

11

Dealing with Uncertainty: Expected Values, Sensitivity Analysis, and the Value of Information

Cost–benefit analysis almost always requires the analyst to predict the future. Whether it is efficient to begin a project depends on what one expects will happen after the project has begun. Yet, analysts can rarely make precise predictions about the future. Indeed, in some cases, analysts can reasonably assume that uncontrollable factors, such as epidemics, floods, bumper crops, or fluctuations in international oil prices, will affect the benefits and costs that would be realized from proposed policies. How can analysts reasonably take account of these uncertainties in CBA?

We focus on three topics relevant to uncertainty: *expected value* as a measure to take account of risks, *sensitivity analysis* as a way of investigating the robustness of net benefit estimates to different resolutions of uncertainty, and the *value of information* as a benefit category for CBA and as a guide for allocating analytical effort. Expected values take account of the dependence of benefits and costs on the occurrence of specific contingencies, or “states of the world,” to which analysts are able to assign probabilities of occurrence. Sensitivity analysis is a way of acknowledging uncertainty about the values of important parameters in prediction; therefore, it should be a component of almost any CBA. When analysts have opportunities for gaining additional information about costs or benefits, they may be able to value the information by explicitly modeling the uncertainty inherent in their decisions. A particular type of information value, called *quasi-option value*, is relevant when assessing currently available alternatives that have different implications for learning about the future.

11.1 Expected Value Analysis

One can imagine several types of uncertainty about the future. At the most profound level, an analyst might not be able to specify the full range of relevant circumstances that may occur. Indeed, the human and natural worlds are so complex that one cannot hope to anticipate every possible future circumstance. Yet, in many situations of relevance to daily life and public policy, it is reasonable to characterize the future in terms of two or more distinct contingencies. For example, in deciding whether to take an umbrella to work, it is reasonable to divide the future into two contingencies: Either it will or it will not rain sufficiently to make the umbrella useful. Of course, other relevant contingencies can be imagined as well – it will be a dry day, but one may or may not be the victim of an attempted mugging in which the umbrella would prove valuable in self-defense! If these additional contingencies are highly unlikely, then it is usually reasonable to ignore them.

Modeling the future as a set of relevant contingencies involves yet another narrowing of uncertainty: How likely are each of the contingencies? If it is feasible to assign probabilities of occurrence to each of the contingencies, then uncertainty about the future becomes a problem of dealing with *risk*. In relatively simple situations, risk can be incorporated into CBA through expected value analysis.

11.1.1 Contingencies and their Probabilities

Modeling uncertainty as risk begins with the specification of a set of *contingencies* that, within a simplified model of the world, are *exhaustive* and *mutually exclusive*. A contingency can be thought of as a possible event, outcome, or state of the world such that one and only one out of the relevant set of possibilities will actually occur. What makes a set of contingencies the basis of an appropriate model for conducting a CBA of a policy?

The set of contingencies ideally should capture the full range of plausible variations in net benefits of the policy. For example, in evaluating the construction and filling of an oil stockpile for use in the event of an oil price shock sometime in the future, the analyst would want to consider at least two contingencies: There will never be another future oil price shock (a situation in which the policy is likely to result in net losses), or there will be some specified major oil price shock (a situation in which the policy is likely to result in net gains).

The analyst should also assess how comprehensively the set of contingencies represents the possible outcomes between the extremes. In some circumstances, the possible contingencies can be listed exhaustively, so that they can be treated as fully representative. More often, however, the selected contingencies sample from an infinite number of possibilities. In these circumstances, each contingency can be thought of as a *scenario*, which is just a description of a possible future. The practical question is: Do the specified contingencies provide a sufficient variety of scenarios to convey the possible futures adequately? If so, then the contingencies are representative.

Figure 11.1 illustrates the representation of a continuous scale with discrete contingencies. The horizontal axis gives the number of inches of summer rainfall in an agricultural region. The vertical axis gives the net benefits of a water storage system, which increase as the amount of rainfall decreases. Imagine that an analyst represents uncertainty about rainfall with only two contingencies: “excessive” rain, and “deficient” rain. The excessive rain contingency assumes 22 inches of rainfall, which would yield zero net benefits from the storage system. The deficient rain contingency assumes zero inches of rainfall, which would yield \$4.4 million in net benefits. If the relationship between rainfall and net benefits follows the straight line labeled *A*, and all the rainfall amounts between 0 and 22 are equally likely, then the average of net benefits over the full continuous range would be \$2.2 million. If the analyst assumed that each of the contingencies were equally likely, then the average over the two contingencies would also be \$2.2 million, so that using two scenarios would be adequately representative.¹

Now assume that net benefits follow the curved line labeled *B*. Again, assuming that all rainfall amounts between 0 and 22 inches are equally likely, the average of net benefits over the full continuous range would only be about \$1.1 million, so that using only these two contingencies would grossly overestimate the average net benefits from the

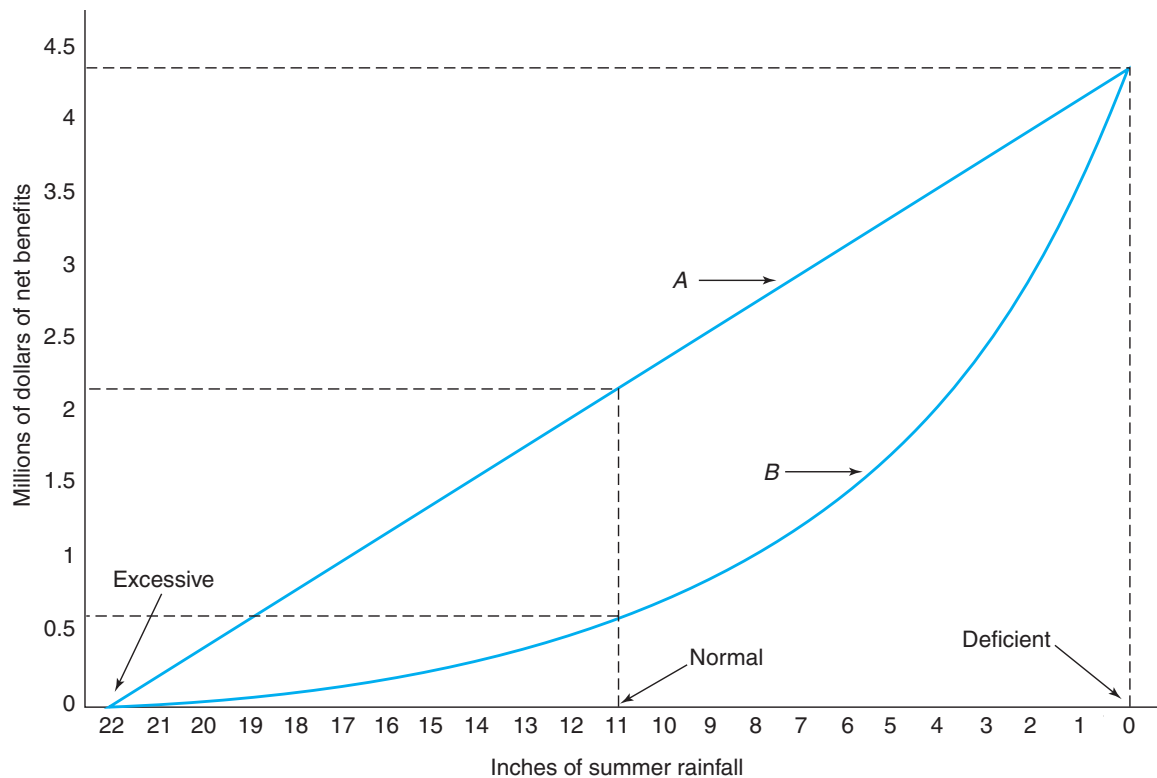


Figure 11.1 Representativeness of contingencies.

storage system.² Adding “normal” rainfall as a contingency that assumes 11 inches of rainfall and averaging net benefits over all three contingencies yields net benefits of \$1.6 million, which is more representative than the average calculated with two contingencies, but still considerably larger than the \$1.1 million predicted over the full continuous range. The inclusion of even more contingencies would be desirable. For example, moving to five equally spaced contingencies gives an average benefit of \$1.3 million, which is much closer to the average over the continuous range.³

Once we have specified a tractable but representative set of contingencies, the next task is to assign probabilities of occurrence to each of them. To be consistent with the logical requirement that the contingencies taken together are exhaustive and mutually exclusive, the probabilities that an analyst assigns must each be non-negative and sum to exactly 1. Thus, if there are three contingencies, C_1 , C_2 , and C_3 , the corresponding probabilities are $p_1 \geq 0$, $p_2 \geq 0$, and $p_3 \geq 0$ such that $p_1 + p_2 + p_3 = 1$.

The probabilities may be based solely on historically observed frequencies or on subjective assessments by clients, analysts, or other experts based on a variety of information and theory or on both history and expertise. For example, return to the contingencies in Figure 11.1: agriculturally “excessive” rain, “normal” rain, and “deficient” rain in a river valley for which a water storage system has been proposed. The national weather service may be able to provide data on average annual rainfall over the last century that allows an analyst to estimate the probabilities of the three specified levels of precipitation from their historical frequencies. If such data were not available, then the

analyst would have to base the probabilities on expert opinion, comparison with similar valleys in the region for which data are available, or other subjective assessment. As such subjective assessments are rarely made with great confidence, it is especially important to investigate the sensitivity of the results to the particular probabilities chosen.

11.1.2 Calculating the Expected Value of Net Benefits

The specification of contingencies and their respective probabilities allows an analyst to calculate the *expected net benefits* of a policy. She does so by first predicting the net benefits of the policy under each contingency and then taking the weighted average of these net benefits over all the contingencies, where the weights are the respective probabilities that the contingencies occur. Specifically, for I contingencies, let B_i be the benefits under contingency i , C_i be the costs under contingency i , and p_i be the probability of contingency i occurring. Then the expected net benefits, $E[NB]$, is given by the formula:

$$E[NB] = p_1(B_1 - C_1) + p_2(B_2 - C_2) + \dots + p_I(B_I - C_I) \quad (11.1)$$

which is just the expected value of net benefits over the I possible outcomes.⁴

Exhibit 11.1

Being explicit about contingencies, their probabilities, and their consequences helps structure complex decision problems. Consider the following letter that President Abraham Lincoln wrote to Major General George B. McClellan on February 3, 1862:

My dear Sir:

You and I have distinct, and different plans for a movement of the Army of the Potomac – yours to be down the Chesapeake, up the Rappahannock to Urbana, and across land to the terminus of the Railroad on the York River – mine to move directly to a point on the Railroad South West of Manassas.

If you will give me satisfactory answers to the following questions, I shall gladly yield my plan to yours.

First. Does not your plan involve a greatly larger expenditure of time and money than mine?

Second. Wherein is a victory more certain by your plan than mine?

Third. Wherein is a victory more valuable by your plan than mine?

Fourth. In fact, would it not be less valuable, in this, that it would break no great line of the enemy's communications, while mine would?

Fifth. In case of disaster, would not a safe retreat be more difficult by your plan than by mine?

Yours truly, Abraham Lincoln

Source: John G. Nicolay and John Hay, editors, *Abraham Lincoln: Complete Works, Volume Two* (New York, NY: The Century Company, 1894), 120.

When facing complicated risk problems, analysts often find it useful to model them as *games against nature*. A game against nature assumes that nature will randomly, and non-strategically, select a particular state of the world. The random selection of a state of the world is according to assumed probabilities. The selection is non-strategic in the sense that nature does not alter the probabilities of the states of the world in response to the action selected by the analysts. A game against nature in *normal form* has the following elements: *states of nature* and their *probabilities of occurrence*, *actions* available to the decision-maker facing nature, and *pay-offs* to the decision maker under each combination of state of nature and action.

Exhibit 11.2

In their evaluation of alternative government oil stockpiling programs in the early 1980s, Glen Sweetnam and colleagues at the US Department of Energy modeled the uncertainty surrounding oil market conditions with five contingencies: *slack market* – oil purchases for the US stockpile of up to 1.5 million barrels per day (mmb/d) could be made without affecting the world oil price; *tight market* – oil purchases increase the world price at the rate of \$3.60 per mmb/d; *minor disruption* – loss of 1.5 mmb/d to the world market (e.g., caused by a revolution in an oil-exporting country); *moderate disruption* – loss of 6.0 mmb/d to the world market (e.g., caused by a limited war in the Persian Gulf); *major disruption* – loss of 12.0 mmb/d to the world market (e.g., caused by a major war in the Persian Gulf). For each of the 24 years of their planning horizon, they assumed that the probabilities of each of the contingencies occurring depended only on the contingency that occurred in the previous year. For each year, they calculated the social surplus in the US oil market conditional on each of the five market contingencies and changes in the size of the stockpile.

The model they constructed allowed them to answer the following questions. For any current market condition and stockpile size, what change in stockpile size maximizes the present value of expected net benefits? How much storage capacity should be constructed? How fast should it be added? The model and the answers it provided were influential in policy debates concerning expansion of the US stockpile, the Strategic Petroleum Reserve.

Sources: Adapted from Glen Sweetnam, “Stockpile Policies for Coping with Oil-Supply Disruptions,” in George Horwich and Edward J. Mitchell, editors, *Policies for Coping with Oil-Supply Disruptions* (Washington, DC: American Enterprise Institute for Public Policy Research, 1982), 82–96. On the role of the model in the policy-making process, see Hank C. Jenkins-Smith and David L. Weimer, “Analysis as Retrograde Action: The Case of Strategic Petroleum Reserves.” *Public Administration Review*, 45(4), 1985, 485–94.

Table 11.1 shows the analysis of alternatives for planetary defense against asteroid collisions as a game against nature in normal form. It considers three possible states of nature over the next 100 years: exposure of Earth to collision with an asteroid larger than 1 km in diameter, which would have enough kinetic energy to impose severe regional or even global effects on society (10 on the Torino Scale); exposure of Earth to collision with an asteroid smaller than 1 km but larger than 20 m in diameter, which would have severe local or regional effects on society (8 or 9 on the Torino Scale); and no exposure of Earth to an asteroid larger than 20 m in diameter. The game shows three alternative actions: Build a forward-based asteroid defense, which would station nuclear devices sufficiently deep in space to attain a good possibility of their timely use in diverting asteroids from collision courses with Earth; build a near-Earth asteroid defense, which would be less expensive but not as effective as the forward-based defense; or do not build any asteroid defense.

Although actually estimating the pay-offs for this game would be both a monumental and a controversial analytical task, Table 11.1 displays some hypothetical figures. The pay-offs, shown as the present value of net costs over the next century, range from \$30 trillion (Earth is exposed to a collision with an asteroid larger than 1 km in diameter in the absence of any asteroid defense) to \$0 (Earth is not exposed to collision with an asteroid larger than 20 m and no defense system is built). Note that estimating the costs of a collision between Earth and an asteroid would itself involve expected value calculations that take account of size, composition, and point of impact of the asteroid. The \$30 trillion figure is about one-third of the world's annual gross domestic product.

The last column of Table 11.1 shows expected values for each of the three alternatives. The expected value for each alternative is calculated by summing the products

Table 11.1 *A Game against Nature: Expected Values of Asteroid Defense Alternatives*

State of nature	Exposure to a collision with an asteroid larger than 1 km in diameter	Exposure to a collision with an asteroid between 20 m and 1 km in diameter	No exposure to collision with an asteroid larger than 20 m in diameter	
Probabilities of states of nature (over next century)	.001	.004	.995	
Actions (alternatives)	Pay-offs (net costs in billions of 2000 dollars)			Expected value
Forward-based asteroid defense	5,060	1,060	60	69
Near-Earth asteroid defense	10,020	2,020	20	38
No asteroid defense	30,000	6,000	0	54

Choose near-Earth asteroid defense: Expected net cost = \$38 billion.

of its pay-off conditional on states of nature with the probabilities of those states. For example, the expected value of pay-offs (present value of net costs) for no asteroid defense is:

$$(0.001)(\$30,000 \text{ billion}) + (0.004)(\$6,000 \text{ billion}) + (0.995)(\$0) = \$54 \text{ billion}$$

Similar calculations yield \$69 billion for the forward-based asteroid defense alternative and \$38 billion for the near-Earth asteroid defense alternative. As the maximization of expected net benefits is equivalent to minimizing expected net costs, the most efficient alternative is near-Earth asteroid defense. Alternatively, one could think of near-Earth asteroid defense as offering expected net benefits of \$16 billion relative to no defense (\$54 billion in expected net costs minus \$38 billion in expected net costs equals \$16 billion in expected net benefits), while forward-based asteroid defense offers negative \$15 billion in expected net benefits relative to no defense (\$54 billion in expected net costs minus \$69 billion in expected net costs equals negative \$15 billion in expected net benefits).

In CBA, it is common practice to treat expected values as if they were certain amounts. For example, imagine that a perfect asteroid defense system would have a present value cost of \$100 billion under each of the states of nature. In this case, assuming accurate prediction of costs, the \$100 billion *would be certain* because it does not depend on which state of nature actually results. CBA generally treats a certain amount such as this as fully commensurate with expected values, even though the latter will not actually result in its expected value. In other words, although the expected net cost of no asteroid defense is \$54 billion, assuming an accurate prediction of pay-offs, the actually realized net cost will be \$30 trillion, \$6 trillion, or \$0. If the perfect defense system cost \$54 billion, then CBA would rank it and no defense as equally efficient.

Treating expected values as if they were certain amounts implies that the person making the comparison has preferences that are *risk-neutral*. A person has risk-neutral preferences when he or she is indifferent between certain amounts and lotteries with the same expected pay-offs. A person is *risk-averse* if he or she prefers the certain amount and is *risk-seeking* if he or she prefers the lottery. Buying insurance, which offers a lower expected pay-off than the certain premium charged, indicates risk-aversion; buying a lottery ticket, which offers a lower expected value than its price, indicates risk-seeking.

Chapter 12 considers the appropriateness of treating expected values and certain equivalents as commensurate (e.g., risk neutrality). Doing so is not conceptually correct in measuring willingness to pay in circumstances in which individuals face uncertainty. Nevertheless, in practice, *treating expected values and certain amounts as commensurate is generally reasonable when either the pooling of risk over the collection of policies, or the pooling of risk over the collection of persons affected by a policy, will make the actually realized values of costs and benefits close to their expected values*. For example, a policy that affects the probability of highway accidents involves reasonable pooling of risk across many drivers (some will have accidents, others will not) so that realized values will be close to expected values. In contrast, a policy that affects the risk of asteroid collision does not involve pooling across individuals (either everyone suffers from the global harm if there is a collision or no one does if there is no collision), so

that the realized value of costs may be very far from their expected value. As discussed in Chapter 12, such unpooled risk may require an adjustment, called *option value*, to expected benefits.

11.1.3 Decision Trees and Expected Net Benefits

The basic procedure for expected value analysis, taking weighted averages over contingencies, can be directly extended to situations in which costs and benefits accrue over multiple years, as long as the risks in each year are independent of the realizations of risks in previous years. Consider, for example, a CBA of a dam with a 20-year life and assuming that the costs and benefits of the dam depend only on the contingencies of below-average rainfall and above-average rainfall in the current year. Additionally, provided the analyst is willing to make the plausible assumption that the amount of rainfall in any year does not depend on the rainfall in previous years, then the analyst can simply calculate the present value of expected net benefits for each year and then calculate the present value of this stream of net benefits in the usual way.

The basic expected value procedure cannot be so directly applied when either the net benefits accruing under contingencies or the probabilities of the contingencies depend on the contingencies that have previously occurred (in other words, they are not independent). For example, above-average rainfall in one year may make the irrigation benefits of a dam less in the next year because of accumulated ground water. In the case of a policy to reduce the costs of earthquakes, the probability of a major earthquake may change each year depending on the mix of earthquakes that occurred in the previous year.

Such situations require a more flexible framework for handling risk than basic expected value analysis. *Decision analysis* provides such a framework.⁵ Though it takes us too far afield to present decision analysis in any depth here, we sketch its general approach and present simple illustrations that demonstrate its usefulness in CBA. A number of book-length treatments of decision analysis are available for those who wish to pursue this topic in more depth.⁶

Decision analysis can be thought of as a *sequential, or extended form, game against nature*. It proceeds in two basic stages. First, one specifies the logical structure of the decision problem in terms of sequences of decisions and realizations of contingencies using a diagram, called a *decision tree*, that links an initial decision (the trunk) to final outcomes (branches). Second, using *backward induction* thinking, one works from final outcomes back to the initial decision, calculating expected values of net benefits across contingencies and pruning dominated branches (i.e., eliminating branches with lower expected values of net benefits).

Consider a vaccination program against a particular type of influenza that involves various kinds of costs.⁷ The costs of the program result from immunization expenditures and possible adverse side effects; the benefits consist of the adverse health effects that are avoided if an epidemic occurs. This flu may infect a population over the next two years before sufficient immunity develops worldwide to stop its spread. Figure 11.2 presents a simple decision tree for a CBA of this vaccination program. The tree should be read from left to right to follow the sequence of decisions, denoted by an open

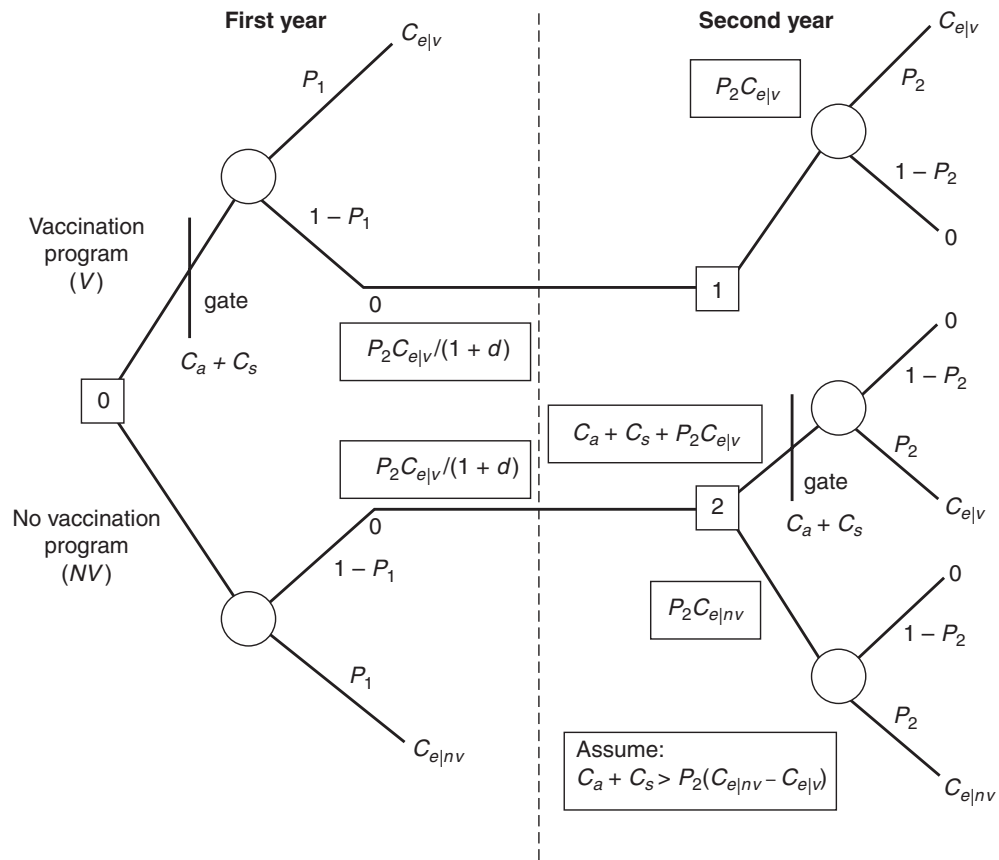


Figure 11.2 Decision tree for vaccination program analysis.

box, and random selections of contingencies, denoted by an open circle. The tree begins with a decision node, the square labeled 0 at the extreme left. The upper bough represents the decision to implement the vaccination program this year; the lower bough represents the decision not to implement the program this year.

Upper Bough: The Vaccination Program. First, follow the upper bough. If the program were implemented, then it would involve direct administrative costs, C_a , and the costs of adverse side effects, such as contracting the influenza from the vaccine itself, suffered by those who are vaccinated, C_s . Note that C_s , like most of the other costs in this example, is itself an expected cost based on the probability of the side effect, the cost to persons suffering the side effect, and the number of persons vaccinated. The solid vertical line on the bough can be thought of as a toll gate at which point the program costs, $C_a + C_s$, are incurred. A chance node, represented by a circle, appears next. Either the influenza infects the population (the upper branch, which occurs with probability P_1 and results in costs $C_{e|v}$, where the subscript should be read as “the epidemic occurs given that the vaccination program has been implemented”), or the influenza does not infect the population (the lower branch, which occurs with probability $1 - P_1$ and results in zero costs at that time). If the influenza does occur, then the population will be immune in the next year. Thus, the upper branch does not continue. If the influenza does not occur,

then there is still a possibility that it might occur in the next year. Therefore, the lower branch continues to the second year, where the square labeled 1 notes the beginning of the second year. It leads directly to another chance node that specifies the two contingencies in the second year: The influenza infects the population (the upper subbranch, which occurs with probability P_2 and results in costs C_{elv}), or the influenza does not infect the population (the lower subbranch, which occurs with probability $1 - P_2$ and results in zero costs).⁸ We assume that P_2 is known at the time of the initial decision.⁹

Lower Bough: No Vaccination Program. We now return to the initial decision node and follow the lower bough that represents no vaccination program in the first year. Initially there is no cost associated with this decision. A chance node follows with two branches: Either the influenza infects the population (the lower branch, which occurs with probability P_1 and results in costs C_{elnv}), or the influenza does not infect the population (the upper branch, which occurs with probability $1 - P_1$ and results in zero costs).¹⁰ If the influenza does occur, then there is no need to consider the next year. If it does not occur, then the tree continues to decision node 2: Either implement the vaccination program in the second year (the upper subbranch crossing the gate where program costs $C_a + C_s$ are incurred) or do not implement it (the lower subbranch).

If the program is implemented, then a chance node occurs: The influenza infects the population (the lower twig, which occurs with probability P_2 and results in costs C_{elv}), or the influenza does not infect the population (the upper twig, which occurs with probability $1 - P_2$ and results in zero costs). One completes the tree by considering the parallel chance node following the decision not to implement the program in the second year: The influenza infects the population (the lower twig, which occurs with probability P_2 and results in costs C_{elnv}), or the influenza does not infect the population (the upper twig, which occurs with probability $1 - P_2$ and results in zero costs).

Solving the Decision Tree. To solve the decision problem, work from right to left, replacing chance nodes with their expected costs and pruning off parallel nodes that are dominated. Consider the chance node following decision node 1. Its expected cost, calculated by the expression $P_2 C_{elv} + (1 - P_2)0$, equals $P_2 C_{elv}$.

Now consider the chance nodes following decision node 2. The lower chance node, following a decision not to implement the vaccination program, has an expected cost of $P_2 C_{elnv}$. The upper chance node has an expected cost of $P_2 C_{elv}$, to which must be added the certain program costs so that the full expected cost of implementing the vaccination program in the second year is $C_a + C_s + P_2 C_{elv}$. The analyst can now compare the expected cost of the two possible decisions at node 2: $P_2 C_{elnv}$ versus $C_a + C_s + P_2 C_{elv}$. To illustrate, assume that program costs are greater than the expected cost reduction from the vaccine, that is, $C_a + C_s > P_2(C_{elnv} - C_{elv})$, then $P_2 C_{elnv}$ is smaller than $C_a + C_s + P_2 C_{elv}$ so that not implementing the program dominates implementing it. (If this were not the case, then the lower branch would be unequivocally dominated by the upper branch.¹¹) The analyst can now prune off the upper subbranch. If we reach decision node 2, then we know that we can obtain expected second-year costs of $P_2 C_{elnv}$.

At decision node 0 the expected costs of implementing the vaccination program (i.e., following the upper bough) consist of direct costs plus the expected costs of following the chance node, which now has the pay-offs C_{elv} if there is an epidemic and the

discounted expected value of node 1, $P_2 C_{elv}/(1 + d)$ if there is not an epidemic. Note that because this latter cost occurs in the second year, it is discounted using rate d . Thus, the present value of expected costs from implementing the vaccination program is given by:

$$E[C_v] = C_a + C_s + P_1 C_{elv} + (1 - P_1) P_2 C_{elv}/(1 + d) \quad (11.2)$$

where the last term incorporates the expected costs from the second year.

The expected costs of not implementing the vaccination program are calculated in the same way: The pay-off if there is not an epidemic becomes the discounted expected costs from decision node 2, $P_2 C_{elnv}/(1 + d)$; the pay-off if there is an epidemic is still C_{elnv} . Therefore, the expression:

$$E[C_m] = P_1 C_{elnv} + (1 - P_1) P_2 C_{elnv}/(1 + d) \quad (11.3)$$

gives the present value of expected costs of not implementing the program.

The final step is to compare the present values of expected costs for the two possible decisions at node 0. We prune the bough with the larger present value of expected costs. The remaining bough is the optimal decision.

As an illustration, suppose that we have gathered data suggesting the following values for parameters in the decision tree: $P_1 = .4$, $P_2 = .2$, $d = .05$, $C_{elv} = .5 C_{elnv}$ (the vaccination program cuts the costs of influenza by half), $C_a = .1 C_{elnv}$ (the vaccination costs 10 percent of the costs of the influenza), and $C_s = .01 C_{elnv}$ (the side-effect costs are 1 percent of the costs of the influenza). For these values, $E[C_v] = .367 C_{elnv}$ and $E[C_m] = .514 C_{elnv}$. Therefore, the vaccination program should be implemented in the first year because $E[C_v] < E[C_m]$.

Calculating Expected Net Benefits of the Vaccination Program. Returning explicitly to CBA, the benefits of the vaccination program are the costs it avoids or simply $E[C_m] - E[C_v]$, which in the numerical example shown in the preceding paragraph equals $0.147 C_{elnv}$. In Chapter 12, we return to the question of the appropriateness of expected net benefits as a generalization of net benefits in CBA.

Extending Decision Analysis. Decision analysis can be used for both public- and private-sector issues, and to structure much more complicated analyses than the CBA of the vaccination program. Straightforward extensions include more than two alternatives at decision nodes, more than two contingencies at chance nodes, more than two periods of time, and different probabilities of events in different periods. Analyses of the US oil stockpiling program typically involve trees so large that they can only be fully represented and solved by computers.¹² *For all problems, whether complex or not, decision analysis can be very helpful in showing how risk should be incorporated into the calculation of expected net benefits.*

11.2 Sensitivity Analysis

Whether or not one structures a CBA explicitly in terms of contingencies and their probabilities, analysts always face some uncertainty about the magnitude of the predicted impacts and the assigned values. Initial analyses usually suppress uncertainty by

using the most likely estimates of unknown quantities. These estimates comprise what is called the *base case*. The purpose of sensitivity analysis is to acknowledge and clarify the underlying uncertainty. In particular, it should convey how sensitive predicted net benefits are to changes in assumptions. If the sign of net benefits does not change when the analyst considers the range of reasonable assumptions, then the results can be considered robust.

The presence of large numbers of unknown quantities is the usual situation in CBA. It makes a brute-force approach of looking at all combinations of assumptions unfeasible. For example, the vaccination program analysis, which is further developed in the next section, requires 17 different uncertain numerical assumptions. If an analyst considered just three different values for each assumption, there would still be over 129 million different combinations of assumptions to assess.¹³ Even if it is feasible to compute net benefits for all these combinations, an analyst would still face the daunting task of sorting through the results and communicating them in an effective way.

Instead, we consider three more manageable approaches to doing sensitivity analysis. First, we demonstrate *partial sensitivity analysis*: How do net benefits change as a single assumption is varied while holding all others constant? Partial sensitivity is most appropriately applied to what the analyst believes to be the most important and uncertain assumptions. It can be used to find the values of numerical assumptions at which net benefits equal zero, or just break even. Second, we consider *worst- and best-case analysis*: Does any combination of reasonable assumptions reverse the sign of net benefits? Analysts are generally most concerned about situations in which their most plausible estimates yield positive net benefits, but they want to know what would happen in a worst case involving the least favorable, or most conservative, assumptions. Third, we consider the use of *Monte Carlo simulation*: What distribution of net benefits results from treating the numerical values of key assumptions as draws from probability distributions? The distribution of net benefits conveys information about the riskiness of the project: its mean (or median) provides a measure of the center of the distribution; its variance, spread around the mean, and the probability of positive net benefits provide information about the riskiness of a policy.

11.2.1 A Closer Look at the Vaccination Program Analysis

We illustrate these techniques by considering a more detailed specification of the costs relevant to the decision analysis of the hypothetical vaccination program presented in Figure 11.2. This program would vaccinate some residents of a county against a possible influenza epidemic.¹⁴

Consider the following general description of the program. Through an advertising and outreach effort by its Department of Health, the county expects to be able to recruit a large fraction of older residents in poor health who are at high mortality risk from influenza, and a much smaller fraction of the general population, for vaccination. As the vaccine is based on a live virus, some fraction of those vaccinated will suffer an adverse reaction that, in effect, converts them to high-risk status and gives them influenza, a cost included in the side effects of the vaccine, C_s . As the vaccine does not always

confer immunity, often because it is not given sufficiently in advance of exposure to the influenza virus, its effectiveness rate is less than 100 percent. Everyone who contracts influenza must be confined to bed rest for a number of days. Analysts can value this loss as the average number of hours of work lost times the average wage rate for the county, although this procedure might overestimate the opportunity costs of time for older persons and underestimate the cost of the unpleasantness of the influenza symptoms for both younger and older persons. They can place a dollar value on the deaths caused by the influenza by multiplying the number of expected deaths times an estimate of the dollar value of life. The various numerical assumptions for the analysis appear in Table 11.2. Notice, for example, that the base case value used for each saved life is \$10 million. That is, it is assumed that people make decisions about how much value they place on small changes in risks of death as if they valued their lives at \$10 million.

Table 11.2 *Base-Case Values for Vaccination Program CBA*

Parameter	Value [range]	Comments
County population (N)	380,000	Total population in the county
Fraction high risk (r)	.06 [.04, .08]	One-half population over age 64
Low-risk vaccination rate (v_l)	.05 [.03, .07]	Fraction of low-risk persons vaccinated
High-risk vaccination rate (v_h)	.60 [.40, .80]	Fraction of high-risk persons vaccinated
Adverse reaction rate (α)	.03 [.01, .05]	Fraction vaccinated who become high-risk
Low-risk mortality rate (m_l) [.000025, .000075]	.00005	Mortality rate for low-risk infected
High-risk mortality rate (m_h) [.0005, .00015]	.001	Mortality rate for high-risk infected
Herd immunity effect (θ)	1.0 [.5, 1.0]	Fraction of effectively vaccinated who contribute to herd immunity effect
Vaccine effectiveness rate (e)	.75 [.65, .85]	Fraction of vaccinated who develop
Hours lost (t)	24 [18, 30]	Average number of work hours lost to illness
Infection rate (i)	.25 [.20, .30]	Infection rate without vaccine
First-year epidemic probability (p_1)	.40	Chance of epidemic in current year
Second-year epidemic probability (p_2)	.20	Chance of epidemic next year
Vaccine dose price (q)	\$9/dose	Price per dose of vaccine
Overhead cost (o)	\$120,000	Costs not dependent on number vaccinated
Opportunity cost of time (w)	\$20/hour	Average wage rate (including benefits) in the county
Value of life (L)	\$10,000,000	Assumed value of life
Discount rate (d)	.035	Real discount rate
Number high-risk vaccinations (V_h)	13,680	High-risk persons vaccinated: $v_h r N$
Number low-risk vaccinations (V_l)	17,860	Low-risk persons vaccinated: $v_l (1 - r) N$
Fraction vaccinated (v)	.083	Fraction of total population vaccinated: $rv_h + v_l (1 - r)$

The benefits of vaccination arise through two impacts. First, those effectively vaccinated are immune to the influenza. Thus, the program targets persons with high mortality risk because they benefit most from immunity. Second, through what is known as the *herd immunity* effect, a positive externality, vaccinated persons reduce the risks of infection to those not vaccinated – this is the reason why some low-risk persons are recruited for vaccination to increase the total fraction of the population that is vaccinated.¹⁵ These two effects cause the expected costs of the epidemic with vaccination, C_{elv} , to be less than the expected costs of the epidemic without the vaccination program, C_{elnv} .

Table 11.3 relates the specific numerical assumptions in Table 11.2 to the parameters in Figure 11.2. From Table 11.3, we see that the direct program costs, C_a , depend on the overhead (i.e., fixed) costs, o , and cost per vaccination, q , times the number of vaccinations given ($V_h + V_l$). The costs of side effects, C_s , depend on the adverse reaction rate, α , the number vaccinated, and the cost per high-risk infection, $wt + m_h L$, where wt is the opportunity cost of lost labor and $m_h L$ is the cost of loss of life. The epidemic's costs without the vaccination program, C_{elnv} , depend on the infection rate, i , the number of high-risk susceptibles, rN , the number of low-risk susceptibles, $(1 - r)N$, and the costs per high- and low-risk infections. Finally, the cost of the epidemic with the vaccination program, C_{elv} , depends on the post-vaccination infection rate, $i - \theta ve$, the number of high-risk individuals remaining susceptible, $rN - eV_h$, the number of low-risk individuals remaining susceptible, $(1 - r)N - eV_l$, and the costs per low- and high-risk infections. Working through these formulas in Table 11.3 yields expected net benefits equal to \$20.3 million for the base-case assumptions presented in Table 11.2.

Partial Sensitivity Analysis. An important assumption in the analysis is the probability that the epidemic occurs. In the base case, we assume that the probability of the epidemic in the next year, given no epidemic in the current year, p_2 , is one-half the probability of the epidemic in the current year, p_1 . To investigate the relationship between net benefits and the probability of epidemic, we vary p_1 (and, hence, p_2) holding all other base-case values constant. Specifically, we vary p_1 from 0 to 0.5 by increments

Table 11.3 *Formulas for Calculating the Net Benefits of Vaccination Program*

Variable	Value (millions of dollars)	Formula
C_a	0.404	$o + (V_h + V_l)q$
C_s	9.916	$\alpha(V_h + V_l)(wt + m_h L)$
C_{elnv}	147.3	$i[rN(wt + m_h L) + (1 - r)N(wt + m_l L)]$
C_{elv}	87.9	$(i - \theta ve)\{(rN - eV_h)(wt + m_h L) + [(1 - r)N - eV_l](wt + m_l L)\}$
EC_v	55.7	$C_a + C_s + p_1 C_{elv} + (1 - p_1)p_2 C_{elv}/(1 + d)$
EC_{nv}	76.0	$p_1 C_{elv} + (1 - p_1)p_2 C_{elv}/(1 + d)$
$E[NB]$	20.3	$EC_{nv} - EC_v$

of 0.05. We thereby isolate the marginal partial effect of changes in probability on net benefits.

The results of this procedure are displayed as the line labeled $L = \$10$ million in Figure 11.3. This label reminds us of another base-case assumption: the value of life equals \$10 million, which we vary next. Because the equations underlying the calculation of net benefits were embedded in a spreadsheet on a personal computer, it was easy to generate the points needed to draw this line by simply changing the values of p_1 and recording the corresponding net benefits.

As expected, this line is upward-sloping: the higher the probability of the epidemic, the larger the net benefits of the vaccination program. For values of p_1 less than about 0.12, net benefits become negative (i.e., the upward-sloping line lies below the solid horizontal line). In other words, if we think that the probability of the epidemic in the current year is less than 0.12, and we are willing to accept the other base-case assumptions, then we should not implement the program. The probability at which net benefits switch sign is called the *breakeven value*. Finding and reporting breakeven values for various parameters is often a useful way to convey their importance.

The line labeled $L = \$5$ million repeats the procedure changing the base-case assumption of the value of life from \$10 million per life to \$5 million per life.¹⁶ The graph thus conveys information about the impact of changes in two assumptions: Each line individually gives the marginal impact of epidemic probability; looking across lines

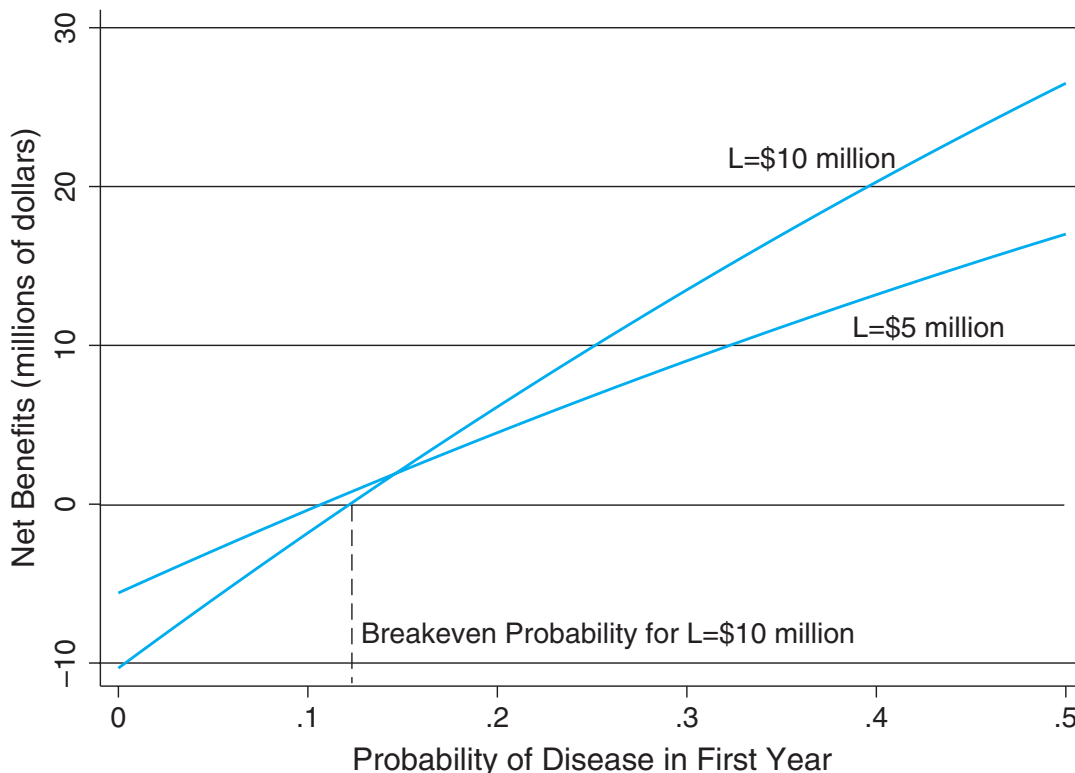


Figure 11.3 Expected net benefits of vaccination for two values of life.

conveys information about the impact of changes in the assumed value of life. As this illustration suggests, we can easily consider the sensitivity of net benefits to changing two assumptions at the same time by constructing families of curves in a two-dimensional graph. Although computers make it feasible to produce graphs that appear three-dimensional, the added information that these graphs convey is often difficult to process visually and therefore should be avoided.

Figure 11.4 considers one more example of partial sensitivity analysis. It repeats the investigation of the marginal impact of epidemic probability on net benefits for two different assumptions about the size of the herd immunity effect, θ . The upper curve is for the base case that assumes a full herd immunity effect ($\theta = 1$). The lower curve assumes that only one half of the effect occurs ($\theta = .5$), perhaps because the population does not mix sufficiently uniformly for the simple model of herd immunity assumed in the base case to apply. (Both cases return to the base-case assumption of \$10 million per life saved.) Now the breakeven probability rises to over 0.16 for the weaker herd immunity effect. Of course, we could instead give primary focus to the herd immunity effect by graphing net benefits against the size of the herd immunity effect, holding epidemic probability constant.

A thorough investigation of sensitivity ideally considers the partial marginal impacts of changes in each of the important assumptions. However, there is a “chicken and egg” problem: Identifying the important assumptions often cannot be done before

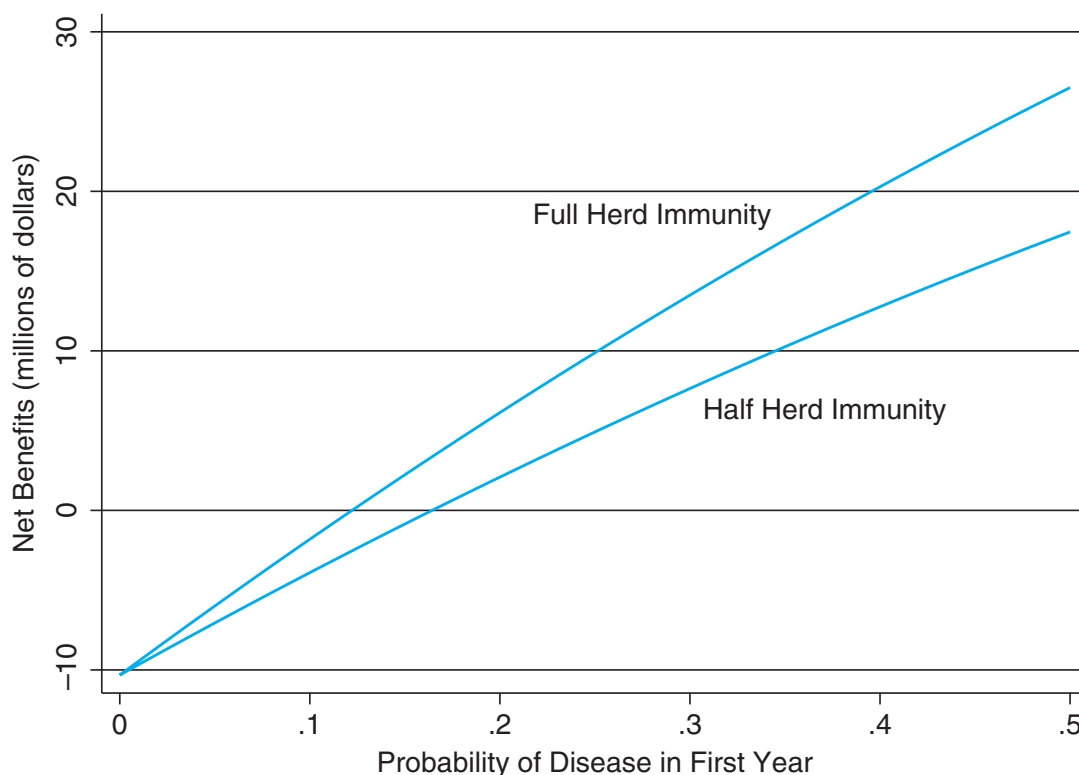


Figure 11.4 Expected net benefits of vaccination.

actually doing the sensitivity analysis because importance depends on the marginal response of net benefits to changes in assumptions, as well as the plausible range of the assumptions. In the analysis of the vaccination program, for example, partial sensitivity analysis might well be warranted for most of the assumptions presented in Table 11.2.

Worst- and Best-Case Analysis. The base-case assumptions, which generally assign the most plausible numerical values to unknown parameters, produce an estimate of net benefits that we think is most likely. In the vaccination program example, these assumptions yield fairly large positive net benefits. We can put a plausible lower bound on net benefits by considering the least favorable of the *plausible range* of values for each of the assumptions. In this way, we can calculate a pessimistic prediction of net benefits. Also, we can calculate an optimistic prediction of net benefits by using the most favorable assumptions. As we discuss later in the chapter, information usually has value in decision-making to the extent it can potentially lead us to make a different choice. Therefore, worst-case analysis is generally most valuable when the base-case expected net benefits are positive; best-case analysis is generally most valuable when the base-case expected net benefits are negative. It should be kept in mind, however, that even if the ranges are plausible, the probability of actually realizing net benefits as extreme as either the worst or the best case gets very small as the number of parameters gets large.

Worst-case analysis acknowledges that society, or specific decision-makers, may be risk-averse. That is, they often care not just about expected net benefits, the appropriate consideration in most cases, but also about the possible “downside.” Furthermore, as we point out in Chapters 1 and 8, there are often cognitive limitations and bureaucratic incentives to generate optimistic forecasts. Worst-case analysis may provide a useful check against these biases.

As a demonstration of worst-case analysis, we take the lower end of each of the ranges presented in Table 11.2 for r , v_h , v_p , m_p , m_h , θ , e , t , and i , and the higher end of the range for α . For example, we assume that r , the fraction of the population at high mortality risk, equals 0.04 rather than the base-case value of 0.06. (For the time being, we keep p_1 , p_2 , q , o , w , L , and d at their base-case values.) With worst-case assumptions, net benefits fall to \$0.11 million. Although still positive, this more conservative estimate is more than two orders of magnitude (10^2) less than under the base-case assumptions.

Return to the question of the sensitivity of net benefits to the probability of epidemic. The breakeven probability rises from about 12 percent under the base-case assumptions to almost 42 percent under the more conservative worst-case assumptions. In other words, expected net benefits would no longer be positive if we assessed the probability of an epidemic to be only 0.4, the assumed value under the base case.

Care must be taken in determining the most conservative assumptions. Under the base-case assumptions, for example, net benefits increase as our assumed value of life increases. Under the conservative assumptions, however, net benefits decrease as the value of life increases. This reversal in the direction of the marginal impact of the value of life occurs because the higher rate of adverse reactions, α , under the conservative case is sufficiently large so that the expected number of deaths is greater with the vaccination program (1.8 deaths) than without it (1.7 deaths).

More generally, caution is warranted when net benefits are a non-linear function of a parameter. In such cases, the value of the parameter that either minimizes or maximizes net benefits may not be at the extreme of its plausible range. Close inspection of partial sensitivity graphs generally gives a good indication of the general nature of the relationship, although these graphs can sometimes be misleading because they depend on the particular assumed values of all other parameters. A more systematic approach is to inspect the functional form of the model used to calculate net benefits. When a non-linear relationship is present, extreme values of assumptions may not necessarily result in extreme values of net benefits. Indeed, inspection of Table 11.3 indicates that net benefits are a quadratic function of vaccination rates v_l and v_h because they depend on C_{elv} , which involves the product of direct effects and the herd effect. Under the base-case assumptions, for instance, net benefits would be maximized if all high-risk persons were vaccinated and 46 percent of low-risk persons were vaccinated. As these rates are well above those that could realistically be obtained by the program, we can reasonably treat the upper and lower bounds of vaccination rates as corresponding to extreme values of net benefits.

If the base-case assumptions generate negative net benefits, then it would have been reasonable to see if more optimistic, or best-case, assumptions produce positive net benefits. If the best-case prediction of net benefits is still negative, then we can be very certain that the policy should not be adopted. If it is positive, then we may want to see if combinations of somewhat less-optimistic assumptions can also sustain positive net benefits.

Monte Carlo Simulations. Partial- and extreme-case sensitivity analyses have two major limitations. First, they may not take account of all the available information about assumed values of parameters. In particular, if we believe that values near the base-case assumptions are more likely to occur than values near the extremes of their plausible ranges, then the worst and best cases are highly unlikely to occur because they require the joint occurrence of a large number of independent low-probability events. Second, these techniques do not directly provide information about the variance, or spread, of the statistical distribution of realized net benefits, which conveys the riskiness of the point estimates. Further, if we cannot distinguish between two policies in terms of expected values of net benefits, then we may be more confident in recommending the one with the smaller variance because it has a higher probability of producing realized net benefits near the expected value.

Monte Carlo simulation provides a way of overcoming these problems. The name derives from the casinos of that famous gambling resort. It is apt because the essence of the approach is playing games of chance many times to elicit a distribution of outcomes. Monte Carlo simulation plays an important role in the investigation of statistical estimators whose properties cannot be adequately determined through mathematical techniques alone. The low opportunity cost of computing makes Monte Carlo simulation feasible for most practicing policy analysts. Because it effectively accounts for uncertainty in complex analyses, it should be in every analyst's tool kit. *Indeed, it should be routinely used in CBA.*

There are three basic steps in performing Monte Carlo simulations. First, the analyst should specify probability distributions for all the important uncertain quantitative assumptions. For the Monte Carlo simulation of the vaccine program, the analysis focuses on the 10 parameters with expressed ranges in Table 11.2. If one does not have theory or empirical evidence that suggests a particular distribution, then it is reasonable to specify a uniform distribution over the range. That is, the most reasonable assumption is that any value between the upper and lower bound of plausible values is equally likely. For example, the analysis assumes that the distribution of the fraction of the population at risk, r , is uniformly distributed between 0.04 and 0.08. Often, though, a more reasonable assumption is that some values near the most plausible estimate should be given more weight. For example, analysts may believe that hours lost due to influenza follow a normal distribution. They could then center it at the best estimate of 24 hours and set the standard deviation at 3.06 so that there is only a 5 percent chance of values falling outside the most plausible range of 18 to 30 hours. (See Appendix 11A for a brief discussion of working with probability distributions.) As discussed in Chapter 4, analysts can sometimes estimate unknown parameters statistically using regression analysis or other techniques. Commonly used regression models allow analysts to approximate the distribution of an unknown parameter as normal with mean and standard deviation given by their empirical estimates.

The second step is to execute a trial by taking a random draw from the distribution for each parameter to arrive at a set of specific values for computing realized net benefits. For example, in the case of the vaccination program analysis, analysts have to determine which contingencies are likely to occur in each of the two periods. To determine if an epidemic occurs in the current year, they take a draw from a Bernoulli distribution with probability p_1 of yielding “epidemic” and $(1 - p_1)$ of yielding “no epidemic.” That is, it is as if one were to flip a coin that has a probability of p_1 of landing with “epidemic” face up. Almost all spreadsheets allow users to take draws from random variables uniformly distributed between 0 and 1 – a draw from the uniform distribution produces an outcome within this range that is as likely to occur as any other outcome in the range. Thus there is a p_1 probability of a value between zero and p_1 occurring. To implement a draw from a Bernoulli distribution that has a probability of p_1 of yielding “epidemic,” one simply compares the draw from the uniform distribution to p_1 : If the random draw from the uniform distribution is smaller (larger) than p_1 , then assume that an epidemic does (not) occur in the current year; if an epidemic does not occur in the current year, then follow a similar procedure to determine if an epidemic occurs in the second year. Three mutually exclusive realizations of net benefits are possible:

$$\text{Epidemic in neither year: } NB = - (C_a + C_s)$$

$$\text{Epidemic in current year: } NB = - (C_a + C_s) + (C_{e|nv} - C_{e|v})$$

$$\text{Epidemic in next year: } NB = - (C_a + C_s) + (C_{e|nv} - C_{e|v}) / (1 + d)$$

where the value of NB depends on the particular values of the parameters drawn for this trial.¹⁷

Exhibit 11.3

Influenza vaccination programs are usually targeted to those in high-risk groups, such as infants, the elderly, and people with compromised immune systems. Is vaccination of healthy workers cost-beneficial? Kristin L. Nichols attempts to answer this question with a cost–benefit analysis. Benefits of vaccination include avoided lost work days, hospitalizations, and deaths. Costs include the costs of the vaccination and lost work days, hospitalizations, and deaths from side effects. She employed Monte Carlo analysis to estimate net benefits. Noting that previous studies reported that managers generally took fewer sick days than other personnel, she built a negative correlation between sick days and wage rate, two of the important parameters, into the Monte Carlo trials. She estimated the mean value of net benefits to be \$18.27 (2010 dollars) with a 95 percent confidence interval ranging from \$44.10 in positive net benefits to \$2.92 in negative net benefits. In order to assess the relative importance of various assumed parameters to net benefits, she regressed the net benefits from each trial on the randomly drawn values of the parameters. Net benefits were most sensitive to the illness rate, the work absenteeism rate due to influenza, and the hourly wages. In addition, a poor match between the vaccine and the circulating virus strain gave negative net benefits. Not surprisingly, the 95 percent confidence interval from the Monte Carlo analysis was much tighter than the best/worst-case range of positive net benefits of \$233.15 to negative net benefits of \$28.45.

Source: Adapted from Kristin L. Nichol, “Cost–Benefit Analysis of a Strategy to Vaccinate Healthy Working Adults Against Influenza.” *Archives of Internal Medicine*, 161(5), 2001, 749–59.

Note that these estimates of NB no longer involve expectations with respect to the contingencies of epidemics, though the cost estimates themselves are expected values.

In the third step, one repeats the trial described in the second step many times – typically a thousand times or more – to produce a large number of realizations of net benefits. The average of these trials provides an estimate of the expected value of net benefits. An approximation of the probability distribution of net benefits can be obtained by breaking the range of realized net benefits into a number of equal increments and counting the frequency with which trials fall into each one. The resulting *histogram* of these counts provides a picture of the distribution. The more trials that go into the histogram, the more likely it is that the resulting picture gives a good representation of the distribution of net benefits. Underlying this procedure is the law of large numbers: as the number of trials approaches infinity, the frequencies will converge to the true underlying probabilities.

Figure 11.5 presents a histogram of 10,000 replications of random draws from the bracketed assumptions in Table 11.2. The assumed distributions are all uniform except that for hours lost, t , which follows a normal distribution, and whether or not the epidemic occurs, which, although a Bernoulli distribution, is implemented with the readily available uniform distribution. The height of each bar is proportional to the number of trials that had net benefits falling in the corresponding increment.

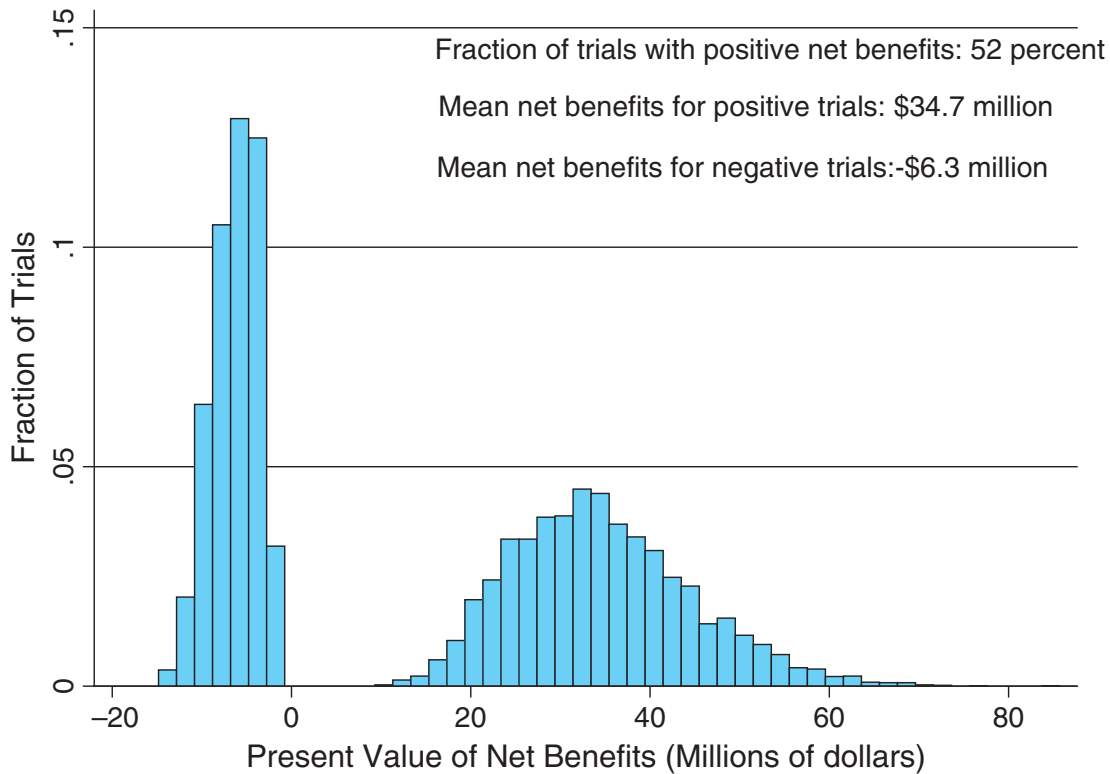


Figure 11.5 Histogram of realized net benefits.

The average of net benefits over the 10,000 trials is \$15.0 million. This differs from our base-case calculation of \$20.3 million because the base-case value of the herd immunity factor, θ , was set at 1 rather than at the middle point of the plausible range. Repeating the Monte Carlo procedure with the herd immunity factor set to 1 yields an average of realized net benefits of \$18.7 million, which is close to the base-case calculation of expected net benefits.¹⁸

The histogram provides a visual display of the entire distribution of net benefits so that its spread and symmetry can be easily discerned. The trials themselves can be used to calculate directly the sample variance, standard deviation, fraction of positive trials, and other summary statistics describing net benefits.

The most striking feature of the histogram is that it reveals a bimodal distribution. If an epidemic occurs in either year, then the vaccination program has positive net benefits and it is as if we are drawing only from the right-most hump of the distribution. If an epidemic occurs in neither year, then the vaccination program has negative net benefits and it is as if we are drawing from the left-most hump of the distribution. The assumed probabilities of epidemic in the two years leads one to expect positive net benefits 52 percent of the time $[p_1 + (1 - p_1)p_2]$, which is close to the 52.8 percent of trials with positive net benefits in the Monte Carlo simulation.

The Monte Carlo results presented in Figure 11.5 treat several parameters as if they are certain. Most importantly, they treat the values of time, the value of a statistical life, and the discount rate as certain. As we explain in Chapter 17, however, these

values are uncertain. Distributions for the values of these parameters could be specified and included in an expanded simulation. Doing so would be most appropriate for the values of time and the value of a statistical life. Another approach, more appropriate for uncertainty about the discount rate, would be to repeat the Monte Carlo simulation for specific values of the parameters rather than including them within a single simulation. For example, as noted in the discussion of the regulatory cases following Chapter 4, the Office of Management and Budget recommends discounting at real rates of 3 and 7 percent. This recommendation could be followed by conducting a separate Monte Carlo simulation at each of these discount rates.

The illustration also assumes that the parameters are independent of one another. In practice, they may be correlated. However, in order to take these correlations into account, one would need to know the variance–covariance matrix of the parameters. Such more sophisticated approaches are beyond the scope of this book.

Sensitivity Analysis Strategy. We recommend the following strategy for conducting sensitivity analysis: First, when there are more than a few uncertain parameters, the most common situation in doing real-world CBA, the analyst should use Monte Carlo simulation as the framework for the analysis. Rather than reporting net benefits based on beliefs about the most likely values of parameters, report the mean value of net benefits from the Monte Carlo trials. To convey the degree of uncertainty about prediction of net benefits, show the histogram of net benefits across trials and report the fraction of trials with positive net benefits.¹⁹ Second, use partial sensitivity analysis to focus attention on how the values of particularly important parameters affect net benefits. The parameters may be important because they have large effects on net benefits, their values are highly uncertain, or because of expectations that specific values of certain parameters, such as the discount rate, be used. Finally, only use worst- and best-case analyses as a fallback expediency when Monte Carlo simulation is impractical.

11.3 Information and Quasi-Option Value

The various analytical techniques developed in the previous sections provide a basis for assessing uncertainty in information about assumed or estimated parameters used in CBA. In this section we demonstrate the use of games against nature to place value on information itself. Use of the normal form illustrates the basic concepts. One then uses decision trees to explicate a particular information value, the quasi-option value, which arises in the context of delaying irreversible decisions, which allows time for the gathering or revelation of information about the future.

11.3.1 *Introduction to the Value of Information*

The value of information in the context of a game against nature answers the following question: by how much would the information increase the expected value of playing the game? As an example of how to answer this question, return to the asteroid defense game presented in Table 11.1. Imagine that scientists have proposed developing a detection device that would allow them to determine with certainty whether the Earth would be exposed to

a collision with a large asteroid (diameter greater than one kilometer) in the next 100 years. What is the maximum investment that should be made to develop this device?

If the device were to be built, then it would reveal which of two possible futures were true: First, with a probability of 0.001, it would show that there would be a collision with a large asteroid. Second, with a probability of 0.999, it would show that there would be no collision with a large asteroid. Each of these two futures implies a different game against nature, as shown in Table 11.4.

Game One, shown on the left side of Table 11.4, results if the detection device indicates that the Earth will be exposed to collision with a large asteroid. Not surprisingly, in this game the best action is to choose forward-based asteroid defense, which has the smallest net costs of the three actions (\$5,060 billion). Game Two, shown on the right side of Table 11.4, results if the detection device indicates that the Earth will not be exposed to collision with a large asteroid. As exposure to collision with a large asteroid is ruled out, the probabilities of the other two possible states of nature are adjusted upward so that they sum to 1 (0.004004 and 0.995996). In this game, the best action is

Table 11.4 *Reformulated Games against Nature: Value of Device for Detecting Large Asteroids*

State of nature	Game One $p = 0.001$		Game Two $p = 0.9999$		
	Exposure to a collision with an asteroid larger than 1 km in diameter	Exposure to a collision with an asteroid between 20 m and 1 km in diameter	No exposure to collision with an asteroid larger than 20 m in diameter		
Probabilities of states of nature (over next century)	1	0.004004	0.995996		
Actions (alternatives)	Pay-offs (net costs in billions of 2000 dollars)	Expected value	Pay-offs (net costs in billions of 2000 dollars)	Expected value	
Forward-based asteroid defense	5,060	5,060	1,060	60	64.01
Near-Earth asteroid defense	10,020	10,020	2,020	20	28.01
No asteroid defense	30,000	30,000	6,000	0	24.02

Game One: Choose forward-based asteroid defense: Expected net cost = \$5,060 billion.

Game Two: Choose no asteroid defense: Expected net cost = \$24.02 billion.

Expected net cost of decision with detection device:

$(0.001)(\$5,060 \text{ billion}) + (0.999)(\$24.02 \text{ billion}) = \$29.06 \text{ billion}.$

Value of information provided by detection device: $\$38 \text{ billion} - \$29.06 \text{ billion} = \$8.94 \text{ billion}.$

to choose no asteroid defense, which has the smallest net costs of the three alternative actions (\$24.02 billion).

Prior to developing the detection device, we do not know which of these two games nature will give us to play. We do know, however, that it will indicate Game One with probability 0.001 and Game Two with probability 0.999. Thus it is possible to compute an expected net cost over the two games as $(0.001)(\$5,060 \text{ billion}) + (0.999)(\$24.02 \text{ billion}) = \29.06 billion . In order to place a value on the information provided by the device, the analyst compares the expected net cost of the optimal choice in the game without it (the \$38 billion shown in Table 11.1) with the expected net cost resulting from optimal choices in the games with it (\$29.06 billion). The difference between these net costs (\$38 billion – \$29.06 billion) equals \$8.94 billion, which is the value of the information provided by the device. Consequently, as long as the detection device costs less than \$8.94 billion, it would be efficient to develop it.

Note that the value of the information derives from the fact that it leads to different optimal decisions. The optimal choice without the device is near-Earth asteroid defense. The optimal choice with the device is either forward-based asteroid defense if collision exposure is confirmed or no asteroid defense if the absence of collision exposure is confirmed.

In practice, analysts rarely face choices requiring them to value perfect information of the sort provided by the asteroid detection device. They do, however, routinely face choices involving the allocation of resources – time, energy, budgets – toward reducing uncertainty in the values of the many parameters used to calculate net benefits. For example, a statistical estimate based on a random sample size of 600 will be much more precise than one based on a sample of 300. How can the analyst determine if the investment in the larger sample size is worthwhile?

In a CBA involving many assumed parameters, Monte Carlo simulation can provide especially useful information. For example, suppose an agency is deciding whether it is worthwhile to invest analytical resources in conducting a study that would reduce the estimate of the variance of hours lost from the influenza described in the previous section. One could replicate the analysis presented in Figure 11.5 with a smaller assumed variance of hours lost and compare the resulting distribution of net benefits to that resulting with the larger variance. A necessary condition for the investment of analytical resources to be worthwhile is a meaningful change in the distribution of realized net benefits.

Exhibit 11.4

Research and development projects typically have very uncertain costs and benefits when they are initiated. Based on an assessment of detailed case studies of six research and development projects (Supersonic Transport, Applications Technology Satellite Program, Space Shuttle, Clinch River Breeder Reactor, Synthetics Fuels from Coal, and Photovoltaics Commercialization), Cohen and Noll concluded: “The final

success of a program usually hinges on a few key technical objectives and baseline economic assumptions about demand or the cost of alternative technologies, or both. The results of the research that addressed the key technical issues, and realizations a few years after that program was started of the key unknown economic parameters, typically made the likely success of a project very clear” (p. 82).

For example, Susan Edelman prepared CBAs of the supersonic transport project with the information that would have been available to conscientious analysts in each of a number of years. She reports that the plausible range of benefit–cost ratios fell from 1.97 to 4.97 in 1963 to 1.32 to 1.84 in 1971. They declined as it became clear that either higher operating costs or reduced loads would result from failures to achieve technical objectives and that operations over land would likely be restricted to reduce the impacts of sonic booms on people (pp. 112–21).

Source: Adapted from Linda R. Cohen and Roger G. Noll, editors, *The Technology Pork Barrel* (Washington, DC: The Brookings Institution, 1991).

11.3.2 Quasi-Option Value

It may be wise to delay a decision if better information relevant to the decision will become available in the future. This is especially the case when the costs of returning to the status quo once a project has begun are so large that the decision is effectively irreversible. For example, consider the decision of whether to develop a virgin wilderness area. Analysts may be fairly certain about the costs and benefits of development to the current generation, but be very uncertain of the opportunity cost to future generations of losing the virgin wilderness. If information revealed over time would reduce uncertainty about how future generations will value the wilderness area, then it may be desirable to delay a decision about irreversible development to incorporate the new information into the decision process. The expected value of information gained by delaying an irreversible decision is called *quasi-option value*.²⁰

Quasi-option value can be quantified by explicitly formulating a multiperiod decision problem that allows for the revelation of information about the value of options in later periods.²¹ Although some environmental analysts see quasi-option value as a distinct benefit category for policies that preserve unique assets such as wilderness areas, scenic views, and animal species, it is more appropriately thought of as a correction to the calculation of expected net benefits through an inappropriate one-period decision problem. As the calculation of quasi-option value itself requires specification of the proper decision problem, *whenever quasi-option value can be quantified, the correct expected net benefits can and should be calculated directly.*

As background for an illustration of quasi-option value, Table 11.5 sets out the parameters for a CBA of alternatives for use of a wilderness area. The value of net benefits from full development (*FD*) and limited development (*LD*) are measured relative to no development (*ND*) for two contingencies. Under the contingency labeled “Low Value,” which will occur with a probability p , future generations place the

Table 11.5 *Benefits and Costs of Alternative Development Policies Assuming No Learning*

	Preservation contingencies	
	Low value	High value
Full development (<i>FD</i>)	B_F	$-C_F$
Limited development (<i>LD</i>)	B_L	$-C_L$
No development (<i>ND</i>)	0	0
Probability of contingency	p	$1 - p$
Expected value of full development:	$E[FD] = pB_F - (1 - p)C_F$	
Expected value of limited development:	$E[LD] = pB_L - (1 - p)C_L$	
Expected value of no development:	$E[ND] = 0$	
Adopt full development if:	$pB_F - (1 - p)C_F > pB_L - (1 - p)C_L$ and $pB_F - (1 - p)C_F > 0$	

same value as current generations on preservation of the wilderness area. Under the contingency labeled “High Value,” which will occur with a probability $1 - p$, future generations place a much higher value than current generations on preservation of the wilderness area. If the Low Value contingency occurs, then *FD* yields a positive present value of net benefits equal to B_F and *LD* yields a positive present value of net benefits equal to B_L . That is, after taking account of all costs and benefits, the present value of net benefits for *FD* and *LD* are the positive amounts B_F and B_L , respectively. If, instead, the High Value contingency occurs, then *FD* yields a negative present value of net benefits equal to $-C_F$ and *LD* yields a negative present value of net benefits equal to $-C_L$, where C_F and C_L are net costs and therefore signed negative to be the present value of net benefits. Assume that $B_F > B_L > 0$ and $C_F > C_L > 0$ so that *FD* yields greater net benefits under the Low Value contingency and greater net costs under the High Value contingency than *LD*.

Imagine that the analyst conducts a CBA assuming that no learning will occur over time. That is, assuming that no useful information will be revealed in future periods. The expected net benefits of *FD* equal $pB_F - (1 - p)C_F$; the expected net benefits of *LD* equal $pB_L - (1 - p)C_L$; and the expected net benefit of *ND* equals 0. One would simply choose the alternative with the largest expected net benefits.

Now consider the case of *exogenous learning*. That is, one assumes that after the first period one discovers with certainty which of the two contingencies will occur. Learning is exogenous in the sense that the information is revealed irrespective of what action is undertaken.

Figure 11.6 presents a decision tree for the exogenous learning situation. The square box at the extreme left-hand side of the figure represents the initial decision. If one selects *FD*, then the result is the same expected value as in the case of no learning – the decision is irreversible and, hence, learning has no value because there is no decision left to make in period 2. If one selects either *LD* or *ND* in the first period, then there is a decision left to make in period 2 once one knows which contingency has occurred. The

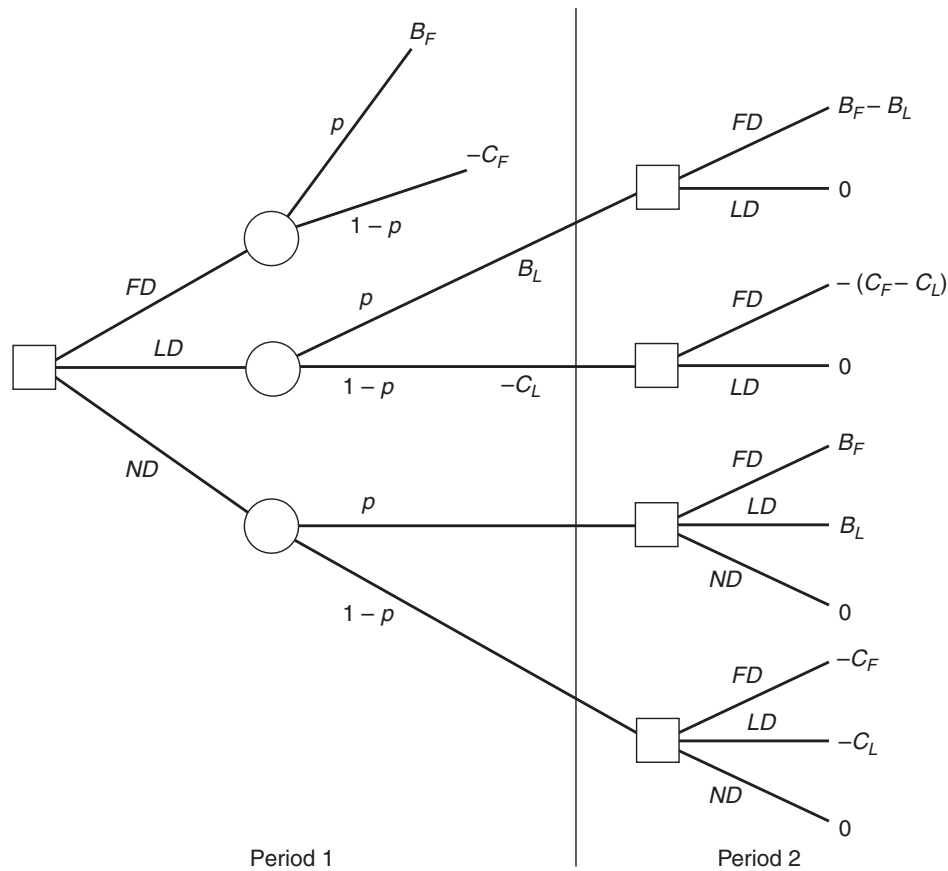


Figure 11.6 Exogenous learning.

expected values of the *LD* and *ND* decisions in period 1 can be found using the backward induction method introduced in the vaccine example developed earlier in the chapter.

First, consider *LD*. If the Low Value contingency is revealed at the beginning of period 2, then the optimal decision will be to complete the development to obtain net benefits $B_F - B_L$. The present value of this amount is obtained by discounting at rate d . It is then added to B_L , the period 1 net benefits, to obtain the net benefits of *LD* conditional on the Low Value contingency occurring. If the High Value contingency is revealed at the beginning of period 2, then the optimal decision is to forgo further development so that the net benefits conditional on the High Value contingency occurring consist only of the $-C_L$ realized in period 1. Multiplying these conditional net benefits by their respective probabilities yields the expected net benefits for limited development in period 1 of $p[B_L + (B_F - B_L)/(1 + d)] - (1 - p)C_L$. Note that it differs from the expected value in the no-learning case by the expected net benefits of the period 2 option, $p(B_F - B_L)/(1 + d)$, which is the quasi-option value of *LD*.

Next consider the decision *ND* in period 1. If the Low Value contingency is revealed at the beginning of period 2, then the optimal decision is *FD*, which has a present value of $B_F/(1 + d)$. If the High Value contingency is revealed at the beginning of period 2, then the optimal decision is *ND*, which has a present value of 0. Consequently,

the expected net benefits from choosing ND in period 1 are $pB_F/(1+d)$, which equal the quasi-option value of ND .

The middle column of Table 11.6 summarizes the expected values of the period 1 alternatives for the case of exogenous learning.

Figure 11.7 presents a decision tree for the endogenous learning situation. Unlike the situation with exogenous learning, information is generated only from development itself. For example, the value placed on preservation by future generations may depend on the risk that development poses to a species of bird that feeds in the wilderness area during its migration. The effect of limited development on the species may provide

Table 11.6 *Expected Values for Decision Problems: Quasi-Option Values (QOV) Measured Relative to No Learning Case*

	No learning	Exogenous learning	Endogenous learning
$E[FD]$	$pB_F - (1-p)C_F$	$pB_F - (1-p)C_F$	$pB_F - (1-p)C_F$
QOV		0	0
$E[LD]$	$pB_L - (1-p)C_L$	$p[B_L + (B_F + B_L)/(1+d)] - (1-p)C_L$	$p[B_L + (B_F - B_L)/(1+d)] - (1-p)C_L$
QOV		$p(B_F - B_L)/(1+d)$	$p(B_F - B_L)/(1+d)$
$E[ND]$	0	$pB_F/(1+d)$	0
QOV		$pB_F/(1+d)$	= 0

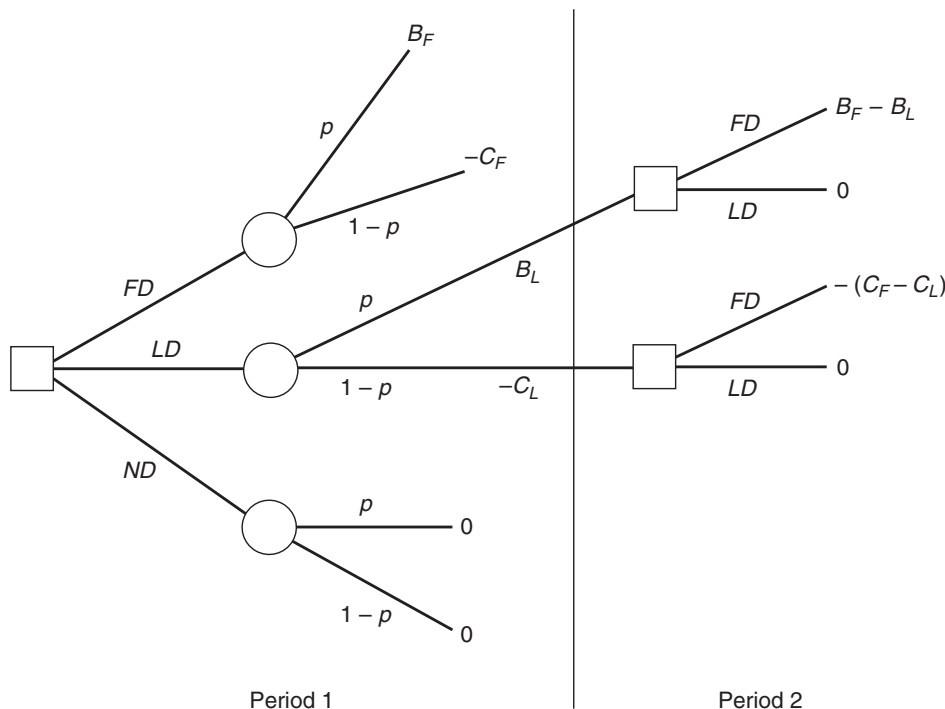


Figure 11.7 Endogenous learning.

enough information to permit a reliable prediction of the effect of full development. If no development is undertaken, then no new information will be available at the beginning of the second period. If full development is undertaken, then new information will be generated, but there will be no decision for it to affect.

As shown in the last column of Table 11.6, the expected net benefits for the *FD* and *LD* alternatives with endogenous learning are identical to those with exogenous learning. The expected net benefits of *ND* are zero, however, because there will be no new information to alter the decision not to develop in the future.

Table 11.7 compares the different learning cases for a specific set of parameter values. If the analyst specifies the decision problem as one of no learning, then *FD* has the largest expected net benefits. Imagine that instead one specifies the decision problem as one of exogenous learning. Now *ND* has the largest expected net benefits. Furthermore, relative to the case of no learning, the quasi-option value of *ND* is \$46.3 million (\$46.3 million – 0) and the quasi-option value of *LD* is \$23.15 million (\$28.15 million – \$5 million). Now instead specify the decision problem as one with endogenous learning. *LD* now has the largest expected net benefits. Relative to the case of no learning, the quasi-option value of *LD* is \$23.15 million (\$28.15 million – \$5 million), and the quasi-option value of *ND* is 0 (0 – 0).

This simple numerical illustration conforms to the common intuition about quasi-option value: *It tends to be large for no development in cases of exogenous learning and large for limited development in cases of endogenous learning.* It is important to keep in mind, however, that the illustration is based on very stylized models of learning. Differently specified models could yield different rankings and different quasi-option values for the alternatives. Even with this simple model, different numerical assumptions could lead to different rankings of alternatives.

Note that the numerical estimates of quasi-option values in the illustration depend on expected values calculated by comparing what was assumed to be the correct two-period decision problem to a one-period decision problem that fails to consider the potential for learning. Of course, if one knows the correct decision problem, then there would be no need to concern ourselves with quasi-option value as a separate benefit

Table 11.7 Numerical Illustration of Quasi-Option Value (millions of dollars)

Expected value	No learning	Exogenous learning	Endogenous learning
$E[FD]$	10.00	10.00	10.00
$E[LD]$	5.00	28.15	28.15
$E[ND]$	0.00	46.30	0.00

Assumptions:

$$B_F = 100 \quad C_F = 80$$

$$B_L = 50 \quad C_L = 40$$

$$p = .5 \quad d = .08$$

category because solving the decision problem would lead to the appropriate calculations of expected net benefits.

11.3.3 *Quasi-Option Value in Practice*

How should analysts treat quasi-option value in practice? Two heuristics seem warranted. First, *quantitative quasi-option values should be based on an explicit decision problem that structures the calculation of the expected net benefits*. An explicit decision problem focuses attention on the key assumptions that determine the magnitude of quasi-option value. It also makes it unnecessary to consider quasi-option value as a distinct benefit category. Second, *when insufficient knowledge is available to formulate a decision problem for explicitly calculating the magnitude of quasi-option value, it should be discussed as a possible source of bias rather than added as an arbitrary quantitative adjustment to expected net benefits*. As with other biases, one can ask the question: How big would quasi-option value have to be to affect the ranking of policies?

11.4 Conclusion

Uncertainty is inherent to some degree in every CBA. Through expected value analysis, the analyst attempts to average over the possible contingencies to arrive at expected net benefits as a plausible prediction of net benefits. In situations not explicitly involving risk, one often assumes parameter values that are more appropriately thought of as draws from probability distributions rather than as certainties. The purpose of sensitivity analysis is to determine how net benefits change if these parameters deviate from their assumed values. Partial sensitivity analysis, the most commonly used approach, focuses attention on the consequences of alternative assumptions about key parameters. Extreme-case analysis examines whether combinations of plausible assumptions exist that reverse the sign of net benefits. Monte Carlo simulation attempts to estimate the distribution of net benefits by explicitly treating assumed parameter values as random variables. It is especially useful when the risk of the policy is of particular concern and the formula for the calculation of net benefits involves the uncertain parameters in other than simple sums; that is, when uncertain parameters are multiplied or divided. While the nature of the policy under consideration and the resources available to the analysts attempting to estimate its benefits and costs determine the appropriate form of sensitivity analysis, every CBA should be subjected to tests of its sensitivity to the assumptions it employs.

Explicit decision analysis frameworks, including games against nature in both normal and extensive form, provide a basis for assessing the value of information in risky circumstances. It allows an explicit calculation of quasi-option value, which is sometimes treated as a separate benefit category in CBAs. Quasi-option values take account of the value of being able to act upon future information. As solving a correctly specified decision problem naturally incorporates quasi-option values, they need not be treated as distinct benefits. Quantitative claims about quasi-option values should be based on an explicit decision problem.

APPENDIX 11A

Monte Carlo Sensitivity Analysis using Commonly Available Software

Two types of generally available software allow analysts to do Monte Carlo simulations easily. Spreadsheets allow it to be done by making rows correspond to trials; statistical packages allow it to be done by making observations correspond to trials. Spreadsheets greatly reduce the labor needed to conduct all the types of sensitivity analysis. Although specialized software is available for doing Monte Carlo analysis, such as Crystal Ball (Decisioneering, Inc.) for use with Excel and DATA (TreeAge, Inc.) for decision analysis, with a bit of effort Monte Carlo simulation can be done with any simple spreadsheet that provides random number generators. Similarly, any statistical package that provides a random number generator can be used. For very complicated simulations, statistical packages that allow structured programming, such as the creation of macros and subprograms, offer greater transparency that helps avoid errors.

Specifying Distributions for Uncertain Parameters

In many situations, analysts are willing to put bounds on the value of uncertain parameters, but unwilling to assume that any values within these bounds are more or less likely than any others. In such cases, uniform distributions are appropriate. Most spreadsheets and statistical packages provide a function for generating random variables that are distributed uniformly from 0 to 1. For example, in Excel, the function `RAND()` returns a draw from this distribution; in the statistical package Stata, `runiform()` does so. To generate uniform random variables with other ranges, one simply multiplies the draw from the random variable uniformly distributed from 0 to 1 by the desired range and then adds the minimum value. So, for example, to get the appropriate random variable for the fraction of high-risk persons in the population, r in Table 11.2, use the following formula: $0.04 + (0.08 - 0.04)z$, where z is the uniform random variable with range 0 to 1.

Some other useful distributions can be generated directly from the uniform distribution for parameters whose values analysts believe fall within bounds, but are more likely to fall closer to the center of the bounds. For example, to obtain a draw from a symmetric triangular distribution between zero and one, simply take one-half of the sum of two independent draws from a uniform distribution over 0 to 1. Generating asymmetric triangular distributions from uniform distributions is a bit more complicated. First, create a variable $t = (\text{mode} - \text{minimum}) / (\text{maximum} - \text{minimum})$, where minimum is the smallest value, maximum is the largest value, and mode is the location of the peak, which falls between the minimum and the maximum.

Second, draw a value, u , from the uniform distribution. Third, if $u < t$, then the value equals

$$\text{minimum} + \sqrt{u(\text{mode} - \text{minimum})(\text{maximum} - \text{minimum})}$$

Fourth, if $u \geq t$, then the value equals

$$\text{maximum} - \sqrt{(1 - u)(\text{maximum} - \text{mode})(\text{maximum} - \text{minimum})}$$

Simply adding together three uniformly distributed random variables produces a distribution bounded between 0 and 3 with much of its density around its center, producing a distribution that looks somewhat like a normal distribution over the range of the bounds.

Assuming actual normal distributions for uncertain parameters is quite common for two reasons. First, the Central Limit Theorem suggests that if the value of the parameter is determined by the sum of many independent random events, then its distribution will be approximately normal. Consistent with the Central Limit Theorem, adding together increasing numbers of independent uniform distributions results in a distribution approximating the normal distribution. Indeed, when spreadsheets and statistical packages only provided uniform random number generators, it was common to approximate normal distributions by summing large numbers of uniform distributions.

Second, analysts often rely on statistical analyses of data for estimates of needed parameters. For example, regression coefficients often serve as point estimates of parameters. Under conventional assumptions for continuous dependent variables, the estimator of the coefficient has approximately a normal distribution. Assuming a normal distribution with a mean equal to the point estimate and a standard deviation equal to the standard error of the estimator naturally follows.

Today spreadsheets and statistical packages provide functions for drawing from normal distributions. For example, in Excel, `NORM.INV(RAND(),μ,σ)` returns a draw from a standard normal distribution (mean equal to μ , and variance equal to σ^2). In Stata, `normal()` provides a standard normal (mean equal to zero; variance equal to 1) that can be converted to a normal distribution with any mean and variance through simple transformations: add a constant equal to the desired expected value and multiply by the square root of the desired variance. A range of 3.92 standard deviations includes 95 percent of the area of the normal distribution. To get the random variable we used in the Monte Carlo simulation for hours lost, t in Table 11.2, we added 24 to the standardized normal and multiplied it by $(30 - 18)/3.92$ so that there was only a 5 percent chance that a value of t would be generated outside the range 18–30.

Table A11.1 summarizes the common applications of the uniform and normal distributions in Monte Carlo simulations for Excel, Stata, and R, an open-source computer language used widely in statistical analysis. Other distributions may also be useful. Most books on mathematical statistics indicate how random variables distributed as chi-square, Student's t , F , and multivariate normal can be generated using combinations of functions of normally distributed random variables. Increasingly, these and other potentially useful distributions, such as the gamma, Poisson, and exponential, are built-in functions in common software packages.

Table A11.1 Common Distributions for Use in Monte Carlo Simulations

Distribution	Excel	Stata	R*
<i>Uniform</i> Minimum = a Maximum = b	+a+ (b - a)*RAND()	gen x = a + (b - a)*runiform()	$x < -\text{runif}(n, a, b)$
<i>Normal</i> Mean = μ Variance = σ^2	+NORM. INV(RAND(), μ, σ)	gen x = $\mu + \sigma * \text{rnormal}()$	$x < -\text{rnorm}(n, \mu, \sigma)$
<i>Symmetric triangular</i> Minimum = a Maximum = b	+a +(b-a)* (RAND()+RAND())/2	gen c = runiform() + runiform() gen x = a + (b - a)*c/2	$x < -(\text{runif}(n, a, b) + \text{runif}(n, a, b))/2$
<i>Asymmetric triangular</i> Minimum = a Maximum = b Mode = c	+RAND() in cell Z + IF(Z < (c - a)/(b - a), a + SQRT(Z*(c - a)*(b - a)), b - SQRT((1 - Z)*(b - c)*(b - a)))	gen t = (c - a)/(b - a) gen u = runiform() gen x = a + sqrt(u*(c - a)*(b - a)) if u < t replace x = b - sqrt((1 - u)*(b - c)*(b - a)) if u >= t	$t < -(c - a)/(b - a)$ $u < -\text{runif}(n, 0, 1)$ $x < -\text{ifelse}(u < t, a + \text{sqrt}(u*(c - a)*(b - a)), b - \text{sqrt}((1 - u)*(b - c)*(b - a)))$
<i>Bounded Normal-like</i> Minimum = a Maximum = b	+a + (b - a)* (+RAND() + RAND()) +(RAND())/3	gen c = runiform() + runiform() + runiform() gen x = a + (b - a)*c/3	$x < -(\text{runif}(n, a, b) + \text{runif}(n, a, b) + \text{runif}(n, a, b))/3$

* R uses arrays so their length, n, the number of trials, must be specified in creating random variables.

Basic Steps Using a Spreadsheet

Once procedures have been developed for generating appropriately distributed random variables, Monte Carlo simulation can be implemented in the following steps.

First, construct a row of appropriate random variables and the formulas that use them to compute net benefits. The last cell in the row should contain net benefits.

Second, copy the entire row a number of times so that the last column of the resulting block contains different realizations of net benefits. Most spreadsheet should be able to handle blocks of more than 1000 rows without memory or speed problems.

Third, analyze the accumulated realizations in the last column along the lines of Figure 11.5, calculating the mean and standard deviation, and plotting them as a histogram.

Basic Steps Using a Statistical Package

In using statistical packages for Monte Carlo simulation, trials are represented as observations. First, create a data set with the number of observations corresponding to the number of trials sought, say 1000, and open a file of the commands that you will employ to create the necessary variables.

Second, give each of the uncertain parameters a variable name and draw values for each observation from its distribution.

Third, combine the parameters in appropriate formulas for costs, benefits, and net benefits. Employing intermediate variables can reduce the chances of errors and facilitate interpretation and modification of the simulation by yourself or others. Indeed, the record of commands provides a transparent record of your calculations. For CBAs that involve complicated calculations, this record is likely to be much easier to interpret than the comparable spreadsheet would be.

Fourth, use available statistical and graphical commands to analyze the variable containing net benefits and display its distribution.

Exercises for Chapter 11

1. The initial cost of constructing a permanent dam (i.e., a dam that is expected to last forever) is \$830 million. The annual net benefits will depend on the amount of rainfall: \$36 million in a “dry” year, \$58 million in a “wet” year, and \$104 million in a “flood” year. Meteorological records indicate that over the last 100 years there have been 86 “dry” years, 12 “wet” years, and 2 “flood” years. Assume the annual benefits, measured in real dollars, begin to accrue at the end of the first year. Using the meteorological records as a basis for prediction, what are the net benefits of the dam if the real discount rate is 5 percent?
2. Use several alternative discount rate values to investigate the sensitivity of the present value of net benefits of the dam in exercise 1 to the assumed value of the real discount rate.
3. The prevalence of a disease among a certain population is 0.40. That is, there is a 40 percent chance that a person randomly selected from the population will have the disease. An imperfect test that costs \$250 is available to help identify those who have the disease before actual symptoms appear. Those who have the disease have a 90 percent chance of a positive test result; those who do not have the disease have a 5 percent chance of a positive test. Treatment of the disease before the appearance of symptoms costs \$2000

and inflicts additional costs of \$200 on those who do not actually have the disease. Treatment of the disease after symptoms have appeared costs \$10,000.

The government is considering the following possible strategies with respect to the disease:

- S1. Do not test and do not treat early.
- S2. Do not test but treat early.
- S3. Test and treat early if positive and do not treat early if negative.

Find the treatment/testing strategy that has the lowest expected costs for a member of the population.

In doing this exercise, the following notation may be helpful: Let D indicate presence of the disease, ND absence of the disease, T a positive test result, and NT a negative test result. Thus, we have the following information:

$$P(D) = .40, \text{ which implies } P(ND) = .60$$

$$P(T|D) = .90, \text{ which implies } P(NT|D) = .10$$

$$P(T|ND) = .05, \text{ which implies } P(NT|ND) = .95$$

This information allows calculation of some other useful probabilities:

$$P(T) = P(T|D)P(D) + P(T|ND)P(ND) = .39 \text{ and } P(NT) = .61$$

$$P(D|T) = P(T|D)P(D) / P(T) = .92 \text{ and } P(ND|T) = .08$$

$$P(D|NT) = P(NT|D)P(D) / P(NT) = .07 \text{ and } P(ND|NT) = .93$$

4. In exercise 3, the optimal strategy involved testing. Does testing remain optimal if the prevalence of the disease in the population is only 0.05? Does your answer suggest any general principle?
5. (Use of a spreadsheet recommended for parts a through e and necessary for part f.) A town with a population of 164,250 persons who live in 39,050 households is considering introducing a recycling program that would require residents to separate paper from their household waste so that it can be sold rather than buried in a landfill like the rest of the town's waste. Two major benefits are anticipated: revenue from the sale of waste paper and avoided tipping fees (the fee that the town pays the owners of landfills to bury its waste). Aside from the capital costs of specialized collection equipment, household containers, and a sorting facility, the program would involve higher collection costs, inconvenience costs for households, and disposal costs for paper that is collected but not sold. The planning period for the project has been set at eight years, the expected life of the specialized equipment.

The following information has been collected by the town's sanitation department.

Waste Quantities: Residents currently generate 3.6 pounds of waste per person per day. Over the last 20 years, the daily per capita amount has grown by about 0.02 pounds per year. Small or no increases in the last few years, however, raise the possibility that levels realized in the future will fall short of the trend.

Capital Costs: The program would require an initial capital investment of \$1,688,000. Based on current resale values, the scrap value of the capital at the end of eight years is expected to be 20 percent of its initial cost.

Annual Costs: The department estimates that the separate collection of paper will add an average of \$6/ton to the cost of collecting household waste. Each ton of paper collected and not sold would cost \$4 to return to the landfill.

Savings and Revenues: Under a long-term contract, tipping fees are currently \$45 per ton with annual increases equal to the rate of inflation. The current local market price for recycled paper is \$22 per ton, but has fluctuated in recent years between a low of \$12 per ton and a high of \$32 per ton.

Paper Recovery: The fraction of household waste made up of paper has remained fairly steady in recent years at 32 percent. Based on the experience of similar programs in other towns, it is estimated that between 60 and 80 percent of paper included in the program will be separated from other waste and 80 percent of the paper that is separated will be suitable for sale, with the remaining 20 percent of the collected paper returned to the waste stream for landfilling.

Household Separation Costs: The sanitation department recognized the possibility that the necessity of separating paper from the waste stream and storing it might impose costs on households. An average of 10 minutes per week per household of additional disposal time would probably be needed. A recent survey by the local newspaper, however, found that 80 percent of respondents considered the inconvenience of the program negligible. Therefore, the department decided to assume that household separation costs would be zero.

Discount Rate: The sanitation department has been instructed by the budget office to discount at the town's real borrowing rate of 6 percent. It has also been instructed to assume that annual net benefits accrue at the end of each of the eight years of the program.

- a. Calculate an estimate of the present value of net benefits for the program.

- b. How large would annual household separation costs have to be per household to make the present value of net benefits fall to zero?
 - c. Assuming that household separation costs are zero, conduct a worst-case analysis with respect to the growth in the quantity of waste, the price of scrap paper, and the percentage of paper diverted from the waste stream.
 - d. Under the worst-case assumptions of part c, how large would the average yearly household separation costs have to be to make the present value of net benefits fall to zero?
 - e. Investigate the sensitivity of the present value of net benefits to the price of scrap paper.
 - f. Implement a Monte Carlo analysis of the present value of net benefits of the program.
6. Imagine that, with a discount rate of 5 percent, the net present value of a hydroelectric plant with a life of 70 years is \$25.73 million and that the net present value of a thermal electric plant with a life of 35 years is \$18.77 million. Rolling the thermal plant over twice to match the life of the hydroelectric plant thus has a net present value of $(\$18.77 \text{ million}) + (\$18.77 \text{ million})/(1 + 0.05)^{35} = \22.17 million .

Now assume that at the end of the first 35 years, there will be an improved second 35-year plant. Specifically, there is a 25 percent chance that an advanced solar or nuclear alternative will be available that will increase the net benefits by a factor of three, a 60 percent chance that a major improvement in thermal technology will increase net benefits by 50 percent, and a 15 percent chance that more modest improvements in thermal technology will increase net benefits by 10 percent.

- a. Should the hydroelectric or thermal plant be built today?
- b. What is the quasi-option value of the thermal plant?

Notes

1. A more realistic assumption (e.g., rainfall amounts closer to the center of the range are more likely) would not change this equality as long as the probability density function of rainfall is symmetric around 11 inches.

2. Assuming that rainfall is distributed uniformly over the range, the expected value of net benefits is simply the area under curve B from 22 inches to 0 inches. See note 4 on how to calculate this area for any distribution of rainfall.

3. The representativeness is very sensitive to the particular shape of the probability density function of rainfall. The use of two contingencies would be even less representative if amounts of rainfall near 11 inches were more likely than more extreme amounts.

4. In the case of a continuous underlying dimension, such as price, the expected value of net benefits is calculated using integration, the continuous analog of addition. Let $NB(x)$ be the net benefits given some particular value of x , the underlying dimension. Let $f(x)$ be the probability density function over x . Then,

$$E[NB] = \int NB(x)f(x)dx$$

where the integration is over the range of x .

5. The term *decision analysis* was originally used to include both choice under risk (statistical decision analysis) and games against strategic opponents (game theory). Now it is commonly used to refer only to the former.

6. We recommend Howard Raiffa, *Decision Analysis: Introductory Lectures on Choices under Uncertainty* (Reading, MA: Addison-Wesley, 1969); Morris H. DeGroot, *Optimal Statistical Decisions* (New York, NY: Wiley Interscience, 2004); Robert L. Winkler, *Introduction to Bayesian Inference and Decision*, 2nd edn (Gainesville, FL: Probabilistic Press, 2003); and Robert D. Behn and James W. Vaupel, *Quick Analysis for Busy Decision Makers* (New York, NY: Basic Books, 1982) as general introductions. For more direct application to CBA, see Miley W. Merkhofer, *Decision Science and Social Risk Management: A Comparative Evaluation of Cost-Benefit Analysis, Decision Analysis, and Other Formal Decision-Aiding Approaches* (Boston, MA: D. Reidel Publishing Company, 1987).

7. Although this example is hypothetical, it captures the essence of the problem that faced public health officials in confronting the N1H1 virus in 2009 and 2010. For an analysis of the issues that arose in the Swine Flu episode in the 1970s, see Richard E. Neustadt and Harvey V. Fineberg, *The Swine Flu Affair: Decision-Making on a Slippery Disease* (Washington, DC: US Government Printing Office, 1978).

8. Note that in this example the probability of an epidemic in the second year is conditional on whether an epidemic occurred in the first year. If an epidemic has occurred in

the first year, then the population gains immunity and there is zero probability of an epidemic in the second year. If an epidemic has not occurred, then there is some probability, p_2 , that one will occur in the second year.

9. Instead, we might have allowed the estimate of P_2 to be adjusted after information was revealed, or gathered, during the first year. If this were the case, then we might use *Bayes' theorem* to update the initial beliefs about P_2 in the face of the new information. Bayes' theorem provides a rule for updating subjective probability estimates on the basis of new information. Let A and B be events. A basic axiom of probability theory is that: $P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$ where $P(A \text{ and } B)$ is the probability of both A and B occurring, $P(A)$ is the probability of A occurring, $P(B)$ is the probability of B occurring, $P(A|B)$ is the conditional probability that A occurs given that B has occurred, and $P(B|A)$ is the conditional probability of B occurring given that A has occurred. It follows directly from the axioms that: $P(A|B) = P(B|A)P(A)/P(B)$ which is the simplest statement of Bayes' rule. Its application is quite common in diagnostic tests. For example, we may know the frequency of a disease in the population, $P(A)$, the probability that a test will yield a positive result if randomly given to a member of the population, $P(B)$, and the conditional probability that, given the disease, the test will be positive, $P(B|A)$. We would thus be able to calculate $P(A|B)$, the conditional probability that someone with a positive test has the disease.

10. Note the assumption that the probability of the influenza reaching the population, p_1 , is independent of whether or not this particular population is vaccinated. This would not be a reasonable assumption if the vaccination were to be part of a national program that reduced the chances that the influenza would reach this population from some other vaccinated population.

11. In this particular problem, it will never make sense to wait until the second year to implement the program if it is going to be implemented at all. If, however, the risk of side effects were expected to fall in the second year, say, because a better vaccine would be available, then delay could be optimal. In terms of the decision tree, we could easily model this alternative scenario by using different values of C_s in the current and next years.

12. For a discussion of the application of decision analysis to the stockpiling problem, see David L. Weimer and Aidan R. Vining, *Policy Analysis: Concepts and Practice*, 6th edn (New York, NY: Routledge, 2017), chapter 17.

13. Calculating the number of combinations: $3^{17} = 129,140,163$.

14. For examples of CBA applied to hepatitis vaccine programs, see Josephine A. Mauskopf, Cathy J. Bradley,

and Michael T. French, "Benefit–Cost Analysis of Hepatitis B Vaccine Programs for Occupationally Exposed Workers." *Journal of Occupational Medicine*, 33(6), 1991, 691–98; Gary M. Ginsberg and Daniel Shouval, "Cost–Benefit Analysis of a Nationwide Neonatal Inoculation Programme against Hepatitis B in an Area of Intermediate Endemicity." *Journal of Epidemiology and Community Health*, 46(6), 1992, 587–94; and Murray Krahn and Allan S. Detsky, "Should Canada and the United States Universally Vaccinate Infants against Hepatitis B?" *Medical Decision Making*, 13(1), 1993, 4–20.

15. Call the basic reproductive rate of the infection R_0 . That is, each primary infection exposes R_0 individuals to infection. If i is the fraction of the population no longer susceptible to infection because of previous infection, then the actual reproductive rate is $R = R_0(1 - i - v)$, where v is the fraction of the population effectively vaccinated. If R falls below 1, then the infection dies out because, on average, each infection generates less than one new infection. Assuming that the population is homogeneous with respect to susceptibility to infection and that infected and non-infected individuals uniformly mix in the population, a rough estimate of the ultimate i for the population is given by the formula $i = 1 - (1/R_0) - v$, where $1 - 1/R_0$ is the estimate of the infection rate in the absence of the vaccine. For an overview, see Roy M. Anderson and Robert M. May, "Modern Vaccines: Immunisation and Herd Immunity." *The Lancet*, 8690(March), 1990, 641–45; and Joseph W. G. Smith, "Vaccination Strategy," in Philip Selby, editor, *Influenza, Virus, Vaccines, and Strategy* (New York, NY: Academic Press, 1976), 271–94.

16. The $L = \$10$ million and $L = \$5$ million lines cross because lives are at risk both from the vaccination side effects and from the epidemic. At low probabilities of epidemic, the expected number of lives saved from vaccination is negative so that net benefits are higher for lower values of life. At higher probabilities of epidemic, the expected number of lives saved is positive so that net benefits are higher for higher values of life.

17. Note that these estimates of NB no longer involve expectations with respect to the contingencies of epidemics, although the cost estimates themselves are expected values. For each random draw, only one combination of contingencies can actually occur. (The epidemic poses a collective risk to the population, while the costs result from the realizations of independent risks to individuals in the population.)

18. In general, if the calculation of net benefits involves sums of random variables, using their expected values yields the expected value of net benefits. If the calculation involves sums and products of random variables, then using their expected values yields the expected value of net benefits only if the random variables are uncorrelated. In the Monte Carlo approach, correlations among variables can be taken into account by drawing parameter values from either multivariate or conditional distributions rather than from independent univariate distributions as in this example. Finally, if the calculation involves ratios of random variables, then even independence (i.e., an absence of correlations) does not guarantee that using their expected values will yield the correct expected value of net benefits. In this latter situation, the Monte Carlo approach is especially valuable because it provides a way of estimating the correct expected net benefits.

19. Aidan R. Vining and David L. Weimer, "An Assessment of Important Issues Concerning the Application of Benefit–Cost Analysis to Social Policy." *Journal of Benefit–Cost Analysis*, 1(1), 2010, 1–38.

20. The concept of quasi-option value was introduced by Kenneth J. Arrow and Anthony C. Fisher, "Environmental Preservation, Uncertainty, and Irreversibility." *Quarterly Journal of Economics*, 88(2), 1974, 312–19.

21. Jon M. Conrad, "Quasi-Option Value and the Expected Value of Information." *Quarterly Journal of Economics*, 44(4), 1980, 813–20; and Anthony C. Fisher and W. Michael Hanemann, "Quasi-Option Value: Some Misconceptions Dispelled." *Journal of Environmental Economics and Management*, 14(2), 1987, 183–90. The concept of quasi-option value also applies to private-sector investments involving large initial costs that cannot be recovered if abandoned. For overviews, see Robert S. Pindyck, "Irreversibility, Uncertainty, and Investment." *Journal of Economic Literature*, 29(3), 1991, 1110–48; Avinash Dixit and Robert Pindyck, *Investment Under Uncertainty* (Princeton, NJ: Princeton University Press, 1994); Gilbert E. Metcalf and Donald Rosenthal, "The 'New' View of Investment Decisions and Public Policy Analysis: An Application to Green Lights and Cold Refrigerators." *Journal of Policy Analysis and Management*, 14(4), 1995, 517–31; and Anthony Edward Boardman and David H. Greenberg, "Discounting and the Social Discount Rate," in Fred Thompson and Mark T. Green, editors, *Handbook of Public Finance* (New York, NY: Marcel Dekker, 1998), 269–318.

Case 11

Using Monte Carlo Simulation: Assessing the Net Benefits of Early Detection of Alzheimer's Disease

Alzheimer's disease (AD), the most common form of dementia, currently affects at least 5 percent of the US population aged 65 years and older. This chronic progressive neurodegenerative disorder increases in prevalence with age, affecting almost half of those persons aged 85 and older. Aging of the US population will likely increase the prevalence of AD from a total of about 5 million people today to 14 million people by 2050.¹ The expected rapid increase in the number of persons with AD will translate into higher public and private long-term care costs paid by households, the Medicaid program, and long-term care insurers. Although studies have estimated a wide range of total annual costs to the US economy resulting from AD, the most plausible estimates now have surpassed two-hundred billion dollars.² Primary care physicians often fail to detect the mild cognitive impairment that is usually apparent in the early stages of AD; they enter diagnoses of AD into medical records even less frequently.³ Nonetheless, fairly inexpensive but effective protocols exist for diagnosing AD⁴ – one well-designed study found that screening those over 70 years old would cost less than \$4,000 per positive diagnosis.⁵ Assuming that 70 percent of those diagnosed participate in treatment or counseling interventions, the cost per treated diagnosis would be about \$5,700. Do the benefits of early detection of AD justify the diagnosis cost?

Early detection of AD could be beneficial in three ways. First, although available drugs can neither reverse nor stop the progression of AD, they can slow it. Use of the drugs during the early stages of the disease can potentially delay the entry of those with AD into expensive long-term care. Second, there is strong evidence that providing support to the caregivers of AD patients can enable them to keep their loved ones in the home longer and with fewer adverse psychological costs for themselves. Third, despite common perceptions to the contrary, most people want to know if they are in the early stages of AD.⁶ This knowledge enables them to participate in planning for their own long-term care as well as to engage in desirable activities, such as travel, while they can still do so.

Early treatment and caregiver support provides large potential benefits by reducing the total time AD patients spend in nursing homes. The median annual nursing home cost in the United States is over \$90,000. Not surprisingly, most AD patients admitted to nursing homes fairly quickly exhaust their financial resources and must rely on public subsidy through the Medicaid program, which now pays for just over half of all nursing home costs in the United States. Avoiding a year of nursing home care thus saves resources for society as well as reduces state and federal Medicaid expenditures. Studies also suggest that at each stage of the disease, AD patients experience a higher quality of life if living in the community rather than in nursing homes. This is offset somewhat by a

lower quality of life for caregivers when AD patients they are caring for live in the community rather than in nursing homes. Treatment and caregiver support have costs that must be taken into account. Further, the time costs of caregivers are substantially larger for AD patients living in the community rather than in nursing homes.

The net benefits of early detection equal the costs without intervention minus the costs with it. In other words, the net benefits of early detection are the avoided costs that it facilitates. Table C11.1 summarizes the cost categories included in a cost–benefit analysis of early detection, which was implanted through the Monte Carlo simulation described below.

Totting up these costs poses a number of analytical challenges. First, as with many public policies, costs precede beneficial impacts. Expenditures on early detection and treatment occur now, but the delayed entry into nursing homes will likely occur years in the future when the disease progresses to a stage that overwhelms caregivers. Second, life-course uncertainties affect whether benefits will actually be realized by any particular patient. People in the relevant age group die from many causes. Consequently, some will die before any benefits from delayed nursing home care can be realized. Third, as is common in analyses of social policies, predicting impacts involves assumptions about uncertain parameters. Most importantly, assumptions about the progression of AD with and without drug treatment and the impacts of caregiver support come from studies with estimation error.

To address these challenges, David Weimer and Mark Sager (henceforth W&S) built a Monte Carlo simulation that tracks patients over their remaining life courses with and without the interventions made possible by early identification of AD.⁷ Each trial of the simulation provides an estimate of net benefits from intervention, sometimes positive as when the patient lives long enough to show a reduced stay in a nursing home, but sometimes negative as when the patient dies before a reduced stay in a nursing home can be realized. The simulation also allowed for the uncertainty about costs displayed in Table C11.1 as well as important uncertainties in the impacts of drug treatment and caregiver

Table C11.1 *Costs Borne by AD Patients, Caregivers, and the Rest of Society*

Cost category	Monetization strategy	Values used (2009 dollars)
Time in nursing home	Annual nursing home cost	66,795/year
Patient utility	Statistical value of life year	93,500 to 187,000 (uniform distribution)
Caregiver utility		
Caregiver time	Median wage and benefit rate	14.69/hour
Drug treatment	Market price	1,825/year
Caregiver support	Wage and benefit rate	35.05/hour
In-home services	Expenditures	0 to \$2,968/year (uniform distribution)

Source: Information extracted from David L. Weimer and Mark A. Sager, “Early Identification and Treatment of Alzheimer’s Disease: Social and Fiscal Outcomes.” *Alzheimer’s & Dementia*, 5(3), 2009, 215–26.

support based on the available estimates in the research literature. For example, the statistical value of a life-year and in-home services were allowed to vary over uniform ranges, although other costs, such as the annual cost of a nursing home, were treated as point estimates. In addition, the impact of caregiver counseling on institutionalization risk was based on the reported confidence interval for the odds ratio for participation in the counseling. In aggregate, the trials provide a distribution of possible outcomes. The average of the trials provides an estimate of the likely net benefit conditional on intervention. If the conditional net benefit is larger than costs per diagnosis, then early detection is efficient.

Modeling the Life Course of AD Patients

A commonly used cognitive metric, the Mini-Mental State Examination (MMSE), plays a central role in W&S modeling the life course of those with AD. The MMSE is a scale ranging from a high of 30 corresponding to normal cognitive ability to 1 for loss of all cognitive ability. Scores of 28–21, 20–11, and 10–1 represent mild, moderate, and severe AD, respectively. Research provides estimates of the rates of decline in MMSE for AD patients both with and without drug treatment. Research also provides estimates of the probability of AD patients being institutionalized as a function of MMSE, age, sex, and marital status (married patients are more likely to have a caregiver).

The general approach adopted by W&S involved separate Monte Carlo simulations for different types of AD patients defined by combinations of initial MMSE, age, sex, marital status, and either an intervention (drug treatment, caregiver counseling, and drug treatment combined with caregiver counseling) or no intervention. Each patient progresses year by year until death. During each year of life, costs of various sorts were incurred: time costs based on estimates from the literature of the time spent by caregivers contingent on whether the patient was in the community or institutionalized and whether the AD was mild, moderate, or severe (monetized at median wage rates); utility costs for both caregivers and patients for mild, moderate, and severe AD (monetized at the value of a life year); nursing home costs for those institutionalized (monetized at average annual cost); and the costs of interventions (monetized at drug prices, counselor wages, and incremental expenditures on in-home services).

After the first year of the simulation, the AD patient has some probability of living to the next year based on survival probabilities for men and women in the general population adjusted for their higher mortality rate. If the AD patient begins with a spouse, then unadjusted survival probabilities are used to determine if the spouse survives to the next year. At the beginning of each subsequent year in which the patient resides in the community, the probability of being institutionalized, based on the updated MMSE, age, and marital status as well as sex and whether caregiver counseling is provided, determines if the patient remains in the community or enters a nursing home. Once the patient enters a nursing home, he or she remains there until death.

W&S modeled the disease progression in two different ways. The first approach was based on estimates that MMSE declines with and without drug treatment. The second approach was based on research suggesting that patients could be divided into slow

and fast decliners and that drug treatment increased the probability of a patient being a slow decliner. Each of these approaches drew parameters from specified distributions. For example, the mean decline model drew declines from a normal distribution with mean 3.5 MMSE points and standard deviation of 1.5 MMSE points for patients not receiving drug treatment.

Simply comparing incurred costs with and without intervention would overestimate the benefits of intervention because some AD patients not identified early would eventually be identified when their symptoms became more obvious. Consequently, the counterfactual to early detection took account of subsequent diagnosis and intervention. In particular, based on available research, it gave each patient not identified initially a 25 percent chance of being diagnosed when MMSE fell to 19 points.

Net benefits for each of the interventions were computed in the following steps: First, the net benefits of intervention in each year were computed as the difference between the costs without the initial intervention (but potentially with it later as specified in the counterfactual) and the costs with it. Second, these differences were discounted back to the present to obtain a present value of net benefits. Third, the first two steps were repeated in 10,000 trials to produce a distribution of the present value of net benefits. Fourth, the mean present value of net benefits was computed by averaging over the trials. The intervention would be assessed as efficient if the average present value of net benefits it produced were larger than the costs of diagnosing an AD patient through screening. A similar method was used to estimate the fiscal impacts of early detection for Wisconsin and the federal government to assess the budgetary implications of early detection.

Table C11.2 shows representative results from the analysis for a married male using the more conservative model of AD progression and drug response. Several general patterns are apparent. First, other things being equal, interventions offer larger net benefits for younger AD patients – younger patients are likely to survive longer and are therefore at greater risk of long stays in nursing homes. Second, drug treatment provides larger net benefits for patients earlier in the AD progression. Slowing disease progression early keeps patients in the mild phase of AD longer, which reduces both caregiver costs and the probability of an early institutionalization. Third, the net benefit from caregiver counseling does not change very much across this range of AD progression. Note that the present values reported in Table C11.2 would be larger for a female patient, but smaller for an unmarried patient.

Policy Implications

The scenarios presented in Table C11.2 all show the net benefits of intervention following early diagnosis of AD to be larger than the \$5,700 cost per treated diagnosis that could be obtained from screening those over 70 years of age. Some caution is warranted in basing policy on the estimates of benefits from drug treatment because the available research from which the estimates were taken typically followed patients for only a few years. Consequently, the drug treatment model in the simulations involves extrapolation

Table C11.2 *Present Value of Net Benefits from Interventions Following Early AD Detection in Married Males (1,000s dollars)*

Age	Intervention	MMSE		
		26	24	22
65	Drug treatment only	94.5	71.4	48.7
	Counseling only	12.0	11.6	11.9
	Drug treatment and counseling	119.3	96.6	72.9
70	Drug treatment only	80.9	62.3	41.5
	Counseling only	9.2	10.5	11.5
	Drug treatment and counseling	103.0	82.2	66.1
75	Drug treatment only	66.5	51.6	34.9
	Counseling only	7.4	11.7	10.7
	Drug treatment and counseling	82.7	69.3	56.8

Source: Calculated using the W&S model.

beyond observed data and may be overly optimistic in assuming that drugs can slow progression over more than a few years. The costs and impacts of caregiver counseling are based on a long-term study with random assignment of caregivers to counseling and a control group, and therefore have a much stronger empirical basis.⁸ As the net benefits from counseling alone are larger than the diagnostic costs, the analysis strongly supports screening combined with caregiver support. W&S also estimated that about half of the net benefits from caregiver counseling would accrue as fiscal savings to the state of Wisconsin, suggesting that screening and caregiver counseling would be fiscally as well as socially beneficial. The W&S analysis was influential in promoting state support for AD screening in Wisconsin and Minnesota.⁹

Accommodating the Different Types of Uncertainty

The W&S analysis shows how Monte Carlo simulation can be used to model three types of uncertainty. First, Monte Carlo simulation can be used to take account of parameter uncertainty, its most common use in CBA. The key is to represent the uncertainty as a probability distribution over possible values of the parameter and then draw values of the parameter from the distribution in the multiple trials. For example, rather than using the point estimate of the reduction in risk of institutionalization reported in the study of caregiver counseling, W&S randomly drew values from the reported confidence interval around the point estimate in their Monte Carlo trials. Second, Monte Carlo simulation can be used to model processes that involve random events by assigning probabilities to them. This capability was especially important in the W&S analysis because of the importance of random events, such as nursing home institutionalization and patient death, to the accruing of costs and benefits. Third, multiple Monte Carlo simulations can

be conducted to take account of model uncertainty. For example, in the W&S analysis, separate Monte Carlo simulations employed two different models of AD progression. Although not done in the W&S analysis, results can be reported as weighted averages across the models.

Exercises for Chapter 11 Case

1. What information would be needed to estimate the net benefits of a state-wide Alzheimer's disease screening program for 65-year-olds?
2. Imagine that you wanted to use a life-course model similar to the Alzheimer's disease model to estimate the net benefits of helping someone quit smoking. What would be the most important similarities and differences?

Notes

1. Centers for Disease Control and Prevention, *Alzheimer's Disease* (www.cdc.gov/aging/aginginfo/alzheimers.htm).
2. Michale D. Hurd, Paco Martorell, Adeline Delavande, Kathleen J. Mullen, and Kenneth M. Langa, "Monetary Costs of Dementia in the United States." *New England Journal of Medicine*, 368, 2013, 1326–34.
3. Alex J. Mitchell, Nicholas Meader, and Michael Pentzek, "Clinical Recognition of Dementia and Cognitive Impairment in Primary Care: A Meta-analysis of Physician Accuracy." *Acta Psychiatrica Scandinavica*, 124(3), 2011, 165–83.
4. Common protocols have three stages: first, a simple screen, such as whether or not the person can name 14 animals in 1 minute; second, for those who screen positive on the animal naming test, the Neurobehavioral Cognitive Status Examination, which requires about 20 minutes to administer; and third, for those who screen positive on the Neurobehavioral Cognitive Status Examination, diagnosis by a physician to rule out other causes of dementia.
5. Malaz Boustani, Christopher M. Callahan, Frederick W. Unverzagt, Mary G. Austrom, Anthony J. Perkins, Bridget A. Fultz, Siu L. Hui, and Hugh C. Hendrie, "Implementing a Screening and Diagnosis Program for Dementia in Primary Care." *Journal of General Internal Medicine*, 20(7), 2005, 572–77.
6. Louise Robinson, Alan Gemski, Clare Abley, John Bond, John Keady, Sarah Campbell, Kritika Samsi, and Jill Manthorpe, "The Transition to Dementia – Individual and Family Experiences of Receiving a Diagnosis: A Review." *International Psychogeriatrics*, 23(7), 2011, 1026–43.
7. David L. Weimer and Mark A. Sager, "Early Identification and Treatment of Alzheimer's Disease: Social and Fiscal Outcomes." *Alzheimer's & Dementia*, 5(3), 2009, 215–26.
8. Mary S. Mittleman, William E. Haley, Olivio J. Clay, and David L. Roth, "Improving Caregiver Well-Being Delays Nursing Home Placement of Patients with Alzheimer Disease." *Neurology*, 67(9), 2006, 1592–99.
9. Alzheimer's Disease Working Group, *Preparing Minnesota for Alzheimer's: Budgetary, Social and Personal Impacts*, Summary of Report to the Legislature, 2010.