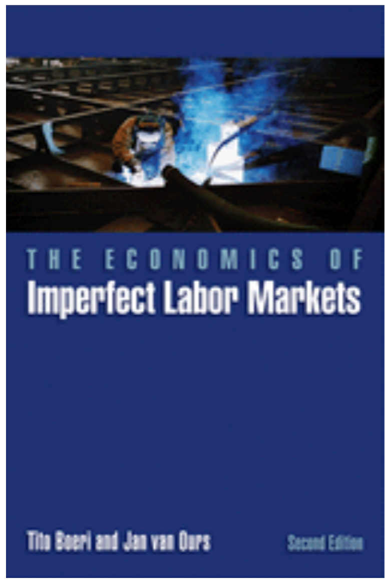
The Economics of Imperfect Labor Markets: Second Edition



Solutions to Exercises

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1 Overview

(11) Andrea's utility function is $U(C,l) = (C-40) \times (l-40)$, where C denotes consumption and l leisure. Andrea earns 10 euros per hour, can at most work 84 hours per week, and has no non-labor income.

(a) Please display Andrea's budget line. Would it be different if she had some non labor income?

(b) What is Andrea's marginal rate of substitution when $l = 50$ and she is on her budget line?

(c) What is Andrea's reservation wage?

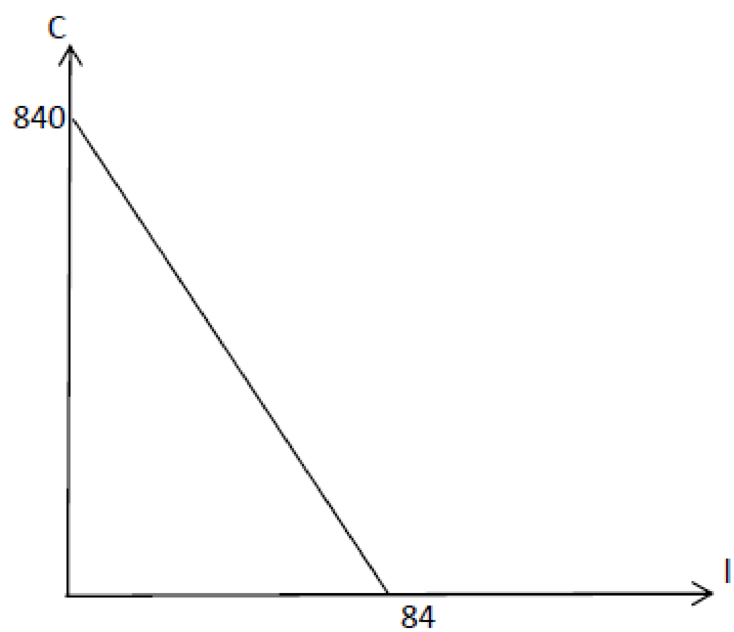
(d) Compute the amount of consumption and leisure, C^* and l^*, that maximize Andrea's utility.

Solution -

(a) Andrea can consume 0 if she does not work. If she works full time she can consume 84×10. Therefore total consumption has to be equal to $10 \times (\textit{hours worked})$ which is $10 \times (84 - l)$. The budget line is given by:

$C = 840 - 10l$

Graphically:



In the case of non-labor income equal to m, Andrea's budget line would be $C = m + w(l_0 - l) = m + 10 \times (84 - l)$. Graphically, it would have a kink in correspondance to $(l = 84, C = m)$ with a vertical segment from $(l = 84, C = m)$ to $(l = 84, C = 0)$, representing the fact that when Andrea is offering 0 hours of work, his total income amounts to his non-labor income m.

(b) The marginal rate of substitution (MRS) at $l = 50$;

$MRS = \frac{\partial U/\partial l}{\partial U/\partial C} = \frac{C-40}{l-40}$

If leisure is 50, there are 34 hours to spent at work. Total consumption (total amount that can be earned at work in the absence of non-labor income) is $34 \times 10 = 340$. Thus $MRS = \frac{340-40}{50-40} = \frac{300}{10} = 30$

(c) The reservation wage is given by the MRS evaluated at the zero hours of work locus $(l = 84, C = 0)$

$$MRS = \frac{0-40}{84-40} = \frac{-40}{44} = -0.9 = w^r$$

The reason why the reservation wage is negative can be found looking at the utility function. Given this preferences, when consumption is lower than 40, the utility becomes negative and an increase in leisure implies a decrease in utility. For this reason Andrea will prefer to work even if the wage is negative. In other words, since leisure implies a disutility, Andrea would be willing to pay to work. Since wage cannot be negative, it is correct to say that Andrea's reservation wage is 0, i.e. she will decide to work regardless the wage offered.

Notice that $w^r < w$, hence the individual will work.

(d) The optimal amount of consumption and leisure is given by the tangency condition between the indifference curves and the budget line. Therefore at the optimal point, the slopes of the two curves will be equal. The slope of the utility curve is the MRS whereas the slope of budget line is the wage. Thus, at optimum;

$MRS = \frac{C-4}{l-40} = 10 \Rightarrow C - 40 = 10l - 400 \Rightarrow C = 10l - 440$

Substituting C into the budget line gives;

$840 - 10l = 10l - 440 \Rightarrow 20l = 1280$

This yields;

$l^* = 64, \quad C^* = (84 - 64)10 = 200$

(12) Mike's preferences over consumption C and leisure l are given by $U(C, l) = Cl$. The hourly wage is 20 euros per hour and there are 168 hours in the week.

(a) Write down Mike's budget constraint and graph it.

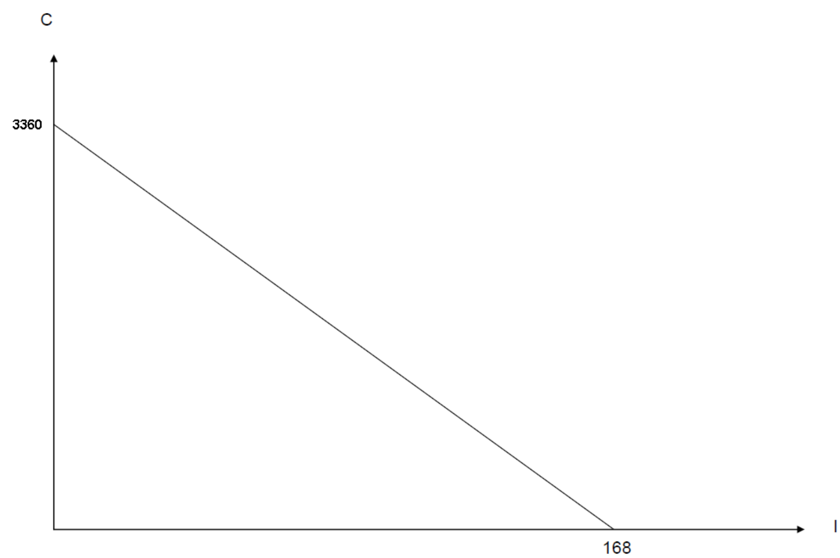
(b) What is Mike's optimal amount of consumption and leisure?

(c) What happens to employment and consumption if Mike receives 200 euros of non-labor income each week?

Solution -

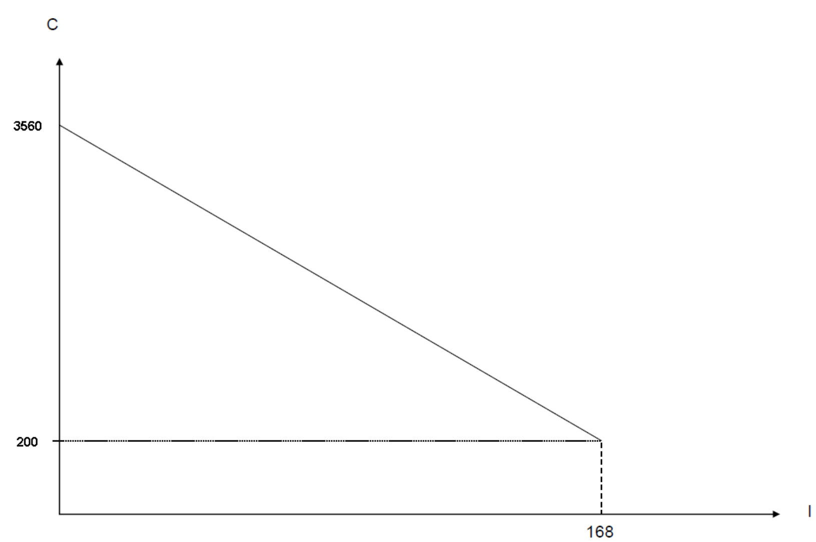
Mike's utility function is given by $U(C, l) = Cl$. There are 168 hours in a week to be split between leisure and work. The after tax wage per hour is 20 euros and there is no non-labor income.

(a) Mike's budget constraint is given by:
$C = w(l_0 - l) = 20 \times (168 - l)$
$C = 3360 - 20l$
a. The budget constraint can be depicted as follows:



(b) The optimal amount of consumption and leisure is given by the condition:
$MRS = \frac{C}{l} = 20 \Rightarrow C = 20l$
Substituting this into the budget constraint yields;
$3360 - 20l = 20l \Rightarrow 40l = 3360 \Rightarrow l^* = 84, \quad h^* = 168 - 84 = 84 \; and \; C^* = 1680$

(c) If there is a non-labor income, the budget line has a kink at $l = 168$, as shown below:



The equation of the budget constraint is now given by $C = 200 + 20 \times (168 - l)$. At the optimum $MRS = w$ so that $C = 20l$ still holds. Substituting this into the new budget constraint yields:

$20l = 200 + 3360 - 20l$

$l^* = 89,\ h^* = 79\ C^* = 1980$

Thus, in the presence of non-labor income reduced hours of work.

(13) (Advanced) Revenues of employers are given by:

$f(L) = \frac{A}{1-\eta} L^{1-\eta}$

with $0 \leq \eta < 1$, in which L is labor input and A and η are constants. Labor supply is specified as: $L^s = w^{\frac{1}{\varepsilon}}$, where ε is a constant.

(a) Show that a government trying to maximize the joint surplus obtains the perfect labor market outcome.

(b) Specify the outcome of Nash bargaining over the market surplus, where β represents the bargaining power of the workers.

(c) Show under which conditions the bargaining outcome is equal to the perfect equilibrium outcome

(d) Assume that employers have all the bargaining power, then illustrate that the magnitude of their power depends on the slope of the labor supply curve.

(e) Provide an intuition for these results.

Solution -

(a) The surplus of employers and employees are given by $\frac{A}{1-\eta} L^{1-\eta} - wL$ and $wL - \frac{L^{\varepsilon+1}}{\varepsilon+1}$ respectively, where $\frac{L^{\varepsilon+1}}{\varepsilon+1} = \int_0^L \mathrm{x}^{\varepsilon}\, dx$.

The joint surplus is then

$\left[\frac{A}{1-\eta} L^{1-\eta} - wL\right] + \left[wL - \frac{L^{\varepsilon+1}}{\varepsilon+1}\right]$

A government maximizing the joint surplus will solve

$$\max_{L} \left(\left[\frac{A}{1-\eta} L^{1-\eta} - wL\right] + \left[wL - \frac{L^{\varepsilon+1}}{\varepsilon+1}\right] \right) = \frac{A}{1-\eta} L^{1-\eta} - \frac{L^{\varepsilon+1}}{\varepsilon+1}$$

Taking the first order condition yields

$\frac{A(1-\eta)}{1-\eta} L^{-\eta} - \frac{\varepsilon+1}{\varepsilon+1} L^{\varepsilon} = 0$ or $AL^{-\eta} = L^{\varepsilon}$.

Therefore

$L^* = A^{\frac{1}{\varepsilon+\eta}}$

which using the labor supply equation (or equivalently the labor demand equation) yields $w^* = A^{\frac{\varepsilon}{\varepsilon+\eta}}$

This corresponds to the perfect labor market equilibrium. Indeed, the competitive outcome has the desirable property of maximizing the total surplus of production over the opportunity cost of employment, or the size of the economic pie generated by the labor market. Since maximization entails equality at the margin of the value of a job for the employer and workers' reservation wages, the competitive outcome also features no welfare loss from unemployment.

(b) With Nash bargaining the problem becomes

$$\max_{w} \quad \left(\left[\frac{A}{1-\eta}L^{1-\eta} - wL\right]^{(1-\beta)}\left[wL - \frac{L^{\varepsilon+1}}{\varepsilon+1}\right]^{\beta}\right)$$

subject to
$L = \left(\frac{A}{w}\right)^{\frac{1}{\eta}}$
Substituting the constraint into the objective function yields

$$\max_{w} \quad \left[w^{\frac{\eta-1}{\eta}}\left(A^{\frac{1}{\eta}}\frac{\eta}{1-\eta}\right)\right]^{1-\beta}\left[A^{\frac{1}{\eta}}w^{\frac{\eta-1}{\eta}} - \frac{1}{\varepsilon+1}A^{\frac{\varepsilon+1}{\eta}}\left(\frac{1}{w}\right)^{\frac{\varepsilon+1}{\eta}}\right]^{\beta}$$

Simple algebra gives us

$$\max_{w} \quad \left[w^{\frac{\eta-1}{\eta}}\left(A^{\frac{1}{\eta}}\frac{\eta}{1-\eta}\right)\right]^{1-\beta}\left[w^{\frac{\eta-1}{\eta}}\left(A^{\frac{1}{\eta}} - \frac{1}{\varepsilon+1}A^{\frac{\varepsilon+1}{\eta}}w^{\frac{-\varepsilon-\eta}{\eta}}\right)\right]^{\beta}$$

Now let's take logarithm of the objective function (a simple monotonic transformation). We can write the function as

$$\max_{w} \ (1-\beta)\log\left[w^{\frac{\eta-1}{\eta}}\right]+(1-\beta)\log\left[\left(A^{\frac{1}{\eta}}\frac{\eta}{1-\eta}\right)\right]+(\beta)\log\left[w^{\frac{\eta-1}{\eta}}\right]+(\beta)\log\left[\left(A^{\frac{1}{\eta}} - \frac{1}{\varepsilon+1}A^{\frac{\varepsilon+1}{\eta}}w^{\frac{-\varepsilon-\eta}{\eta}}\right)\right]$$

Deriving the FOC and using some algebra yields $\frac{\eta-1}{\eta}\frac{1}{w} + \beta\frac{\frac{1}{\varepsilon+1}A^{\frac{\varepsilon+1}{\eta}}\frac{\varepsilon+\eta}{\eta}w^{-\frac{\varepsilon+2\eta}{\eta}}}{A^{\frac{1}{\eta}} - \frac{1}{\varepsilon+1}A^{\frac{\varepsilon+1}{\eta}}w^{-\frac{\varepsilon+\eta}{\eta}}} = 0$

We can further simplify this equation
$\beta\frac{1}{\varepsilon+1}A^{\frac{\varepsilon+1}{\eta}}\frac{\varepsilon+\eta}{\eta}w^{-\frac{\varepsilon+2\eta}{\eta}} = \frac{1-\eta}{\eta}\frac{1}{w}A^{1\eta} - \frac{1-\eta}{\eta(\varepsilon+1)}A^{\frac{\varepsilon+\eta}{\eta}}w^{-\frac{\varepsilon+2\eta}{\eta}}$
which yields $w^{-\frac{\varepsilon+\eta}{\eta}} = \frac{1-\eta}{\eta}A^{-\frac{\varepsilon}{\eta}}\left[\frac{\beta(\varepsilon+1)+(1-\eta)}{\eta(\varepsilon+1)}\right]^{-1}$
$w = \frac{\eta}{1-\eta}^{\frac{\eta}{\varepsilon+\eta}}A^{\frac{\varepsilon}{\varepsilon+\eta}}\left[\frac{\beta(\varepsilon+\eta)}{\eta(\varepsilon+1)} + \frac{1-\eta}{\eta(\varepsilon+1)}\right]^{\frac{\eta}{\varepsilon+\eta}}$
Finally we can write the wage offer as
$w = A^{\frac{\varepsilon}{\varepsilon+\eta}}\left(\left[\frac{\beta(\varepsilon+\eta)}{\varepsilon+1} + \frac{1-\eta}{\varepsilon+1}\right]\frac{1}{1-\eta}\right)^{\frac{\eta}{\varepsilon+\eta}}$
Therefore
$w = \mu^{\frac{\eta}{\varepsilon+\eta}}w^{*}$ where w^{*} is the perfect equilibrium wage and
$\mu = \left[\frac{\beta(\varepsilon+\eta)}{\varepsilon+1} + \frac{1-\eta}{\varepsilon+1}\right]\frac{1}{1-\eta}$

(c) The bargaining outcome is equal to the perfect equilibrium outcome if the markup $\mu^{\frac{\eta}{\varepsilon+\eta}} = 1$. This holds if $\eta = 0$, that is, labor demand is infinitely elastic (horizontal), or $\varepsilon = \infty$, that is, labor supply is vertical. The markup is one unit also when $\mu = 1$, that is $\frac{1-\beta}{\beta} = \frac{\eta}{1-\eta}\frac{1+\varepsilon}{\varepsilon}$

(d) If $\beta = 0$ then $\mu = \frac{1}{1+\varepsilon}$, which clearly depends on ε.

(e) The problem becomes:

$$\max_L \frac{AL^{1-\eta}}{1-\eta} - wL$$

subject on being on the labor supply, that is, $w = L^{\varepsilon}$

If employers have all the bargaining power (pure monopsony case) then the markup is inversely related to the elasticity of labor supply. As ε decreases, the slope of L^s increases. As a result, the markup imposed by collective bargaining decreases and the wage offer approaches the competitive level. The reason is that as labor supply become more elastic, a change in wages causes a larger change in supply.

2 Minimum Wage

(11) Suppose that w is the wage and L is employment. The supply curve of low wage workers is given by $w = 10 + 2L$. The demand curve is given by $w = 70 - 2L$.

1. What are the equilibrium levels of wage, employment and unemployment?
2. What happens to employment and unemployment if a minimum wage of 40 euros is introduced?
3. What happens to employment and unemployment if a minimum wage of 60 euros is introduced?

Solution -
Labor supply is given as $w = 10 + 2L$. Labor demand is $w = 70 - 2L$.

(a) Equilibrium
$Supply = Demand \ \Rightarrow \ 10 + 2L = 70 - 2L$
which yields
$L = 15, \quad w = 40$
Since labor supplied is equal to labor demanded, there is no unemployment.

(b) If there is a mandatory minimum wage at 40 Euros, then the labor demanded will be
$40 = 70 - 2l \ \Rightarrow \ 2l = 30$
which yields
$L = 15, \quad w = 40$
This is not surprising since the minimum wage is equal to the prevailing market equilibrium wage; neither labor demand nor labor supply will be affected. There is still no unemployment in the labor market.

(c) If there is a mandatory minimum wage at 60 Euros, then the labor demanded will be
$60 = 70 - 2l \ \Rightarrow \ 2l = 10$
which yields
$L = 5, \quad w = 60$
Since at the $w = 60$ labor supply is 25 ($60 = 10 + 2L$), there is unemployment. In particular, the unemployment rate is $\frac{20}{25} = 0.80$.

(12) A firm faces a perfectly elastic demand for its product at a price of 10 euros per unit. The firm is also confronted with an upward-sloping labor supply curve specified as $w = 10 + 2L$, where L is the number of workers hired per hour and w is the hourly wage. Each hour of labor produces 5 products. Assume that the firm only uses labor to produce its products. Also assume that the firm is profit-maximizing.

(a) How many workers should the firm hire each hour?

(b) What wage will the firm pay and how much profit does it make?

(c) What happens to employment and profits if a minimum wage of 40 is introduced?

d) What happens to profits if the firm does not adjust employment in response to the minimum wage?

Solution -
Labor supply is given by $w = 10 + 2L$.

(a) The labor supply implies that total costs are given by $wL = 2L^2 + 10L$. Therefore the marginal labor cost (MLC) function is;

$MLC = 4L + 10$

The value of the marginal product of labor is ($price \times each\ hour\ production = 5 \times 10$) 50. At the equilibrium, the marginal product and marginal cost of labor are equalized. Therefore the profit-maximizing amount of labor is given by;

$50 = 4L + 10 \ \Rightarrow \ L = 10$

(b) The equilibrium wage is, then, given by $w = 10 + 2 \times 10$ or $w = 30$

(c) The minimum wage is clearly binding as, without the minimum wage, the firm would pay $w = 30$. Since the minimum wage is set below the competitive equilibrium wage (that must equal the horizontal labor demand at 50), we are in the region where labor supply is the short side of the market. It follows that employment will read on the labor supply. Solving the labor supply equation for L we obtain that L^s is $\frac{w-10}{2}$ and, given the minimum wage at 40, we have that the new level of employment will be 15, which is higher than without the minimum wage, and there is no unemployment.

In a more general case, one has to impose that employment is on the short-side of the market. The maximization problem that the firm faces should be written as

$Max\ (50 - w)L \quad such\ that \quad w \geq 40 \ \ and \ \ L(demanded) \leq L(supplied)$

We can write the Lagrangian of the maximization problem as follows

$$\Pi = (50 - w)L + \lambda_1(w - 2L - 10) + \lambda_2(w - 40) \tag{1}$$

The corresponding Kuhn-Tucker conditions are

$\Pi_w = -L + \lambda_1 + \lambda_2 = 0 \qquad w \geq 0$

$\Pi_L = 50 - w - 2\lambda_1 = 0 \qquad L \geq 0$

$\Pi_{\lambda_1} = w - 2L - 10 \geq 0 \qquad \lambda_1 \geq 0$

$\Pi_{\lambda_2} = 40 - w \geq 0 \qquad \lambda_2 \geq 0$

Case(1):$\lambda_1 > 0$ and $\lambda_2 > 0$

Since both constraints are binding, we have

$-L + \lambda_1 + \lambda_2 = 0$

$50 - w - 2\lambda_1 = 0$

$w - 2L - 10 = 0$

$40 - w = 0$

Solving 4 equations with 4 unknowns yields

$w = 40$, $L = 15$, $\lambda_1 = 5$ and $\lambda_2 = 10$
This gives a profit of
$Profits = (50 - w)L = 150$
which is clearly lower than without the minimum wage, where $Profits = (50-30)10 = 200$.

(d) If the firm does not adjust its employment level after the introduction of the minimum wage, then its profits will be even lower, that is,
$Profits = (50 - w)L = 10 * 10 = 100$

(13) (Advanced) Consider the problem of a pure monopsonist choosing wages to maximize profits: $\pi = (p - w)G(w)$, where $p > w$ denotes value of the marginal product (the firm operates under constant returns to scale), w is the wage, and $G(w)$ is the aggregate labor supply.

(a) Derive the first-order condition for wages as a function of the relevant elasticities.

(b) How does this wage equation react to changes in productivity?

Suppose further that labor supply is given by $G(w) = (w - b)^2$, where b is the value of leisure (inclusive of any UB), and clearly $w > b$.

(c) Derive the wage equation under this specialization of labor supply and interpret the results.

(d) What happens if a minimum wage of b is introduced?

Solution -
The profit function is given by $\Pi = (p - w)G(w)$.

(a) The first order condition;
$\frac{\partial \pi}{\partial w} = 0 \Rightarrow p\frac{\partial G}{\partial w} = \left(G + \frac{\partial G}{\partial w}w\right)$
By dividing both sides by G and multiplying by w, we obtain
$p\frac{w}{G}\frac{\partial G}{\partial w} = \left(\frac{w}{G}G + \frac{\partial G}{\partial w}\frac{w}{G}w\right)$
Which yields

$$p\frac{w}{G}\frac{\partial G}{\partial w} = w\left(1 + \frac{1}{\varepsilon}\right)$$

$$p\frac{1}{\varepsilon} = w\left(1 + \frac{1}{\varepsilon}\right)$$

$$w = \frac{p}{\varepsilon + 1}$$

Thus the wage is lower than p, the less elastic is the labor supply (the larger is ε), the lower the wage is.

(b) Since p denotes the value of the marginal product, an increase in productivity can be reflected as an increase in p. Therefore as p increases, we can see that wages increase.

(c) Now the labor supply is given as $G(w) = (w - b)^2$.
From (a) we know that FOC is;

$$p\frac{\partial G}{\partial w} = \left(G + \frac{\partial G}{\partial w}w\right)$$

Therefore

$$p2(w - b) = \left((w - b)^2 + 2(w - b)w\right)$$

$$2p - 2w = (w - b)$$

Which yields

$$w = \frac{2}{3}p + \frac{b}{3}$$

As one can see, now the wage is not only a function of the productivity but also of the unemployment benefit. One can think of the unemployment benefit an outside option for employees, so that they can better negotiate in wage bargaining process. As this power increases, the wage offer they receive eventually increases.

(d) Since labor supply exists as long as the wages are higher than b, a minimum wage at b will not affect the results.

3 Unions

(10) Wages in Kumbekistan are set via national agreements, in spite of large within country disparities in economic and labor market performance. In Eastern Kumbekistan labor demand is given by: $L_E^d = 1,000,000 - 20w$ where w is the annual wage, while in Western Kumbekistan is given by $L_W^d = 800,000 - 20w$. Labor supply is the same in each region and there is no interregional mobility of the workforce $L^s = 700,000 + 10w$. Suppose that collective bargaining, involving mainly Eastern workers and employers, impose the wage that clears the market in Eastern Kumbekistan.

(a) What would be the employment and unemployment level in the two regions?

(b) Suppose that there is a labor supply shock, e.g., brought about by migration to the richest region, and hence labor supply in the East is now $L_E^s = 790,000 + 10w$ and national wage contracts are revised accordingly. What happens to employment and unemployment levels in the two regions?

(c) Finally suppose that wage setting is decentralized and workers and firms in the West are allowed to set wages clearing the regional labor market. What would be in such case the wage differential between the two regions? And how large should be the flow of workers from the Western to the Eastern regions to bring this wage differential to zero?

Solution -

(a) Collective bargaining imposes a wage that clears the Eastern region market: $L_E^d = L^s$
$1,000,000 - 20w = 700,000 + 10w$
$w^* = 10,000$
Therefore, in the Eastern region, $w_E^* = 10,000$ and there are 800,000 persons employed and there is no unemployment. On the other hand, in the Western region, wage is the same, but $L_W^s(w_W = 10,000) = 800,000$ and $L_W^d = 600,000$, therefore there are 200,000 unemployed persons in the Western region.

(b) Consequently to the labor supply shock in the Eastern region, the clearing market wage becomes:
$1,000,000 - 20w = 790,000 + 10w$
$w^* = 7,000$
In the Eastern market, there are 860,000 people employed and there's no unemployment; in the Western labor market there are 770,000-660,000=110,000 persons unemployed: with respect to the previous situation, the number of unemployed people decreased.

(c) Now that the bargaining is decentralized, we need to calculate the equilibrium in the Western market:
$L_W^s = L_W^d$
$700,000 + 10w = 800,000 - 20w$
$w_W^* = 3,333$ and $L_w^* = 733,330$. At the same time, in the Eastern region, $w_E^* = 10,000$ and $L_E^* = 800,000$. Hence, we have that the wage differential between the two regions is $\Delta w = 6,667$.

In order to find the number of workers who need to move from West to East to bring the wage differential to zero, we need to set the following system of equations, where x is the number of migrants from the Western to the Eastern region, and solve for x.

$$\begin{cases} w_W = w_E & \\ 700,000 - x + 10w_W = 8000 - 20w_W & \text{equilibrium in the Western region} \\ 700,000 + x + 10w_E = 1,000,000 - 20w_E & \text{equilibrium in the Eastern region} \end{cases}$$

Which yields $x = 100,000$.

(11) Assume that the firm's labor demand curve is given by $w = 120 - 0.02L$, where w is the hourly wage, and L is the level of employment. Assume further that the union's utility objective is given by: $U = wL$

(a) What wage would a monopoly union impose?

(b) How many workers would be employed under the union contract?

Consider now a different objective function for the union. Suppose that the union's utility function is given by: $U = (w - w^*)L$, where w^* is the competitive wage equal to 50 euros per hour.

(c) What wage would a monopoly union demand?

(d) How many workers will be employed under the union contract?

(e) Is your answer different than with the previous specialization of the objective function of the union? Why?

Solution -
Labor demand is $w = 120 - 0.02L$.

(a) We can explain labor demand as function of the wage, that is $L^d = 6,000 - 50w$. Substituting this into the objective function of the union gives $6,000w - 50(w^2)$.
Therefore the union will maximize the utility $U = 6,000w - 50w^2$ with respect to w.
The first order condition is then
$6,000 - 100w = 0 \;\Rightarrow\; w = 60$

(b) Substituting this wage into the labor demand equation, we obtain that employment is 3000.

(c) Now the objective function is $U = (w - w^*)L$. At the optimum, MRS will be equal to the slope of labor demand curve. $0.02 = \frac{w-w^*}{L}$
Since $w^* = 50$ and $L = 6,000 - 50w$, we can write the condition given above as
$0.02 = \frac{w-50}{6,000-50w}$
Which yields $w = 85$

(d) Substituting this wage into the labor demand function yields $L = 1,750$.

(e) The wage and employment levels are different under the specifications for the objective function. The reason for this is that in the second specification the marginal utility of employment to union is diminished at any wage level by w^*. This means that for any given labor level, a union demands a higher wage to attain the same amount of utility. A higher wage, on the other hand, decreases the labor demanded by the firms reducing employment.

1. Strikes reduce the profits of a firm π according to the function $\pi = (60 - 2s)(20 - w)$, where s denotes strike duration and w wages. The resistance of unions, hence strike duration, is a decreasing function of the wage: $s = 40 - w$. Supposing that the employer knows exactly the resistance of unions, which wage should she offer?

   Solution -
   Let's substitute s into the profit function and then maximize profit with respect to wages.

   $\pi = (60 - 2(40 - w))(20 - w) = (-20 + 2w)(20 - w)$

   The first order condition is given by

   $60 - 4w = 0$

   which yields

   $w = 15, \quad s = 25$

(13) (Advanced) Consider a firm that faces a constant per unit price of 1,500 Euros for its output. The firm hires L workers from a union at a daily wage of w, to produce output q, where the production function is $q = \sqrt{L}$, so the marginal product of labor implied by this production function is $1/\sqrt{L}$. There are 324 workers in the union. Any union worker who does not work for the firm can find a nonunion job paying 50 euros per day. The union wants to maximize total earnings for its members.

   (a) What is the firm's labor demand function?
   (b) If the firm is allowed to specify w and the union is then allowed to provide as many workers as it wants (up to 324) at the daily wage of w, what wage will the firm set? How many workers will the union provide? Calculate the output, the profit of the firm, and the total income of the 324 union workers.
   (c) If the union is allowed to specify w and the firm is then allowed to hire as many workers as it wants (up to 324) at the daily wage of w, what wage will the union set to maximize the total income of all 324 workers? How many workers will the firm hire? Calculate the output, the profit of the firm, and the total income of the 324 union workers. Compare this with the result of the previous question.

   Solution -
   The production function is given by $q = \sqrt{L}$.

   (a) The profit of the firm can be written as;
   $\Pi = pq - wL = 1500\sqrt{L} - wL$
   Profit maximization yields; $\frac{\partial \Pi}{\partial L} = 1500\frac{1}{2}\frac{1}{\sqrt{L}} - w = 0$
   which yields $L = \left(\frac{750}{w}\right)^2$

(b) Since the firm will take the first action and determine w, it will consider how many workers the union will provide for this wage level. Let $\bar{w}$ denote the wage offer made by the firm.

Unions utility is then

$U = (\bar{w} - 50)L$

Obviously for any wage above 50, the union will provide all of its workers. If the wage is lower than 50, then no worker will be provided. Since the firm knows this, it will offer the lower wage at which the union offer workers, that is 50 Euros. Union then provides 324 workers.

Output is then $q = \sqrt{324} = 18$

The equilibrium price is 1500.

At wage=96 firm will want to hire $L = \left(\frac{750}{50}\right)^2 = 225$ workers.

Then the profit is $1500 \times 18 - 225 \times 50 = 15,750$.

Total income of workers is $50 \times (225 + 99) = 16,200$

(c) Now the sequence of decisions change so that the union moves first. Let $\bar{w}$ denote the wage offered by the union. Then the firm will want to hire $\left(\frac{750}{\bar{w}}\right)^2$ workers.

Union will maximize the total income of its workers which is given by

$\bar{w}L + (324 - L) \times 50$

where L is the number of workers hired by the firm.

Substituting labor demand in the objective function of the union yields:

$\bar{w}\left(\frac{750}{\bar{w}}\right)^2 + (324 - \left(\frac{750}{\bar{w}}\right)^2) \times 50$

$\frac{750^2}{\bar{w}} + 324 \times 50 - \frac{50 \times (750)^2}{\bar{w}^2}$

Then taking the first order condition yields

$-\frac{750^2}{\bar{w}^2} + \frac{50 \times 2 \times 750^2}{\bar{w}^3} = 0$

yielding

$\bar{w} = 100$

Then the firm will want to hire $\left(\frac{750}{100}\right)^2 = 56.25 \approx 56$ workers.

Output is then $q = \sqrt{56} = 7.5$.

Price again 1500.

Then the profit is $1500 \times 7.5 - 56 \times 100 = 5,650$.

Total income of workers is $56 \times 100 + 50 \times (324 - 56) = 19,000$

4 Discrimination

(10) A firm can use either white or black workers to produce output. The firm faces the following production function: $q = 10L_w + 10L_b$, where q is output, L_w is the number of white workers and L_b is the number of black workers.

(a) If the market wage for white workers is $W_w = 40$, market wage of black workers is $W_b = 35$, and the output produced by these workers sells for a price of 6, how many workers of each type will the firm hire if it expects to produce 100 units of output and is a cost minimizer?

(b) If the firm is discriminatory and has a discrimination coefficient equal to 0.2, how many workers of each type would it hire?

(c) What is the cost of the firm's discriminatory behavior in dollars?

Solution -

Production function is given as $q = 10L_w + 10L_b$

(a) If the firm is a cost minimizer it will hire only black workers because the wage is lower for them. Since there is no difference in term of productivity of the workers the only way to increase the profit is to reduce costs by reducing the wage. $100 = 10 \times L_b$ indicates that only 10 black workers will be hired.

(b) If there is a discrimination coefficient towards black workers then the perceived wage costs will change. A coefficient of 0.2 indicates that perceived wage costs of black workers will be $(1.2) \times 35 \times L_b$ which is equal to $42 \times L_b$. This means that now only white people will be hired because, due to discrimination, the perceived costs of a black worker exceeds that of a white worker. Thus the firm will hire 10 white workers.

(c) In the no-discrimination case, the profit of the firm is $100 \times 6 - 10 \times 35 = 250$. In the case of discrimination, the profits are reduced to $100 \times 6 - 10 \times 40 = 200$. Therefore the cost to the firm of the discriminatory behavior is 50.

(11) The production function of a firm is specified as $q = 10\sqrt{L_m + L_f}$, where L_m and L_f are the number of male workers and female workers employed by the firm. Suppose that the market wage for female workers is 4, the market wage for male workers is 5, and the price unit of output is 8. The firm only needs labor to produce.

(a) How many female workers and male workers would a non-discriminatory firm hire? How much profit does this firm earn?

(b) A firm that discriminates against female workers with a discrimination coefficient of 0.2, how many female workers and male workers will this firm hire?

(c) At what discrimination coefficient would a firm be indifferent between female workers and male workers?

(d) A firm that discriminates against female workers with a discrimination coefficient of 0.5, how many female workers and male workers will this firm hire?

Solution -
The production function is given by $q = 10\sqrt{L_m + L_f}$

The derivative of the production function with respect to L_m and L_f gives the marginal product of labor.

$\frac{\partial q}{\partial L_m} = MP_M = \frac{5}{\sqrt{L_m+L_f}}$

Similarly;

$\frac{\partial q}{\partial L_f} = MP_F = \frac{5}{\sqrt{L_m+L_f}}$

(a) Since the market wage for female workers is lower, the firm will hire only female workers. Profit maximization implies:
$Max\ 80\sqrt{L_f} - 4L_f$
The first order condition is
$\frac{40}{\sqrt{L_f}} = 4 \ \Rightarrow \ L_f = 100$
Then the profit is given by
$\Pi = 80\sqrt{100} - 4 \times 100 = 400$

(b) Discrimination against female workers with a coefficient of 0.20 indicates that the perceived wage cost for a female worker is $(1.20)4 = 4.8$. Since the wage cost is still lower than that of a male worker (which is 5), the firm will hire only female workers.
$Max\ 80\sqrt{L_f} - 4.8L_f$
The first order condition is:
$\frac{40}{\sqrt{L_f}} = 4.8 \ \Rightarrow \ L_f \approx 70$
Then the profit is given by
$\Pi = 80\sqrt{70} - 4.8 \times 70 \approx 333$

(c) The firm will be indifferent between female and male workers if the perceived wage of a female worker is equal to the actual wage of a male worker. This condition is satisfied by $(x)4 = 5$ which gives $x = 1.25$. The firm will be indifferent if the coefficient of discrimination is equal to 0.25.

(d) Discrimination against female workers with a coefficient of 0.50 indicates that perceived wage cost for a female worker is $(1.50)4 = 6$. Since the wage cost is now higher than that of a male worker, the firm will hire only male workers.
$Max\ 80\sqrt{L_m} - 5L_m$
The first order condition is:
$\frac{40}{\sqrt{L_m}} = 5 \ \Rightarrow \ L_m = 64$
Then the profit is given by
$\Pi = 80\sqrt{64} - 5 \times 64 = 320$

(12) Wages for males (w_m) and females (w_f) depend on years of schooling s and years of experience e:

$w_m = 200 + 10s + 5e$

$w_f = 200 + 5s + 3e$

Men have on average 10 years of schooling and 14 years of experience. Women have on average 9 years of schooling and 10 years of experience.

(a) How big is the gender wage gap?

(b) Use the Blinder-Oaxaca decomposition to calculate what share of the gender wage gap is due to discrimination.

(c) What share of the gender wage gap would be due to discrimination if we ignore experience?

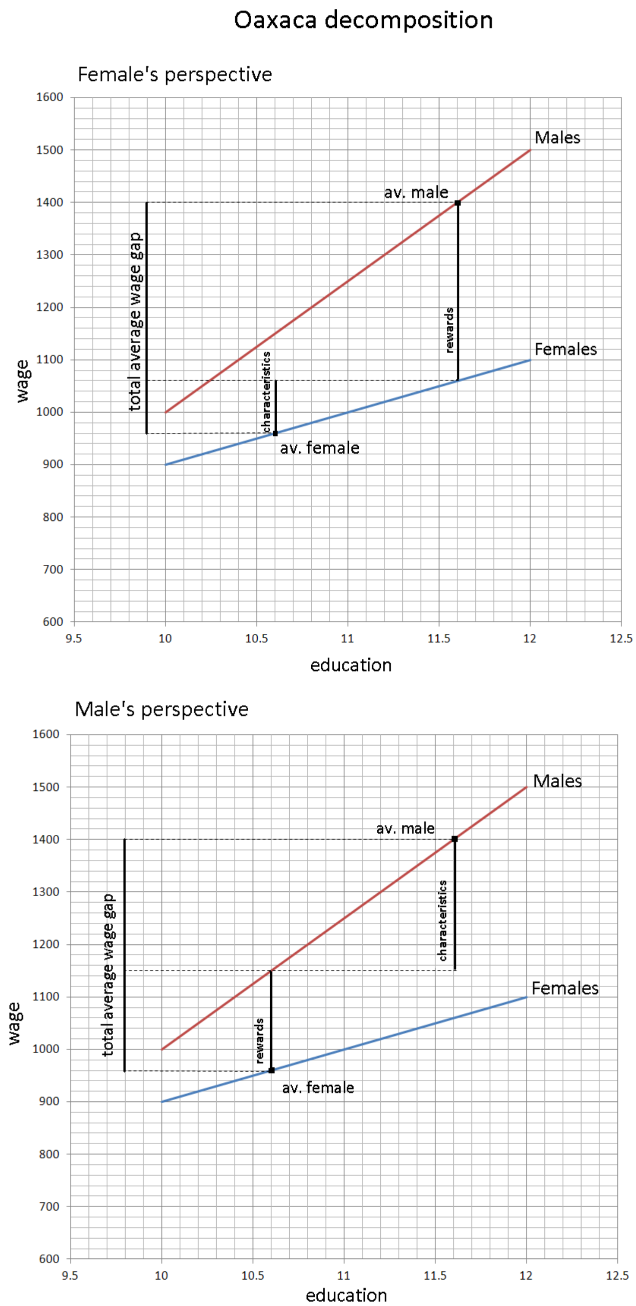
Solution -
Wage profiles for males and females are respectively given by

$w_m = 200 + 10s + 5e$

$w_f = 200 + 5s + 3e$

(a) The male-female wage gap is given by $w_m - w_f$
This is equal to
$w_m - w_f = 5(2s_m - s_f) + 5e_m - 3e_f$
Considering two persons with average characteristics of men and women, we see that the average gap is
$w_m - w_f = 5(20 - 9) + 70 - 30 = 95$

(b) Oaxaca decomposition
$w_m - w_f = (s_m - s_f)10 + (e_m - e_f)5 + 5s_f + 2e_f$
The first two terms are the differences in characteristics and the last two the differences in rewards.
We see that even though a man and a woman have the same years of schooling and experience, their wages differ by $5s_f + 2e_f$ which is solely due to the discrimination. For an average person this is equal to 65 which is 68 percent of the total wage gap (95).



Alternatively, the wage gap can be decomposed from male's perspective as follows,

$$w_m - w_f = (s_m - s_f)5 + (e_m - e_f)3 + 5s_m + 2e_m$$

Which for an average male and female is

$$w_m - w_f = (10 - 9)5 + (14 - 10)3 + 5(10) + 2(14)$$

Once again, the first two terms are the differences in characteristics and the last two the differences in rewards. In this case the wage gap explained by the difference in characteristics is only 17, which is 18 percent of the total gap (95).
The provided graphical representation may help to understand why the two alternative decompositions lead to different results.

(c) If we ignore the experience part of decomposition, the discrimination for an average person will be 45 which is 82 percent of the total wage gap (55).

(13) (Advanced) Suppose that an employer has monopsony power over women but not over men. The labor supply curve of women is given by $L_f = w_f^{\frac{1}{\varepsilon f}}$. The labor supply curve of men is flat. In labor demand ($L^d = (\frac{A}{w})^{\frac{1}{\eta}}$) men and women are perfect substitutes.

(a) Determine the equilibrium level of employment and wages for men and women.
(b) Show that both employment and wages for women go up if the wage for men go up.
(c) What happens to employment and wages for women if the monopsony power over women goes up?

Solution -

The labor supply curve for women is given by $L_f = w_f^{\frac{1}{\varepsilon f}}$.

(a) The equilibrium level of total employment is given by the intersection between male labor supply and labor demand, that is, isolating the wage in labor demand and equating it to the the (flat) labor supply of men, $L_m^s = w_m$:

$$w_m = \frac{A}{L^\eta}$$

For women, labor supply is $L_f^s = w^{\frac{1}{\varepsilon}}$. This gives $w_f = (L_f^s)^\varepsilon$. The marginal cost curve for female workers is, then, $(\varepsilon + 1)(L_f^s)^\varepsilon$. Equilibrium employment of women is given by intersection of the flat male labor supply function and marginal cost of labor for females. This implies:

$w^* = (\varepsilon + 1)(L_f)^\varepsilon$ or $L_f = \left(\frac{w^*}{\varepsilon+1}\right)^{\frac{1}{\varepsilon}}$

Then the wage for females is obtained as $w_f = (L_f^s)^\varepsilon$. This implies $w_f = (\left(\frac{\bar{w}}{\varepsilon+1}\right)^{\frac{1}{\varepsilon}})^\varepsilon$ which gives

$w_f = \frac{w^*}{\varepsilon+1}$

(b) If male wages go up, employment of females increases. Indeed

$$\frac{\partial L_f}{\partial w^*} = \frac{1}{\varepsilon}\left[\frac{w^*}{\varepsilon+1}\right]^{\frac{1-\varepsilon}{\varepsilon}} > 0$$

while wages of women:

$$\frac{\partial w_f}{\partial \bar{w}} = \frac{1}{\varepsilon + 1} \;>\; 0$$

(c) Now assume that monopsony power goes up. The latter can be measured in terms of the inverse elasticity of the labor supply or the parameter ε. The derivative of women wage with respect to ε is negative:

$$\frac{\partial w_f}{\partial \varepsilon} = -\frac{w^*}{(\varepsilon + 1)^2} \;<\; 0$$

While employment of women:

$$\frac{\partial L_f}{\partial \varepsilon} = \left(-\frac{1}{\varepsilon^2}\right) \times L_f \times ln\left(\frac{w^*}{\varepsilon + 1}\right) + \frac{1}{\varepsilon}\left(\frac{w^*}{\varepsilon + 1}\right)^{-1}\left(-\frac{w^*}{(\varepsilon + 1)^2}\right) \times L_f \;<\; 0$$

5 Working Hours

(12) Suppose that hours and workers in a firm are combined in such a way that the isolabor function is $y = L\sqrt{h}$, where L denotes the number of workers and h is the number of hours.

(a) Display the iso-labor curve and compute the elasticities of labor with respect to L and h.

Suppose further that labor costs are given by $C = L(F + wh)$, where w is the hourly wage, and F denotes fixed costs per worker.

(b) Obtain the cost minimizing choice of hours and workers for a given level of output, say $\bar{y}$

(c) Interpret the results, notably the relationship that the two factor demand functions have with the scale of production.

Solution -

(a) The elasticity of output with respect to workers is 1, while the elasticity with respect to hours is $\frac{\partial y}{\partial h}\frac{h}{y} = \frac{1}{2}\frac{L}{\sqrt{h}}\frac{h}{L\sqrt{h}} = \frac{1}{2}$

(b) The cost minimizing h and L choices are given by $h^* = \frac{F}{w}$ and $L^* = yh^{*-\frac{1}{2}} = \frac{y}{\sqrt{\frac{F}{w}}}$.

Notice that the demand for hours is independent of y, unlike the demand for workers. This implies that cost-minimizing employers facing a reduction of the scale of production (y), e.g. during a recession period, will tend to reduce the number of workers rather than the hours of work.

(13) (Advanced) Let the isolabor curve be $y = g(L, h) = Lh^\alpha$ where L denotes workers and h hours, while y is an arbitrary constant. There is an overtime premium, $\omega > w$, so that the cost of production is given by

$$C = \begin{cases} (F + wh)L & \text{if } h \leq \bar{h} \\ (F + \omega(h - \bar{h}) + \bar{h}w)L & \text{if } h > \bar{h} \end{cases}$$

where $\bar{h}$ denotes normal hours.

(a) Please derive the optimal levels of h and L if there is no overtime premium.

(b) Please derive the optimal level of h and L when $\omega > w > 0$.

(c) Please display graphically the cases $h^* < \bar{h}$, $h^* = \bar{h}$ and $h^* > \bar{h}$.

Solution -

(a) If there is no overtime premium, $\omega = w$. Therefore the minimizing problem is min $(F + wh)L$ with respect to h and L, such that $\bar{y} = Lh^{\alpha}$. The cost minimizing h and L levels are given by:

$h^* = \frac{\alpha}{1-\alpha}\frac{F}{w}$

$L^* = yh^{*-\alpha} = y(\frac{\alpha}{1-\alpha}\frac{F}{w})^{-\alpha}$

(b) Assume now that $\omega > w$, the cost minimizing problem is min $(F + wh)L$ with respect to h and L, such that $\bar{y} = Lh^{\alpha}$.

In the case where $h^* < \bar{h}$, the cost minimizing choice of hours does not involve overtime work, hence the solution is the same as in (a):

Consider now the case where instead $h^* > \bar{h}$. The cost minimizing problem is min $(F + \omega(h - \bar{h}) + \bar{h}w)L$ with respect to h and L, such that $\bar{y} = Lh^{\alpha}$.

If we substitute L from the constraint into the objective function we obtain $(F + \omega(h - \bar{h}) + \bar{h}w)\frac{y}{h^{\alpha}}$.

Taking the first order condition with respect to h yields

$h^* = \frac{\alpha}{1-\alpha}\frac{F - w\bar{h} + \omega\bar{h}}{w}$

and

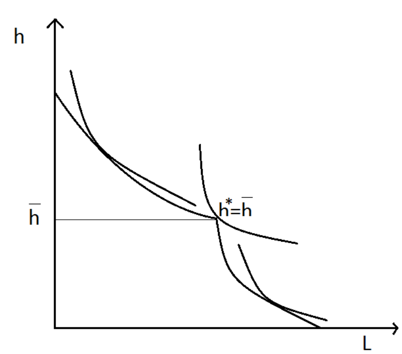
$L^* = y(h^*)^{-\alpha}$

Notice finally that the condition $h^* > \bar{h}$ implies that

$h^* = \frac{\alpha}{1-\alpha}\frac{F - w\bar{h} + \omega\bar{h}}{w} > \bar{h}$.

or

$\frac{\alpha F}{w - \alpha\omega} > \bar{h}$



(c)

6 Retirement

(11) Joe has worked until reaching the age of 60. He now has two options. The first is to work for another 5 years, earning 40,000 euros per year, retire at age 65, and collect a pension of 10,000 euros per year for the following 15 years. The second option is to retire immediately and collect a yearly pension of X euros for the next 20 years. Suppose that a euro received today is worth 1.05 euros received next year.

(a) What value of X gives the worker the same total income (earnings and retirement benefits) in net present value terms in the two options?

(b) What value of X gives the worker the same pension wealth in the two options?

(c) Consider a state-provided medical insurance which is provided for free to persons as long as they continue to work up to the age of 65. Those not working under 65 years old can purchase this health insurance for 5,000 euros per year. If Joe values retiring at age 60 over age 65 at 200,000 euros, for what values of X would he retire at age 60?

Solution -
Note that the discount rate is 5 percent and constant over time, therefore the discount factor is 0.95.

(a) It is the value of X such that:
$\sum_{t=0}^{4} 40000 \times 0.95^t + \sum_{t=5}^{19} 10000 \times 0.95^t = \sum_{t=0}^{19} X \times 0.95^t$
Using the geometric series properties, it can be rewritten as:
$40000(\frac{1-0.95^5}{1-0.95}) + 10000(\frac{0.95^5-0.95^{20}}{1-0.95}) - X(\frac{1-0.95^{20}}{1-0.95}) = 0$
Which yields
$X \approx 20640$ euros.

(b) It is the value of X such that
$\sum_{t=5}^{19} 10000 \times 0.95^t = \sum_{t=0}^{19} X \times 0.95^t$
Applying geometric series properties:
$10000(\frac{0.95^5-0.95^{20}}{1-0.95}) - X(\frac{1-0.95^{20}}{1-0.95}) = 0$
Which yields $X \approx 6475$ euros.

(c) It is the value of X such that
$\sum_{t=0}^{4} 40000 \times 0.95^t + \sum_{t=5}^{19} 10000 \times 0.95^t = \sum_{t=0}^{19} X \times 0.95^t - \sum_{t=0}^{4} 5000 \times 0.95^t + 200000$
Which yields
$X \approx 91189$ euros.

(12) Consider a worker who is currently 65 years old and expects to live until the age of 85. Her current and future wage is 25,000 euros per year. She has the option to retire immediately or postpone retirement up to the age of 70. If she does retire immediately, she gets a yearly pension of 10,000 euros. If she postpones retirement until the age of 66, she can obtain a higher pension, say 12,000 euros per year. Retiring at the age of 67 she receives 14,000 euros, at 68 16,000 euros, at 69 18,000 euros, and at 70 (when she is, in any event, compelled to retire) she can get 20,000 euros per year. Assume that after retirement, the pension is the

only source of income and that the utility function of the individual is given by $U = R^{\alpha} Y^{1-\alpha}$ where R denotes the total number of years of retirement, Y is the net present value of her (remaining) lifetime income and $0 < \alpha < 1$. Assume for simplicity that the market (and subjective) discount factor is one.

(a) Tabulate the net present value of income and the pension wealth for this individual at different levels of R.

(b) Display the leisure (retirement) income trade-off for this individual, putting the retirement age on the horizontal axis.

(c) At which levels of α does the individual decide to retire at the age of 65? At which levels of α will she retire at 70?

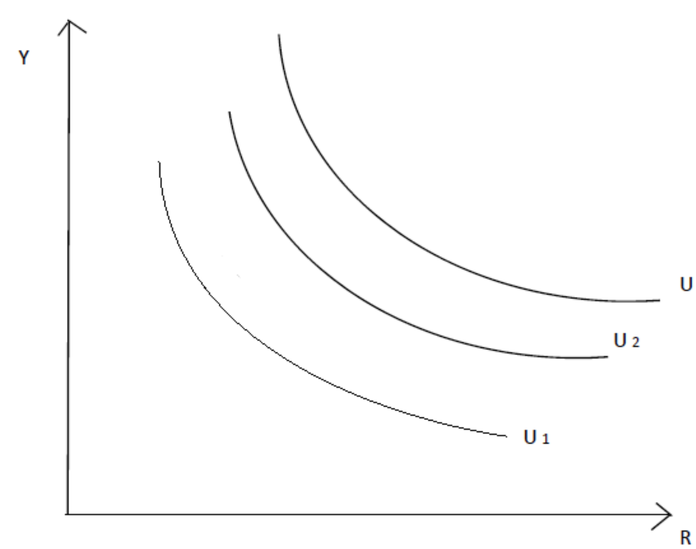
(d) Supposing that $\alpha = .9$, which accrual rate of pensions (increase in the amount of yearly pension if the individual postpones retirement by one year) would be required to induce the worker to postpone retirement until the age of 70?

Solution -
The utility function is given by $U = R^{\alpha} Y^{1-\alpha}$.

(a) Pension Wealth at different ages of retirement:

Retirement Age	R	Years in Work	Yearly Wage	Total Wage	Yearly Pension	Total Pension	Y
65	20	0	25000	0	10000	200000	200000
66	19	1	25000	25000	12000	228000	253000
67	18	2	25000	50000	14000	252000	302000
68	17	3	25000	75000	16000	272000	347000
69	16	4	25000	100000	18000	288000	388000
70	15	5	25000	125000	20000	300000	425000

(b) Leisure (Years in retirement) and total earnings



(c) Given the utility function of the worker, she will prefer to retire immediately if:
$20^\alpha(200000)^{1-\alpha} \geq 19^\alpha(253000)^{1-\alpha}$
Taking the natural logarithm of both sides and simplifying the inequality yield
$0.12\alpha \geq 0.10$
Therefore she will retire at the age of 65 for
$\alpha \geq 0.82$
A similar condition can be written for retirement at the age of 70
$15^\alpha(425000)^{1-\alpha} \geq 16^\alpha(388000)^{1-\alpha}$
Again, taking the natural logarithm of both sides and simplifying:
$\alpha \leq 0.57$

(d) The worker will postpone her retirement until 70 if the utility she gets if she retires at 70 is higher than the utility she gets if she retires at 69.
For an accrual rate of r, the total income she gets if she retires at 70 (69) is a yearly wage of 25000 for 5 years and a pension of $10000(1+r)^5$ for 15 years (a yearly wage of 25000 for 4 years and a pension of $10000(1+r)^4$ for 16 years), the years of pension (R) are 15 (16).
Given her preferences, with $\alpha = 0.9$, she will prefer to retire at 70 if

$$(15)^{0.9}(15 \cdot 10000(1+r)^5 + 5 \cdot 25000)^{0.1} \geq (16)^{0.9}(16 \cdot 10000(1+r)^4 + 4 \cdot 25000)^{0.1}$$

or

$$r \geq 93.2\%$$

This accrual rate would imply that the pension benefit almost doubled for each additional year spent working, it is therefore too high to be realistic. The reason is that with $\alpha = 0.9$ the years of leisure are deemed very important with respect to the total income.

(13) (Advanced) In 1996 Italy and Sweden both introduced a NDC pension system. There are important design differences between the two regimes. The so-called transformation coefficients are higher in Sweden than in Italy. Moreover, Italian pensions are kept constant in real terms, independently of wage growth, whereas in Sweden they are not fully indexed to inflation when wage growth falls below a certain level (right now it is 1.6 percent).

(a) In your view what is the rationale behind such differences in the design of pension systems?

(b) What are their implications for the induced age of retirement?

(c) How do they affect the pension expenditure to GDP ratio over the business cycle?

Solution

(a) The transformation coefficients in a NDC system convert the capitalized contributions into a flow of pension benefits that depend on the life expectation at retirement and provide for a return related to the potential growth of the economy. This return is front loaded, that is, provided immediately as it is included in the initial pension benefits.

Once established the annuity, the pensions to be paid until death, this amount is adjusted each year to take into account of inflation and possibly changes in the tax base of pensions, the wages earned by the current contributors.
In the Swedish NDC scheme, pension benefits are indexed to inflation by a factor that depends on the growth of average real wages. If real wages (and hence presumably productivity) increase more than a certain level (1.6 per cent, considered the potential growth rate), then pensions are more than fully indexed to inflation. In other words, when wages (output) grow more than the potential, pensions take a fraction of the increase in real wages. When instead real wages growth falls short of 1.6 per cent, then pensions are allowed to decline in real terms. This happened, for instance, during the Great Recession. This approach is consistent with the fact that the growth of potential output has already been taken into account when defining the annuity. In Italy, instead, pensions are fully indexed to inflation, that is, they are constant in real terms, independently of the growth rate of the economy and the dynamics of the labor market. Thus, pensions have not been reduced during the Great Recession. Moreover, as the potential output growth of Italy was in the past quite systematically overestimated, this approach allowed for a growth of the ratio of average pensions over average wages. The differences between the two systems are likely to reflect the different representation of pensioners in the political process in the two countries. In Italy, pensioners represent more than 50 per cent of union members.

(b) The indexation to prices only in Italy opens up the possibility that two subsequent cohorts of pensioners are treated much differently, the phenomenon of the so-called "vintage pensions". In Sweden, instead, pensions are not fully insulated from output growth and labour market developments and hence the level of pensions cannot differ toomuch among subsequent generations. This makes the system more neutral as to the age of retirement. In a country like Italy, a perspective low growth of the economy, induces individual to retire as early as possible, to get a higher pension benefit.

(c) Both in Italy and Sweden, the pension expenditure to GDP ratio tends to increase during the recession, as the denominator falls more than the numerator. However, in Italy, the increase is more market as the value of pensions is always preserved in real terms.

7 Family Policies

(12) Consider a couple ranking purchased goods and services C and home-produced goods and services D as follows:

$$U = CD, \tag{2}$$

where D is produced via a decreasing returns-to-scale technology $f(h_D) = \sqrt{h_D}$, using as input time h_D subtracted from market work h. Suppose further that the individuals can allocate 100 hours per week to either market work or home production.

(a) What is the optimal allocation of time between market and home production when the wage is 10 euros per hour?

(b) Suppose that now the couple has a child and, given the extra value of the time spent with the child, the joint utility becomes

$$U = CD^2. \tag{3}$$

At the same time, any hour spent away from the child involves a cost of 5 euros to be paid to a babysitter. How does this affect the allocation of time of the family?

(c) Would your answer differ if home production technologies improve, (e.g., as a result of the introduction of disposable diapers and microwaves ovens), so that household production becomes $f(h_D) = h_D^{.8}$?

Solution -

(a)

$$U = CD \text{ and } D = \sqrt{h_D} \text{ therefore } U = C\sqrt{h_D}$$

$$C = wh + m$$

Using the time constraint $h = l_0 - h_D$, and imposing $m = 0$,

$$C = 10(l_0 - h_D)$$

or, since $l_0 = 100$

$$C = 1000 - 10h_D$$

$$MRS = \frac{0.5C/\sqrt{h_D}}{\sqrt{h_D}} = \frac{0.5C}{h_D}$$

Imposing that the MRS equals the hourly wage and then exploiting the budget constraint

$$\frac{0.5C}{h_D} = 10$$

$$h_D = 33 \text{ and } h = 67$$

(b)

$$U = CD^2 \text{ and } D = {h_D}^{0.5} \text{ therefore } U = Ch_D$$

$$C = 10(100 - h_D) - 5h = 500 - 5h_D$$

$$MRS = w \text{ so } \frac{C}{h_D} = 5 \text{ so } 5h_D = 500 - 5h_D$$

$$h_D = 50 \text{ and } h = 50$$

(more hours dedicated to home production, less hours to labor market activities)

(c)

$$U = CD^2 \text{ and } D = {h_D}^{0.8} \text{ therefore } U = C{h_D}^{1.6}$$

$$MRS = w \text{ so } \frac{1.6C{h_D}^{0.6}}{{h_D}^{1.6}} = 5$$

and plugging C into the budget constraint

$$h_D = 61.5 \text{ and } h = 38.5$$

(13) (Advanced) Consider a household where the husband can earn a market wage of 25 euros per hour or produce at home the equivalent of 10 euros. The wife can instead earn 20 euros per hour or produce domestically the equivalent of 15 euros. Both members of the household have a total allocation of time to either market work or home production of 50 hours per week.

(a) Does it make sense for both members of the couple to work for pay?

(b) Would an increase of the wife's wage affect her decision? Would it affect the decision of the husband?

Suppose that now that the productivity of the husband and the wife are dependent on the amount of time spent in home production by the other partner. How does this affect their allocation of time if:

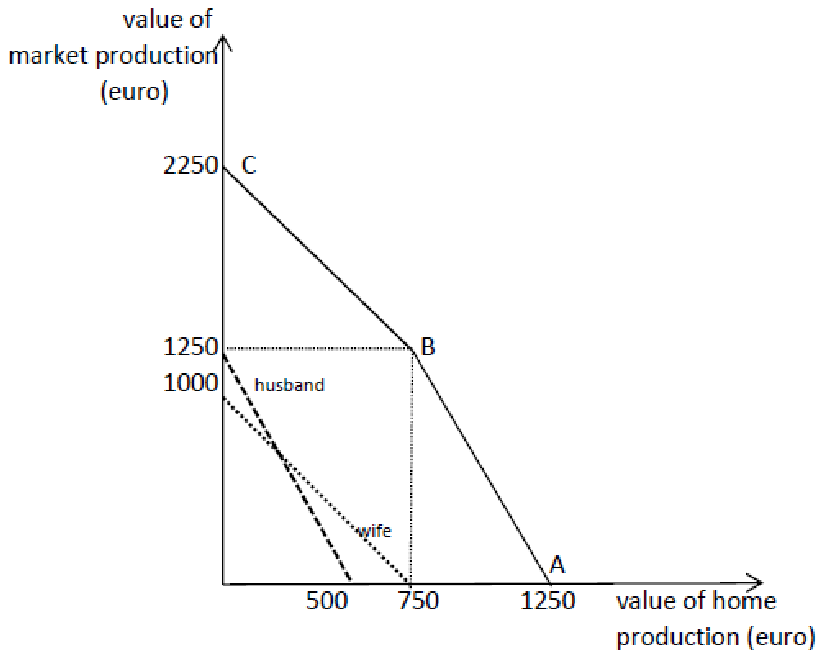
(a) The husband and wife are substitutes in home production?

(b) The husband and wife are complements in home production?

(c) Their decision is based on bargaining within the household?

Solution -

a) The husband has a competitive advantage in market work while the wife in home production. This can be observed by depicting the family's opportunity frontier (segment ABC), that is, the combination of husband and wife's feasibility sets. This family set of consumption opportunities is composed by two segments: AB, along which the wife devotes all her time to home production, while the husband allocates time to both market and home production, and BC, along which the husband devotes all his time to market production while the wife allocates some time to market work and some to home production.

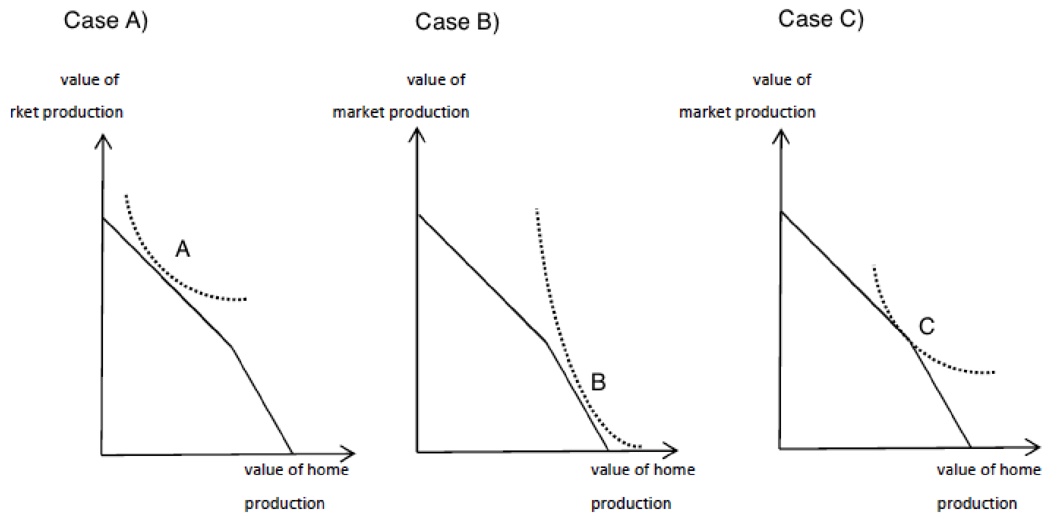


Depending on preferences of the household, there are three possible outcomes:

- Case A: the husband devotes all his time to work, the wife works in the market *and* at home

- Case B: the husband works in the market *and* at home, the wife devotes all his time to the home production

- Case C: we have a complete specialization, the husband does only work in the market, and the wife works at home.



(b) Specialization in the market is driven by the wage differential between the spouses. Even if the two were equally productive at home, the family could expand his set of opportunities if the lowest salary spouse devoted the majority of her/his time to home production. If the salary of the wife increases, the wage gap is reduced, leading to less specialization within the household.

(a) When the husband and the wife are substitutes in home production, then spouses will “specialize" regardless of their preferences. That is, both spouses will not allocate time to both home production and the market sector.

(b) On the other hand, complementarity of husband and wife in home production means that the degree of substitution between the two decrease, until being null when they are perfect complements, implying a smaller and smaller degree of specialization within the household.

(c) Intra-household bargaining refers to negotiations that occur between the spouses in order to arrive at decisions regarding the allocation of time and consumption within the household. In such models, the outcome mainly depends on the bargaining power of the spouses, that can be represented as the value of their outside option (divorce), that in turn depends on their own endowment.

8 Education and Training

(10) Suppose that Andrea's wage-schooling locus is given by:

Years of schooling	Earnings
6	12,000
7	15,360
8	19,200
9	22,200
10	24,420
11	26,400
12	27,720
13	28,680
14	28,800

(a) Derive the marginal rate of return schedule. When will Andrea quit school if his discount rate is 5 percent? What if it is 15 percent?

(b) Suppose now that the government imposes an income tax of 25 percent on both labor earnings and interest income. What is the effect of this income tax on Andrea's educational attainment?

Solution -

(a) The Marginal Rate of Return (MRR) schedule is given below

Years of Schooling	Earnings	Change in Earnings	MRR	MRR$\times$ 100
6	12000			
7	15360	3360	0.28	28.0
8	19200	3840	0.25	25.0
9	22200	3000	0.16	15.6
10	24420	2220	0.10	10.0
11	26400	1980	0.08	8.1
12	27720	1320	0.05	5.0
13	28680	960	0.03	3.5
14	28800	120	0.00	0.4

If his discount rate is 5%, Andrea will quit school after 12 years of schooling. If his discount rate is 15%, Andrea will quit the school after 9 years.

(b) If the tax is imposed on both interest incomes and the wage earnings, Andrea will not change his educational attainment. The reason is that the tax is imposed on both income channels and will make no difference on Andrea's incentives. What would make a difference in Andrea's choice, for example, is a higher tax on labor earnings. In this case Andrea could stop receiving education earlier and use the money to earn interest instead of using it in education because labor earnings would be taxed more heavily than interest earnings.

(11) Tom, a 18-year-old boy, has to decide how many more years to study. He knows that, if he stops studying now, he will get a wage $w_0 = 1200$ euros; if, instead, he decides to invest s more years in education, his wage will be $w^n = w_0 + 600log(1 + s)$. However, if Tom decides to study, he knows he will have to pay a yearly school fee of 120 euros.

(a) Given the above data, how many more years will Tom stay in school? Given this choice, what is his expected wage?

Suppose that a reform of the education system is implemented in Tom's country: school fees are fixed to a lower level of 75 per year, but education becomes lower quality, so that the expected wage after n years of school is now $w^n = w_0 + 300log(1 + s)$.

(b) How will the reform affect Tom's decision on his optimal investment in years of education?

(c) How many more years will he spend in school?

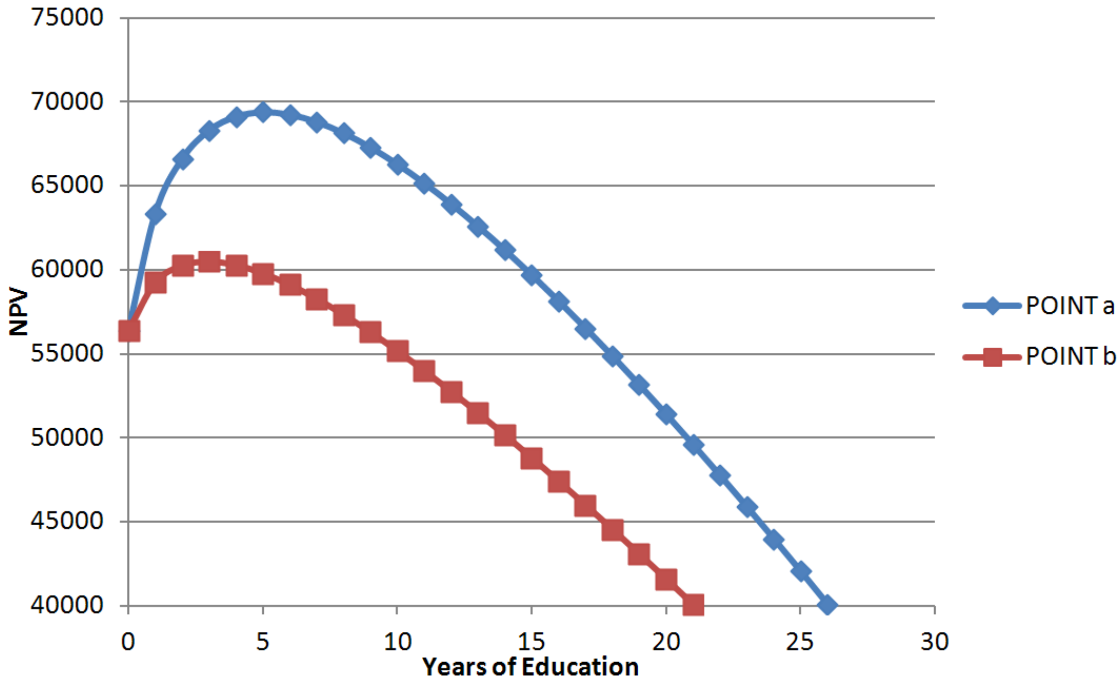
(d) What is his expected wage now?

Solution -

(a) Assuming that he will work until the age of 65 and that the interest rate is 0, it is possible to find out the optimal choice. The present value of its total earnings is given by the sum of the wages for the whole working period ($65 - 18 - s$ years) minus the total cost of schooling ($120s$).

$$NPV(s) = (65 - 18 - s) \cdot (1200 + 600log(1 + s)) - (120s)$$

The diagram below shows the NPV as a function of s. Since $NPV(4) = 69153.4$, $NPV(5) = 69409.4$ and $NPV(6) = 69269.4$, $s = 5$ is a maximum at least in its neighborhood. Since the second derivative is always negative in the domain, the equation is concave and only has one (global) maximum that corresponds to the previously identified local maximum s=5.



(b-c) The answer can be found applying the same reasoning of the previous point,

$$NPV(s) = (65 - 18 - s) \cdot (1200 + 300log(1 + s)) - (75s)$$

The maximum, as the diagram shows, corresponds to 4 years of schooling.

(d) In point a wage was $1200 + 600 * log(1 + 5) = 1666.9$, while in point b it was $1200 + 300 * log(1 + 4) = 1409.7$.

(12) Suppose that there are two types of workers in the economy: low productivity workers (proportion 0.75 of the population) and high productive workers (proportion 0.25). Low productive individuals have a lifetime productivity of 300 euros, while highly productive workers have a lifetime productivity of 500 euros. The employer knows the two proportions and the relative productivity, but, at hiring, cannot distinguish a high productivity from a low productivity worker.

(a) Which wage would a competitive employer pay in this context?

Consider now schooling. For low productivity workers, a year of education costs 25 euros, while for an highly productive individual it costs only 16 euros. Education has no effect on productivity.

(b) What is the minimum educational attainment (in terms of years of schooling) that an employer can impose, in order to be able make sure that only high productivity workers would apply for these jobs?

(c) Suppose that also jobs not requiring any educational attainment are offered. Who loses and who gains from this policy of the employer?

Solution -

(a) If productivity is visible, workers are paid a wage that equals their lifetime productivity, therefore, $w_L = 300$ (low productivity workers) and $w_H = 500$ (high productivity workers). If the employer doesn't know the workers' productivity, he will offer a weighted average of the two productivities, where weights are given by the proportions of the two groups in the population:

$500 \times 0.25 + 300 \times 0.75 = 350$

(b) Since the new contract will exclude low productivity workers, the pay will be equal to the productivity of the high productivity workers, i.e. 500 euros. With this wage, the minimum educational attainment that ensures that only high productivity will apply has to be such that a low productivity worker would not earn any surplus from that contract, hence, since the outside option of the low productivity workers is 0,

$500 - 25s = 0$, or $s = 20$

High productivity workers will earn a surplus of

$500 - 16s = 180$

Low productivity won't work and won't study, their surplus will therefore be 0.

(c) In this case, the employer will offer to low productivity workers a contract with no educational requirement and will pay them their productivity, i.e. $s_L = 0$ and $w_L = 300$. The high productivity workers will still be paid their productivity ($w_H = 500$), but now the education required to ensure that low productivity workers won't apply for this contract will be lower because their outside option is now 300, s_H should be such that

$500 - 25s_H = 300$,or $s_H = 8$

The surplus of high productivity will now be

$500 - 16 \cdot 8 = 372$

And low productivity workers will now earn a surplus of 300, they will therefore be both better off.

(13) (Advanced) John is a 16 years old student and has to decide whether or not to leave school and start working or to study for another year. If he leaves school, he will earn 45,000 Euro per year for the rest of his working life. If he increases his level of education with one additional year of studies, he would earn 50,000 Euro per year for the rest of his working life. Attending school for one more year will cost him 30,000 Euro. John plans to work until reaching the age of 70. Assume that John uses a discount rate of 5 percent per year:

(a) Show that it is profitable for John to attend an extra year of education

(b) Suppose that further years of education would increase John's annual earnings by 5,000 Euro per year of education. However, the costs of additional years of education are also increasing according to $15,000s^2$, where s denotes the additional years of education. How many additional years of education should John acquire?

Solution -

The discount rate is 5 percent.

(a) If he leaves the school he will earn

$45000 + \frac{45000}{1.05} + \frac{45000}{1.05^2} + ... + \frac{45000}{1.05^{53}}$

Using the properties of geometric series:

$\sum_{t=0}^{53} \frac{45000}{1.05^t} = 45000\frac{1-0.95^{54}}{1-0.95} \approx 843,595$

If he studies one more year

$-30000 + \sum_{t=1}^{53} \frac{50000}{1.05^t} = -30000 + 50000\frac{0.95-0.95^{54}}{1-0.95} \approx 857,328 > 843,595$

(b) Now the earning profile changes. s years of education will yield $45000 + 5000s$. The total cost of receiving s years of education is $15000s^2$.

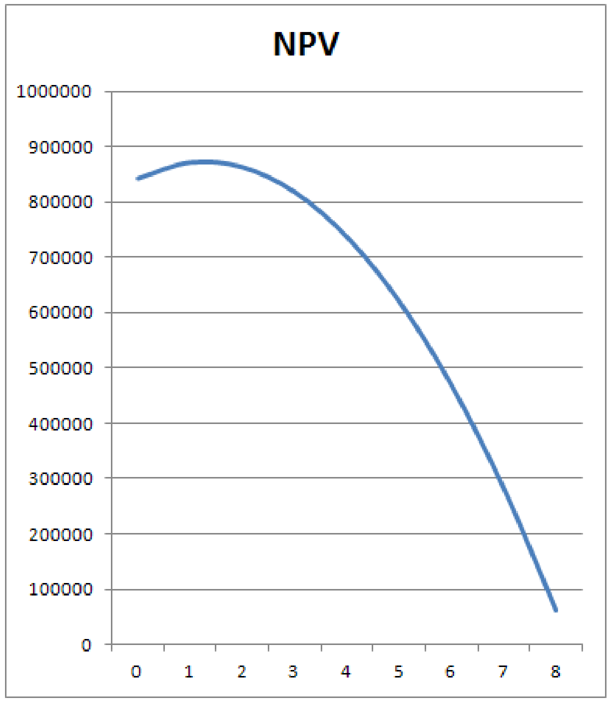
Therefore John will chose s such that it maximizes the net present value of its earnings

$NPV = -15,000s^2 + \sum_{t=s}^{53} \frac{45,000+5,000s}{1.05^t}$

Due to the presence of non-linearities, there is not a simple analytic solution. Solving the problem numerically, one obtains:

s	NPV
0	843,595
1	872,328
2	863,811
3	818,643
4	737,384
5	620,552
6	468,630
7	282,064
8	61,272
...	...

Therefore, given the yearly schooling costs, the expected wage, and the length of his working life, John will choose to acquire 1 additional year of education.


NPV
1000000
900000
800000
700000
600000
500000
400000
300000
200000
100000
0
0
1
2
3
4
5
6
7
8

9 Migration

(11) A family - husband, wife, and two young children - reside in southern Italy: the husband currently earns 30,000 euros per year, whereas the wife does not work. The couple is considering moving to northern Italy, where the husband could find a new job paying him 35,000 euros per year, and the wife could start working at 15,000 euros per year. Nonetheless, the family likes the South much better: living there has a monetary utility in present value terms of 35,000 euros. Moreover, should the wife start to work, the family would have to pay childcare costs equal to 10,000 euros per year.

(a) If the family is supposed to live in northern Italy for the current and the next 3 years and has a discount rate of 12 percent, is it convenient for the family to (immediately) migrate to the north?

(b) Would your answer be different if childcare was cheaper, e.g. equal to 9,000 euros per year?

(c) At which discount rate will the family change opinion about moving to Northern Italy?

Solution -
Note that there is no moving costs in the question.

(a) The earnings for two options are
No migration:
$30000 + 35000 + \frac{30000}{1.12} + \frac{30000}{1.12^2} + \frac{30000}{1.12^3} = 137053$
Immediate migration:
$40000 + \frac{40000}{1.12} + \frac{40000}{1.12^2} + \frac{40000}{1.12^3} = 136072$
The family will not migrate to the northern Italy.

(b) If childcare costs are 9000 euros per year instead of 10000, the earnings in the case of immediate migration are $139473 > 137053$: the family immediately migrates.

(c) Family will choose to move to the North if the expected earnings from moving is higher than not moving. Using the earning given in (a), this condition can be written as:
$65000 + \frac{30000}{x} + \frac{30000}{x^2} + \frac{30000}{x^3} \leq 40000 + \frac{40000}{x} + \frac{40000}{x^2} + \frac{40000}{x^3}$
Where r is the discount rate and $x = 1 + r$.
Simple rearranging yields
$2.5x^3 \leq x^2 + x + 1$
This inequality holds for $x \leq 1.097$ indicating that the family chooses to move to the North if the discount rate (r) is lower than 9.8%.

(12) Suppose that a worker with an annual discount rate of 10 percent resides in the Netherlands and is considering whether to stay there or to move to Italy. there are three work periods left in her working life and pensions are independent of earnings. If the worker remains in The Netherlands, he will earn 40,000 euros per year in each of the three periods. If he moves to Italy, he will earn 44,000 in each of the three periods. What is the highest cost of migration that the worker is willing to incur and still migrate?

Solution -
There are only three periods;

No-migration has a payoff

$40000 + \frac{40000}{1.1} + \frac{40000}{1.1^2} = 109421$ Migration with a cost of C has a payoff

$(44000 - C) + \frac{44000}{1.1} + \frac{44000}{1.1^2} = 120363 - C$

Therefore the highest cost would be $C = 120363 - 109421$ which is 10942.

(13) (Advanced) John is a young worker who faces a decision on whether or not to migrate from The Netherlands to Italy. If he stays in The Netherlands he will earn 45,000 euros per year for the rest of his life. If he moves to Italy he will earn 50,000 euros per year for the rest of his life. Moving from the Netherlands to Italy will cost him 50,000 euros.

(a) If John uses a discount rate of 5 percent per year, show that it is profitable for John to move to Italy.

(b) At what discount rate would John be indifferent between staying and moving?

(c) At what level of level of moving costs would John be indifferent between staying in the Netherlands or moving to Italy?

(d) Suppose that in The Netherlands (in Italy) there is a 4% (2%) probability of losing a job per year and that those losing a job earn for the rest of their life a social assistance of 10,000 euros (5,000 euros). Would John migrate under these conditions?

(e) Under these conditions, at which probability of losing a job in Italy would John be indifferent between moving or remaining in The Netherlands?

Solution -
Assuming that John's working life is sufficiently long, we can use the properties of geometric series.

(a) The net present value of migration in this case is $\frac{w_F - w_H}{i} - C_0 = \frac{50{,}000 - 45{,}000}{0.05} - 50,000 = 50,000 > 0$, therefore it is profitable for John to migrate.

(b) It is the interest rate i such that NPV of migrating equals 0. Therefore $\frac{50{,}000 - 45{,}000}{i} - 50,000 = 0$, which yields $i = 10\%$.

(c) It is the moving cost C_0 such that moving gains equals moving costs, that is $\frac{50{,}000 - 45{,}000}{0.05} = C_0$, which yields $C_0 = 100,000$.

(d) Calling p_N the probability of losing a job in the Netherlands, p_I the probability of losing a job in Italy, SA_N and SA_I the social assistance in the two countries, the NPV of migrating is calculated as:

$NPV = \frac{w_I(1-p_I) + SA_I p_I - w_N(1-p_N) + SA_N p_N}{i} - C_0 =$

$= \frac{50{,}000(0.98) + 5{,}000(0.02) - 45{,}000(0.96) + 10{,}000(0.04)}{0.05} - 50,000 = 76,000 > 0$

John is still moving to Italy.

(e) We need to find p_I such that

$$\frac{50{,}000(1-p_I)+5{,}000(p_I)-45{,}000(0.96)+10{,}000(0.04)}{0.05} - 50,000 = 0$$

Which yields

$p_I = 10.44\%$

10 EPL

(11) Consider a country in which firms produce output (assumed to be the numeraire good) using labor L as the only production factor, with the technology $Y = f(A^i, L)$ where A^i is a technology parameter that fluctuates with the economy. It can take the value $A^b = 100$ in bad times, which occur with probability $p = 2/3$, and value $A^g = 300$ in good times, which occur with probability $1 - p = 1/3$. In the labor market wages are rigid and fixed to be $w = 10$. Assume no type of employment protection is in place in the country, so that firms can adjust their stock of labor at any time by hiring and firing workers at will. Compute the equilibrium levels of employment, wages and profits in good and bad times, and their averages, for each of the following specifications for the production function:

(a) $Y = A^i \times \log L$

Assume that employment protection is introduced: it is now unboundedly costly, for firms, to adjust the stock of labor.

(b) How do employment and wages change?

(c) Which of the two scenarios (no EPL versus EPL) is more profitable for firms?

(d) And by how much?

(e) Interpret these results

Solution -
The production function is given by $Y = f(A^i, L)$.

(a) For $Y = A^i \times \log L$;

Firms can adjust immediately employment to changes in technological conditions. In good times;

$Max_L\ 300 \log L - wL$ The first order condition yields $L_g = 30$

$Profits(\Pi_g) = 300 \log 30 - 10 \times 30 = 720$

In bad times;

$Max\ 100 \log L - wL$

The first order condition yields

$L_b = 10$

$Profits(\Pi_b) = 100 \log 10 - 10 \times 10 = 130$

Averages: $\bar{L} = 16.6$, $\bar{\Pi} = 327$ $\bar{w} = 10$

(b) In this case the firm cannot adjust employment to shocks. Hence, it will maximize the expected profit

$Max_L \left(\frac{1}{3}300 + \frac{2}{3}100\right) \log L - 10L$

The first order condition yields

$\left(\frac{500}{3}\frac{1}{L}\right) - 10 = 0$

which yields

$L^R = 50/3 = 16.6$

$Profit(\Pi)^R = \frac{500}{3} \log 16.6 - 166 = 302$

Note that $L^R = \bar{L}$ and $\Pi^R = \bar{\Pi}$. However profits are lower.
For $Y = A^i \times L$; L is still 100.

(d) In the no-EPL case the average profits are 327, in the EPL-case the profits are 302. The difference in profits is therefore 7.6%.

(e) The no-EPL case is better for the firm as it is able to adjust to changing economic circumstances.

(12) (Advanced) Consider a firm operating in an imperfect labor market, where any job generates a surplus $\sigma = y - w^r$, so that a risk-neutral employee can be paid more than her reservation wage (w^r) and the firm can realize some profits ($y - w$), where y is the value of the marginal product, even when the product market is competitive. Wages are set with a rent-sharing scheme, assigning a share β of the surplus to the worker.

(a) Express the equilibrium wage as a weighted average of the outside options of the employer and the worker.

Suppose now that a mandatory severance pay S is introduced that is paid to the worker by the employer if the job is discontinued.

(b) Write out the two outside options under these circumstances.

(c) Would the equilibrium wage w be affected by the severance scheme? And what about the wage net of the severance?

(d) What happens if instead of a severance scheme, a firing tax is introduced that is paid by the employer to a third party, say, a lawyer? Would the equilibrium wage w be affected by the firing tax? And what about the wage net of the firing tax? Please interpret these results.

Solution -

(a) Surplus to employee: $y - w$. Surplus to employer: $w - w^r$. Total surplus
$(y - w) + (w - w^r) = y - w^r$.
Rent-sharing solves the following Nash bargaining problem
$\max_w (y - w)^{(1-\beta)}(w - w^r)^\beta$
Taking logs (monotonic transformation)
$(1 - \beta)log(y - w) + \beta log(w - w^r)$
First order condition yields
$\frac{-1+\beta}{y-w} + \frac{\beta}{w-w^r} = 0$
$\rightarrow w = \beta y + (1 - \beta)w^r$
Substituting this rent sharing rule in the two surpluses we get that the surplus of employers is $y - \beta y - (1 - \beta)w^r$ while the surplus of workers is $\beta y - \beta w^r$.

(b) If there is a severance payment,
Surplus to employer: $y - w - (-S)$.
Surplus to employee: $w - w^r - S$.
With given rent-sharing scheme,

$\max_w \, [y - w + S]^{(1-\beta)}[w - w^r - S]^\beta$

Taking the logs, the first order condition reads $\frac{-(1-\beta)}{y-w+S} + \frac{\beta}{w-w^r-S} = 0$.

Therefore the equilibrium wage is $w = \beta y + (1-\beta)w^r + S$ and the net surplus of the employer and of the employee is unaffected since $y - w - (-S) = y - \beta y - (1-\beta)w^r - S + S = y - \beta y - (1-\beta)w^r$ $w - w^r - S = \beta y - \beta w^r - S + S = \beta y - \beta w^r$.

(c) $\frac{\partial w}{\partial S} = 1 \rightarrow$ the equilibrium wage responds fully to the introduction of the severance, leaving the two equilibrium surpluses unaffected according to Lazear neutrality result.

(d) If there is a firing tax, say F, which is not a transfer to employer but a payment to third parties, then F does not any longer appear in the surplus equation of employees:

$\max_w \, [y - w + F]^{(1-\beta)}[w - w^r]^\beta$ which yields

$w = \beta y + (1-\beta)w^r + \beta F$ and now the surplus of the employees increases by βF while that of employers declines by the same amount. Thus, we longer have the neutrality result.

Notice further that

$\frac{\partial w}{\partial S} = \beta$

The higher the share of the surplus of the workers, the stronger the effect of the severance on the wage.

(13) (Advanced) Consider an employed worker who has a two-period job. The worker discount the future at a rate $\delta < 1$. In the first period the wage is given by $w_1 = w(1-\gamma)$ while in the second period it is given by $w_2 = w(1+\gamma)$. In other words, γ is a parameter that encodes the wage tenure profile of the job. At the end of the first period the worker faces an involuntary separation with probability λ. If a λ shock hits the worker, he loses the second-period wage. Conditional on a λ shock, the worker faces a probability α of finding another job. Obviously, if he finds a new job will have to get the first-period wage of $w(1-\gamma)$. The outside option in the second period is the unemployment benefit b.

(a) Write down the present discounted value of a job.

(b) Which restrictions are needed for the labor market to operate?

Assume that the worker faces an increase in mobility, which can be recorded either by an increase in λ or an increase in α.

(c) What happens to the value of the job when job destruction increases?

(d) And when it does α increase?

(e) How do the previous two answers change when the increase in mobility is also associated with an increase in the premium to tenure γ?

Solution -

(a) $PDV = w(1-\gamma) + \frac{\lambda(\alpha w(1-\gamma)+(1-\alpha)b)+(1-\lambda)w(1+\gamma)}{1+\delta}$

(b) Necessary restrictions are

$b \leq w(1-\gamma)$

(c) To find the effect of a change in λ on PDV, let's find the derivatives

$\frac{\partial PDV}{\partial \lambda} = w\frac{\alpha-1-\lambda(\alpha+1)}{1+\delta} + \frac{(1-\alpha)b}{1+\delta}$

Since $w,\ 1+\delta,\ 1-\alpha$ (therefore $\alpha - 1 < 0$), b are all positive

$\frac{\partial PDV}{\partial \lambda} < 0$

(b) Similarly

$\frac{\partial PDV}{\partial \alpha} = \frac{\lambda(w(1-\gamma)-b)}{1+\delta}$

Since all separate terms are positive;

$\frac{\partial PDV}{\partial \alpha} > 0$

(c) From the derivatives above:

Sign of $\frac{\partial PDV}{\partial \alpha}$ does not depend on γ. However as γ increases effect gets smaller (notice that derivative becomes smaller).

On the other hand as γ increases $\frac{\alpha-1}{\alpha+1} > \gamma$ will be relaxed. (At some level the inequality will not hold.) In this case $\frac{\partial PDV}{\partial \lambda} < 0$.

11 Unemployment Benefits

(11) A worker is looking for a job. His marginal revenue from the job search is $MR = 50 - 1.5w$, where w is the wage offer at hand, whereas his marginal cost of job search (in the presence of UBs) is $MC = 5 + w$.

(a) Provide an interpretation for these MR and MC curves: why is MR a negative function of the wage at hand? What does the intercept of MC represent? And its slope?

(b) What is the worker's reservation wage?

(c) Suppose UBs are cut, so that the marginal cost of search increases to $MC = 20 + w$. What is the new reservation wage? Will the worker accept a job offer at 15 euros?

Solution -

(a) The marginal revenue from the job search decreases as the wage offer at hand increases. The reason is that as wage offer at hand increases, the probability of receiving a higher wage offer in the future decreases (assuming random draws of wage offers). This means that the marginal revenue of job search will be lower. The marginal cost of search, on the other hand, increases since, as the wage offer at hand increases, the cost of looking for another offer increases. The intercept shows that even if the wage offer at hand is zero, there is a marginal cost of searching which is not the opportunity cost but a pure search or effort cost. The slope of the MC curve, on the other hand, indicates that this cost depends on the wage offer at hand. Looking for an incremental change in the wage offer causes an increase in the marginal cost at a 1-to-1 rate.

(b) The reservation wage is the one that equates MR to MC, i.e. $w = 18$.

(c) With the new marginal cost, the reservation wage will be 12. Therefore the worker accepts the offer of 15.

(12) Mike's utility function for consumption C and leisure l is:

$U(C, l) = C \times L$.

There are 168 hours in the week, he earns 15 euros per hour.

(a) Write down Mike's budget constraint and graph it.

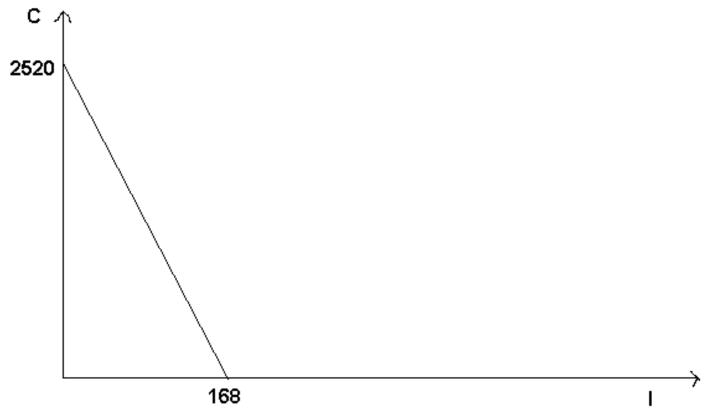
(b) What is Mike's optimal amount of consumption and leisure?

(c) What happens to employment and consumption if Mike receives 320 euros of unemployment benefits each week?

(d) Which is the value of unemployment benefits that would make Mike indifferent between working and not working?

Solution -

(a) The budget constraint is given by $C = w(l_0 - l) = 15(168 - l) = 2520 - 15l$. It can be depicted as follows:



(b) Optimal amount of consumption and leisure: $MRS = w$. Therefore
$\frac{C}{l} = w \rightarrow C = 15l$
Plugging it into the budget constraint it yields
$l^* = 84$ and $C^* = 1,260$

(c) We need to compare Mike's utility in the two cases:
If Mike works his optimal number of hours, his utility is $U(C^*; l^*) = U(1,260; 84) = 1,260 \times 84 = 105,840$.
If Mike doesn't work and receives the UBs, his utility is $U(320; 168) = 320 \times 168 = 53,760$.
Therefore, Mike decides to offer his optimal number of hours of work $168 - l^* = 84$ and consume $C^* = 1,260$.

(d) We need to find the amount of UBs such that $U(C^*; l^*) = U(UB; l_0)$. Therefore $U(1,260; 84) = U(UB; 168) \rightarrow 105,840 = UB \times 168 \rightarrow UB^* = \frac{105,840}{168} = 630 euros$.

(13) (Advanced) In a job search model the flow value of employment can be written as:

$$\rho V_e(w) = w + q(V_u - V_e(w))$$

where ρ is the discount factor, w is the wage, q is the job separation rate and V_e and V_u are the value of employment and unemployment respectively. The flow value of unemployment is:

$$\rho V_u = z + \lambda \int_x^\infty [V_e(w) - V_u] dH(w)$$

where z is the flow value of leisure, λ is the job offer arrival rate, $H(w)$ is the wage distribution and x is the reservation wage.

(a) Explain the intuition of both equations.

(b) Derive the reservation wage equation.

(c) Derive the average duration of unemployment.

(d) Show that the reservation wage increases with the job offer arrival rate.

(e) Discuss why in theory the relationship between the job offer arrival rate and the duration of unemployment is ambiguous.

Solution -

(a) The equation for the flow value of employment indicates that this is given by the wage minus the capitalized surplus from working over being unemployed, weighted by the probability that a job-match is dissolved. Similarly the flow value of unemployment is given by unemployment benefits plus the capitalized surplus from being employed over the outside option, times the job finding rate probability.

(b) The flow value of employment can be re-written as

$$V_e(w) = \frac{1}{\rho}\left(w + q(V_u - V_e(w))\right)$$

This yields

$$V_e(w) = \frac{w + qV_u}{\rho + q}$$

The reservation wage is implicitly given by $V_e(w^r) = V_u$. Thus

$$V_e(w^r) = \frac{w^r + qV_u}{\rho + q} = V_u$$

or

$$w^r + qV_u = (\rho + q)V_u$$

$$w^r = \rho V_u$$

The flow value of unemployment then can be written as

$$w^r = \rho V_u = z + \lambda \int_{w^r}^{\infty} [V_e(w) - V_u] dH(w)$$

While the term in square brackets

$$V_e(w) - V_u = \frac{w + \rho V_u}{\rho + q} - V_u = \frac{w + qV_u - \rho V_u - qV_u}{\rho + q} = \frac{w - \rho V_u}{\rho + q}$$

and since $w^r = \rho V_u$:

$$w^r = z + \frac{\lambda}{\rho + q} \int_{w^r}^{\infty} [w - w^r] dH(w)$$

(c) The average duration of unemployment is given by the inverse of the probability of leaving unemployment. A person will leave unemployment if s/he receives a job offer which is higher than the reservation wage. Therefore the probability of leaving unemployment is given by

$\lambda\,(1 - H(w^r))$

and the average duration of unemployment

$D = \frac{1}{\lambda(1-H(w^r))}$

(d) In order to evaluate how the reservation wage reacts to changes in the job offer arrival rate, we need to use the Implicit Function Theorem (IFT). Let

$$G(w^r, z, \lambda, \rho, q, w) = w^r - z - \frac{\lambda}{\rho + q} \int_{w^r}^{\infty} [w - w^r] dH(w) = 0$$

Then

$$\frac{\partial w^r}{\partial \lambda} = -\frac{\frac{\partial G}{\partial \lambda}}{\frac{\partial G}{\partial w^r}}$$

It is easy to show that $\frac{\partial G}{\partial \lambda} < 0$, since $\frac{\partial G}{\partial \lambda} = -\frac{1}{\rho+q} \int_{w^r}^{\infty} [w - w^r] dH(w) < 0$,

while the denominator,

$$\frac{\partial G}{\partial w^r} = 1 - \frac{\partial \left[\frac{\lambda}{\rho+q} \int_{w^r}^{\infty} [w - w^r] dH(w) \right]}{\partial w^r}$$

Note that we have to take the derivative with respect to w^r where w^r also appears in the boundry of integration. This yields

$$\frac{\partial G}{\partial w^r} = 1 - \frac{\lambda}{\rho + q} \left(-w^r h(w^r) - \int_{w^r}^{\infty} dH(w) \right)$$

Therefore

$$\frac{\partial G}{\partial w^r} = 1 + \frac{\lambda}{\rho + q} w^r h(w^r) + \frac{\lambda}{\rho + q} \int_{w^r}^{\infty} dH(w) \geq 0$$

Therefore

$$\frac{\partial w^r}{\partial \lambda} = -\frac{\ominus}{\oplus} > 0$$

(e) Direct effect: an increase in λ directly increases the exit rate from unemployment, so it decreases the average unemployment duration (notice the λ in denominator of average unemployment duration equation). Indirect effect: from (d) we know that an increase in λ increases the reservation wage, so it increases the average unemployment duration (note that as w^r increases $H(w^r)$ increases). Therefore, the effect of job arrival rate on unemployment duration is ambiguous.

12 ALMP

(11) Consider a matching function specified as $M = \lambda U^\alpha V^{1-\alpha}$, in which M is the number of matches per time period, U is the stock of unemployed workers, V is the stock of vacancies and λ represents the efficiency of the matching process. The job separation rate equals δ and there is a constant labor force normalized for convenience to one.

(a) Show that the Beveridge curve shifts outward if δ increases.

(b) Show that if the government invests in increasing the match efficiency the Beveridge curve shifts inward.

Solution -

(a) Given the assumption of a constant labor force, the flow of workers from employment to unemployment is given by $F_{in}^U = \delta(1-U)$. At the steady, the inflow into unemployment F_{in} equals the outflow from unemployment M, i.e.:
$\delta(1-U) = \lambda U^\alpha V^{1-\alpha}$ or
$\frac{\delta}{\lambda} = (\frac{U}{1-U})^\alpha(\frac{V}{1-U})^{1-\alpha} \approx u^\alpha v^{1-\alpha}$
where u is the unemployment rate and v is the vacancy rate. The Beveridge curve represents the relationship between u and v when δ and λ are constant. Therefore, if δ increases, the hyperbole shifts outwards. An outward Beveridge Curve shift can be generally due to a rise in the separation rate (as in this case), an increase in sectoral mismatch, a reduction in the aggregate matching efficiency due to compositional change, a reduction in recruiting or search intensities.

(b) From the same equation, we can see that if the matching efficiency (λ) increases, $\frac{\delta}{\lambda}$ decreases, therefore shifting the Beveridge curve inward.

(12) (Advanced) Assume the following system of equations in a situation in which unemployed workers may be confronted by a benefit sanction:

$$\rho V_u = \max_{0\leq s\leq 1} \left[b - \gamma(s) + \mu s(V_e - V_u) + \phi(1-s)(V_s - V_u)\right], \tag{4}$$

$$\rho V_s = \max_{0\leq s\leq 1} \left[(1-p_s)b - \gamma(s) + \mu s(V_e - V_s)\right], \tag{5}$$

where ρ is the discount rate, V_u is the value of being unemployed, b is the level of benefits, s is the intensity of search, $\gamma(s) = \frac{1}{2}s^2$ represents the search cost function, μs is the job finding rate, V_e is the value of being employed, V_s is the value of being unemployed after receiving a benefit sanction, $\phi(1-s)$ is the monitoring intensity, p_s is the penalty, s_u is the search intensity before a benefit sanction is imposed, and s_s is the search intensity after a benefit sanction is imposed. Jobs are assumed to last forever.

(a) Interpret the two equations.

(b) How large is the ex ante effect of a benefit sanction?

(c) How large is the ex post effect of a benefit sanction?

(d) Under which conditions is $s_u > s_s$?

(e) Provide an intuition for these results.

Solution -

$$
\begin{aligned}
\rho V_u &= \max_{0 \le s \le 1} \left[b - \gamma(s) + \mu s (V_e - V_u) + \phi(1-s)(V_s - V_u) \right], \\
\rho V_s &= \max_{0 \le s \le 1} \left[(1-p_s) b - \gamma(s) + \mu s (V_e - V_s) \right],
\end{aligned}
$$

(a) The first equation indicates that flow value of unemployment when the sanction is not imposed has three components. These three components are 1) the inflow of income as unemployment benefits minus the search cost ($b - \gamma(s)$), 2) potential income variation in case a job is created ($\mu s(V_e - V_u)$) and 3) income change in case of a benefit sanction in the future ($\phi(1-s)(V_s - V_u)$). Similarly, the flow value of unemployment where a sanction is imposed has two components: the flow value of unemployment benefits after the sanction is imposed minus the search costs ($(1-p_s)b - \gamma(s)$) and potential change in income in case of a job finding ($\mu s(V_e - V_s)$).

(b) Let's first see how we can obtain optimal search intensities.

If we take the first order condition for both of the equations given above (by taking derivatives with respect to s) we find

$$
\begin{aligned}
&\gamma'(s_u) = \mu(V_e - V_u) - \phi(V_s - V_u) \\
&\gamma'(s_s) = \mu(V_e - V_u) + \mu(V_u - V_s),
\end{aligned}
$$

These two equations characterize the optimal search strategy. If there was no monitoring and no benefit sanctions we would obtain;

$$
\gamma'(s) = \mu(V_e - V_u)
$$

As γ is increasing in s and $V_s < V_u$ tells us that a positive monitoring rate increases search effect of those unemployed workers who have not been sanctioned (ex ante effect).

(c) A sanction also increases search effort of the sanctioned unemployed because the difference between the value of employment and the value of unemployment increases after the sanction (ex post effect)

(d) Thus, the ex-ante effect is larger than ex-post effect if $\phi > \mu$ and $s_u < 1$.

(e) The ex-ante effect is larger if monitoring/sanction rate is intensive. This means that, under a high sanction rate, an individual with no sanction tries harder to prevent a possible sanction.

(13) (Advanced) Use the same system of equations as in the previous question. Show under which conditions:

(a) s_u increases with p_s.

(b) s_s increases with p_s.

(c) s_u increases with ϕ.

(d) s_s increases with ϕ.

Solution -

(a) $\frac{\partial s_u}{\partial p_s} > 0$

If the penalty increases then the expected loss increases. Therefore an agent with no sanction increases search intensity to prevent any sanction.

(b) $\frac{\partial s_s}{\partial p_s} > 0$

If the penalty increases, the value of unemployment with a sanction decreases (relative to the value of employment). Therefore a sanctioned unemployed increases the search intensity because employment seems to be more valuable.

(c) $\frac{\partial s_u}{\partial \phi} > 0$

If the sanction rate increases, an unemployed without any sanction increases his search intensity because higher sanction rates make search more effective in avoiding a possible sanction.

(d) $\frac{\partial s_s}{\partial \phi} < 0$

If the sanction rate increases, the difference between non-sanctioned and sanctioned values of unemployment decreases. Therefore search intensity decreases because there is less to gain from finding a job.

13 Payroll Taxes

(10) The utility function of an individual is: $U(c, l) = \sqrt{l \times c}$. The price of the consumption good is p, the wage rate is w, and non-labor income is m. The individual has T hours to work (h) or consume leisure (l).

(a) Write down the utility maximization problem. Given $p = 1$, $w = 5$, $m = 20$, and $T = 16$, what is the optimal labor supply? What is the optimal level of consumption?

(b) Assume that V increases to $m = 30$. What is the new labor supply and consumption?

(c) An EITC program is introduced that provides a \$1 tax credit for every hour of work. The effective wage rate is now $w = 5 + 1 = 6$. Calculate optimal labor supply and consumption under this program. Explain the result.

Solution -

(a) Given the utility function, we can write the optimization problem as:

$$\max_{l,c} \sqrt{lc} \;\; \textit{subject to} \quad c = m + w(T - l) \rightarrow c = 20 + 5(16 - l) = 100 - 5l$$

Substituting the budget constraint into the objective function and taking the first order condition yields

$$\frac{100 - 10l}{2\sqrt{100l - 5l^2}} = 0$$

Solving the above equation gives $l^* = 10$. Therefore, from the budget constraint, $c^* = 50$.

(b) If $m = 30$, the budget constraint becomes $c + 5l = 110$. Maximization of the same utility function under the new constraint yields $l^* = 11$ and $c^* = 55$.

(c) With the new wage, the budget constraint becomes $c + 6l = 116$. Maximization of the same utility function with the new constraint yields $l^* = 9.66$ and $c^* = 58$. Hence labor supply increases (the substitution effect dominates in this case).

(11) Suppose that, in the market for blue collars workers, labor supply is equal to $w = 20 + 8L$, while labor demand is given by $w = 80 - 12L$.

(a) Characterize the labor market equilibrium (wage w, employment L, unemployment U) without taxes.

(b) How do these equilibrium levels change with the introduction of a payroll tax on take-home pay of 50 percent of total wage costs, to be paid by the employers?

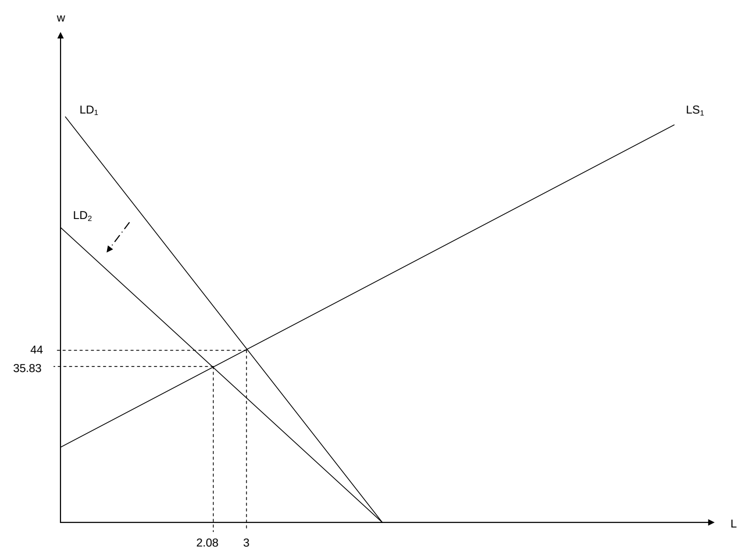
(c) Assume now that supply becomes more rigid. Is the effect of an employer-paid payroll tax stronger or weaker on wages? And what about employment?

(d) What happens if supply is unchanged, but demand becomes more rigid?

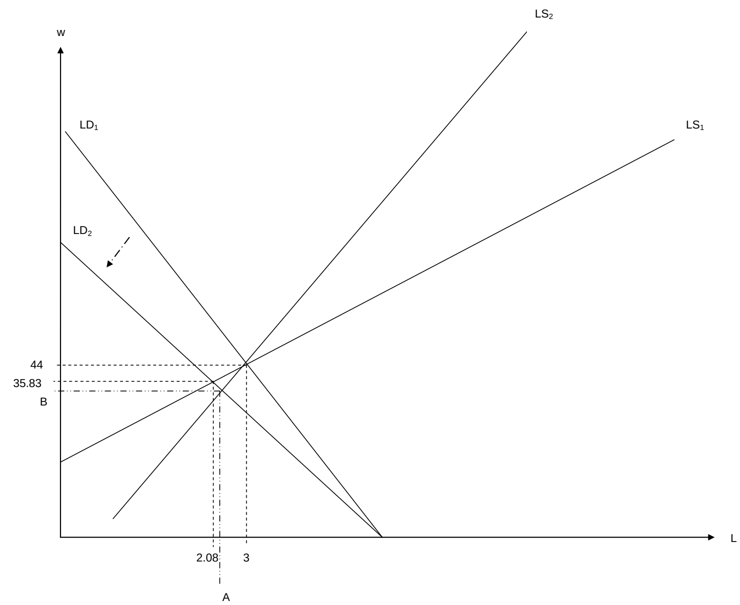
Solution -

(a) Supply and demand are equalized at $w^* = 44$. This yields $L^* = 3$. There is no unemployment.

(b) Denote labor demand and labor costs (wage) for the employers (demand size of the labor market) by the superscript d and those of employees (supply side of the labor market) by the superscript s:
$L^d = \frac{80-w^d}{12} = \frac{80-w^s(1+0.5)}{12}$
Thus equalizing L^d and L^s we have
$\frac{w^s-20}{8} = \frac{80-1.5w^s}{12}$
$w^s = \frac{880}{24} = 36.67 < w^*$ and $L^* = \frac{36.67-20}{8} = 2.08$
The change in the wages and employment because of this tax can be shown on the following graph;



(c) If the supply is more rigid then the decrease in wages will be larger. On the other hand, the effect on employment will be smaller. As can be shown in the graph below, now the change in wages occurs from the initial point to B, whereas the change in employment occurs from the initial point to A.



(d) If the demand is more rigid, then the size of the decrease in wages (also in employment) depends on how rigid the demand is. If the demand function becomes almost inelastic then we might end up with a much lower after-tax wage level. On the other hand if the change in slope of demand line is small then the change in wage can be small as well.

(12) Suppose the supply curve of fast-food employees is given by: $w = 10 + 5L$, while the demand curve is given by: $w = 50 - 3L$.

(a) Compute the equilibrium levels of wage w, employment L, and unemployment U.

(b) How do these levels change with the introduction of a payroll tax of 25 percent, to be paid by employers?

(c) How do these levels change if the same payroll tax is instead paid by employees, on wages?

Solution -

(a) Similar to (11), supply and demand are equalized at $w = 35$. This yields $L = 5$. There is no unemployment.

(b) With payroll tax the labor demand becomes $w(1 + 0.25) = 50 - 3L$. Equalization of supply and demand in this case yields $w = 30.3$ and $L = 4.05$.

(c) With payroll tax paid by employees labor supply becomes $w(1 - 0.25) = 10 + 5L$. Equalization of supply and demand in this case yields $w = 38.5$ and $L = 3.8$.

(13) (Advanced) Consider the general equilibrium model of the labor market of technical annex 13.9.2. Evaluate the effects of the following institutional interactions:

(a) A *flexicurity reform* type 1 reducing employment protection while increasing the generosity of UBs.

(b) A *flexicurity reform* type 2 reducing employment protection while increasing employment-conditional incentives.

(c) A *flexicurity reform* type 3 reducing employment protection while increasing hiring subsidies.

(d) A revenue neutral increase in the progressiveness of taxation combined with an increase in employment conditional-incentives.

Solution -

We provide first a qualitative description of the results. Next we show how to obtain them applying Cramer's rule to the job creation and job destruction conditions.

a) Reducing the employment protection causes an increase in the job destruction rate because it becomes less costly to fire any worker. As the reservation productivity at which jobs are destroyed increases, average productivity increases. If workers do not have a significant bargaining power, the effect of the reduction in employment protection dominates that of the increase in productivity in the rent-sharing rule, implying a lower average wage. This, in turn, reduces the unemployment benefit, hence moderates the effect of the reform on the job loss rate. The surplus of employers increases, inducing a higher job creation rate. If the reform is accompanied by an increase in the unemployment benefit, then there will be a stronger effect on job destruction and a lower effect on job creation, increasing the likelihood that a rise in unemployment is observed.

(b) In this case the increase in the job finding rates due to the reduction in employment protection will be amplified by an increase in employment-conditional incentives, which also partly counteract the effect of the reduction in employment protection on the job destruction margin. Thus, unlike in flexicurity type I, unemployment may not rise or even decline.

c) If the reduction in the employment protection is accompanied with an increase in hiring subsidies, then the increase in the job finding rates will be amplified as in case (b). However, hiring subsidies increase job destruction. Thus, also this flexicurity type III reform is likely to induce a rise in unemployment.

d) When unemployment benefits are fully indexed to the net wage, then employment conditional incentives have no effects on job destruction. Hence, the increase of the progressiveness of taxation will increase the effects of hiring subsidies on the job creation margin, improving the effects of a flexicurity type III reform on employment/unemployment.

Take the job creation and destruction conditions:

$$G : (\rho + \lambda)\frac{sk}{m(\theta)} - (1-\beta)(y - R) = 0$$

$$F : sR(1-\beta) + s\lambda \int_R^1 (z - R)dF(z) - \frac{\beta sk\theta + (1-\beta)(b+a)}{1-t} = 0$$

We apply the implicit function theorem and take the derivatives of the two implicit functions with respect to the two endogenous variables of the model, (θ and R), obtaining the following Jacobian:

$$J = \begin{bmatrix} \frac{\partial G}{\partial \theta} & \frac{\partial G}{\partial R} \\ \frac{\partial F}{\partial \theta} & \frac{\partial F}{\partial R} \end{bmatrix} = \begin{bmatrix} -\frac{(\rho+\lambda)skm'(\theta)}{m^2(\theta)} & 1-\beta \\ -\frac{\beta sk}{1-t} & s\lambda(1-2R) + s(1-\beta) \end{bmatrix}$$

Applying Cramer's rule, it is possible to find the marginal effect of one parameter on any of the two endogenous variables substituting into the matrix J the vector of the partial derivatives of the two implicit functions with respect to the parameter in the Jacobian. The marginal effect is then obtained as the ratio of the two determinants of the matrices.
For instance, it is possible to compute the marginal effect of the parameter b on R^* as follows:

$$\frac{\partial R}{\partial b} = \frac{\begin{vmatrix} \frac{\partial G}{\partial \theta} & -\frac{\partial G}{\partial b} \\ \frac{\partial F}{\partial \theta} & -\frac{\partial F}{\partial b} \end{vmatrix}}{\begin{vmatrix} \frac{\partial G}{\partial \theta} & \frac{\partial G}{\partial R} \\ \frac{\partial F}{\partial \theta} & \frac{\partial F}{\partial R} \end{vmatrix}} = \frac{|J^*|}{|J|}$$

Now, the vector of partial derivative with respect to the parameter b in the implicit functions is given by:

$$\begin{bmatrix} -\frac{\partial G}{\partial b} \\ -\frac{\partial F}{\partial b} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1-\beta}{1-t} \end{bmatrix}$$

substituting this into the second column of the Jacobian (the one with the partial derivatives with respect to the variable R) it is possible to write the augmented Jacobian and compute its determinant:

$$|J^*| = \begin{vmatrix} -\frac{(\rho+\lambda)skm'(\theta)}{m^2(\theta)} & 0 \\ -\frac{\beta sk}{1-t} & \frac{1-\beta}{1-t} \end{vmatrix} = -\frac{(1-\beta)(\rho+\lambda)skm'(\theta)}{(1-t)m^2(\theta)}$$

Which, since $m'(\theta) < 0$, is always positive.
It is also possible to compute the determinant of the original Jacobian:

$$|J| = -\frac{(s\lambda(1-2R) + S(1-\beta))((\rho+\lambda)skm'(\theta))}{m^2(\theta)} + \frac{(1-\beta)\beta sk}{1-t}$$

For $R < 1$ (when R=1 the labour market is shut down) this is positive since $m'(\theta) < 0$.

Thus

$$\frac{\partial R^*}{\partial b} = \frac{|J^*|}{|J|} = \frac{\oplus}{\oplus} = \oplus$$

In words, the marginal effect of b on R^* is positive.

The same procedure can be implemented with reference to θ.

$$|J^*| = \begin{vmatrix} 0 & 1-\beta \\ \frac{1-\beta}{1-t} & s\lambda(1-2R)+S(1-\beta) \end{vmatrix} = -\frac{(1-\beta)^2}{1-t}$$

Which is always negative, hence

$$\frac{\partial \theta^*}{\partial b} = \frac{|J^*|}{|J|} = \frac{\ominus}{\oplus} = \ominus$$

Hence the marginal effect of b on θ^* is negative.