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Lecture note

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[*Supplementary reading materials includes GdR Chapters 5 and 6. In addition, also consult the "Veileder i samfunnsøkonomiske analyser" section 3.5*]

Discounting future impacts

We will now address several practical considerations that are essential for calculating the Net Present Value (NPV) of a project. First, we assume that a discount rate has been provided to us. Subsequently, we will delve into the theoretical aspects related to choosing an appropriate discount rate.

Basics of discounting

Discounting takes place over periods not years. However, for expositional simplicity, we assume that each period is a year. Consider an investment that lasts for one year and yields return at an annual interest rate of i.

The future value in one year of an amount Xinvested at interest rate i is: $Y = X(1+i)$.

Then the present value (PV) of this future aggregate fund of Y must be the same as X, which gives us a formula for PV as

$PV \ = \ X = Y/(1+i)$

We can extend this line of argument to investments that yield returns over multiple years.

The present value, PV, of an amount Y received in $T-$th years, with interest compounded annually at rate i is:

$PV = \frac{Y}{(1+i)^T}$.

This formulation presents a case of exponential discounting where the T-th period discount factor is given by $\delta^T = 1/(1+i)^T$.

The present value for a stream of benefits $\underline{B} = (B_0, B_1, ..., B_T)$ and costs $\underline{C} = (C_0, C_1, ..., C_T)$ over T years (here 0 denoting the current year) *is*:

$PV(\underline{B}) = \sum_{t=0}^{T} \frac{B_t}{(1+i)^t}$ and $PV(\underline{C}) = \sum_{t=0}^{T} \frac{C_t}{(1+i)^t}$

Net Present Value (NPV)

Net present value (NPV) of a project with a stream of benefits $\underline{B} = (B_1, ..., B_T)$ and costs $\underline{C} = (C_1, ..., C_T)$ over T years is

$NPV(\underline{B}, \underline{C}) = \sum_{t=0}^{T} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{T} \frac{C_t}{(1+i)^t} = \sum_{t=0}^{T} \frac{B_t - C_t}{(1+i)^t}$.

See, for example, Figure 1 and Figure 2, which illustrates the timeline of a project with streams of benefits and costs accrued over 5 years, and consider a discount/interest rate of 0.04.

Year	Event	Annual benefits	Annual costs	Annual net social benefits
0	Purchase and install	0	500,000	–500,000
1	Annual benefits and costs	150,000	25,000	125,000
2	Annual benefits and costs	150,000	25,000	125,000
3	Annual benefits and costs	150,000	25,000	125,000
4	Annual benefits and costs	150,000	25,000	125,000
5	Annual benefits and costs	150,000	25,000	125,000
	PV	667,773	611,296	56,478

Figure 1: Streams of benefits and costs of a project

Inspecting the formula for NPV, we can see that for a project that typically yields benefits in later years and incurs higher costs in the current year, NPV can decrease with the discount rate i. If we set $i = 0$, the formula simply adds the net benefits (benefits minus costs) over the years.

For example, the project described in Figure 1 will have an aggregate (non-discounted) benefit of $750,000$ and an aggregate (non-discounted) cost of $625,000$, thus generating a net figure of $125,000$. For large values of i, the future benefits are less valuable in today's terms and will reduce the NPV.

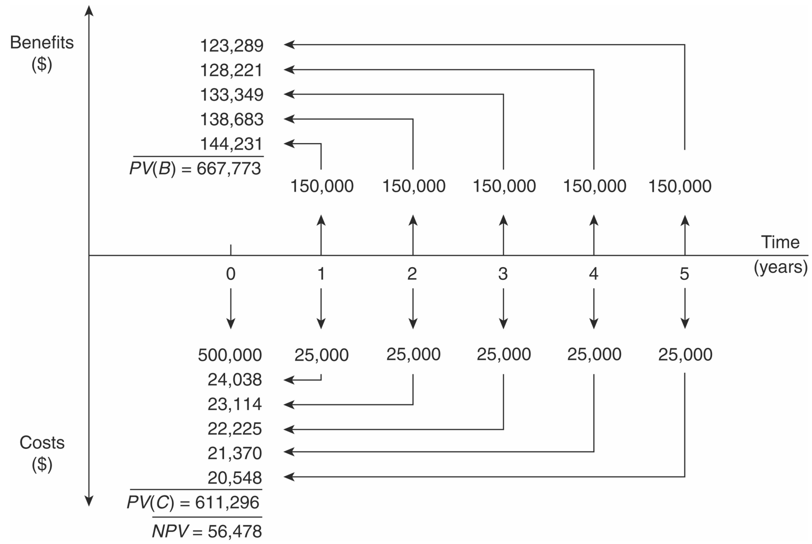

Figure 2: A project with streams of future benefits and costs

Internal Rate of Return (IRR)

The Internal Rate of Return (IRR) of a project equals the discount rate at which the project's NPV equals zero. Sometimes, IRR provides a good indication of the desirability of a project. IRR is the highest discount rate that leaves the project profitable.

Compounding over multiple subperiods

All the previous formulas considered the length of the periods to be one year and the rate of interest/discount to be an annual one. However, it might be possible that benefits or costs are realized at different intervals, such as semi-annually or quarterly, etc. In such cases, the interest/discount rate needs to be adjusted accordingly.

The conversion formula is given by

$(1 + i_s)^s = (1 + i)$,

where s is the number of subperiods (for example, 2 in the case of half-yearly, 4 in the case of quarterly), and i_s denotes the corresponding interest/discount rate.

Solving for i_s gives

$i_s = (1 + i)^{\frac{1}{s}} - 1$.

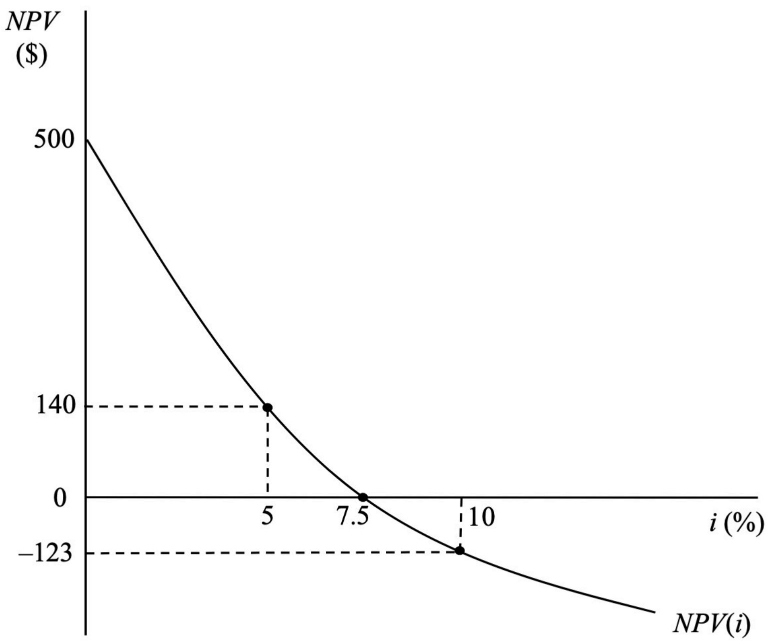

Figure 3: Internal rate of return

Some useful formulae

If future returns are constant across periods, then we can use the following formula to represent the sum of a finite geometric series.

$a + ad + ad^2 + ... + ad^n = a\frac{1-d^{n+1}}{1-d}$ where $a > 0$, and $d \neq 1$.

Also note that as n approaches infinity, the absolute value of d must be less than one for the series to converge. The sum then becomes

$a + ad + ad^2 + ... = \frac{a}{1-d}$ where $0 < d < 1$.

Using the formula for the sum of a finite number of terms in a geometric series

$\sum_{t=0}^{T} \frac{V}{(1+i)^t} = V[\frac{1+i-(1+i)^{-T}}{i}]$, and

$\sum_{t=1}^{T} \frac{V}{(1+i)^t} = V[\frac{1-(1+i)^{-T}}{i}]$.

Observe that the difference between the two expressions above stems from the different the number of periods considered in the two series, one starts from period 0 and the other starts from period 1.

Annuity and perpetuity

An annuity is an equal, fixed amount received (or paid) each year for a number of years. A perpetuity is an indefinite annuity. Many CBAs contain annuities or perpetuities. Fortunately, there are some simple formulas for calculating their PVs.

The present value of an annuity of V per annum (with payments received at the end of each year) for$T$ years starting from year 1 with interest at $i$percent is given by

$PV = \sum_{t=1}^{T} \frac{V}{(1+i)^t} = V[\frac{1-(1+i)^{-T}}{i}]$

and the present value of the present value of an amount V received at the end of each year (starting from year 1) in perpetuity is

$PV = \sum_{t=1}^{\infty} \frac{V}{(1+i)^t} = \frac{V}{i}.$

Equivalent Annual Net Benefits (EANB)

It is sometimes useful to convert the NPV of a project in terms of an Annutiy; for example, find V such that a project with a stream of benefits $\underline{B} = (B_0, B_1, ..., B_T)$ and costs $\underline{C} = (C_0, C_1, ..., C_T)$ satisfies the following:

$NPV(\underline{B}, \underline{C}) = \sum_{t=1}^{T} \frac{V}{(1+i)^t} = V \sum_{t=1}^{T} \frac{1}{(1+i)^t} = V[\frac{1-(1+i)^{-T}}{i}].$

The value of V refers to the annuity payment (also called *Equivalent Annual Net Benefits* or EANB) and the term $(1-(1+i)^{-T})/i$ is called the $T-$period annuity factor, which equals the present value of an annuity payment of 1kr over T periods at an annual interest rate of i. Thus,

$EANB = \frac{NPV}{annuityfactor}$

Comparing returns in nominal versus real terms

A relevant question when quantifying future benefits and costs is whether these measurements should be expressed in real or nominal rates.

Fortunately, as long as we maintain consistency in expressing benefits/costs and the discount rate in the same manner, both methods will yield the same comparative assessment. We should either measure benefits and costs in real dollars and discount them using a real discount rate, or measure them in nominal dollars and discount using a nominal discount rate.

Expressing benefits and costs in real terms is sometimes more intuitive because it takes into account the potential effects of inflation on future benefits and costs. However, when doing so, we must convert the nominal interest rate to a real discount rate using the following formula, where i, r, and m represent the annual nominal interest rate, the annual real interest rate, and the annual inflation rate, respectively:

$(1 + r)(1 + m) = (1 + i)$,

which gives

$r = \frac{i-m}{1+m}$.

Expressing figures in real terms would also grant us greater flexibility when selecting variable inflation rates for future periods. Future inflation rate forecasts are typically obtainable from statistical bureaus.

Project selection

After discounting the benefits and costs, we require a decision criterion for accepting or rejecting a project and for selecting one if there are multiple acceptable projects. NPV can serve as a key indicator for this assessment purpose.

When faced with the decision of whether to accept or reject a single project, it is logical to apply the decision rule:
Accept if $NPV > 0$; reject if $NPV < 0$.

This is because we have already expressed the benefits and costs of the public project under consideration in monetary terms. Consequently, we can interpret the project as an investment project. If we could invest funds in a project that would yield returns at an interest rate equal to the discount rate, then its NPV would be precisely zero. We can regard such a project as the opportunity cost of the public project.

A project with a positive NPV implies that the funds invested in the project generate benefits exceeding its opportunity cost. This not only enables the recovery of the invested funds along with interest payments but also yields additional benefits.

This NPV-based decision rule can also be applied to choose among multiple projects. If the impacts of multiple, mutually exclusive alternative projects are calculated relative to the status quo, one should choose the project with the *highest* NPV, as long as this project's $NPV > 0$. If the $NPV < 0$ for all projects, one should maintain the status quo.

Another indicator for assessment is the IRR. Recall that IRR is the value of the interest rate that makes NPV equal to zero.

A commonly used decision rule, when there is only one alternative, is to invest in the project if its IRR is greater than the discount rate.

Similarly, when comparing two projects with different IRRs, one might prefer the one with the higher IRR, especially when one project consistently dominates the other in terms of NPV.

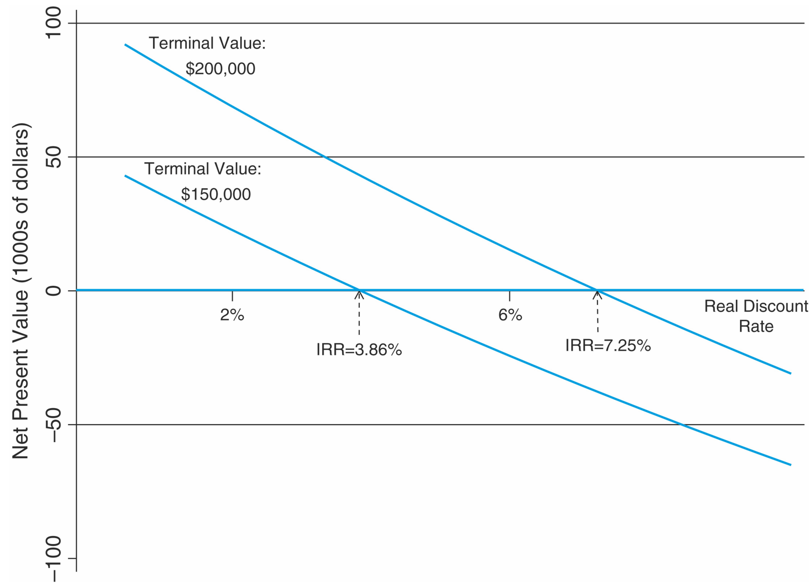

Figure 4: Two projects with different IRR

Although IRR conveys useful information, there are some problems associated with using IRR as a decision rule. For instance, IRR may not be unique; that is, there may be more than one discount rate at which the NPV is zero. Second, IRRs are expressed as percentages (ratios), not in monetary values. Therefore, they should not be used to select one project from a group of projects that differ in size.

Comparing projects with different time frames

Choosing one project over another solely based on the NPV of each project is problematic if the two projects have different time frames. Such projects are not directly comparable.

Two appropriate methods to evaluate projects with different life spans are:

Roll over method

If project A spans n times the number of years as project B, then assume that project B is repeated n times and compare the NPV of n repeated project Bs to the NPV of (one) project A.

For example, if project A lasts 30 years and project B lasts 15 years, compare the NPV of project A to the NPV of 2 back-to-back project Bs, where the latter is computed:

$NPV = x + \frac{x}{(1+i)^{15}}$,

where x is the NPV of one 15-years lasting project B.

Equivalent Annual Net Benefits Method

Recall that $EANB = NPV/(annuity factor)$, which is the amount received each year for the life of the project that has the same NPV as the project itself.

Comparisons can be drawn based on EANBs, which provide more comparable per-year figures. However, it does not take into account that the two projects have different lifespans.

Selection of discount rate

So far, we have presented our analysis for a given discount rate. This discount rate effectively assigns varying weights to the benefits and costs accrued in different periods. When assessing government policies or projects, we must determine the suitable weights to apply to policy impacts occurring in different years.

The choice of the appropriate discount rate is equivalent to determining the proper set of weights, also known as social discount rates (SDR).

Various approaches have been employed to determine the Social Discount Rate (SDR). Two prominent methods are the Rate of Time Preference method and the Opportunity Cost of Capital method.

An Individual's Marginal Rate of Time Preference (MTRP)

An individual's MRTP measures how much additional consumption she would accept in the future to be willing to postpone one unit consumption in the current year.

Equality of MRTPs and market interest rate in perfect capital market

In a perfectly competitive capital market, an individual's MRTP equals the market interest rate i, as shown in Figure 5.

The observation of the equality between MRTP and the market interest rate is significant. It implies that even if individuals have varying time preferences, their consumption choices may differ, but their MRTPs will remain identical to the prevailing market interest rate. We can therefore uniquely characterize the common market interest rate as the social rate of time preference.

Equality of the Social Rate of Time Preference and the Return on Investment (ROI) in perfect capital market

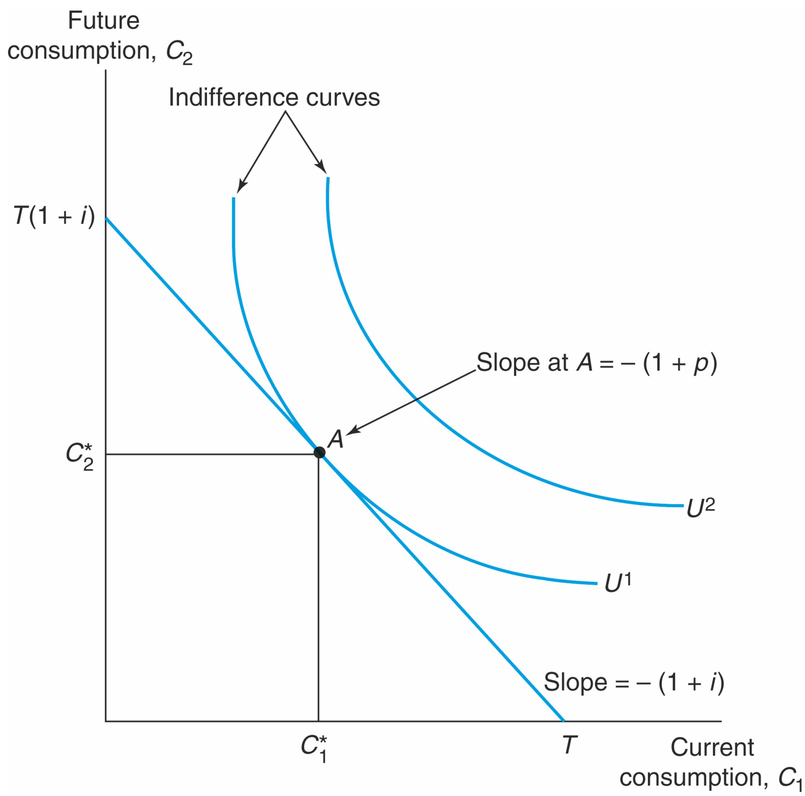

Figure 5: MRTP equals the market interest rate

Economists have extended the analysis to to incorporate production within a closed economy. The analysis shows that if there are no market failures or taxes, and that there are no transaction costs associated with borrowing and lending, then the MRTP and the market interest rate coincides with the marginal rate of return on investment (ROI). In such a closed economy (with the assumption of perfect capital market), the common interest rate would be the obvious choice for the SDR.

Social opportunity cost of capital method

The social opportunity cost of capital (SOC) method builds on influential work by Arnold Harberger. Harberger analyzed a closed domestic market for investment and savings. He assumed that any new government project would be funded by domestic government borrowing which would raise the interest rate and result in a fall in private investment and an increase in savings.

Harberger argues that the social discount rate should reflect the social opportunity cost of capital, which can be obtained by weighting the marginal rate of return on private investment and the after-tax marginal return on savings (or foregone consumption) with respective weights given by the size of the relative contributions that investment and consumption would make toward funding the project.

Ramsey model: Social time preference method

Frank Ramsey suggested an approach for determining the SDR that does not rely on market rates of interest. He proposed a model with infinite periods in which society (or a single representative individual) attempts to maximize a social welfare function that reflects the values society places on per capita consumption over time.

Following this model, the society's *marginal rate of time preference* (STP), would equal the sum of two components: one that reflects the reality of impatience and the other that reflects society's preference for smoothing consumption over time:

$STP = \rho + \mu g,$

where g is the percentage change in per capita consumption (i.e. consumption growth) and is the absolute value of the elasticity of the marginal utility of consumption with respect to changes in consumption, and ρ, g, $\mu \geq 0$. This equation is known as the Ramsey formula.

Discounting Intergenerational projects

So far we've discussed only constant (time-invariant) SDRs. Time invariant SDR may be appropriate for projects affecting the same group of people over different years. There are reasons, however, to suggest the use of a time-declining SDR for projects that might affects different generations.

Empirical evidence suggests that people use lower discount rates for events that occur farther into the future. Long-term environmental and health consequences have very small present values when discounted using a constant rate, often implying that spending a relatively small amount today to avert a costly disaster several centuries in the future is not cost-beneficial.

Constant rates do not appropriately take into account the preferences of future, as yet unborn, generations.

Constant rates do not appropriately allow for uncertainty as to market discount rates in the future.

To address these issues, a time-declining rate schedule is often recommended.

Current practice in Norway

Current discounting practices for public projects vary depending on the nature and scope of the projects; see the Norwegian guideline in the review committee's report on cost benefit analysis (NOU 2012: 16, pp. 66-67). Here I am quoting some relevant parts of it (from page 13):

> – A real risk-adjusted discount rate of 4 percent will be reasonable for use in the cost-benefit analysis of an ordinary public measure, such as a transportation measure, for effects in the first 40 years from the date of analysis. – Beyond 40 years, it is reasonable to assume that one will be unable to secure a long-term rate in the market, and the discount rate should accordingly be determined on the basis of a declining certainty equivalent rate as the interest rate risk is supposed to increase with the time horizon. A rate of 3 percent is recommended for the years from 40 to 75 years into the future. A discount rate of 2 percent is recommended for subsequent years.