Chapter 2: Intro to Relational Model

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Database System Concepts, 7th Ed.

Course material repository

- https://github.com/uisf-course/DB

Outline

- Structure of Relational Databases
- Database Schema
- Keys
- Schema Diagrams
- Relational Query Languages
- The Relational Algebra

Relational model

- There is a close correspondence between the concept of *table* and the mathematical concept of *relation*
- In mathematical terminology, a *tuple* is simply a sequence (or list) of values
- A relationship between *n* values is represented mathematically by an *n-tuple* of values, which corresponds to a row in a table
- in the relational model the term relation is used to refer to a table, while the term tuple is used to refer to a row. Similarly, the term attribute refers to a column of a table.

Example of a *Instructor* Relation

attributes
(or columns)

ID	*name*	*dept_name*	*salary*
10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

tuples
(or rows)

Example of a *Course* Relation

course_id	*title*	*dept_name*	*credits*
BIO-101	Intro. to Biology	Biology	4
BIO-301	Genetics	Biology	4
BIO-399	Computational Biology	Biology	3
CS-101	Intro. to Computer Science	Comp. Sci.	4
CS-190	Game Design	Comp. Sci.	4
CS-315	Robotics	Comp. Sci.	3
CS-319	Image Processing	Comp. Sci.	3
CS-347	Database System Concepts	Comp. Sci.	3
EE-181	Intro. to Digital Systems	Elec. Eng.	3
FIN-201	Investment Banking	Finance	3
HIS-351	World History	History	3
MU-199	Music Video Production	Music	3
PHY-101	Physical Principles	Physics	4

Example of a *prereq* Relation

course_id	*prereq_id*
BIO-301	BIO-101
BIO-399	BIO-101
CS-190	CS-101
CS-315	CS-101
CS-319	CS-101
CS-347	CS-101
EE-181	PHY-101

Example of a *Department* Relation

dept_name	*building*	*budget*
Biology	Watson	90000
Comp. Sci.	Taylor	100000
Elec. Eng.	Taylor	85000
Finance	Painter	120000
History	Painter	50000
Music	Packard	80000
Physics	Watson	70000

Relation Schema and Instance

- $A_1, A_2, \ldots, A_n$ are *attributes*
- $R = (A_1, A_2, \ldots, A_n)$ is a *relation schema*

  Example:

  instructor = (*ID, name, dept_name, salary*)
- A relation instance *r* defined over schema *R* is denoted by *r* (*R*).
- The current values a relation are specified by a table
- An element ***t*** of relation ***r*** is called a *tuple* and is represented by a *row* in a table

Attributes

- The set of allowed values for each attribute is called the **domain** of the attribute
- Attribute values are (normally) required to be **atomic**; that is, indivisible
  - For e.g. multiple phone-number for an attribute is not atomic
  - For e.g. if phone number consist of country-code, …
- The special value ***null*** is a member of every domain. Indicated that the value is “unknown”
- The null value causes complications in the definition of many operations

Relations are Unordered

- Order of tuples is irrelevant (tuples may be stored in an arbitrary order)
- Example: *instructor* relation with unordered tuples

ID	*name*	*dept_name*	*salary*
22222	Einstein	Physics	95000
12121	Wu	Finance	90000
32343	El Said	History	60000
45565	Katz	Comp. Sci.	75000
98345	Kim	Elec. Eng.	80000
76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
58583	Califieri	History	62000
83821	Brandt	Comp. Sci.	92000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
76543	Singh	Finance	80000

Database Schema

- Database schema -- is the logical structure of the database.
- Database instance -- is a snapshot of the data in the database at a given instant in time.
- Example:
  - schema: *instructor* (*ID, name, dept_name, salary*)
  - Instance:

ID	*name*	*dept_name*	*salary*
22222	Einstein	Physics	95000
12121	Wu	Finance	90000
32343	El Said	History	60000
45565	Katz	Comp. Sci.	75000
98345	Kim	Elec. Eng.	80000
76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
58583	Califieri	History	62000
83821	Brandt	Comp. Sci.	92000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
76543	Singh	Finance	80000

Duplication is not coincidence

- attribute *dept_name* appears in both the *instructor* schema and the
- *department* schema
- using common attributes in relation schemas is one way of relating tuples of distinct relations
- For e.g. : we wish to find the information about all the instructors who work in the Watson building

Keys

- How to distinguish the tuples in a relation?
- The values of the attribute values of a tuple must be such that they can *uniquely identify* the tuple.
  - no two tuples in a relation are allowed to have exactly the same value for all attributes
- A superkey is a set of one or more attributes that, taken collectively, allow us to identify uniquely a tuple in the relation
  - The {ID} attribute of the relation *instructor ?*
  - The {name} attribute of *instructor* ?
  - What about {ID, name}
- *Formal :*
  - *R* denote the set of attributes in the schema of relation *r*
  - a subset *K* of *R* is a *superkey* for *r*
  - That is, if t_1 and t_2 are in *r* and $t_1 \neq t_2$, then $t_1.K \neq t_2.K$.

Keys

- If *K* is a superkey, then so is any superset of *K*.
- Superkey *K* is a **candidate key** if *K* is minimal
  - no proper subset is a superkey
    Example: {*ID*} is a candidate key for *Instructor*
- One of the candidate keys is selected to be the **primary key**.
  - Which one?
  - Primary key attributes are also underlined.
  - Primary key constraint

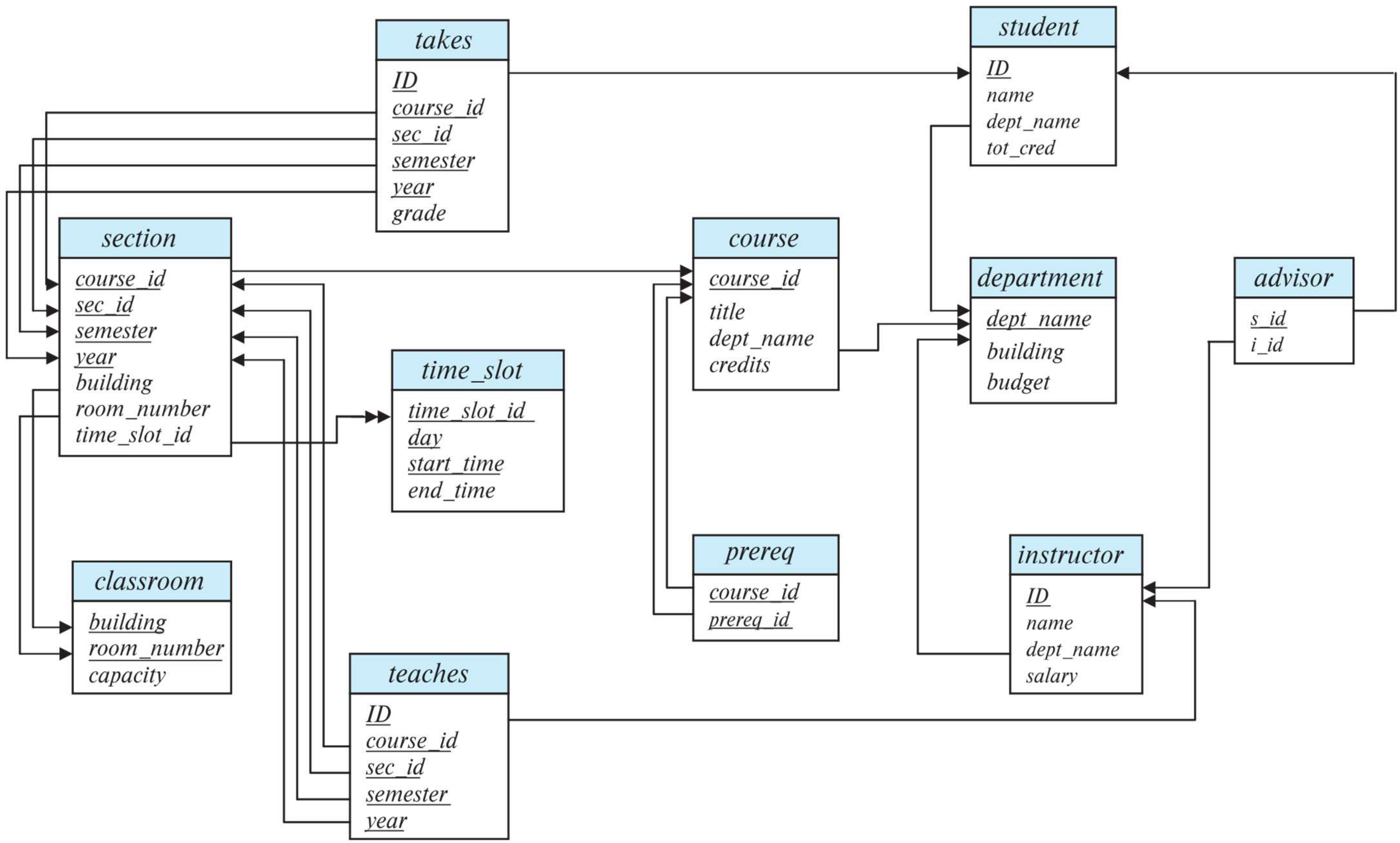
Keys

- **Foreign key** constraint: Value in one relation must appear in another
  - **Referencing** relation
  - **Referenced** relation
  - Example: *dept_name* in i*nstructor* is a foreign key from *instructor* referencing *department*
    - in any database instance, given any tuple, say t_a, from the *instructor* relation, there must be some tuple, say t_b, in the *department* relation such that the value of the *dept_name* attribute of t_a is the same as the value of the primary key, *dept_name*, of t_b.
- in a foreign-key constraint, the referenced attribute(s) must be the primary key of the referenced relation.A
- referential integrity constraint :
  - requires that the values appearing in specified attributes of any tuple in the referencing relation also appear in specified attributes of at least one tuple in the referenced relation.
  - as an example, consider the values in the *time_slot _id* attribute
  - foreign-key constraints are a *special case* of referential integrity constraints

Schema of university database

- course (course_id, title, dept_name, credits)
- student(ID, name, dept_name, tot_cred)
- department(dept_name, building, budget)
- instructor(ID, name, dept_name, salary)
- section(course_id, sec_id, semester, year, building, room number, time_slot_id)
- teaches(ID, course_id, sec_id, semester, year)
- takes(ID, course_id, sec_id, semester, year, grade)
- classroom(building, room number, capacity)
- advisor(s_ID, i_ID)
- time_slot(time_slot_id, day, start_time, end_ime)
- prereq(course_id, prereq_id)

Schema Diagram for University Database



Relational Algebra

- The relational algebra forms the theoretical basis of the SQL query language.
- consisting of a set of operations that take one or two relations as input and produce a new relation as their result.
- operators
  - select: σ
  - project: $\prod$
  - union: $\cup$
  - Set-Intersection : $\cap$
  - set difference: –
  - Cartesian product: x
  - rename: ρ
- select, project, and rename operations, are called *unary* operations
- union, Cartesian product, and set difference, operate on pairs of relations and are, therefore, called *binary* operations

Select Operation

- The **select** operation selects tuples that satisfy a given predicate.
- Notation: $\sigma_p(r)$
- *p* is called the **selection predicate**
- Example: select those tuples of the *instructor* relation where the instructor is in the "Physics" department.
  - Query

$$\sigma_{dept\_name=\text{"Physics"}}(instructor)$$

  - Result

ID	*name*	*dept_name*	*salary*
22222	Einstein	Physics	95000
33456	Gold	Physics	87000

Instructor Relation

ID	*name*	*dept_name*	*salary*
22222	Einstein	Physics	95000
12121	Wu	Finance	90000
32343	El Said	History	60000
45565	Katz	Comp. Sci.	75000
98345	Kim	Elec. Eng.	80000
76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
58583	Califieri	History	62000
83821	Brandt	Comp. Sci.	92000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
76543	Singh	Finance	80000

Select Operation (Cont.)

- We allow comparisons using

  $=, \neq, >, \geq. <. \leq$

  in the selection predicate.
- We can combine several predicates into a larger predicate by using the connectives:

  $\wedge$ (**and**), $\vee$ (**or**), $\neg$ (**not**)
- Example: Find the instructors in Physics with a salary greater $90,000, we write:

  $$\sigma_{dept\_name=\text{“Physics”} \wedge salary > 90{,}000}(instructor)$$

- The select predicate may include comparisons between two attributes.
  - Example, find all departments whose name is the same as their building name:
  - $\sigma_{dept\_name=building}(department)$

Department Relation

dept_name	*building*	*budget*
Biology	Watson	90000
Comp. Sci.	Taylor	100000
Elec. Eng.	Taylor	85000
Finance	Painter	120000
History	Painter	50000
Music	Packard	80000
Physics	Watson	70000

Project Operation

- A unary operation that returns its argument relation, with certain attributes left out.
- Notation:

$$\Pi_{A_1, A_2, A_3 \ldots . A_k}(r)$$

  where A_1, A_2, …, A_k are attribute names and r is a relation name.
- The result is defined as the relation of k columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets

Project Operation Example

- Example: eliminate the *dept_name* attribute of *instructor*
- Query*:*

$$\Pi_{ID,\ name,\ salary}\ (instructor)$$

- Result:

ID	*name*	*salary*
10101	Srinivasan	65000
12121	Wu	90000
15151	Mozart	40000
22222	Einstein	95000
32343	El Said	60000
33456	Gold	87000
45565	Katz	75000
58583	Califieri	62000
76543	Singh	80000
76766	Crick	72000
83821	Brandt	92000
98345	Kim	80000

Composition of Relational Operations

- The result of a relational-algebra operation is relation and therefore of relational-algebra operations can be composed together into a **relational-algebra expression**.
- Consider the query -- Find the names of all instructors in the Physics department.

$$\Pi_{name}(\sigma_{\ dept\_name\ =\text{“}Physics\text{”}}\ (instructor))$$

- Instead of giving the name of a relation as the argument of the projection operation, we give an expression that evaluates to a relation.

Cartesian-Product Operation

- The Cartesian-product operation (denoted by X) allows us to combine information from any two relations.
- Example: the Cartesian product of the relations *instructor* and *teaches* is written as:

  instructor X *teaches*

- We construct a tuple of the result out of each possible pair of tuples: one from the *instructor* relation and one from the *teaches* relation (see next slide)
- Since the instructor *ID* appears in both relations we distinguish between these attribute by attaching to the attribute the name of the relation from which the attribute originally came.
  - *instructor.ID*
  - *teaches.ID*

The *instructor* x *teaches* table

instructor.ID	*name*	*dept_name*	*salary*	*teaches.ID*	*course_id*	*sec_id*	*semester*	*year*
10101	Srinivasan	Comp. Sci.	65000	10101	CS-101	1	Fall	2017
10101	Srinivasan	Comp. Sci.	65000	10101	CS-315	1	Spring	2018
10101	Srinivasan	Comp. Sci.	65000	10101	CS-347	1	Fall	2017
10101	Srinivasan	Comp. Sci.	65000	12121	FIN-201	1	Spring	2018
10101	Srinivasan	Comp. Sci.	65000	15151	MU-199	1	Spring	2018
10101	Srinivasan	Comp. Sci.	65000	22222	PHY-101	1	Fall	2017
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
12121	Wu	Finance	90000	10101	CS-101	1	Fall	2017
12121	Wu	Finance	90000	10101	CS-315	1	Spring	2018
12121	Wu	Finance	90000	10101	CS-347	1	Fall	2017
12121	Wu	Finance	90000	12121	FIN-201	1	Spring	2018
12121	Wu	Finance	90000	15151	MU-199	1	Spring	2018
12121	Wu	Finance	90000	22222	PHY-101	1	Fall	2017
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
15151	Mozart	Music	40000	10101	CS-101	1	Fall	2017
15151	Mozart	Music	40000	10101	CS-315	1	Spring	2018
15151	Mozart	Music	40000	10101	CS-347	1	Fall	2017
15151	Mozart	Music	40000	12121	FIN-201	1	Spring	2018
15151	Mozart	Music	40000	15151	MU-199	1	Spring	2018
15151	Mozart	Music	40000	22222	PHY-101	1	Fall	2017
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
22222	Einstein	Physics	95000	10101	CS-101	1	Fall	2017
22222	Einstein	Physics	95000	10101	CS-315	1	Spring	2018
22222	Einstein	Physics	95000	10101	CS-347	1	Fall	2017
22222	Einstein	Physics	95000	12121	FIN-201	1	Spring	2018
22222	Einstein	Physics	95000	15151	MU-199	1	Spring	2018
22222	Einstein	Physics	95000	22222	PHY-101	1	Fall	2017
...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...

Join Operation

- The Cartesian-Product

  instructor X *teaches*

  associates every tuple of instructor with every tuple of teaches.

  - Most of the resulting rows have information about instructors who did NOT teach a particular course.

- To get only those tuples of "*instructor* X *teaches* " that pertain to instructors and the courses that they taught, we write:

  $\sigma_{\,instructor.id = \, teaches.id} \; (instructor \; \text{x} \; teaches \,))$

  - We get only those tuples of "*instructor* X *teaches"* that pertain to instructors and the courses that they taught.

- The result of this expression, shown in the next slide

Join Operation (Cont.)

- The table corresponding to:

$$\sigma_{\,instructor.id = \ teaches.id}\,(instructor \ \times \ teaches))$$

instructor.ID	*name*	*dept_name*	*salary*	*teaches.ID*	*course_id*	*sec_id*	*semester*	*year*
10101	Srinivasan	Comp. Sci.	65000	10101	CS-101	1	Fall	2017
10101	Srinivasan	Comp. Sci.	65000	10101	CS-315	1	Spring	2018
10101	Srinivasan	Comp. Sci.	65000	10101	CS-347	1	Fall	2017
12121	Wu	Finance	90000	12121	FIN-201	1	Spring	2018
15151	Mozart	Music	40000	15151	MU-199	1	Spring	2018
22222	Einstein	Physics	95000	22222	PHY-101	1	Fall	2017
32343	El Said	History	60000	32343	HIS-351	1	Spring	2018
45565	Katz	Comp. Sci.	75000	45565	CS-101	1	Spring	2018
45565	Katz	Comp. Sci.	75000	45565	CS-319	1	Spring	2018
76766	Crick	Biology	72000	76766	BIO-101	1	Summer	2017
76766	Crick	Biology	72000	76766	BIO-301	1	Summer	2018
83821	Brandt	Comp. Sci.	92000	83821	CS-190	1	Spring	2017
83821	Brandt	Comp. Sci.	92000	83821	CS-190	2	Spring	2017
83821	Brandt	Comp. Sci.	92000	83821	CS-319	2	Spring	2018
98345	Kim	Elec. Eng.	80000	98345	EE-181	1	Spring	2017

Join Operation (Cont.)

- The **join** operation allows us to combine a select operation and a Cartesian-Product operation into a single operation.
- Consider relations *r* (*R*) and *s* (*S*)
- Let "theta" be a predicate on attributes in the schema R "union" S. The join operation r $\bowtie_\theta$ s is defined as follows:

$$r \bowtie_\theta s = \sigma_\theta\,(r \times s)$$

- Thus

$$\sigma_{\textit{instructor.id} = \textit{teaches.id}}\,(\textit{instructor} \times \textit{teaches})$$

- Can equivalently be written as

$$\textit{instructor} \bowtie_{\textit{Instructor.id} = \textit{teaches.id}} \textit{teaches}.$$

Union Operation

- The union operation allows us to combine two relations
- Notation: $r \cup s$
- For $r \cup s$ to be valid.
  1. *r, s* must have the *same* **arity** (same number of attributes)
  2. The attribute domains must be **compatible**
     - the types of the *i* th attributes of both input relations must be the same, for each *I*
- Such relations are referred to as compatible relations
  - it would not make sense to take the union of the *instructor* and *section* relations
- Example: to find all courses taught in the Fall 2017 semester, or in the Spring 2018 semester, or in both

$$\Pi_{course\_id}\,(\sigma_{\,semester=\text{“Fall”}\ \wedge\ year=2017}\,(section))\ \cup$$
$$\Pi_{course\_id}\,(\sigma_{\,semester=\text{“Spring”}\ \wedge\ year=2018}\,(section))$$

Union Operation (Cont.)

- Result of:

$$\Pi_{course\_id} (\sigma_{semester=\text{“Fall”} \land year=2017} (section)) \cup$$
$$\Pi_{course\_id} (\sigma_{semester=\text{“Spring”} \land year=2018} (section))$$

course_id
CS-101
CS-315
CS-319
CS-347
FIN-201
HIS-351
MU-199
PHY-101

Course Relation

course_id	*title*	*dept_name*	*credits*
BIO-101	Intro. to Biology	Biology	4
BIO-301	Genetics	Biology	4
BIO-399	Computational Biology	Biology	3
CS-101	Intro. to Computer Science	Comp. Sci.	4
CS-190	Game Design	Comp. Sci.	4
CS-315	Robotics	Comp. Sci.	3
CS-319	Image Processing	Comp. Sci.	3
CS-347	Database System Concepts	Comp. Sci.	3
EE-181	Intro. to Digital Systems	Elec. Eng.	3
FIN-201	Investment Banking	Finance	3
HIS-351	World History	History	3
MU-199	Music Video Production	Music	3
PHY-101	Physical Principles	Physics	4

Figure 2.2 The *course* relation.

Set-Intersection Operation

- The set-intersection operation allows us to find tuples that are in both the input relations.
- Notation: $r \cap s$
- Assume:
  - r, s have the *same arity*
  - attributes of r and s are compatible
- Example: Find the set of all courses taught in both the Fall 2017 and the Spring 2018 semesters.

$$\Pi_{course\_id} (\sigma_{semester=\text{“Fall”} \land year=2017} (section)) \cap$$
$$\Pi_{course\_id} (\sigma_{semester=\text{“Spring”} \land year=2018} (section))$$

  - Result

course_id
CS-101

Set Difference Operation

- The set-difference operation allows us to find tuples that are in one relation but are not in another.
- Notation *r* – *s*
- Set differences must be taken between **compatible** relations.
  - *r* and *s* must have the same arity
  - attribute domains of *r* and *s* must be compatible
- Example: to find all courses taught in the Fall 2017 semester, but not in the Spring 2018 semester

$$\Pi_{course\_id} (\sigma_{semester=\text{“Fall”} \land year=2017}(section)) -$$
$$\Pi_{course\_id} (\sigma_{semester=\text{“Spring”} \land year=2018}(section))$$

course_id
CS-347
PHY-101

The Rename Operation

- The results of relational-algebra expressions do not have a name that we can use to refer to them. The rename operator, ρ, is provided for that purpose
- The expression:

  $$\rho_x(E)$$

  returns the result of expression E under the name x
- Another form of the rename operation:

  $$\rho_{x(A1,A2,\ ..\ An)}(E)$$

The Rename Operation

- Find the ID and name of those instructors who earn more than the instructor whose ID is 12121

$$\Pi_{i.ID,i.name}\ ((\sigma_{i.salary\ >\ w.salary}(\rho_i(instructor)\ \times\ \sigma_{w.id=12121}(\rho_w\ (instructor)))))$$

Equivalent Queries

- There is more than one way to write a query in relational algebra.
- Example: Find information about instructors in the Physics department with salary greater than 90,000
- Query 1

$$\sigma_{\textit{dept\_name}=\text{"Physics"} \wedge \textit{salary} > 90{,}000}(\textit{instructor})$$

- Query 2

$$\sigma_{\textit{dept\_name}=\text{"Physics"}}(\sigma_{\textit{salary} > 90.000}(\textit{instructor}))$$

- The two queries are not identical; they are, however, equivalent -- they give the same result on any database.

Equivalent Queries

- There is more than one way to write a query in relational algebra.
- Example: Find information about courses taught by instructors in the Physics department
- Query 1

$$\sigma_{dept\_name=\text{“Physics”}}(instructor \bowtie_{instructor.ID\,=\,teaches.ID} teaches)$$

- Query 2

$$(\sigma_{dept\_name=\text{“Physics”}}(instructor)) \bowtie_{instructor.ID\,=\,teaches.ID} teaches$$

- The two queries are not identical; they are, however, equivalent -- they give the same result on any database.

course_id	*sec_id*	*semester*	*year*	*building*	*room_number*	*time_slot_id*
BIO-101	1	Summer	2017	Painter	514	B
BIO-301	1	Summer	2018	Painter	514	A
CS-101	1	Fall	2017	Packard	101	H
CS-101	1	Spring	2018	Packard	101	F
CS-190	1	Spring	2017	Taylor	3128	E
CS-190	2	Spring	2017	Taylor	3128	A
CS-315	1	Spring	2018	Watson	120	D
CS-319	1	Spring	2018	Watson	100	B
CS-319	2	Spring	2018	Taylor	3128	C
CS-347	1	Fall	2017	Taylor	3128	A
EE-181	1	Spring	2017	Taylor	3128	C
FIN-201	1	Spring	2018	Packard	101	B
HIS-351	1	Spring	2018	Painter	514	C
MU-199	1	Spring	2018	Packard	101	D
PHY-101	1	Fall	2017	Watson	100	A

Figure 2.6 The *section* relation.

ID	*course_id*	*sec_id*	*semester*	*year*
10101	CS-101	1	Fall	2017
10101	CS-315	1	Spring	2018
10101	CS-347	1	Fall	2017
12121	FIN-201	1	Spring	2018
15151	MU-199	1	Spring	2018
22222	PHY-101	1	Fall	2017
32343	HIS-351	1	Spring	2018
45565	CS-101	1	Spring	2018
45565	CS-319	1	Spring	2018
76766	BIO-101	1	Summer	2017
76766	BIO-301	1	Summer	2018
83821	CS-190	1	Spring	2017
83821	CS-190	2	Spring	2017
83821	CS-319	2	Spring	2018
98345	EE-181	1	Spring	2017

Figure 2.7 The *teaches* relation.

End of Chapter 2