Noen matematiske regler

1. Noen algebraiske regler

$$x^1 = x$$

$$x^{-1} = \frac{1}{x^1} \qquad \rightarrow x^{-\alpha} = \frac{1}{x^\alpha}$$

$$x^0 = 1$$

$$x^\alpha \cdot x^\beta = x^{\alpha+\beta} \qquad \rightarrow \frac{x^\alpha}{x^\beta} = x^\alpha \cdot x^{-\beta} = x^{\alpha-\beta}$$

$$(x^\alpha)^\beta = x^{\alpha\cdot\beta} \qquad \rightarrow (x^\alpha)^{1/\beta} = x^{\alpha/\beta}$$

$$\log(x+z) = \log(x+z)$$

$$\log(x \cdot z) = \log(x) + \log(z)$$

$$\log(x/z) = \log(x) - \log(z)$$

$$\log(x^z) = \mathrm{z} \cdot \log(x)$$

$$\log(e) = 1$$

$$\log(e^x) = x \cdot \log(e) = x$$

2. Regler for derivering

2.1 Eksponentregeln (power rule)

$$\frac{dx^\alpha}{dx} = \alpha \cdot x^{\alpha-1}$$

Eksempel:

$$y = x^2$$

$$\frac{dy}{dx} = 2 \cdot x^{2-1} = 2 \cdot x^1 = 2x$$

$$y = x^{-2}$$

$$\frac{dy}{dx} = -2 \cdot x^{-2-1} = -2 \cdot x^{-3} = \frac{-2}{x^3}$$

2.1 Derivasjon av additive funksjoner

$$\frac{d\big(f(x)+g(x)\big)}{dx}=\frac{df(x)}{dx}+\frac{dg(x)}{dx}$$

Eksempel: $$y=2x+x^2\rightarrow f(x)=2x,\ \ g(x)=x^2$$

$$\frac{dy}{dx}=2+2x$$

2.2 Produktregeln

$$\frac{d\big(f(x)\cdot g(x)\big)}{dx}=\frac{df(x)}{dx}\cdot g(x)+\frac{dg(x)}{dx}\cdot f(x)$$

Eksempel: $$y=2x\cdot x^2\rightarrow f(x)=2x,\ \ g(x)=x^2$$

$$\frac{dy}{dx}=2\cdot x^2+2x\cdot 2x=2x^2+4x^2=6x^2$$

Notere at vi hadde kunnet bruke de algebraiske reglene for å gjøre dette enklere

$$y=2x\cdot x^2\rightarrow y=2x^{1+2}=2x^3$$

$$\frac{dy}{dx}=3\cdot 2x^{3-1}=6x^2$$

2.3 Kjederegeln (chain rule)

$$\frac{d\Big(f\big(z(x)\big)\Big)}{dx}=\frac{df\big(z(x)\big)}{dz(x)}\cdot\frac{dz(x)}{dx}$$

Eksempel

$$y=z^2,\qquad z=\frac{1}{x}$$

$$\frac{dy}{dx}=2\cdot\left(\frac{1}{x}\right)^{2-1}\cdot\left(-\frac{1}{x^2}\right)=-\frac{2}{x}\cdot\frac{1}{x^2}=-\frac{2}{x^3}$$

Notere at vi hadde kunnet skrive dette enklere ved bruk av noen algebraiske regler

$$y=z^2,\qquad z=\frac{1}{x}$$

$$\rightarrow y=(x^{-1})^2=x^{-2}$$

$$\frac{dy}{dx}=-2\cdot x^{-2-1}=-2\cdot x^{-3}=-\frac{2}{x^3}$$

2.4 Derivasjon av den naturlige logaritmen

$$\frac{d\log(x)}{dx} = \frac{1}{x}$$

$$\frac{d\log(f(x))}{dx} = \frac{1}{f(x)} \cdot \frac{df(x)}{dx}$$

Eksempel

$$y = \alpha \cdot x + x^{\beta} \rightarrow \log(y) = \log(\alpha \cdot x + x^{\beta})$$

$$\frac{d\log(y)}{dx} = \left(\frac{1}{\alpha \cdot x + x^{\beta}}\right) \cdot \left(\alpha + \beta \cdot x^{\beta-1}\right)$$

2.5 Derivasjon funksjoner av det naturlige tallet e

$$\frac{de^{f(x)}}{dx} = \frac{df(x)}{dx} \cdot e^{f(x)}$$

Eksempel:

$$y = e^{\alpha \cdot x}$$

$$\frac{dy}{dx} = \alpha \cdot x^{1-1} \cdot e^{\alpha \cdot x} = \alpha \cdot e^{\alpha \cdot x}$$