Løsningsforslag oppgave 1 – mappeoppgave 2 vår 2025

Linear Hotelling Model with Two Firms

Setup:

- The market is the unit interval $[0, 1]$ of consumers.
- Firm 1 is located at 0, Firm 2 is located at 1.
- Transportation cost is $t > 0$ per unit distance.
- Firms have constant marginal costs $c_1 > c_2$.
- Each firm sets a price p_i to maximize profit.
- Consumers buy from the firm offering the lower *delivered* price (price + travel cost).

Consumer Indifference Point: A consumer at $x \in [0, 1]$ is indifferent if

$$p_1 + t\,x \;=\; p_2 + t\,(1 - x),$$

which solves to

$$x^* \;=\; \frac{p_2 - p_1 + t}{2\,t}.$$

Then

$$Q_1 \;=\; x^*, \quad Q_2 \;=\; 1 - x^*.$$

Profits:

$$\pi_1 \;=\; (p_1 - c_1)\,Q_1, \quad \pi_2 \;=\; (p_2 - c_2)\,Q_2.$$

Del 2 a) Sequential Price Setting (Stackelberg) - Firm 2 Leads and Firm 1 follows

Firm 1's Best Response: If Firm 2 sets p_2, then

$$p_1 = \frac{p_2 + t + c_1}{2}.$$

Firm 2's Profit (Anticipating p_1):

$$Q_2 = \frac{p_1 + t - p_2}{2t} \quad \text{with} \quad p_1 = \frac{p_2 + t + c_1}{2} \implies Q_2 = \frac{-p_2 + 3t + c_1}{4t}.$$

$$\pi_2 = (p_2 - c_2)\, Q_2 = (p_2 - c_2)\, \frac{-p_2 + 3t + c_1}{4\,t}.$$

Firm 2's Optimal Price:

$$\frac{d}{dp_2}\big[(p_2 - c_2)\,(-p_2 + 3t + c_1)\big] = 0 \implies p_2^* = \frac{3t + c_1 + c_2}{2}.$$

Firm 1's Price in Subgame-Perfect Equilibrium:

$$p_1^* = \frac{p_2^* + t + c_1}{2} = \frac{\frac{3t+c_1+c_2}{2} + t + c_1}{2} = \frac{5t + 3c_1 + c_2}{4}.$$

Hence

$$\boxed{p_2^* = \frac{3t + c_1 + c_2}{2}, \quad p_1^* = \frac{5t + 3c_1 + c_2}{4}.}$$

Note that

$$p_1^* - p_2^* = \frac{c_1 - c_2 - t}{4}$$

so that

- If $c_1 - c_2 > t$ then $p_1^* > p_2^*$.
- If $c_1 - c_2 < t$ then $p_1^* < p_2^*$.

The relative size of the cost difference and the travel cost determines which firm has the higher price.

Del 2 b) Simultaneous Price Setting

Demands:

$$Q_1 = \frac{p_2 - p_1 + t}{2\,t}, \quad Q_2 = \frac{p_1 + t - p_2}{2\,t}.$$

Profits:

$$\pi_1 = (p_1 - c_1)\,\frac{p_2 - p_1 + t}{2\,t}, \quad \pi_2 = (p_2 - c_2)\,\frac{p_1 + t - p_2}{2\,t}.$$

Best Responses:

$$\text{Firm 1:} \quad p_1 = \frac{p_2 + t + c_1}{2},$$

$$\text{Firm 2:} \quad p_2 = \frac{p_1 + t + c_2}{2}.$$

Nash Equilibrium: Solve simultaneously:

$$\boxed{p_1^* = \frac{3\,t + 2\,c_1 + c_2}{3}, \quad p_2^* = \frac{3\,t + c_1 + 2\,c_2}{3}.}$$

Note that $c_1 > c_2$ implies $p_1^* > p_2^*$ so that the high cost firm sets the higher price in Nash equilibrium.