Computational Data Mining

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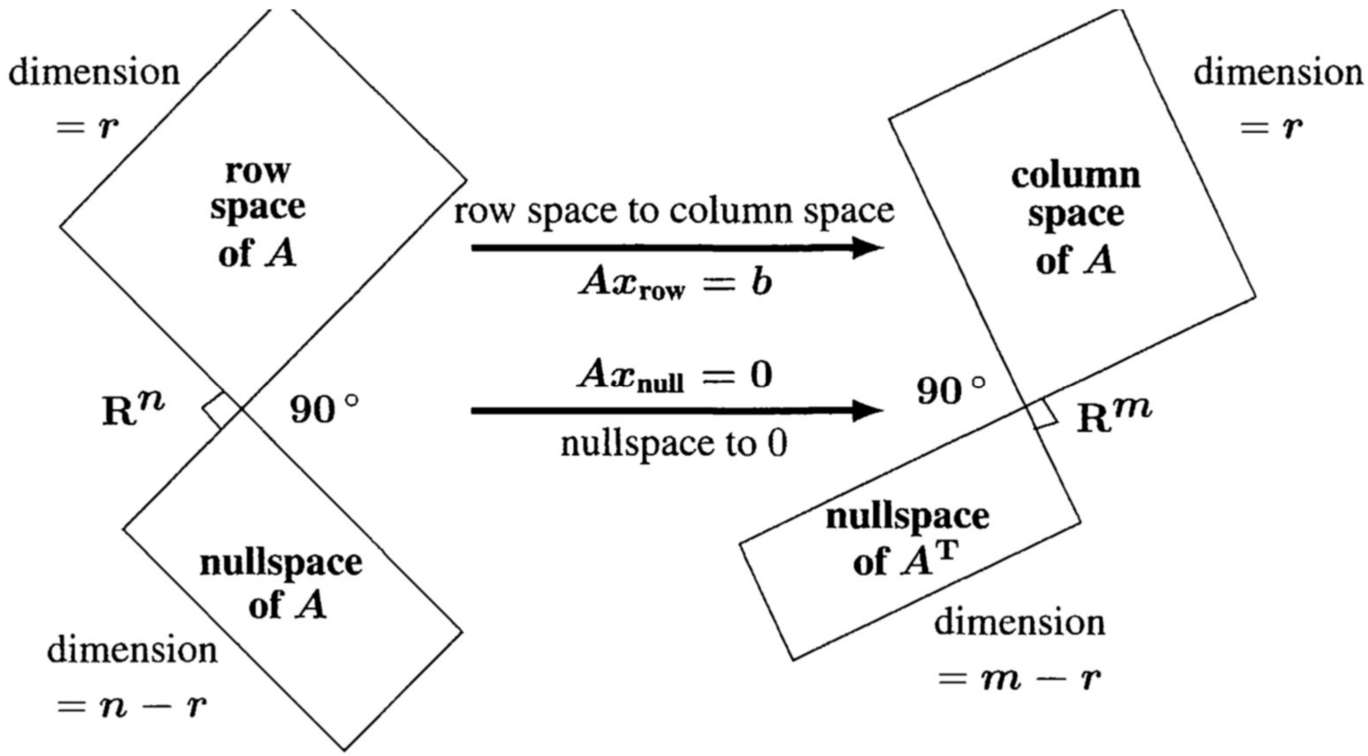
Orthogonal subspaces



Example 1 $A = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{6} \end{bmatrix} = \boldsymbol{u}\boldsymbol{v}^{\mathrm{T}}$ has $m = 2$ and $n = 2$. We have subspaces of $\mathbf{R}^2$.

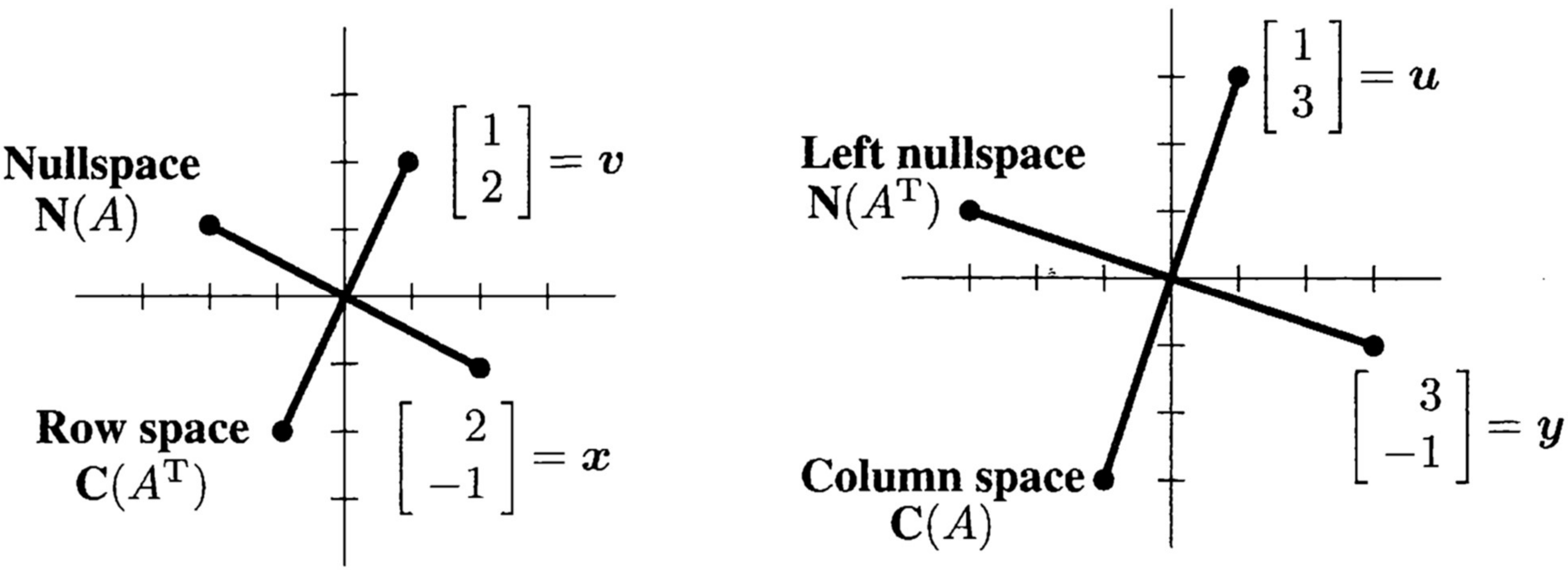
The column space $\mathbf{C}(A)$ line through $\boldsymbol{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

The row space $\mathbf{C}(A^{\mathrm{T}})$ line through $\boldsymbol{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

The nullspace $\mathbf{N}(A)$ line through $\boldsymbol{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

The left nullspace $\mathbf{N}(A^{\mathrm{T}})$ line through $\boldsymbol{y} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

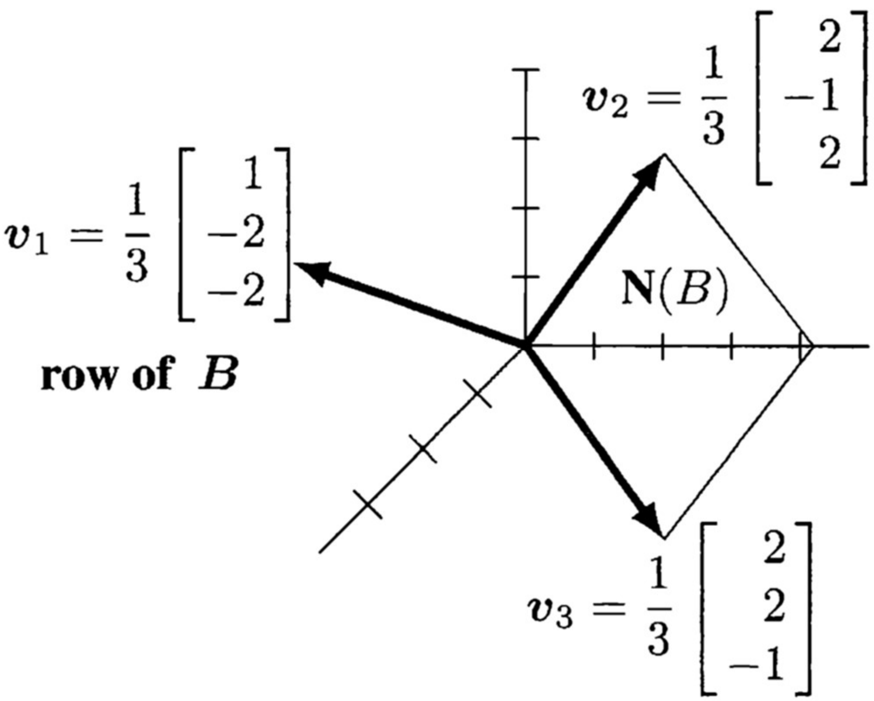
4 sub space



Nullspace
$\mathbf{N}(A)$
$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \boldsymbol{v}$
Row space
$\mathbf{C}(A^{\mathrm{T}})$
$\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \boldsymbol{x}$
$\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \boldsymbol{u}$
Left nullspace
$\mathbf{N}(A^{\mathrm{T}})$
$\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \boldsymbol{y}$
Column space
$\mathbf{C}(A)$

Example 2 $B = \begin{bmatrix} 1 & -2 & -2 \\ 3 & -6 & -6 \end{bmatrix}$ has $m = 2$ and $n = 3$.

$$\boldsymbol{x}_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \textbf{ and } \boldsymbol{x}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$



Row space = infinite line through $\boldsymbol{v}_1$

Nullspace = infinite plane of $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$

$n = 3$ columns of B

$r = 1$ independent column

$$\begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \boldsymbol{v}_3 \end{bmatrix} = \begin{matrix}\text{orthonormal} \\ \text{basis for } \mathbf{R}^3\end{matrix}$$

Some definition

- Row-echelon form :
  - all its non-zero rows have an entry such that all the entries to its left and below it are equal to zero
  - the first nonzero entry of each row is called a pivot
  - columns in which pivots appear are called pivot columns

$$\begin{bmatrix} 1 & a_0 & a_1 & a_2 & a_3 \\ 0 & 0 & 2 & a_4 & a_5 \\ 0 & 0 & 0 & 1 & a_6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Reduced row echelon form
  - row canonical form
  - leading entry in each nonzero row is 1
  - Each column containing a leading 1 has zeros in all its other entries

$$\begin{bmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{bmatrix}$$

Compute Bx = 0

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 8 & 11 & 14 \\ 1 & 3 & 5 & 8 & 11 \\ 4 & 10 & 16 & 23 & 30 \end{pmatrix},$$

$$B_{\text{rref}} = \begin{pmatrix} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\mathbf{x} = x_3 \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -1 \\ 2 \\ 0 \\ -2 \\ 1 \end{pmatrix}.$$

$$\begin{aligned} x_1 &= x_3 - x_5, \\ x_2 &= -2x_3 + 2x_5, \\ x_4 &= -2x_5. \end{aligned}$$

Zero eigen value

- By definition, we have $Ax=\lambda x$
- In the special case when A have dependent column
  - if $\lambda=0$ it becomes $Ax=0$, have solution other than $x = 0$
  - null space of A is the space that is spanned by the eigenvectors of 0 eigenvalue.

Question

- Show that each vector in null space of A is perpendicular to each vector in row space of A