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Lecture note

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[*Supplementary reading materials includes GdR Chapters 5 and 6. In addition, also consult the "Veileder i samfunnsøkonomiske analyser" section 3.5*]

Discounting future impacts

We will now address several practical considerations that are essential for calculating the Net Present Value (NPV) of a project. First, we assume that a discount rate has been provided to us. Subsequently, we will delve into the theoretical aspects related to choosing an appropriate discount rate.

Basics

Discounting takes place over periods not years. However, for expositional simplicity, we assume that each period is a year. Consider an investment that lasts for one year and yields return at an annual interest rate of i.

The future value in one year of an amount Xinvested at interest rate i is: $Y = X(1+i)$.

Then the present value (PV) of this future aggregate fund of Y must be the same as X, which gives us a formula for PV as

$PV \;=\; X = Y/(1+i)$

We can extend this line of argument to investments that yield returns over multiple years.

The present value, PV, of an amount Y received in $T-$th years, with interest compounded annually at rate i is:

$PV = \frac{Y}{(1+i)^T}$

The present value for a stream of benefits $B = (B_0, B_1, ..., B_T)$ and costs $C = (C_0, C_1, ..., C_T)$ over n years (here 0 denoting the current year) *is*:

$PV(B) = \sum_{t=0}^{T} \frac{B_t}{(1+i)^t}$ and $PV(C) = \sum_{t=0}^{T} \frac{C_t}{(1+i)^t}$

Net Present Value (NPV) of a Project

NPV of a project with a stream of benefits $(B_1, ..., B_T)$ and costs $(C_1, ..., C_T)$ over n years is

$NPV = \sum_{t=0}^{T} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{T} \frac{C_t}{(1+i)^t} = \sum_{t=0}^{T} \frac{B_t - C_t}{(1+i)^t}$.

See, for example, Figure 1 and Figure 2, which illustrates the timeline of a project with streams of benefits and costs accrued over 5 years, and consider a discount/interest rate of 0.04.

Year	Event	Annual benefits	Annual costs	Annual net social benefits
0	Purchase and install	0	500,000	–500,000
1	Annual benefits and costs	150,000	25,000	125,000
2	Annual benefits and costs	150,000	25,000	125,000
3	Annual benefits and costs	150,000	25,000	125,000
4	Annual benefits and costs	150,000	25,000	125,000
5	Annual benefits and costs	150,000	25,000	125,000
	PV	667,773	611,296	56,478

Figure 1: Streams of benefits and costs of a project

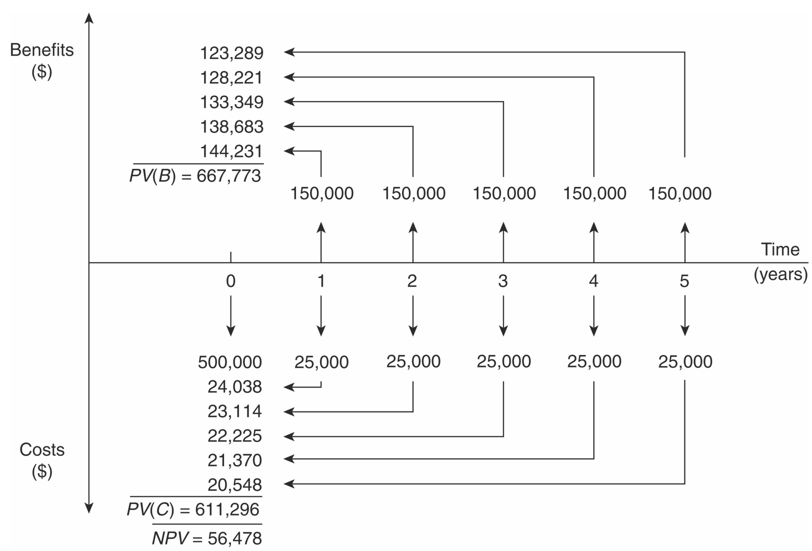

Figure 2: A project with streams of future benefits and costs

Inspecting the formula for NPV, we can see that for a project that typically yields benefits in later years and incurs higher costs in the current year, NPV can decrease with the discount rate i. If we set $i = 0$, the formula simply adds the net benefits (benefits minus costs) over the years.

For example, the project described in Figure 1 will have an aggregate (non-discounted) benefit of $750,000$ and an aggregate (non-discounted) cost of $625,000$, thus generating a net figure of $125,000$. For large values of i, the future benefits are less valuable in today's terms and will reduce the NPV.

The Internal Rate of Return (IRR) of a project equals the discount rate at which the project's NPV equals zero. Sometimes, IRR provides a good indication of the desirability of a project. IRR is the highest discount rate that leaves the project profitable. Similarly, when comparing two projects with different IRRs, one might prefer the one with the higher IRR, especially when one project consistently dominates the other in terms of NPV. Later, when we introduce the concept of a social discount rate, we will see that a commonly used decision rule, when there is only one alternative to the status quo, is to invest in the project if its IRR is greater than the social discount rate.

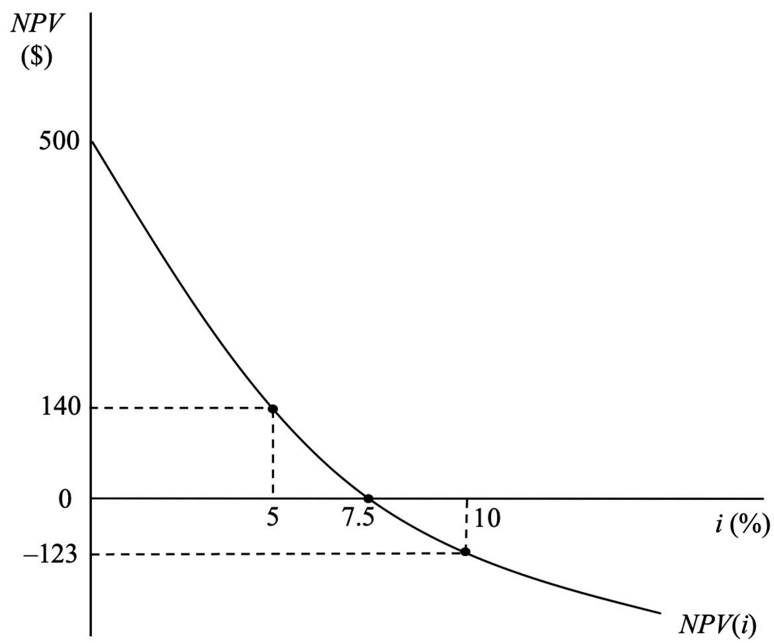

Figure 3: Internal rate of return

Although IRR conveys useful information, there are some problems associated with using IRR as a decision rule. For instance, IRR may not be unique; that is, there may be more than one discount rate at which the NPV is zero. Second, IRRs are expressed as percentages (ratios), not in monetary values. Therefore, they should not be used to select one project from a group of projects that differ in size.

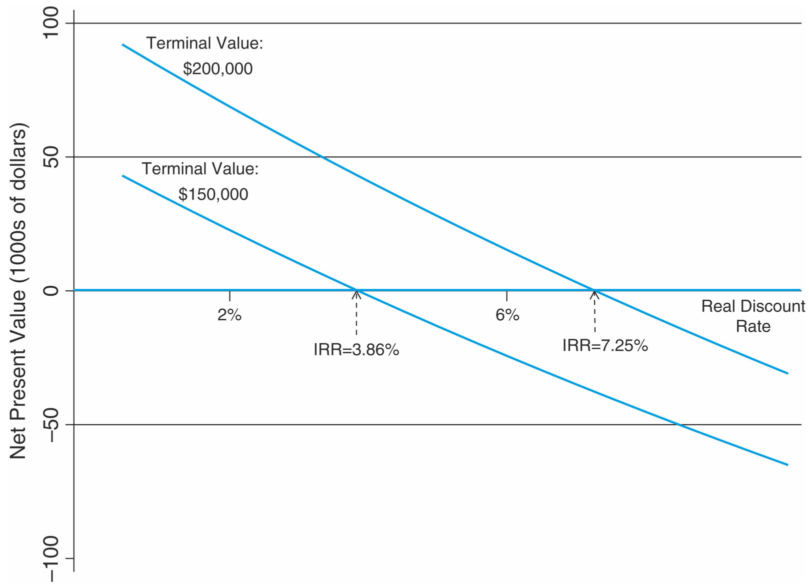

Figure 4: Two projects with different IRR

Some useful formulae

If future returns are constant across periods, then we can use the following formula to represent the sum of a finite geometric series.

$a + ad + ad^2 + ... + ad^n = a\frac{1-d^{n+1}}{1-d}$ where $a > 0$, and $d \neq 1$.

Also note that as n approaches infinity, the absolute value of d must be less than one for the series to converge. The sum then becomes

$a + ad + ad^2 + ... = \frac{a}{1-d}$ where $0 < d < 1$.

Using the formula for the sum of a finite number of terms in a geometric series

$\sum_{t=0}^{T} \frac{V}{(1+i)^t} = V[\frac{1+i-(1+i)^{-T}}{i}]$, and

$\sum_{t=1}^{T} \frac{V}{(1+i)^t} = V[\frac{1-(1+i)^{-T}}{i}]$.

Annuity and perpetuity

An annuity is an equal, fixed amount received (or paid) each year for a number of years. A perpetuity is an indefinite annuity. Many CBAs contain annuities or perpetuities. Fortunately, there are some simple formulas for calculating their PVs.

The present value of an annuity of V per annum (with payments received at the end of each year) for$T$ years starting from year 1 with interest at $i$percent is given by

$PV = \sum_{t=1}^{T} \frac{V}{(1+i)^t} = V[\frac{1-(1+i)^{-T}}{i}]$

and the present value of the present value of an amount V received at the end of each year (starting from year 1) in perpetuity is

$PV = \sum_{t=1}^{\infty} \frac{V}{(1+i)^t} = \frac{V}{i}$.

(To be updated)...