INTRODUCING ADVANCED MACROECONOMICS

GROWTH AND BUSINESS CYCLES

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The real interest rate: *ex ante* versus *ex post*

We are now ready to derive the relationship between the inflation rate and the aggregate demand for goods and services. This relationship, called the aggregate demand (AD) curve, will be one of the two cornerstones of our model of the macro economy.

The first step in our derivation of the AD curve is the specification of the relationship between the nominal interest rate, the real interest rate and inflation. We have previously used the popular definition according to which the real interest rate is given by $r = i - \pi$, but now we need to be more precise. For a saver or a borrower the actual real interest rate r^a earned or paid between the current period and the next one is given by:

$$1 + r^a \equiv \frac{1+i}{1+\pi_{+1}}. \tag{27}$$

The reasoning behind (27) is this: if the current price level is P, giving up one unit of consumption today will enable you to invest the amount P in the capital market. Your nominal wealth one year from now will then be $P(1 + i)$. With an inflation rate π_{+1} between the current and the next period, a unit of consumption tomorrow will cost you $P(1 + \pi_{+1})$, so the purchasing power of your wealth one year from now will be only $P(1 + i)/P(1 + \pi_{+1}) = (1 + i)/(1 + \pi_{+1})$. Thus your *real* rate of return is $r^a = (1 + i)/(1 + \pi_{+1}) - 1$, which is just another way of writing (27).

The variable r^a is called the *ex post* real interest rate, because it measures the real interest rate implied by the *actual* rate of inflation, measured *after* the relevant time period has passed ('*ex post*'). However, since saving and investment decisions must be made *'ex ante'*, *before* the future price level is known with certainty, the real interest rate affecting aggregate demand for goods is the so-called *ex ante* real interest rate (r) which is based on the rate of inflation π^e_{+1} expected to prevail over the next period:

$$1 + r \equiv \frac{1+i}{1+\pi^e_{+1}}. \tag{28}$$

You may easily verify that:

$$r = \frac{i - \pi^e_{+1}}{1+\pi^e_{+1}} \approx i - \pi^e_{+1}, \tag{29}$$

where the latter approximation holds as long as π^e_{+1} does not deviate too much from zero. In the special case of static inflation expectations where agents assume that the rate of price increase over the next period will correspond to the rate of inflation experienced between the previous and the current period, we have $\pi^e_{+1} = \pi$. It then follows from (29) that the *ex ante* real interest rate may be proxied by $r = i - \pi$, corresponding to the popular definition of the real interest rate. Still, you should keep in mind that the more correct specification of the real interest rate influencing saving and investment decisions is given by (29).