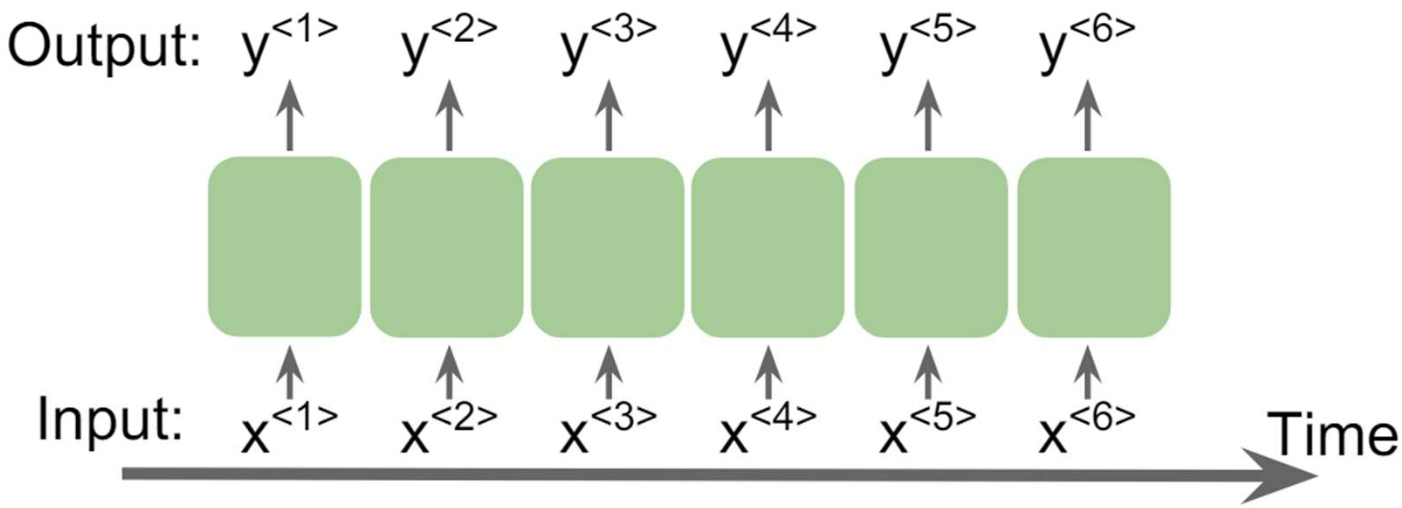
RNN

Sequence data: order matters

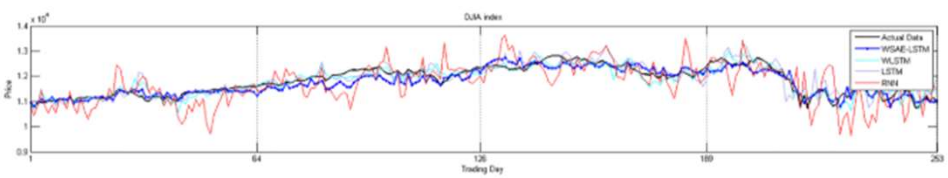
The movie my friend has **<u>not</u>** seen is good
The movie my friend has seen is **<u>not</u>** good

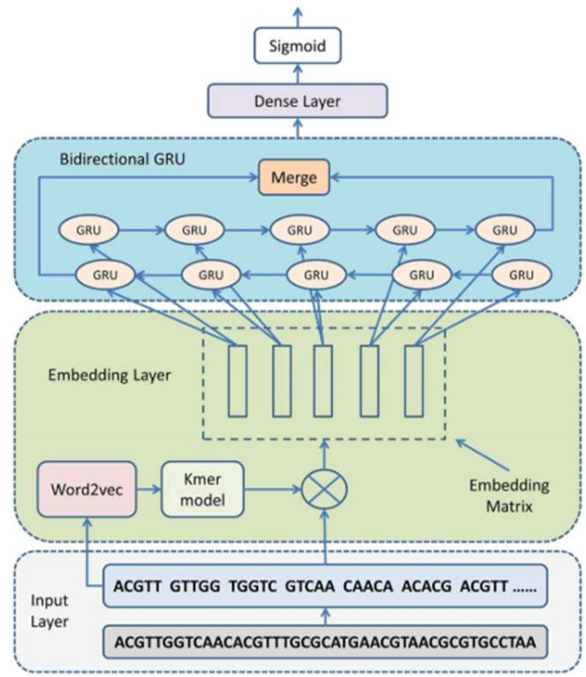


Applications: working with Sequential Data

- Text classification
- Speech recognition (acoustic modeling)
- language translation
- ...

Stock market predictions





Shen, Zhen, Wenzheng Bao, and De-Shuang Huang. "[Recurrent Neural Network for Predicting Transcription Factor Binding Sites]." *Scientific reports* 8, no. 1 (2018): 15270.

DNA or (amino acid/protein) sequence modeling

Applications: Speech Recognition

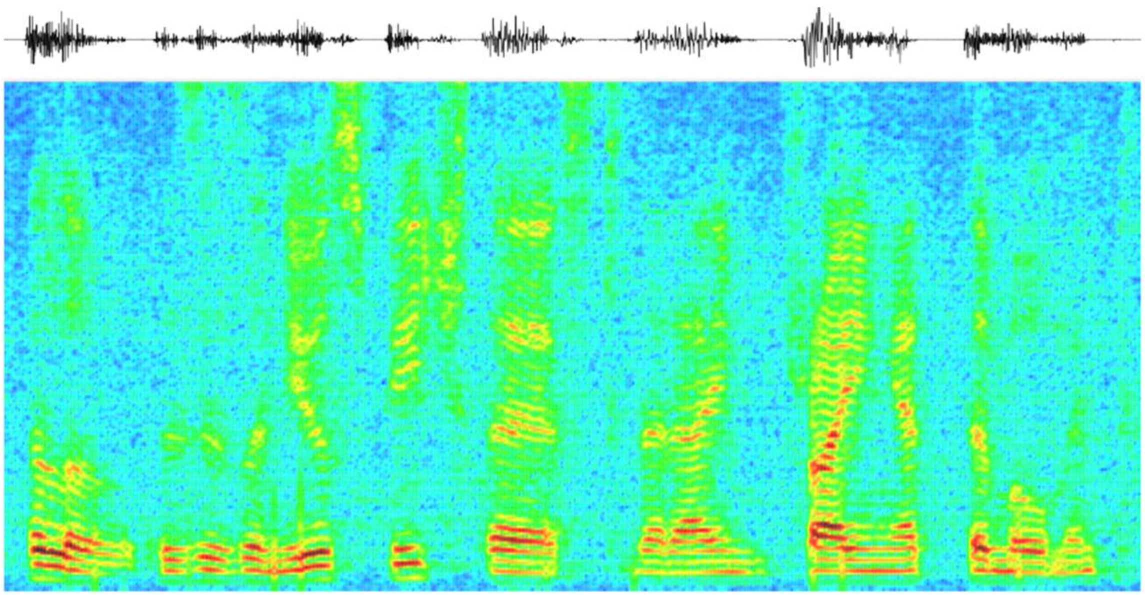


Figure: source

- Speech Recognition
  - Analyze a series of spectral vectors, determine what was said.
- Note: Inputs are sequences of vectors. Output is a classification result.

Application : Text analysis

Stephen Curry scored 34 points and was named the NBA Finals MVP as the Warriors claimed the franchise's seventh championship overall. And this one completed a journey like none other, after a run of five consecutive finals, then a plummet to the bottom of the NBA, and now a return to greatness just two seasons after having the league's worst record.

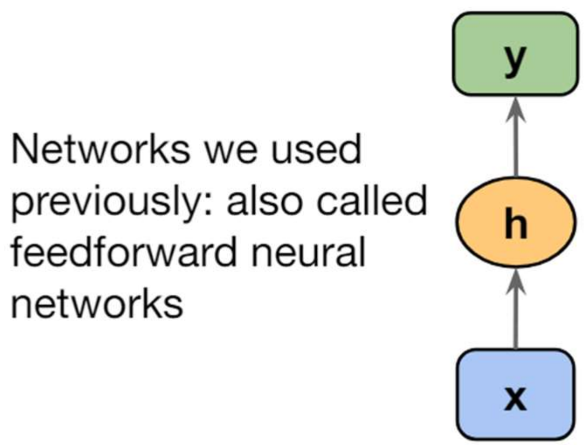
- Football or Basketball?
- Text Analysis
  - E.g. analyze document, identify topic
    - Input series of words, output classification output
  - E.g. read English, output Persian
    - Input series of words, output series of words

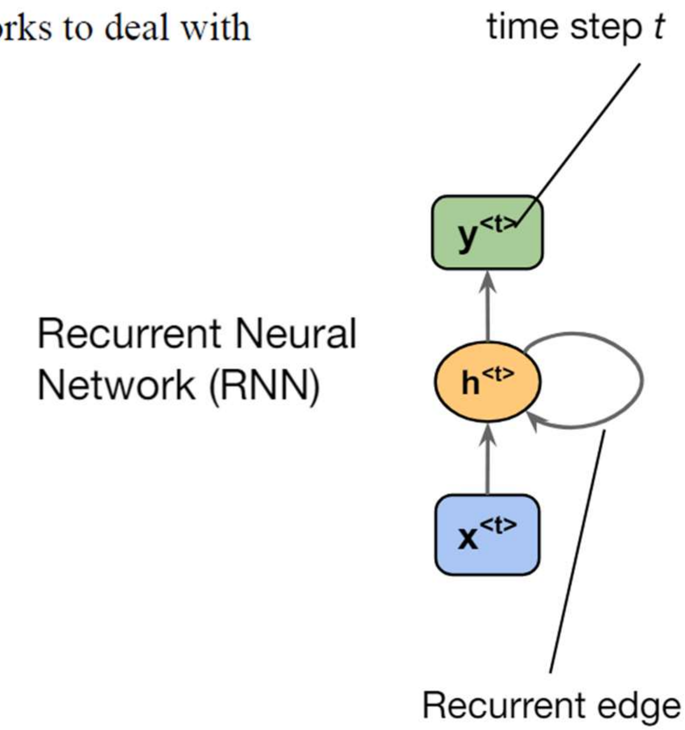
Application: Stock Market prediction

- Stock Market Prediction
  - Should I invest, vs. should I not invest in X?
  - Decision must be taken considering how things have fared over time.
- Note: Inputs are sequences of vectors. Output may be scalar or vector.

Recurrent Neural Network

- A variant of the conventional feed-forward artificial neural networks to deal with sequential data
- Hold the knowledge about the past (Have memory!)





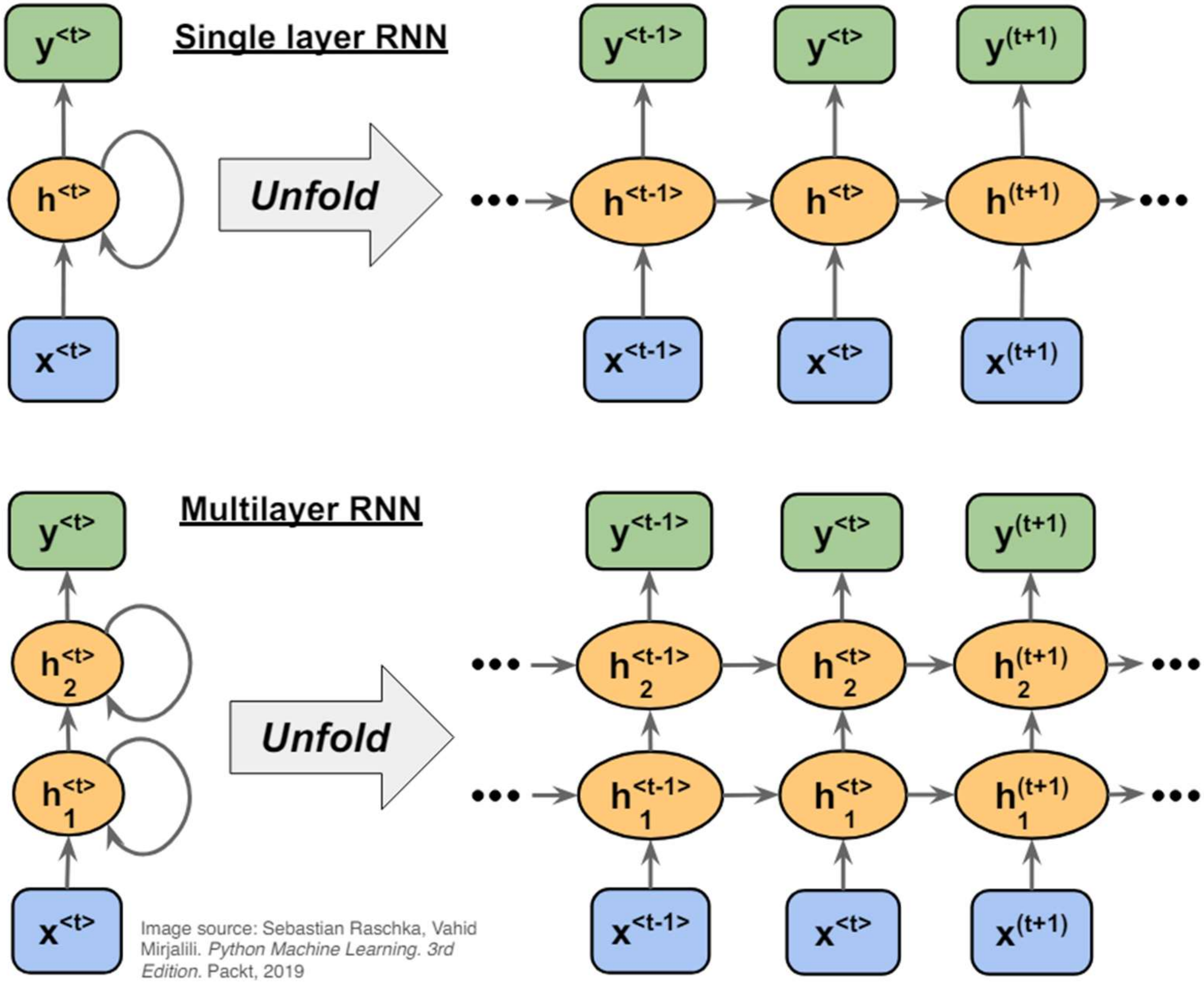
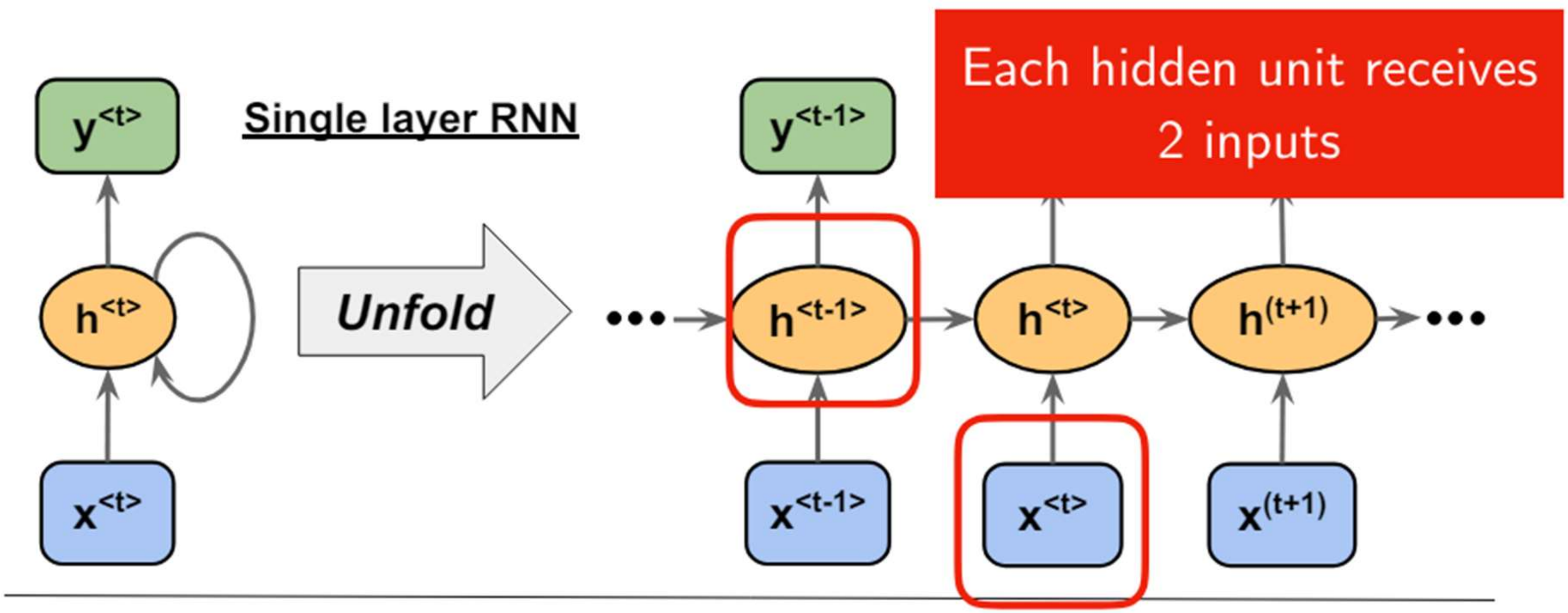
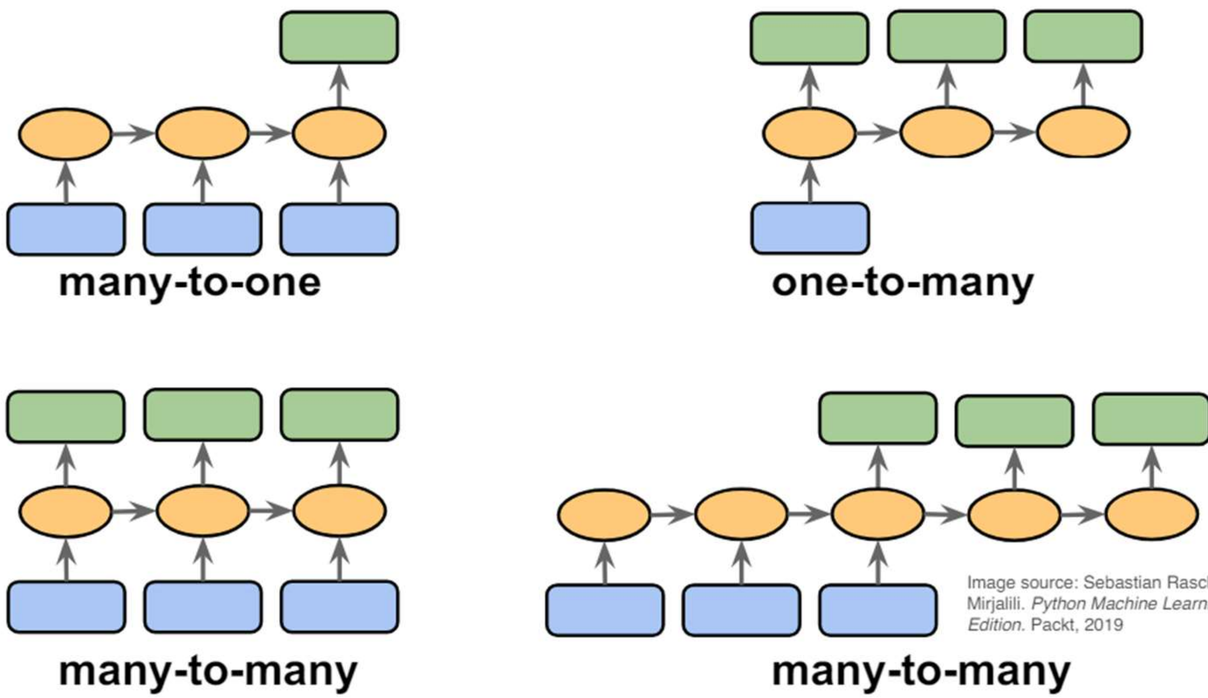


Image source: Sebastian Raschka, Vahid Mirjalili. *Python Machine Learning. 3rd Edition.* Packt, 2019



Single layer RNN
Unfold
Each hidden unit receives 2 inputs
y<t>
h<t>
x<t>
y<t-1>
h<t-1>
h<t>
h(t+1)
x<t-1>
x<t>
x(t+1)

Different type of sequence modeling tasks



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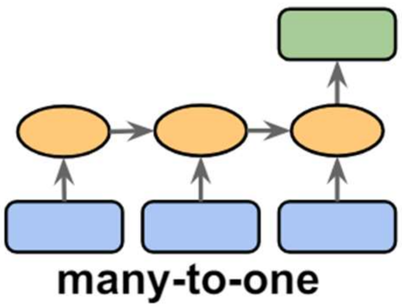
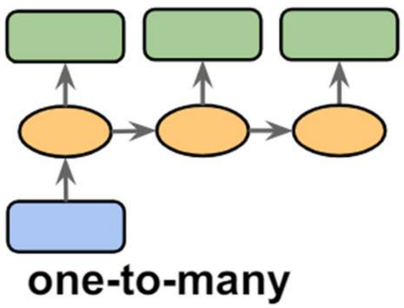
Different type of sequence modeling tasks



- Many-to-one: The input data is a sequence, but the output is a fixedsize vector, not a sequence.

- Ex.: sentiment analysis, the input is some text, and the output is a class label.

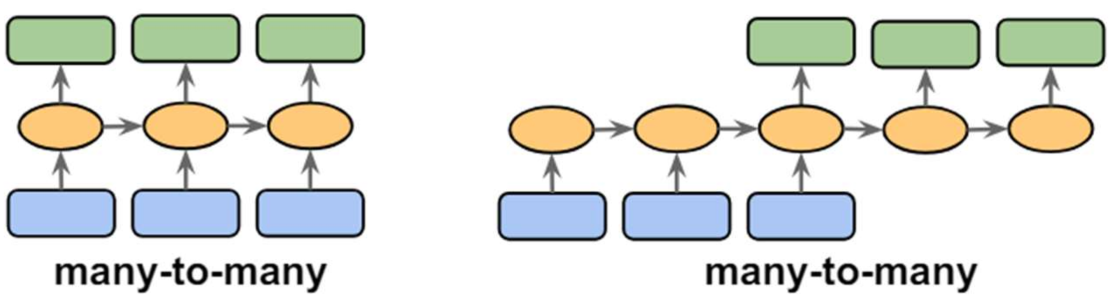
Different type of sequence modeling tasks



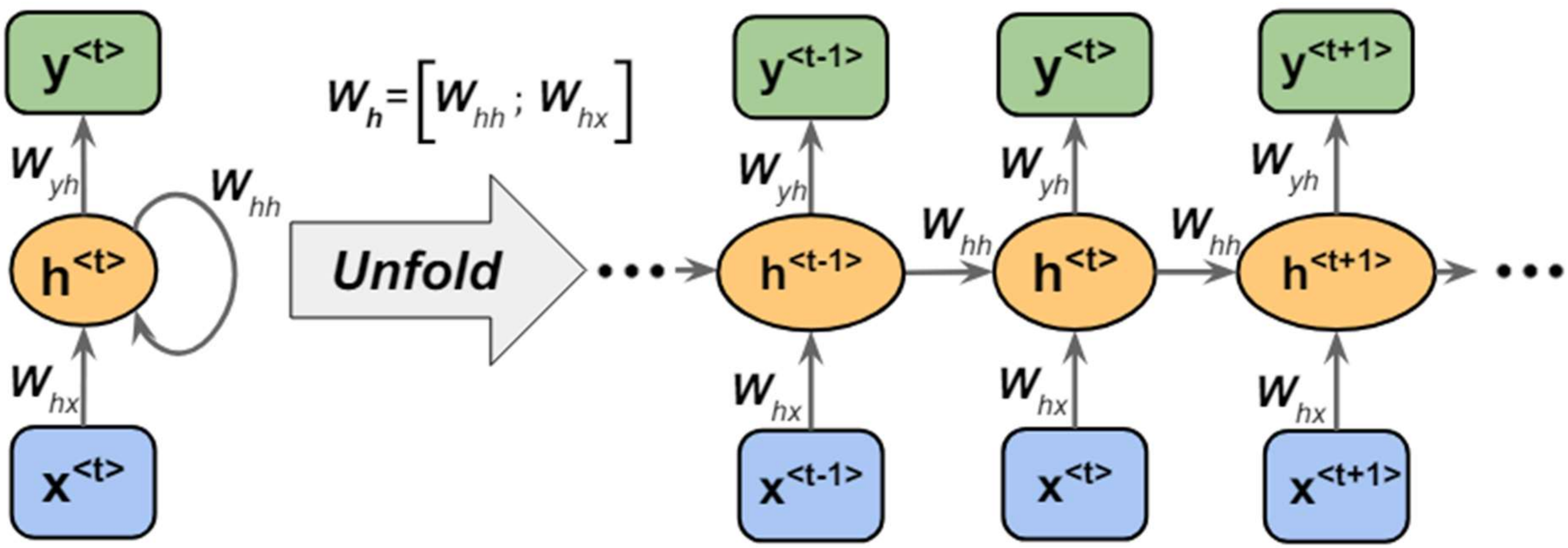
- One-to-many: Input data is in a standard format (not a sequence), the output is a sequence.
- Ex.: Image captioning, where the input is an image, the output is a text description of that image

Different type of sequence modeling tasks

- Many-to-many: Both inputs and outputs are sequences. Can be direct or delayed.
- Ex.: Video-captioning, i.e., describing a sequence of images via text (direct).
- Translating one language into another (delayed)



Weight matrices in a single-hidden layer RNN



Weight matrices in a single-hidden layer RNN

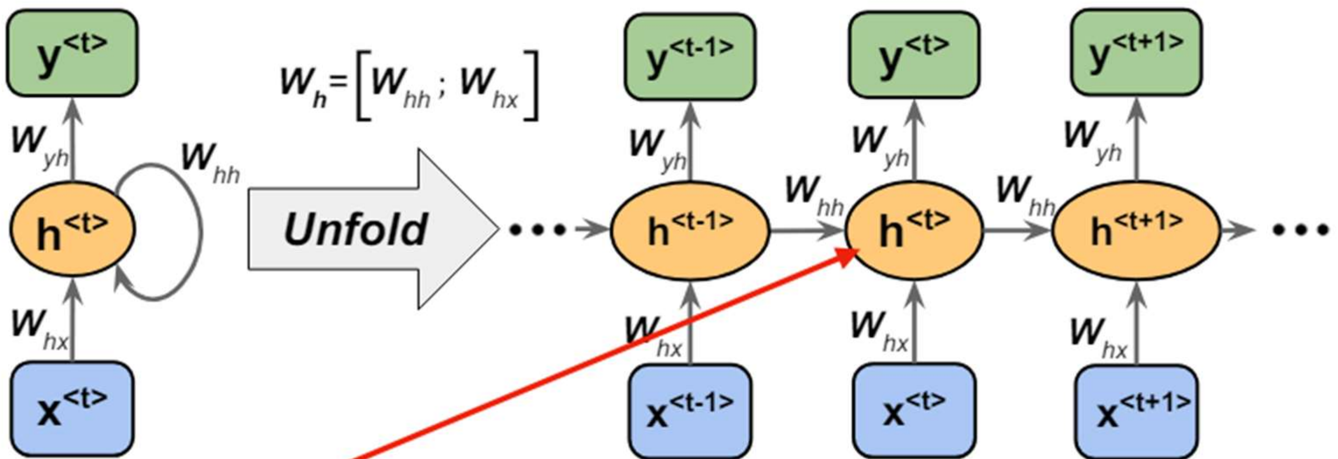


Image source: Sebastian Raschka, Vahid Mirjalili. *Python Machine Learning. 3rd Edition*. Packt, 2019

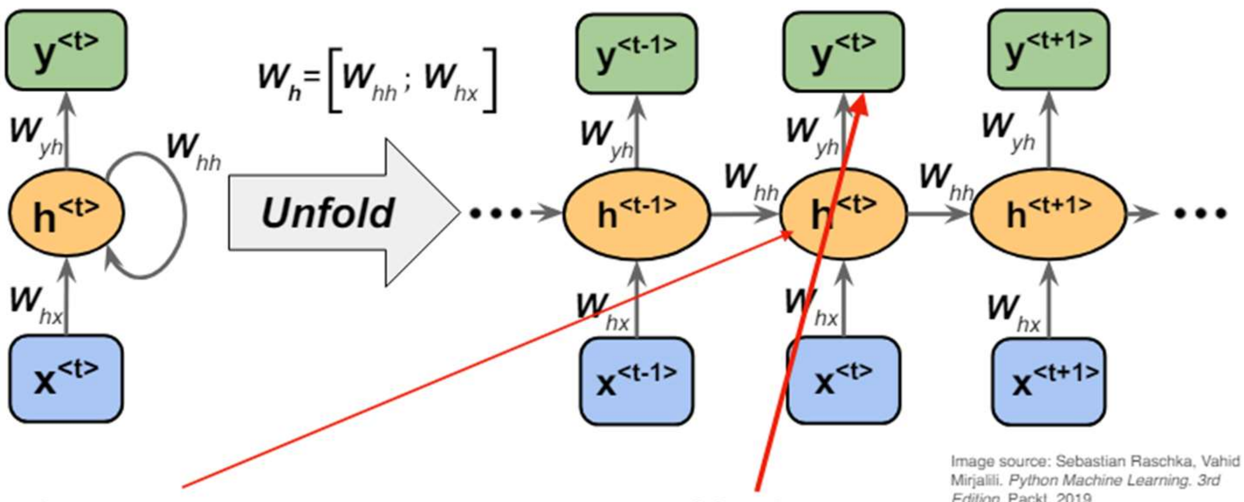
Net input:

$$\mathbf{z}_h^{\langle t\rangle} = \mathbf{W}_{hx}\mathbf{x}^{\langle t\rangle} + \mathbf{W}_{hh}\mathbf{h}^{\langle t-1\rangle} + \mathbf{b}_h$$

Activation:

$$\mathbf{h}^{\langle t\rangle} = \sigma_h\left(\mathbf{z}_h^{\langle t\rangle}\right)$$

Weight matrices in a single-hidden layer RNN



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Activation:

$$\mathbf{h}^{\langle t \rangle} = \sigma_h\left(\mathbf{z}_h^{\langle t \rangle}\right)$$

Net input:

$$\mathbf{z}_y^{\langle t \rangle} = \mathbf{W}_{yh}\mathbf{h}^{\langle t \rangle} + \mathbf{b}_y$$

Output:

$$\mathbf{y}^{\langle t \rangle} = \sigma_y\left(\mathbf{z}_y^{\langle t \rangle}\right)$$

RNN

$$\underbrace{\boxed{h_t}}_{\text{new state}} = \underbrace{\boxed{f_W}}_{\text{some function with parameters W}}(\underbrace{\boxed{h_{t-1}}}_{\text{old state}}, \underbrace{\boxed{x_t}}_{\text{input vector at some time step}})$$

Figure: RNN formula, source

- We can process a sequence of vectors x by applying a recurrence formula at every time step
- The same function and the same set of parameters are used at every time step.

RNN: forward pass

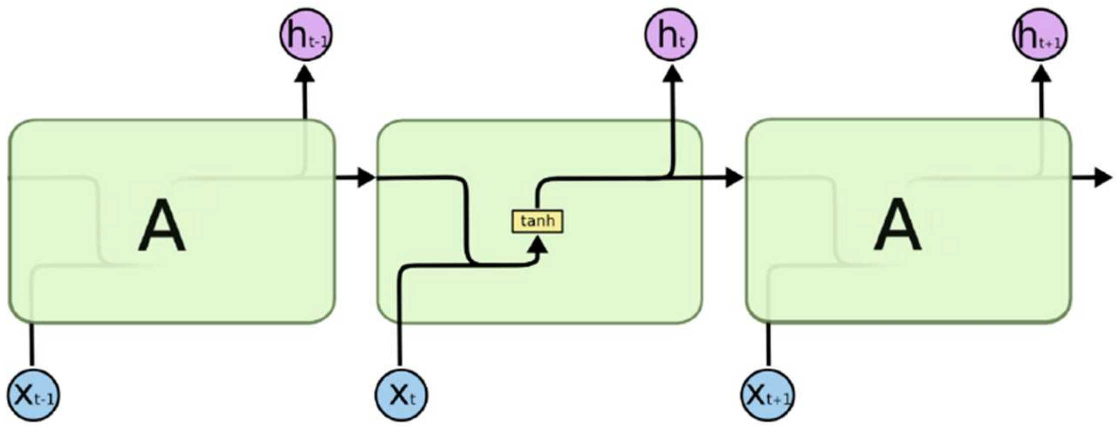
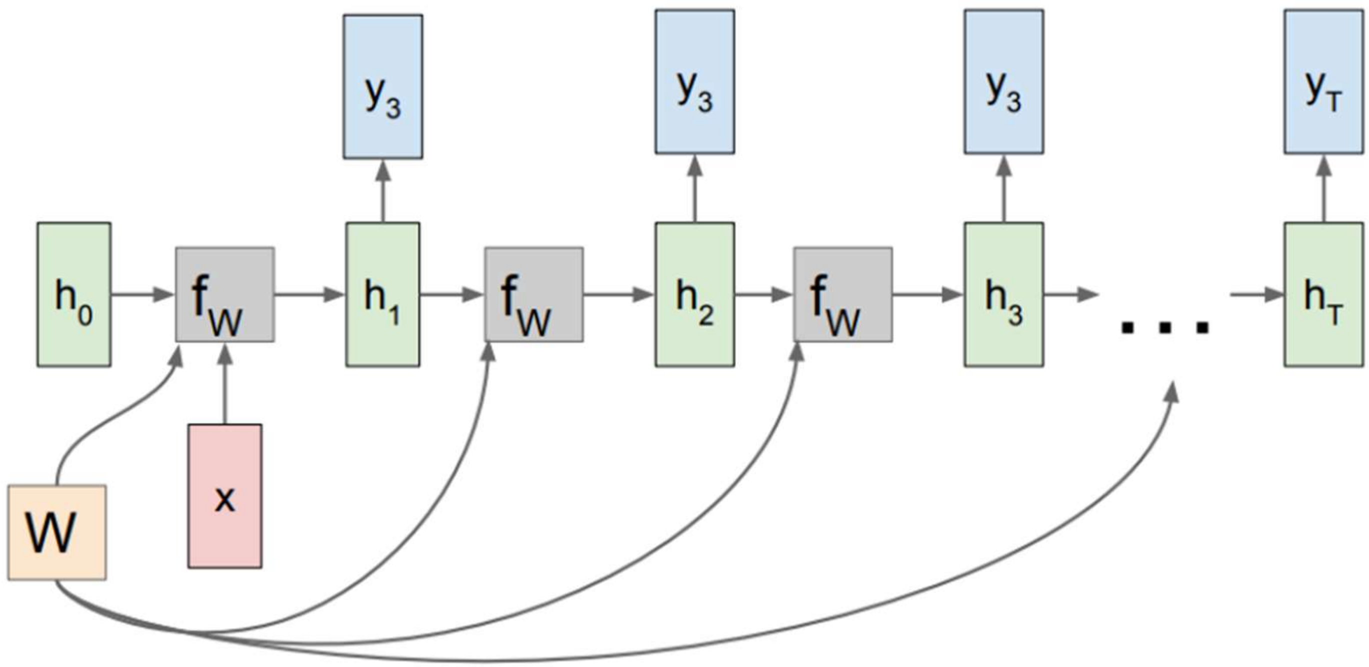


Figure: The repeating module in a standard RNN contains a single layer, source

$$
\begin{aligned}
h_t &= \tanh(W_{hh}h_{t-1} + W_{hx}x_t + b_h) \\
&= \tanh((W_{hh}W_{hx})\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h) \\
&= \tanh(W\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h)
\end{aligned}
$$

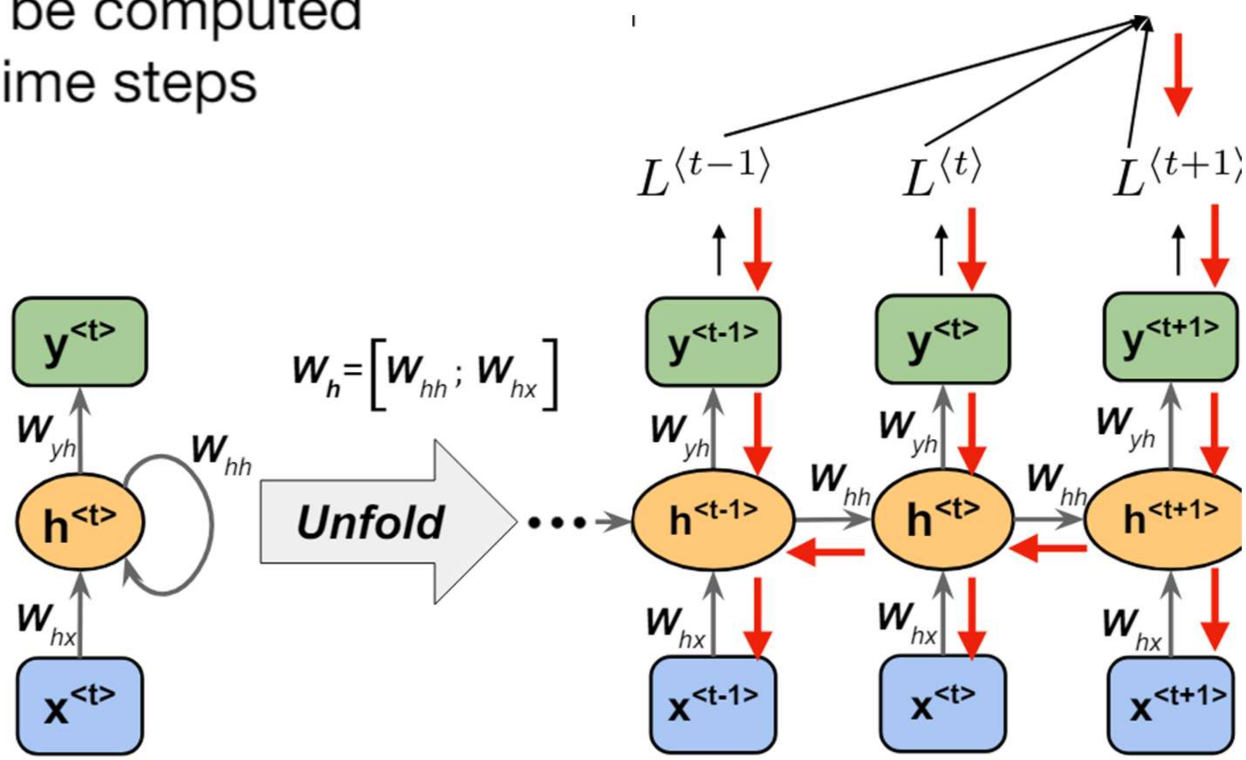
RNN: computational Graph

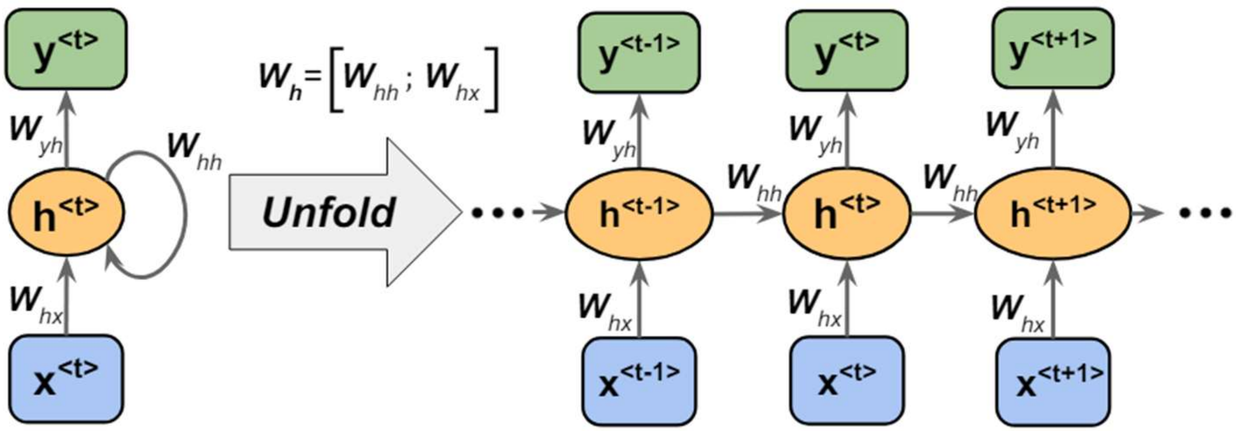


RNN: Backpropagation Through Time

The overall loss can be computed as the sum over all time steps

$$L = \sum_{t=1}^{T} L^{\langle t \rangle}$$



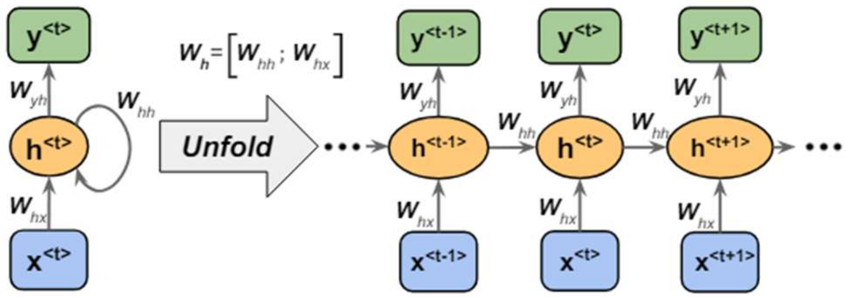


Werbos, Paul J. "Backpropagation through time: what it does and how to do it." *Proceedings of the IEEE* 78, no. 10 (1990): 1550-1560.

$$L = \sum_{t=1}^{T} L^{(t)}$$

$$\frac{\partial L^{(t)}}{\partial \mathbf{W}_{hh}} = \frac{\partial L^{(t)}}{\partial y^{(t)}} \cdot \frac{\partial y^{(t)}}{\partial \mathbf{h}^{(t)}} \cdot \left(\sum_{k=1}^{t} \frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}} \cdot \frac{\partial \mathbf{h}^{(k)}}{\partial \mathbf{W}_{hh}} \right)$$

Backpropagation through time



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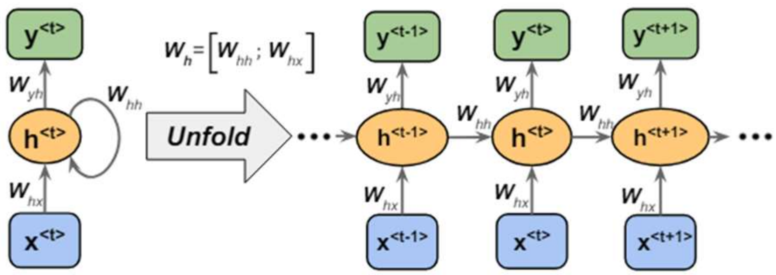
$$\frac{\partial L^{(t)}}{\partial \mathbf{W}_{hh}} = \frac{\partial L^{(t)}}{\partial y^{(t)}} \cdot \frac{\partial y^{(t)}}{\partial \mathbf{h}^{(t)}} \cdot \left(\sum_{k=1}^{t} \boxed{\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}}} \cdot \frac{\partial \mathbf{h}^{(k)}}{\partial \mathbf{W}_{hh}} \right)$$

computed as a multiplication of adjacent time steps:

$$\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}} = \prod_{i=k+1}^{t} \frac{\partial \mathbf{h}^{(i)}}{\partial \mathbf{h}^{(i-1)}}$$

Backpropagation through time



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$$L = \sum_{t=1}^{T} L^{(t)} \qquad \frac{\partial L^{(t)}}{\partial \mathbf{W}_{hh}} = \frac{\partial L^{(t)}}{\partial y^{(t)}} \cdot \frac{\partial y^{(t)}}{\partial \mathbf{h}^{(t)}} \cdot \left(\sum_{k=1}^{t} \boxed{\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}}} \cdot \frac{\partial \mathbf{h}^{(k)}}{\partial \mathbf{W}_{hh}} \right)$$

computed as a multiplication of adjacent time steps:

This is very problematic:
Vanishing/Exploding gradient problem!

$$\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}} = \prod_{i=k+1}^{t} \frac{\partial \mathbf{h}^{(i)}}{\partial \mathbf{h}^{(i-1)}}$$