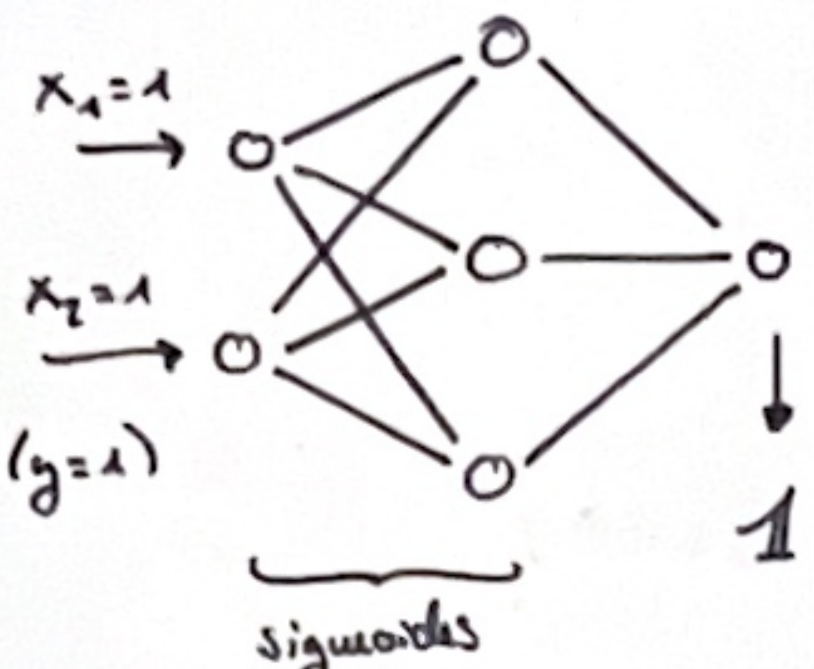
Boletín 4. Redes neuronales

Ejercicio 1.



$$W^{(1)}=\begin{pmatrix}-2 & 1\\ 1 & -1\\ 3 & -1\end{pmatrix};\quad b^{(1)}=\begin{pmatrix}0\\-1\\1\end{pmatrix}$$

$$W^{(2)}=(2\ \ 3\ \ 1);\quad b^{(2)}=(0)$$

$$f(z)=\frac{1}{1+e^{-z}}$$

a) $z^{(l+1)}=W^{(l)}a^{(l)}+b^{(l)}$; $a^{(l+1)}=f(z^{(l+1)})$

$$z^{(2)}=W^{(1)}\underset{x}{\underset{\shortparallel}{a^{(1)}}}+b^{(1)}=\begin{pmatrix}-2 & 1\\ 1 & -1\\ 3 & -1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}0\\-1\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\2\end{pmatrix}+\begin{pmatrix}0\\-1\\1\end{pmatrix}=\boxed{\begin{pmatrix}-1\\-1\\3\end{pmatrix}}$$

$$a^{(2)}=f(z^{(2)})=\begin{pmatrix}f(-1)\\f(-1)\\f(3)\end{pmatrix}\underset{\text{sigmoide}}{=}\begin{pmatrix}\frac{1}{1+e}\\ \frac{1}{1+e}\\ \frac{1}{1+e^{-3}}\end{pmatrix}\approx\boxed{\begin{pmatrix}0.2689\\0.2689\\0.9526\end{pmatrix}}$$

$$z^{(3)}=W^{(2)}a^{(2)}+b^{(2)}=(2\ \ 3\ \ 1)\begin{pmatrix}\frac{1}{1+e}\\ \frac{1}{1+e}\\ \frac{1}{1+e^{-3}}\end{pmatrix}+(0)=\frac{5}{1+e}+\frac{1}{1+e^{-3}}\approx\boxed{2.2973}$$

$$a^{(3)}=\overset{1}{f}(z^{(3)})=z^{(3)}\approx\boxed{2.2973}$$

b) $\delta^{(n_l)}=(a^{(n_l)}-y)\circ f'(z^{(n_l)})$ / $f'(z)=\begin{cases}1 & \text{si } f(z)=1\\ \frac{e^{-z}}{(1+e^{-z})^2} & \text{si } f(z)=\frac{1}{1+e^{-z}}\end{cases}$

Salida: $\delta^{(3)}=(a^{(3)}-y)\circ\overset{1}{\cancel{f'(z^{(3)})}}=\frac{5}{1+e}+\frac{1}{1+e^{-3}}-1\approx\boxed{1.2973}$

Capa oculta: $\delta^{(2)}=[(W^{(2)})^T\delta^{(3)}]\circ f'(z^{(2)})=\begin{pmatrix}2\left(\frac{5}{1+e}+\frac{1}{1+e^{-3}}-1\right)\\ 3\left(\frac{5}{1+e}+\frac{1}{1+e^{-3}}-1\right)\\ \frac{5}{1+e}+\frac{1}{1+e^{-3}}-1\end{pmatrix}\circ\begin{pmatrix}\frac{e}{(1+e)^2}\\ \frac{e}{(1+e)^2}\\ \frac{e^{-3}}{(1+e^{-3})^2}\end{pmatrix}=$

$$=\begin{pmatrix}2\left(\frac{5}{1+e}+\frac{1}{1+e^{-3}}-1\right)\frac{e}{(1+e)^2}\\ 3\left(\frac{5}{1+e}+\frac{1}{1+e^{-3}}-1\right)\frac{e}{(1+e)^2}\\ \left(\frac{5}{1+e}+\frac{1}{1+e^{-3}}-1\right)\frac{e^{-3}}{(1+e^{-3})^2}\end{pmatrix}\approx\boxed{\begin{pmatrix}0.5101\\0.7652\\0.0586\end{pmatrix}}$$

c) Calculamos primero los gradientes $\nabla_{W^{(l)}} J = \delta^{(l+1)} (a^{(l)})^T$; $\nabla_{b^{(l)}} J = \delta^{(l+1)}$

$$\boxed{\nabla_{W^{(1)}} J = \delta^{(2)} (a^{(1)})^T \approx \begin{pmatrix} 0.5101 \\ 0.7652 \\ 0.0586 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix} = \begin{pmatrix} 0.5101 & 0.5101 \\ 0.7652 & 0.7652 \\ 0.0586 & 0.0586 \end{pmatrix}}$$

$$\boxed{\nabla_{b^{(1)}} J = \delta^{(2)} \approx \begin{pmatrix} 0.5101 \\ 0.7652 \\ 0.0586 \end{pmatrix}}$$

$$\boxed{\nabla_{W^{(2)}} J = \delta^{(3)} (a^{(2)})^T \approx 1.2973 \begin{pmatrix} 0.2689 & 0.2689 & 0.9526 \end{pmatrix} \approx \begin{pmatrix} 0.3488 & 0.3488 & 1.2358 \end{pmatrix}}$$

$$\boxed{\nabla_{b^{(2)}} J = \delta^{(3)} \approx 1.2973}$$

Fijamos $\Delta W^{(l)} = \Delta b^{(l)} = 0$ y $\begin{cases} \Delta W^{(l)} := \overset{0}{\cancel{\Delta W^{(l)}}} + \nabla_{W^{(l)}} J \\ \Delta b^{(l)} := \Delta b^{(l)} + \nabla_{b^{(l)}} J \end{cases}$

Actualizamos los pesos y bias con $\alpha = 1/2$, $\lambda = 1$ y $m = 1$ (un batch)

$$W^{(l)} = W^{(l)} - \alpha \left[\frac{1}{m} \Delta W^{(l)} + \lambda W^{(l)} \right] = W^{(l)} - \frac{1}{2} \left[\nabla_{W^{(l)}} J + W^{(l)} \right]$$

$$b^{(l)} = b^{(l)} - \alpha \left[\frac{1}{m} \Delta b^{(l)} \right] = b^{(l)} - \frac{1}{2} \nabla_{b^{(l)}} J$$

$$\boxed{W^{(1)} \approx \begin{pmatrix} -2 & 1 \\ 1 & -1 \\ 3 & -1 \end{pmatrix} - \frac{1}{2} \left[\begin{pmatrix} 0.5101 & 0.5101 \\ 0.7652 & 0.7652 \\ 0.0586 & 0.0586 \end{pmatrix} + \begin{pmatrix} -2 & 1 \\ 1 & -1 \\ 3 & -1 \end{pmatrix} \right] \approx \begin{pmatrix} -1.2551 & 0.2449 \\ 0.1174 & -0.8826 \\ 1.4707 & -0.5293 \end{pmatrix}}$$

$$\boxed{b^{(1)} \approx \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0.5101 \\ 0.7652 \\ 0.0586 \end{pmatrix} = \begin{pmatrix} -0.2551 \\ -1.3826 \\ 0.9707 \end{pmatrix}}$$

$$\boxed{W^{(2)} \approx \begin{pmatrix} 2 & 3 & 1 \end{pmatrix} - \frac{1}{2} \left[\begin{pmatrix} 0.3488 & 0.3488 & 1.2358 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 1 \end{pmatrix} \right] \approx \begin{pmatrix} 0.8256 & 1.3256 & -0.1179 \end{pmatrix}}$$

$$\boxed{b^{(2)} \approx 0 - \frac{1}{2} \cdot 1.2973 \approx -0.6487}$$