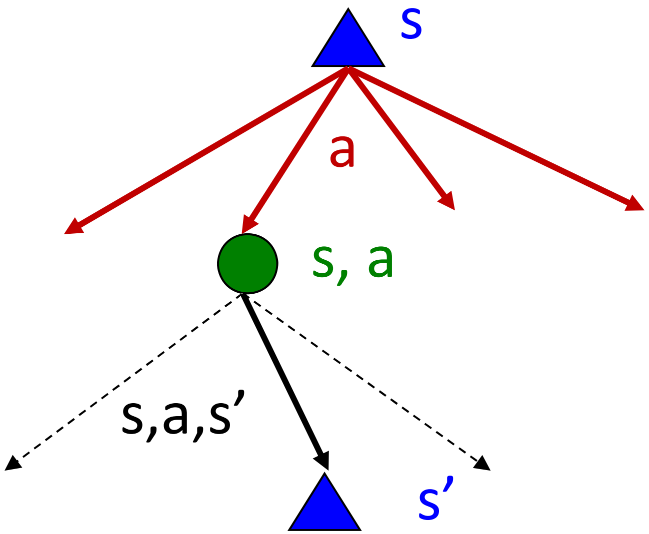


Policy Evaluation

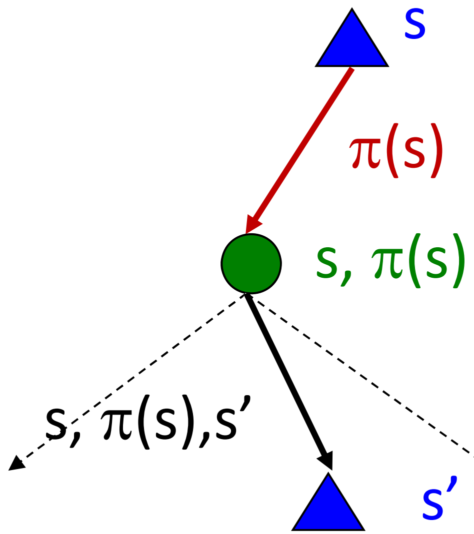


Fixed Policies

Do the optimal action



Do what π says to do



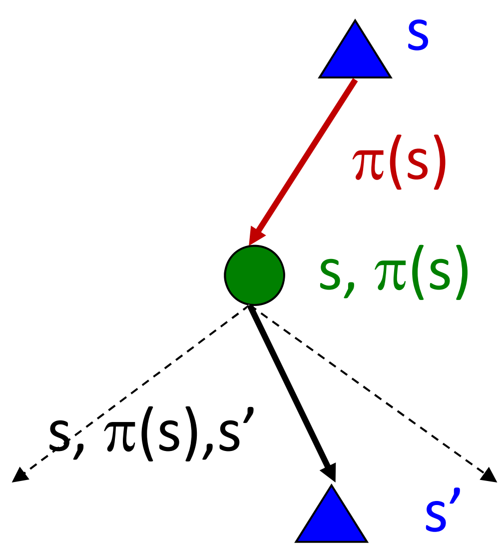
Expectimax trees max over all actions to compute the optimal values

If we fixed some policy $\pi(s)$, then the tree would be simpler – only one action per stat

- … though the tree's value would depend on which policy we fixed

Utilities for a Fixed Policy

Another basic operation: compute the utility of a state s under a fixed (generally non-optimal) policy



Define the utility of a state s, under a fixed policy π:

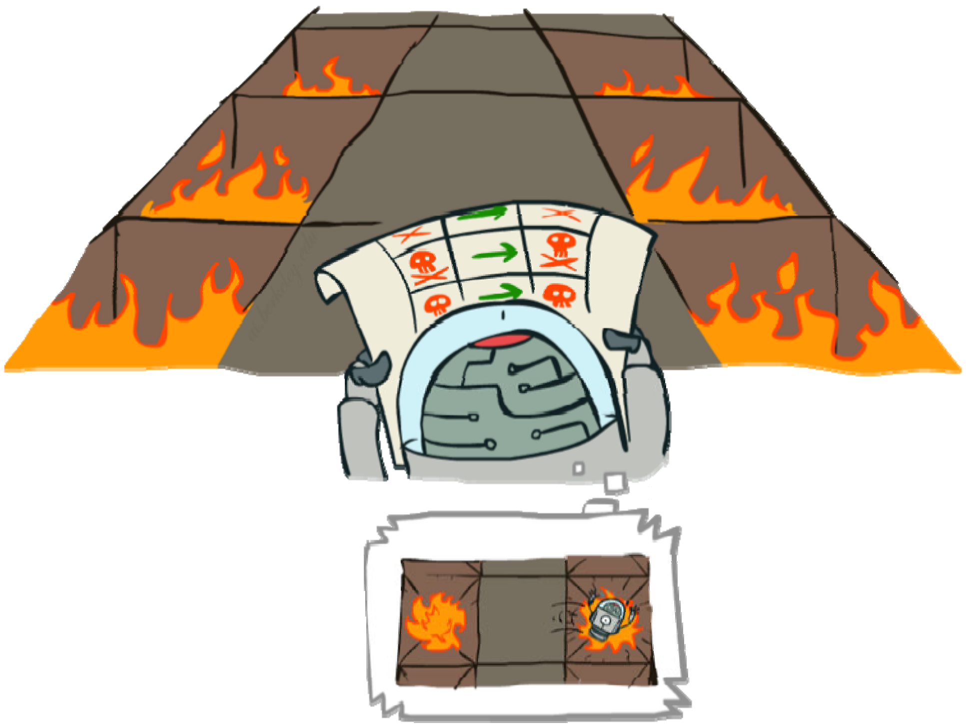
$V^{\pi}(s)$ = expected total discounted rewards starting in s and following π

Recursive relation (one-step look-ahead / Bellman equation):

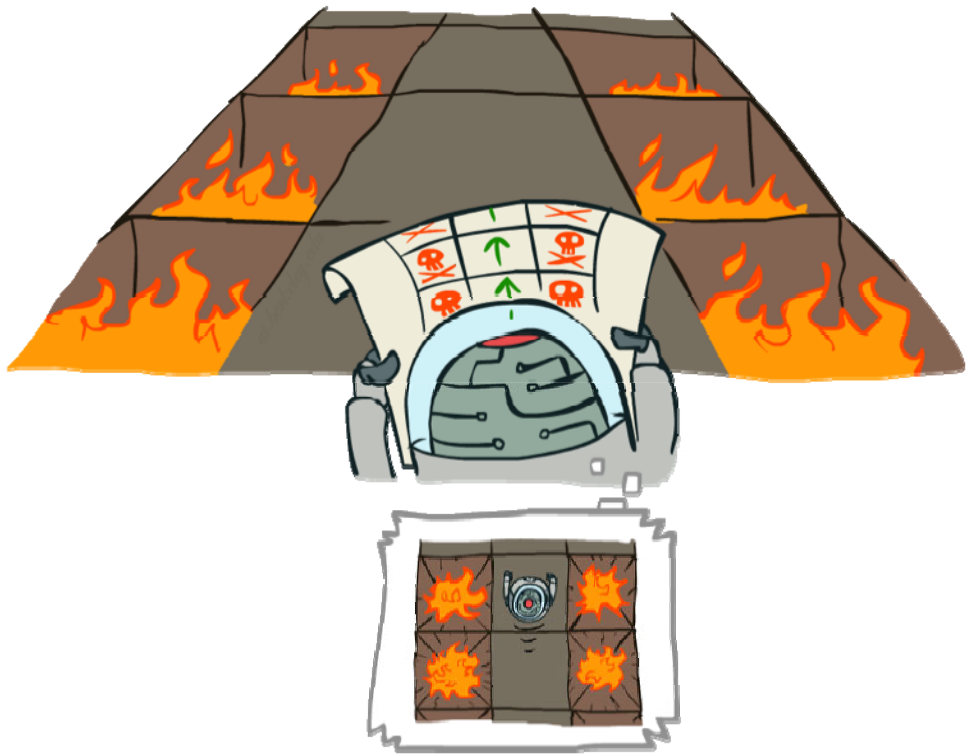
$$V^{\pi}(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^{\pi}(s')]$$

Example: Policy Evaluation

Always Go Right



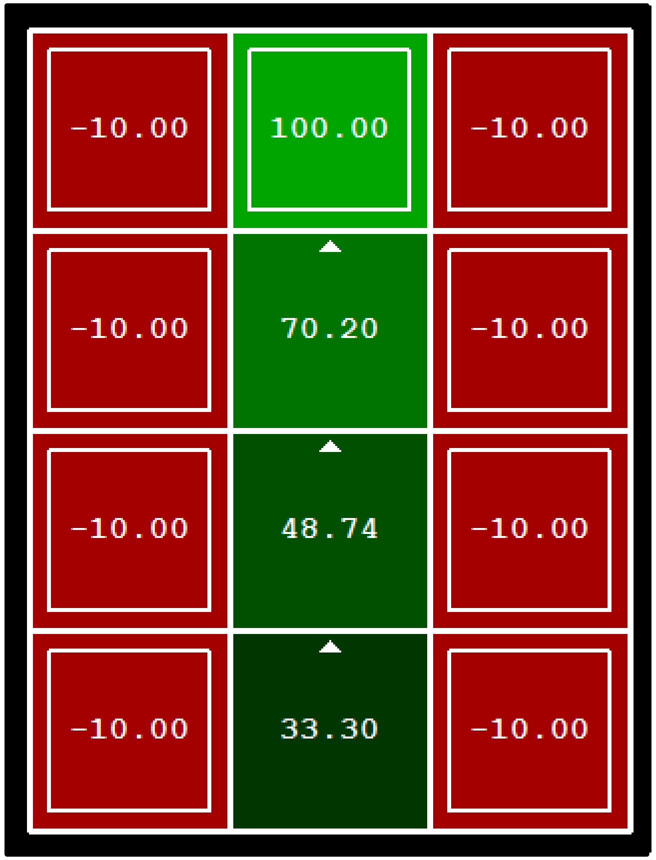
Always Go Forward



Example: Policy Evaluation

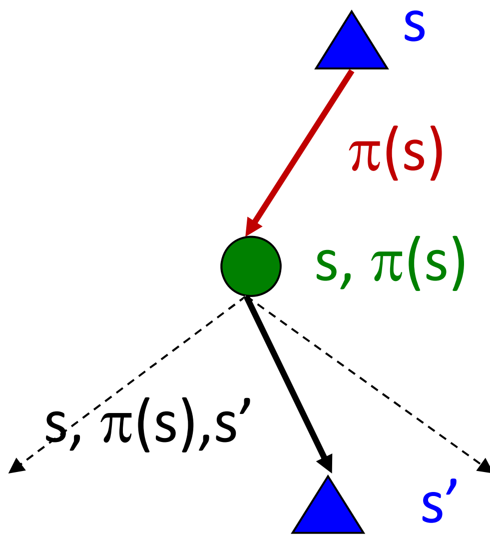
Always Go Right

Always Go Forward



Policy Evaluation

How do we calculate the V's for a fixed policy π?



Idea 1: Turn recursive Bellman equations into updates (like value iteration)

$$V_0^{\pi}(s) = 0$$

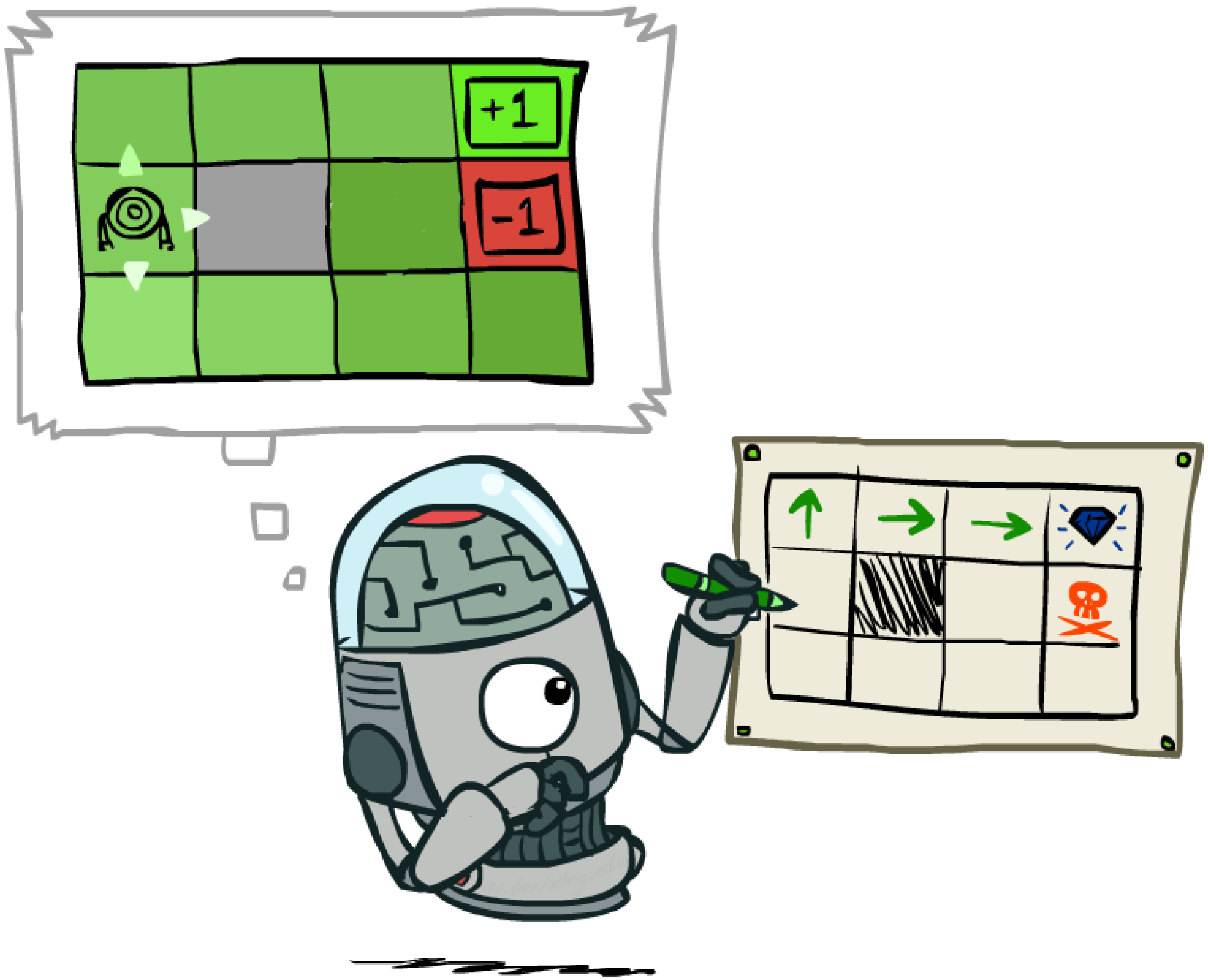
$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

Efficiency: $O(S^2)$ per iteration

Idea 2: Without the maxes, the Bellman equations are just a linear system

- Solve with Matlab (or your favorite linear system solver)

Policy Extraction



Computing Actions from Values

Let's imagine we have the optimal values V*(s)

How should we act?

- It's not obvious!

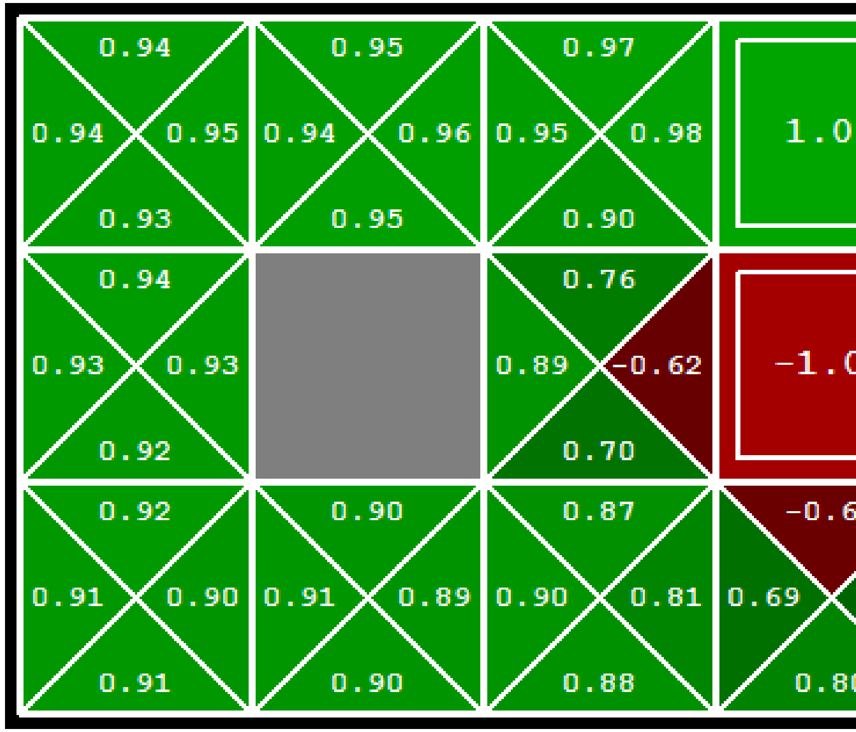
We need to do a mini-expectimax (one step)

$$\pi^*(s) = \arg\max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

This is called policy extraction, since it gets the policy implied by the value

Computing Actions from Q-Values

Let's imagine we have the optimal q-values:



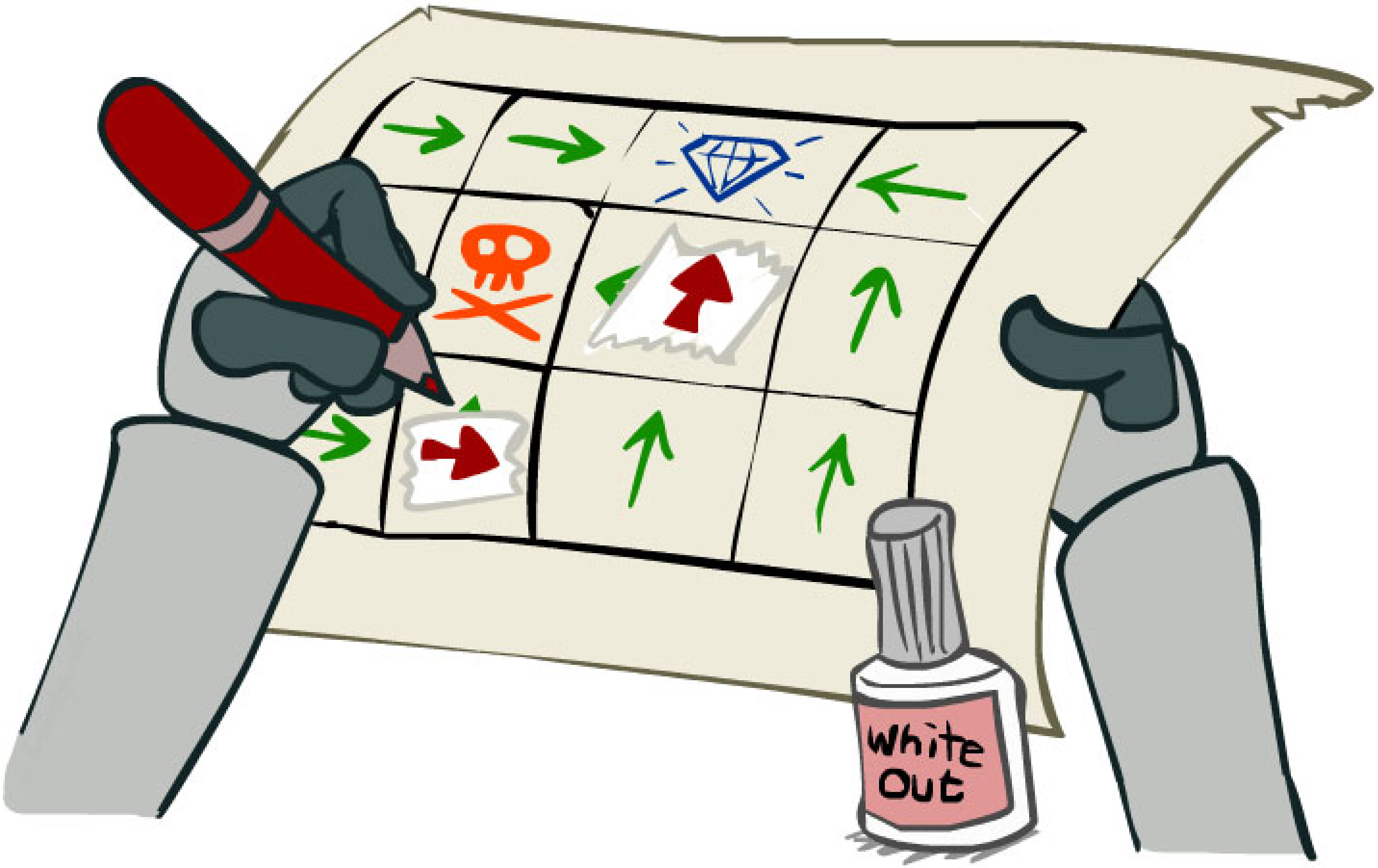
How should we act?

- Completely trivial to decide!

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

Important lesson: actions are easier to select from q-values than values!

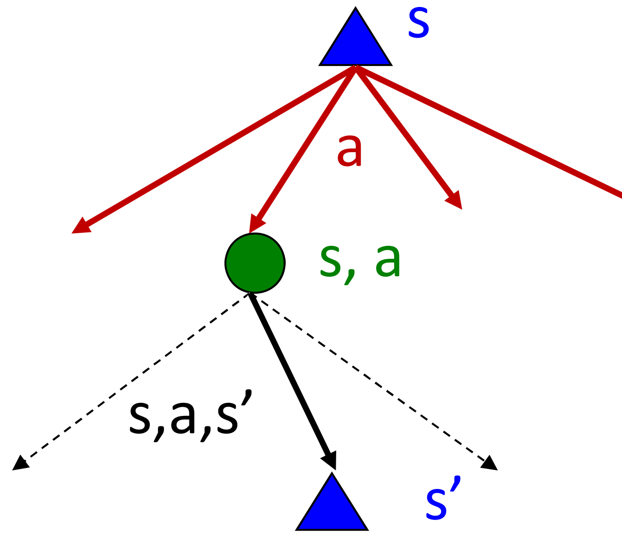
Policy Iteration



Problems with Value Iteration

Value iteration repeats the Bellman updates:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$



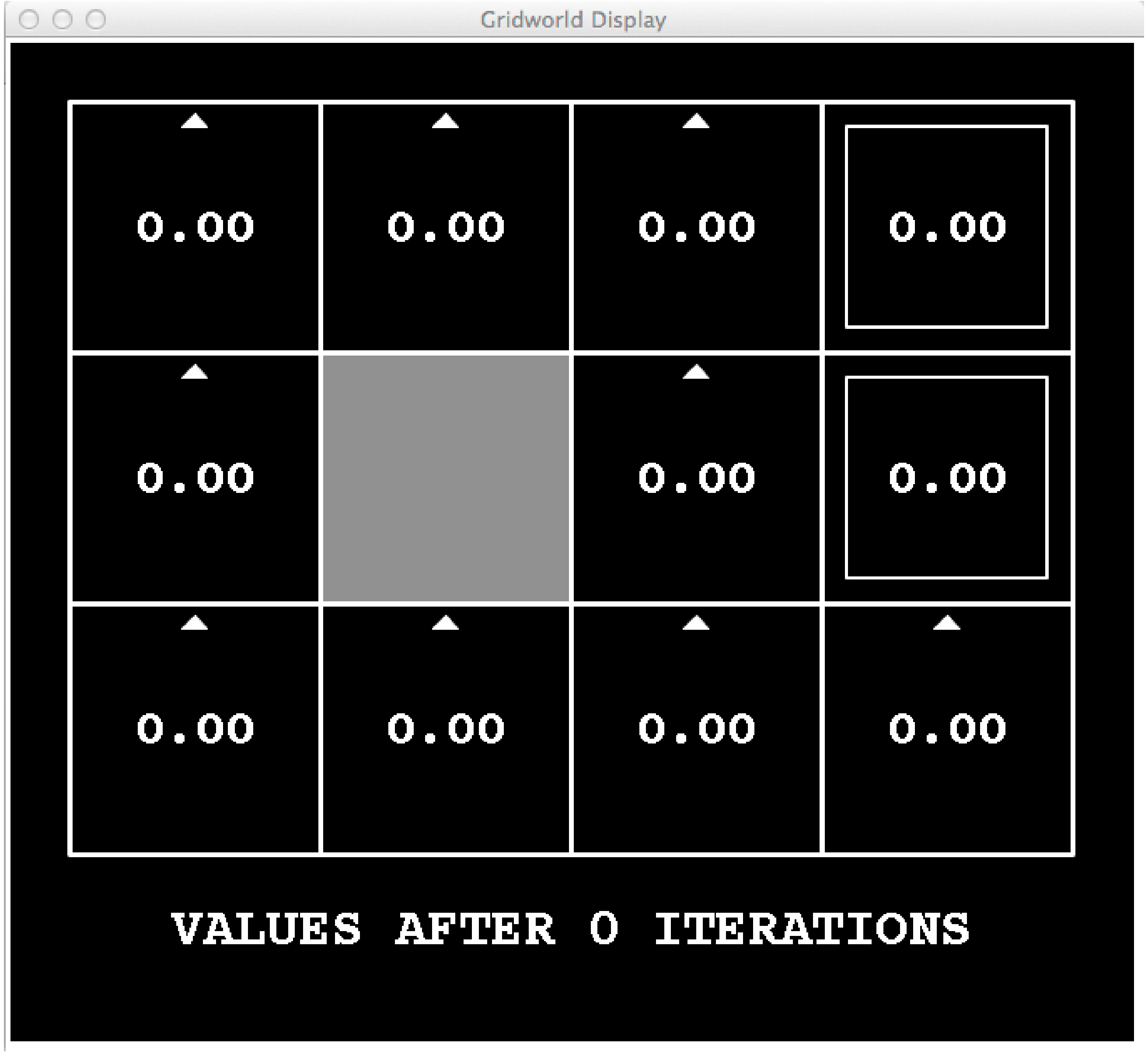
Problem 1: It's slow – $O(S^2A)$ per iteration

Problem 2: The "max" at each state rarely changes

Problem 3: The policy often converges long before the values

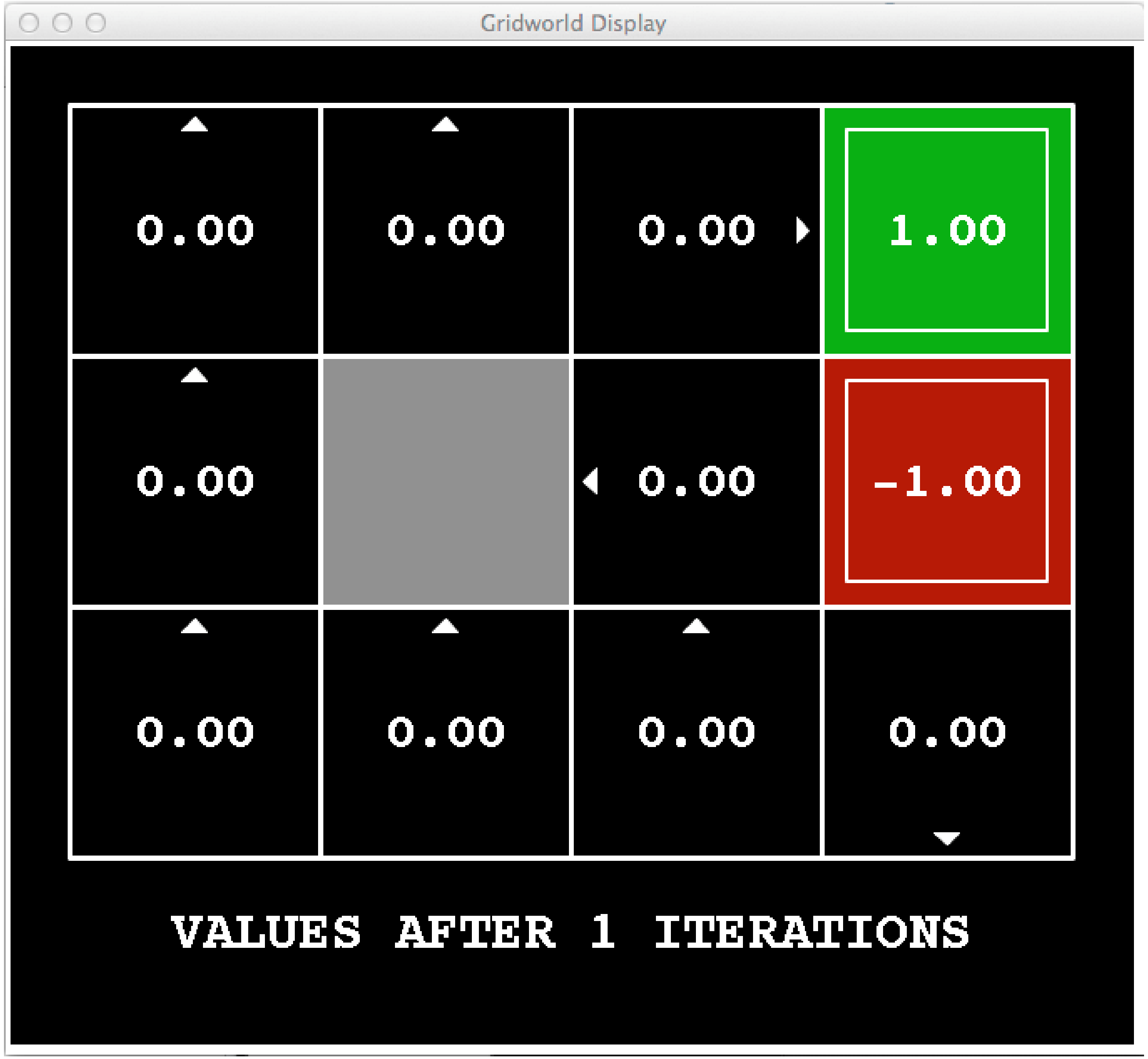
[Demo: value iteration (

k=0



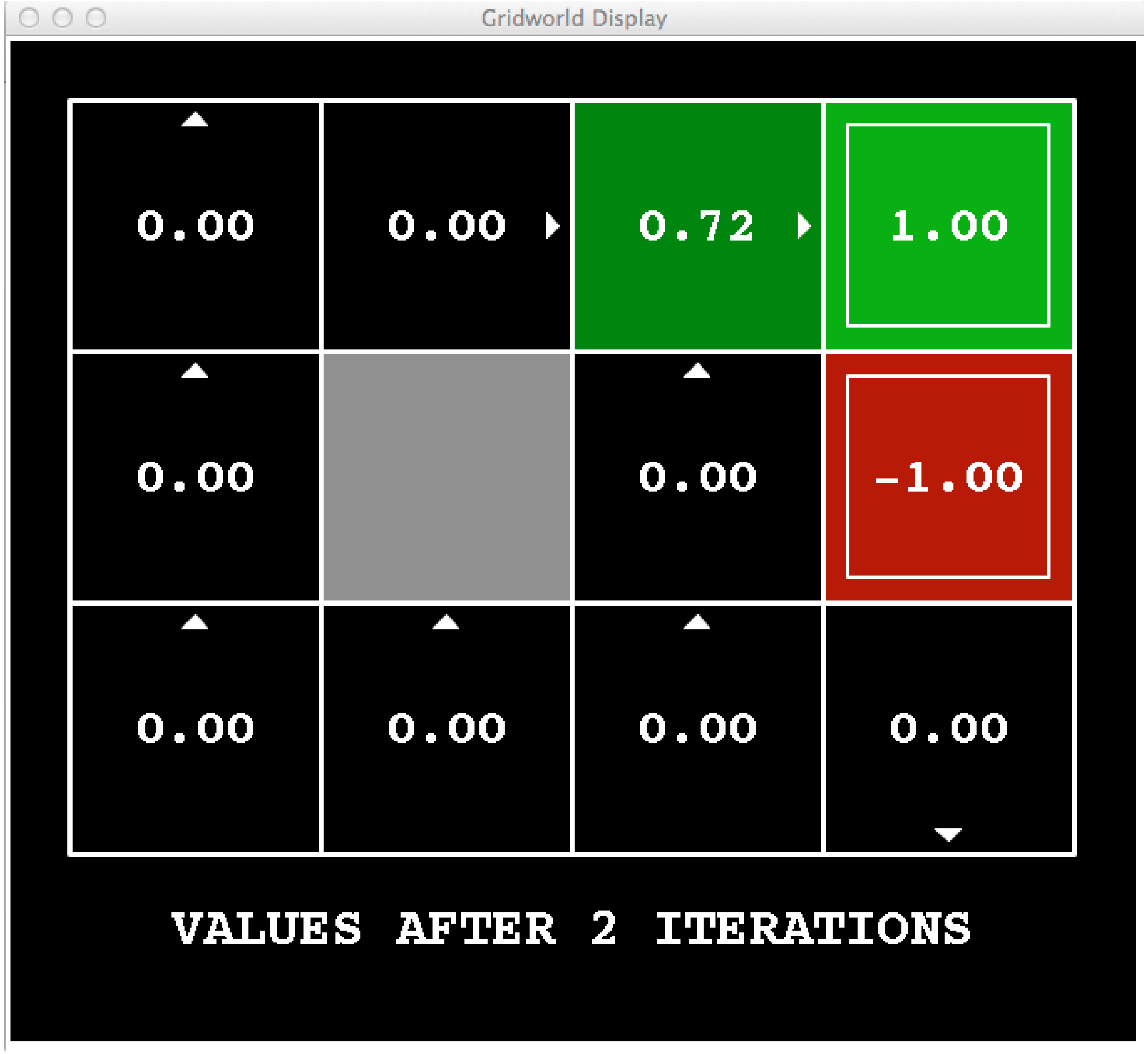
Noise = 0.2
Discount = 0.9
Living reward =

k=1



Noise = 0.2
Discount = 0.9
Living reward =

k=2

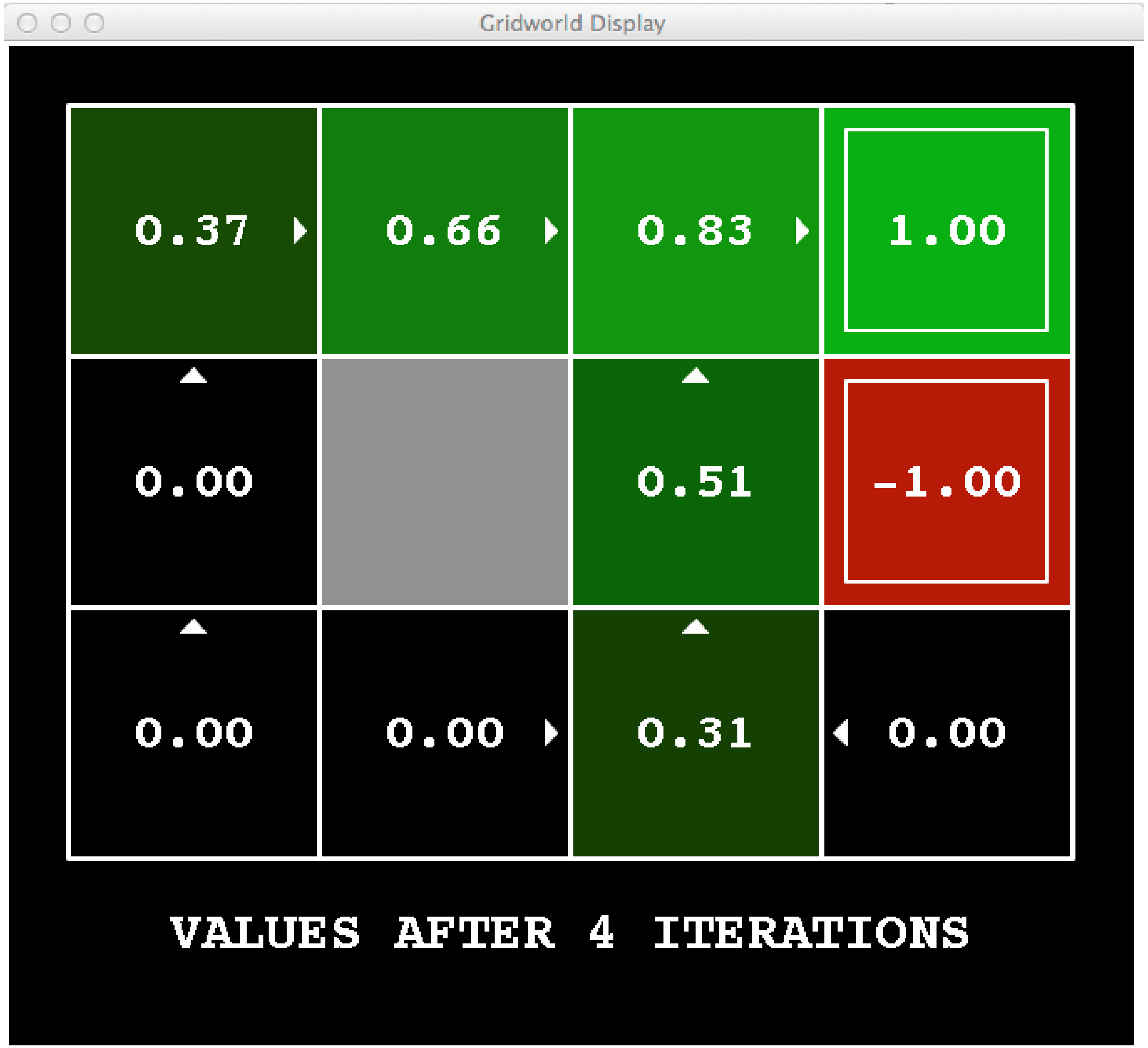


Noise = 0.2
Discount = 0.9
Living reward =

k=3

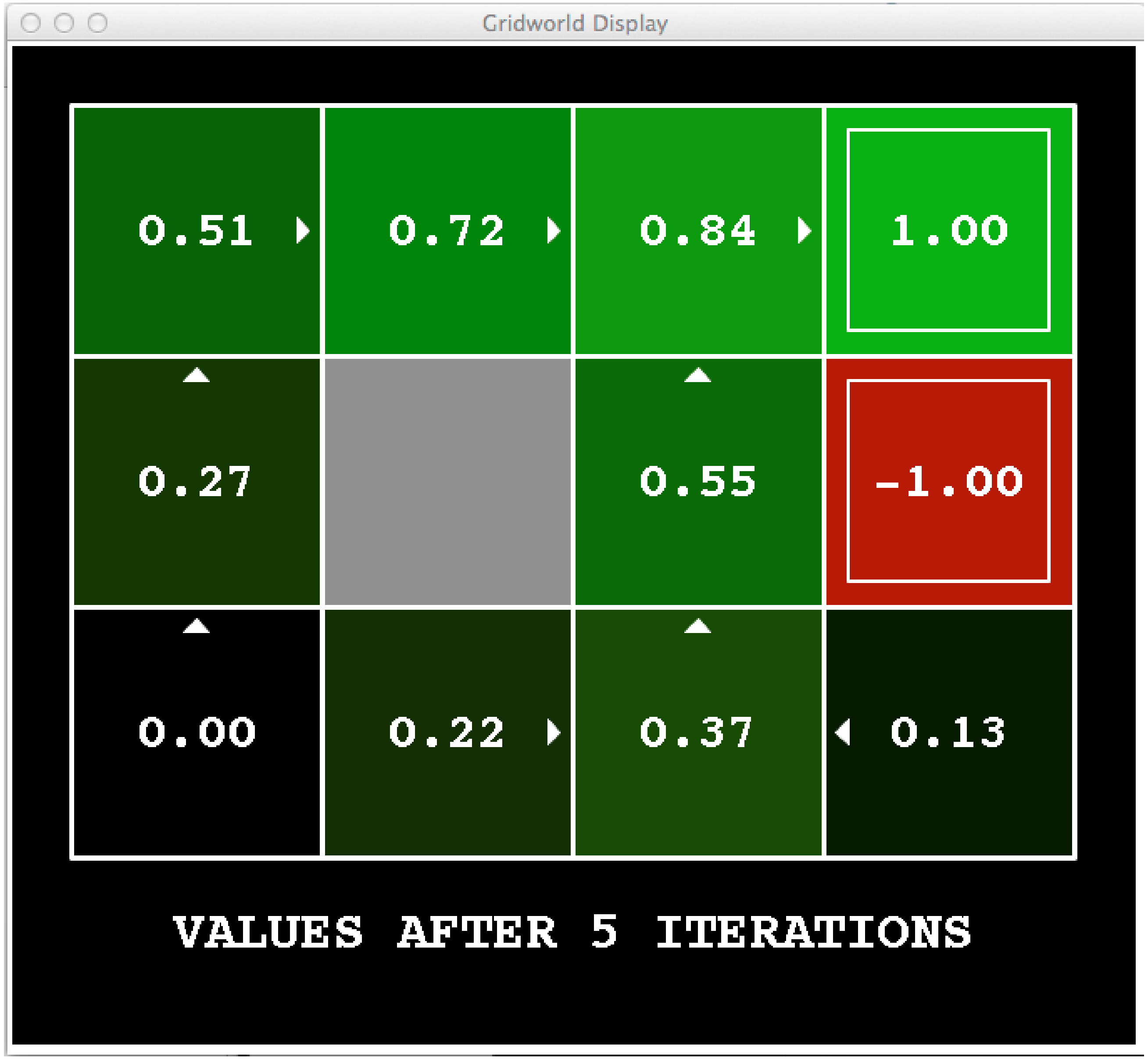
Noise = 0.2
Discount = 0.9
Living reward =

k=4



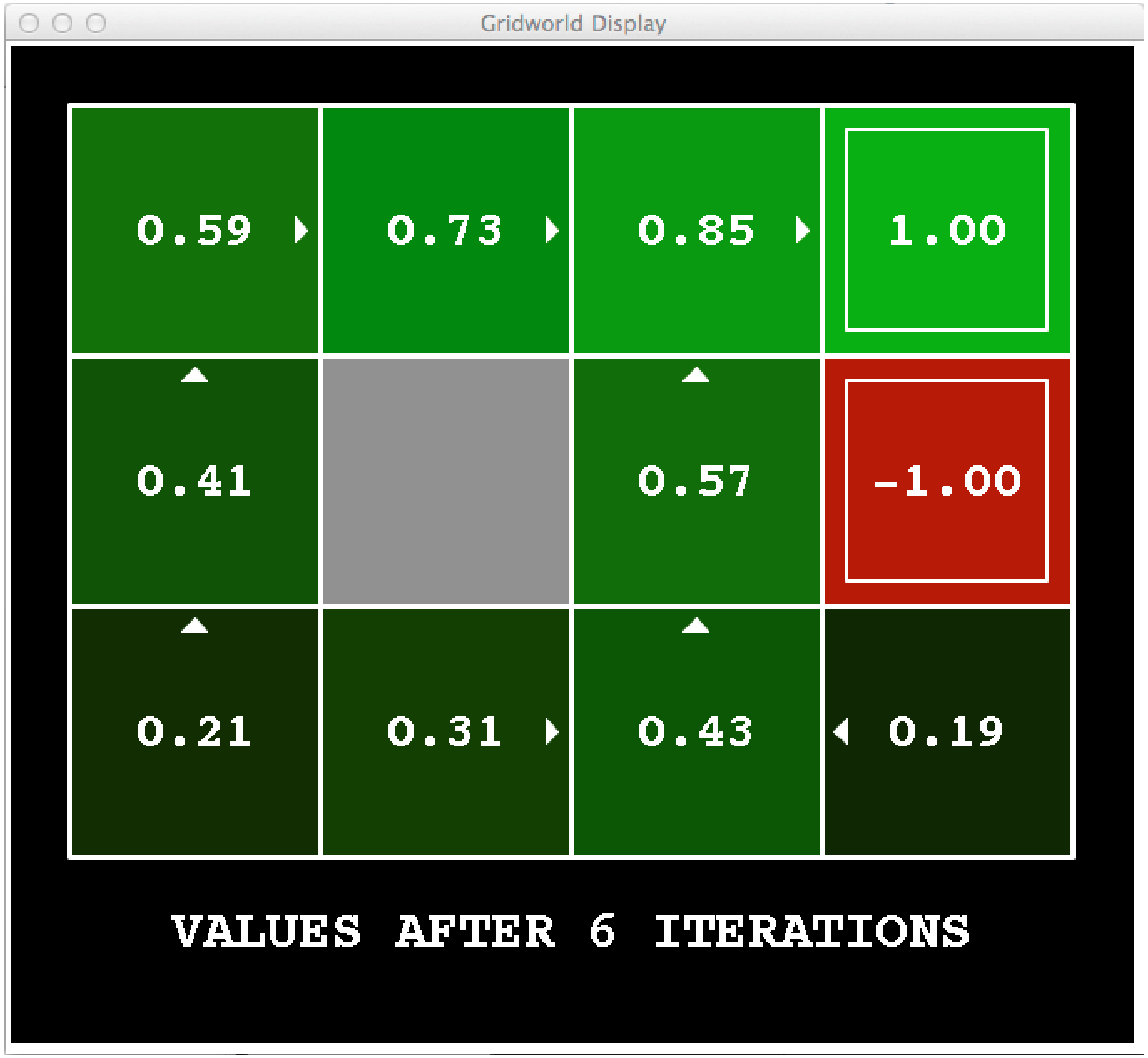
Noise = 0.2
Discount = 0.9
Living reward =

k=5



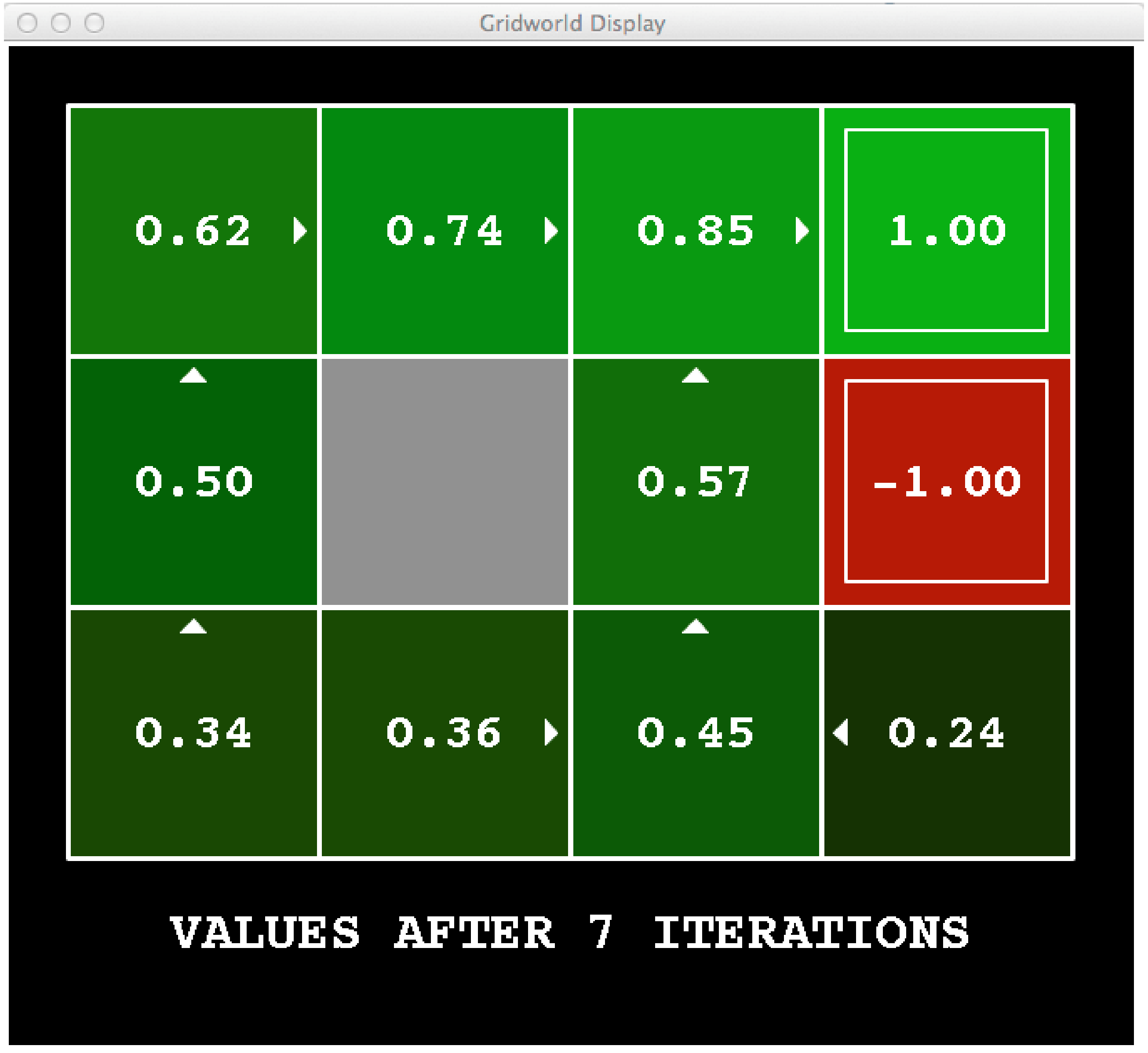
Noise = 0.2
Discount = 0.9
Living reward =

k=6



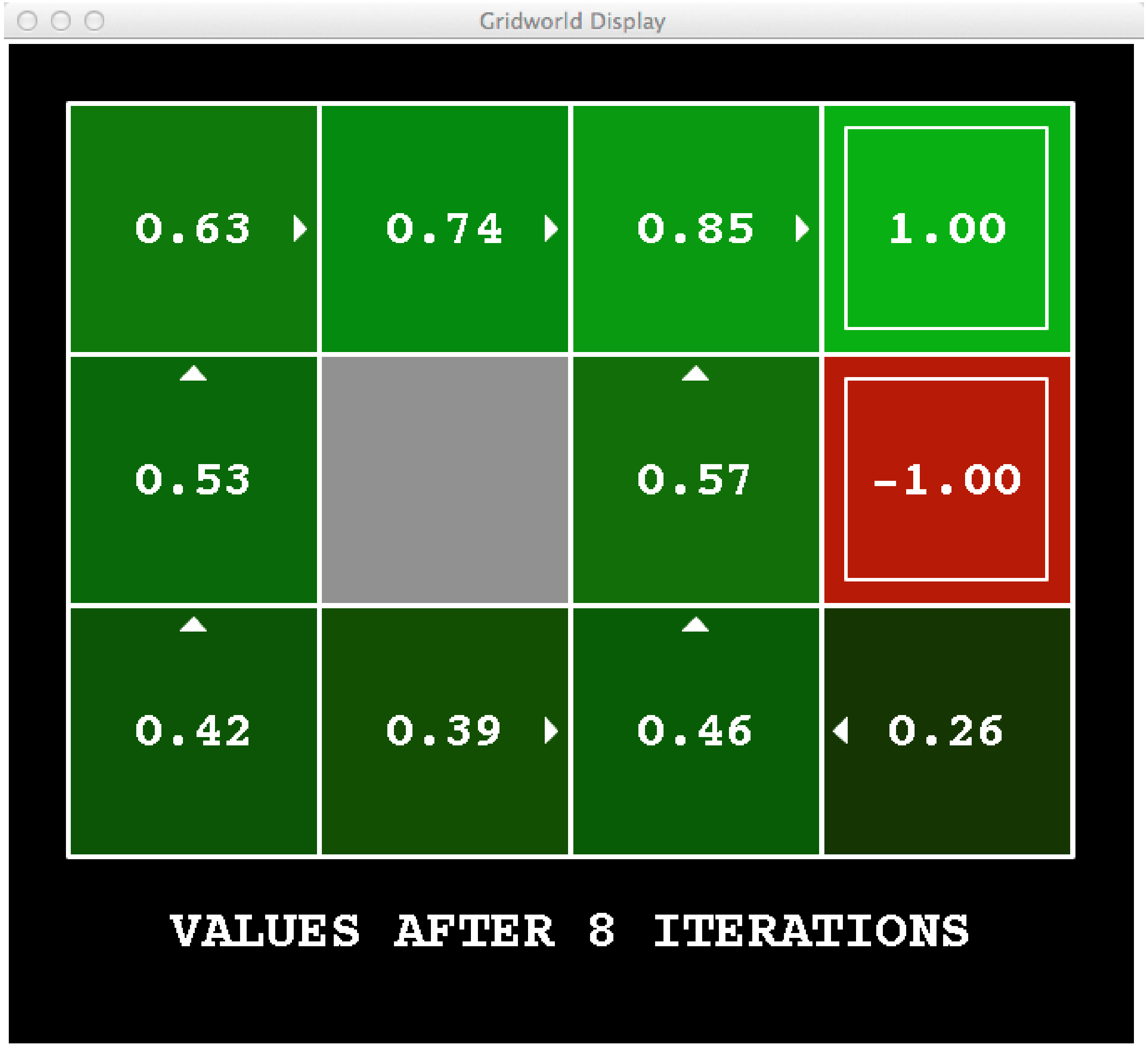
Noise = 0.2
Discount = 0.9
Living reward =

k=7



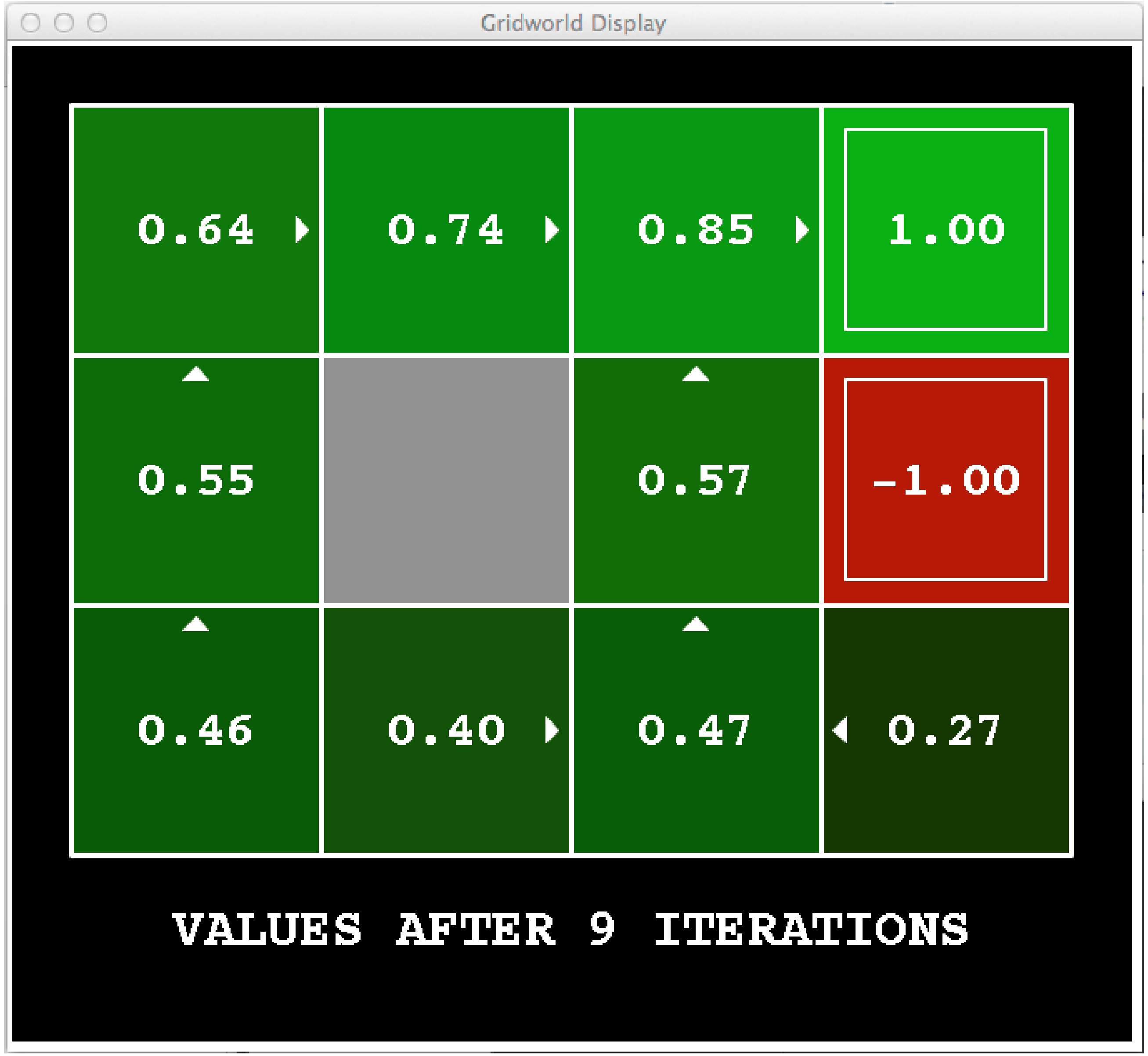
Noise = 0.2
Discount = 0.9
Living reward =

k=8



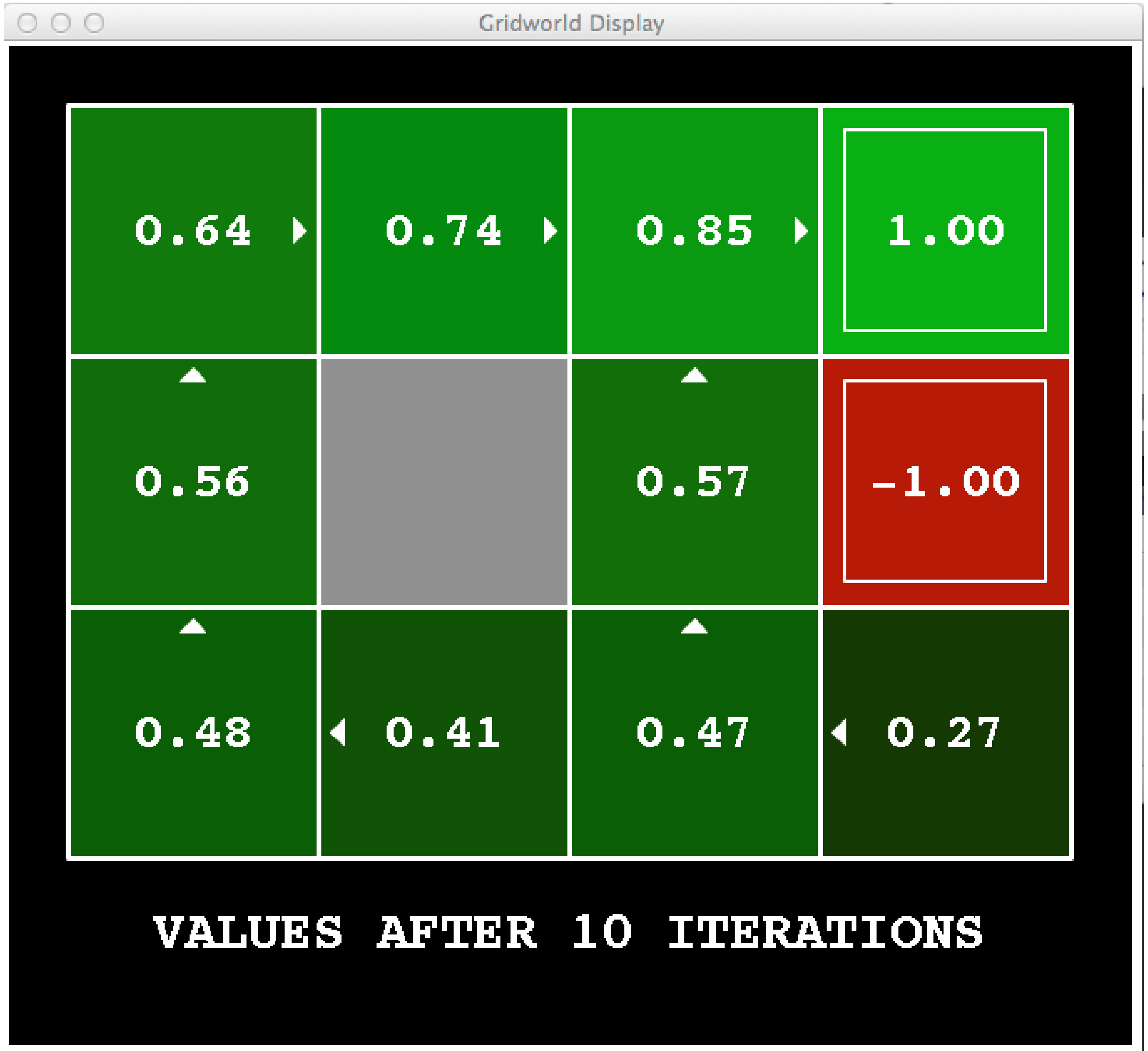
Noise = 0.2
Discount = 0.9
Living reward =

k=9



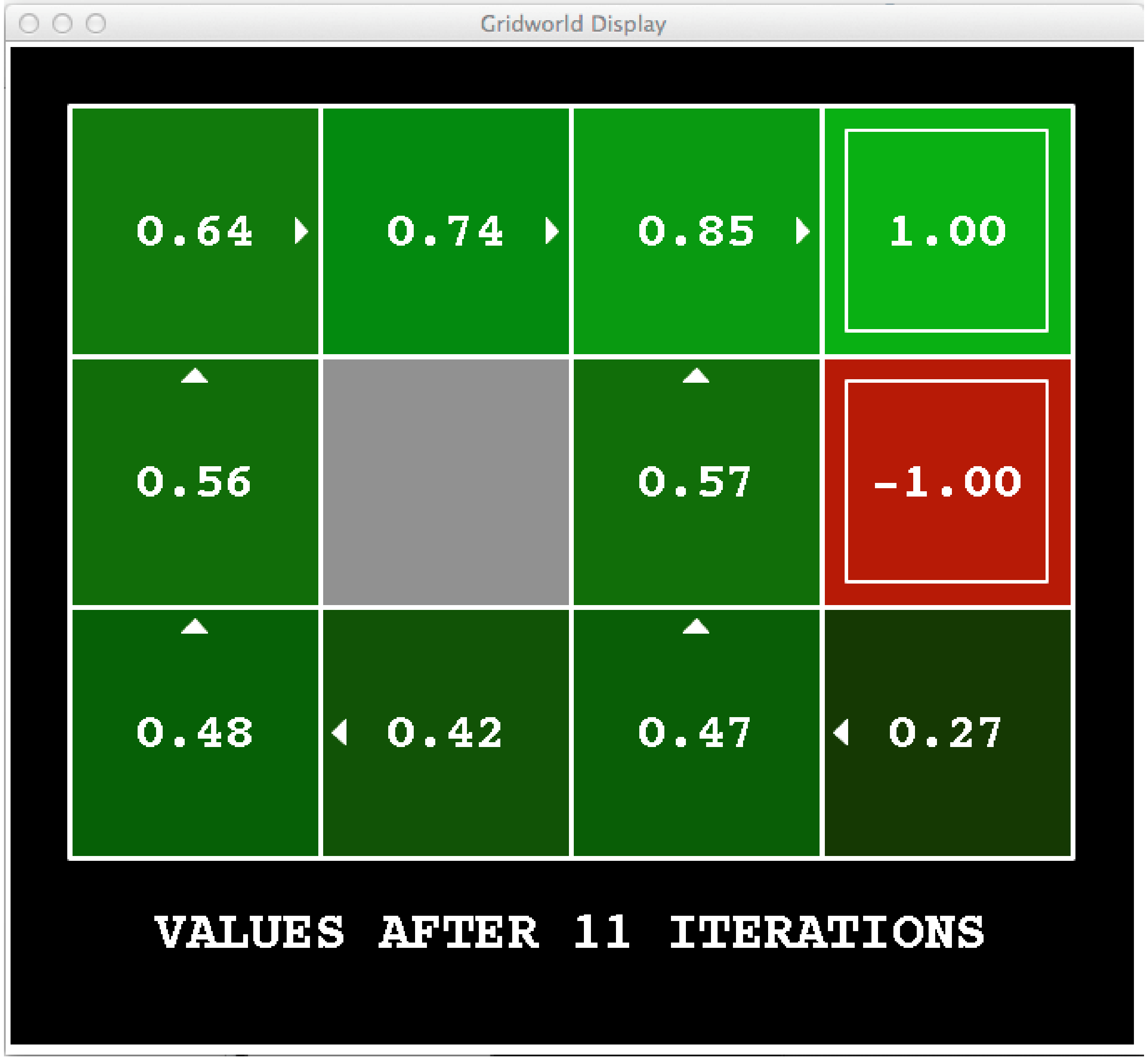
Noise = 0.2
Discount = 0.9
Living reward =

k=10



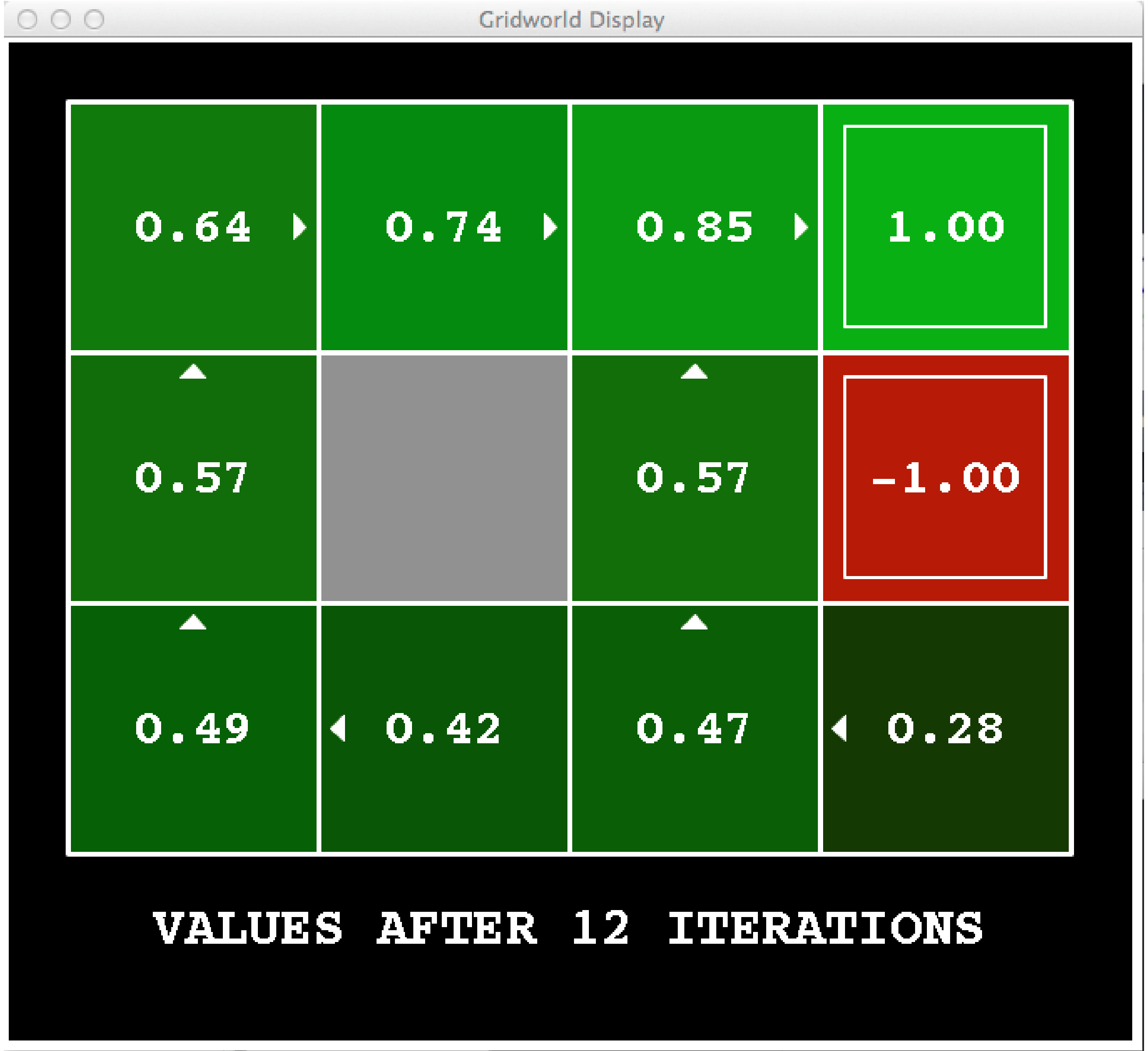
Noise = 0.2
Discount = 0.9
Living reward =

k=11



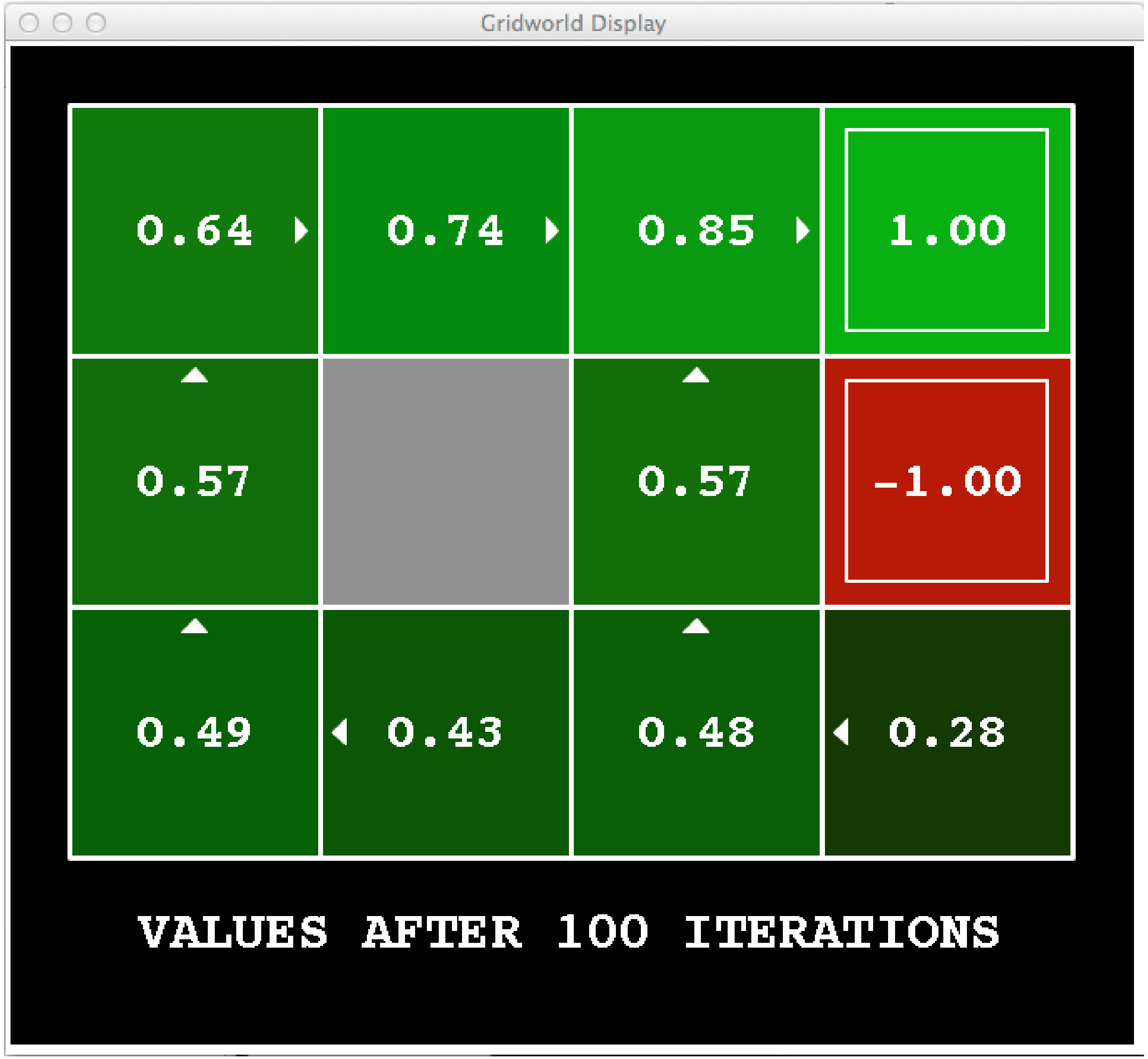
Noise = 0.2
Discount = 0.9
Living reward =

k=12



Noise = 0.2
Discount = 0.9
Living reward =

k=100



Noise = 0.2
Discount = 0.9
Living reward =

Policy Iteration

Alternative approach for optimal values:

- Step 1: Policy evaluation: calculate utilities for some fixed policy (not optimal utilities!) until convergence
- Step 2: Policy improvement: update policy using one-step look-ahead with resultin converged (but not optimal!) utilities as future values
- Repeat steps until policy converges

This is policy iteration

- It's still optimal!
- Can converge (much) faster under some conditions

Policy Iteration

Evaluation: For fixed current policy π, find values with policy evaluation:

- Iterate until values converge:

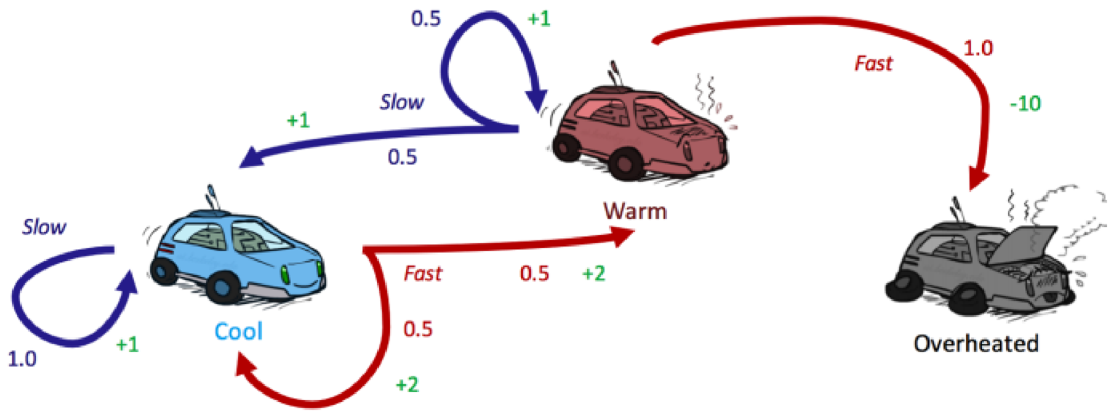
$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s') \left[R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')\right]$$

Improvement: For fixed values, get a better policy using policy extraction

- One-step look-ahead:

$$\pi_{i+1}(s) = \arg\max_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi_i}(s')\right]$$

Question



	cool	warm	overheated
π_0	*slow*	*slow*	–

$$\gamma = 0.5.$$

Comparison

Both value iteration and policy iteration compute the same thing (all optimal values)

In value iteration:

- Every iteration updates both the values and (implicitly) the policy
- We don't track the policy, but taking the max over actions implicitly recomputes it

In policy iteration:

- We do several passes that update utilities with fixed policy (each pass is fast because we consider only one action, not all of them)
- After the policy is evaluated, a new policy is chosen (slow like a value iteration pass)
- The new policy will be better (or we're done)

Both are dynamic programs for solving MDPs

Summary: MDP Algorithms

So you want to….

- Compute optimal values: use value iteration or policy iteration
- Compute values for a particular policy: use policy evaluation
- Turn your values into a policy: use policy extraction (one-step lookahead)

These all look the same!

- They basically are – they are all variations of Bellman updates
- They all use one-step lookahead expectimax fragments
- They differ only in whether we plug in a fixed policy or max over actions